

Determination of the starting Solution

In any transportation model we determine a starting BFS and then iteratively move towards the optimal solution which has the least shipping cost.

There are three methods to determine a starting BFS. As mentioned earlier, any BFS will have only $m+n-1$ basic variables (which may assume non-zero = positive values) and the remaining variables will all be non-basic and so have zero values. In any transportation tableau, we only indicate the values of basic variables. The cells corresponding to non-basic variables will be blank.

Degenerate BFS

If in a cell we find a zero mentioned, it means that that cell corresponds to a basic variable which assumes a value of zero. In simplex language, we say that we have a degenerate BFS.

NORTH-WEST Corner Method for determining a starting BFS

The method starts at the north-west corner cell (i.e. cell (1,1)).

Step 1. We allocate as much as possible to the selected cell and adjust the associated amounts of supply and demand by subtracting the allocated amount.

Step 2. Cross out the row (column) with zero supply (zero demand) to indicate that no further assignments can be made to that row(column).

If both a row and a column are simultaneously satisfied then

if exactly one row or column is left uncrossed make the obvious allocations and stop. Else cross out one only (either the row or the column) and leave a zero supply(demand) in the uncrossed out row(column).

Step 3. If no further allocation is to be made, stop. Else move to the cell to the right (if a column has just been crossed out) or to the cell below if a row has just been crossed out. Go to Step 1.

Consider the transportation tableau:
Destination

		1	2	3	4	Supply
Source	1	3 <u>3</u>	7 <u>2</u>	6 <u></u>	4 <u></u>	5
	2	2 <u></u>	4 <u>1</u>	3 <u>1</u>	2 <u></u>	2
	3	4 <u></u>	3 <u></u>	8 <u>1</u>	5 <u>2</u>	3
	Demand	3	4	2	2	

Total shipping cost = 48

2. LEAST COST Method of determining the starting BFS.

In this method we start assigning as much as possible to the cell with the least unit transportation cost (ties are broken arbitrarily) and the associated amounts of supply and demand are adjusted by subtracting the allocated amount.

Cross out the row (column) with zero supply (zero demand) to indicate that no further assignments can be made to that row(column).

If both a row and a column are simultaneously satisfied then

if exactly one row or column is left uncrossed make the obvious allocations and stop. Else cross out one only (either the row or the column) and leave a zero supply(demand) in the uncrossed out row(column).

Next look for the uncrossed out cell with the smallest unit cost and repeat the process until no further allocations are to be made.

Consider the transportation tableau:
Destination

		1	2	3	4	Supply
Source	1	3 ①	7 ①	6 ②	4 ②	3 4 2
	2	2 ②	4	3	2	2
	3	4	3 ③	8	5	3
	Demand	3 1	3 0	2	2	

The tableau shows the following data:

- Source 1:** Allocations of 1 unit to Destination 1 and 2 units to Destination 4. A green path starts at (1,4), goes left to (1,3), then down to (3,2), and finally left to (1,1). A red arrow points from (1,4) to (1,3).
- Source 2:** Allocation of 2 units to Destination 1.
- Source 3:** Allocation of 3 units to Destination 2. An orange arrow points from (3,2) to (1,4).
- Demands:** Destination 1 has a demand of 1 (with 3 crossed out). Destination 2 has a demand of 0 (with 3 crossed out). Destinations 3 and 4 have demands of 2 each.
- Supply:** Source 1 has a supply of 4 (with 3 crossed out). Source 2 has a supply of 2. Source 3 has a supply of 3.

Total shipping cost = 36

3. Vogel's approximation method (VAM)

Step 1. For each row (column) remaining under consideration, determine a penalty by subtracting the smallest unit cost in the row (column) from the next smallest unit cost in the same row(column). (If two unit costs tie for being the smallest unit cost, then the penalty is 0).

Step2. Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the cell with the least unit cost in the selected row or column.(Again break the ties arbitrarily.) Adjust the supply and demand and cross out the satisfied row or column.

If both a row and a column are simultaneously satisfied then

if exactly one row or column is left uncrossed make the obvious allocations and stop. Else cross out one only (either the row or the column) and leave a zero supply(demand) in the uncrossed out row(column). (**But omit that row or column for calculating future penalties**).

Step 3. If all allocations are made, stop. Else go to step 1.

Total shipping cost = 32