

### 5 variable K-map ( $A, B, C, D, E$ )

		DE				DE							
		00	01	11	10	00	01	11	10	0'	1	0'	1
		BC											
		00	0	1	3	2	00	16	17	19	18		
		01	4	5	7	6	01	20	21	23	22		
		11	12	13	15	14	11	28	29	31	30		
		10	8	9	11	10	10	24	25	27	26		

$A=0$

$A=1$

$$\Rightarrow f(A, B, C, D, E) = \sum m(4, 5, 6, 7, 9, 11, 20, 21, 22, 23, 25, 27, 29, 31)$$

		DE				DE							
		00	01	11	10	00	01	11	10	00	01	11	10
		BC	BC	BC	BC	BC	BC	BC	BC	BC	BC	BC	BC
		00	0	1	3	2	00	16	17	19	18		
		01	1	5	7	6	01	20	21	23	22		
		11	12	13	15	14	11	28	29	31	30		
		10	8	(19)	11	10	10	24	25	27	26		

$A=0$

$B\bar{C}E$

$A=1$

$ACE$

$$f = ACE + \bar{B}C + B\bar{C}E$$

### 6 variable K-map ( $A, B, C, D, E, F$ )

$AB$

$\rightarrow 00$   
 $\rightarrow 01$   
 $\rightarrow 10$   
 $\rightarrow 11$

		EF				EF							
		00	01	11	10	00	01	11	10	00	01	11	10
		CD											
		00	0	1	3	2	00	16	17	19	18		
		01	4	5	7	6	01	20	21	23	22		
		11	12	13	15	14	11	28	29	31	30		
		10	8	9	11	10	10	24	25	27	26		

$A=0, B=0$

$A=0, B=1$

		EF			
		00	01	11	10
		CD	CD	CD	CD
		00	32	33	35
		01	36	37	39
		11	44	45	47
		10	40	41	43

$A=1, B=0$

		EF			
		00	01	11	10
		CD	CD	CD	CD
		00	48	49	51
		01	52	53	55
		11	60	61	63
		10	56	57	59

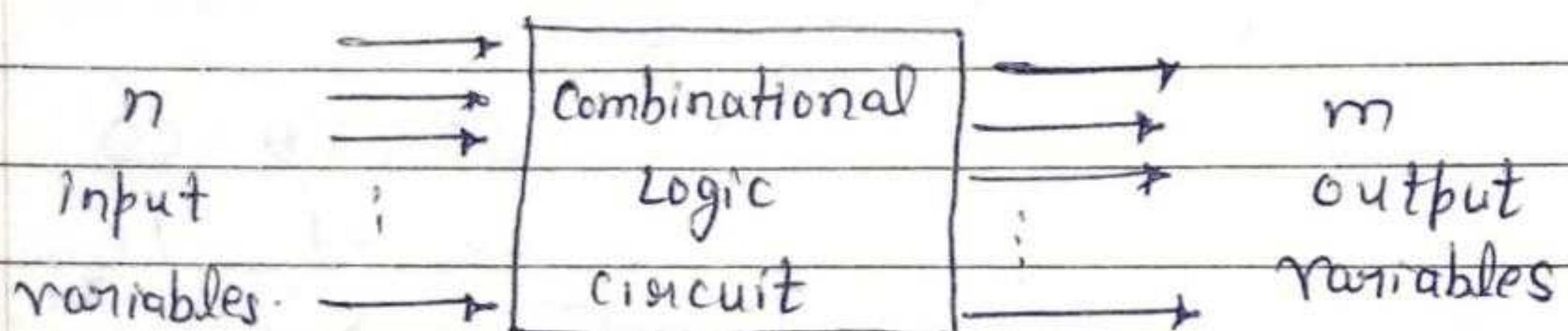
$A=1, B=1$

## "UNIT - 3"

### "COMBINATIONAL CIRCUITS"

#### Combinational circuits \*

- \* A combinational circuit is one in which the present state of the combination of the logic inputs decides the output.
- \* The term combination logic mean combining of two or more logic gates to form a required function where the output at a given time depends only on the input.
- The required output data is obtained from this process by transforming the binary information given at the input.



- A combinational circuit operates in three steps-
  - It accepts  $n$ -different inputs.
  - The combination of gates operates on the inputs.
  - " $m$ " different output are produced as per requirement.

#### Characteristics \*

1. The output of combinational circuit at any instant of time, depends only on the level present at input terminals.

2. The combinational circuit do not use any memory.  
The previous state of input does not have any effect on the present state of the circuit.
3. A combinational circuit can have an  $n$  number of inputs and  $m$  number of outputs.

### Classifications +

#### combinational Logic Gate

Arithmetic and logical function	Data Transmission	Code Convertors
Adders	Multiplexers	Binary
Subtractors	Demultiplexers	BCD
Comparitors	Encoders	7-Segment
PLD's	Decoders	

"Adder" → An adder is a digital circuit that performs addition of two numbers-

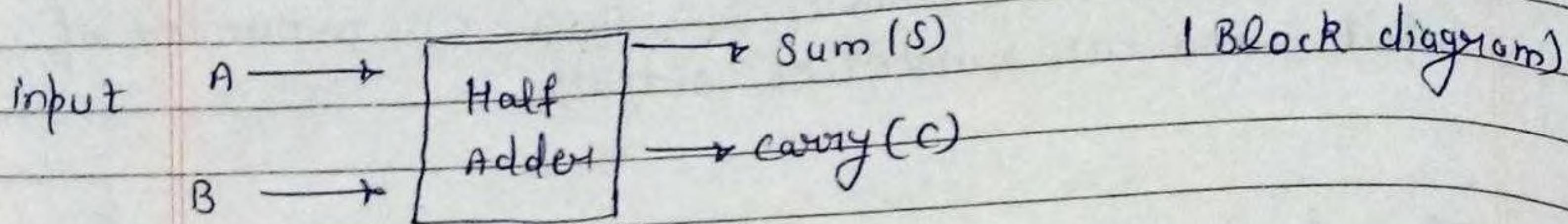
1. Half Adder
2. Full Adder

#### "Half Adder"

Half adder is a combinational logic circuit with two inputs and two outputs. The half adder is designed to add two single bit binary number  $A$  and  $B$ .

It is the basic building block for addition of two single bit numbers.

This circuit has two outputs carry and sum.



Truth Table →

A	B	$\bar{B}$	C	$\bar{C}$	B
0	0	1	0	1	0
0	1	1	1	0	1
1	0	0	1	0	0
1	1	0	0	1	1

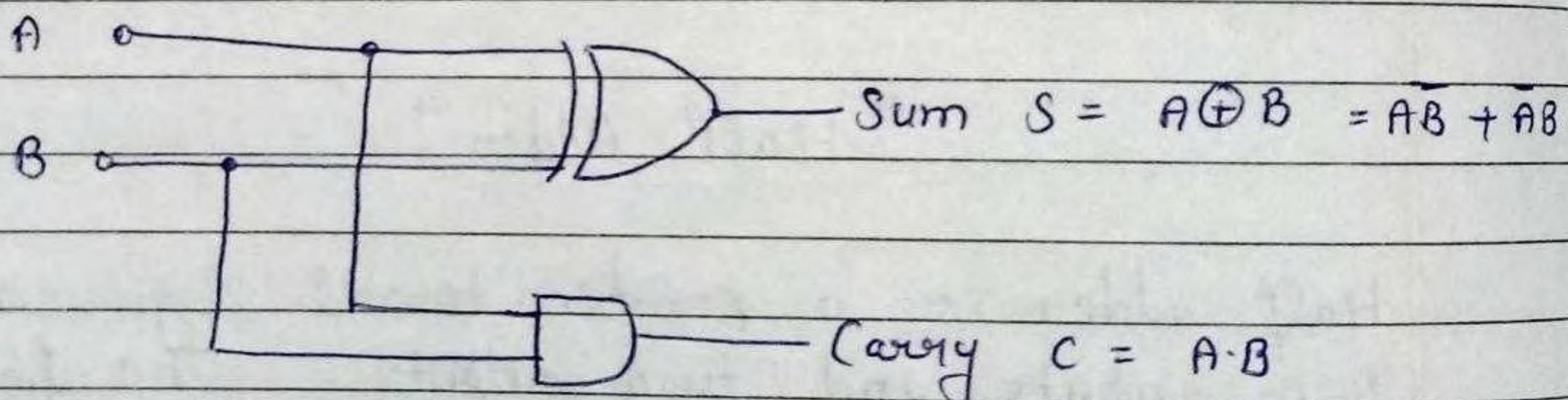
Inputs		Outputs	
A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

K-map for sum output

$$\text{Sum} = A\bar{B} + \bar{A}B \\ = A \oplus B$$

K-map for carry output

$$\text{Carry} = AB$$

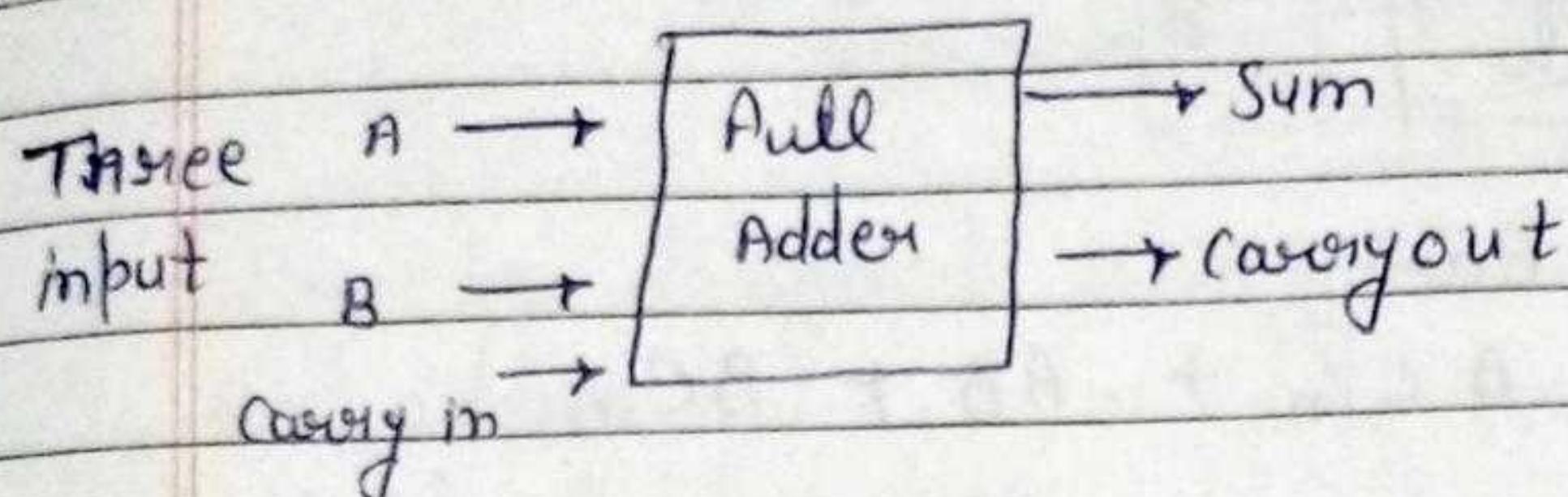


Circuit Diagram

## "Full Adder"

To overcome the drawback of Half adder circuit a, 3 single bit adder circuit called Full Adder is developed.

It can add two one-bit numbers A and B and carry  $C_{in}$ . The full adder is a three input and two output combinational circuit.



(Block diagram)

Truth Table -

Inputs			Outputs	
A	B	$C_{in}$	S	$C_o$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

	$A$	$B$	$C_{in}$	$\bar{B}C_{in}$	$B\bar{C}_{in}$	$\bar{B}\bar{C}_{in}$	$BC_{in}$
$\bar{A}$	0	0	0	0	1	1	0
A	1	0	1	1	0	0	1

K-map for sum  $\rightarrow$

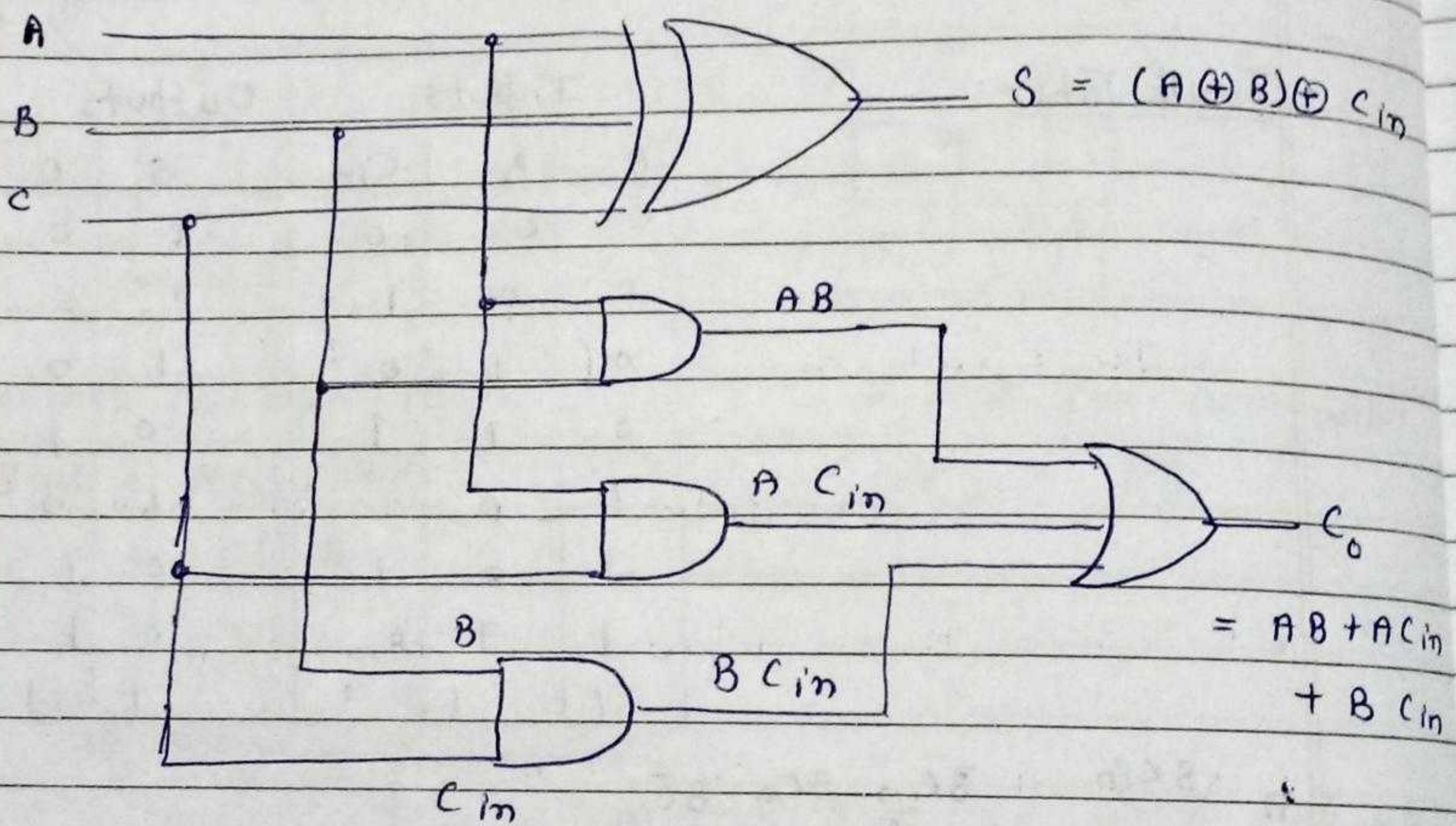
$$\begin{aligned}
 \text{Sum} &= \bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}C_{in} + AB\bar{C}_{in} \\
 &= \bar{B}(A\bar{C}_{in} + \bar{A}C_{in}) + B(A\bar{C}_{in} + \bar{A}\bar{C}_{in}) \\
 &= \bar{B}(A \oplus C_{in}) + B(A \odot C_{in}) \\
 &= \bar{B}(A \oplus C_{in}) + B(\bar{A} \oplus C_{in}) \\
 &= A \oplus B \oplus C_{in}
 \end{aligned}$$

$A$	$B$	00	01	11	10
$\bar{A}$	0	0	0	1	0
$A$	1	0	1	0	0

$A$	$B$	00	01	11	10
$\bar{A}$	0	0	0	1	0
$A$	1	0	1	0	0

K-map for Carry  
 $\text{Carry} = A C_{in} + AB + BC_{in}$



Logic Diagram

## Application of Full Adder +

1. The full adder acts as the basic building block of the 4 bit / 8 bit binary / BCD adder ICs such as 7483.
2. It is used in the digital diary.
3. It is used in digital computers.
4. It is used in arithmetic logic unit (ALU).

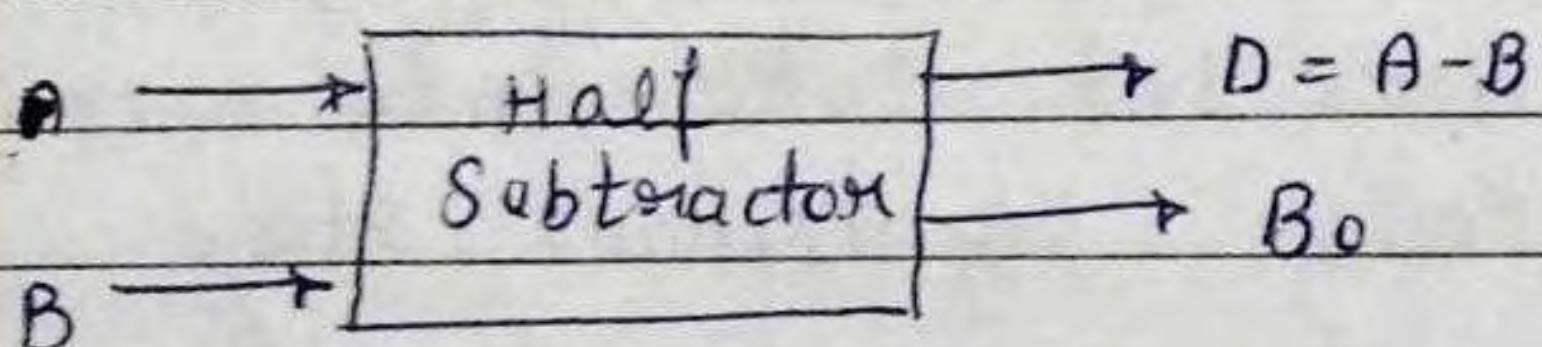
## "Subtractor"

In electronic, a subtractor can be designed using the same approach as that of an adder.

1. Half Subtractor
2. Full Subtractor.

## "Half Subtractor"

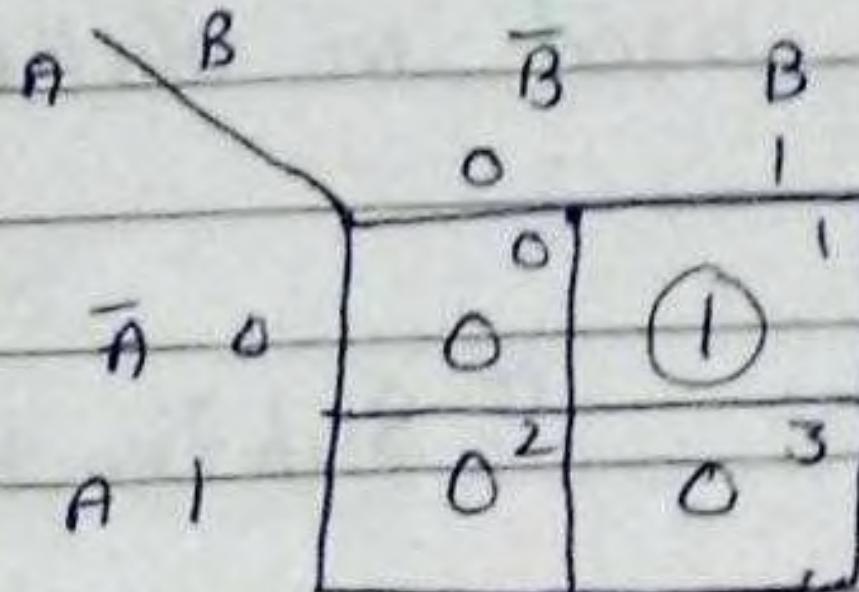
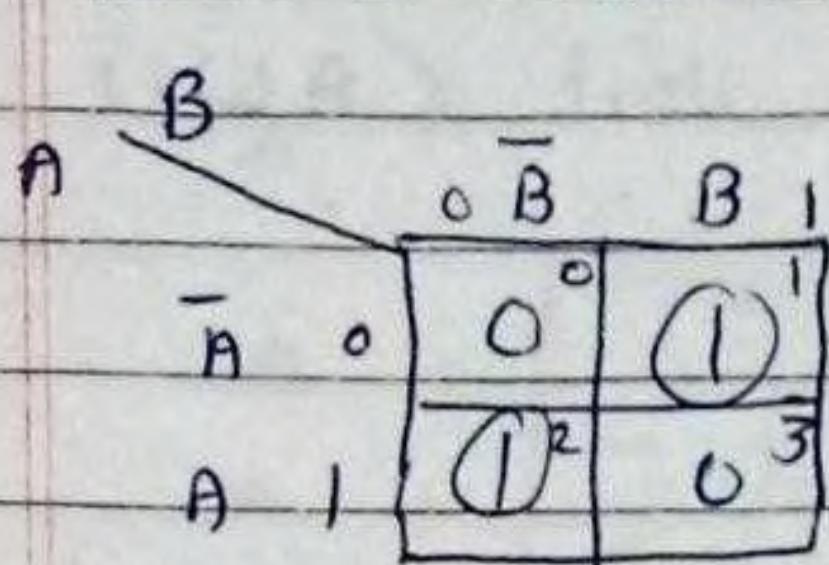
1. Half subtractor is a combination circuit with two inputs and two output (difference and borrow).
2. It produces the difference b/w the two binary bits at the input and also produces a output (Borrow) to indicate if a 1 has been borrowed in the subtraction ( $A - B$ ), A is called as Minuend bit and B is called Subtrahend bit.



(Block diagram)

Truth table -

Inputs		Outputs	
A	B	Difference D	Borrow B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



K-map for difference -

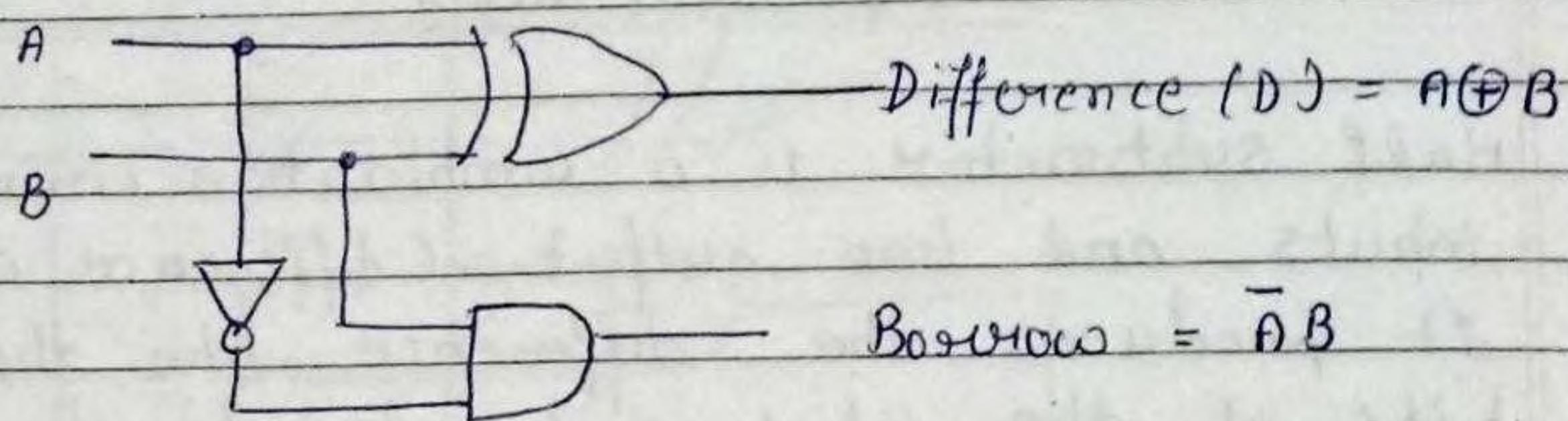
$$\text{Diff} = A\bar{B} + \bar{A}B$$

$$= A \oplus B$$

K-map for Borrow.

$$\text{Borrow} = \bar{A}B$$

Logic Diagram -



Disadvantage of Half subtractor -

Half subtractor can only perform the subtraction of two binary bits. But while performing the subtraction, it does not take into account the borrow of the lower significant stage.

## "Full subtractor"

The disadvantage of a half subtractor is overcome by full subtractor.

The full subtractor is a combinational circuit with three inputs  $A, B, C$  and two output  $D$  and  $C'$ .  $A$  is the minuend,  $B$  is subtrahend,  $C$  is the borrow produced by the previous stage,  $D$  is the difference output and  $C'$  is the borrow output.

