PH699L course: SPECIAL TOPICS IN PHYSICS - I

A brief introduction to optical frequency comb

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Abstract

Frequency comb, a revolutionary tool in modern photonics and spectroscopy, has transformed various fields of science and technology since its discovery. Originating from the Nobel Prize-winning work of Professor Theodor Hänsch and Dr. John L. Hall, frequency combs are generated by mode-locked lasers, producing a spectrum of evenly spaced frequency lines resembling the teeth of a comb. This report provides a concise introduction to the fundamental principles of frequency combs, including their generation, characteristics, and applications. It explores different pulsed laser generation techniques including cavity damping and Q-switching, but their pulsed width is restricted to ps(10^{-12}s), and the most important pulsed laser generation technique by "mode-locking". The revolutionary titanium-sapphire crystal-based mode-locked laser produced < 100 fs(10^{-15}s) pulses, due to the high gain bandwidth (650-1100 nm) of this medium[2]. We will explore the underlying physics behind mode-locked lasers and describe how the comb-like spectrum is formed. Additionally, the report discusses the diverse applications of frequency combs, ranging from precision spectroscopy and metrology to telecommunications and optical clocks.

Contents

1 Introduction

Imagine a ruler not for measuring distance, but for precisely measuring the colours of light, each colour corresponding to a specific frequency. This ruler of light, with its distinct "teeth" of equally spaced, ultra-stable frequencies, is the essence of a frequency comb[6, 22, 20]. Generated by specially designed pulsed lasers, frequency combs offer unparalleled precision and control over light, leading to breakthroughs in various scientific and technological fields. This introduction will delve into the fascinating world of frequency combs, exploring their unique properties, how they are generated, and the diverse applications that are transforming our understanding and manipulation of light.

The frequency comb is a pulsed laser with stabilized carrier offset and repetition frequency[19], throughout this report we will understand each important word of its definition, what is pulsed laser and its generation technique, the two important frequency parameters carrier offset frequency and repetition frequency and the most important part is to stabilize the frequency comb for the accurate measurement of the unknown frequencies. This report aims to provide a comprehensive introduction to frequency combs. Specifically, we aim to explore the significance of optical frequency combs in the measurement of unknown frequencies and their integral role as components of optical clocks, serving as ultimate tools for optical frequency metrology.

The applications of optical frequency comb span a wide array of disciplines[6, 17]. From building ultra-accurate atomic clocks to analyzing the light from distant stars, including fundamental physics, atomic spectroscopy[4], astronomy, and metrology. Beyond their scientific significance, these standards play a crucial role in establishing benchmarks for other physical quantities, enhancing precision in various applications such as satellite-based navigation and telecommunications networks[21].

This report delves into the fascinating world of frequency combs. We'll start on a journey that begins with a historical overview on frequency standards (Section 2). This foundation sets the stage for understanding how frequency combs have transformed how we measure frequencies, as we'll explore how frequency chain were used for the measurement of frequency (sec(2.1)). Since the heart of a frequency comb lies in pulsed lasers, particularly mode-locked lasers, we'll delve into various methods for generating such pulses (Section 3). This includes techniques like cavity dumping(3.1), Q-switching(3.2), and the crucial concept of mode-locking[12](3.3). With a solid understanding of pulsed lasers, we'll then shift our focus to the star of the show: the frequency comb itself(sec(4)). We'll explore what it is, how it works, and the origin of a critical parameter known as the carrier offset frequency (sec(4.2)) (throughout, we'll consistently use this term for offset frequency instead). Additionally, we'll cover techniques for measuring this offset frequency using a self-referencing approach (Sec 4.3).

Having established the core principles of frequency combs, we'll move on to the crucial aspect of stabilization(5). We'll explore methods for stabilizing both the offset frequency (f_0) and the repetition frequency (f_{rep}). This is essential for ensuring the comb's exceptional precision. Finally, the document will showcase the true power of frequency combs by demonstrating how they are used to measure unknown frequencies with unmatched accuracy (Section 6). We'll also explore the diverse applications of this remarkable technology, highlighting its impact on various scientific and technological fields.

By following this step-by-step approach, we'll gain a comprehensive understanding of frequency combs and their transformative role in the world of precision measurement.

2 Historical overview

Measuring frequency in the lower RF or microwave range is relatively simple, thanks to the availability of counters and synthesizers. Techniques like heterodyning further expand the measurable frequency range, enabling the measurement of microwave frequencies up to several hundred gigahertz.

However, measuring optical frequencies poses a significant challenge due to the limited number of accurately known frequencies in the optical spectrum. Additionally, establishing a known frequency requires complex methods to bridge the gap between optical and microwave frequencies, as absolute frequency measurements are anchored in the SI second.

Various techniques have been developed over the past four and a half decades to address these challenges, including frequency chains and frequency division methods. Despite these efforts, the landscape changed dramatically with the introduction of femtosecond frequency combs, which were honored with the Nobel Prize in 2005[7, 8]. These combs offer a revolutionary solution for measuring arbitrary optical frequencies.

2.1 Frequency multiplication using Harmonic frequency chain

Frequency multiplication using harmonic frequencies is a technique to generate a higher frequency output signal based on a lower frequency input signal[11]. These harmonics are multiples of the original signal's frequency and arise when the signal is fed through a non-linear element like a diode or transistor. This element distorts the signal, creating a cocktail of frequencies including the original and its harmonics. A bandpass filter then acts as a picky listener, allowing only the desired harmonic to pass through, essentially multiplying the original frequency. Frequency multipliers are key components in radio frequency systems, helping generate the high frequencies needed for radio waves.

Historically, the first measurements of laser frequency occurred in the late 1960s and early 1970s using harmonic mixing techniques. For instance, in 1967, the frequencies of single-mode emission from the NCH laser were measured with high precision by mixing laser frequencies with high-order harmonics of a microwave signal[9]. The first measurement of frequency of the visible radiation, specifically of at 520THz, was done in 1979[1]. Subsequent advancements led to frequency measurements extending into the terahertz and visible regions, with continuous improvements in accuracy over the years.

Frequency measurements can be based on primary or secondary standards, with phase-coherent measurements linking the unknown frequency to the primary standard to minimize uncertainties. The development of frequency chains facilitated accurate frequency metrology, contributing to improved uncertainty in the speed of light and the redefinition of the meter in 1983.

While significant progress has been made in frequency metrology, bridging large frequency gaps remains challenging. Conventional methods of optical frequency synthesis from microwaves are complex and expensive, typically targeting specific optical frequencies. Harmonic frequency chains offer a promising approach but require substantial effort to construct and operate, with each chain focusing on a specific optical frequency.

3 Pulsed laser generation techniques

Continuous lasers emit a steady stream of light, certain applications require short, intense bursts. This is where pulsed lasers come in, and they're the beating heart of frequency combs. Imagine a laser cavity where light bounces back and forth, building up its intensity. In a normal laser, some light escapes to create a continuous beam. However, in a pulsed laser, all the light waves inside the cavity are precisely synchronized, forming a powerful pulse that gets released periodically. This creates short bursts of light with much higher intensity compared to a continuous beam, even if the average power remains the same.

But generating these pulses isn't magic. Techniques like cavity dumping, Q-switching, and mode-locking are used for the generation of the pulsed laser. Understanding pulsed lasers, particularly mode-locked lasers, is vital for appreciating the power and functionality of frequency combs, which we'll explore next.

3.1 Cavity dumping

Imagine a laser beam – a continuous stream of light. But what if scientists needed short, intense bursts instead? Cavity dumping is one of the methods of generating pulsed laser. In a laser cavity, light bounces back and forth, building up in intensity. Normally, some light escapes to create the beam. Cavity dumping uses a special switch. First, light builds up within the cavity, thanks to special mirror coatings that trap it. Then, the switch, like a quick dam release, opens for a very short time. This allows a powerful pulse of light to escape, while the rest remains trapped. The switch closes, and the cycle repeats, generating another pulse. Cavity dumping creates short pulses with much higher intensity than a continuous beam, even with the same average power. These ultrashort bursts (think trillionths of a second!) find uses in material processing for delicate tasks, high-resolution distance measurements, and even studying the building blocks of our world at the atomic level. Overall, cavity dumping is a clever technique for squeezing intense light pulses from lasers, opening doors to exciting advancements in science and technology.

