A BRIEF INTRODUCTION TO OPTICAL FREQUENCY COMB

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CONTENT

HISTORICAL OVERVIEW

- The first measurement of laser frequency was in 1967, when the frequencies of single mode emission of the NCH laser at 890 GHz.
- In 1979 the first frequency measurement of visible radiation(520THz).

Frequency multiplication using harmonic frequency chains

The idea behind such a chain is to start from a low frequency and generate higher harmonics of this frequency using different non-linear devices. A harmonic of the precisely known low frequency is generated and used to determine an unknown higher frequency by measuring their beat frequency.

1) LO Hocker, A Javan, D Ramachandra Rao, L Frenkel, and T Sullivan. Absolute frequency measurement and spectroscopy of gas laser transitions in the far infrared. Applied Physics Letters, 10(5):147–149, 1967.
2) KM Baird, Kenneth M Evenson, GR Hanes, DA Jennings, and FR Petersen. Extension of absolute-frequency measurements to the visible: frequencies of ten hyperfine components of iodine. Optics letters, 4(9):263–264, 1979.

WHY FREQUENCY COMB

Optical atomic clocks rely on the natural oscillation of atoms, which occurs at much higher frequencies than standard microwave-based atomic clocks.

$$\frac{v_{optical}}{v_{microwave}} = \frac{10^{15}}{10^{10}}$$

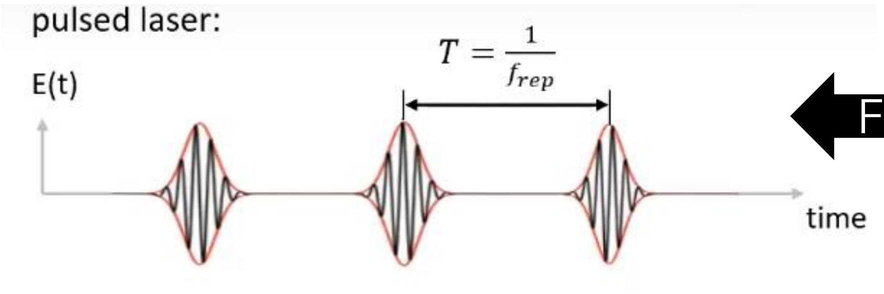
How will one measure it?

A harmonic frequency chain covering a large frequency span becomes a highly complex, large-scale, and expensive system which involves significant efforts to build and operate.

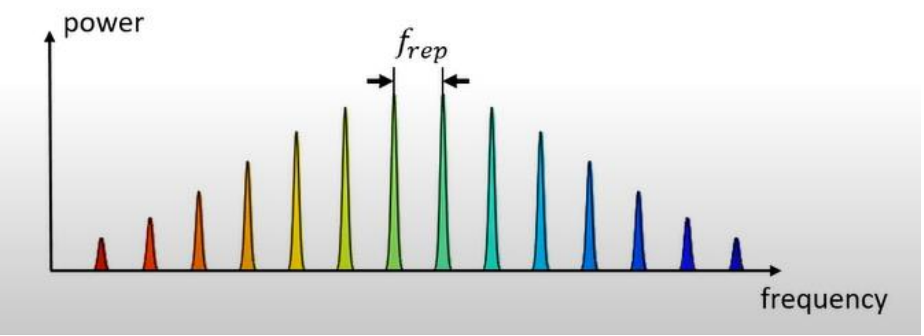
INTRODUCTION TO FREQUENCY COMB

An optical frequency comb is a remarkable tool that can serve as an optical ruler for measuring unknown frequencies.

An optical frequency comb consists of an equidistant comb lines and these lines are generated by mode-locked lasers, which produce a regular train of ultrashort pulses.



Fourier transform



Time domain

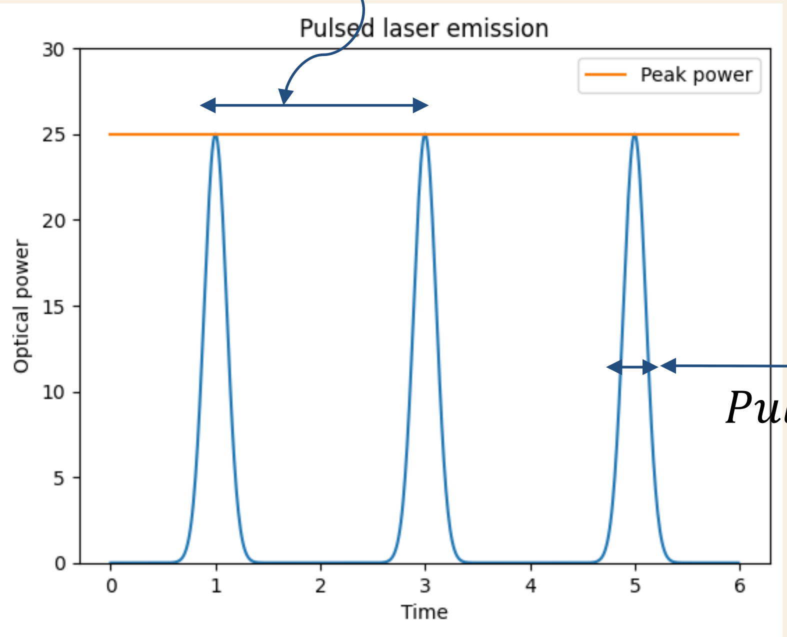
Frequency domain

Image credit: Menlo system wepage

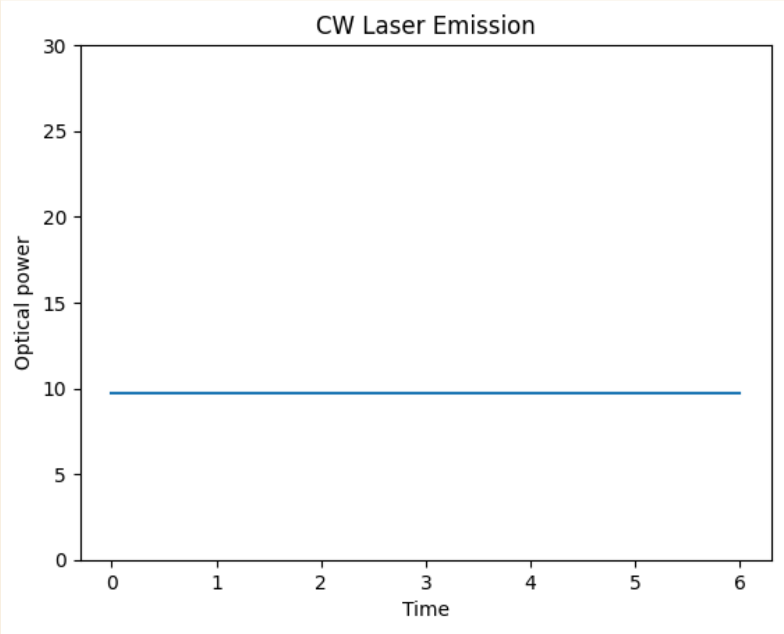
PULSED LASER

- A type of laser system that emits lights in short pulse rather than a continuous wave

$Pulse\ duration(T)$



$Pulse\ width(\tau)$



The average power of pulsed and CW laser is almost equal.