Truth Table -

Inputs		B <sub>in</sub>	Outputs (A - B - B <sub>in</sub> )	B <sub>o</sub>
A	B			
(minuend)	(subtrahend)	0	0	0
0	0	0	1	1
0	1	1	1	1
0	1	0	0	1
1	0	1	1	0
1	0	0	0	0
1	1	0	1	1
1	1	1	0	0

B <sub>Bin</sub>	$\bar{B}B_{in}$		$\bar{B}B_{in}$	
	00	01	11	10
$\bar{A}$	0	1	3	2
$A$	1	0	1	0

$$\bar{B}B_{in} + A\bar{B}B_{in} + \bar{A}\bar{B}B_{in} + AB\bar{B}_{in}$$

K-map for difference =  $\bar{B}(A\bar{B}_{in} + \bar{A}B_{in}) + B(\bar{A}B_{in} + \bar{A}\bar{B}_{in})$

$$\bar{B}(A \oplus B_{in}) + B(A \odot B_{in})$$

$$\bar{B}(A \oplus B_{in}) + B(A \oplus B_{in})$$

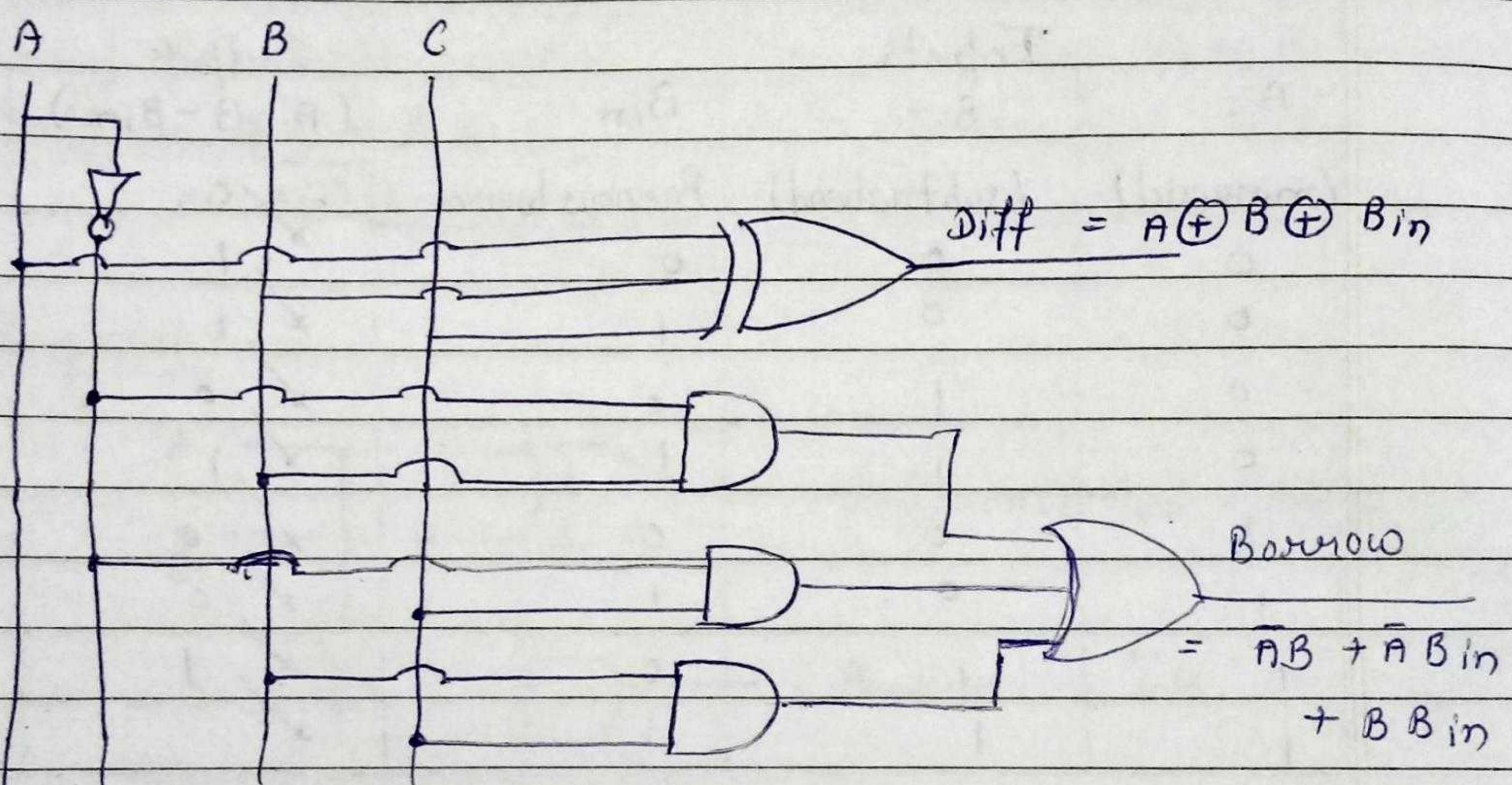
$$A \oplus B \oplus B_{in}$$

$A$	$B\bar{B}_{in}$	$\bar{B}\bar{B}_{in}$	$\bar{B}B_{in}$	$BB_{in}$	$B\bar{B}_{in}$
$\bar{A}$	0	0	1	1	0
$A$	1	0	0	1	0
	4	5	7	6	2

K-map for Borrow -

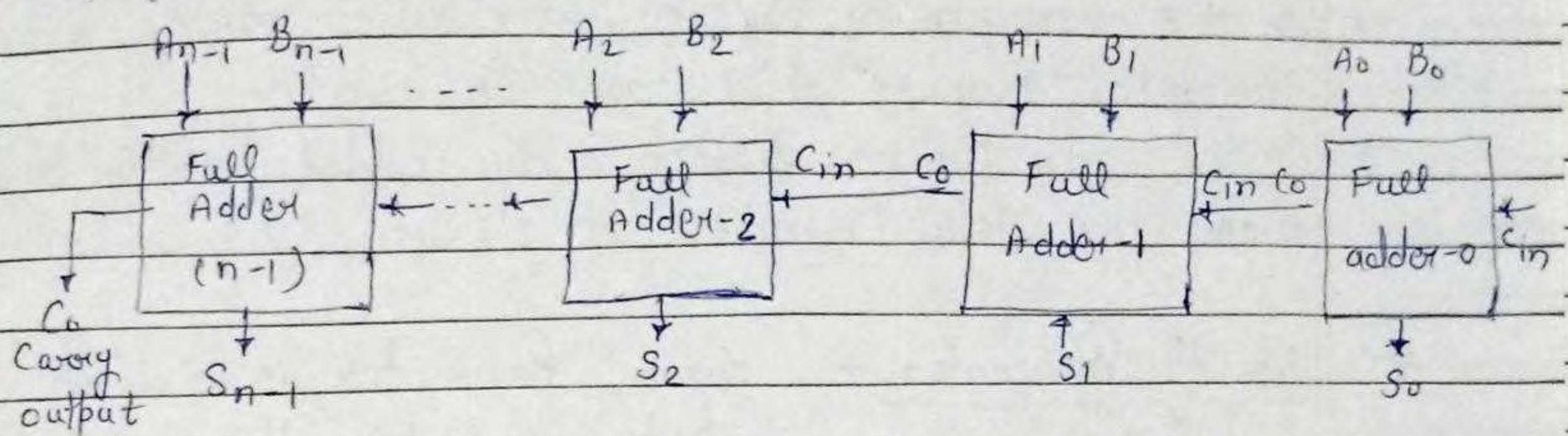
$$\text{Borrow} = \bar{A}B_{in} + \bar{A}B + BB_{in}$$

Logic Diagram →



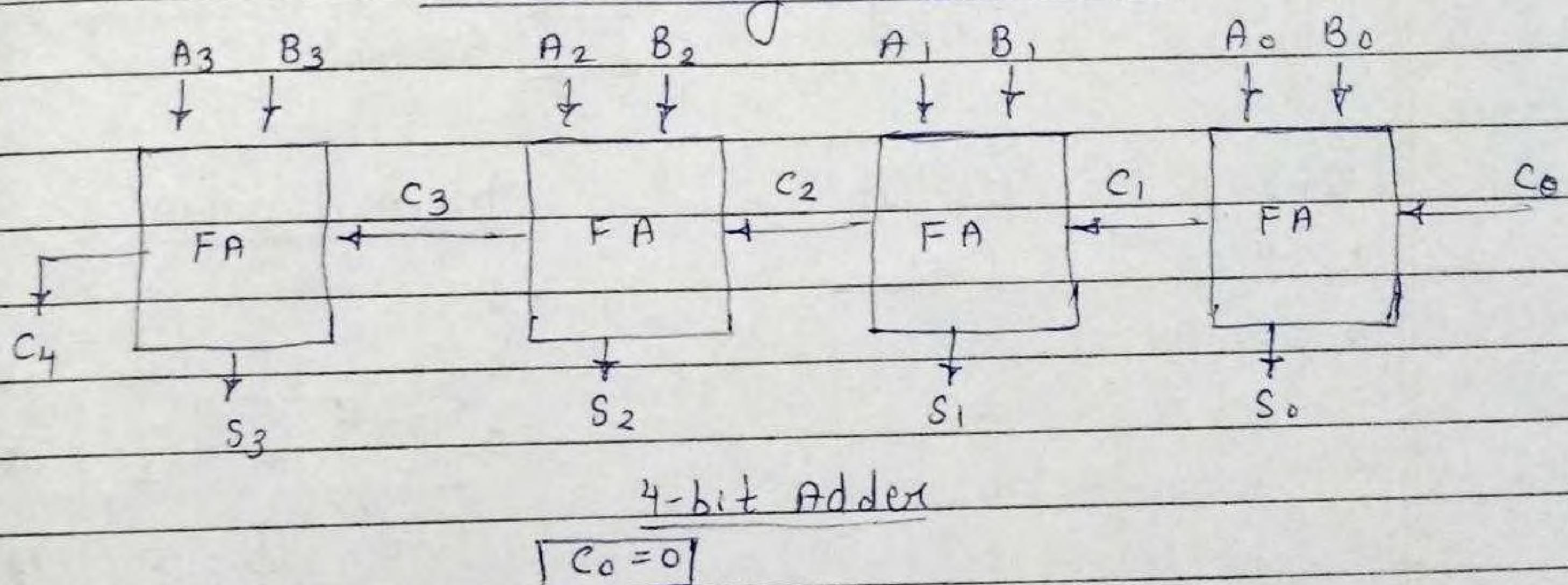
### "n-bit Parallel Adder"

- The full adder is capable of adding only two single digit binary numbers along with a carry output.
- To add two n-bit binary numbers we need to use the n-bit parallel adder.
- It uses a number of full adders in cascade. The carry output of the previous full adder is connected to carry input of the next full adder.



Block diagram of n-bit parallel adder

### "4-bit Binary Parallel Adder"



- $A_0$  and  $B_0$  represent the LSB of the four bit words  $A$  and  $B$ .

## Binary Adder

This is also called  
Ripple Carry adder,  
because of the construction  
with full adders are  
connected in cascade.

Subscript i :	3	2	1	0	
Input carry	0	0	1	0	$C_i$
	1	0	1	1	$A_i$
	0	0	1	1	$B_i$
Sum	1	1	1	0	$S_i$
Output carry	0	0	1	1	$C_{i+1}$

## Magnitude Comparator

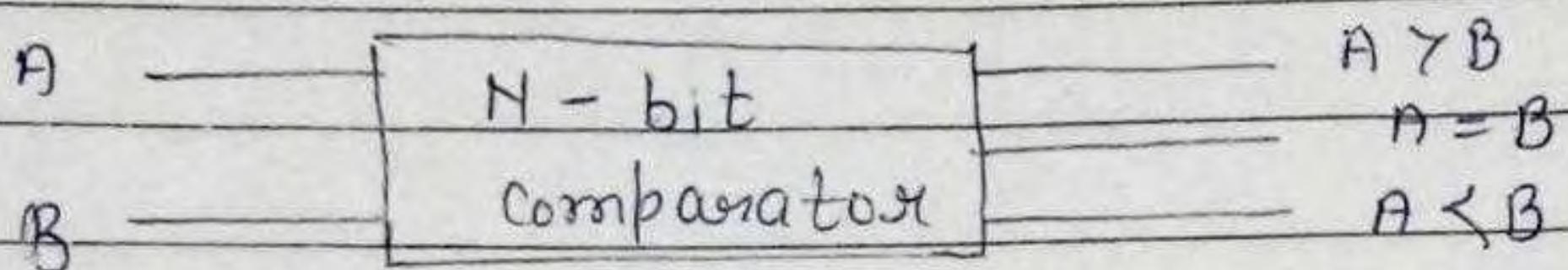
It is a combinational circuit that compares two numbers and determine their relative magnitude.

The output of Comparator is usually 3 binary variables indicating

$$A > B$$

$$A = B$$

$$A < B$$



### 2-Bit Magnitude Comparator

4 bit comparator used to compare two binary numbers each of two bits is called 2-bit magnitude comparator. It consists of four inputs and three outputs to generate less than, equal to and greater than between two binary numbers.

2-Bit Magnitude comparator ( $2^4 = 16$ )

	Input				Output			$A \geq B$	$A = B$	$A < B$
	$A_0$	$A_1$	$B_0$	$B_1$						
0	0	0	0	0	0	1	0	00 = 00		
1	0	0	0	1	0	0	1	00 < 01		
2	0	0	1	0	0	0	1	00 < 10		
3	0	0	1	1	0	0	1	00 < 11		
4	0	1	0	0	1	0	0	01 > 00		
5	0	1	0	1	0	1	0	01 = 01		
6	0	1	1	0	0	0	1	01 < 10		
7	0	1	1	1	0	0	1	01 < 11		
8	1	0	0	0	1	0	0	10 > 00		

$A_1$	$A_0$	$B_1$	$B_0$	$A > B$	$A = B$	$A < B$	
0	0	1		1	0	0	$10 > 01$
0	1	0		0	1	0	$10 = 10$
1	0	1	1	0	0	1	$10 < 11$
1	1	0	0	1	0	0	$11 > 00$
1	1	1	0	1	0	0	$11 > 01$
1	1	1	1	0	0	0	$11 > 10$
1	1	1	1	0	1	0	$11 = 11$

$A_1 A_0 \backslash B_1 B_0$	$\bar{B}_1 \bar{B}_0$	$\bar{B}_1 B_0$	$B_1 \bar{B}_0$	$B_1 B_0$	$A_1 A_0 \backslash B_1 B_0$	$\bar{B}_1 \bar{B}_0$	$\bar{B}_1 B_0$	$B_1 \bar{B}_0$	$B_1 B_0$
00	00	01	11	10	00	00	01	11	10
$\bar{A}_1 \bar{A}_0$	00	00	01	03	00	00	01	03	02
$\bar{A}_1 A_0$	01	14	05	07	00	04	15	07	06
$A_1 A_0$	11	12	11	13	015	012	013	015	014
$A_1 \bar{A}_0$	10	08	09	011	010	08	09	011	010

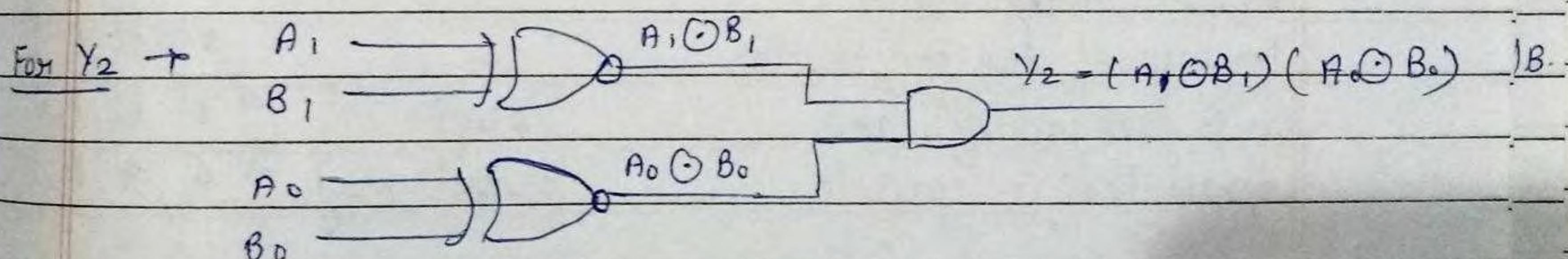
$$Y_1(A > B) = A_1 \bar{B}_1 + A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 \bar{B}_0$$

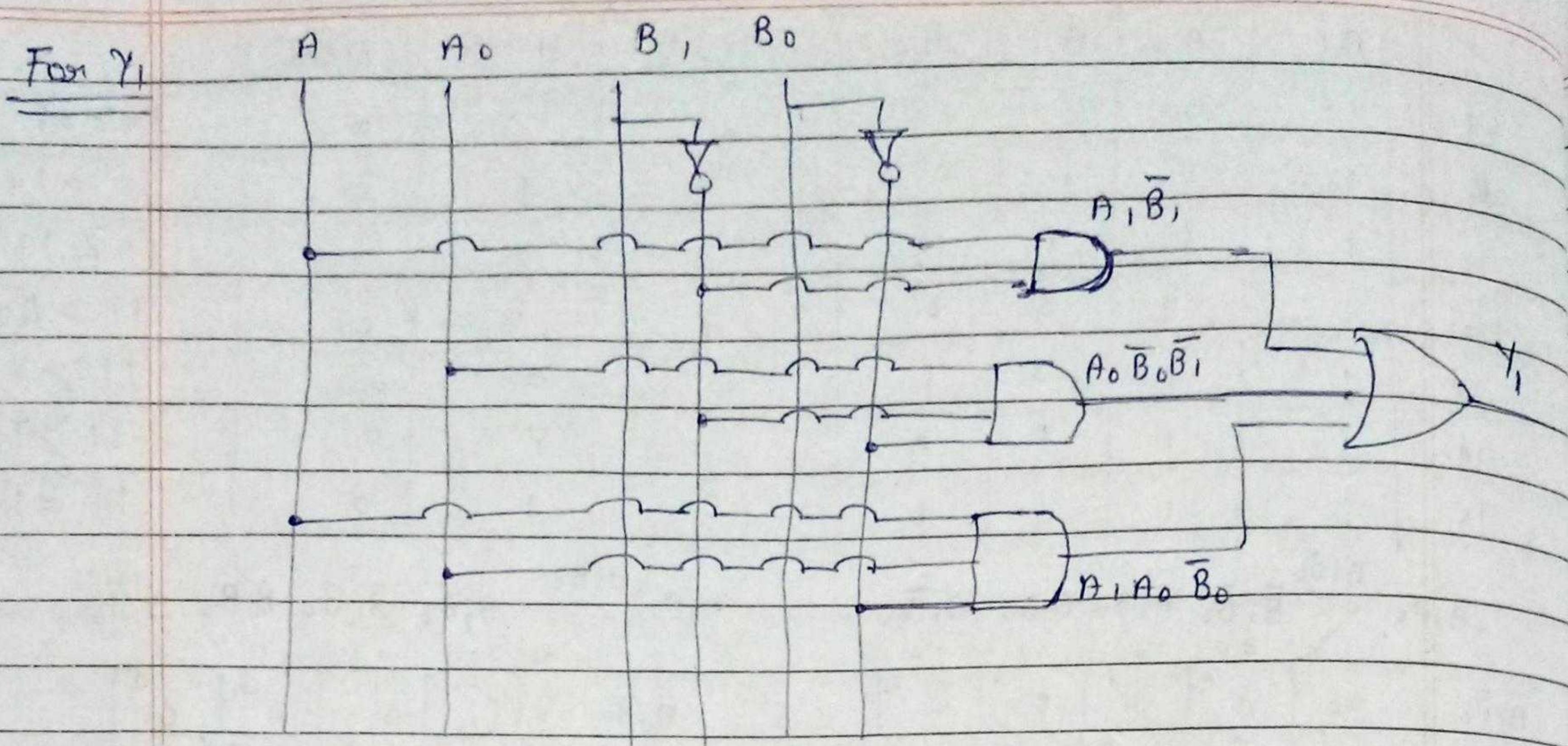
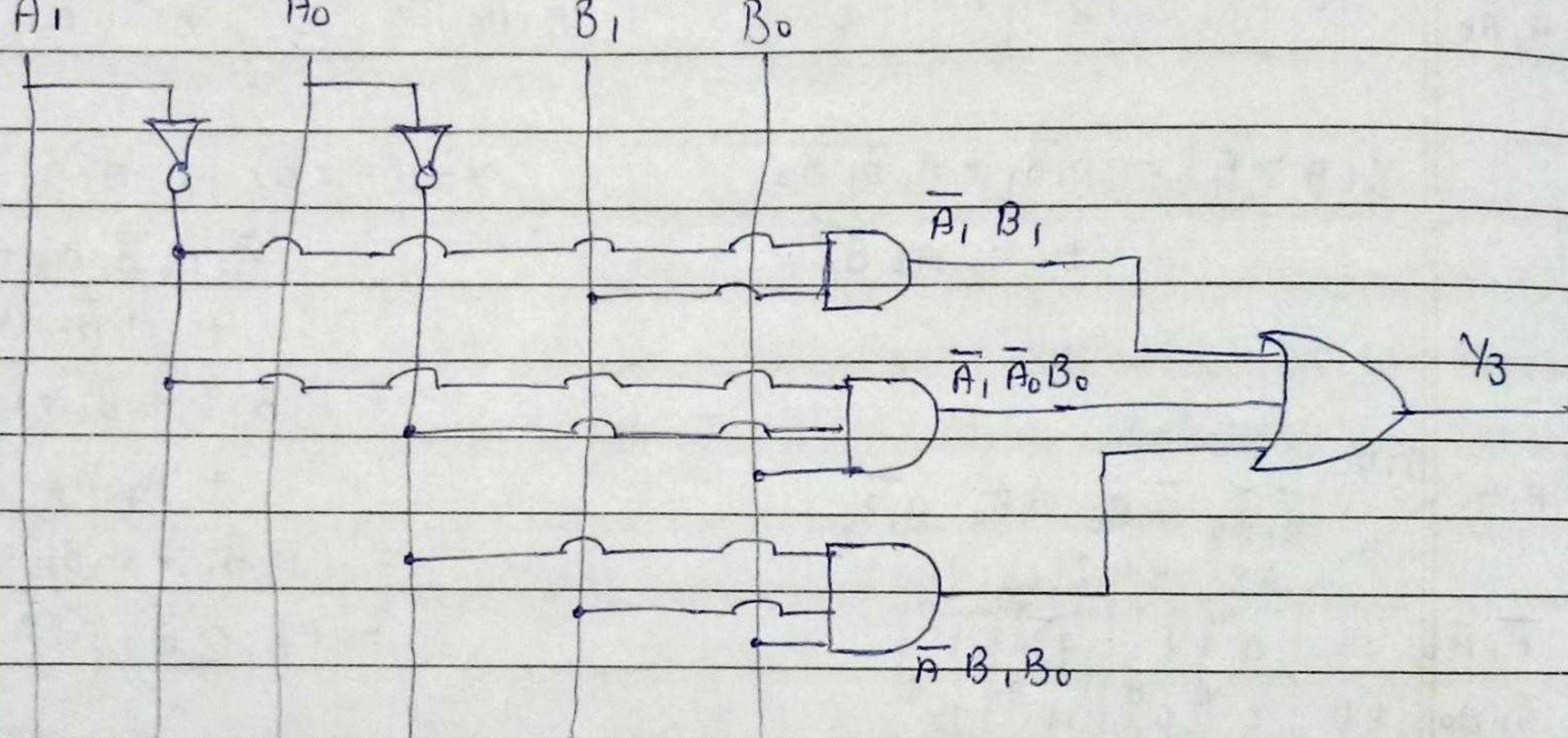
$$Y_2(A = B) = \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + \bar{A}_1 A_0 \bar{B}_1 B_0 + A_1 \bar{A}_0 B_1 \bar{B}_0 + A_1 A_0 B_1 B_0 = \bar{A}_1 \bar{B}_1 (\bar{A}_0 \bar{B}_0 + A_0 B_0)$$

$$+ A_1 B_1 (A_0 B_0 + \bar{A}_0 \bar{B}_0) = (\bar{A}_1 \bar{B}_1 + A_1 B_1) (\bar{A}_0 \bar{B}_0 + A_0 B_0) = (A_1 \odot B_1) (A_0 \odot B_0)$$

$A_1 A_0 \backslash B_1 B_0$	$\bar{B}_1 \bar{B}_0$	$\bar{B}_1 B_0$	$B_1 \bar{B}_0$	$B_1 B_0$
00	00	01	11	10
$\bar{A}_1 \bar{A}_0$	00	00	01	03
$\bar{A}_1 A_0$	01	04	05	07
$A_1 A_0$	11	012	013	015
$A_1 \bar{A}_0$	10	08	09	011

$$Y_3(A < B) = \bar{A}_1 B_1 + \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_0 B_1 B_0$$



For  $Y_1$  $A \quad A_0 \quad B_1 \quad B_0$ For  $Y_3$  $A_1 \quad A_0 \quad B_1 \quad B_0$ 

## "Multiplexer" (Data Selector)

A multiplexer is a digital circuit which selects one of the  $n$  data inputs and routes to the output. The selection of one of  $n$  inputs is done by the select inputs.

Usually there are  $2^n$  input lines and  $n$  selection

lines whose bit combinations determine which input line is selected.

For Ex- for 2 to 1 multiplexer if selection S is zero then  $I_0$  has the path to output and if S is one  $I_1$  has the path to output.

### Advantage of Multiplexer -

1. It reduces the number of wires.
2. So it reduces the circuit complexity and cost.
3. We can implement many combinational circuit using MUX.
4. It simplifies the logic design.
5. It does not need the K-map and simplification.