3.2 Q-switching

Q-switching is a technique used in laser technology to produce short[23, 14], high-energy pulses of light. It stands as one of the fundamental methods for generating pulsed lasers, alongside cavity dumping and mode-locking. The essence of Q-switching lies in temporarily preventing the laser cavity from oscillating by introducing a "loss" element, typically a fast-acting optical switch, into the optical resonator. Q stand for quality in the Q-switching and mathematically it can be represented as

$$Q = 2\pi \frac{Energy\ stored\ in\ cavity}{Energy\ loss\ per\ optical\ cycle} \tag{1}$$

During the Q-switching process, the active medium of the laser is pumped to a high-energy state, preparing it for lasing. However, the cavity remains in a condition of high losses, preventing lasing action from occurring. As the population inversion in the gain medium builds up, the energy stored in the system increases. Once the desired energy level is reached, the loss element is rapidly switched out of the cavity, allowing the stored energy to be released as a short, intense pulse of laser light. This sudden release of energy results in a pulse with a high peak power and short duration, making Q-switched lasers ideal for applications requiring precise and powerful bursts of energy, such as laser machining, laser ranging, and medical procedures. Despite its simplicity, Q-switching plays a crucial role in various fields, contributing to advancements in science, technology, and medicine.

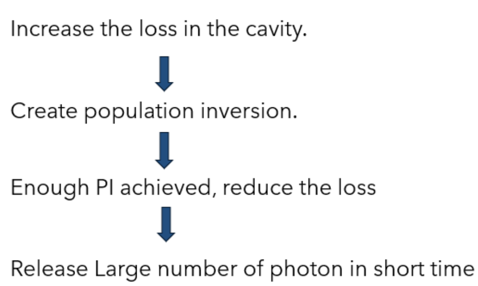

Figure 1: The procedure for generating short pulses through Q-switching

There are two types of Q-switching one is active Q-switching in which an externally variable attenuator like acousto-optic or electro-optic modulator is used and another one is passive Q-switching which involves a saturable absorber like dye inside the resonator.

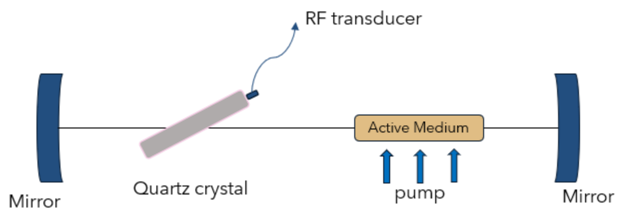

Figure 2: Q-switching using electro-optic modulator

In the Q-switching method, energy is stored inside the active medium, unlike cavity dumping technique. Inside the resonator (2), the electro-optic effect within the quartz crystal alters the light's properties. This essentially increases the losses within the cavity, preventing laser action from starting. This high-loss state allows the active medium to accumulate energy, building up a population inversion. when RF is on the beam deflected out of the cavity, yielding high loss and when RF is off the beam transits the cavity with low loss Some of the famous Q-switched lasers are the Nd-yag laser, Yb-yag laser and ruby laser[14].

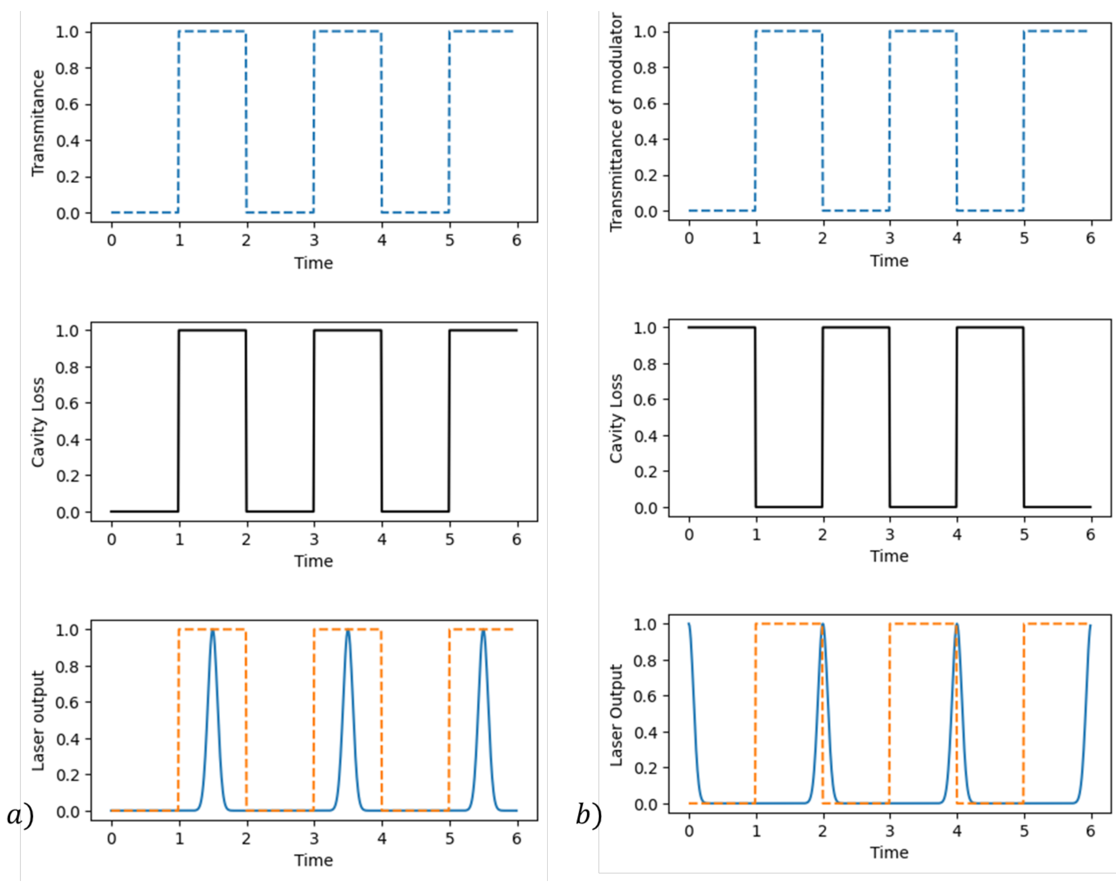

Figure 3: Comparision of the output pulses of from the different techniques a) cavity dumping, b) q-switching

3.3 Mode-locking

Mode-locked lasers are instrumental in creating light pulses with incredibly short durations[18, 15]. They stand out as the preferred method for generating optical frequency combs due to their capability of producing pulses ranging from picoseconds (10^{-12}) to femtoseconds (10^{-15}). While Q-switched lasers can generate pulses on the order of nanoseconds (10^{-9}), One of the key distinctions between Q-switched and mode-locked lasers lies in their active medium. While the emission linewidth of mode-locked lasers tends to be broad, accommodating a vast number of longitudinal modes beneath the envelope (on the order of 10^6), Q-switched lasers exhibit a narrower emission linewidth. In a Q-switched laser, only a few longitudinal mode typically exist within the cavity, especially in systems like the He-Ne laser with a cavity length of 10cm only one longitudinal mode exists. The process of mode-locking involves precisely controlling the optical cavity to ensure that the oscillation of the laser modes becomes synchronized in phase. This synchronization results in the constructive interference of light waves within the cavity, leading to the formation of ultrashort pulses of light.

