MA 374 – Financial Engineering Lab

Lab – 1

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1 QUESTION - 1:

The initial option prices for the European Call Option and European Put Option are:

Number of Sub-intervals (M)	European Call Option	European Put Option
1	38.16764	19.94172
5	34.90653	16.68061
10	33.62502	15.39910
20	33.85945	15.63353
50	33.98118	15.75527
100	34.01116	15.78524
200	34.01958	15.79366
400	34.01913	15.79321

Binomial Pricing Algorithm:

1. At time $t = t_i$ ($= i.\delta t$), there are i + 1 possible asset prices, i.e,

$$S_n^i = d^{i-n} u^n S_0 , \qquad 0 <= n <= i$$

2. Since continuous compounding convention is used, gross return is $R = e^{r.\delta t}$.
3. The probability (p) of an upward return in price is $\frac{R-d}{u-d}$.
4. At expiry, i.e, t = T, we calculate the price of the option using the respective payoff function for both the call and put option, i.e,

$$C_n^M = \max(S_n^M - K, 0) , \qquad 0 <= n <= M$$
$$P_n^M = \max(K - S_n^M, 0) , \qquad 0 <= n <= M$$

where, C_n^M is the nth possible price of the call option for the Mth interval, and P_n^M is the nth possible price of the put option for the Mth interval

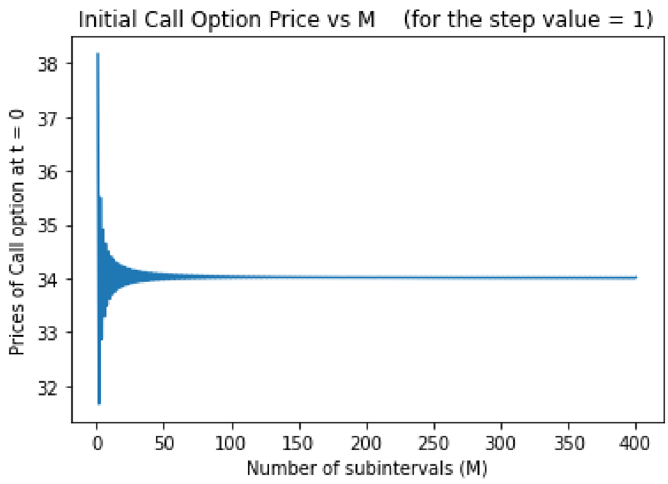
5. Now, we continuously apply **Backward Induction** to find out the option price at t = 0 by using following relation:

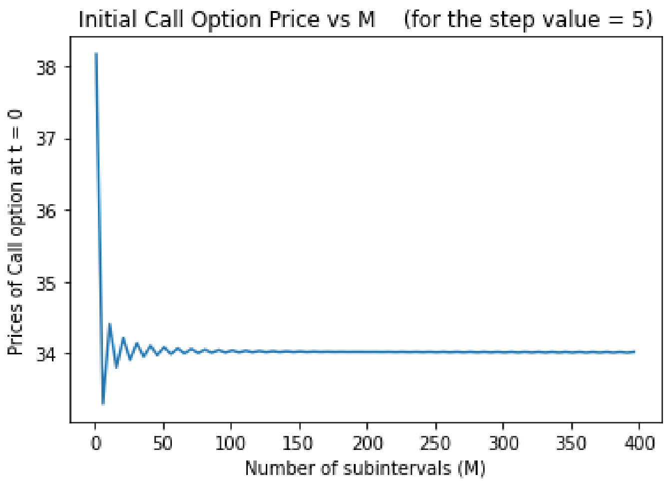
$$C_n^i = (1-p) . C_{n+1}^{i+1} + p . C_n^{i+1} \qquad , 0 <= n <= i \ \& \ 0 <= i <= M - 1$$
$$P_n^i = (1-p) . C_{n+1}^{i+1} + p . P_n^{i+1} \qquad , 0 <= n <= i \ \& \ 0 <= i <= M - 1$$

6. C_0^0 and P_0^0 are the required values, i.e, initial option prices.

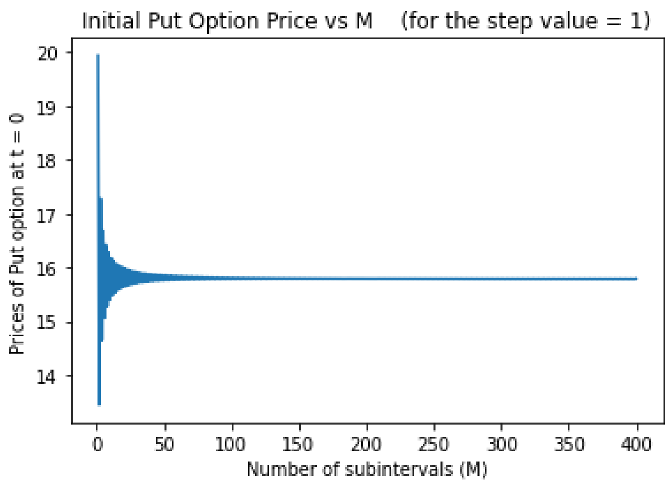
2 QUESTION - 2:

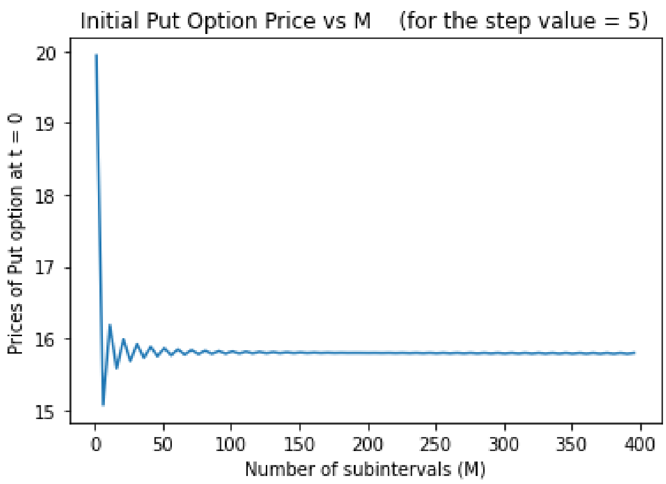
The plots for the Call Option are:





The plots for the Put Option are:





Observation:

1. From all the plots, we observe that the initial Call Option price converges to 34.02 (approx.) while the initial Put Option price converges to 15.79 (approx.).
2. The convergence of the plot is faster when step value is 5. The deviations from the convergence value is higher when the value of M is less.
3. The convergence is not perfect in the sense that the values tend to oscillate around the specified values (*in point 1*) even if the sub-intervals number is increased beyond 400. But this oscillation is normal for such numerical algorithms and gives a correct approximation of the required value since fluctuations tend to happen at 3rd/4th decimal place onwards.
4. We can say that the convergence of the values is fast enough as not too many iterations are required to approximately attain the converged value.

3 QUESTION – 3 :

The required values of the **call options** at given time stamps for M = 20 are:

Time points / Options	t = 0	t = 0.50	t = 1	t = 1.50	t = 3	t = 4.50
1.	33.86	59.96	100.66	160.61	519.10	1419.42
2.	X	31.89	57.70	98.44	359.93	1024.99
3.	X	15.10	29.80	55.30	242.03	732.79
4.	X	X	13.47	27.57	154.84	516.32
5.	X	X	5.15	11.77	91.19	355.96
6.	X	X	X	4.12	46.98	237.16
7.	X	X	X	1.13	19.73	149.15
8.	X	X	X	X	6.15	83.95
9.	X	X	X	X	1.24	36.25
10.	X	X	X	X	0.12	8.15
11.	X	X	X	X	0.00	0.00
12.	X	X	X	X	0.00	0.00
13.	X	X	X	X	0.00	0.00
14.	X	X	X	X	X	0.00
15.	X	X	X	X	X	0.00
16.	X	X	X	X	X	0.00
17.	X	X	X	X	X	0.00
18.	X	X	X	X	X	0.00
19.	X	X	X	X	X	0.00

Observations:

1. At t = 0.25*i, we will get i + 1 different values of the options, since i + 1 different asset prices are available according to the Binomial Model.

2. The same observation holds for the Put Option.

The required values of the **put options** at given time stamps for M = 20 are:

Time points / Options	t = 0	t = 0.50	t = 1	t = 1.50	t = 3	t = 4.50
1.	15.63	8.48	3.50	0.94	0.00	0.00
2.	X	15.49	8.00	3.00	0.00	0.00
3.	X	24.67	15.27	7.44	0.01	0.00
4.	X	X	24.98	14.96	0.17	0.00
5.	X	X	35.97	25.27	1.24	0.00
6.	X	X	X	36.97	4.96	0.00
7.	X	X	X	48.30	13.22	0.00
8.	X	X	X	X	25.96	0.00
9.	X	X	X	X	40.53	0.60
10.	X	X	X	X	53.85	8.28
11.	X	X	X	X	64.43	26.64
12.	X	X	X	X	72.36	46.28
13.	X	X	X	X	78.23	60.83
14.	X	X	X	X	X	71.60
15.	X	X	X	X	X	79.59
16.	X	X	X	X	X	85.50
17.	X	X	X	X	X	89.88
18.	X	X	X	X	X	93.13
19.	X	X	X	X	X	95.53

No-arbitrage Condition:

In order for no arbitrage opportunity to exist, following relation must exist:

$$d < R < u$$

where,

$$R = e^{r.\delta t}$$

$$d = e^{-\sigma\sqrt{\delta t} + \left(r - \frac{1}{2}\sigma^2\right)\delta t}$$

$$u = e^{\sigma\sqrt{\delta t} + \left(r - \frac{1}{2}\sigma^2\right)\delta t} \quad \text{, where } \delta t = \frac{T}{M}$$