### Necessity of Multiplexer +

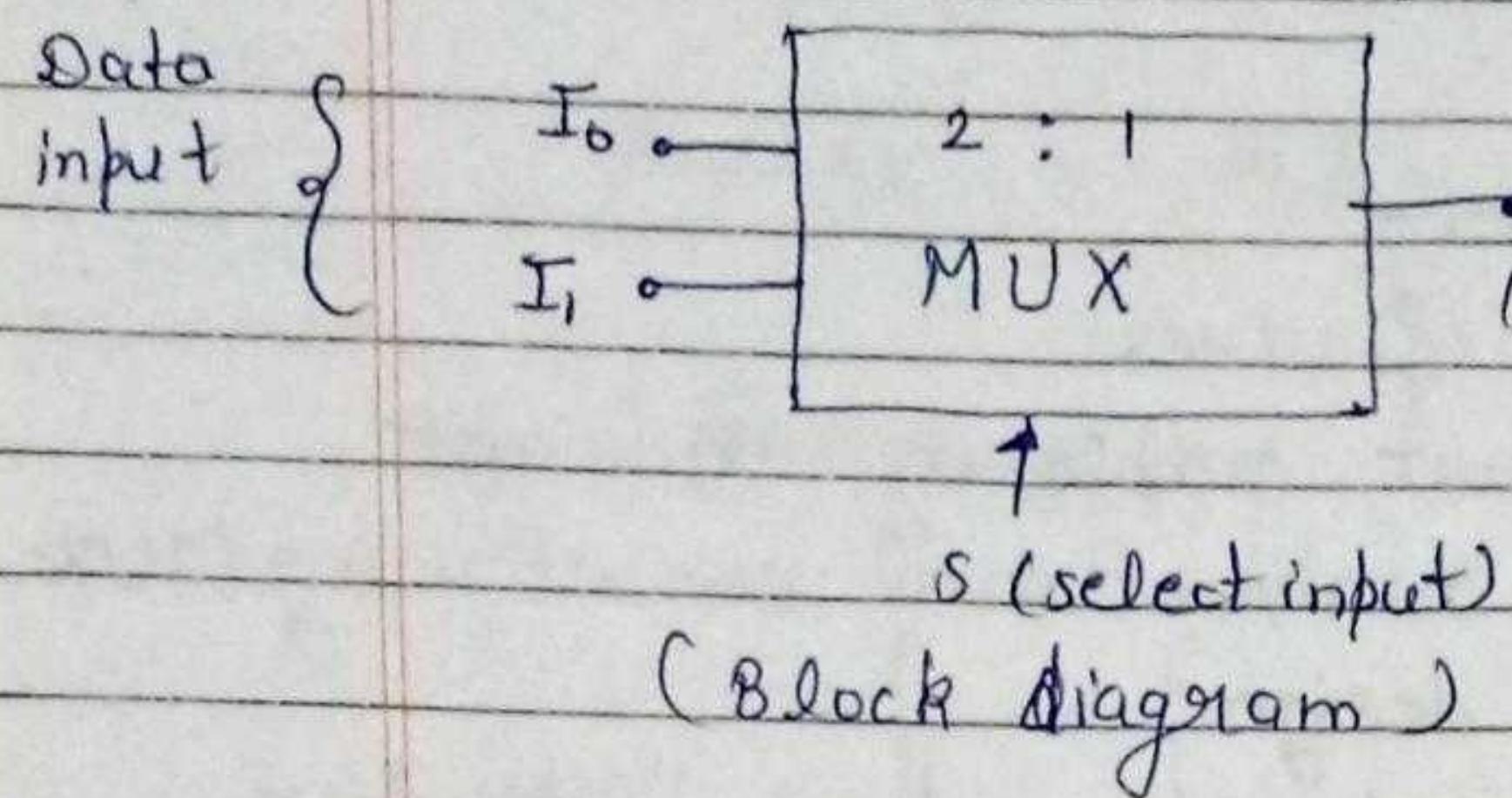
- + In most of the electronic system, the digital data is available on more than one lines. It is necessary to route this data over a single line.
- + Multiplexer ~~and~~ improves the modifiability of the digital circuits because it reduces the number of external wired connections.

### Types of Multiplexer →

1. 2:1 multiplexer
2. 4:1 multiplexer
3. 8:1 multiplexer
4. 16:1 multiplexer
5. 32:1 multiplexer

"2:1 multiplexer" ( $2^1 = 1$  select line)

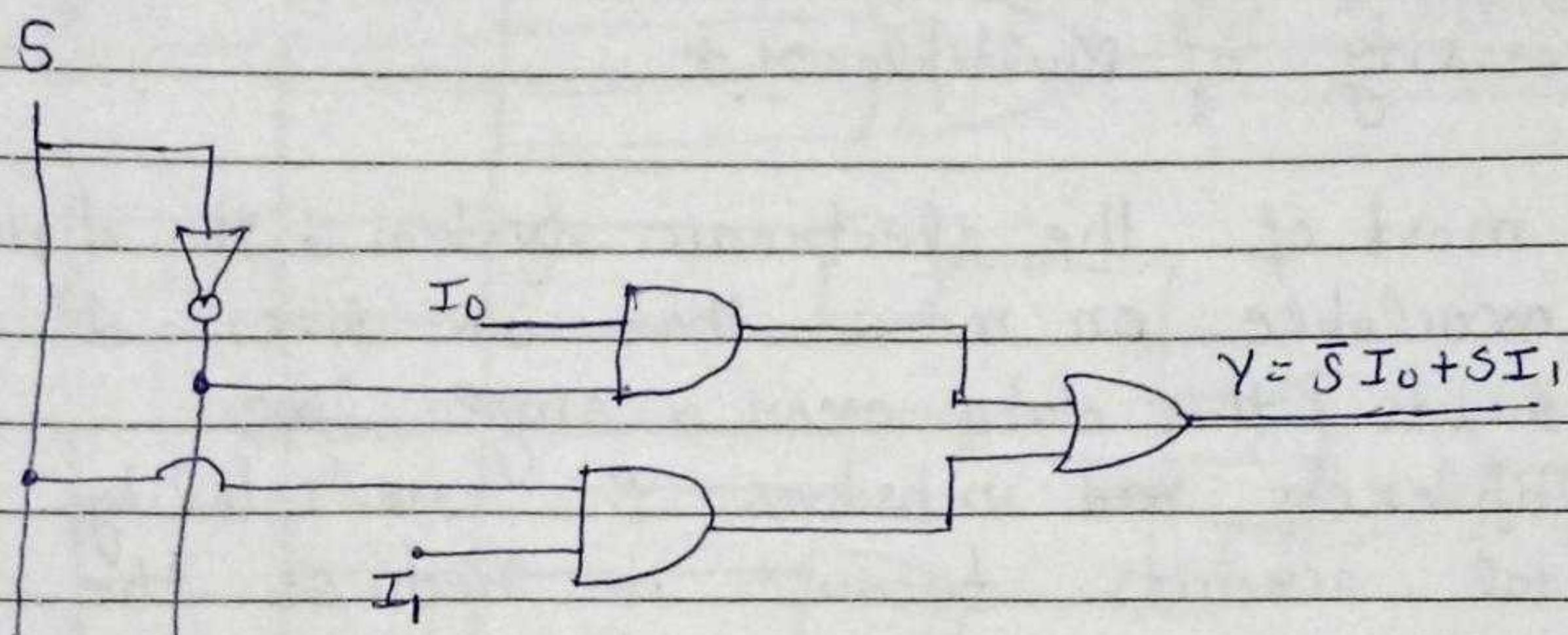
It has two data inputs  $I_0$  and  $I_1$ , one select input  $s$  and one output.



Truth Table

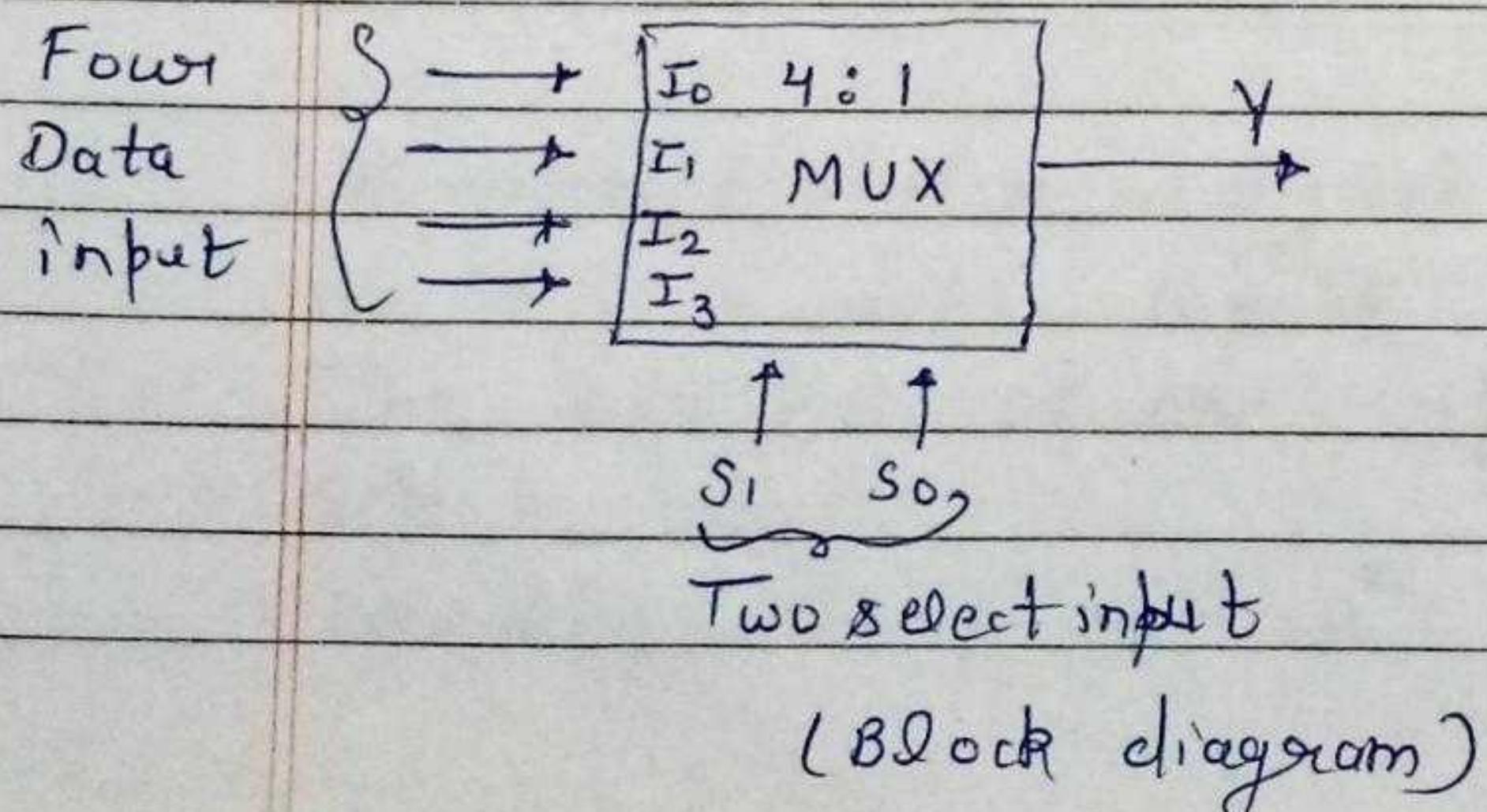
Select Input	Output
0	$I_0$
1	$I_1$

$$Y = \bar{s}I_0 + sI_1$$



Realization of 2:1 MUX using gates.

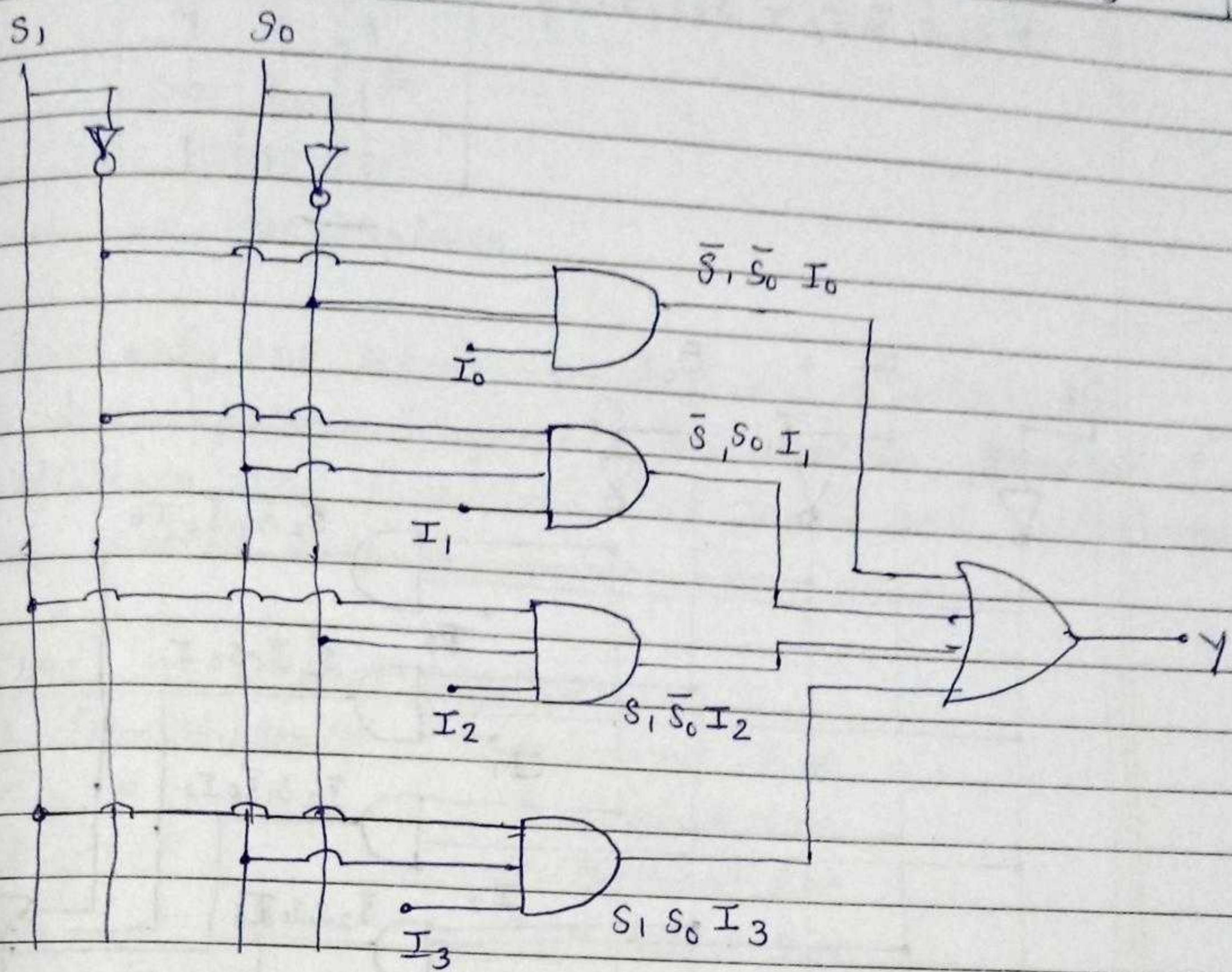
"4:1 multiplexer" ( $2^2 = 2$  select line)



Truth Table +

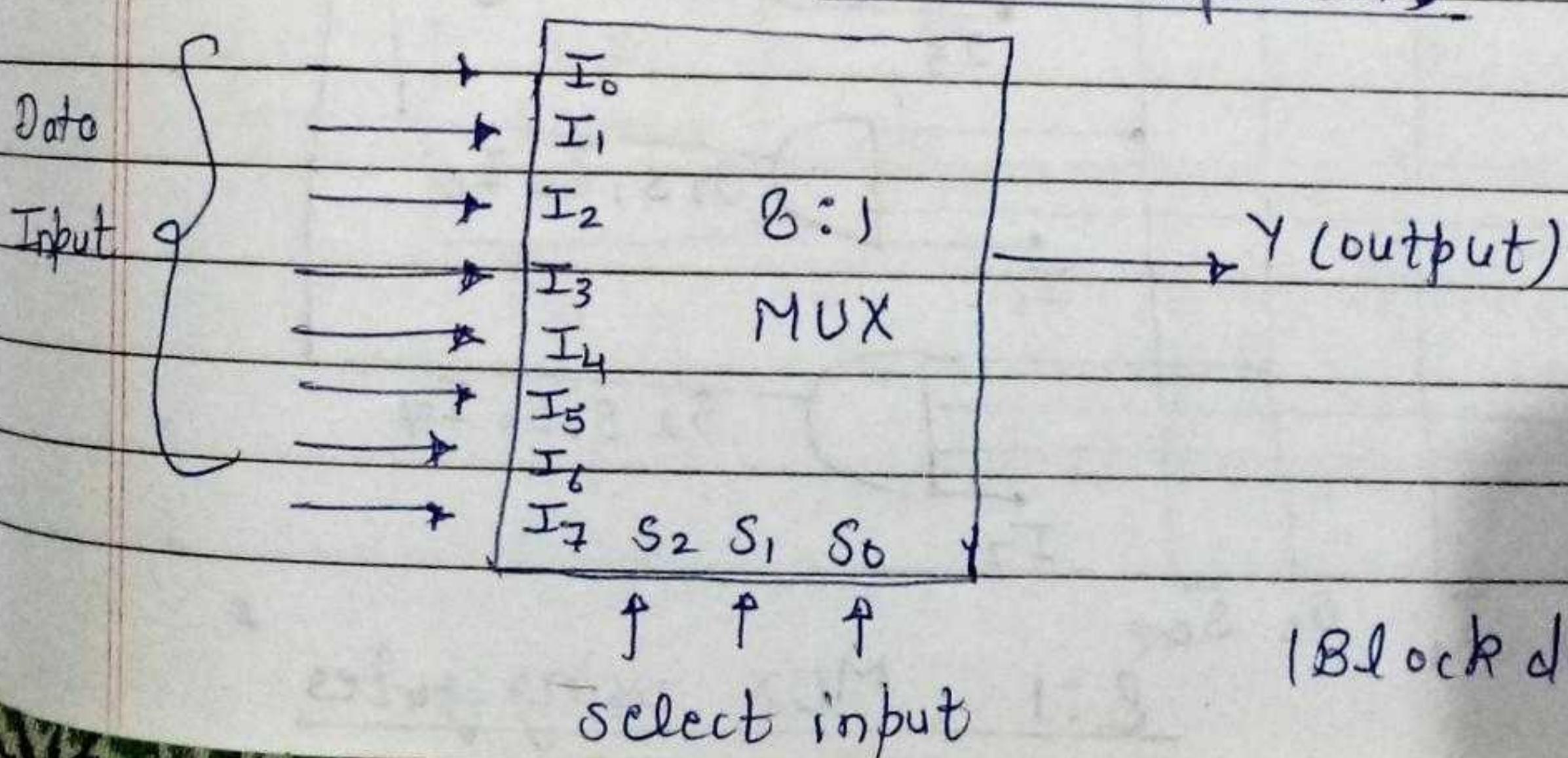
$$y = \bar{s}_1 \bar{s}_0 I_0 + \bar{s}_1 s_0 I_1 \\ + s_1 \bar{s}_0 I_2 + s_1 s_0 I_3$$

Select input	Output select
0, 0	y
0, 1	$I_0$
1, 0	$I_1$
1, 1	$I_2$
	$I_3$



Realization of 4:1 multiplexer using basic gates

"8:1 multiplexer" ( $2^3 = 3$  select lines)

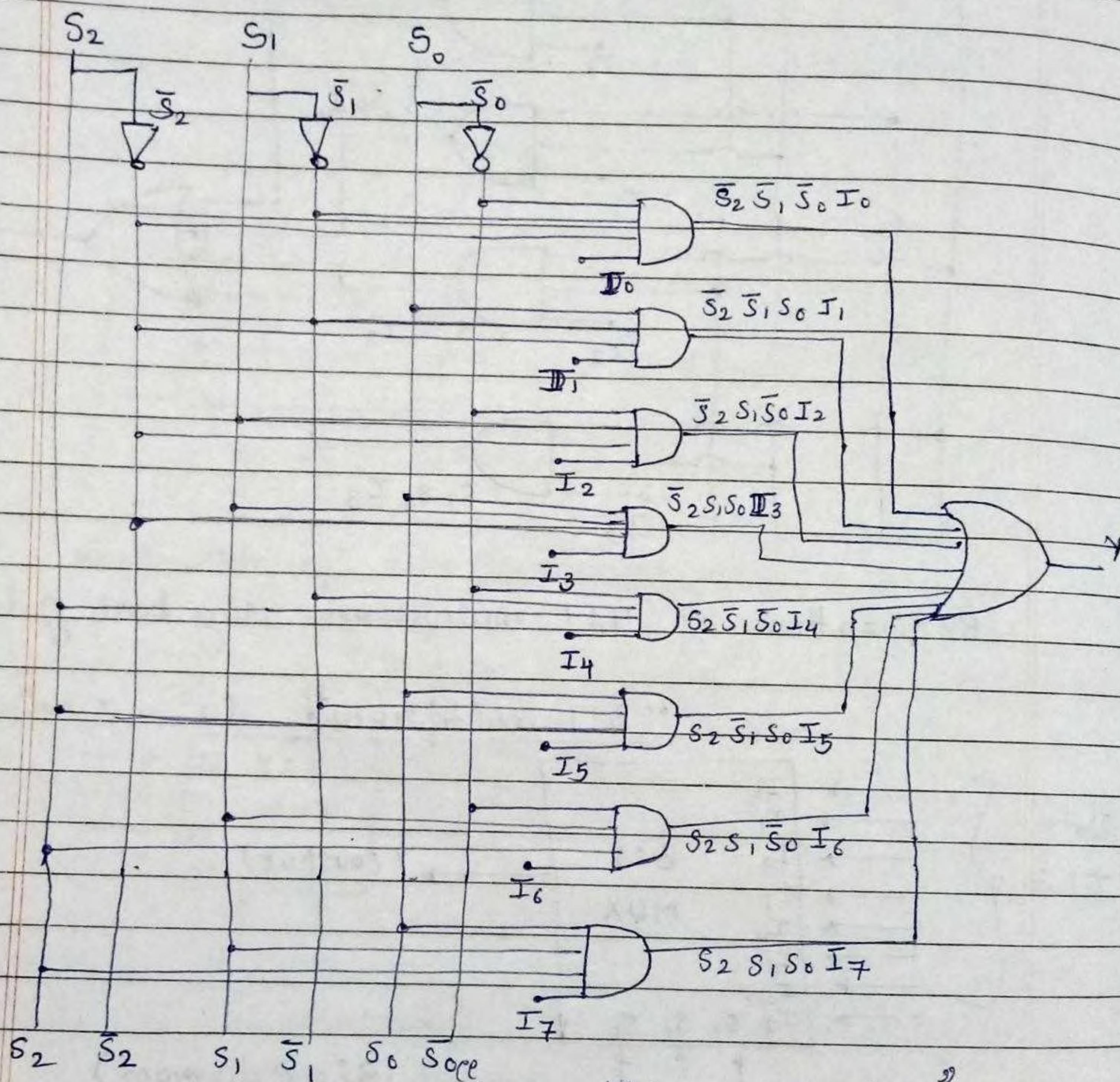


(Block diagram)

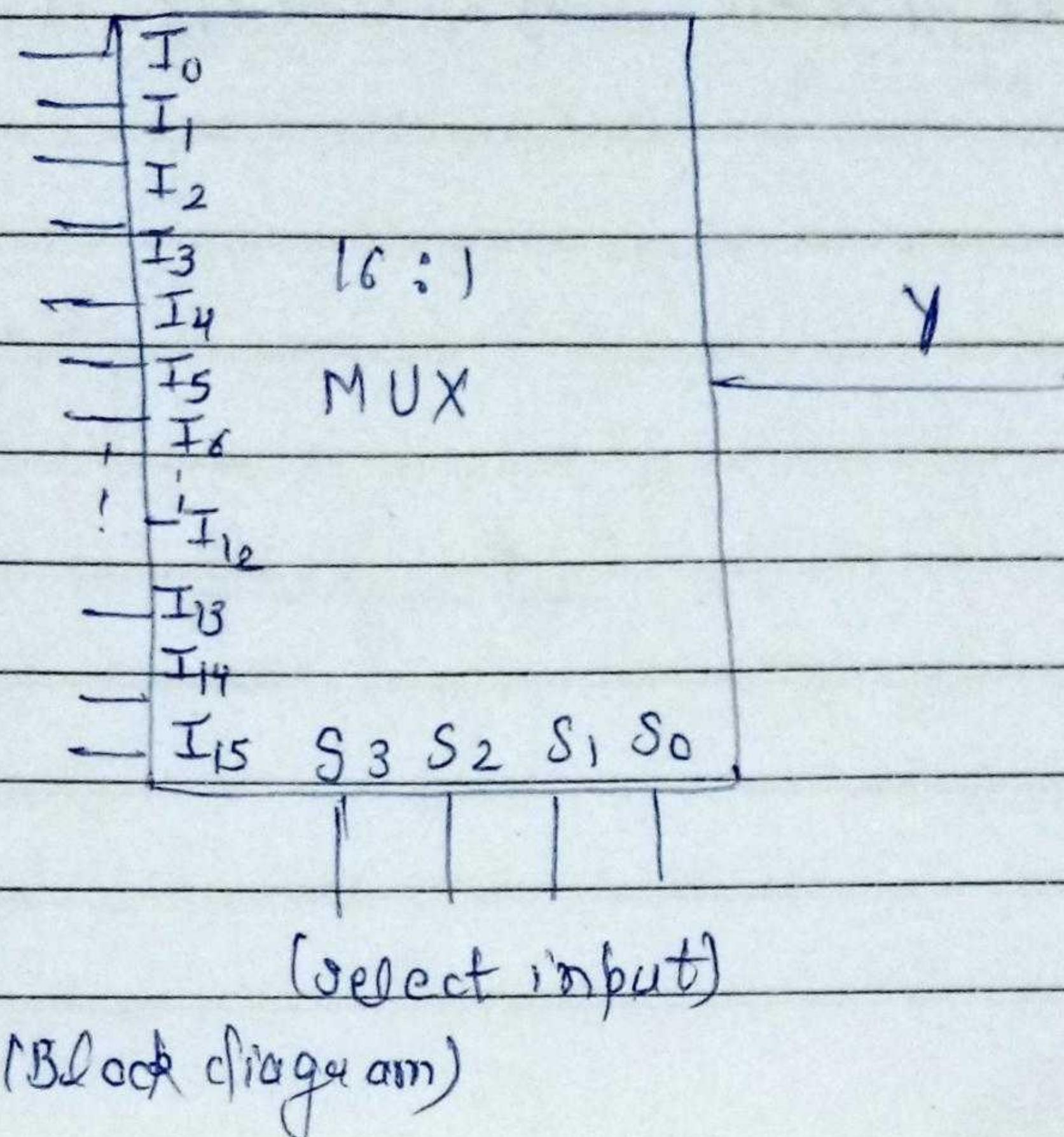
Truth Table →

$$Y = \bar{S}_2 \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_2 \bar{S}_1 S_0 I_1 + \bar{S}_2 S_1 \bar{S}_0 I_2 + \bar{S}_2 S_1 S_0 I_3 + S_2 \bar{S}_1 \bar{S}_0 I_4 + S_2 \bar{S}_1 S_0 I_5 + S_2 S_1 \bar{S}_0 I_6 + S_2 S_1 S_0 I_7$$