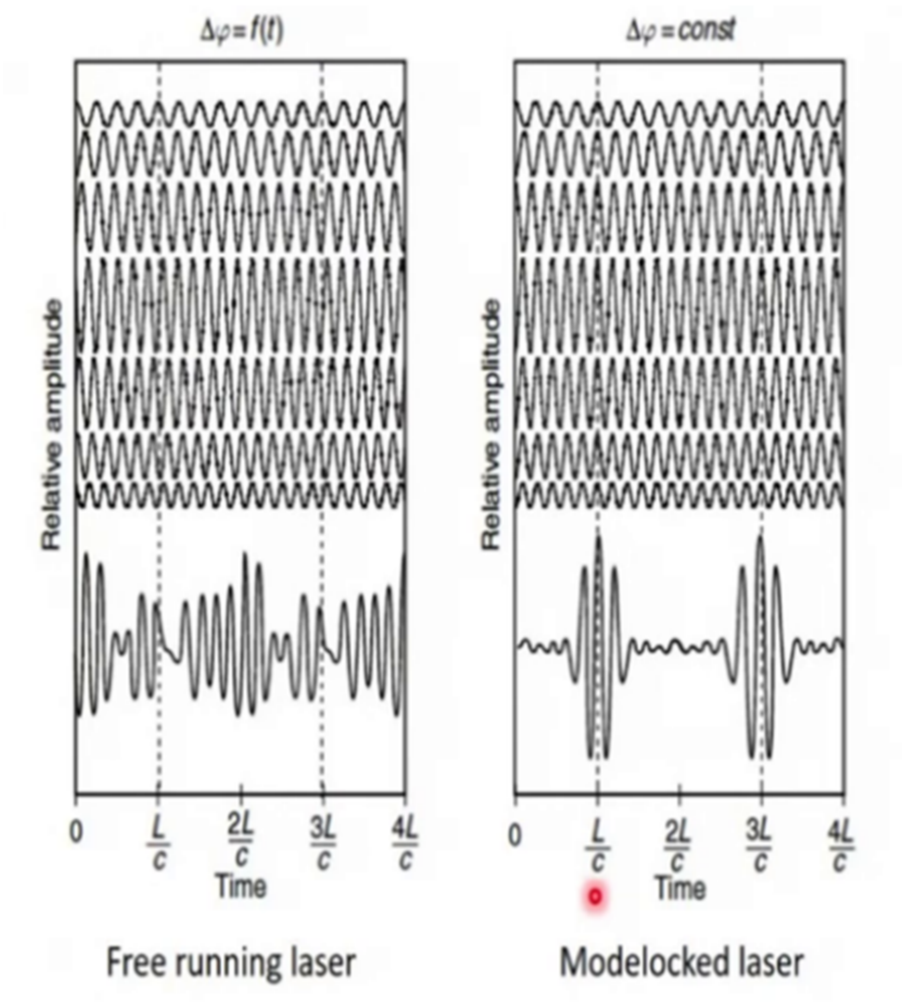

Figure 4: Image credit: https://archive.nptel.ac.in/courses/104/101/104101122/

In the absence of constant phase$\Delta\phi$(4), the longitudinal modes of the gain medium may or may not constructively interfere (Free-running laser). However, after mode-locking, there is a significantly higher likelihood that the modes will align constructively, resulting in the generation of short pulses, possibly even in the femtosecond range(Modelocked laser).

The bandwidth of the active medium is quite broad due to which large number of longitudinal modes present and each of these longitudinal modes can oscillate independently to other, let the electric field of nth mode as

$$E(t) = \sum_{n=0}^{N-1} E_n e^{\iota 2\pi f_n t - \iota\phi_n} \tag{2}$$

Where, $f_n = f_0 + nf_{rep}$ is the frequency of the n^{th} mode, f_{rep} is the repetition frequency, ϕ_n is the initial phase, N is the total number of the modes and v_0 is the frequency of the first mode.

if we consider the amplitude of all the longitudinal mode as equal then,

$$E(t) = E_0 \sum_{n=0}^{N-1} e^{\iota 2\pi f_n t - \iota\phi_n} \tag{3}$$

The difference between the modes is given by $\Delta v = v_{n+1} - v_n = \frac{c}{2nl}$ Where l is the length of the cavity and c is the speed of the light, the total intensity of the longitudinal modes is

$$I(t) = |E(t)|^2 = \sum_{m=0}^{N-1} E_m^* e^{-\iota 2\pi f_m t + \iota\phi_m} + \sum_{n=0}^{N-1} E_n e^{\iota 2\pi f_n t - \iota\phi_n} \tag{4}$$

$$I(t) = \sum_n |E_0|^2 + \sum_n \sum_m E_n E_m^* e^{\iota 2\pi (f_n - f_m)t - \iota(\phi_n - \phi_m)} \tag{5}$$

$$I(t) = N|E_0|^2 + |E_0|^2 \sum_n \sum_m e^{\iota 2\pi (f_n - f_m)t - \iota(\phi_n - \phi_m)} \tag{6}$$

The intensity is equal to N times the intensity of individual modes, if we do not lock the phase or if the initial phases are different for all the longitudinal modes, because the first term $N|E_0|^2$ is much larger due to large value of N than the second term which is small preodic temporal fluctuation thus,

$$I(t) = N|E_0|^2 \tag{7}$$

This value can vary if a few modes randomly phase together, but for a large value of N, it doesn't vary significantly from the average value.

$$I(t) = N|E_0|^2 \pm |E_0|^2 \tag{8}$$

Let's see what will happen if we lock the phases of all the modes together, mode locking is simply all about creating phase relation somehow and maintaining it. let

$$\phi_n = \phi_0 \ for \ all \ value \ of \ n \tag{9}$$

then the combined total amplitude of the modes can be expressed as

$$\begin{aligned} E(t) &= E_0 e^{\iota\phi_0} \sum_{n=0}^{N-1} e^{\iota 2\pi n f_n t} \\ &= E_0 e^{\iota\phi_0} e^{\iota 2\pi f_0 t} \sum_{n=0}^{N-1} e^{\iota 2\pi n f_{rep} t} \\ &= E_0 e^{\iota\phi_0} e^{\iota 2\pi f_0 t} \sum_{n=0}^{N-1} e^{\iota n\delta t} \end{aligned} \tag{10}$$

Where $\delta = 2\pi f_{rep}$

$$E(t) = E_0 e^{\iota\phi} e^{\iota 2\pi f_0 t} \sum_{n=0}^{N-1} e^{\iota n\delta t} \tag{11}$$

The sum geometric series for the total electric field will be,

$$E(t) = E_0 e^{\iota\phi} e^{\iota 2\pi f_0 t} [1 + e^{\iota\delta} + e^{2\iota\delta} + \cdot + e^{\iota(N-1)\delta}] \tag{12}$$

The summation of the finite sum geometric series will be,

$$\frac{[1 - e^{\iota N\delta}]}{1 - e^{\iota\delta}} \tag{13}$$

The total electric field will be,

$$E(t) = E(t) = E_0 e^{\iota\phi} e^{\iota 2\pi f_0 t} \frac{e^{\frac{\iota N\delta}{2}} [e^{\frac{-\iota N\delta}{2}} - e^{\frac{\iota N\delta}{2}}]}{e^{\frac{\iota\delta}{2}} [e^{\frac{-\iota\delta}{2}} - e^{\frac{\iota\delta}{2}}]} \tag{14}$$

The total intensity $|E(t)|^2$ can be expressed as,

$$I(t) = |E(t)|^2 = |E_0|^2 \frac{|sin\left(\frac{N\delta}{2}\right)|^2}{|sin\left(\frac{\delta}{2}\right)|^2} \tag{15}$$

case-1: Minima occur at-

$$\frac{N\delta}{2} = m\pi$$

intensity become zero(I(t) →0) at the value of

$$\delta = \frac{2m\pi}{N}$$

where, $m = 1, 2, 3, 4, 5, ...$
case-2: Maxima occur at-

$$\frac{\delta}{2} = m\pi$$

in this case, intensity becomes the maximum

$$I(t) = N^2|E_0|^2$$

There is a significantly large difference in the intensity value $I(t)$ when calculated calculations conducted with and without modelocking, due to a large value of N that is of the order of 10^6. Modelocking facilitates the generation of remarkably narrow pulses, on the scale of picoseconds and femtoseconds. These ultrashort pulses are unattainable through methods like Q-switching and cavity dumping, primarily due to limitations imposed by the gain medium.

There are mainly two types of mode-locking that are commonly used: active and passive mode-locking. In active modelocking, an external modulation source, such as an acousto-optic modulator (AOM) or an electro-optic modulator (EOM), is used to modulate the intensity or phase of light inside the laser cavity. This modulation imposes a frequency comb structure on the laser spectrum, leading to the generation of ultrashort pulses. Active modelocking offers precise control over the pulse characteristics and can produce pulses with durations ranging from picoseconds to femtoseconds. Unlike active modelocking, passive modelocking relies on intracavity elements to induce modelocking without the need for external modulation sources. Typically, a saturable absorber or a semiconductor saturable absorber mirror (SESAM) is placed inside the laser cavity to passively modulate the intracavity light intensity. Passive modelocking is simpler and more robust than active modelocking, making it suitable for various applications, including ultrafast spectroscopy and microscopy.

