MA 374 – Financial Engineering Lab

Lab – 10

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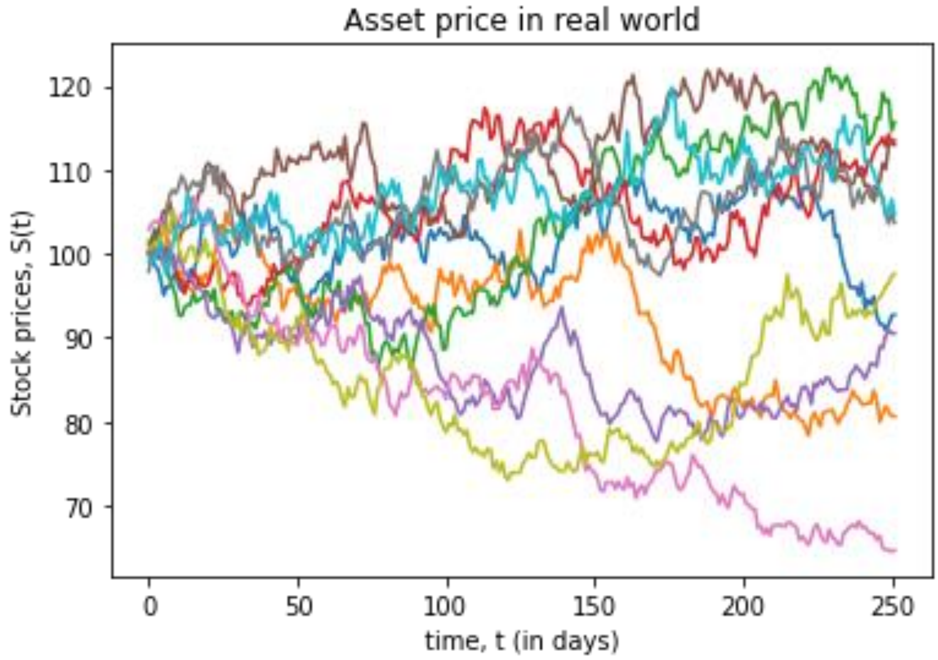
1 QUESTION - 1:

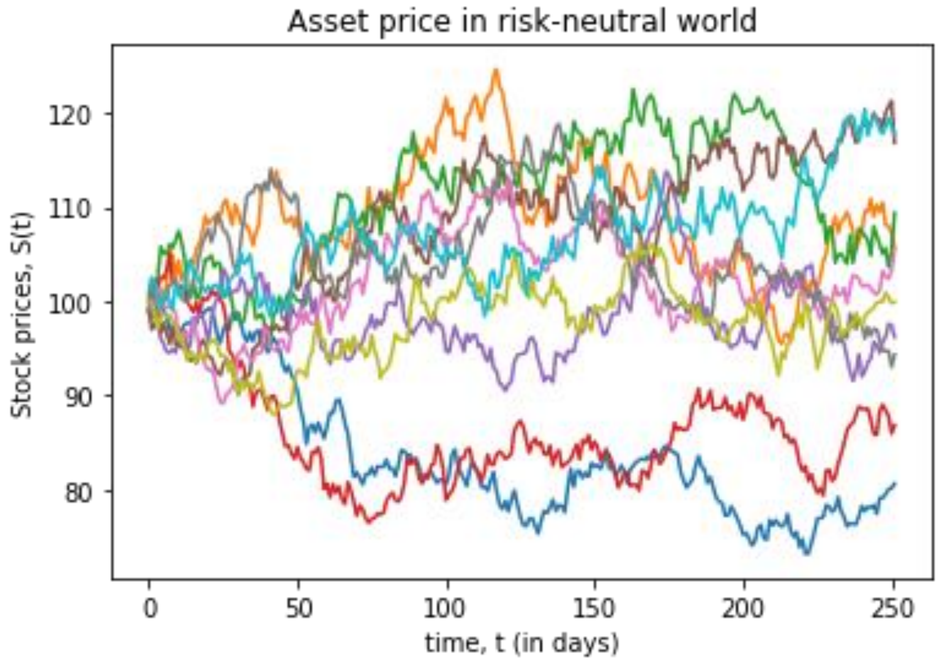
i. 10 different paths of the asset price making use of GBM in real world is:



The evolution of the asset price in real world is governed by following differential equation:

$$dS = \mu S dt + \sigma S dW(t)$$

ii. 10 different paths of the asset price making use of GBM in risk-neutral world is:



The evolution of the asset price in the risk-neutral world is governed by following differential equation:

$$dS = rSdt + \sigma SdW^*(t)$$

where,

W* is a Brownian motion under risk-neutral probability

The prices of a six month fixed-strike Asian option with various strike prices are:

i. **For K = 90**

Asian call option price	=	10.862624901856938
Variance in Asian call option price	=	58.39469992793498
Asian put option price	=	0.32468226004584866
Variance in Asian put option price	=	2.066430081106349

ii. **For K = 105**

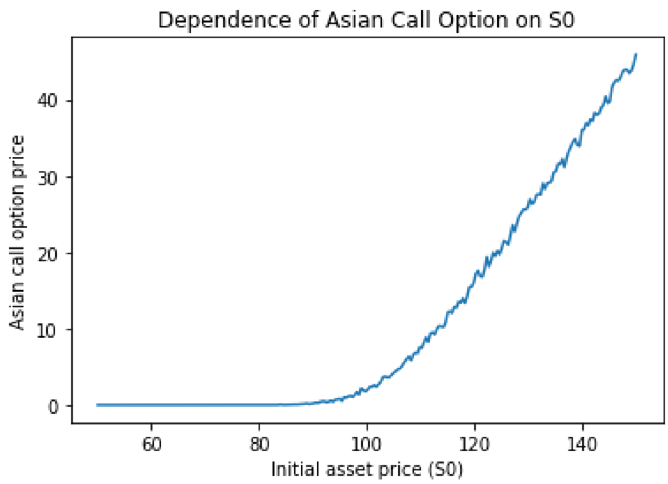
Asian call option price	=	1.5584630969412054
Variance in Asian call option price	=	12.677753955004992
Asian put option price	=	5.788439211068758
Variance in Asian put option price	=	34.399597488272555

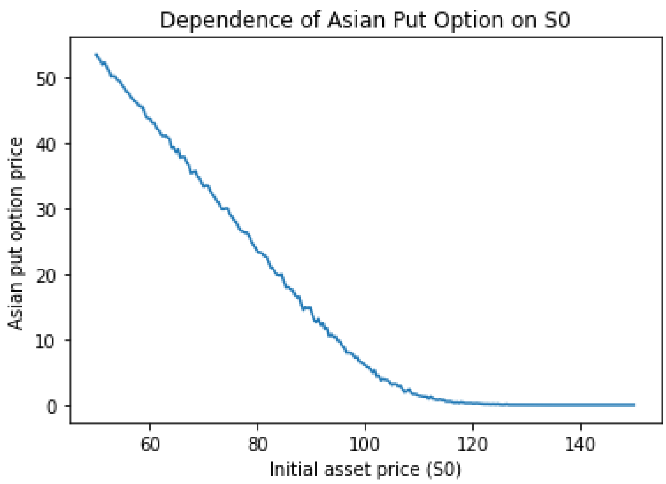
iii. **For K = 110**

Asian call option price	=	0.6309095063091923
Variance in Asian call option price	=	5.034227817317501
Asian put option price	=	9.658016088261482
Variance in Asian put option price	=	46.92782023133578