Select input	Output select		
$S_2$	$S_1$	$S_0$	$\gamma$
0	0	0	$I_0$
0	0	1	$I_1$
0	1	0	$I_2$
0	1	1	$I_3$
1	0	0	$I_4$
1	0	1	$I_5$
1	1	0	$I_6$
1	1	1	$I_7$

8:1 MUX using gates

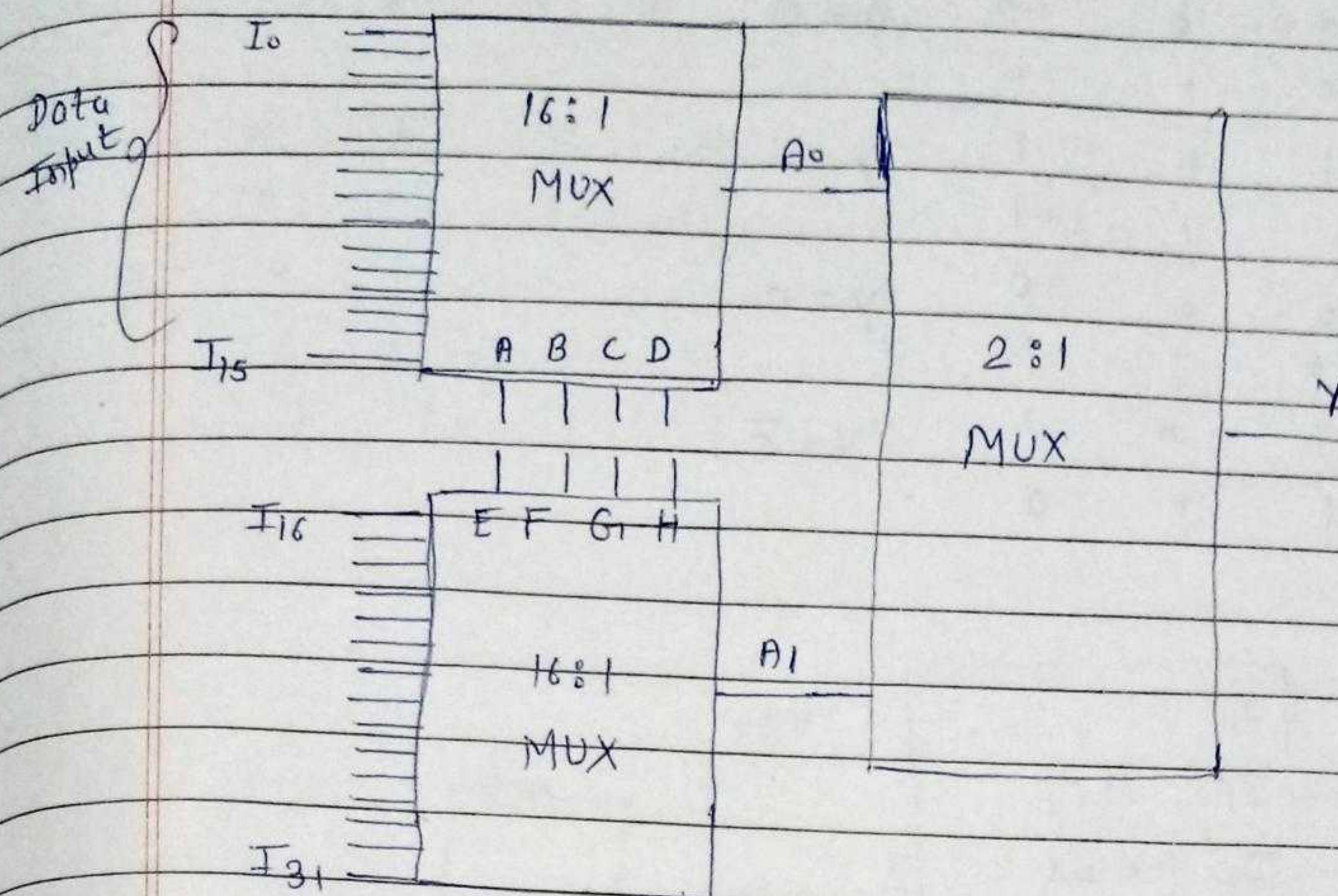
<sup>66</sup> "16:1 Multiplexer"  $2^4 \rightarrow 4$  select lines



Select input				Output select
<u><math>S_3</math></u>	<u><math>S_2</math></u>	<u><math>S_1</math></u>	<u><math>S_0</math></u>	<u><math>\gamma</math></u>
0	0	0	0	$I_0$
0	0	0	1	$I_1$
0	0	1	0	$I_2$
0	0	1	1	$I_3$
0	1	0	0	$I_4$
0	1	0	1	$I_5$
0	1	1	0	$I_6$
0	1	1	1	$I_7$
1	0	0	0	$I_8$
1	0	0	1	$I_9$
1	0	1	0	$I_{10}$
1	0	1	1	$I_{11}$
1	1	0	0	$I_{12}$
1	1	0	1	$I_{13}$
1	1	1	0	$I_{14}$
1	1	1	1	$I_{15}$

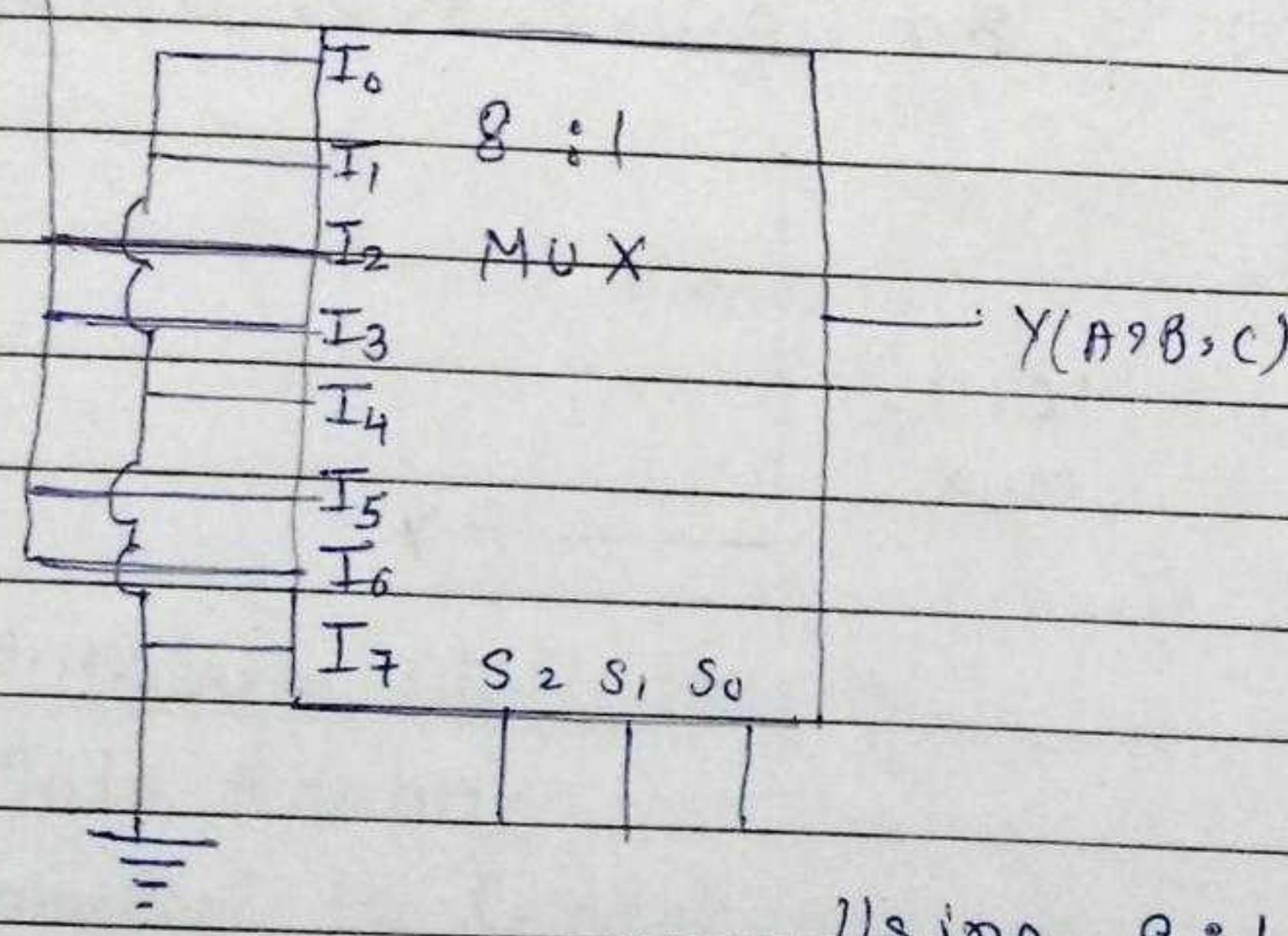
$$\begin{aligned}
 \gamma = & \bar{S}_3 \bar{S}_2 \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_3 \bar{S}_2 \bar{S}_1 S_0 I_1 + \bar{S}_3 \bar{S}_2 S_1 \bar{S}_0 I_2 + \bar{S}_3 \bar{S}_2 S_1 S_0 I_3 + \\
 & \bar{S}_3 S_2 \bar{S}_1 \bar{S}_0 I_4 + \bar{S}_3 S_2 \bar{S}_1 S_0 I_5 + \bar{S}_3 S_2 S_1 \bar{S}_0 I_6 + \bar{S}_3 S_2 S_1 S_0 I_7 + \\
 & S_3 \bar{S}_2 \bar{S}_1 \bar{S}_0 I_8 + S_3 \bar{S}_2 \bar{S}_1 S_0 I_9 + S_3 \bar{S}_2 S_1 \bar{S}_0 I_{10} + S_3 \bar{S}_2 S_1 S_0 I_{11} + \\
 & S_3 S_2 \bar{S}_1 \bar{S}_0 I_{12} + S_3 S_2 \bar{S}_1 S_0 I_{13} + S_3 S_2 S_1 \bar{S}_0 I_{14} + S_3 S_2 S_1 S_0 I_{15}
 \end{aligned}$$

# Design a 32:1 Multiplexer using 16:1 Multiplexer and 2:1 multiplexer



# Implement the following using 8:1, 4:1 multiplexer.

$$Y(A, B, C) = \sum m(2, 3, 5, 6)$$



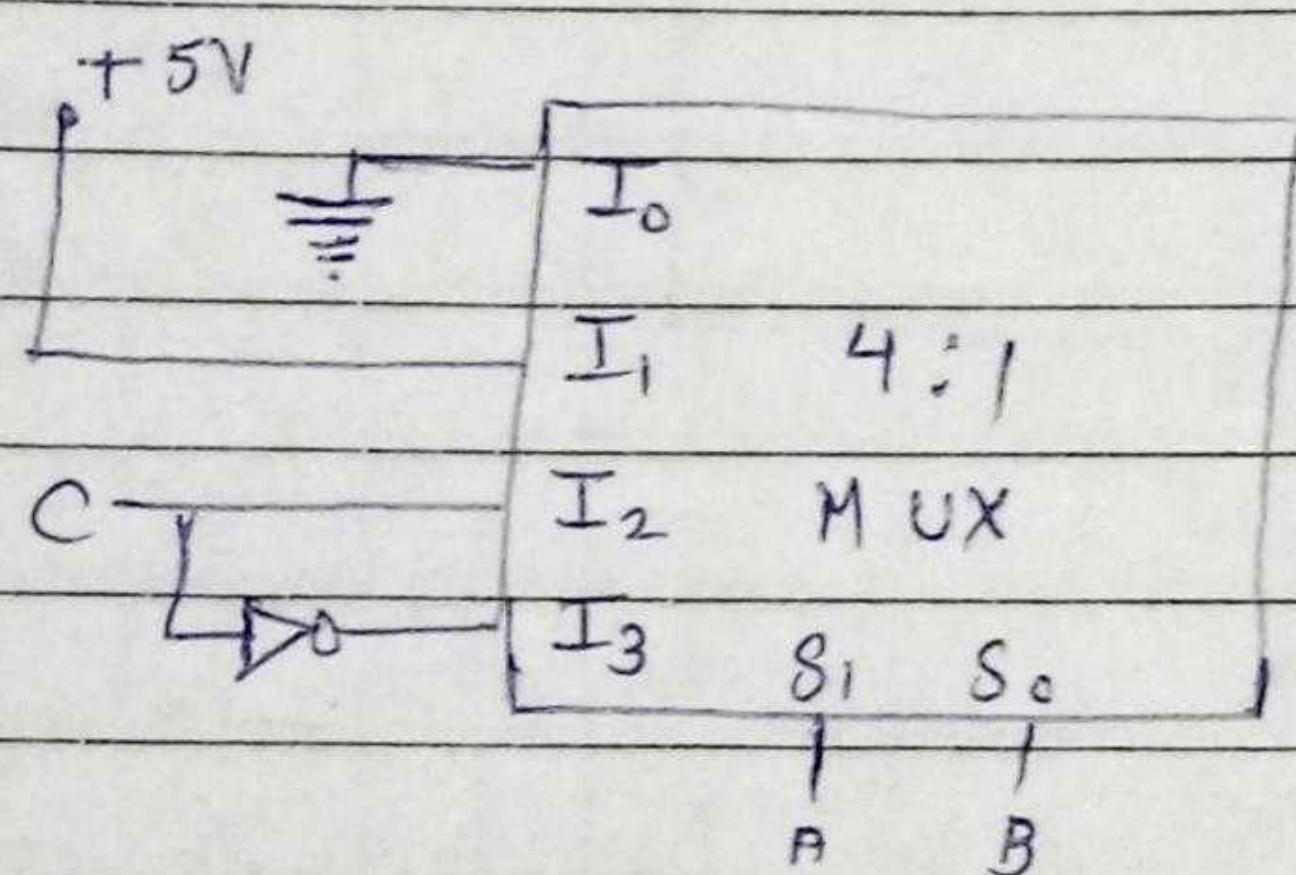
$$I = I_2, I_3, \bar{I}_5, I_6 = +5V$$

$$O = I_0, I_1, I_4, I_7 = \underline{\underline{I}}$$

Using 8:1

Using 4:1

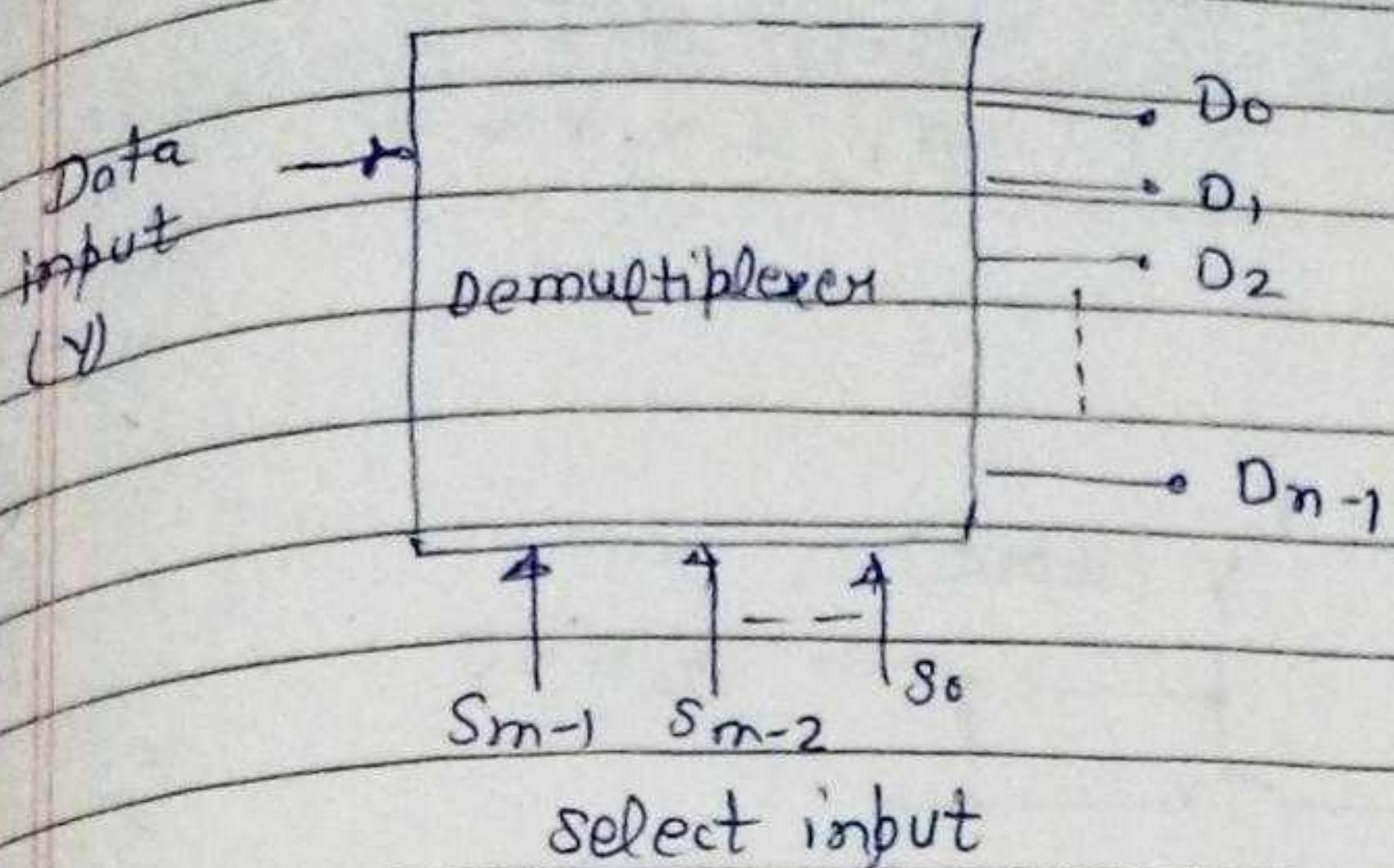
	A	B	C	y	
0	0	0	0	0	$y = 0$
1	0	0	1	0	
2	0	1	0	1	$y = 1$
3	0	1	1	1	
4	1	0	0	0	$y = c$
5	1	0	1	1	
6	1	1	0	1	$y = \bar{c}$
7	1	1	1	0	



## Multiplexer Application +

1. Data Routing
2. Parallel to Serial converter  
(Many input to one output)
3. Logic Junction Generation.

## "Demultiplexers"



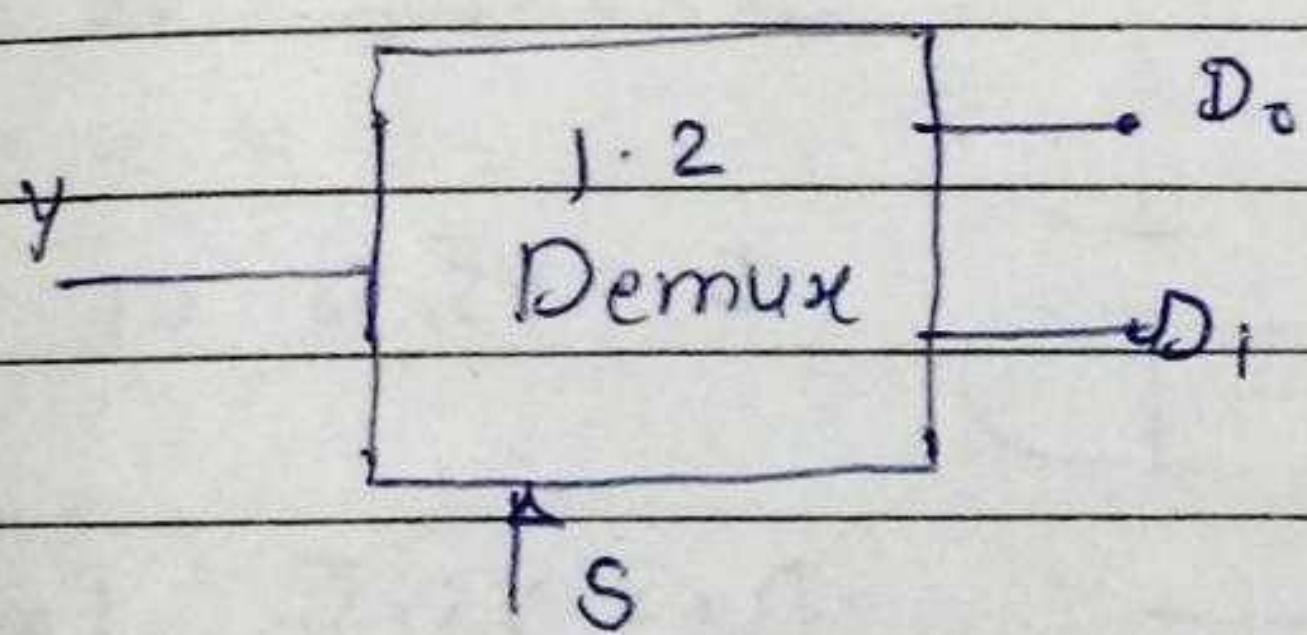
(a) 1 : n demultiplexer

- It has only one input, n output and m select input.
- + A demultiplexer performs the reverse operation of a multiplexer i.e. it receives one input and distributes it over several outputs.

