MA 374 – Financial Engineering Lab

Lab – 7

Name - Vishisht Priyadarshi Roll No - 180123053

1 Question - 1:

The price of European Call and Put Option given by BSM framework obtained after solving Black-Scholes-Merton PDE is:

$$C(x,t) = xN(d_1) - Ke^{-r(T-t)}N(d_2)$$
$$P(x,t) = Ke^{-r(T-t)}N(-d_2) - xN(-d_1)$$

where,

$$d_1 = \frac{\log\left(\frac{x}{K}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\log\left(\frac{x}{K}\right) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

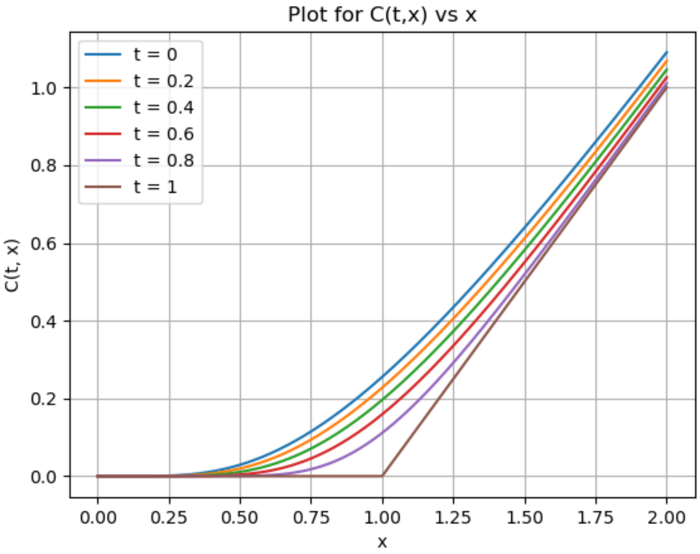
$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}y^2}\,dy$$

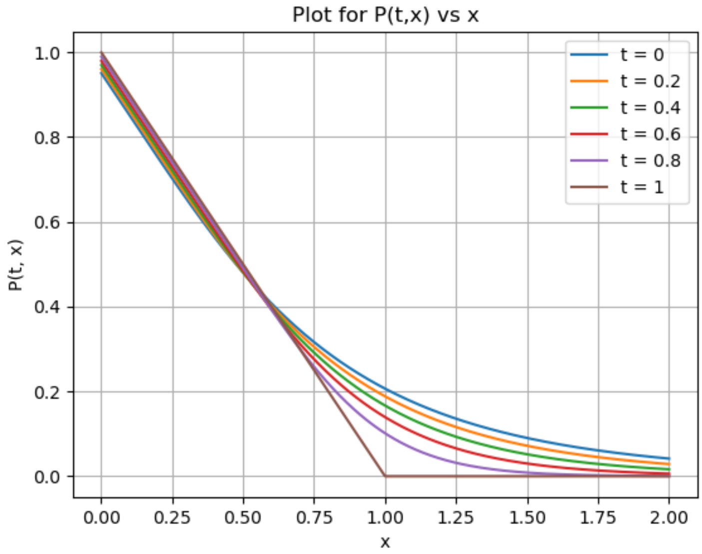
The program for computing the price of European call and put options at time t in the classical BSM framework is as follows:

```python
import numpy as np
import math
import matplotlib.pyplot as plt
from scipy.stats import norm


def BSM_model(x, t, T, K, r, sigma):
  d1 = ( math.log(x/K) + (r + 0.5 * sigma * sigma) * (T - t) ) / ( sigma * math.sqrt(T - t) )
  d2 = ( math.log(x/K) + (r - 0.5 * sigma * sigma) * (T - t) ) / ( sigma * math.sqrt(T - t) )


  call_price = x * norm.cdf(d1) - K * math.exp( -r * (T - t) ) * norm.cdf(d2)
  put_price = K * math.exp( -r * (T - t) ) * norm.cdf(-d2) - x * norm.cdf(-d1)

  return call_price, put_price


def main():
  C, P = BSM_model(10, 0.2, 1, 1, 0.05, 0.6)
  print("Call Price =", C)
  print("Put Price =", P)


if __name__=="__main__":
  main()
```

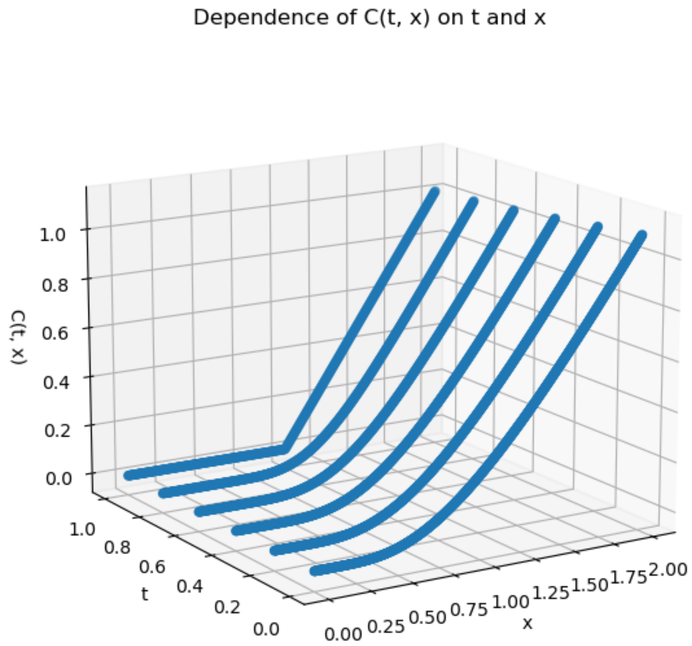
2 QUESTION - 2:

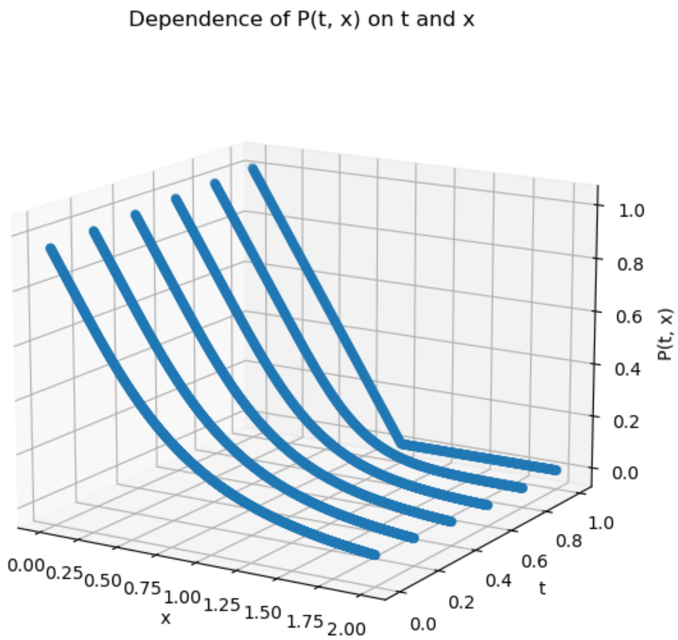
The plot of C(t, x) and P(t, x) as a function of x is as follows:





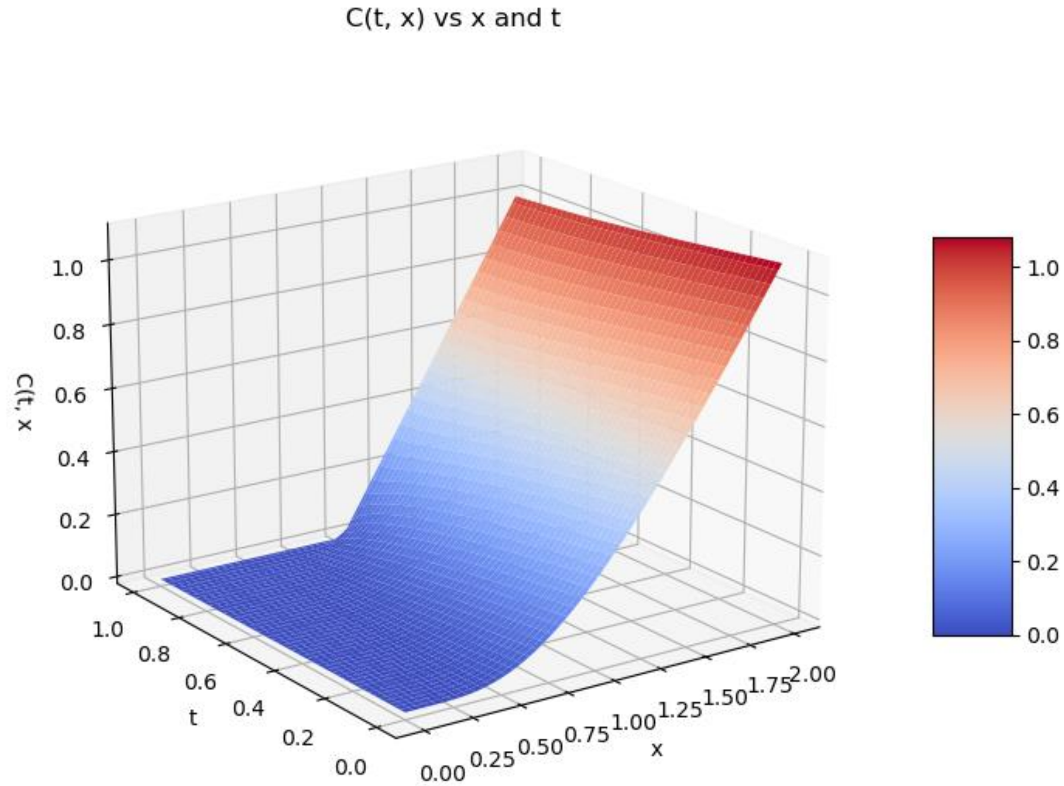
The 3-dimensional plots for C(t, x) & P(t, x) are:

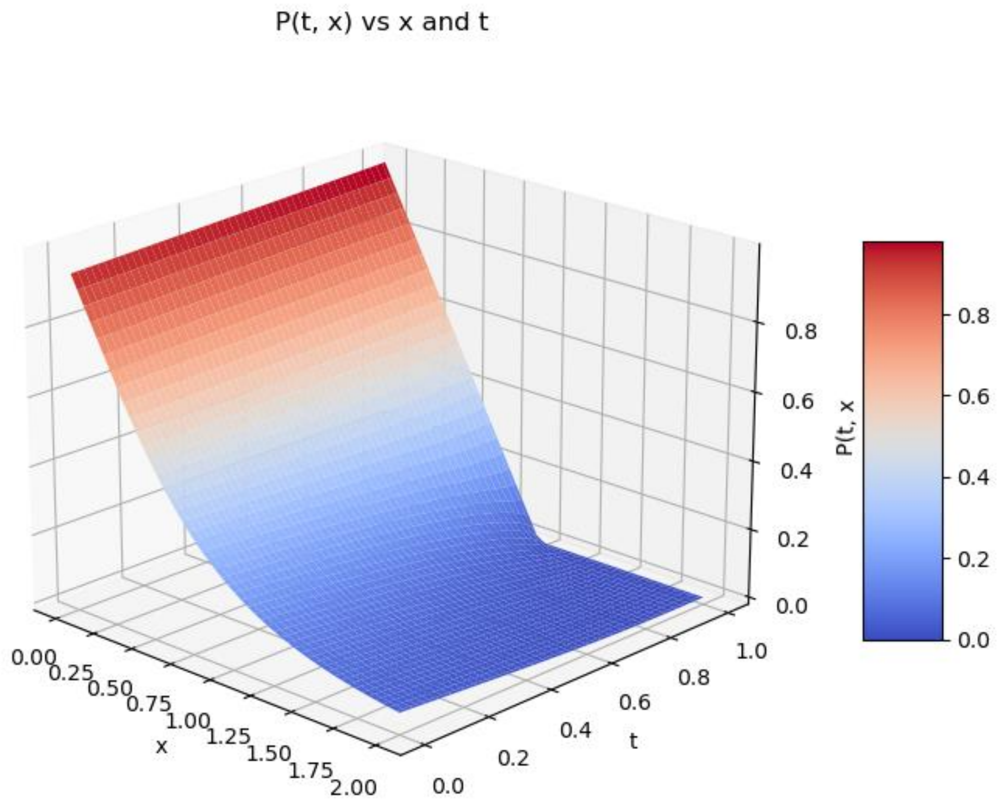




3 QUESTION – 3 :

The plots for C(t, x) and P(t, x) as smooth surfaces above the (t, x) plane are:





4 QUESTION – 4: SENSITIVITY ANALYSIS

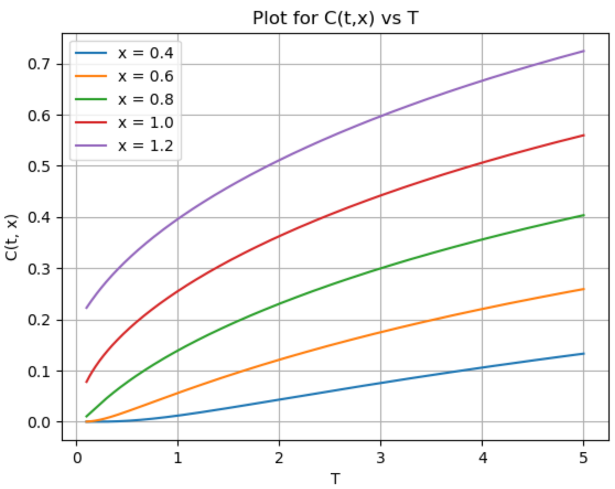
- The parameters values are varied accordingly, and where some particular values of parameters are required, they are taken from the following:

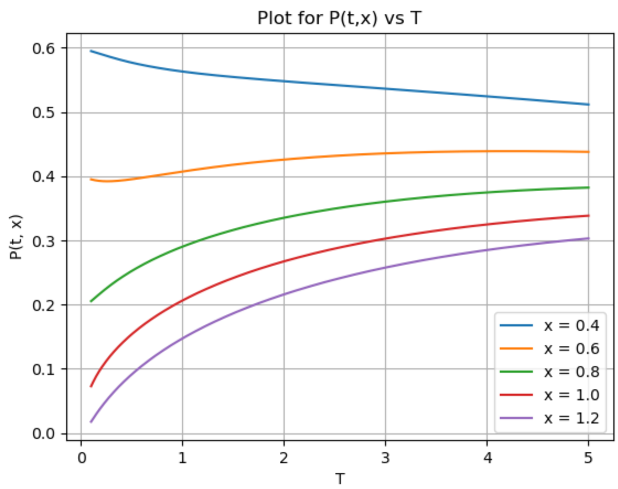
$$x = 0.8, \quad t = 0, \;\; T = 1, \;\; K = 1, \;\; r = 0.05, \;\; \sigma = 0.6$$

i. Variation of C and P with stock price x:

The plots for this case was done in Q2 and Q3.

ii. Variation of C and P with expiration time T:

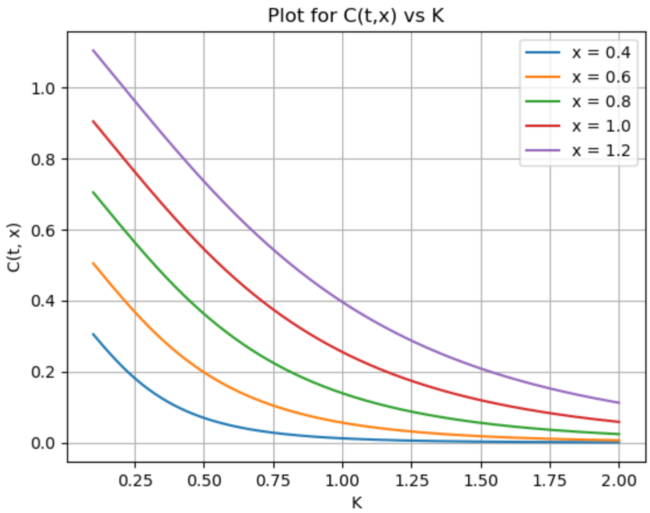


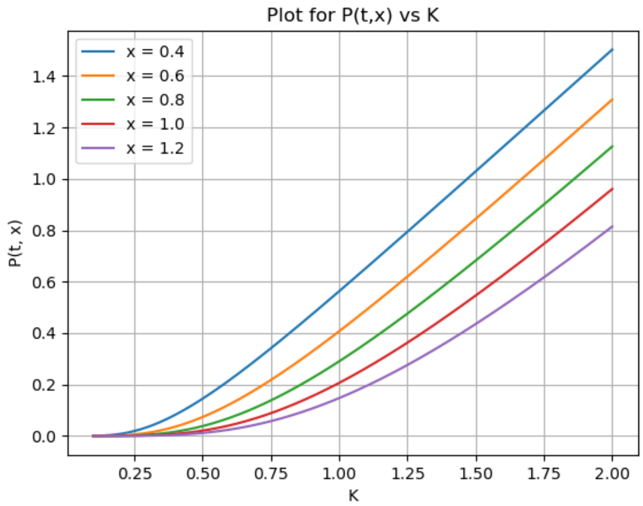


Some of the values are: (with parameters as x = 0.8, t = 0, K = 1, r = 0.05 and σ = 0.6)

Sl No.	T	C(t, x)	P(t, x)
1.	0.1	0.0104876	0.2055
2.	0.590982	0.0897777	0.260661
3.	1.08196	0.147911	0.29525
4.	1.57295	0.195092	0.319458
5.	2.06393	0.235421	0.33737
6.	2.55491	0.270874	0.350952
7.	3.04589	0.302594	0.361329
8.	3.53687	0.331314	0.369225
9.	4.02786	0.357547	0.375138
10.	4.51884	0.381665	0.379429

iii. Variation of C and P with strike price K:

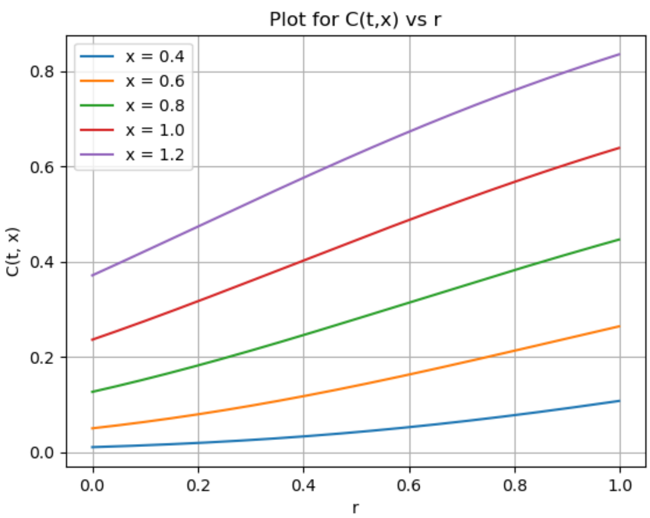


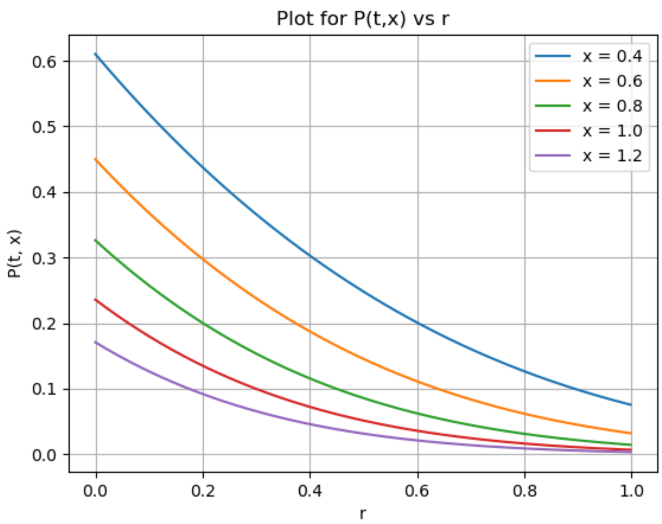


Some of the values are: (with parameters as $x = 0.8$, $t = 0$, $T = 1$, $r = 0.05$ and $\sigma = 0.6$)

