MA 323 - Monte Carlo Simulation Assignment - 1

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1 QUESTION - 1:

- **Sequence of x_i for a = 6, b = 0, m = 11 :**

Sequence / Seeds(x_0)	1st value	2nd value	3rd value	4th value	5th value	6th value	7th value	8th value	9th value	10th value
0	0	0	0	0	0	0	0	0	0	0
1	6	3	7	9	10	5	8	4	2	1
2	1	6	3	7	9	10	5	8	4	2
3	7	9	10	5	8	4	2	1	6	3
4	2	1	6	3	7	9	10	5	8	4
5	8	4	2	1	6	3	7	9	10	5
6	3	7	9	10	5	8	4	2	1	6
7	9	10	5	8	4	2	1	6	3	7
8	4	2	1	6	3	7	9	10	5	8
9	10	5	8	4	2	1	6	3	7	9
10	5	8	4	2	1	6	3	7	9	10

  - When seed (x_0) = 0, only 1 distinct value appears (0) which goes on repeating. For seeds (x_0) = 1 to 10, 10 distinct values from 1 to 10 appear for each seed before the sequence starts repeating intself ,i.e, period length = m – 1 = 10.

- **Sequence of x_i for a = 3, b = 0, m = 11 :**

Sequence / Seeds(x0)	1st value	2nd value	3rd value	4th value	5th value
0	0	0	0	0	0
1	3	9	5	4	1
2	6	7	10	8	2
3	9	5	4	1	3
4	1	3	9	5	4
5	4	1	3	9	5
6	7	10	8	2	6
7	10	8	2	6	7
8	2	6	7	10	8
9	5	4	1	3	9

10	8	2	6	7	10

- When seed (x_0) = 0, only 1 distinct value appears (0) which goes on repeating. For seeds (x_0) = 1 to 10, 5 distinct values from 1 to 10 appear for each seed before the sequence starts repeating intself ,i.e, period length = 5.

Best Choice:

- The largest possible period length of linear congruence generator is m – 1. This value is achieved when a = 6 (full period), while period length for a = 3 is only 5. So the linear congruence generator with a = 6 is preferred over a = 3 as it has higher period length. This is because there will be more randomness in the generated numbers as there are more numbers in the sequence.
  And x_0 (seed) should be a non-zero value, as x_0 = 0 has no randomness in it.