The average power, $P_{avg} = P_{peak} * \tau/T$

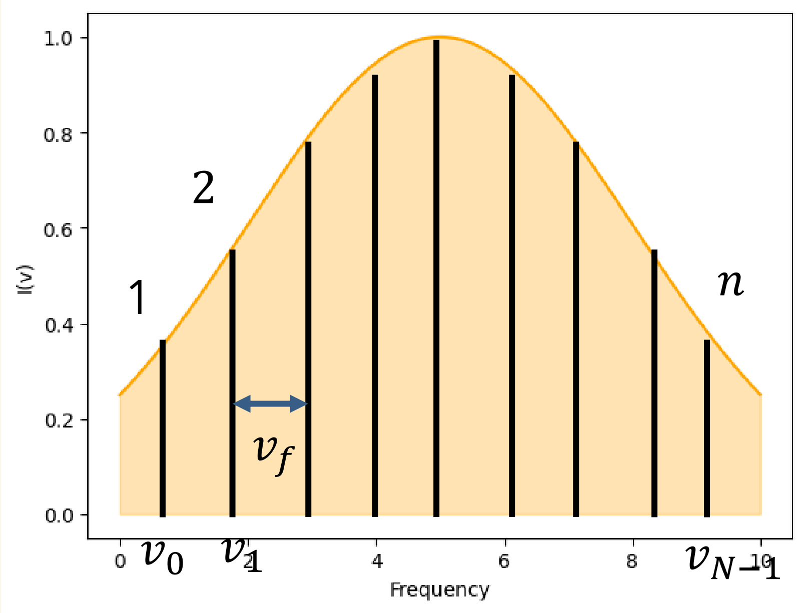
Energy per pulse, $E = P_{avg}T = P_{peak}\tau$

MODE LOCKING

This technique allows the generation of optical pulses as short fs, ultrashort pulses).

Mode locking is all about creating a phase relation somehow and maintaining it.

