INDIAN INSTITUTE OF TECHNOLOGY ROPAR
Department of Mathematics
Second Semester of Academic Year 2015-2016
MAL 114 - Linear Algebra
Lab tutorial sheet I

---

(1) By elementary row operations, find the rank of the following matrices

(a) $\begin{pmatrix} 2 & 3 & 1 & 0 & 4 \\ 3 & 1 & 2 & -1 & 1 \\ 4 & -1 & 3 & -2 & -2 \\ 5 & 4 & 3 & -1 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 0 & 2 \\ 0 & -2 & 5 \\ 2 & -1 & 4 \\ 4 & -2 & 6 \end{pmatrix}$ (c) $\begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$

(2) Determine which system has (i) unique solution (ii) infinitely many solutions and (iii) no solution using row reduced echelon form.

(a) $x + 2y + 3z = 6,\ x + y + z = 3,\ 2x + 3y + 4z = 10$
(b) $x - y + z + 3w = 1,\ 2z + y + 5z = 0,\ x + 3y + 5z - 5w = 2$
(c) $x_1 + x_2 + x_3 = 2,\ 2x_1 + x_3 = 1,\ x_1 + 2x_2 = 5,\ x_1 + 3x_2 + 2x_3 = 5$
(d) $2x + 2y + z = 5,\ x - y + z = 1,\ 3x + y + 2z = 4$

(3) Determine the following set of vectors in S are linearly independent or dependent in V

(a) $S = \{$(3 -2 4 5), (0 2 3 -4), (1 0 2 7), (7 -10 1 29)$\}$, $V = \mathbb{R}^4$

(b) $S = \{$(1 2 1 3 2), (1 3 3 5 3), (3 8 7 1 3 8), (1 4 6 9 7), (5 13 13 25 19)$\}$, $V = \mathbb{R}^5$

(c) $S = \left\{ \begin{pmatrix} -1 & 0 & 1 \\ 4 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 6 & 3 & 0 \\ 1 & 2 & 0 \end{pmatrix}, \begin{pmatrix} -7 & 8 & 14 \\ 26 & -11 & 17 \end{pmatrix}, \begin{pmatrix} -1 & -2 & 0 \\ -3 & 4 & -1 \end{pmatrix}, \begin{pmatrix} -14 & -4 & 3 \\ 13 & -5 & 1 \end{pmatrix} \right\}$, $V = \mathbb{M}_{2\times 3}(\mathbb{R})$

(d) $S = \left\{ \begin{pmatrix} 2+3i \\ 4-5i \\ 1+i \end{pmatrix}, \begin{pmatrix} 2-4i \\ 1-i \\ 2i \end{pmatrix}, \begin{pmatrix} 6+2i \\ 9-11i \\ 2+4i \end{pmatrix} \right\}, V = \mathbb{C}^3$

(4) Let P_2 be a vector space of polynomials of degree 2 or less. Determine whether the following vectors in P_2 are linearly independent or dependent

(a) $p = 6t^2 + 8t + 2$ and $q = 3t^2 + 4t + 1$

(b) $p = t^2 + 3t - 1$, $q = 2t^2 + 7t + 5$ and $r = 7$.

(5) (a) Write $v =$(1 3 -3 5) as linear combination of vectors $v_1 =$(1 2 -1 0), $v_2 =$(2 -1 1 -1), $v_3 =$(2 -1 3 1) and $v_4 =$(3 -1 1 0) in $\mathbb{R}^4$.

(b) Find a basis for the column space of V spanned by vectors $\{v_1, v_2, v_3, v_4\}$.

(6) (a) Determine the following set of vectors (S) forms a basis in $\mathbb{R}^4$, where $S = \{$(1 0 0 4), (3 2 5 0), (2 2 4 4), (3 4 9 -4)$\}$

(b) If not, find the basis and the dimension of the subspace spanned by S.

(c) Extend S to the basis of $\mathbb{R}^4$.