- **Output Screenshot:**

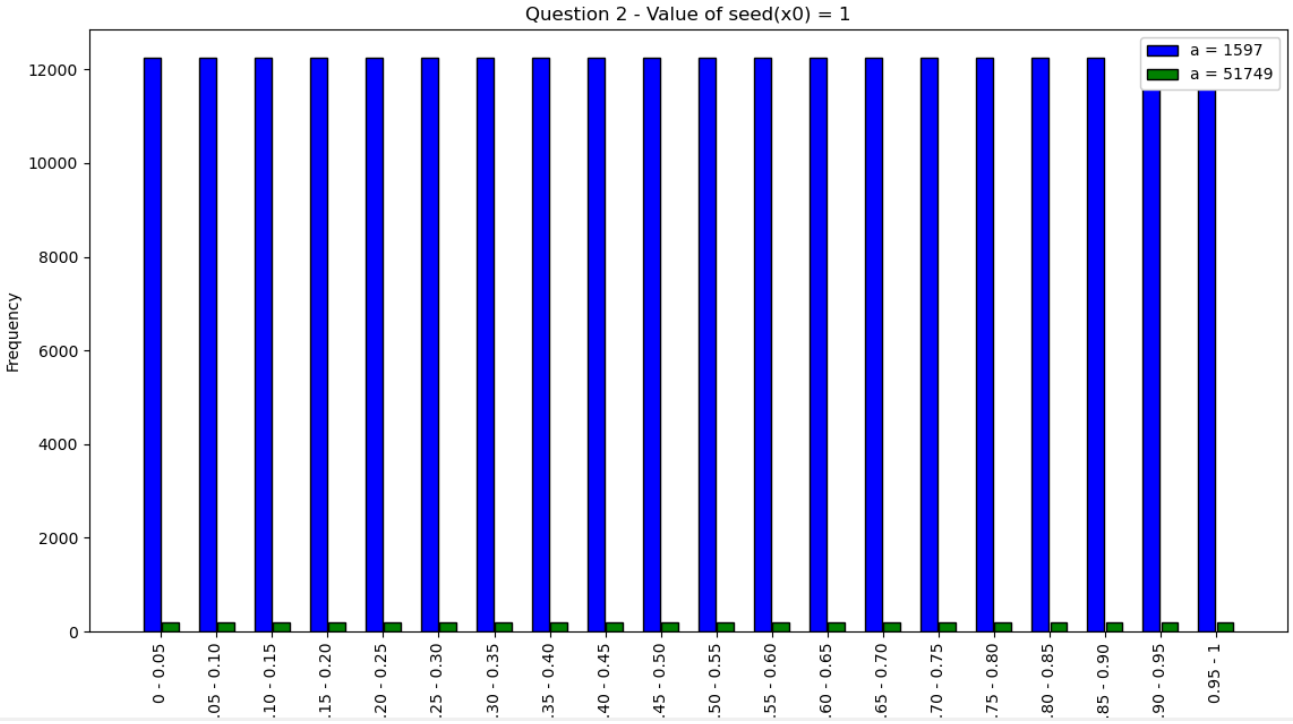
```
a = 6, b = 0, m = 11:
seed = 0, Numbers = [0]
seed = 1, Numbers = [6, 3, 7, 9, 10, 5, 8, 4, 2, 1]
seed = 2, Numbers = [1, 6, 3, 7, 9, 10, 5, 8, 4, 2]
seed = 3, Numbers = [7, 9, 10, 5, 8, 4, 2, 1, 6, 3]
seed = 4, Numbers = [2, 1, 6, 3, 7, 9, 10, 5, 8, 4]
seed = 5, Numbers = [8, 4, 2, 1, 6, 3, 7, 9, 10, 5]
seed = 6, Numbers = [3, 7, 9, 10, 5, 8, 4, 2, 1, 6]
seed = 7, Numbers = [9, 10, 5, 8, 4, 2, 1, 6, 3, 7]
seed = 8, Numbers = [4, 2, 1, 6, 3, 7, 9, 10, 5, 8]
seed = 9, Numbers = [10, 5, 8, 4, 2, 1, 6, 3, 7, 9]
seed = 10, Numbers = [5, 8, 4, 2, 1, 6, 3, 7, 9, 10]

a = 3, b = 0, m = 11:
seed = 0, Numbers = [0]
seed = 1, Numbers = [3, 9, 5, 4, 1]
seed = 2, Numbers = [6, 7, 10, 8, 2]
seed = 3, Numbers = [9, 5, 4, 1, 3]
seed = 4, Numbers = [1, 3, 9, 5, 4]
seed = 5, Numbers = [4, 1, 3, 9, 5]
seed = 6, Numbers = [7, 10, 8, 2, 6]
seed = 7, Numbers = [10, 8, 2, 6, 7]
seed = 8, Numbers = [2, 6, 7, 10, 8]
seed = 9, Numbers = [5, 4, 1, 3, 9]
seed = 10, Numbers = [8, 2, 6, 7, 10]
```