### Types of Demultiplexers-

- (i) 1:2 Demultiplexer → 1 select line
- (ii) 1:4 " → 2 select line
- (iii) 1:8 " → 3 select line
- (iv) 1:16 " → 4 select line
- (v) 1:32 " → 5 select line

## "1 : 2 Multiplexer"



Truth Table

	Selected input	Output
S	D <sub>1</sub> , D <sub>0</sub>	D <sub>1</sub> D <sub>0</sub>
0	0	0 1
1	1	1 0

$$D_0 = Y\bar{S} \quad D_1 = Y \cdot S$$

if  $S = 0$

$$D_0 = Y \cdot \bar{0} = Y$$

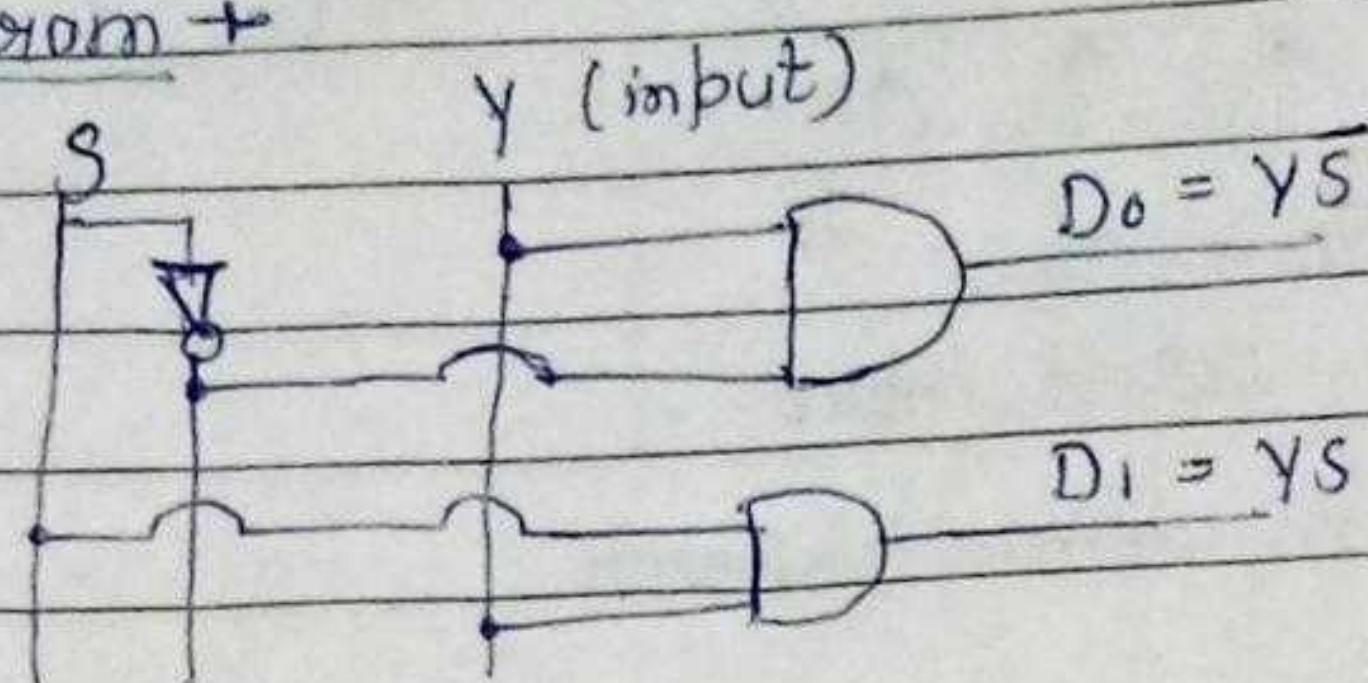
$$D_1 = Y \cdot 0 = 0$$

if  $S = 1$

$$D_0 = Y \cdot \bar{1} = 0$$

$$D_1 = Y \cdot 1 = Y$$

combinational Diagram +



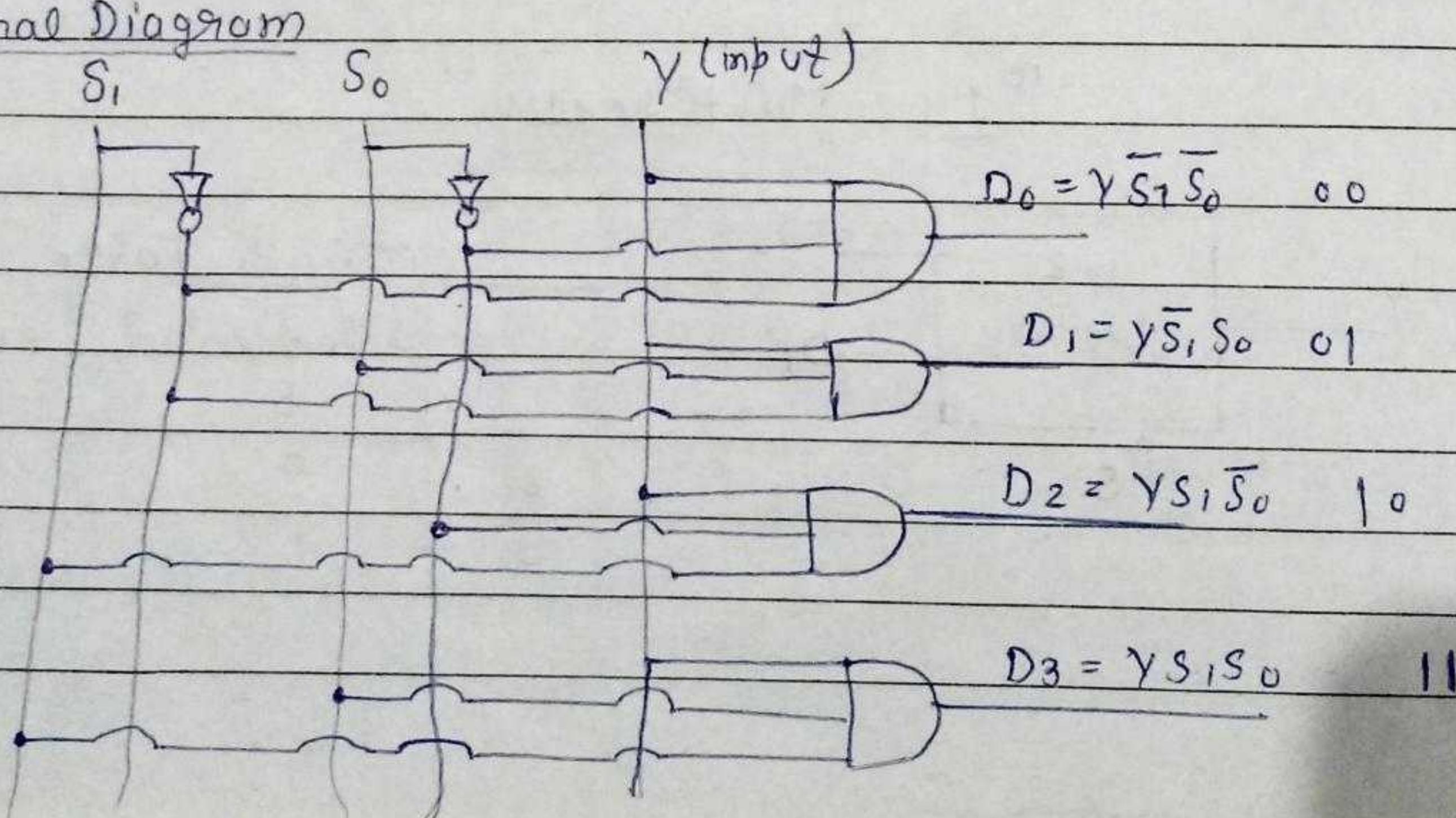
1 : 4 Demultiplexer

select inputs		output					
$S_1$	$S_0$	$D_3$	$D_2$	$D_1$	$D_0$		
0	0	0	0	0	0	1	
1	0	1	0	0	1	0	$Y \rightarrow$ 1:4 DEMUX
2	1	0	0	1	0	0	
3	1	1	1	0	0	0	

$S_0 \quad S_1$

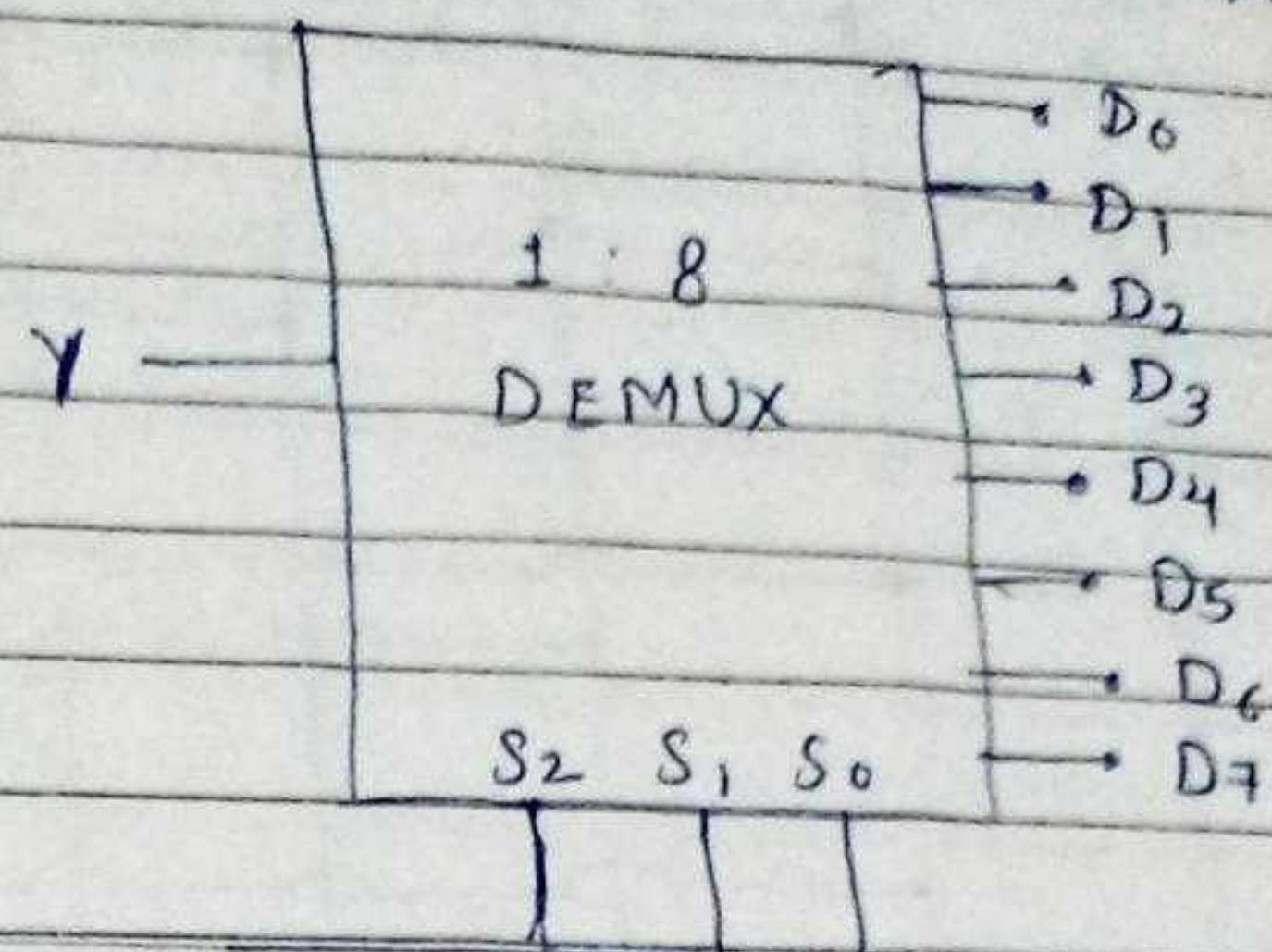
$$D_0 = Y \bar{S}_1 \bar{S}_0, \quad D_1 = Y \bar{S}_1 S_0, \quad D_2 = Y S_1 \bar{S}_0, \quad D_3 = Y S_1 S_0$$

combinational Diagram



1:8 Demultiplexer

It has one data input , eight outputs , three select inputs.



select input			output							
S <sub>2</sub>	S <sub>1</sub>	S <sub>0</sub>	D <sub>7</sub>	D <sub>6</sub>	D <sub>5</sub>	D <sub>4</sub>	D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>
0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	1	0	0	0	0	0	0	1	0	0
0	1	1	0	0	0	0	1	0	0	0
1	0	0	0	0	0	1	0	0	0	0
1	0	1	0	0	1	0	0	0	0	0
1	1	0	0	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0

$$D_0 = Y \bar{S}_2 \bar{S}_1 \bar{S}_0$$

$$D_4 = Y S_2 \bar{S}_1 \bar{S}_0$$

$$D_1 = Y \bar{S}_2 \bar{S}_1 S_0$$

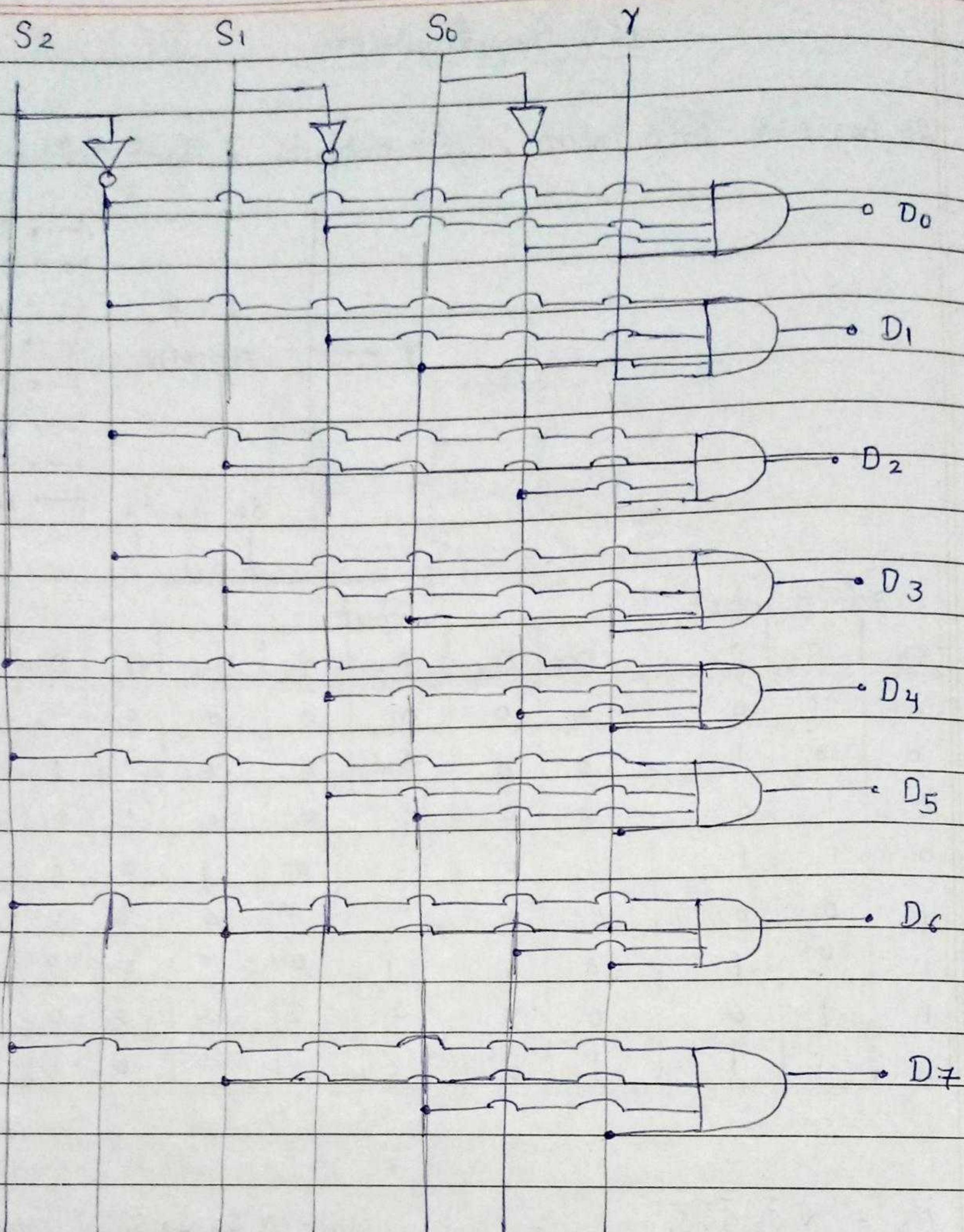
$$D_5 = Y S_2 \bar{S}_1 S_0$$

$$D_2 = Y \bar{S}_2 S_1 \bar{S}_0$$

$$D_6 = Y S_2 S_1 \bar{S}_0$$

$$D_3 = Y \bar{S}_2 S_1 S_0$$

$$D_7 = Y S_2 S_1 S_0$$

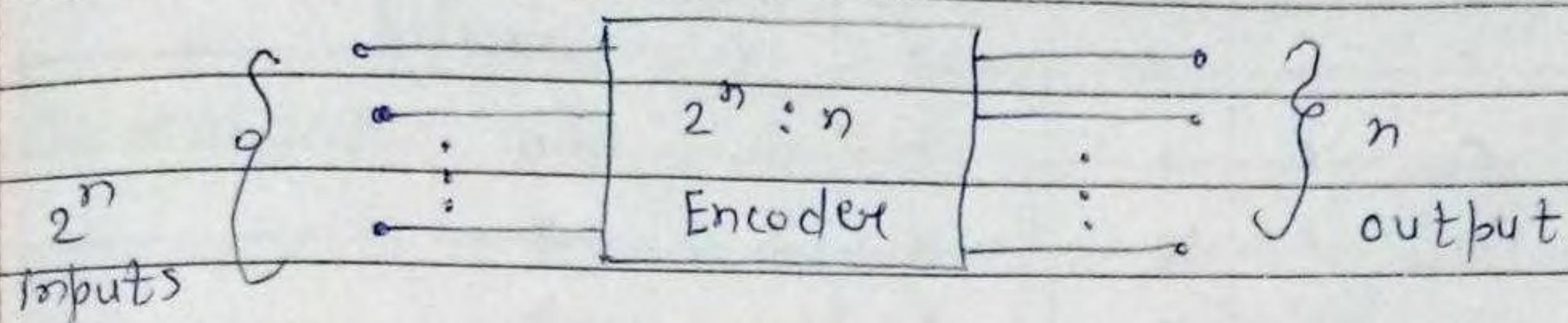


Combinational Diagram.

## "Encoders"

An Encoder is a combinational circuit that performs the inverse operation of Decoder. It has maximum  $2^n$  input lines and  $n$  output lines. It will produce a binary code equivalent to the input, which is active high. Therefore, the encoder encodes  $2^n$  input lines with  $n$  bits.

$2^n : n$   
Input      Output



Block diagram of Encoder.

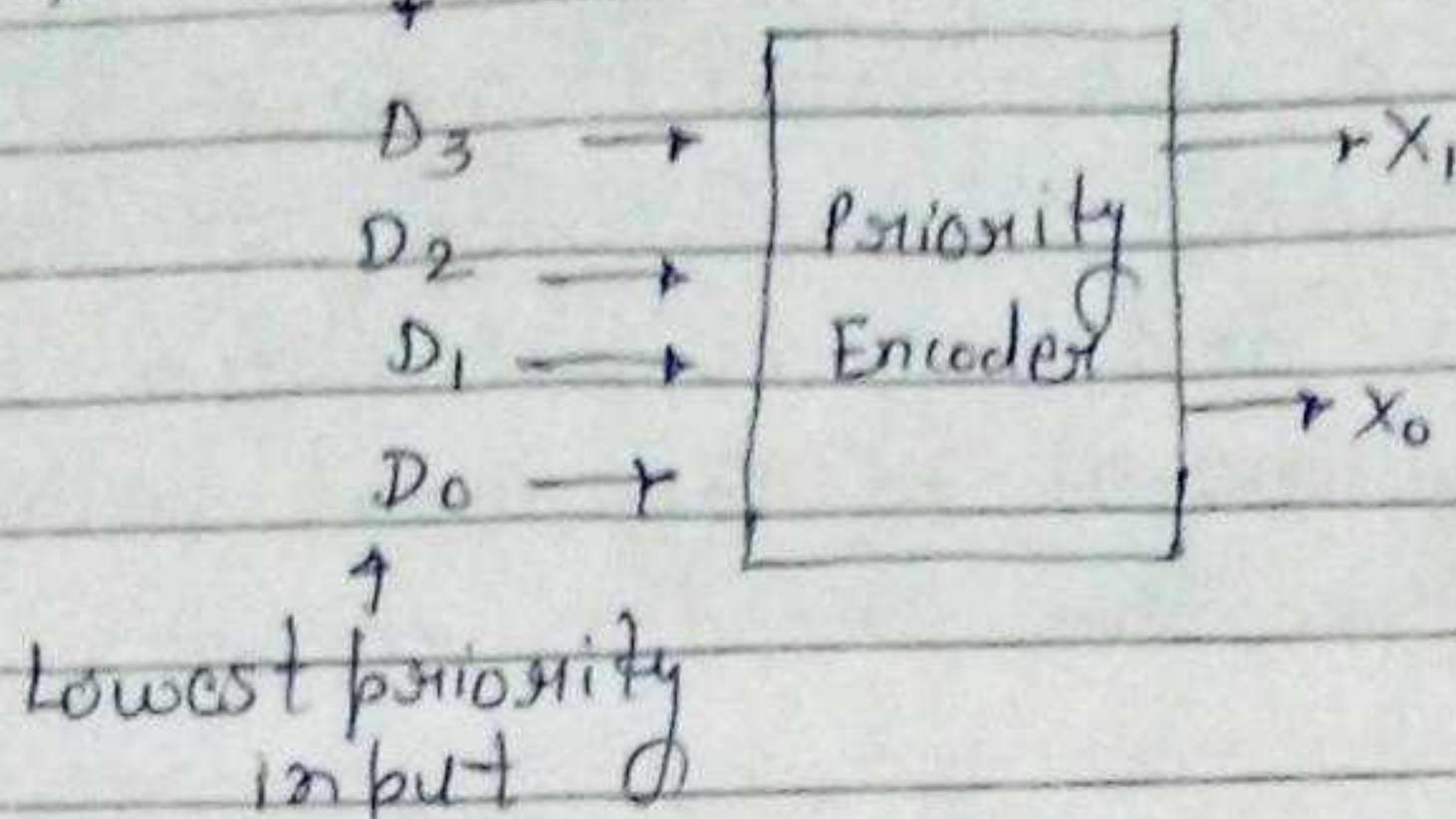
### Types of Encoders +

- (1) Priority Encoders
- (2) Decimal to BCD encoders
- (3) Octal to binary encoders
- (4) Hexadecimal to binary encoder.

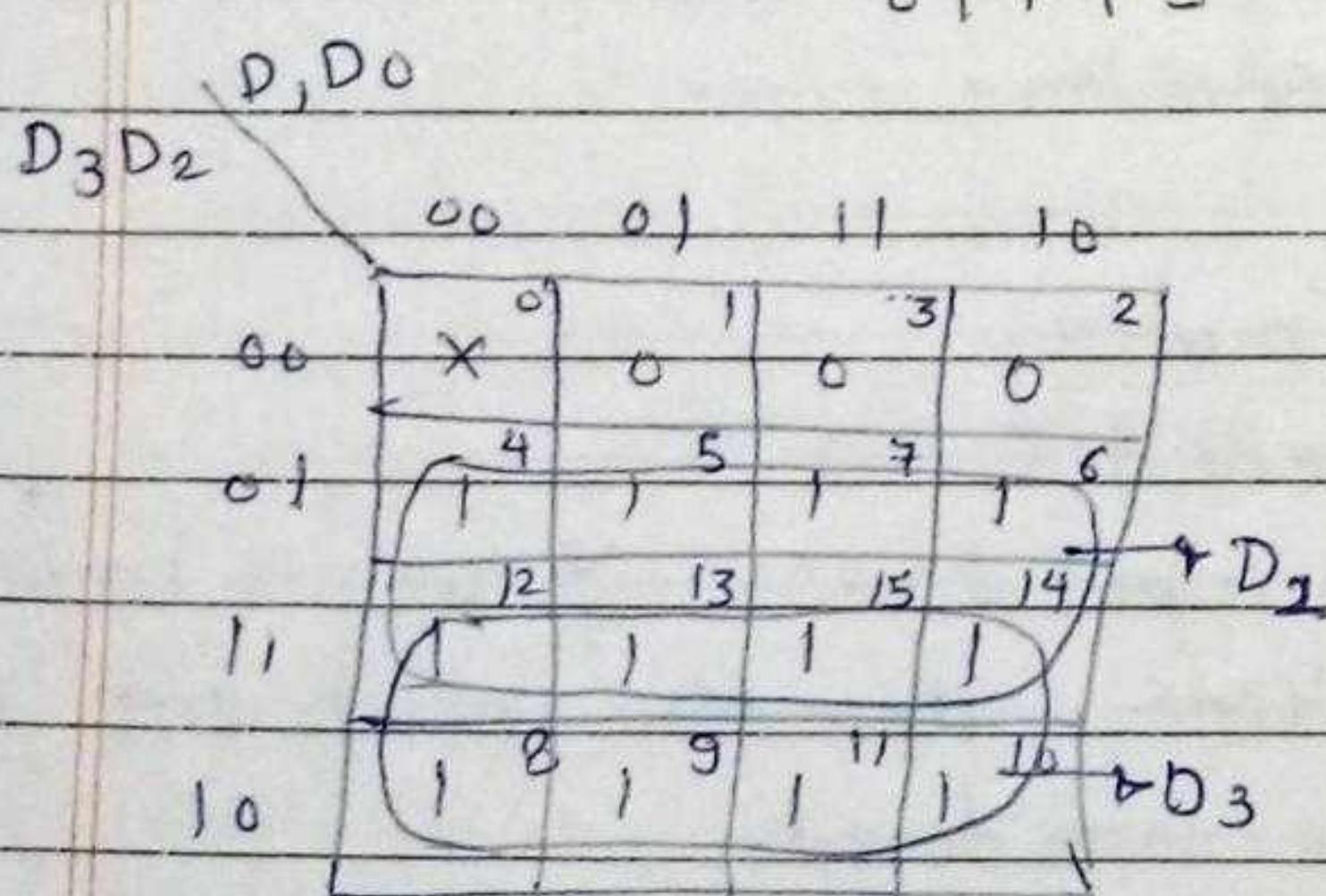
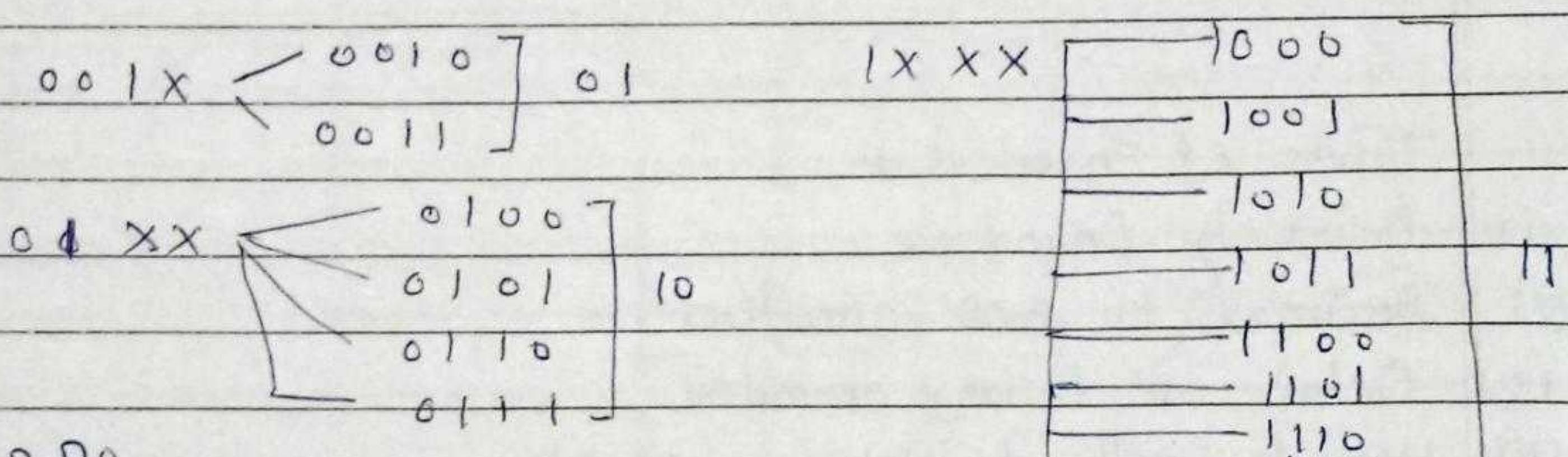
### Priority Encoder

- This is a special type of encoder
- Priorities are given to input lines. If two or more input lines are 1 at the same time then the input line with highest priority will be considered.
- There are four inputs  $D_0$  through  $D_3$ , and two output  $X_1$  and  $X_0$ , out of these  $D_3$  has highest priority and  $D_0$  is lowest priority.