3.3.1 Active mode-locking

Active modelocking is a dynamic technique employed in laser systems to generate ultrashort optical pulses by actively modulating the laser cavity's properties. Unlike passive modelocking, which relies on nonlinear optical effects within saturable absorbers, active modelocking involves external modulation of the laser cavity parameters, such as the gain or losses, to achieve pulse formation.

In active modelocking, an external modulation source, such as an electro-optic modulator (EOM) or an acousto-optic modulator (AOM), is introduced into the laser cavity. These modulators allow for precise control over the phase and amplitude of the intracavity optical field. By actively modulating the cavity parameters at a frequency corresponding to the desired pulse repetition rate, the laser output can be coerced into generating a train of ultrashort pulses.

The modulation applied by the external source introduces a periodic modulation to the intracavity optical field, effectively imposing a frequency comb structure on the laser output. Through careful adjustment of the modulation parameters, such as modulation depth and frequency, the cavity dynamics can be engineered to favor the formation of ultrashort pulses via mode-locking.

Let's delve into the mathematical framework of active mode-locking by examining the electric field of a single mode.

$$E(t) = E_0 cos 2\pi v_1 t \tag{16}$$

Electric field after the modulation,

$$E(t) = (E_0 + E_0 cos 2\pi v_f t) cos 2\pi v_1 t \tag{17}$$

$$E(t) = E_0 cos 2\pi v_1 t + \frac{E_0}{2}(cos 2\pi (v_1 + v_f) + cos 2\pi (v_1 + v_f)) \tag{18}$$

Modulating the frequency of the longitudinal modes induces the creation of sidebands, with each sideband frequency corresponding to that of its adjacent longitudinal mode. These generated sidebands exhibit synchronized phases, leading to the modulation of other modes in phase coherence. This synchronization effect propagates across multiple modes, driving them to oscillate coherently and facilitating the phenomenon of mode-locking.

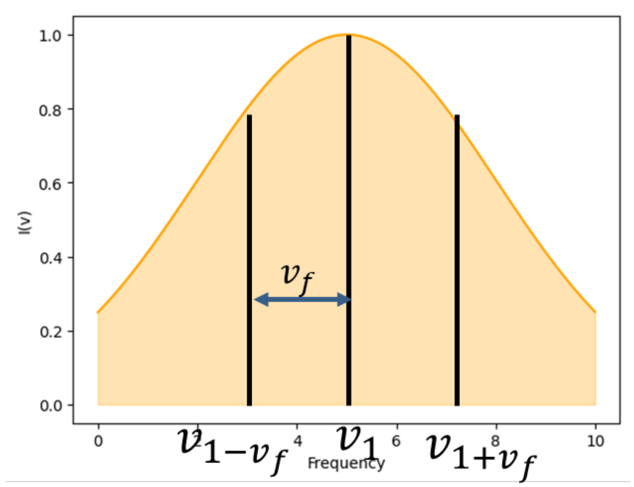

Figure 5: Active mode-locking results in the emergence of sidebands at individual frequencies. These sidebands constructively interfere with the existing frequencies within the bandwidth of the gain medium.

Active mode-locking offers several advantages, including flexibility and tunability. Unlike passive techniques that rely on intrinsic cavity properties, active mode-locking allows for precise control over the pulse characteristics by adjusting the modulation parameters. This flexibility enables the generation of pulses with tailored properties, such as pulse duration, repetition rate, and spectral bandwidth, to suit specific applications.

Additionally, active mode-locking can facilitate synchronization with external systems or devices, making it suitable for applications requiring precise timing or synchronization, such as ultrafast spectroscopy or optical communications. However, active mode-locking systems can be more complex and require careful optimization of the modulation parameters to achieve stable and reliable pulse generation. Moreover, the introduction of external modulators adds complexity and potential sources of noise to the laser system.

3.3.2 *Passive mode-locking*

Modelocking using saturable absorbers is a passive technique employed in laser systems to generate ultrashort optical pulses. At the heart of this method lies the nonlinear optical properties of certain materials known as saturable absorbers. These absorbers exhibit a unique behavior where their absorption of light decreases as the intensity of the incident light increases. This phenomenon, termed optical saturation, allows saturable absorbers to selectively transmit high-intensity pulses while attenuating the continuous-wave background. This nonlinear behavior forms the basis of mode-locking using saturable absorbers. In practical implementations, saturable absorbers are strategically placed inside the laser cavity where they interact with the circulating optical field. Typically positioned near the gain medium or at a cavity focus, the saturable absorber modulates the intracavity light intensity. As light circulates within the cavity, the saturable absorber preferentially transmits high-intensity pulses, effectively shaping the intracavity optical field. some of the famous saturable absorber are Methylen blue, and Crypto cyanine.

One of the notable advantages of mode-locking using saturable absorbers is its inherent simplicity and stability. Unlike active mode-locking techniques that rely on external modulation sources and complex feedback systems, saturable absorbers act as passive modulators within the laser cavity. This simplicity contributes to the robustness and reliability of the mode-locking process, making it suitable for various laser configurations and applications. Furthermore, saturable absorbers offer versatility across a broad range of wavelengths, enabling their use in diverse ultrafast optics and photonics applications. Their compatibility with different laser systems and straightforward operation make them valuable tools for generating ultrashort pulses essential in fields such as spectroscopy, microscopy, telecommunications, and laser processing.

4 Frequency comb

The concept of a frequency comb[3] originated from the pioneering work of Dr. John L. Hall and Dr. Theodor W. Hänsch in the 1990s, who were awarded the Nobel Prize in Physics in 2005 for their contributions to the development of precision spectroscopy, including the frequency comb technique.

Frequency combs are typically generated using mode-locked lasers[13] as discussed in the earlier sections, which produce ultrashort pulses of light with extremely narrow linewidths. These ultrashort pulses are evenly spaced in time (i.e., they repeat at regular intervals), their frequency spectrum forms a comb-like structure in the optical domain.

The fundamental characteristic of a frequency comb lies in its ability to partition an optical frequency, such as 200 THz, into discrete components represented by integer multiples of the repetition rate combined with an offset frequency. Both the repetition rate and the offset frequency are typically expressed in the megahertz domain, allowing for precise quantification and manipulation of the optical spectrum.

$$\underset{\underset{200THz}{\uparrow}}{f_n} = n\underset{\underset{250MHz}{\uparrow}}{f_{rep}} + \underset{\underset{35MHz}{\uparrow}}{f_0}$$

The frequency comb is a mode-locked laser with stabilized offset and repetition frequency then only one can measure any unknown frequency with high accuracy. The remarkable property of frequency combs lies in their ability to link optical frequencies with microwave frequencies, enabling highly accurate frequency measurements and precise timekeeping. This is achieved through a process called frequency synthesis, where the optical frequencies of the comb are stabilized and referenced to an atomic clock or a frequency standard, such as a hydrogen maser.

One of the most significant applications of frequency combs is in optical frequency metrology, where they serve as an invaluable tool for measuring optical frequencies with unprecedented precision. They enable the calibration of optical atomic clocks, the characterization of atomic and molecular transitions, and the development of optical atomic clocks based on optical transitions, offering the potential for even higher precision than traditional microwave-based atomic clocks.

4.1 Measurement of repetition frequency (f_{rep})

Determining the repetition frequency of pulses is a straightforward process. By directing the output of the mode-locked laser to a fast detector capable of measuring the intensity of incoming pulses, we can infer the frequency. This method involves analyzing the intervals between successive pulses captured by the detector.

4.2 Origin of offset frequency (f_0)

A mode-locked laser produces a train of short pulses each separated by a time interval of round trip time T,

$$T = \frac{2l}{c} \tag{19}$$

where l is the length of the cavity and c is the speed of the light. Inside the cavity, dispersion causes a distinction between the group velocity and phase velocity. This discrepancy in velocity induces a phase shift for each pulse within the cavity.

$$\Delta\phi = \left(\frac{1}{v_g} - \frac{1}{v_p} l\omega\right) \tag{20}$$

where v_p is the phase velocity inside the cavity, it refers to the speed at which the carrier wave (the fast sinusoidal oscillation) of a light pulse propagates through a medium and v_g is the group velocity, it represents the speed at which the wave packet (the envelope) containing the pulse's energy moves through the medium and ω is the carrier frequency. The phase shift experienced by the pulses results in the generation of an offset frequency in the frequency domain. This offset frequency needs to be accurately measured and stabilized to enhance the precision of frequency measurements achieved with the frequency comb.

$$f_0 = \frac{\Delta\phi f_r ep}{2\pi} \tag{21}$$

Where $f_{rep} = 1/T$ is the repetition frequency, The connection between representations in the time and frequency domains is illustrated in Figure (6).