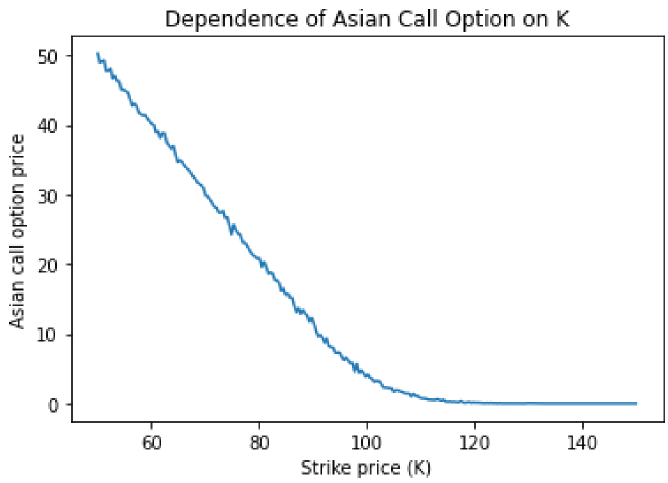
Sensitivity Analysis –

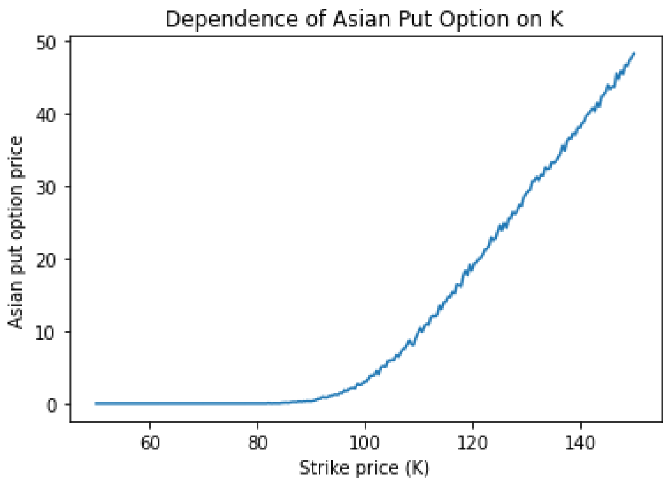
1. Variation of Option prices with S0:



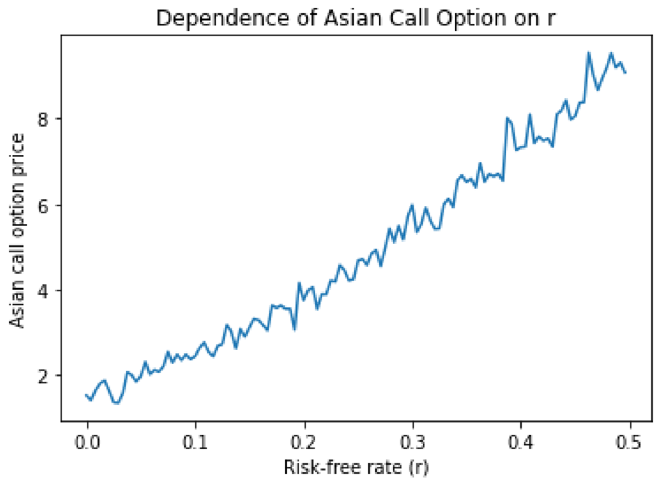


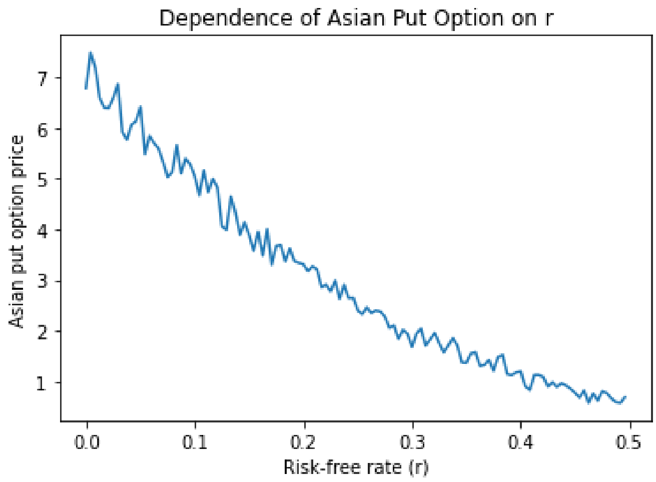
2. Variation of Option prices with K:



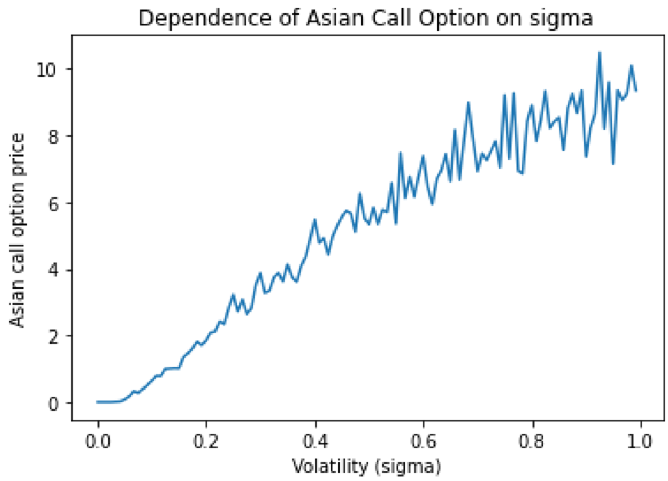


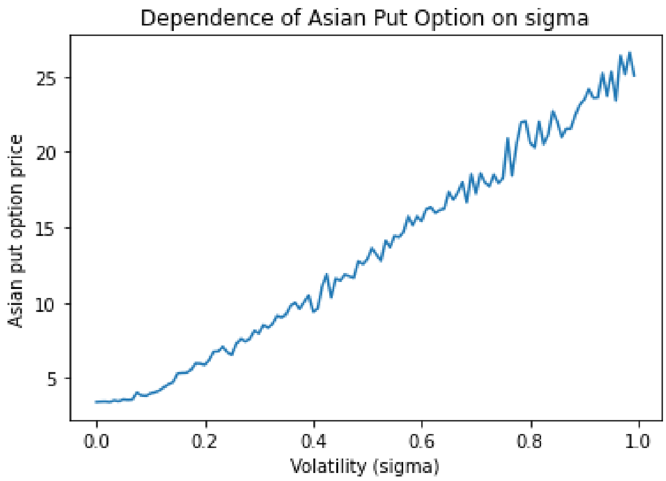
3. Variation of Option prices with r:





4. Variation of Option prices with σ:





Observations:

1. The price of the call option increases while that of the put option decreases, with an increase in the initial asset price, S0.
2. The price of the call option decreases while that of the put option increases, with an increase in the strike prices, K.
3. The price of the call option increases while that of the put option decreases, with an increase in the risk free interest, r
4. The price of both call and put option increases with an increase in the volatility.
5. There appears to be some fluctuations in the plots, which we try to minimise using the variance reduction schemes, in the next question.

2 QUESTION - 2:

The prices of a six month fixed-strike Asian option with various strike prices, after performing variance reduction are:

i. **For K = 90**

Asian call option price	=	10.777407638423961
Variance in Asian call option price	=	42.437611690317354
Asian put option price	=	0.24120825759450176
Variance in Asian put option price	=	0.9802493017216785

ii. **For K = 105**

Asian call option price	=	1.5839608139406547
Variance in Asian call option price	=	9.695739993819949
Asian put option price	=	5.733840752085572
Variance in Asian put option price	=	24.764831506907765

iii. **For K = 110**

Asian call option price	=	0.6194635242053073
Variance in Asian call option price	=	4.21611018568922
Asian put option price	=	9.347443148494298
Variance in Asian put option price	=	34.9947487975462

Observations –

The price of both call and put options obtained using both with and without variance reduction, are comparable. The respective variances are compared in the following table:

i. For Call Option:

SI No.	Strike Price (K)	Variance (without reduction)	Variance (with reduction)
1.	95	58.39469992793498	42.437611690317354
2.	105	12.677753955004992	9.695739993819949
3.	110	5.034227817317501	4.21611018568922

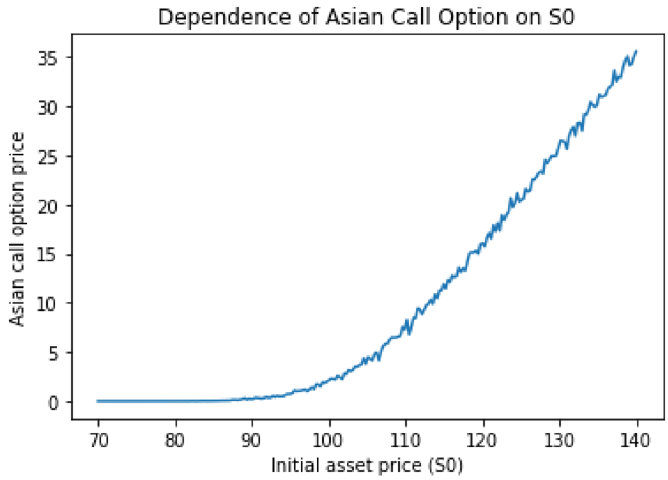
ii. For Put Option:

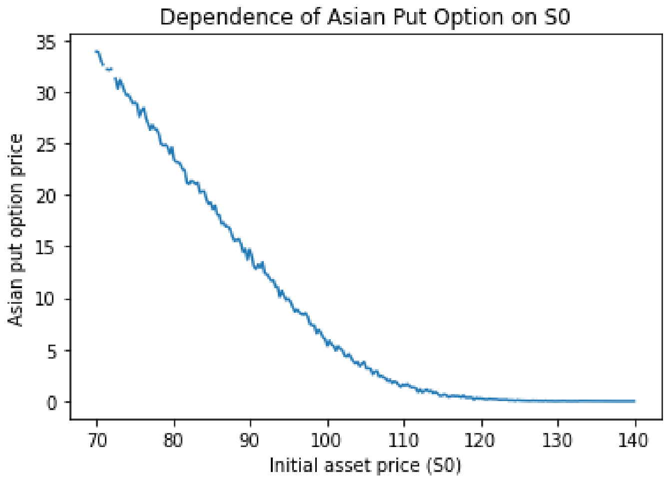
SI No.	Strike Price (K)	Variance (without reduction)	Variance (with reduction)
1.	95	2.066430081106349	0.9802493017216785
2.	105	34.399597488272555	24.764831506907765
3.	110	46.92782023133578	34.9947487975462

So, we can clearly observe that the variance reduction is successful, and we have reduced the variance in calculating the option prices.