Highest priority  
input ↓



Input				Output		
D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>	X <sub>1</sub>	X <sub>0</sub>	
0	0	0	0	X	X	→ inverted
0	0	0	1	0	0	-10
0	0	1	X	0	1	-1
0	1	X	X	1	0	-2
1	X	X	X	1	1	-3

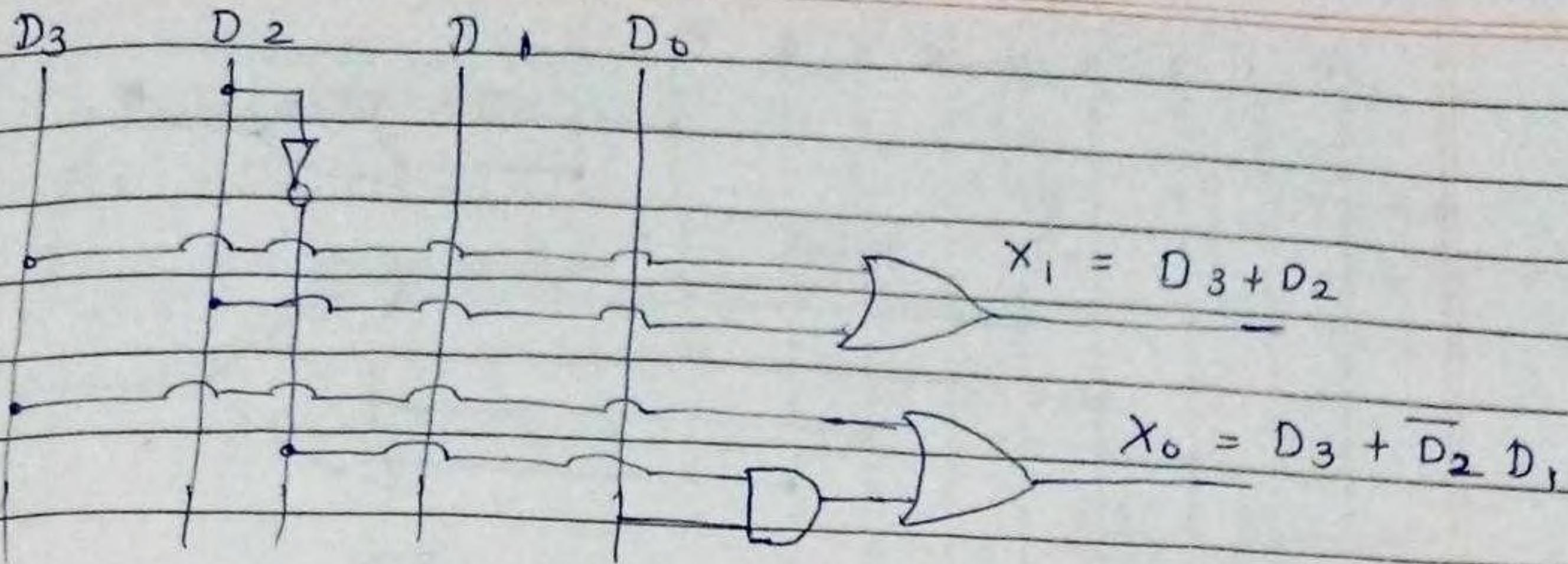


$$X_1 = D_3 + D_2$$

K-map for X<sub>1</sub>

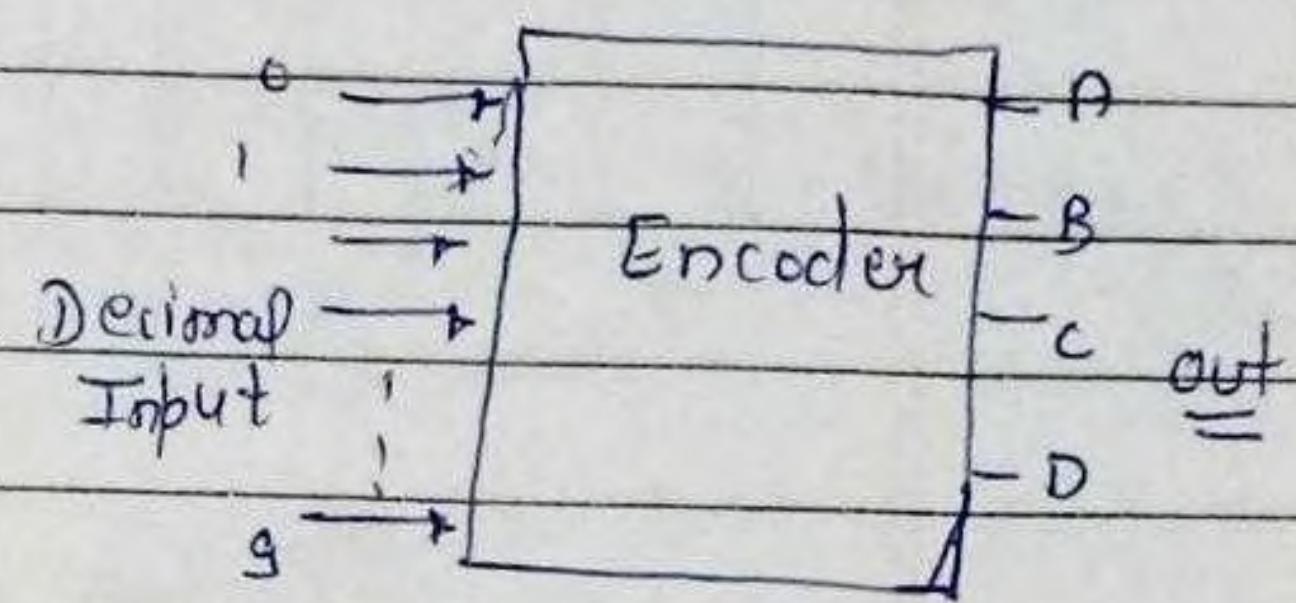
$$X_0 = D_3 + \overline{D}_2 D_1$$

K-map for X<sub>0</sub>



### "Decimal to BCD Encoder"

Decimal Input	BCD Output			
	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1



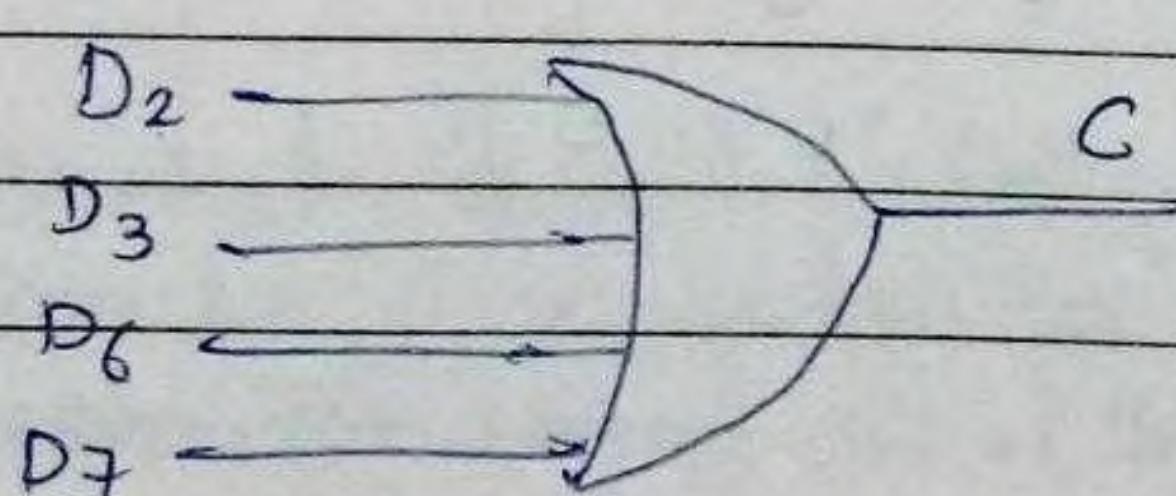
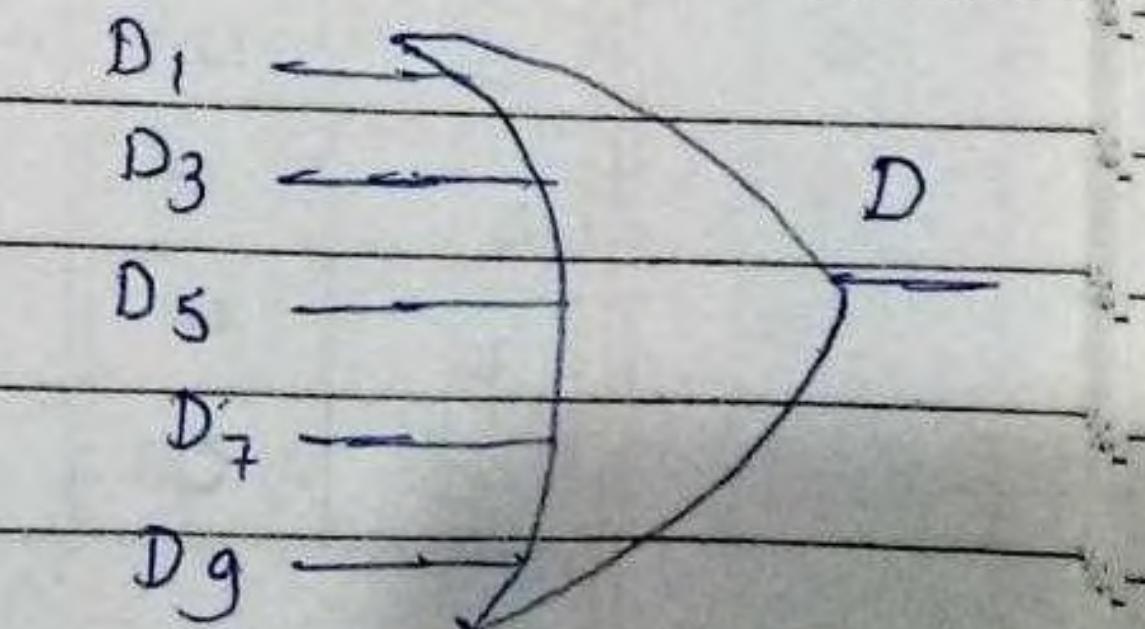
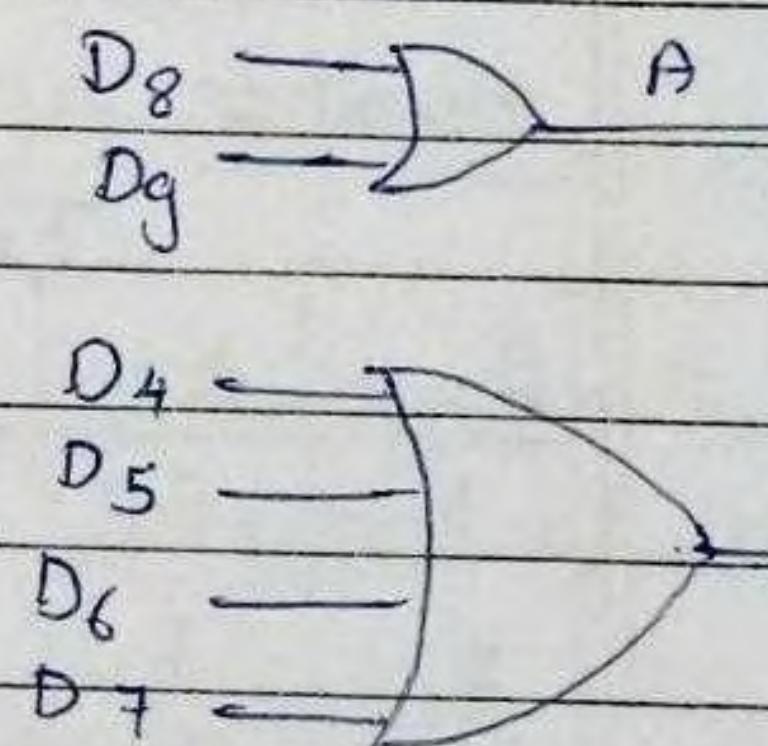
Block diagram

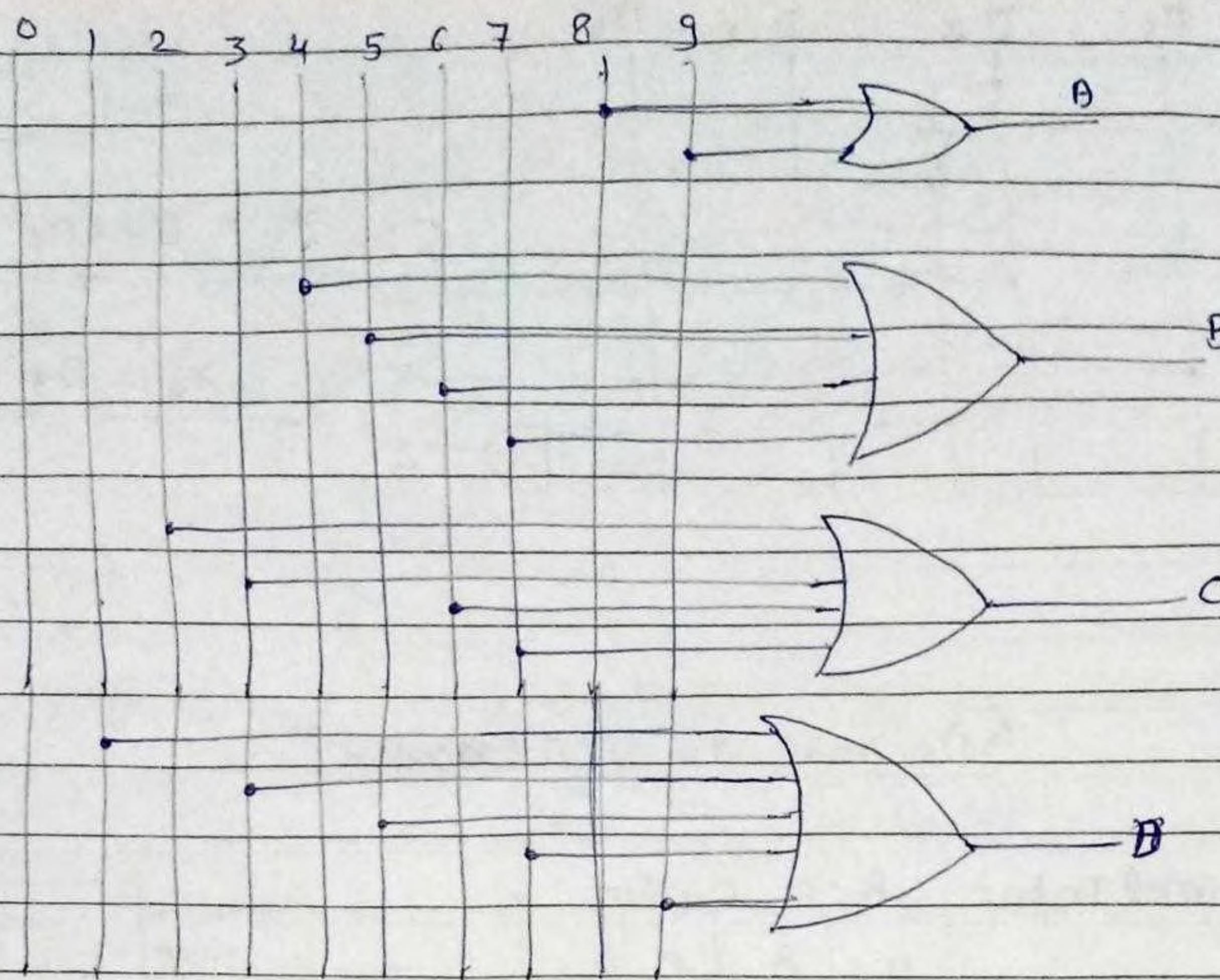
$$A = D_8 + D_9$$

$$B = D_4 + D_5 + D_6 + D_7$$

$$C = D_2 + D_3 + D_6 + D_7$$

$$D = D_1 + D_3 + D_5 + D_7 + D_9$$





"Octal to Binary Encoder"

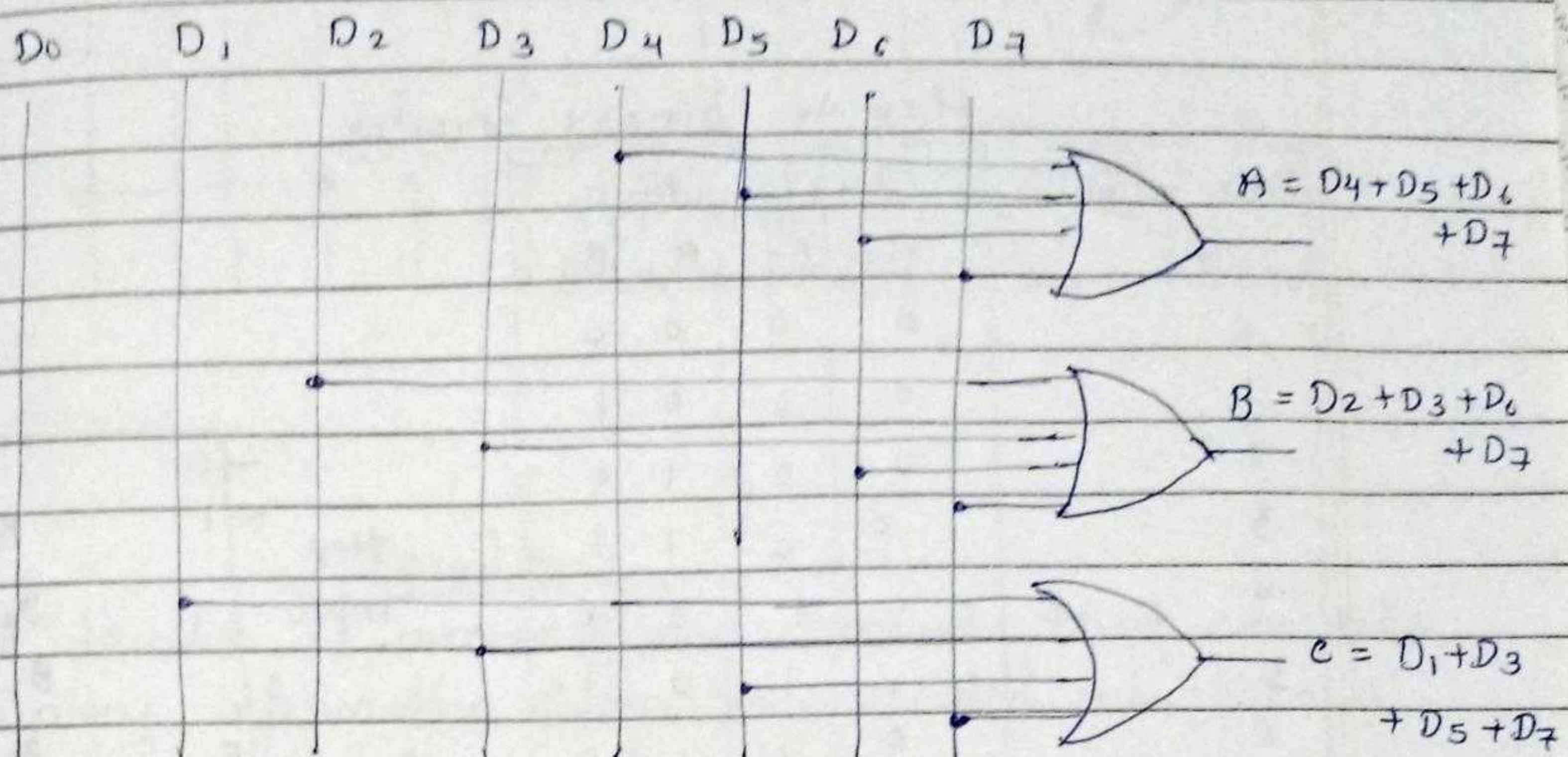
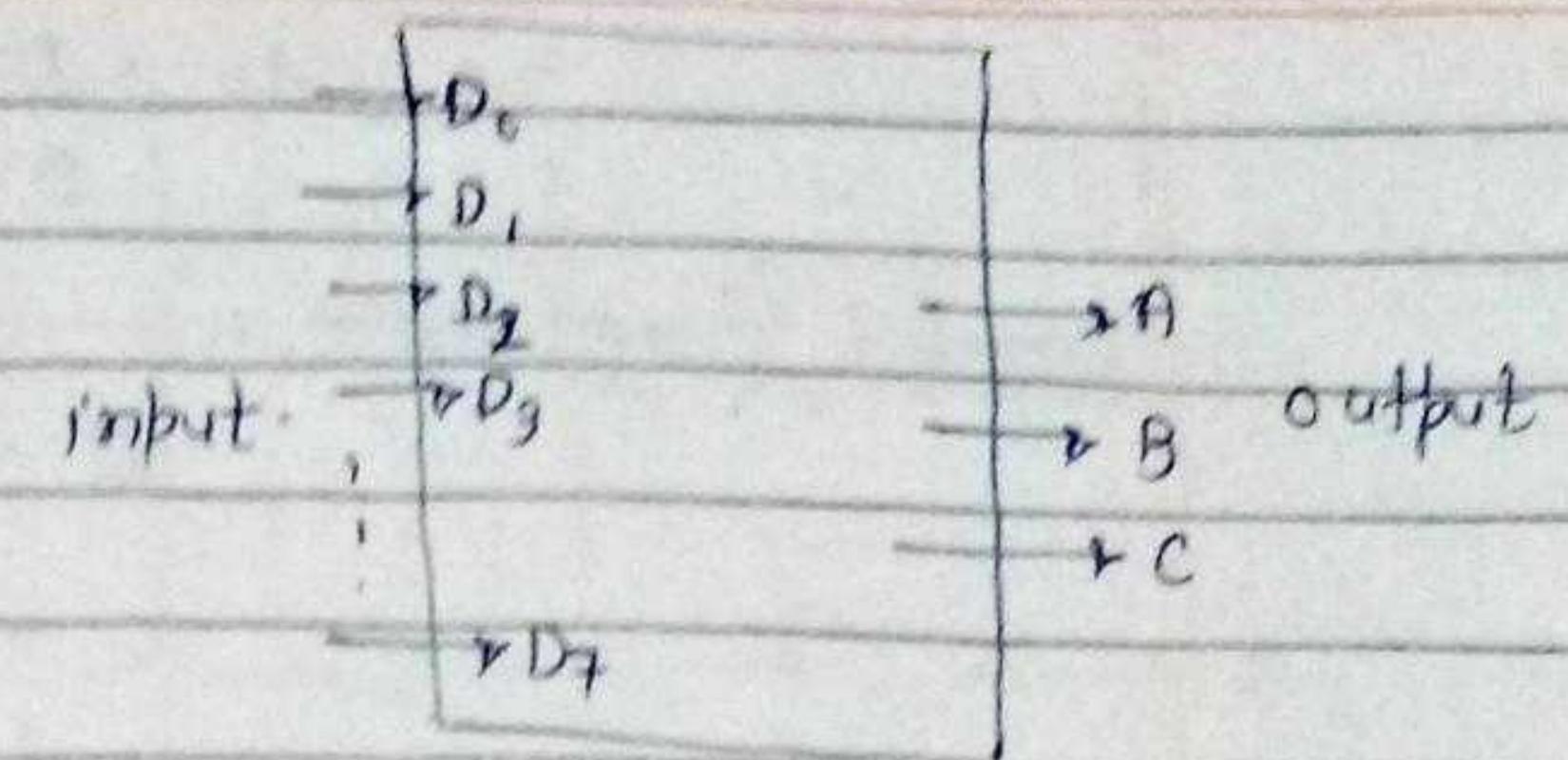
The octal to binary encoder has 8-input lines and 3-output lines. Corresponding to the eight input octal number we get three bit binary output

Input								Output		
D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>	A	B	C
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

$$A = D_4 + D_5 + D_6 + D_7$$

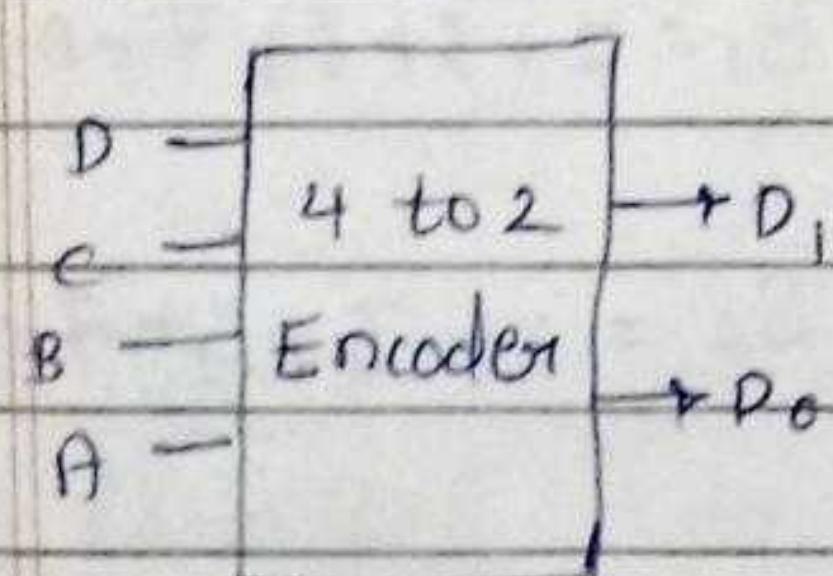
$$B = D_2 + D_3 + D_6 + D_7$$

$$C = D_1 + D_3 + D_5 + D_7$$



### 4 to 2 Encoder

Let 4 to 2 Encoder has four inputs, D, C, B, A and two output D<sub>1</sub> and D<sub>0</sub>. The block diagram of 4 to 2 Encoder -