2 QUESTION - 2:

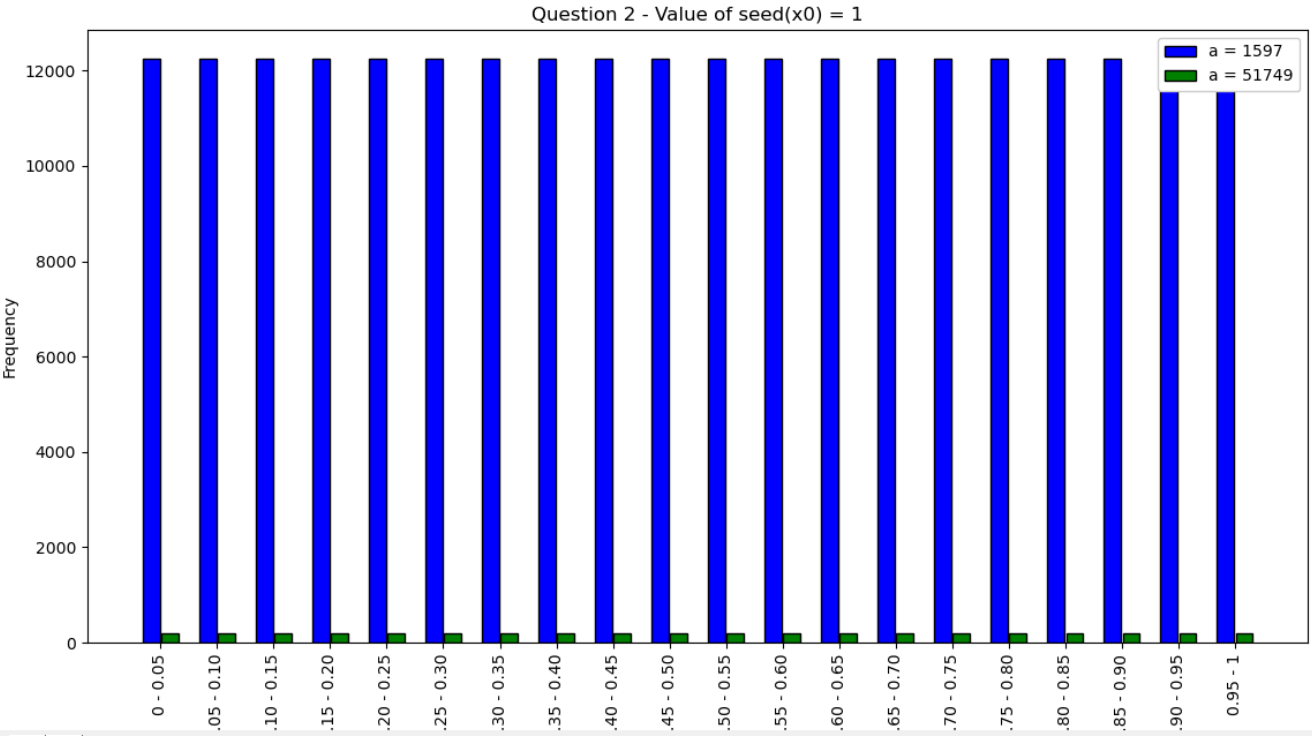
- **Tabulated Data of Frequency of numbers (u_i) in different intervals:**