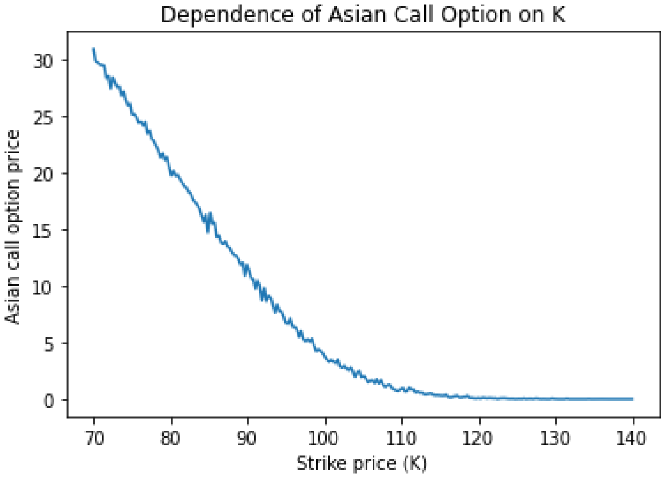
Sensitivity Analysis after performing Variance Reduction –

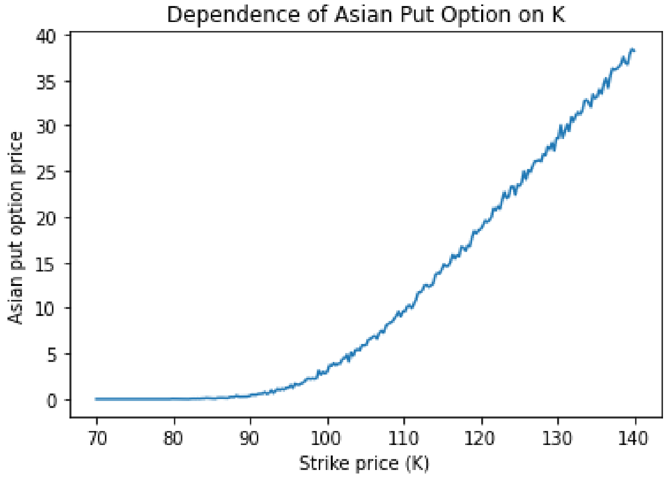
1. Variation of Option prices with S0:



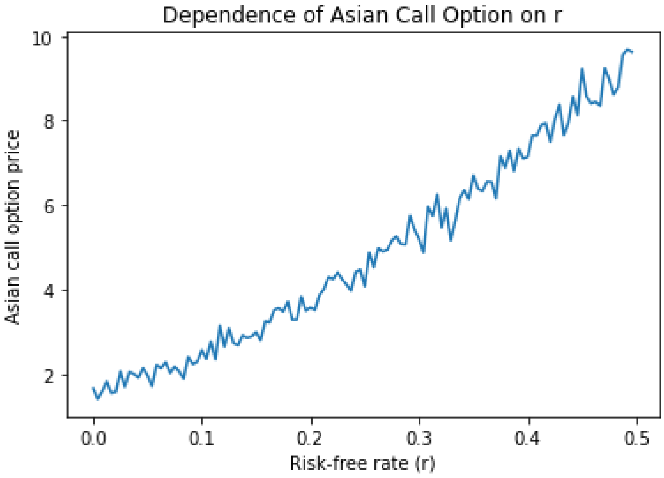


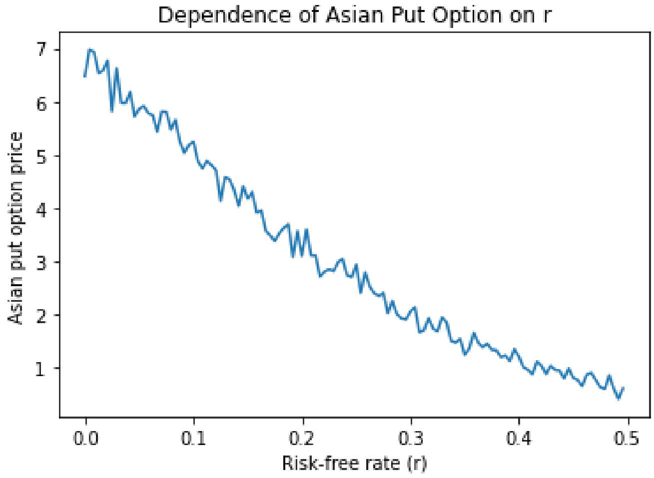
2. Variation of Option prices with K:



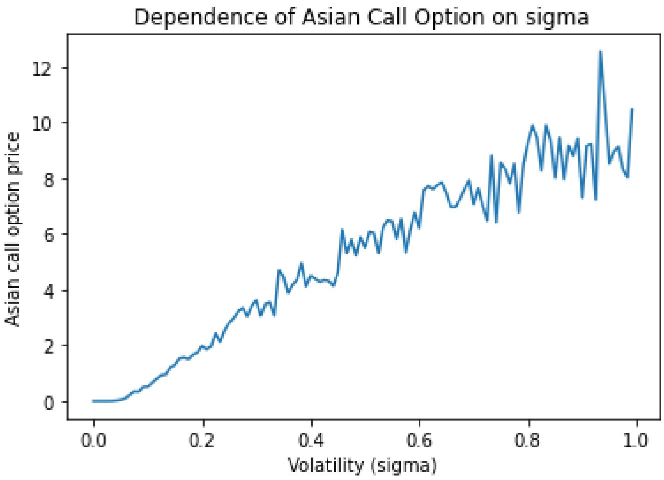


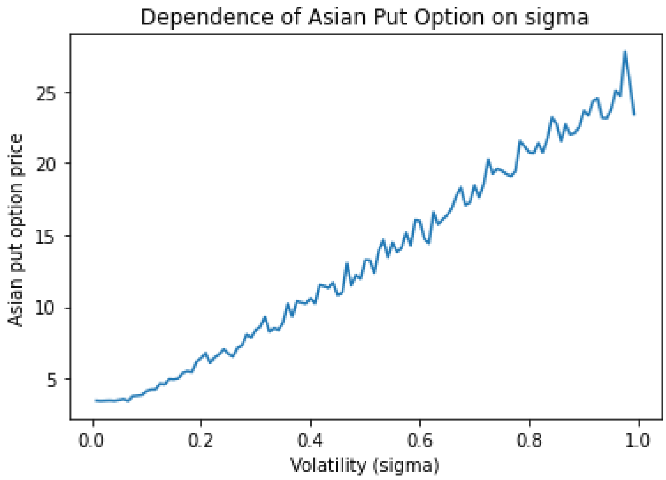
3. Variation of Option prices with r:





4. Variation of Option prices with σ:





Variance Reduction Scheme:

The **method of Control Variates** is used as the variance reduction technique. This method exploits the information about the errors in estimation of known quantities to reduce the error in the estimation of the unknown quantity.
The price of a standard European Put Option, with the payoff of max[(K – S(T), 0)], and a standard European Call Option, with the payoff of max[(S(T) – K, 0)], is taken as the **Control Variable** respectively.

Let Y_1, Y_2, … Y_n be the output from the n replications of the simulations, and X_1, X_2, … X_n be the corresponding output using the control variable.

For any fixed b, we calculate following:

$$Y_i(b) = Y_i - b(X_i - E(X))$$

We can show that the estimator obtained in this manner, called **Control Variate Estimator**, is an unbiased estimator.

Now, we calculate using an optimal value of b, which minimizes the variance of our estimator. We calculate following term:

$$b_n = \frac{\sum_1^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_1^n (X_i - \bar{X})^2}$$

Using strong law of large numbers, we can conclude that b* is our required value, where

$b_n \to$ b*   , with probability 1

Hence using this optimal value of b, we can calculate the Control Variate Estimator, which achieves variance reduction. The variance reduction ratio depends on the correlation between the quantity Y and the control X.

Observations:

1. Earlier, we have quantitatively demonstrated that the variance reduction is achieved. This claim is even more supported by the constructed plots.
2. On careful analysis, the fluctuations in the plots seem to be less than the case when variance reduction was not applied. So, the scheme achieves its goal.
3. The nature of the plots is consistent with our expectations, which is explained in the last question.