(7) Let $A = \begin{pmatrix} 1 & 2 & 1 & 3 & 1 \\ 2 & 1 & 5 & -3 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 3 & 0 & 9 & -9 & 1 \\ 1 & 1 & 2 & 0 & 1 \end{pmatrix}$. Find

(a) a basis for the row space of A consisting vectors of A only
(b) a basis for the column space of A consisting vectors of A only
(c) basis for the solution space of $Ax = 0$.

(8) Let W_1 and W_2 are subspaces of $\mathbb{R}^4$ spanned by vectors (1 1 1 1), (1 1 -1 1), (1 -1 1 -1) and (1 1 0 0), (1 -1 1 0), (2 -1 1 -1) respectively. Find

(a) a basis of $W_1 + W_2$.

(b) dimension of $W_1 \cap W_2$

(9) Let $V = \mathbb{R}^4$ and $u = (1\ 1\ 0\ 2)$, $v = (2\ 0\ 1\ 1)$, $w = (1\ -1\ 1\ -1)$ vectors in $\mathbb{R}^4$. If $W = span\{u, v, w\}$, then find a basis of $W^{\perp}$.

(10) $S = \{v_1, v_2, v_3, v_4\} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ forms a basis in $\mathbb{R}^4$. Find the coordinate vectors $u_{[S]}, v_{[S]}, (u+v)_{[S]}$ and $u_{[S]} + v_{[S]}$, where $u = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$.

(11) The vectors $u = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, w = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ forms a basis S of $\mathbb{R}^3$. Find:

(a) The change of basis matrix P from the usual basis $E = \{e_1, e_2, e_3\}$ to S.

(b) The change of basis matrix Q from S to E.

(12) Suppose the x-axis and y-axis in the plane $\mathbb{R}^2$ are rotated counterclockwise 45° so that the new x'-axis and y'-axis are along the line $y = x$ and the line $y = -x$, respectively.

(a) Find the change of basis matrix P.

(b) Find the coordinates of the point A(5,6) under the given rotation.

(13) Let $\mathbb{F}$ be a field of complex numbers and T be a linear transformation from $\mathbb{F}^3$ into $\mathbb{F}^3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3,\ 2x_1 + x_2 - x_3,\ -x_1 - 2x_2)$. Find dimension of range and kernel of T.

(14) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y, z, w) = (x - y + z + w,\ x + 2z - w,\ x + y + 3z - 3w)$. Then find the basis and dimension of range space and null space of T.

(15) Let the projection of the vector u onto a vector v is defined by $\text{proj}_v u = \frac{<u,v>}{\|u\|\|v\|}$ using standard inner product.

(a) Find the $\text{proj}_v u$, where $u = (1\ -3\ 1\ 2)$ and $v = (2\ -1\ 1\ 0)$.
(b) Find the projection of u onto the space W spanned by the vectors $\{(-1\ 0\ -1\ 3), (2\ 1\ 4\ 2)\}$.
(c) Find a vector w which is orthogonal to the space W.

(16) Apply the Gram-Schmidt Process to obtain an orthogonal basis and then an orthonormal basis for the subspace W of $\mathbb{R}^7$ spanned by $v_1 = (1\ 3\ 4\ 5\ 3)$, $v_2 = (4\ 5\ 6\ 7\ 2), v_3 = (7\ 9\ 0\ 2\ 1), v_4 = (5\ 4\ 6\ 7\ 3)$ and $v_5 = (1\ 4\ 6\ 8\ 0)$.

(17) $S = \{v_1 = (1, 2, 3, -1)^t,\ v_2 = (1, -1, 1, -2)^t,\ v_3 = (-1, 5, 3, 0)^t,\ v_4 = (-1, 2, 1, -1)^t\}$. If the above set of vectors are linearly independent, then find the orthonormal basis. Otherwise find linearly independent vectors from the set S and orthonormalize it.

(18) Find an orthogonal basis for the vector space V that contains the vectors (1 2 -1 0) and (1 0 1 3) in $\mathbb{R}^4$,