Pulse train(6) generated from the mode-locked laser resulting in the phase shift $\Delta\phi$ due to dispersion inside the cavity in the time domain. In the frequency domain, the lines of a comb consist of pulse modes evenly spaced, with each line offset by a constant factor f_0 (analogous to the $\Delta\phi$i n the time domain). The frequency of any line is determined by multiplying the mode index of the line by the repetition rate and adding the frequency offset.

Each line of the frequency comb represents a mode of the pulse, evenly spaced in the frequency domain, with each mode offset by a constant factor f_0 (equivalent to $\Delta\phi$in time). The frequency of any given line is determined by multiplying the mode index of the line by the repetition rate and adding the frequency offset.

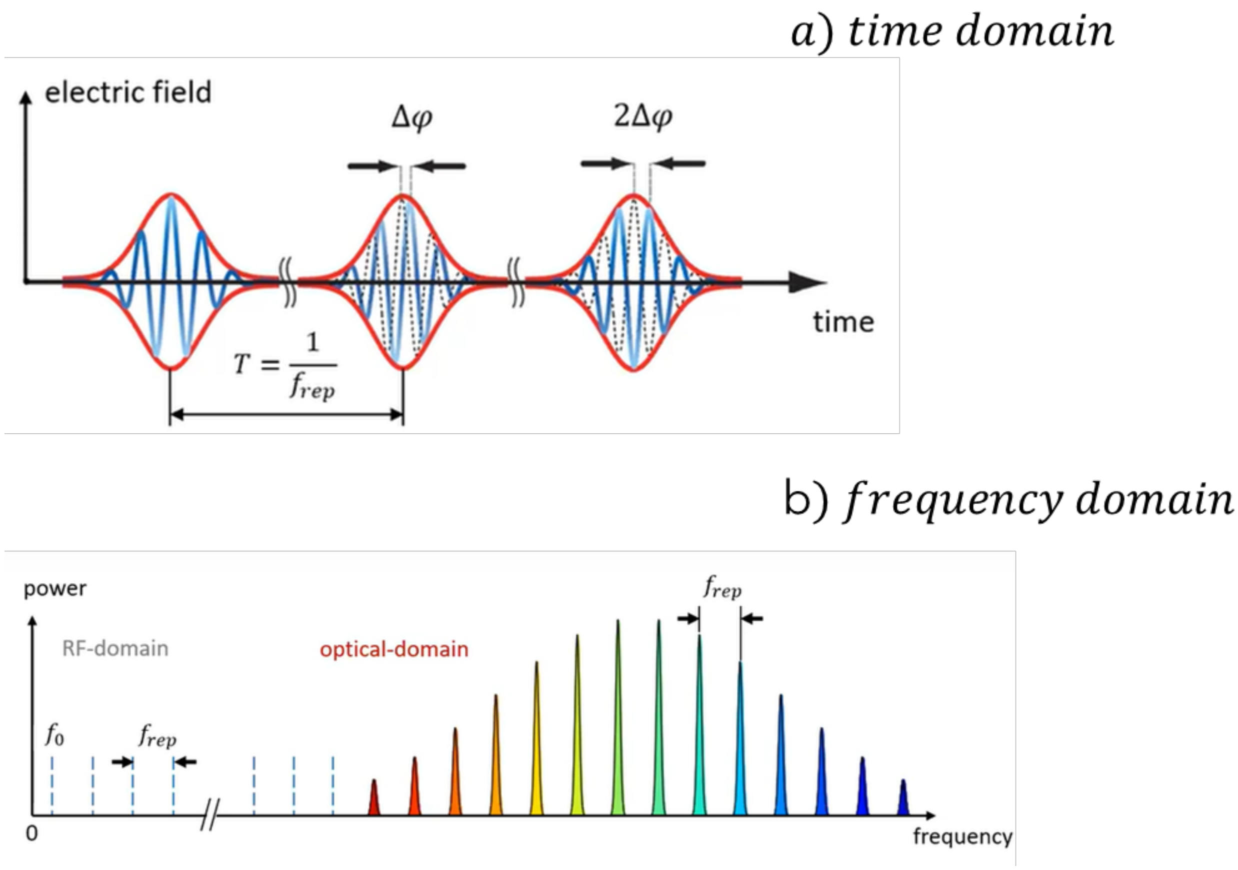

Figure 6: a)In the time domain, the pulse train is generated from the mode-locked laser, b) In the frequency domain, the Fourier transform of the time domain results in a comb-like structure. Image credit: Menlo systems, *https://www.youtube.com/watch?v=UgiBYZ2oZT4&t=1379s*

4.3 Measurement of Offset frequency

Measuring the offset frequency poses a challenge, and a key prerequisite is ensuring that the laser possesses octave-spanning characteristics, such as a broad gain bandwidth like that found in a Ti-Sapphire laser. A frequency comb is deemed octave-spanning when its spectral coverage extends across a frequency range that is twice as wide as the lowest frequency present within the comb. Essentially, this means that the highest frequency in the spectrum should be at least double the value of the lowest frequency.

Direct measurement of the offset frequency is challenging because intensity measurements alone do not contain information about phase. Moreover, the frequency offset is directly linked to the phase offset $\Delta\phi$ (20), which itself is non-deterministic. However, a technique known as self-referencing[19] using $f - 2f$ interferometer leverages the octave spectrum of the comb and second harmonic generation(SHG). In this technique, a frequency f on the lower side of the comb is passed through a second harmonic crystal, effectively doubling its frequency. This action shifts all frequencies of the comb to twice their original values, with the new f frequency coinciding with the older $2f$ frequency.

It's important to note that the offset is also doubled in this process.

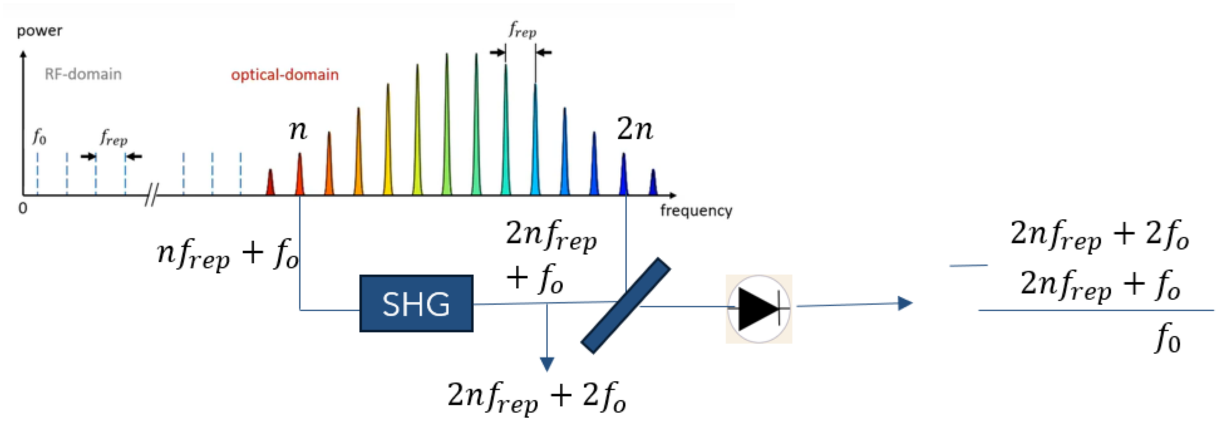

Figure 7: Measurement of offset frequency using self-referencing technique

Hence, by utilizing second harmonic generation (SHG) to double the frequency and comparing it with the original frequency, we can derive the offset frequency. It's important to note that we cannot selectively target any specific n^{th} comb line of the frequency comb; SHG will double the entire spectrum of the frequency comb. Consequently, measuring the beat frequency in the spectrum analyzer provides us with the value of the offset frequency.