E.g. Ti: Sapphire laser



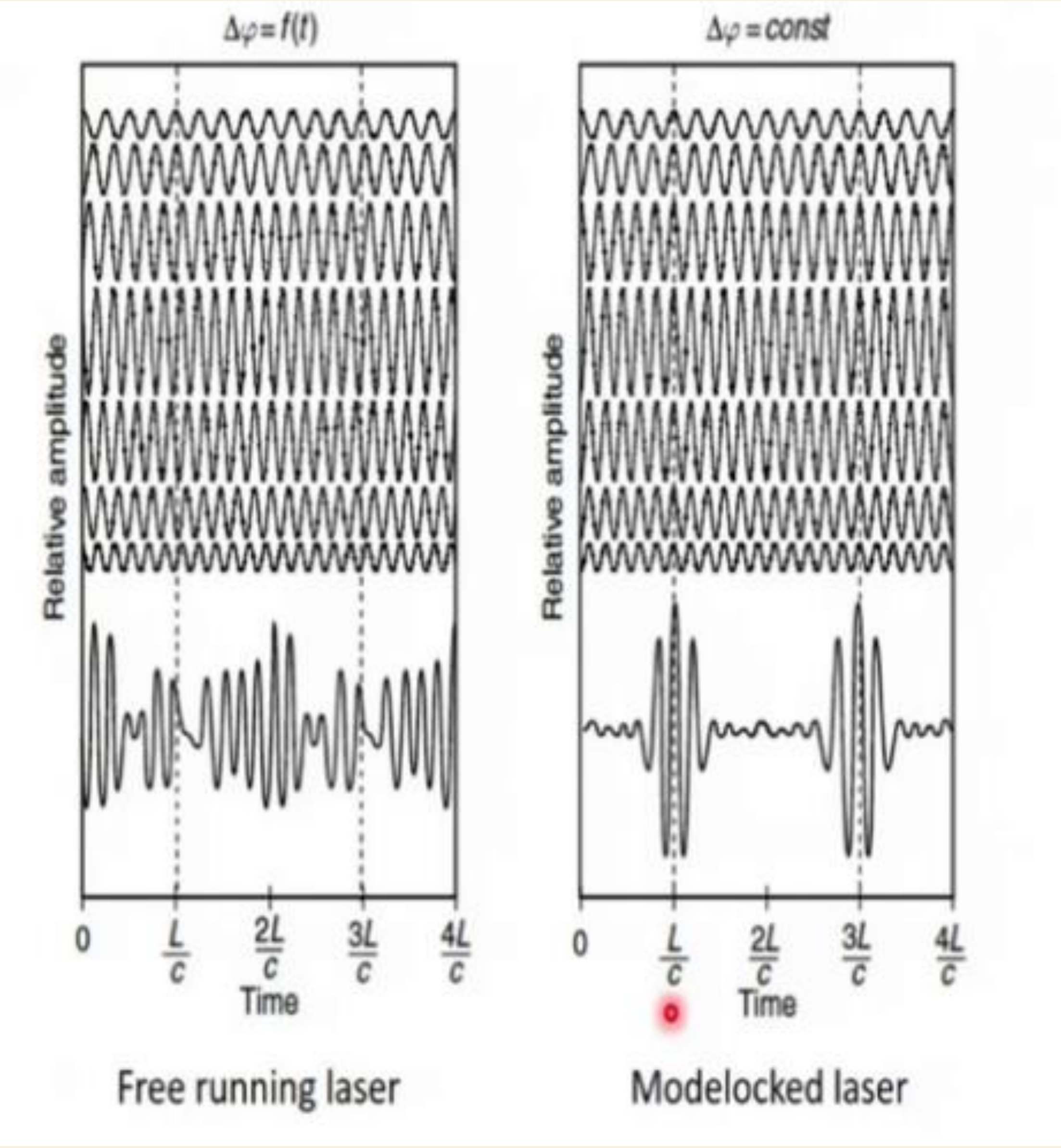


Image credit: https://archive.nptel.ac.in/courses/104/101/104101122/

MODE LOCKING

The amplitude of the nth mode of Electric field can be written as

$$E(t) = \sum_{n=0}^{N-1} E_n e^{\iota 2\pi \nu_n t - \iota \phi_n}$$

ν_n is the frequency and ϕ_n is the initial phase

$\nu_n = \nu_0 + n\nu_f$

ν_0 is the frequency of the first mode

The combined total amplitude of the modes can be expressed as

$$E(t) = E_0 \sum_{n=0}^{N-1} e^{\iota 2\pi \nu_n t - \iota \phi_n}$$

MODE LOCKING

The difference in frequency between the modes is given by

$$v_{n+1} - v_n = \Delta v = \frac{c}{2nd}$$

The total intensity of the multi-longitudinal modes output is

$$I(t) = |E(t)|^2 = \sum_{m=0}^{N-1} E_m e^{\iota 2\pi \nu_m t - \iota \phi_n} \sum_{n=0}^{N-1} E_n^* e^{-\iota 2\pi \nu_n t + \iota \phi_n}$$

$$I(t) = \sum_n |E_0|^2 + \sum_n \sum_m E_n E_m^* e^{\iota 2\pi (v_n - v_m) t - \iota (\phi_n - \phi_m)}$$

MODE LOCKING

$$I(t) = N|E_0|^2 + |E_0|^2 \sum_n \sum_m e^{\iota 2\pi(v_n - v_m)t - \iota(\phi_n - \phi_m)}$$

The intensity is equal to N times the intensity of individual modes.

$$I(t) = N|E_0|^2$$

This value can vary if a few modes randomly phase together, But for a large value of N, it doesn’t vary significantly from the average value.

$$I(t) = N|E_0|^2 \pm |E_0|^2$$

MODE LOCKING

Let's see what will happen if we lock the phases of all the modes together; this is mode-locking.

$$\Phi_n = \phi_0 \, for \, all \, value \, of \, n$$

Then the combined total amplitude of the modes can be expressed as

$$E(t) = E_0 e^{\iota\phi_0} \sum_{n=0}^{N-1} e^{\iota 2\pi \nu_n t}$$

Since, $v_n = v_0 + n v_f$

$$E(t) = E_0 e^{\iota\phi_0} e^{\iota 2\pi \nu_o t} \sum_{n=0}^{N-1} e^{\iota 2\pi n \nu_f t}$$

MODE LOCKING

$$E(t) = E_0 e^{\iota\phi_0} e^{\iota 2\pi v_o t} \sum_{n=0}^{N-1} e^{\iota n\delta t}$$

$where, \delta = 2\pi v_f$

$$E(t) = E_0 e^{\iota\phi_0} e^{\iota 2\pi v_o t}[1 + e^{\iota\delta} + e^{\iota 2\delta} + \ldots + e^{\iota(N-1)\delta}]$$

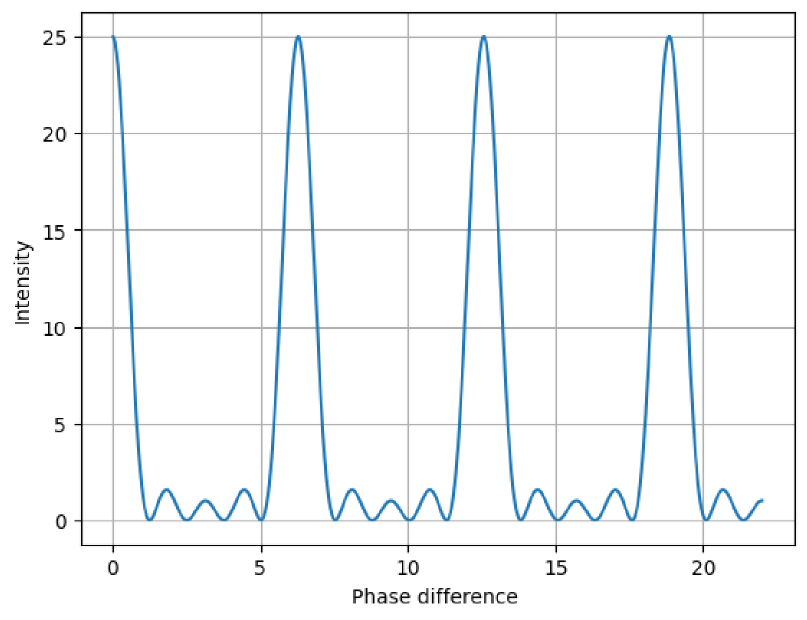
$$E(t) = E_0 e^{\iota\phi_0} e^{\iota 2\pi v_o t} \frac{[1 - e^{\iota N\delta}]}{[1 - e^{\iota\delta}]}$$

$$E(t) = E_0 e^{\iota\phi_0} e^{\iota 2\pi v_o t} \frac{e^{\frac{\iota N\delta}{2}}\left[e^{-\frac{\iota N\delta}{2}} - e^{\frac{\iota N\delta}{2}}\right]}{e^{\frac{\iota\delta}{2}}\left[e^{-\frac{\iota\delta}{2}} - e^{\frac{\iota\delta}{2}}\right]}$$

MODE LOCKING

The total intensity can be expressed as

$$I(t) = |E_0|^2 \frac{|\sin\left(\frac{N\delta}{2}\right)|^2}{|\sin\left(\frac{\delta}{2}\right)|^2}$$



Minima occur at: $\dfrac{N\delta}{2} = m\pi$

$\longrightarrow$ $I = 0 \; at \; \delta = \dfrac{2m\pi}{N}$ where m = 1,2,3 ...

Maxima occur at: $\dfrac{\delta}{2} = m\pi$

$\longrightarrow$ By l hospital's rule $\boxed{I(t) = N^2|E_0|^2}$

METHOD OF MODE LOCKING

Active mode-locking **Passive mode-locking**

- Time gap between pulses or round trip time, $T = \frac{1}{\Delta v} = \frac{2L}{C},$
- Pulse width $\tau = \frac{1}{N\Delta v}$
- Where N is the number of modes, and by increasing the no. of modes, one can create shorter and shorter pulses.