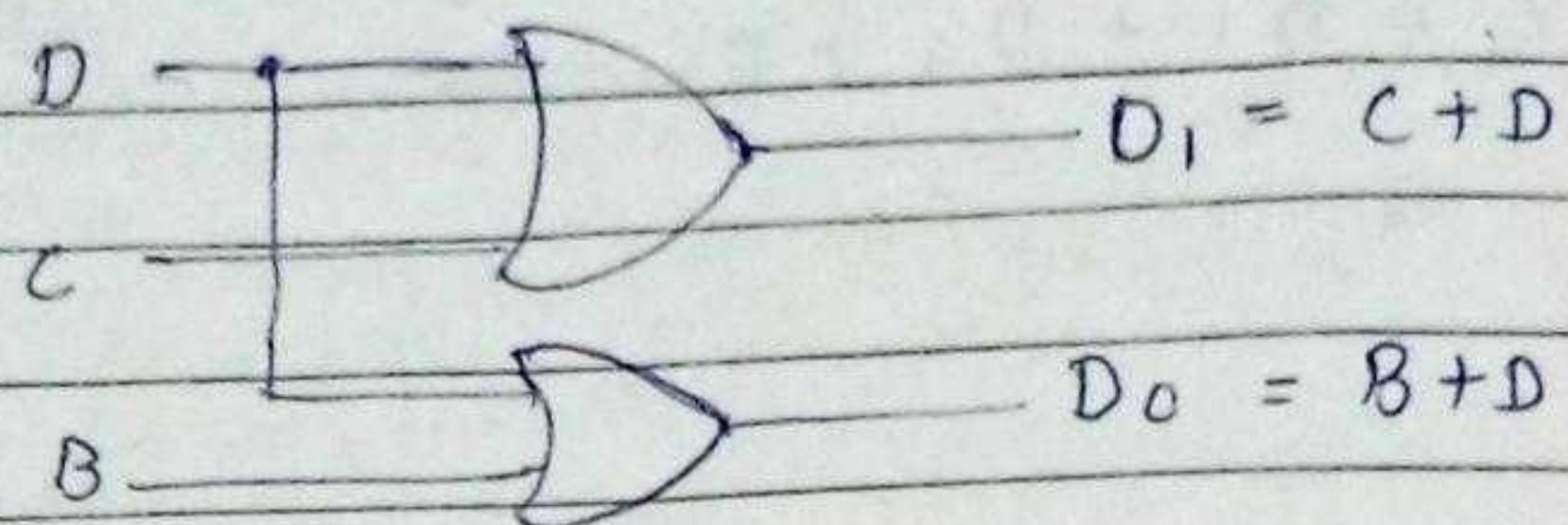
Input				Output	
D	C	B	A	D <sub>1</sub>	D <sub>0</sub>
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

at any time, only one of these 4 inputs can be '1' in order to get the respective binary code at the output.

$$D_0 \quad D_1 = C + D$$

$$D_0 = B + D$$

Circuit diagram

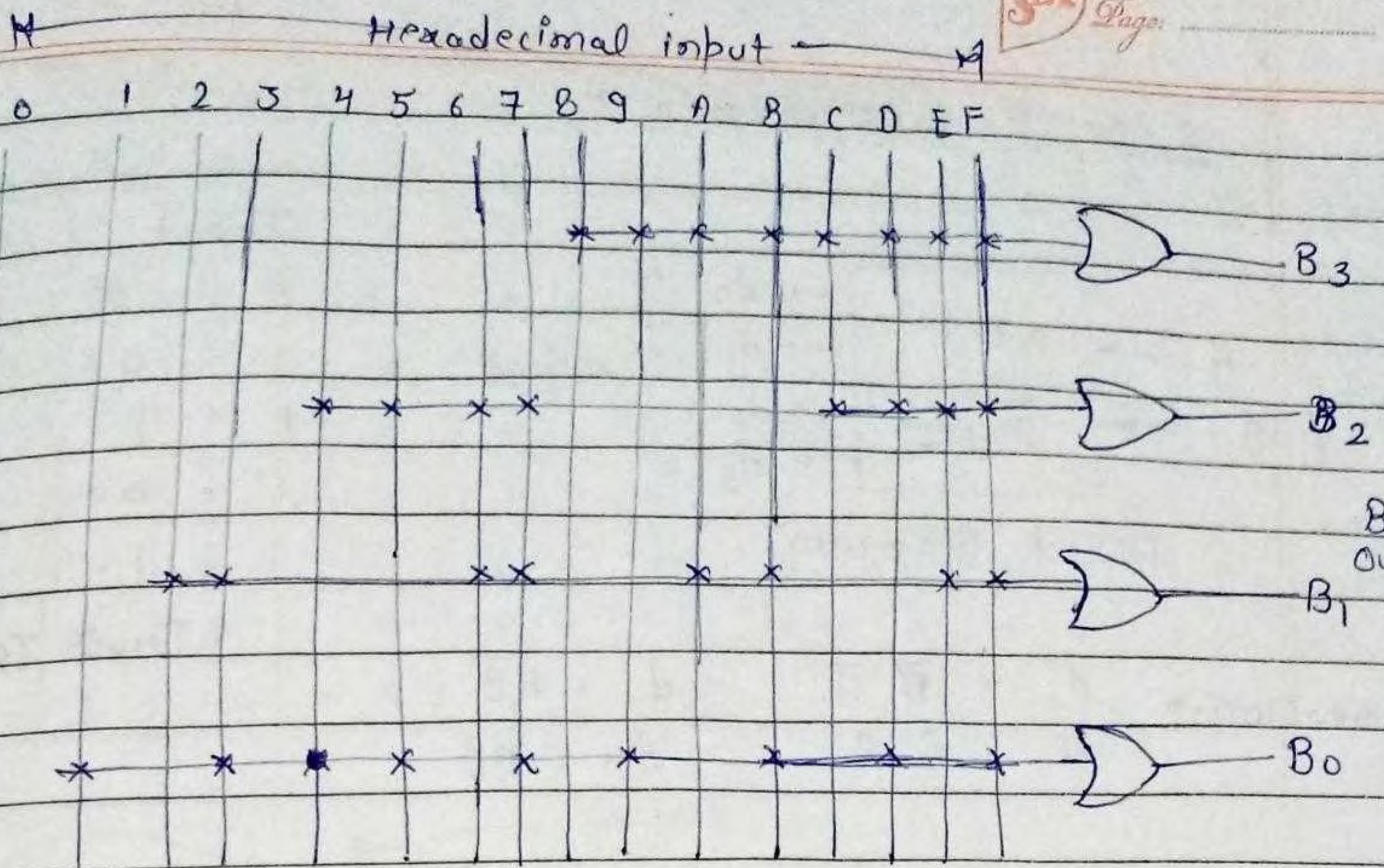


### Hex to binary Encoder

Hex input	B <sub>3</sub>	B <sub>2</sub>	B <sub>1</sub>	B <sub>0</sub>
H				
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
A	1	0	1	0
B	1	0	1	1
C	1	1	0	0
D	1	1	0	1
E	1	1	1	0
F	1	1	1	1

Hex input      B<sub>3</sub>      B<sub>2</sub>      B<sub>1</sub>      B<sub>0</sub>

B<sub>3</sub> = 8 + 9 + A + B + C + D + E  
 + F  
 B<sub>2</sub> = 4 + 5 + C + 7 + C + D + E + F  
 B<sub>1</sub> = 2 + 3 + 6 + 7 + A + B + E + F  
 B<sub>0</sub> = 1 + 3 + 5 + 7 + 8 + D + F

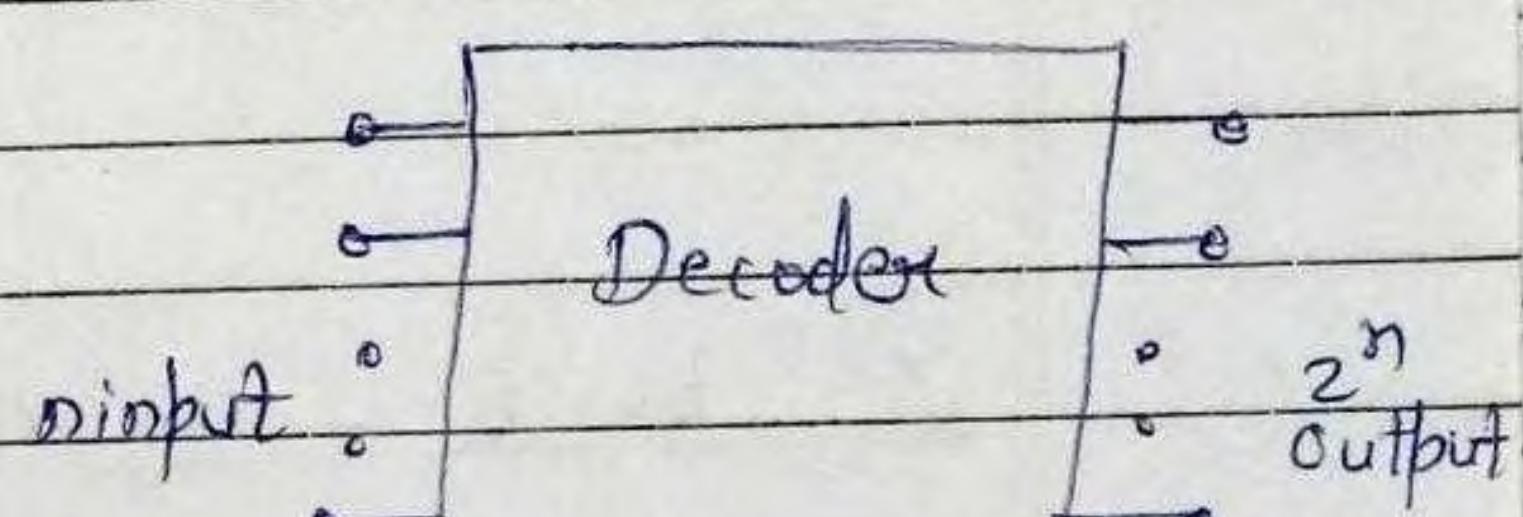


### Decoder

A decoder is combinational circuit that converts binary information from the coded inputs to a maximum of  $2^n$  unique outputs.

### Typical applications -

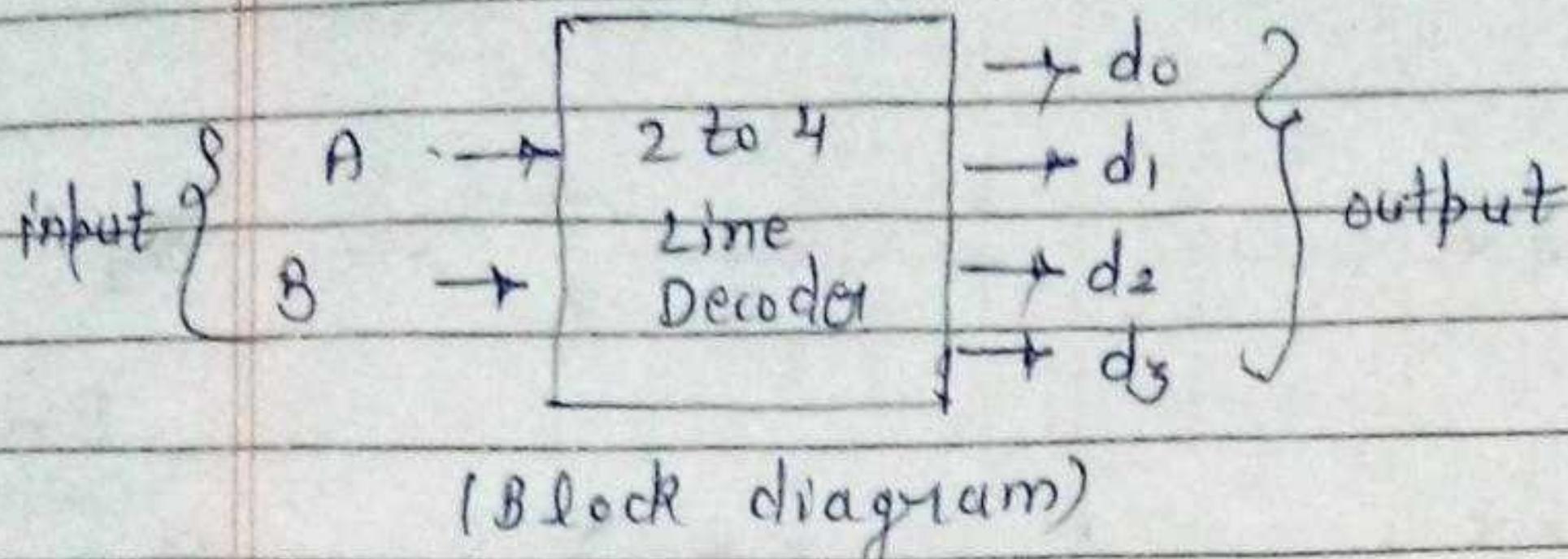
1. Code converters
2. BCD to seven segment decoders
3. Nixie tube decoders
4. Relay actuators.



Block diagram.

### Types -

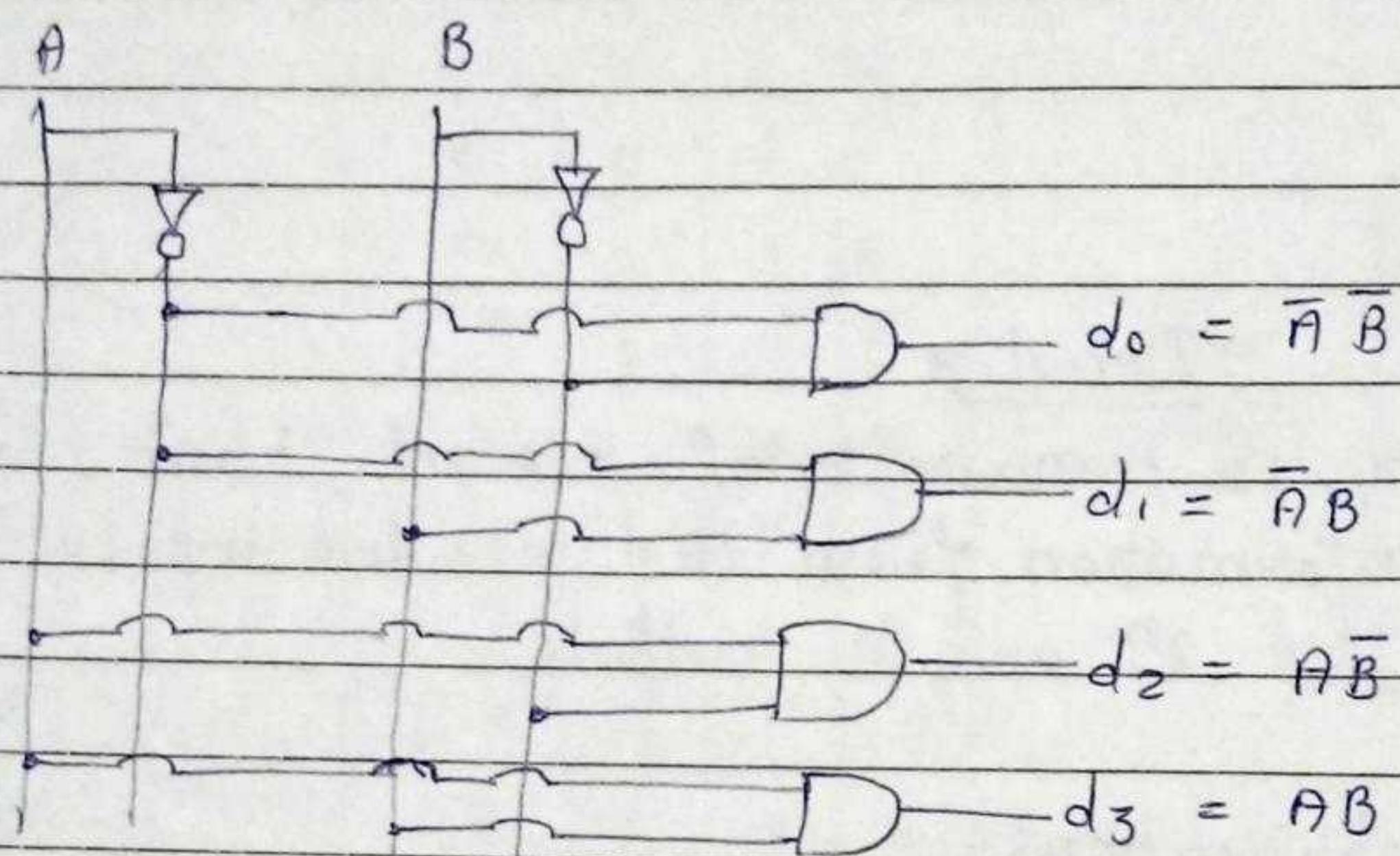
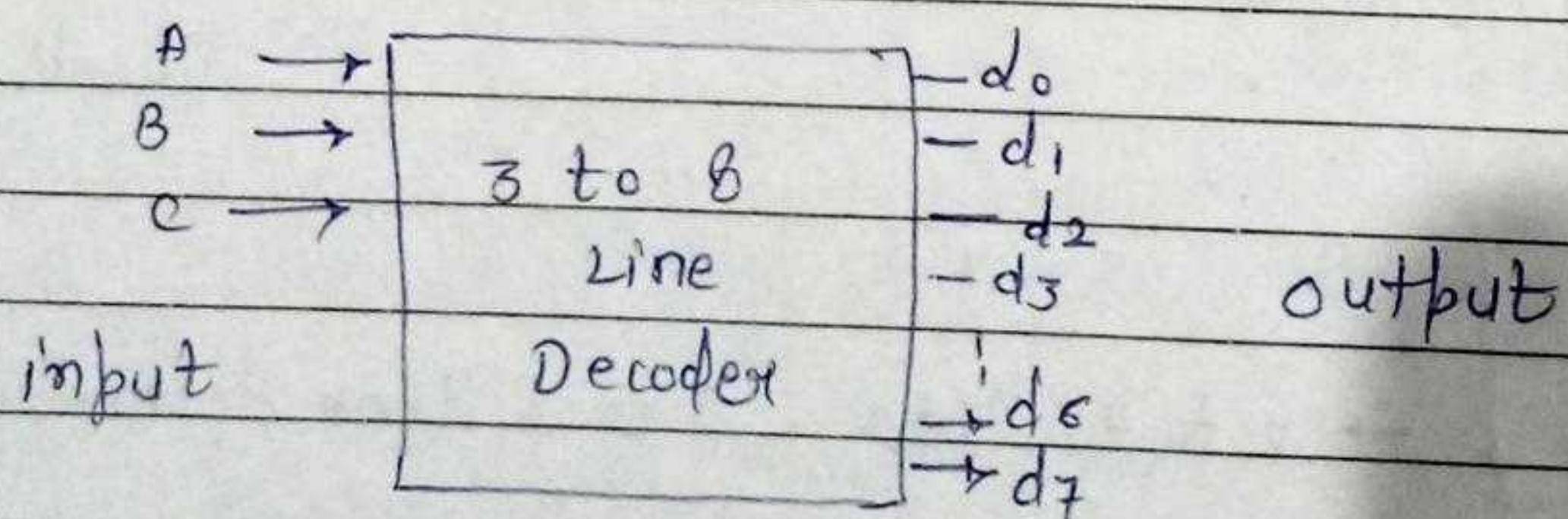
1. Parallel  $\rightarrow$  2 to 4 line, 3 to 8 Line decoder and so on.
2. Tree
3. Balanced

2 to 4 Line decoder →

Input		
A	B	Output
0	0	d <sub>0</sub>
0	1	d <sub>1</sub>
1	0	d <sub>2</sub>
1	1	d <sub>3</sub>

(Truth Table)

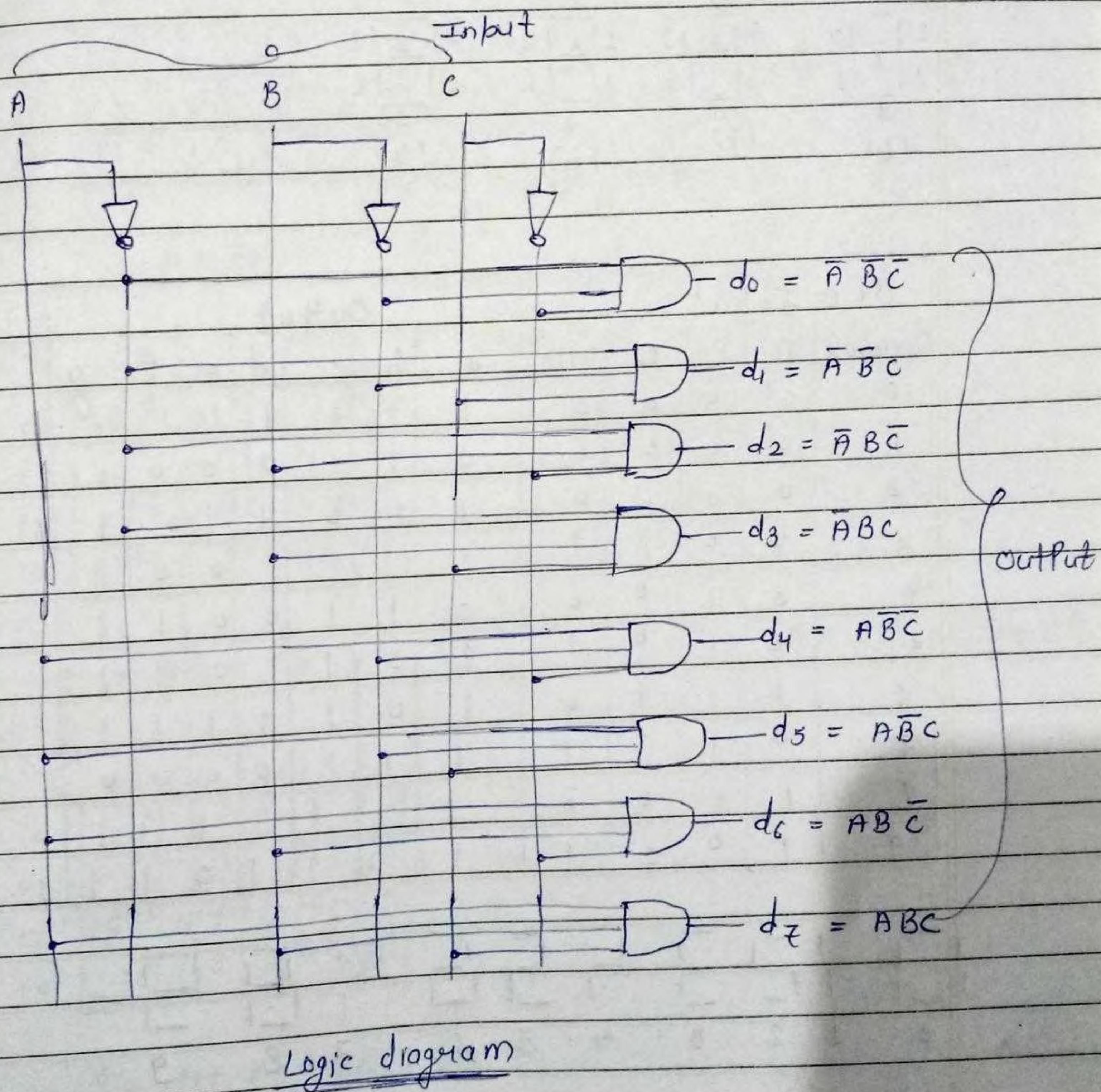
Expression →  $d_0 = \bar{A} \bar{B}$        $d_2 = A \bar{B}$   
 $d_1 = \bar{A} B$        $d_3 = AB$

3. to 8 Line decoder →

(Block diagram)

## Truth Table

Input			Output								
A	B	C	$d_0$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$	$d_7$	Exp
0	0	0	1	0	0	0	0	0	0	0	$\bar{A}\bar{B}\bar{C}$
0	0	1	0	1	0	0	0	0	0	0	$\bar{A}\bar{B}C$
0	1	0	0	0	1	0	0	0	0	0	$\bar{A}B\bar{C}$
0	1	1	0	0	0	1	0	0	0	0	$\bar{A}BC$
1	0	0	0	0	0	0	1	0	0	0	$A\bar{B}\bar{C}$
1	0	1	0	0	0	0	0	1	0	0	$A\bar{B}C$
1	1	0	0	0	0	0	0	0	0	1	$ABC$
1	1	1	0	0	0	0	0	0	0	1	$ABC$



# "BCD to 7-Segment decoder"

a  
b  
c  
d

common cathode  
(1)