5 Stabilizing the frequency comb

5.1 Stabilizing the frequency comb to a radio frequency reference

Stabilizing a frequency comb typically involves two main approaches, with the simplest method being to synchronize the frequency comb to a radio frequency reference. In this setup, a figure(8), along with a pre-amplifier, detects the repetition rate of the comb by shining light onto a photodiode. The detected repetition rate is then fed back into the locking electronics, which include a phase detector. This phase detector compares the repetition rate, typically around 250 megahertz, with the reference signal. The locking electronics subsequently adjust the frequency comb to lock it to the reference signal. The feedback system comprises both slow and fast feedback loops. Additionally, the f-$2f$ interferometer, which is crucial for achieving octave-spanning characteristics, provides the beat signal. This beat signal, used to measure the offset frequency, is also fed back into the locking electronics for precise control and stabilization of the frequency comb.

In frequency space, locking both the repetition rate and the frequency offset ensures that all parameters are fixed. The offset frequency f_0 of the comb is defined by the reference signal, while the spacing between comb lines is also fixed. This stabilization ensures the precision and accuracy of the frequency comb for various applications in metrology, spectroscopy, and telecommunications.

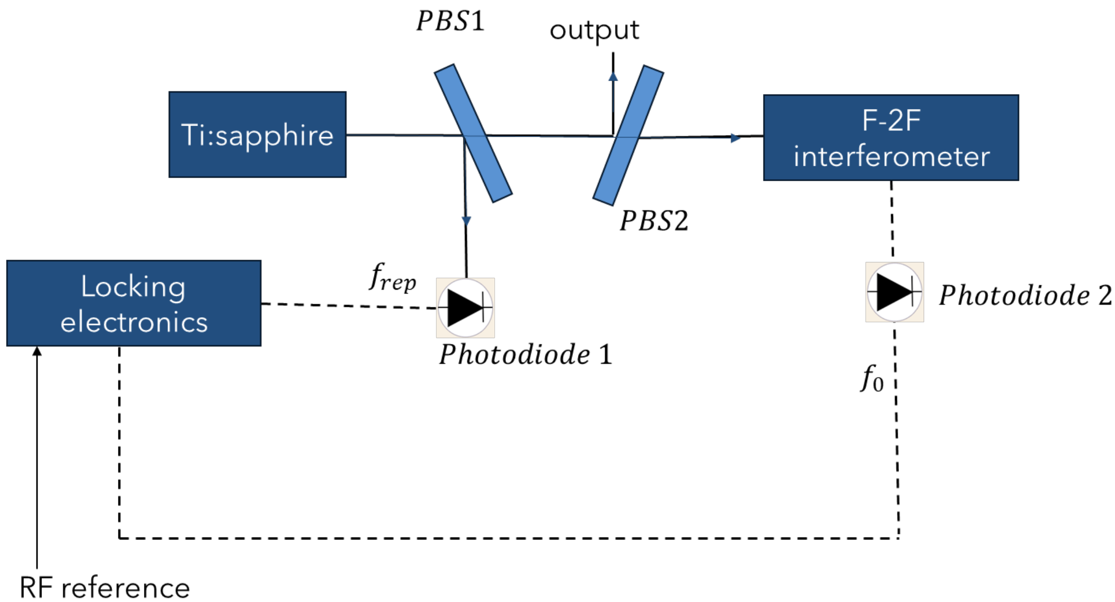

Figure 8: Stabilizing The frequency comb to a radio frequency reference, where both f_{rep} and f_0 are locked

The advantages of this setup are evident in its simplicity and the stability it provides to all comb lines in absolute frequencies. By deriving the radio frequency reference from an atomic clock, there's the potential to link the frequency comb to the SI second, resulting in exceptional long-term stability. However, there are also some disadvantages to consider. As indicated by the formula, the optical comb modes are determined by n times the repetition rate, where n is typically on the order of one million. Consequently, if the radio frequency reference is noisy, this noise is upconverted to the optical domain, with the scaling factor being n^2. This implies that it's challenging to tightly lock the frequency comb to a radio frequency reference due to the upconversion of RF noise.

5.2 Stabilizing the comb to an ultra-stable CW laser

An alternative approach involves using a continuous wave (CW) laser and leveraging the beat frequency between the frequency comb and the CW laser. This method offers even greater dynamic stability compared to simply locking the repetition rate f_{rep}. A portion of the pre-amplified light is diverted to a B detection unit, which essentially functions as a combiner and filter. In the speed detection unit, the light from the oscillator is combined with continuous wave (CW) laser light from a stable cavity, typically operating in the C band. The beat signal between the frequency comb and the CW laser is then detected on the photodiode. In terms of frequency space, there's no change in f_0. However, with the introduction of the ultra-stable laser, we now lock the distance between the ultra-stable CW laser and one mode of the frequency comb. Unlike before, we do not lock the repetition rate in this setup. It's crucial to grasp that the feedback mechanism here provides the distance between the frequency comb and the CW laser, which is the parameter we lock. Consequently, if the cavity is detuned or experiences slight movement, the frequency comb adjusts its repetition rate to maintain the beat frequency between the frequency comb and the CW laser.

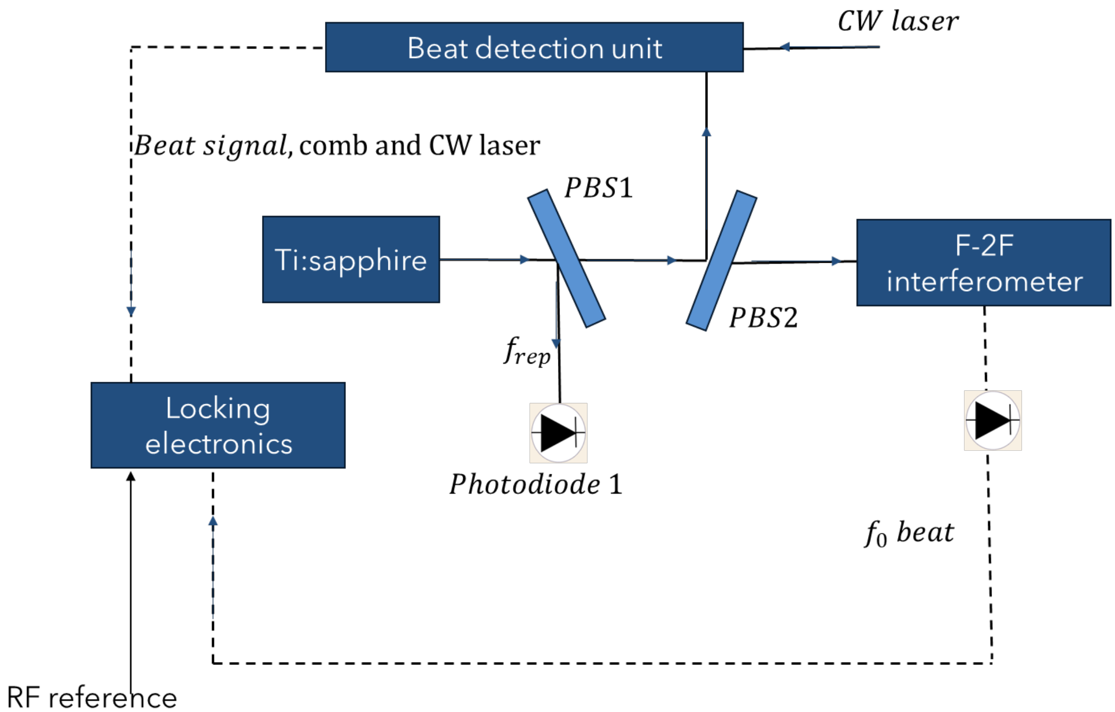

Figure 9: Stabilizing the frequency comb using another CW laser instead of locking the repetition frequency f_{rep}

The advantages of this approach are twofold. Firstly, it enables spectral purity transfer, which means that the sub-Hertz linewidth of the optical reference system is transferred to every comb line. Secondly, when detecting the light from the frequency comb, which is now locked to the ultra-stable cavity, using a fast photodiode, ultra-stable microwaves are generated. This is in contrast to the previous approach where upconversion of RF noise was a concern. Here, the optical reference already exhibits very little phase noise. By locking the frequency comb to this reference and detecting the light on a fast photodiode, not only is the optical frequency divided down to the repetition rate of the comb, but the noise of the optical reference is also divided by a factor of n, roughly 1 million. As a result, the noise is significantly reduced, leading to record-breaking ultra-stable microwaves.