Before mode locking: $I = NE_0^2$

After Mode locking: $I = N^2E_0^2$

SPECTRUM OF PULSED LASER

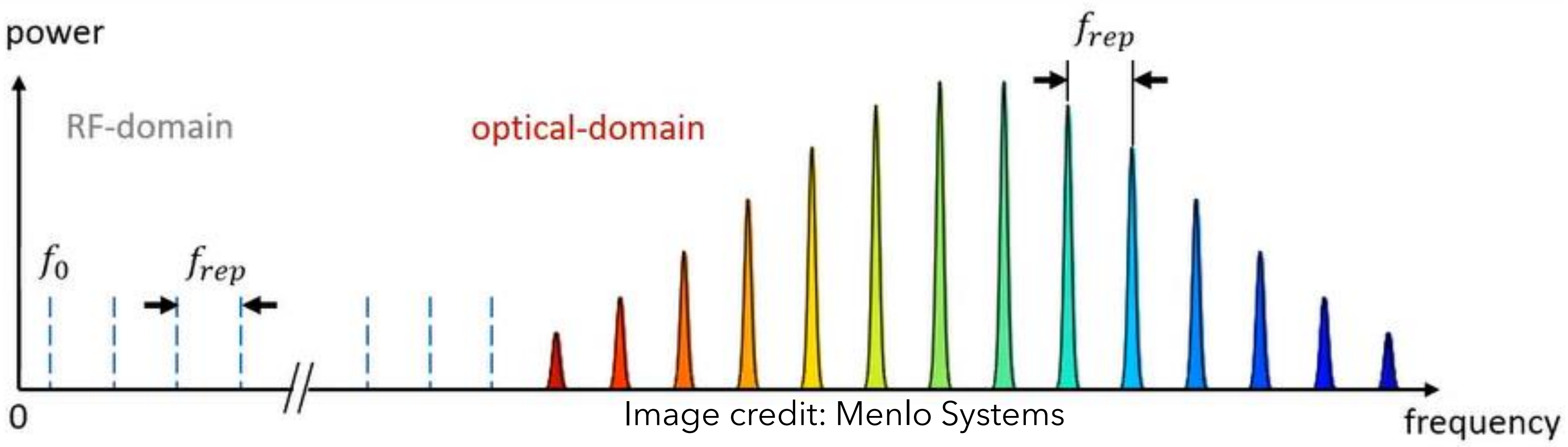


Image credit: Menlo Systems

Two important parameters f_{rep} (repetition rate) and f_0 (offset frequency)

- f_{rep} = is it constant over the entire spectrum?
- Origin of offset frequency f_0?
- How to measure and stabilize f_0?
- How to measure any unknown frequency?

CARRIER ENVELOPE PHASE

The CEP represents the phase difference between the carrier wave (the oscillation at the laser's central frequency) and the position of the intensity envelope of the pulse.

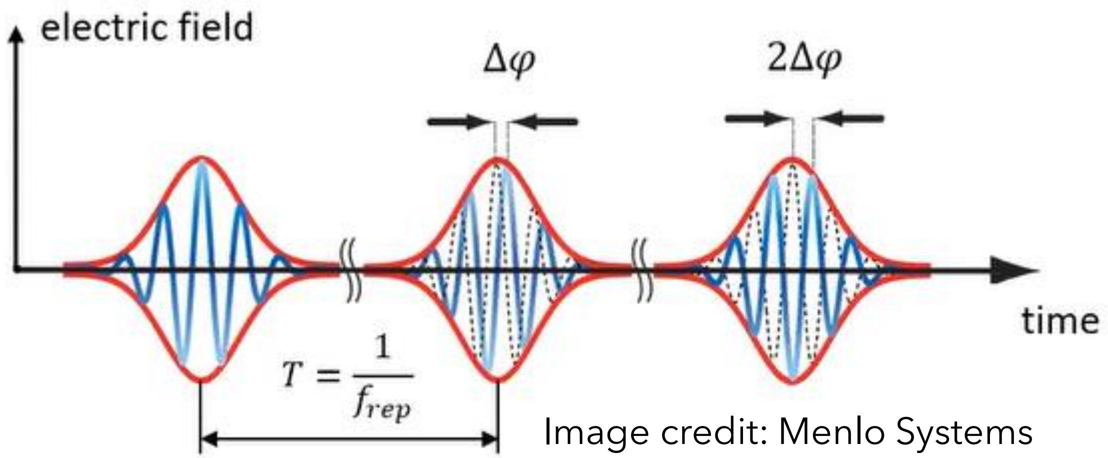


Image credit: Menlo Systems

In simpler terms, it quantifies how the carrier wave aligns with the pulse's shape.

ORIGIN OF OFFSET FREQUENCY

Due to dispersion inside the cavity the group and phase velocities are different. This difference leads to a phase shift $\Delta\phi$.

The pulse-to-pulse change in the phase can be expressed as

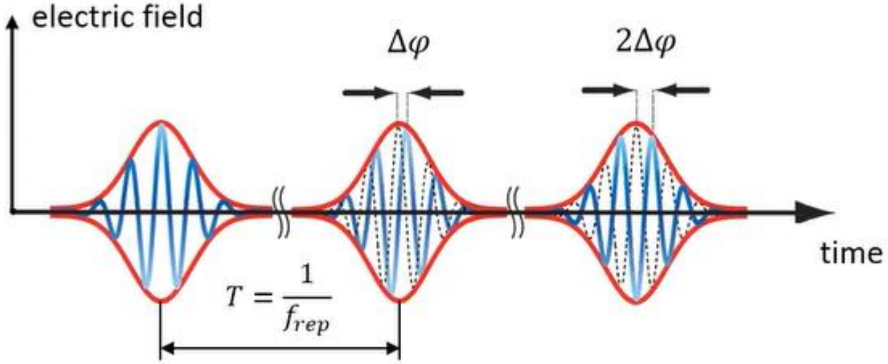
$$\Delta\phi = (\frac{1}{v_g} - \frac{1}{v_p}) l_c \omega_c$$