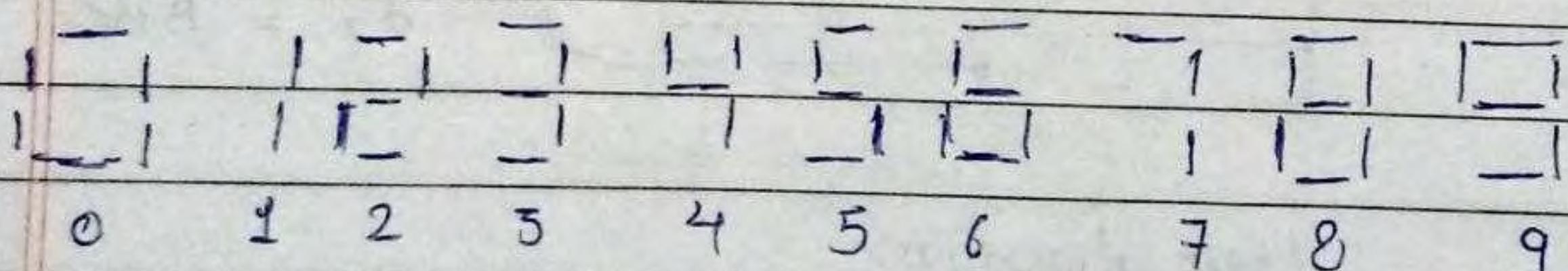
common anode  
(0)

LED GLOW  $\rightarrow$  1

NOT GLOW  $\rightarrow$  0

<u>0-9</u>	<u>11011</u>	<u>01011</u>	<u>01112</u>	<u>01111</u>	<u>11111</u>	<u>11110</u>
	<u>11112</u>	<u>01111</u>	<u>11100</u>	<u>01111</u>	<u>01110</u>	<u>01111</u>
(0)	(1)	(2)	(3)	(4)	(5)	
	<u>11110</u>	<u>01111</u>	<u>11111</u>	<u>11111</u>		
	<u>11111</u>	<u>01110</u>	<u>11111</u>	<u>01111</u>		
(6)	(7)	(8)	(9)			

BCD Input				Output							
Decimal	A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1



AB CD

	00	01	11	10
00	1°	0' 1	1 3	1 2
01	0 4	1 5	1 7	1 6
11	x 12	x 13	x 15	x 14
10	1 8	1 9	x 11	x 10

AB CD

	00	01	11	10
00	1°	1' 1	1 3	1 2
01	1 4	1 5	1 7	1 6
11	x 12	x 13	x 15	x 14
10	1 8	1 9	x 11	x 10

$$G = A + C + BD + \bar{B}\bar{D}$$

$$b = \bar{B} + \bar{C}\bar{D} + CD$$

AB CD

	00	01	11	10
00	1°	1' 1	1 3	1 2
01	1 4	1 5	1 7	1 6
11	x 12	x 13	x 15	x 14
10	1 8	1 9	x 11	x 10

AB CD

	00	01	11	10
00	1°	0' 1	1 3	1 2
01	0 4	1 5	0 7	1 6
11	x 12	x 13	x 15	x 14
10	1 8	1 9	x 11	x 10

$$C = B + \bar{C} + D$$

$$d = A + CD + \bar{D}C + \bar{C}\bar{D} + B\bar{C}\bar{D}$$

AB CD

	00	01	11	10
00	1°	0' 1	1 3	1 2
01	0 4	0 5	0 7	1 6
11	x 12	x 13	x 15	x 14
10	D 8	0 9	x 11	x 10

AB CD

	00	01	11	10
00	1°	0' 1	0 3	0 2
01	1 4	1 5	0 7	1 6
11	x 2	x 3	x 15	x 14
10	1 8	1 9	x 11	x 10

$$e = \bar{B}\bar{D} + CD$$

$$f = A + \bar{C}\bar{D} + B\bar{C}\bar{D} + B\bar{D}$$

AB CD

	00	01	11	10
00	1°	0' 1	1 3	1 2
01	1 4	1 5	0 7	1 6
11	x 12	x 13	x 15	x 14
10	1 8	1 9	x 11	x 10

$$g = A + CD + BC + \bar{B}\bar{C}$$

"Code conversion"

1. Binary to Gray code converter.
2. Gray to Binary code converter
3. BCD to Excess 3
4. Excess 3 to BCD
5. Gray to Excess 3
6. Binary to BCD converter
7. BCD to Binary converter
8. BCD to 7 segment code converter.

"Binary to Gray code converter."

In Gray code only one bit changes at a time.

Decimal	Binary inputs				Gray output			
	B <sub>3</sub>	B <sub>2</sub>	B <sub>1</sub>	B <sub>0</sub>	G <sub>3</sub>	G <sub>2</sub>	G <sub>1</sub>	G <sub>0</sub>
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	1
3	0	0	1	1	0	0	1	0
4	0	1	0	0	0	1	1	0
5	0	1	0	1	0	1	1	1
6	0	1	1	0	0	1	0	1
7	0	1	1	1	0	1	0	0
8	1	0	0	0	1	1	0	0
9	1	0	0	1	1	1	0	1
10	1	0	1	0	1	1	1	1
11	1	0	1	1	1	1	1	0
12	1	1	0	0	1	0	1	0
13	1	1	0	1	1	0	1	1
14	1	1	1	0	1	0	0	1
15	1	1	1	1	1	0	0	0

$B_3B_2$	$\bar{B}_3\bar{B}_2$	$\bar{B}_1\bar{B}_0$	$\bar{B}_1B_0$	$B_1B_0$	$B_1\bar{B}_0$
$B_3B_2$	00	01	11	10	
$\bar{B}_3\bar{B}_2$	00	01	03	02	
$B_3B_2$	01	04	05	03	06
$\bar{B}_3\bar{B}_2$	11	12	13	15	14
$B_3B_2$	10	18	19	11	10

$B_3B_2$	$\bar{B}_3\bar{B}_2$	$\bar{B}_1\bar{B}_0$	$\bar{B}_1B_0$	$B_1B_0$	$B_1\bar{B}_0$
$B_3B_2$	00	01	11	10	
$\bar{B}_3\bar{B}_2$	00	01	03	02	
$B_3B_2$	01	14	15	17	16
$\bar{B}_3\bar{B}_2$	11	012	013	015	014
$B_3B_2$	10	18	19	11	10

K-map for  $G_3 \rightarrow$

$$G_3 = B_3$$

K-map for  $G_2 \rightarrow$

$$G_2 = \bar{B}_3B_2 + B_3\bar{B}_2$$

$$G_2 = B_3 \oplus B_2$$

$B_3B_2$	$\bar{B}_3\bar{B}_2$	$\bar{B}_1\bar{B}_0$	$\bar{B}_1B_0$	$B_1\bar{B}_0$	
$B_3B_2$	00	01	11	10	
$\bar{B}_3\bar{B}_2$	00	01	03	02	
$B_3B_2$	01	14	15	0	0
$\bar{B}_3\bar{B}_2$	11	12	13	15	14
$B_3B_2$	10	08	09	11	10

$B_3B_2$	$\bar{B}_3\bar{B}_2$	$\bar{B}_1\bar{B}_0$	$\bar{B}_1B_0$	$B_1\bar{B}_0$	
$B_3B_2$	00	01	11	10	
$\bar{B}_3\bar{B}_2$	00	01	03	02	
$B_3B_2$	01	04	15	0	0
$\bar{B}_3\bar{B}_2$	11	012	013	015	014
$B_3B_2$	10	08	09	11	10

K-map for  $G_1 \rightarrow$

$$G_1 = B_2\bar{B}_1 + \bar{B}_2B_1$$

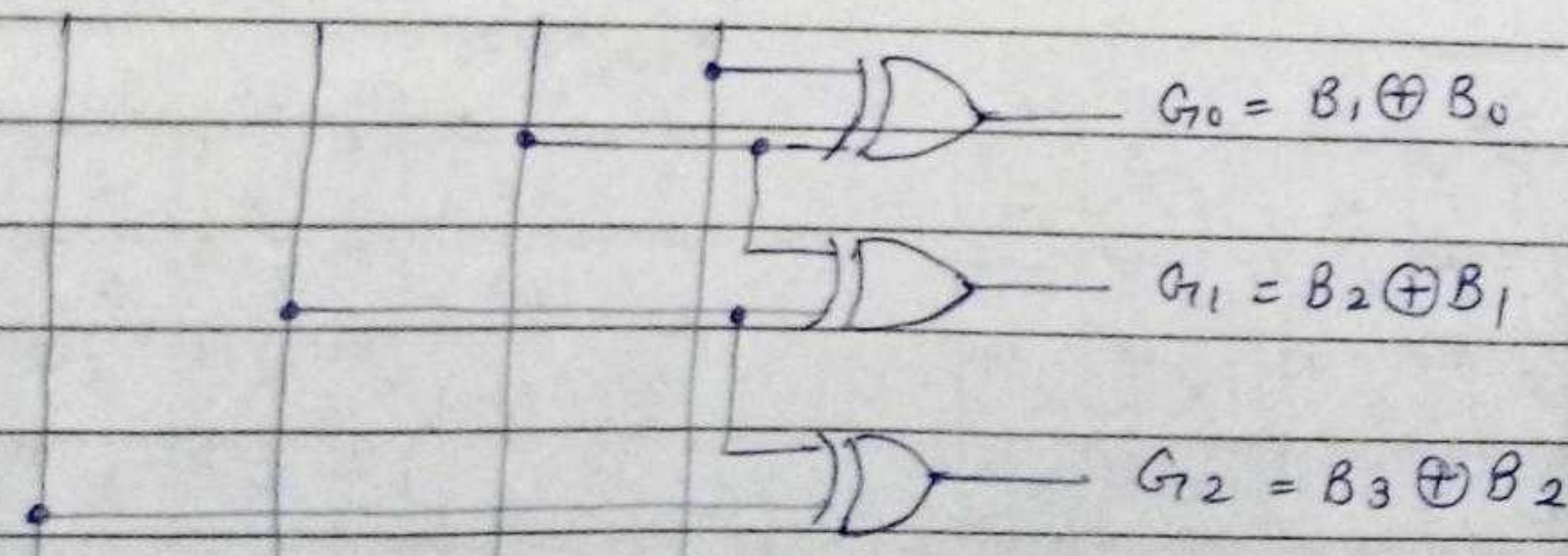
$$= B_2 \oplus B_1$$

K-map for  $G_0 \rightarrow$

$$G_0 = \bar{B}_1B_0 + B_1\bar{B}_0$$

$$= B_1 \oplus B_0$$

$B_3 \quad B_2 \quad B_1 \quad B_0$

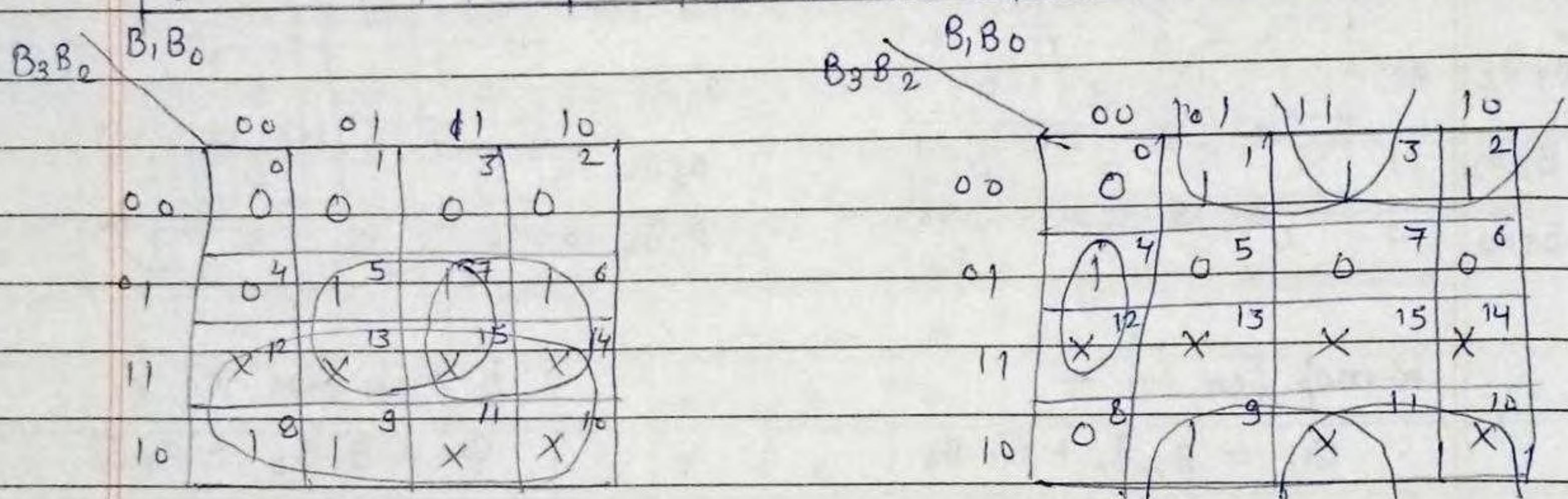


$$G_3 = B_3$$

Diagram

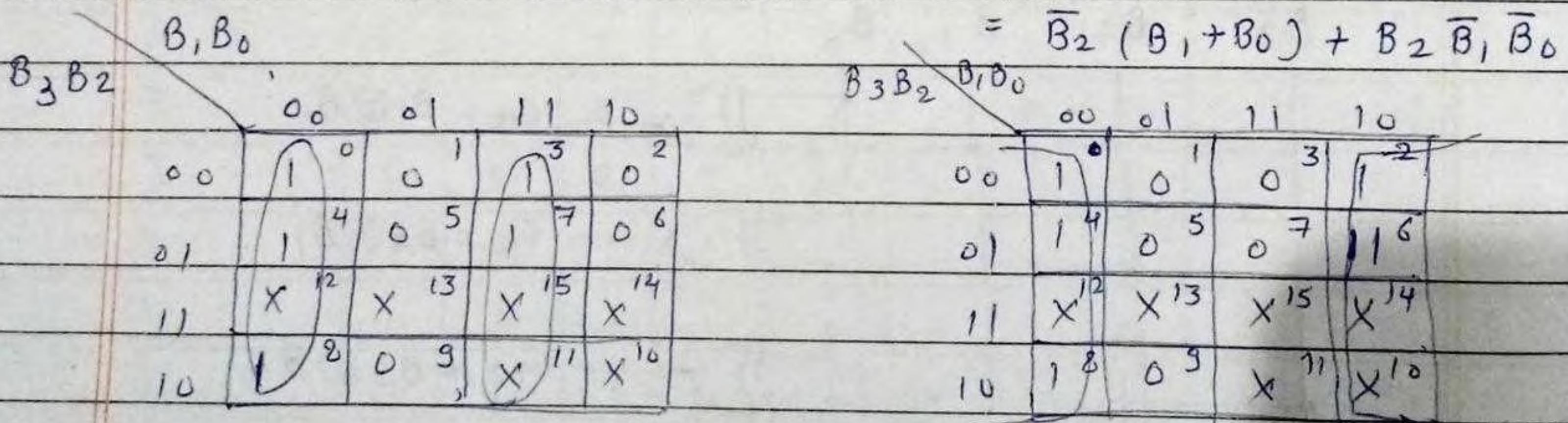
"BCD to Excess-3"

Decimal	BCD Inputs				Excess-3 outputs			
	$B_3$	$B_2$	$B_1$	$B_0$	$E_3$	$E_2$	$E_1$	$E_0$
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0



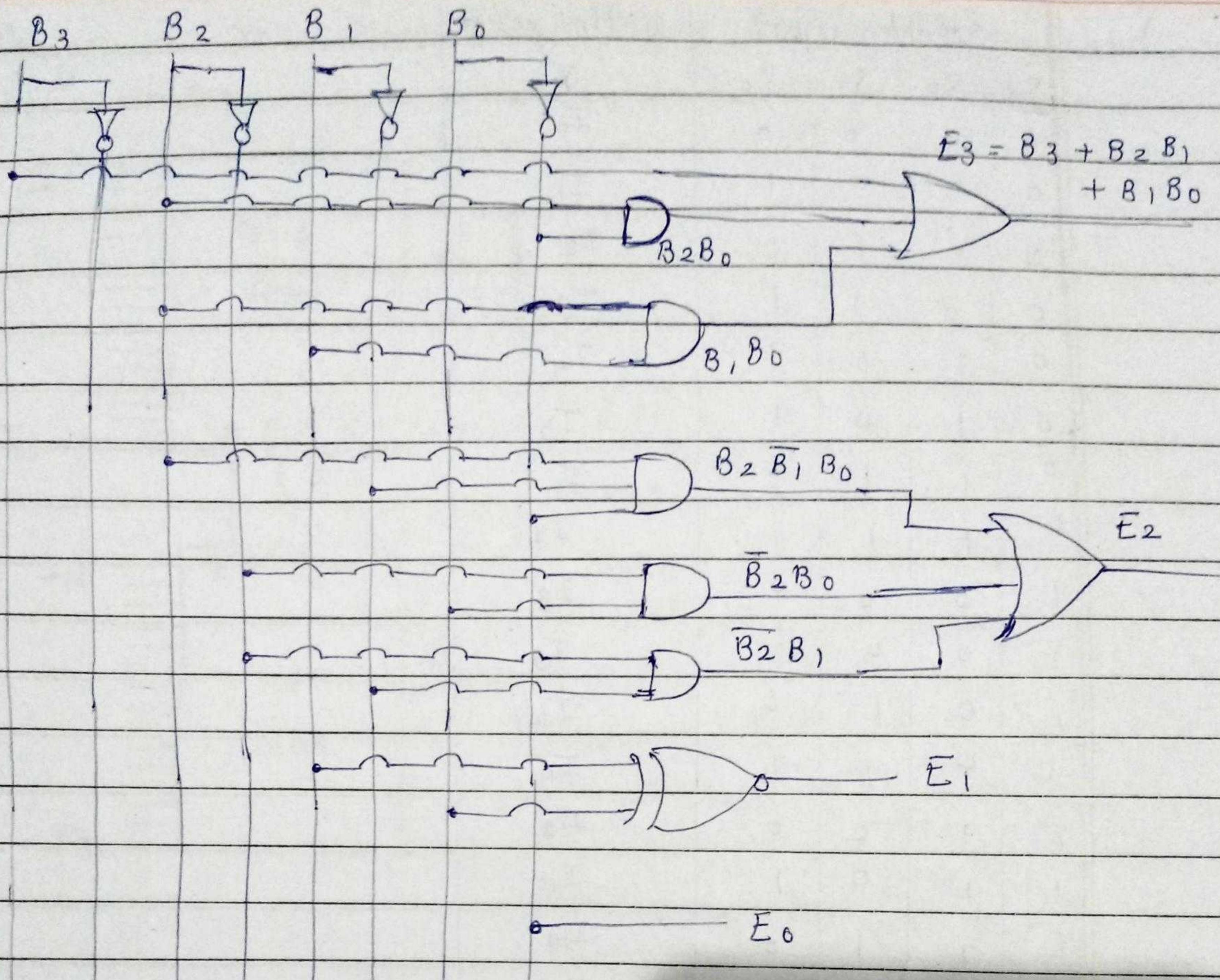
$$\text{K-map for } E_3 = B_3 + B_2 B_0 + B_2 B_1 \\ = B_3 + B_2 (B_0 + B_1)$$

$$\text{K-map for } E_2 = \\ = \overline{B}_2 B_1 + \overline{B}_2 B_0 + B_2 \overline{B}_1 \overline{B}_0 \\ = \overline{B}_2 (B_1 + B_0) + B_2 \overline{B}_1 \overline{B}_0$$



$$\text{K-map for } E_1 = \overline{B}_1 \overline{B}_0 + B_1 B_0 \\ = (B_1 \odot B_0)$$

$$\text{K-map for } E_0 \\ E_0 = \overline{B}_0$$



## \* Parity Generators / checkers

Parity bit 1 0 1 0 1 1 0 0 0 0 1

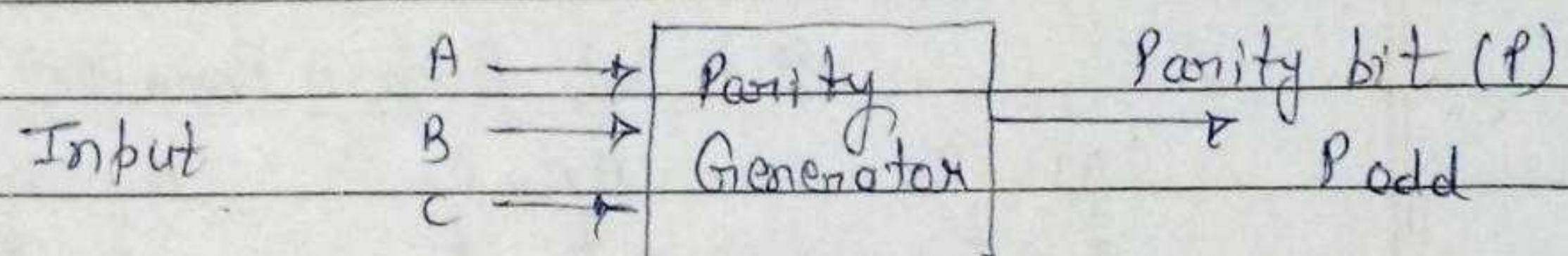
Even Parity  $\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline \text{Parity bit} & \downarrow & & & & & & & \\ \hline \end{array}$  Number of 1's = 2

Odd Parity  $\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline \text{Parity bit} & \downarrow & & & & & & & \\ \hline \end{array}$  Number of 1's = 1

→ In the even parity system, the added parity bit will make the total number of 1's an even number.

→ In the odd parity system, the added parity bit will make the total number of 1's an odd number.

\* Parity Generator + Parity generator is a logic circuit which generates the parity bits for even parity or odd parity.



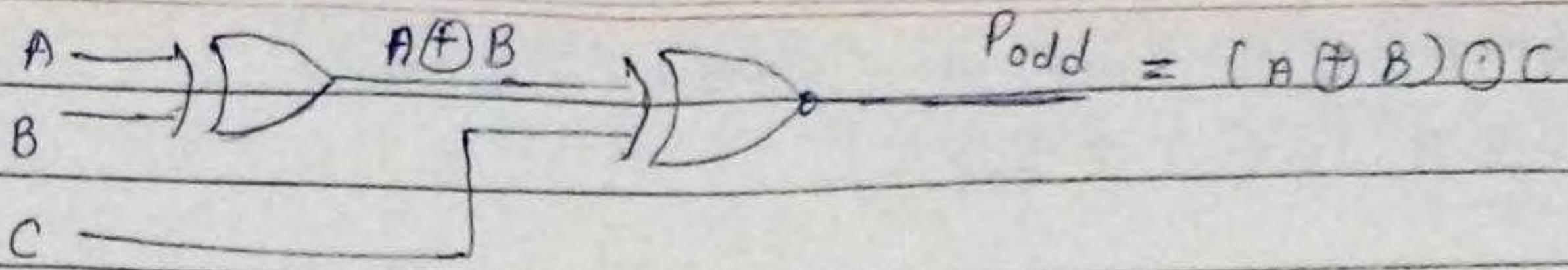
Input			Output
A	B	C	P <sub>odd</sub>
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Block diagram

Input			Output		
A	B	C	00	01	11
0	0	0	1	0	0
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	0
1	1	0	1	1	0
1	1	1	0	0	1

$$\begin{aligned}
 P_{\text{odd}} &= \bar{A}\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC + ABC \\
 &= \bar{C}(\bar{A}\bar{B} + AB) + C(A\bar{B} + \bar{A}B) \\
 &= \bar{C}(A \oplus B) + C(A \oplus B) \\
 &= (A \oplus B) \oplus C
 \end{aligned}$$

Table



odd Parity Generator

Parity checker →

→ The Parity checker circuit is at the receiver.

→ The output of Parity checker circuit is denoted by Parity error output PEO.

$PEO = 1$  --- if error is present i.e. if the received four bit word has an even parity

$PEO = 0$  --- if there is no error i.e. if the received four bit word has an odd parity.

Input			Output	
A	B	C	$P_{\text{odd}}$	$P_c(\text{odd})$
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	1	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

K-map +							
A	B	C	$P_{\text{odd}}$	00	01	11	10
0	0	0	1	1	0	0	0
0	0	1	0	0	1	1	1
0	1	0	1	0	1	0	1
0	1	1	0	1	1	1	0
1	0	0	0	0	0	1	1
1	0	1	1	1	0	0	0
1	1	0	1	0	1	0	1
1	1	1	0	1	0	1	0

$$\begin{aligned}
 P_c(\text{odd}) &= \bar{A}\bar{B}\bar{C}\bar{P}_{\text{odd}} + \bar{A}\bar{B}C P_{\text{odd}} \\
 &\quad + \bar{A}B\bar{C}P_{\text{odd}} + \bar{A}BC\bar{P}_{\text{odd}} \\
 &\quad + AB\bar{C}\bar{P}_{\text{odd}} + ABCP_{\text{odd}} \\
 &\quad + A\bar{B}\bar{C}P_{\text{odd}} + A\bar{B}C\bar{P}_{\text{odd}} \\
 &= \bar{A}\bar{B}(\bar{C}\bar{P}_{\text{odd}} + C P_{\text{odd}}) \\
 &\quad + \bar{A}B(\bar{C}P_{\text{odd}} + C\bar{P}_{\text{odd}}) + \\
 &\quad AB(\bar{C}\bar{P}_{\text{odd}} + C P_{\text{odd}}) + \\
 &\quad A\bar{B}(C\bar{C}P_{\text{odd}} + C\bar{P}_{\text{odd}})
 \end{aligned}$$

$$\bar{x}\bar{y} + xy = x \oplus y$$

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$$= (A \odot B) (C \odot P_{\text{odd}}) + (A \oplus B) (C \oplus P_{\text{odd}})$$

$$P_C(\text{odd}) = (A \oplus B) \odot (C \oplus P_{\text{odd}})$$

Logic diagram -