	a = 1597, b = 1, m = 244944					a = 51749, b = 1, m = 244944				
Seeds / Frequency	$X_0 = 1$	$X_0 = 2$	$X_0 = 3$	$X_0 = 4$	$X_0 = 6$	$X_0 = 1$	$X_0 = 2$	$X_0 = 3$	$X_0 = 4$	$X_0 = 6$
0.00 – 0.05	12247	12247	12247	12247	12247	195	195	194	195	195
0.05 – 0.10	12247	12247	12247	12247	12247	194	194	193	194	194
0.10 – 0.15	12248	12248	12248	12248	12248	195	195	196	194	194
0.15 – 0.20	12247	12247	12247	12247	12247	193	193	193	195	194
0.20 – 0.25	12247	12247	12247	12247	12247	195	195	196	194	195
0.25 – 0.30	12247	12247	12247	12247	12247	195	195	194	195	195
0.30 – 0.35	12247	12247	12247	12247	12247	194	194	193	194	194
0.35 – 0.40	12247	12247	12247	12247	12247	194	194	196	194	194
0.40 – 0.45	12248	12248	12248	12248	12248	194	194	193	195	194
0.45 – 0.50	12247	12247	12247	12247	12247	195	195	196	194	195
0.50 – 0.55	12247	12247	12247	12247	12247	195	195	194	195	195
0.55 – 0.60	12247	12247	12247	12247	12247	194	194	193	194	194
0.60 – 0.65	12248	12248	12248	12248	12248	195	195	196	194	194
0.65 – 0.70	12247	12247	12247	12247	12247	193	193	193	195	194
0.70 – 0.75	12247	12247	12247	12247	12247	195	195	196	194	195
0.75 – 0.80	12247	12247	12247	12247	12247	195	195	194	195	195
0.80 – 0.85	12247	12247	12247	12247	12247	194	194	193	194	194
0.85 – 0.90	12247	12247	12247	12247	12247	194	194	196	194	194
0.90 – 0.95	12248	12248	12248	12248	12248	194	194	193	195	194
0.95 – 1.00	12247	12247	12247	12247	12247	195	195	196	194	195

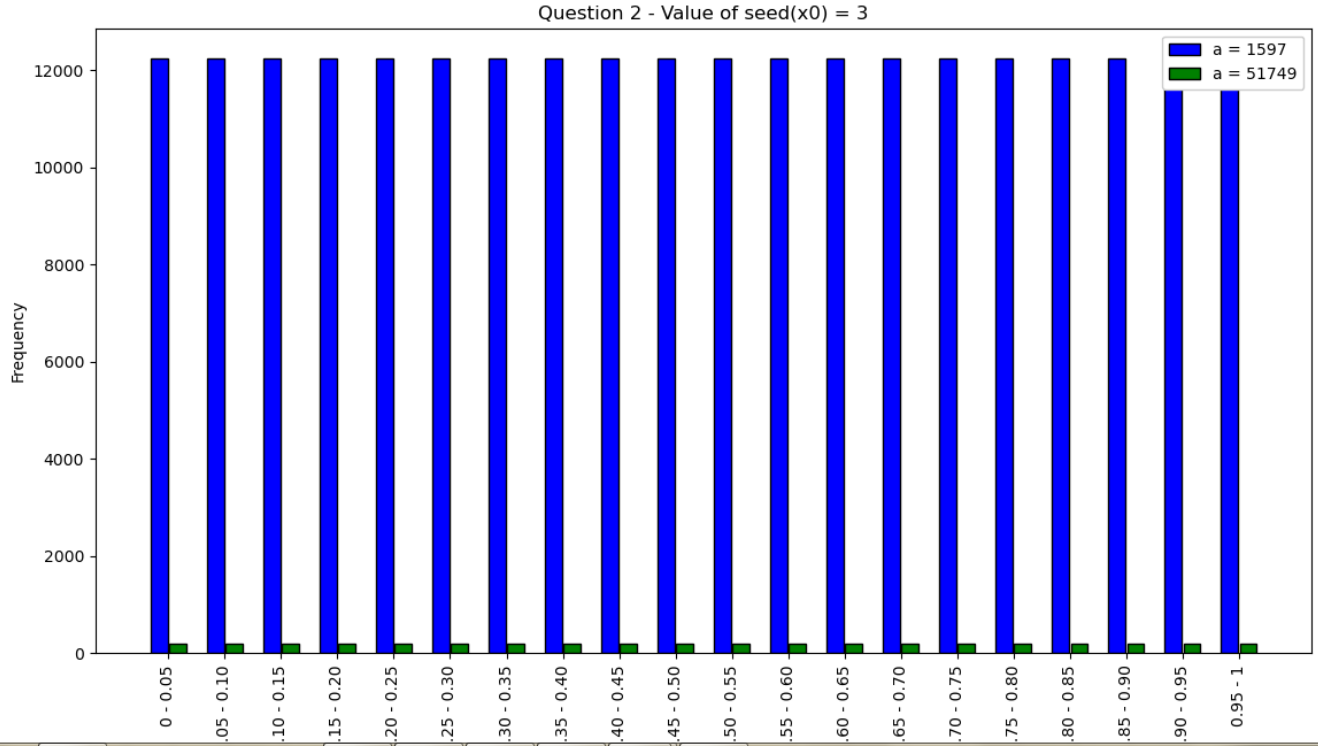
- **Bar Diagram for a = 1597,51749, b = 1, m = 244944, seed (x_0) = 1:**