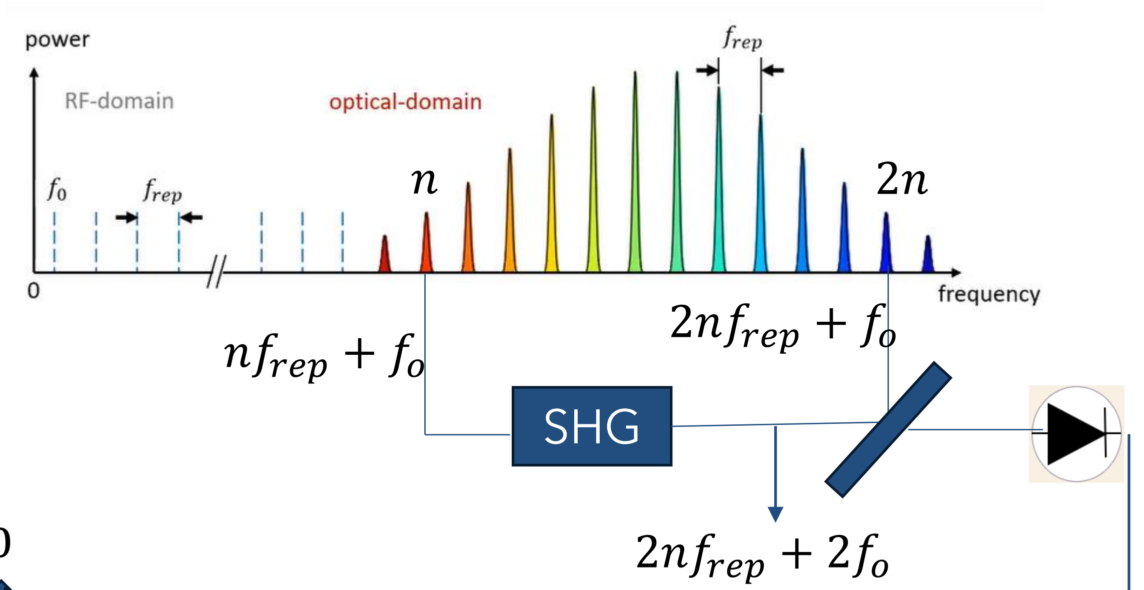
Where v_p *and* v_g is the phase and group velocity inside the cavity respectively, ω_c is the carrier frequency and l_c is the round trip length of the cavity

This pulse-to-pulse change in phase leads to the generation of some initial offset frequency

$$f_0 = \frac{\Delta\phi}{2\pi T}$$

WHAT IS FREQUENCY COMB?





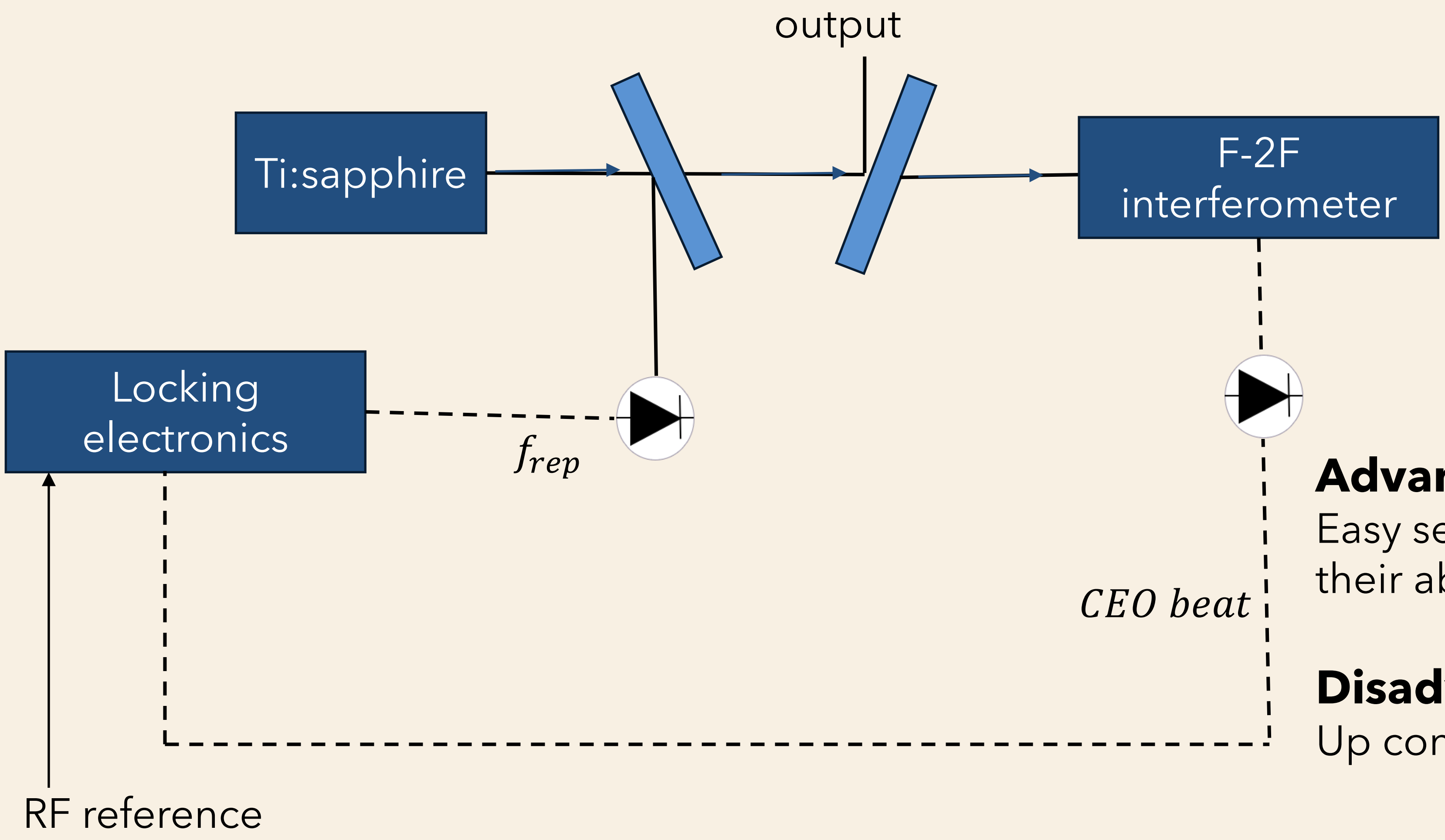
$$f_n = nf_{rep} + f_0$$

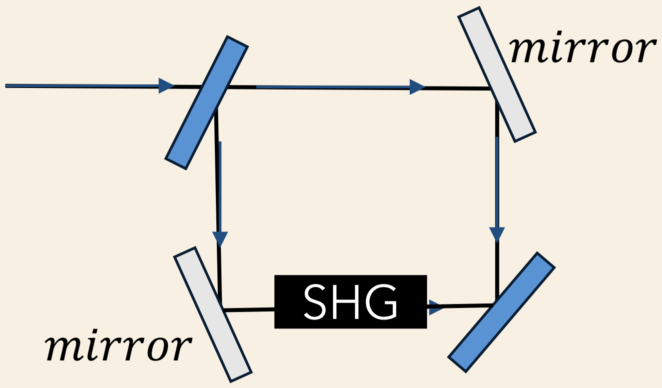
$200THz$ $250MHz$ $35MHz$

$$\begin{array}{r} 2nf_{rep} + 2f_o \\ -\quad 2nf_{rep} + f_o \\ \hline f_0 \end{array}$$