However, a disadvantage of this method is that it requires an ultra-stable CW laser. While complete systems are available from certain vendors, they may come at a higher cost compared to standalone frequency comb systems. Nevertheless, these systems offer exceptional performance and stability, making them highly desirable for various precision applications.

6 Measurement of unknown frequency

Frequency comb spectroscopy[16, 4] is a sophisticated technique employed to measure unknown frequencies with exceptional precision. Frequency comb technology facilitates the precise measurement of unknown frequencies by comparing them to the evenly spaced and stable frequencies generated by the comb. This technique enables accurate determination of unknown frequencies through spectral analysis and calibration against the known comb frequencies.

Accurately measuring an unknown frequency using a frequency comb requires several key pieces of information. First and foremost, you need to know the set values of the offset frequency(f_0) and repetition frequency. Additionally, the frequency comb must be stabilized to ensure precise measurements. The process begins by measuring the beat frequency(f_{beat}) between the unknown frequency and the frequency comb. This beat frequency represents the difference between the unknown frequency and the frequency of the n^{th} comb line f_n. Mathematically, the unknown frequency can be calculated as the sum of n^{th} comb line frequency and beat frequency.

$$f_{un} = f_n + f_{beat} \tag{22}$$

where f_{un} is the unknown frequency, f_n is the frequency of the n^{th} comb line and f_{beat} is the beat frequency of unknown frequency and frequency comb. The value of the beat frequency typically falls within the range $0 \leq f_{beat} \leq \frac{f_{rep}}{2}$, Once you have measured the beat frequency, determining the value of f_n is straightforward. This can be calculated easily if you know the set values of f_{rep} and f_0,

$$f_n = nf_{rep} + f_0 \tag{23}$$

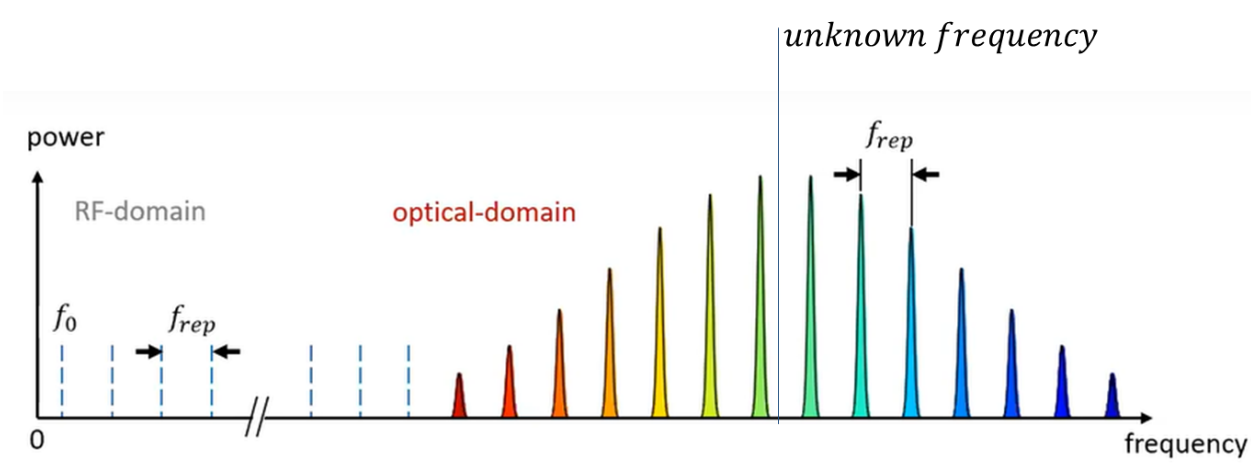

Figure 10: Measurement of unknown frequency f_{un} using frequency comb, Image credit: Menlo Systems, $https://www.youtube.com/watch?v = UgiBYZ2oZT4\&t = 1379s$

the only thing remain unknown is the index of the comb line n, To determine the comb line index n, there are several methods available. One straightforward approach is to measure the unknown frequency using a wavemeter with reasonable accuracy, ensuring it is better than half the repetition rate. With this frequency measurement, you can then use the formula provided to calculate n. While the resulting n may not be an integer, rounding it to the nearest integer will provide the correct mode you're measuring at.

Alternatively, you can measure the beat frequency between the unknown frequency and the repetition rate f_{rep} for significantly different values. For example, you can detune the repetition rate from 248 to 252 megahertz and measure the beat frequency in both cases. From these measurements, n can be calculated.

$$f_{un} = nf_{rep} + f_0 + f_{beat} \tag{24}$$

Let say measure the f_{beat} for two different value of the f_{rep} will give,

$$f_{un} = nf_{rep1} + f_0 + f_{beat1} \tag{25}$$

$$f_{un} = nf_{rep2} + f_0 + f_{beat2} \tag{26}$$

the difference between eq(25) and 26 will give the value of the n,

$$n = \frac{\Delta f_{beat}}{\Delta f_{rep}} \tag{27}$$

If you have access to two frequency combs, it offers an alternative approach. By conducting simultaneous measurements of beat signals with significantly different repetition rates, precise determination of the n, becomes feasible, which ensures the accurate measurement of the unknown frequency.

7 Applications of frequency comb

7.1 Optical atomic clock

Optical frequency combs have sparked a revolution in atomic clocks and timekeeping. Unlike traditional atomic clocks, which rely on the oscillations of atoms measured in microwave frequencies, optical atomic clocks track atomic oscillations at incredibly high frequencies—around 500,000 billion times per second. However, existing electronic systems struggle to directly measure these optical frequencies.

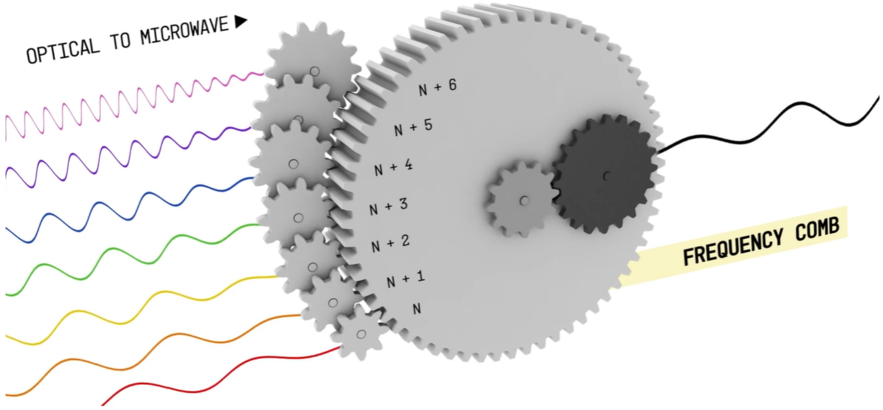

Figure 11: Frequency comb acts like gear, breakdown the optical frequency to microwave frequency, image credit: NIST, https://www.nist.gov/topics/physics/optical-frequency-combs

Optical frequency combs serve as versatile tools akin to gears in a clock, facilitating the seamless translation of high-frequency optical signals to lower-frequency microwaves(11) and vice versa[5]. Moreover, they enable scientists to effortlessly convert between various optical frequencies, enhancing flexibility and precision in a wide range of applications.

This is where frequency combs come into play, acting as a crucial bridge in the heart of the clock. Imagine a frequency comb as a set of precisely spaced gears. It takes the high-frequency light interacting with the atoms in the clock and converts it into a lower-frequency signal[10]. This allows electronics to finally "count" these rapid ticks, similar to how gears in a traditional clock translate the rapid swings of a pendulum into a slower, countable movement.