- **Bar Diagram for a = 1597, 51749, b = 1, m = 244944, seed (x_0) = 2:**

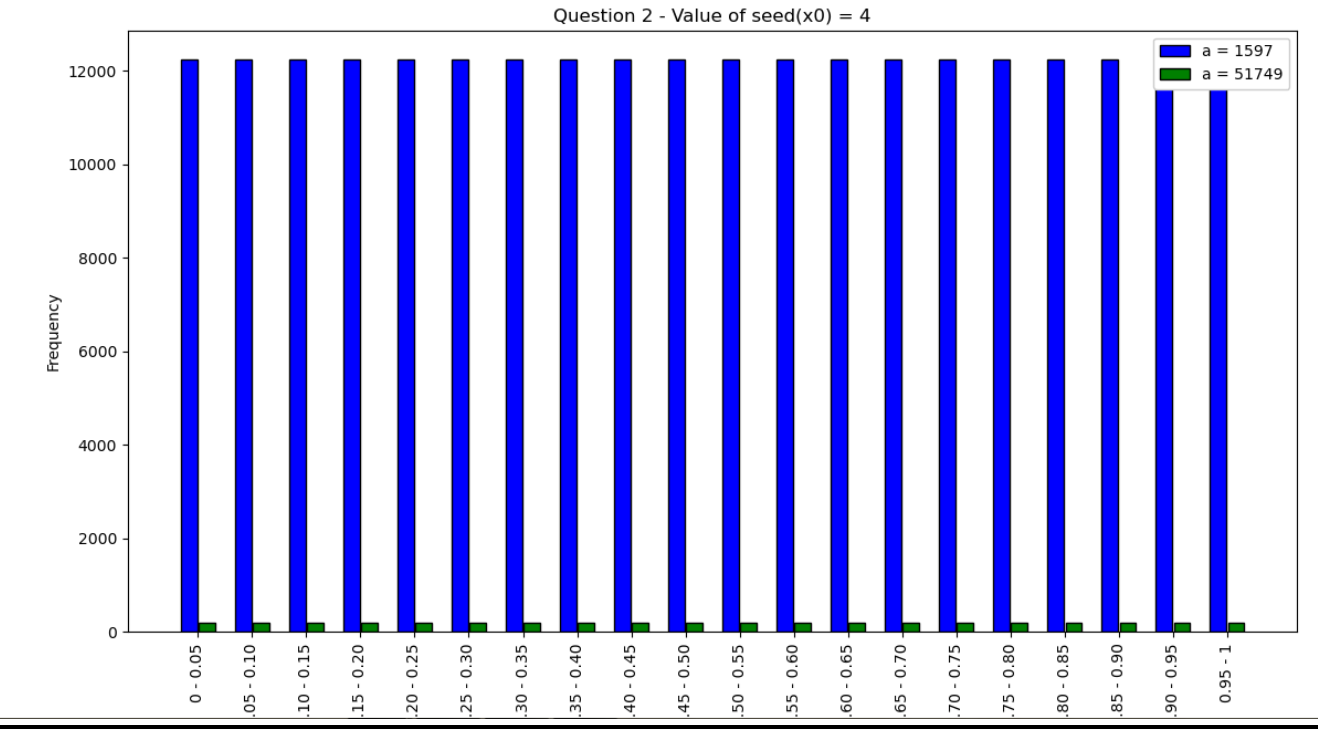


- **Bar Diagram for a = 1597, 51749, b = 1, m = 244944, seed (x_0) = 3:**



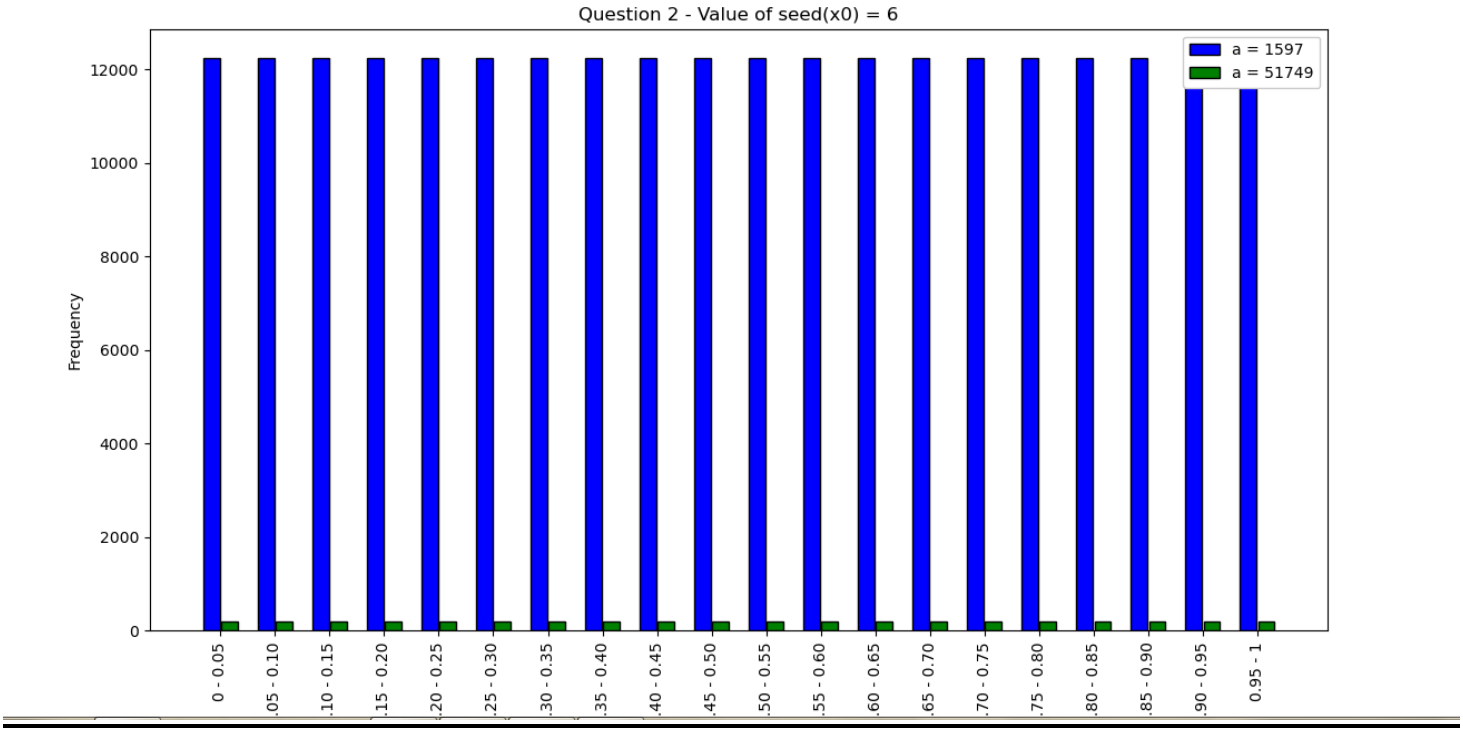
Frequency Range

- **Bar Diagram for a = 1597, 51749, b = 1, m = 244944, seed (x_0) = 4:**



Frequency Range

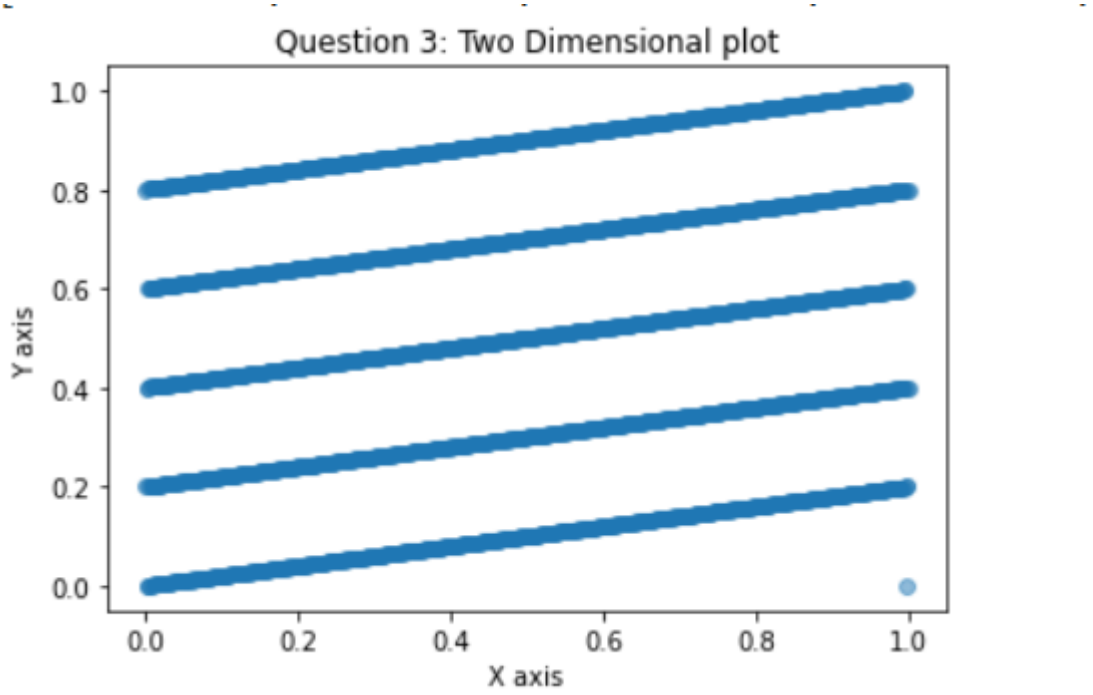
- **Bar Diagram for a = 1597, 51749, b = 1, m = 244944, seed (x_0) = 6:**



- **Observations:**
  1. The numbers are uniformly generated between 0 – 1. The frequency of different numbers lying in same length intervals are almost same. So, the random number generator follows the property of generation of numbers uniformly.
  2. For different value of seed (x_0), the frequencies are almost identical, and so the nature of bar graphs is same.
  3. When a = 1597, b = 1, m = 244944, the Linear Congruence Generator has its full period, i.e. m – 1. But when a = 51749, b = 1, m = 244944, the Linear Congruence Generator does not achieve its full period.

3 QUESTION – 3 :

- **Scatter plot for co-ordinates (u_{i-1}, u_i) with a = 1229, b = 1, m = 2048 and seed(x_0) = 1 :**



- **Observations:**
  1. The scatter plot contains 5 almost parallel lines originating at different y – coordinates.
  2. There is an outlier present at x = 1.0 (approx.). I believe this is present due to the precision issues while taking modulus in Python code (which is a bit different from the standard notion of modulus operation in other programming languages).
  3. I found that this plot helps in what is known as "Spectral Test". LCGs have a property that when plotted in 2 dimensions, lines will form, on which all possible outputs can be found. The spectral test compares the distance between these planes; the further apart they are, the worse the generator is.