An optical frequency comb is a pulsed laser with a stabilized repetition rate and carrier-envelope offset frequency

STABILIZING THE FREQUENCY COMB





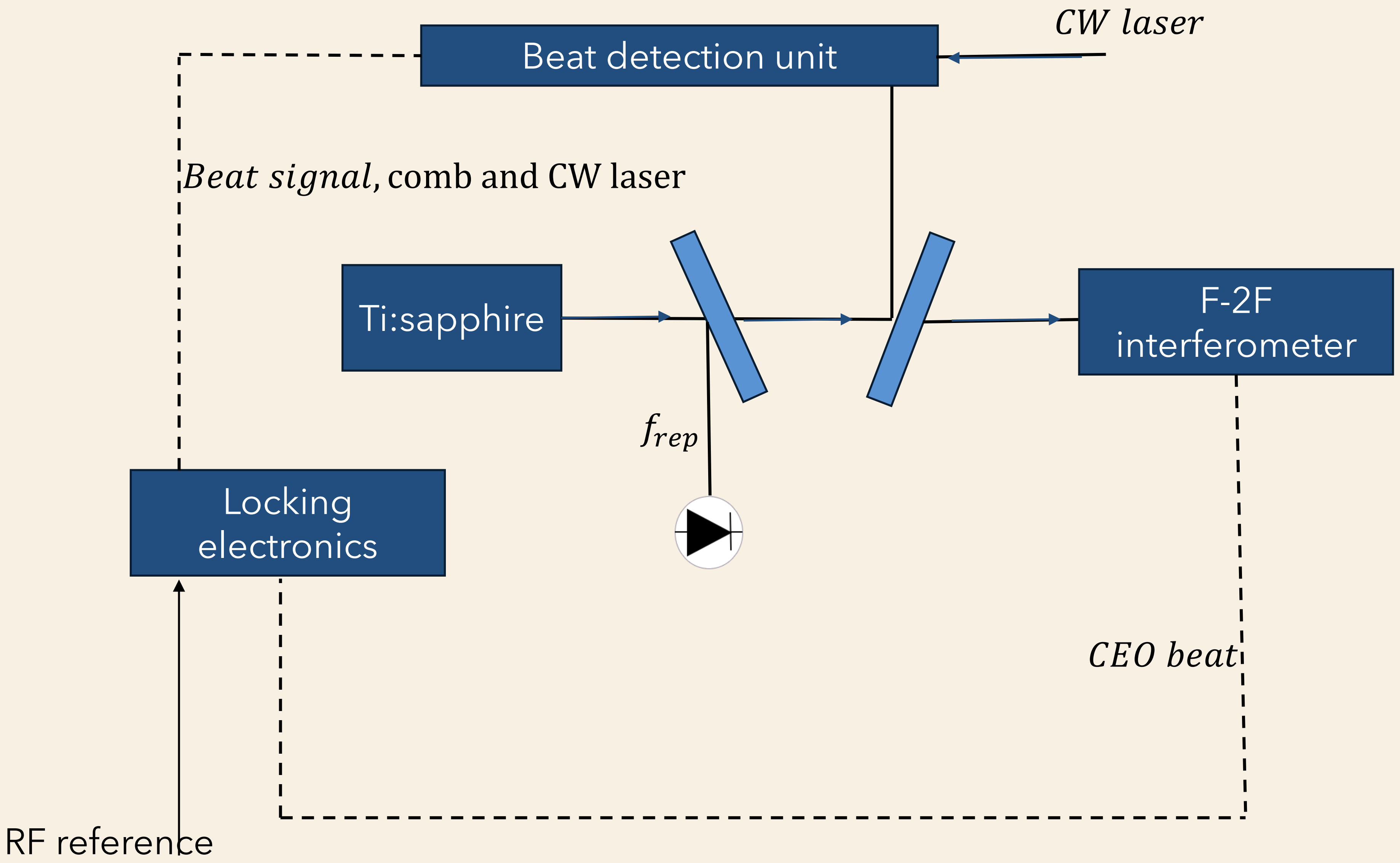
Advantage

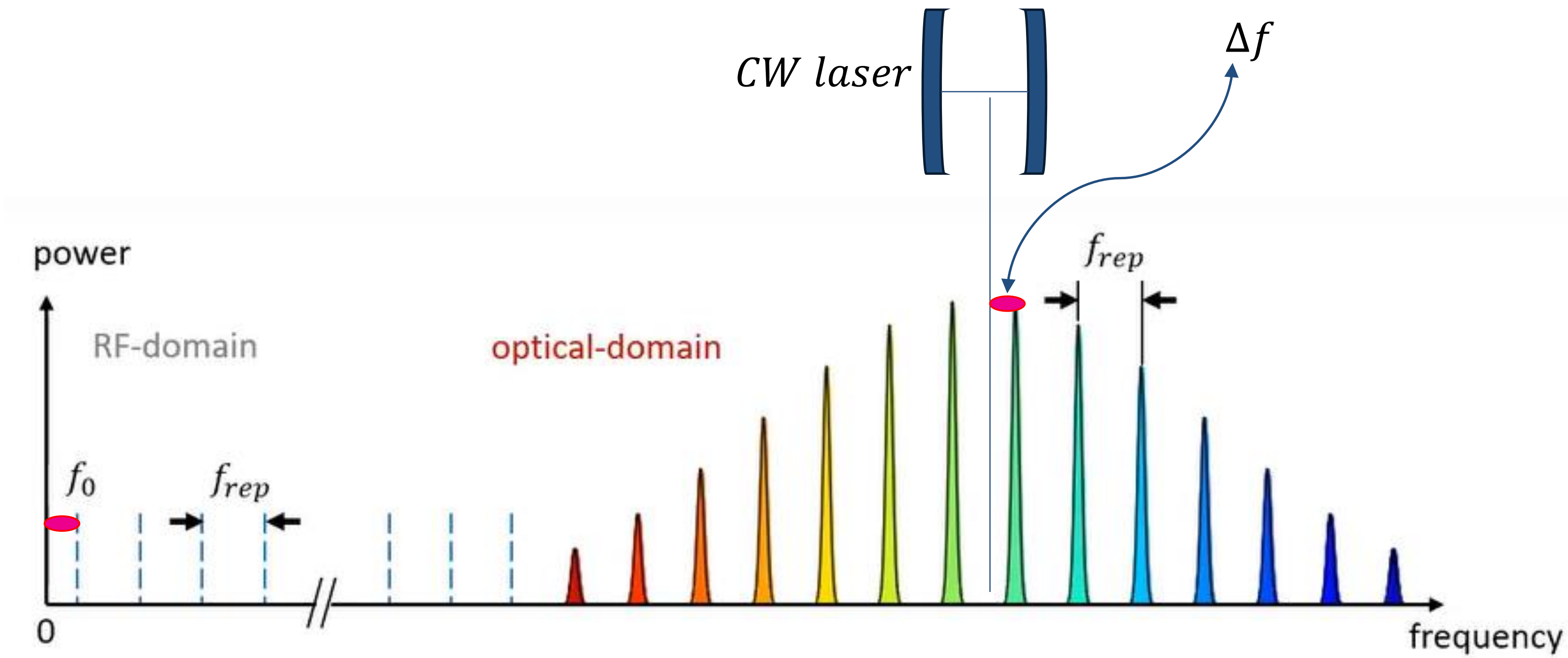
Easy setup, all comb lines are stable in in their absolute frequencies.