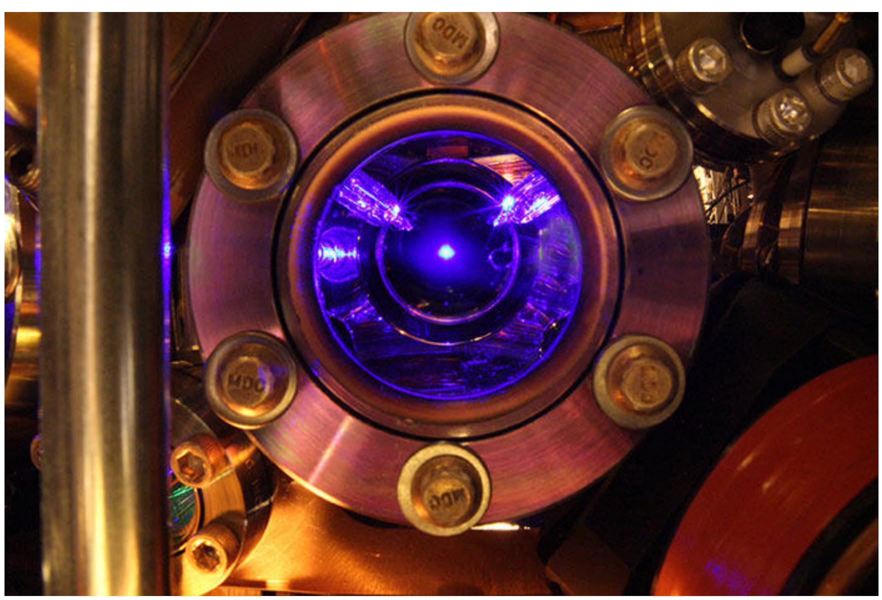

Figure 12: JILA's strontium optical atomic clock was made possible by the frequency comb. Image credit: The Ye group and Brad Baxley, JILA,

However, the story doesn't end there. Frequency combs offer another crucial benefit: maintaining the stability of the clock. While the atomic transitions themselves are incredibly stable, the process of preparing and interacting with the atoms can be disruptive. Frequency combs help to mitigate this by keeping the light source (laser) synchronized with the atoms, even during these preparation cycles. This ensures the clock continues to operate with exceptional accuracy.

7.2 Distance measurement

The utilization of optical frequency combs in long-distance applications, such as lidar (Light Detection and Ranging), represents a significant technological advancement with diverse practical implications. NIST's patenting of lidar in 2013 underscores the transformative potential of this technology in measuring distances with unprecedented precision.

In essence, lidar systems employing optical frequency combs analyze the reflected light from objects to precisely determine their distance. This innovation holds promise across various fields, with notable applications already underway in research settings. For instance, NIST's fire research laboratory has leveraged frequency comb-based lidar to penetrate flames, enabling the identification of melting objects even amidst the chaos of a fire. Additionally, these lidar systems have been instrumental in generating intricate 3D maps, showcasing their versatility in spatial analysis.

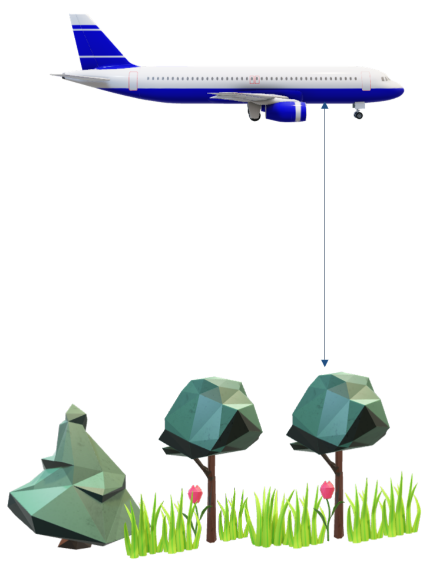

Figure 13: The distance between the surface of the earth and plane can be precisely measured using frequency comb based lidar(Light Detection and Ranging)

Looking ahead, the potential applications of lidar utilizing optical frequency combs extend beyond terrestrial domains. There is burgeoning interest in employing this technology in space missions, where the ability to maintain tight formations of satellites and other space instruments is crucial. By acting as a unified instrument, lidar systems based on optical frequency combs could enhance coordination and efficiency in space exploration endeavours.

7.3 Frequency comb in atmospheric science

These combs generate millions of precisely spaced frequencies in short pulses, offering a powerful tool for efficiently studying the quantity, structure, and dynamics of atoms and molecules.

One notable application is in pollution research. Scientists at JILA have utilized optical frequency combs to investigate short-lived molecules associated with fossil fuel combustion and air pollution. Additionally, these combs enable the exploration of the intricate structures and behaviors of large and complex molecules.
In the detection of trace gases within oil and gas fields(14), a mobile dual-frequency comb laser spectrometer plays a pivotal role. Positioned at the center of a circular arrangement, this spectrometer is surrounded by retroreflecting mirrors. When laser light emitted from the spectrometer, depicted as a yellow line, encounters a gas cloud, it penetrates through the cloud and subsequently strikes the retroreflector. The retroreflector then redirects the light back along its original path, precisely returning it to the point of its emission. This process facilitates the collection of crucial data utilized in identifying leaking trace gases, with a particular focus on methane, along with pinpointing the locations of leaks and quantifying their emission rates,

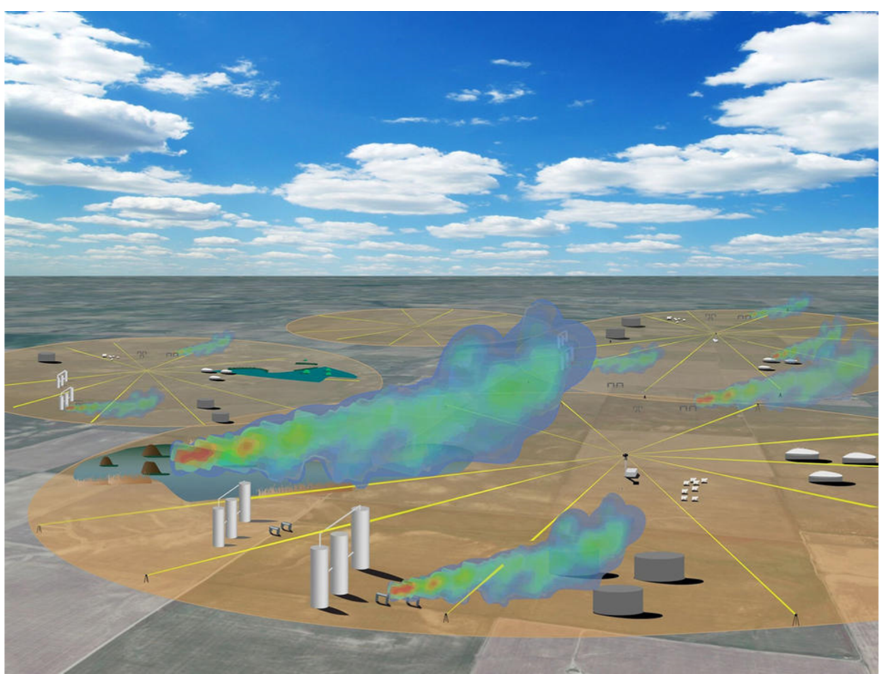

Figure 14: Frequency comb at the center of the gas field to identify leaking trace gases. image credit:Stephanie Sizemore and Ian Coddington/NIST

Moreover, researchers are pioneering the use of optical frequency combs for detecting trace amounts of various molecules in gases. In a notable development in 2019, a collaboration between NIST, the University of Colorado Boulder, and LongPath Technologies resulted in the creation of a portable spectroscopy system employing dual-comb technology. This innovative system facilitates the detection of minute methane emissions from oil and gas fields, showcasing the potential of optical frequency combs in environmental monitoring and emissions control.

8 Conclusion

This report has provided a comprehensive overview of frequency combs and their applications in precision frequency measurements. We have explored the fundamental concepts underlying frequency combs, including their generation using mode-locked lasers and the formation of equally spaced comb lines in the optical spectrum.

The report delved into the significance of octave-spanning spectral coverage in frequency combs, highlighting its importance in enabling precise frequency measurements across a wide range. We discussed the measurement of offset frequency and the challenges associated with accurately determining comb line indices, emphasizing the need for precise knowledge of the offset frequency and repetition frequency.

Furthermore, we examined various techniques for stabilizing frequency combs, such as locking to a radio frequency reference or a continuous wave laser. Each approach offers unique advantages and considerations, ultimately contributing to the overall stability and accuracy of frequency measurements. The report discussed practical methods for utilizing frequency combs in measuring unknown frequencies, outlining the process of measuring beat frequencies and determining the frequency of comb lines. These techniques provide valuable insights into accurately characterizing unknown frequencies with high precision.

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