Disadvantage

Up conversion of RF noise, scaling factor: n^2

STABILIZING THE FREQUENCY COMB

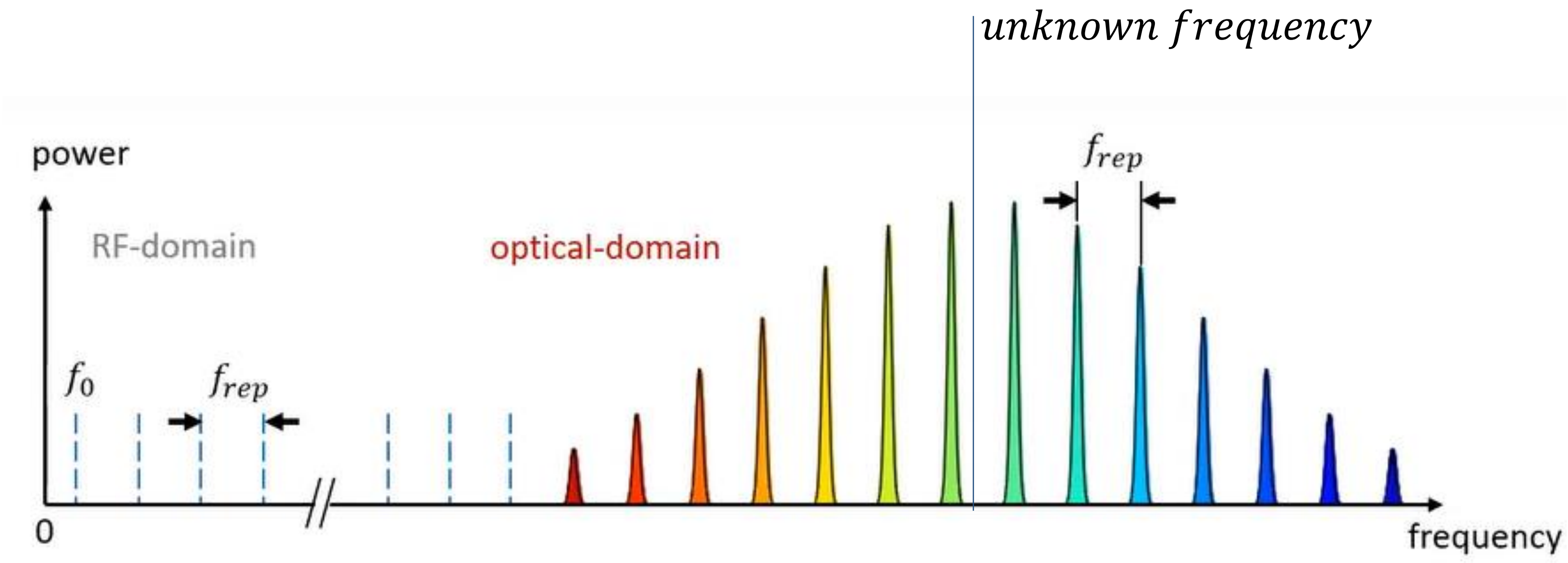




Advantage
Noise reduction

Disadvantage
You need a ultra-stable CW laser.

MEASUREMENT OF UNKNOWN FREQUENCY



$f_{un} = f_n + f_{beat}$, where $0 \leq |f_{beat}| \leq \frac{f_{rep}}{2}$

$f_n = nf_{rep} + f_0$

How to determine comb line index n:

- Measure f_{un} with a wavemeter.
- Measure f_{rep} and f_{beat} for vastly different repetition rates and then calculate n.

$$n = \frac{\Delta f_{beat}}{\Delta f_{rep}}$$

WHAT A FREQUENCY COMB CAN DO?

Optical atomic clock

The frequency comb takes the high-frequency light interacting with the atoms in the clock and converts it into a lower-frequency signal. This allows electronics to finally "count" these rapid ticks, similar to how gears in a traditional clock translate the rapid swings of a pendulum into a slower, countable movement.

Astronomy and cosmology

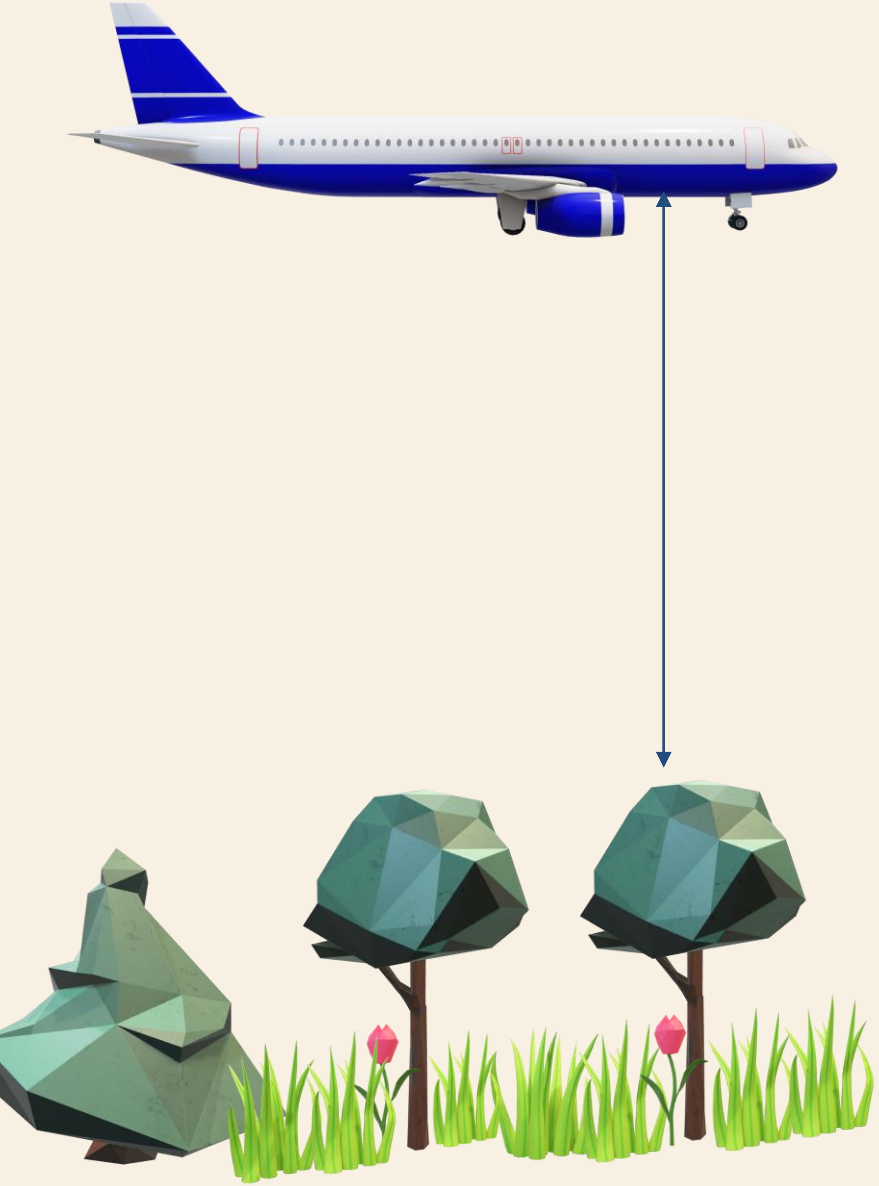
Optical frequency combs also help scientists search for exoplanets around distant stars. By tracking the exact colors of light from these stars.

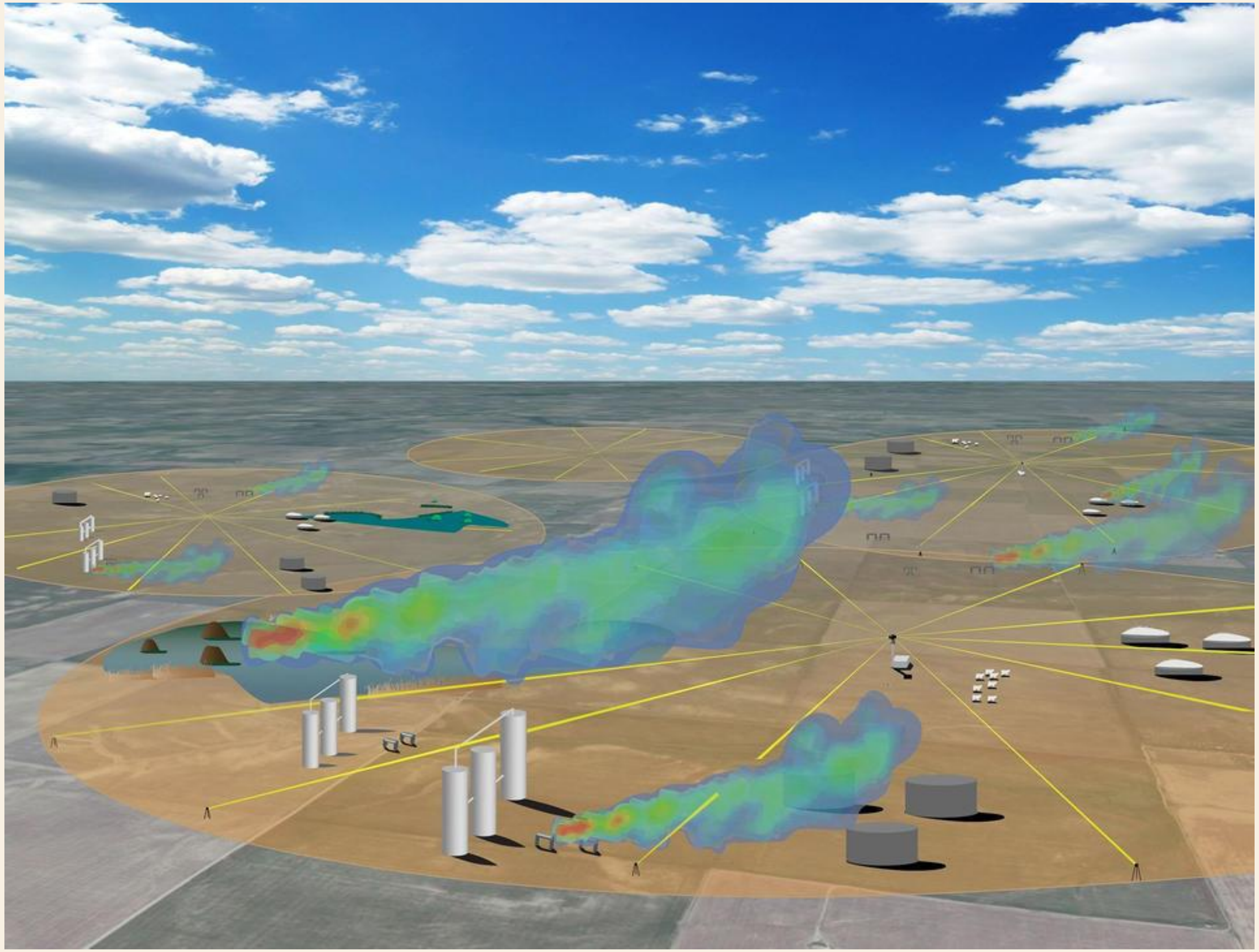
Precise distance measurement

NIST patented lidar, a light detecting and ranging system that utilizes optical frequency combs to measure the distance to an object by analyzing light reflected from it.

Atmospheric science and greenhouse gases

Optical frequency combs generate millions of frequencies in short pulses; they can be used to quickly and efficiently study the quantity, structure, and dynamics of various molecules and atoms.





Credit: Stephanie Sizemore and Ian Coddington/NIST

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- Femtosecond combs for optical frequency metrology
- Femtosecond Optical Frequency Comb: Principle, Operation, and Applications
- Colloquium: Femtosecond optical frequency combs

THANK YOU