Sl No.	K	C(t, x)	P(t, x)
1.	0.1	0.704885	7.64124e-06
2.	0.290381	0.527951	0.00417013
3.	0.480762	0.376097	0.0334112
4.	0.671142	0.261841	0.100251
5.	0.861523	0.181355	0.200861
6.	1.0519	0.126098	0.3267
7.	1.24228	0.0883989	0.470096
8.	1.43267	0.0625982	0.625392
9.	1.62305	0.0448062	0.788695
10.	1.81343	0.0324182	0.957403

iv. Variation of C and P with rate of interest r:

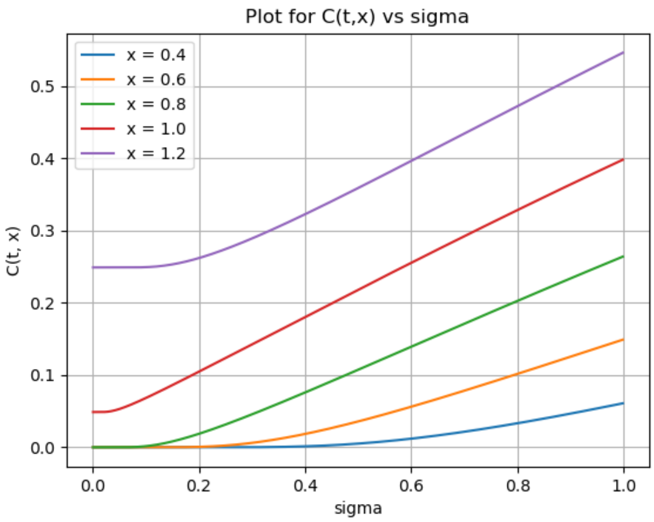


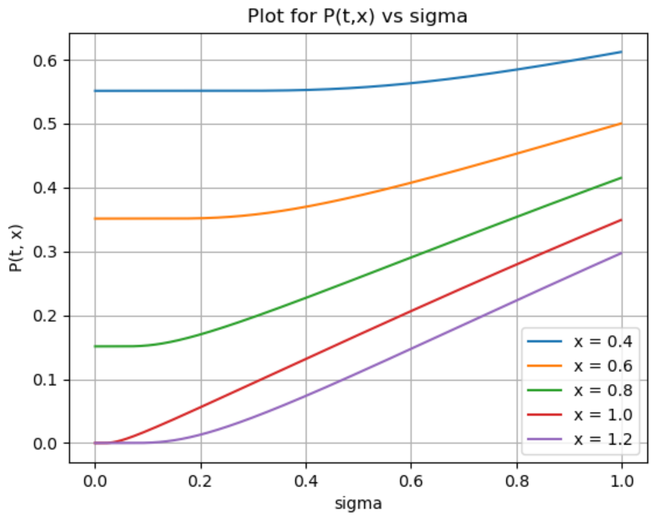


Some of the values are: (with parameters as $x = 0.8$, $t = 0$, $T = 1$, $K = 1$ and $\sigma = 0.6$)

Sl No.	r	C(t, x)	P(t, x)
1.	0	0.126249	0.326249
2.	0.1	0.152689	0.257526
3.	0.2	0.181639	0.20037
4.	0.3	0.212714	0.153533
5.	0.4	0.24544	0.11576
6.	0.5	0.279282	0.0858129
7.	0.6	0.313685	0.0624963
8.	0.7	0.348098	0.0446838
9.	0.8	0.382015	0.0313436
10.	0.9	0.414987	0.0215565

v. Variation of C and P with Volatility σ:

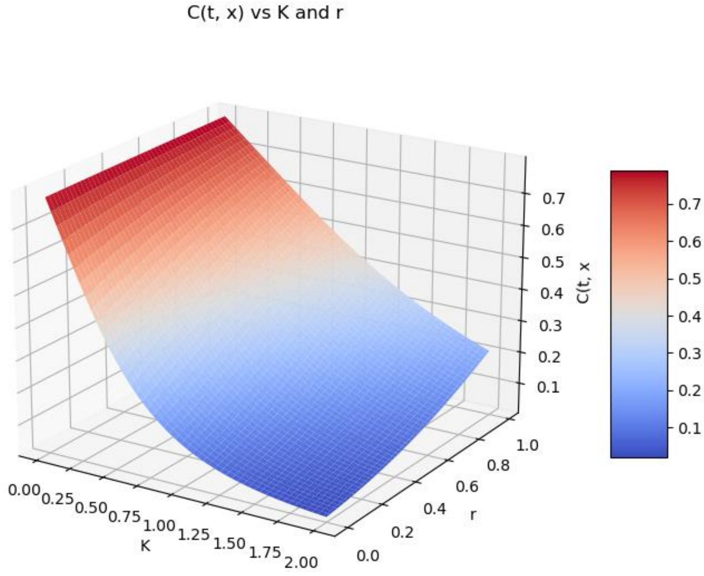


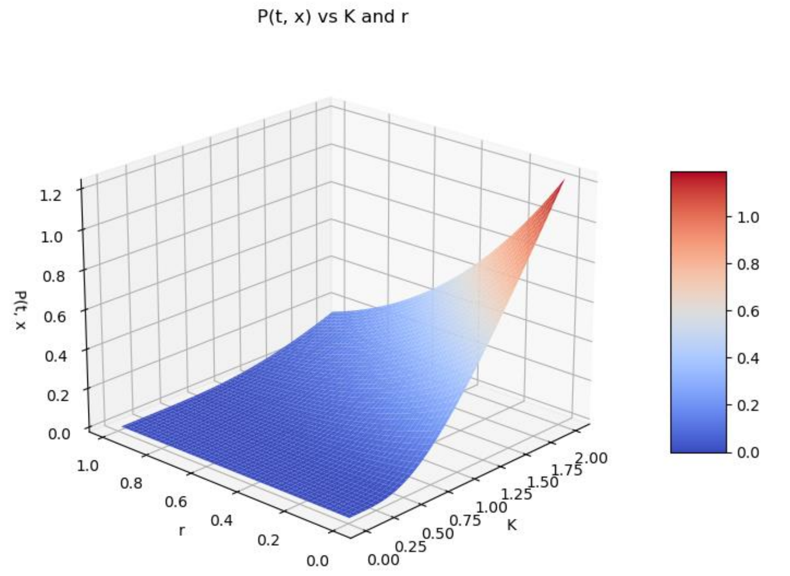


Some of the values are: (with parameters as x = 0.8, t = 0, T = 1, K = 1 and r = 0.05)

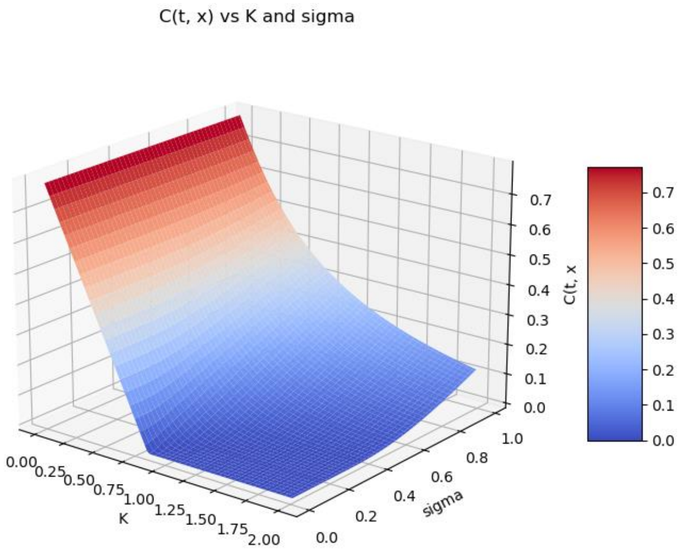
Sl No.	σ	C(t, x)	P(t, x)
1.	0.001	0	0.151229
2.	0.1009	0.00154652	0.152776
3.	0.2008	0.0187849	0.170014
4.	0.3007	0.0457362	0.196966
5.	0.4006	0.0759687	0.227198
6.	0.5005	0.10742	0.258649
7.	0.6004	0.139262	0.290492
8.	0.7003	0.171081	0.32231
9.	0.8002	0.202627	0.353856
10.	0.9001	0.233733	0.384963

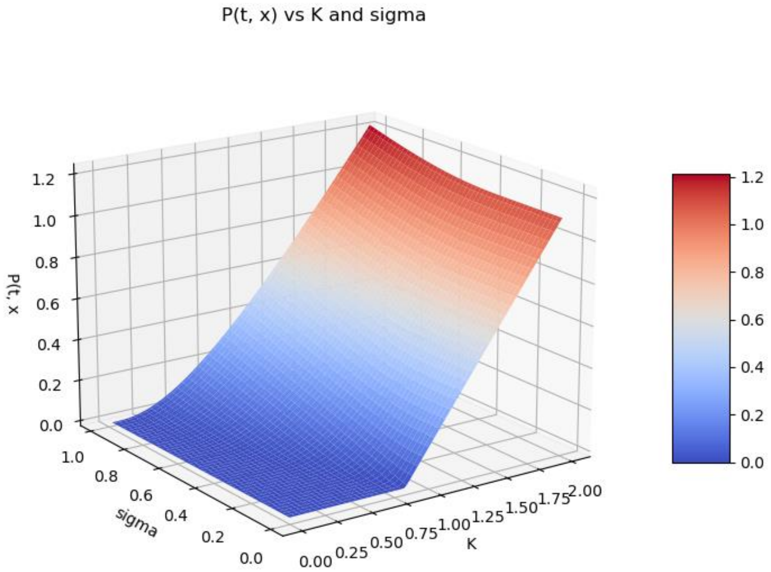
vi. Variation of C and P with K & r:



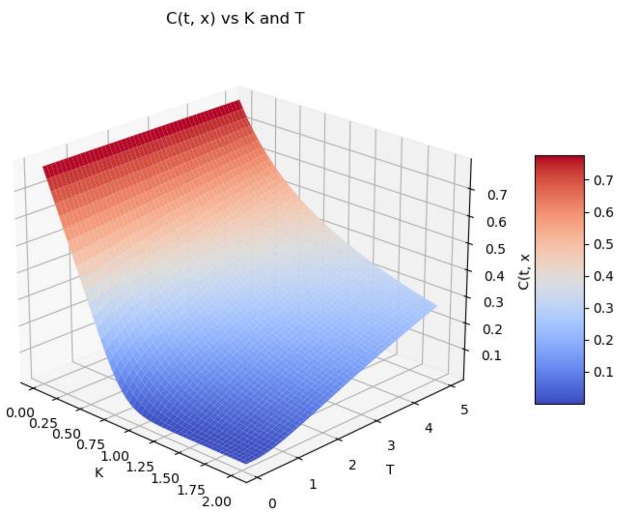


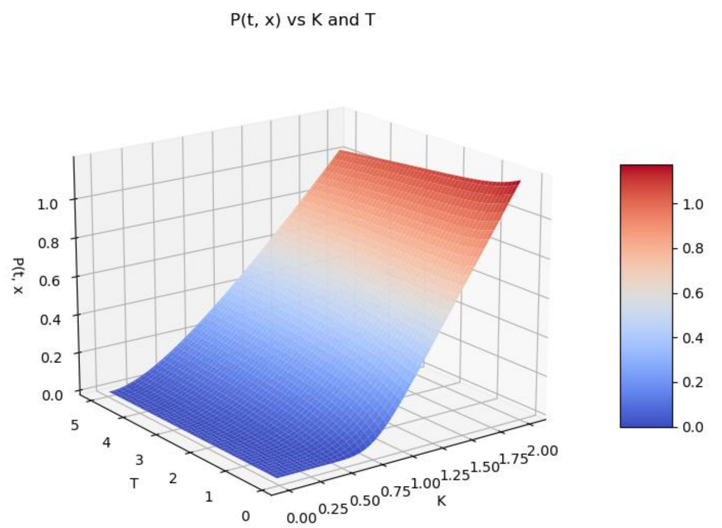
vii. Variation of C and P with K & σ:



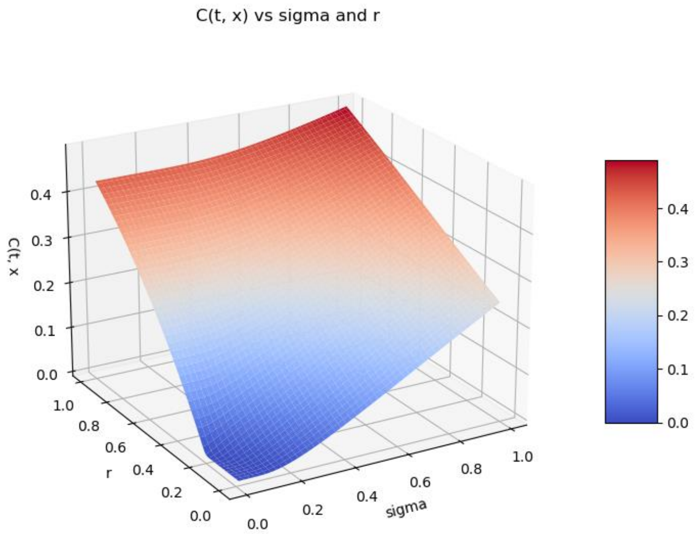


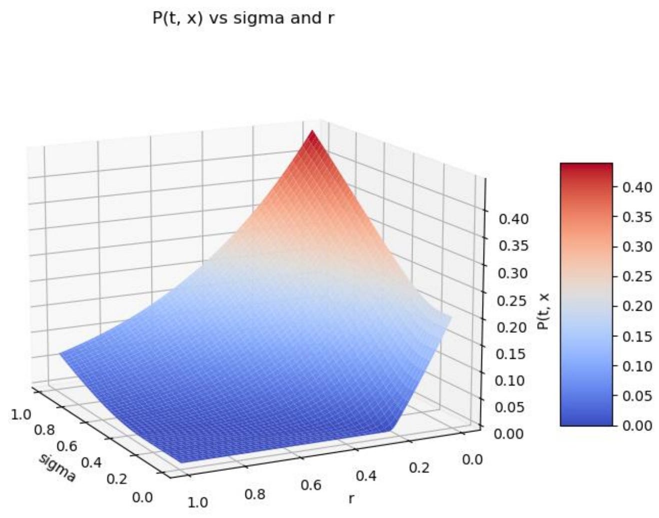
viii. Variation of C and P with K & T:



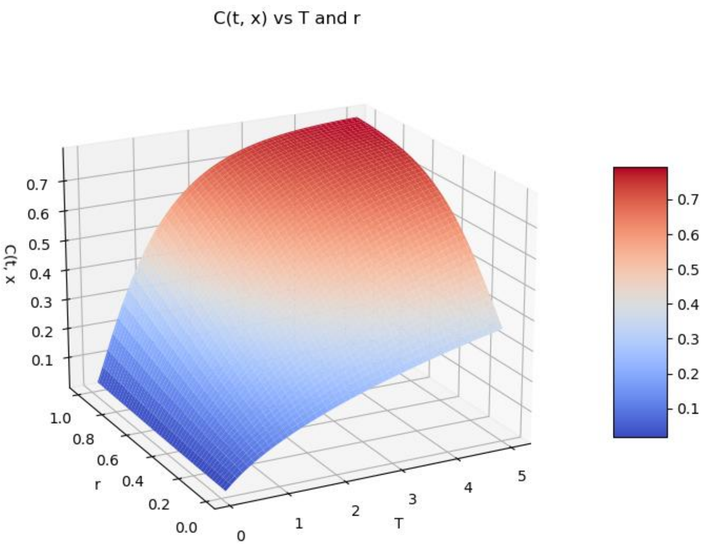


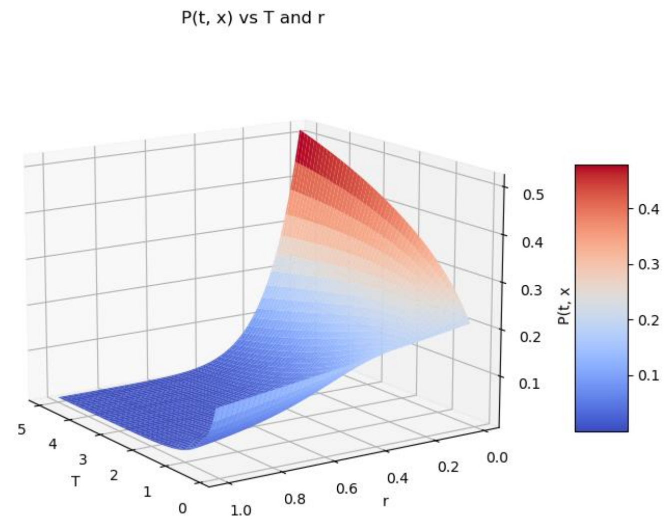
ix. Variation of C and P with r & σ:



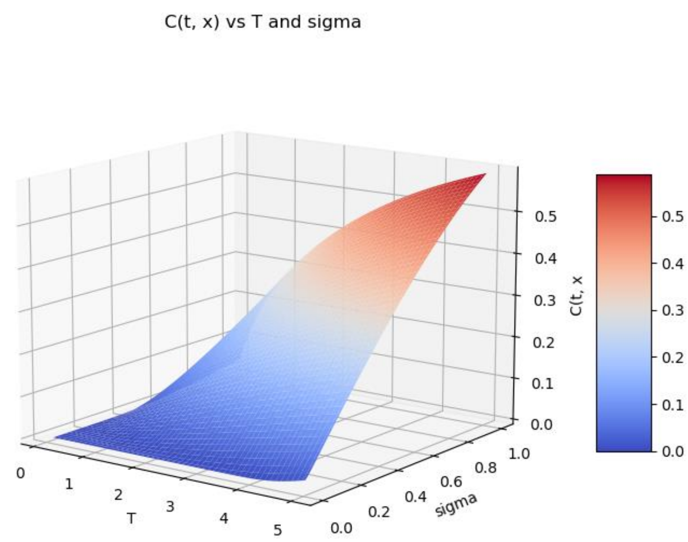


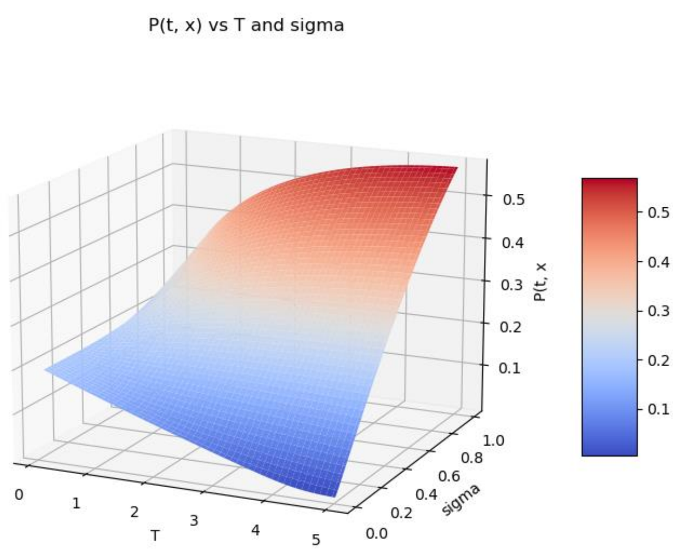
x. Variation of C and P with T & r:



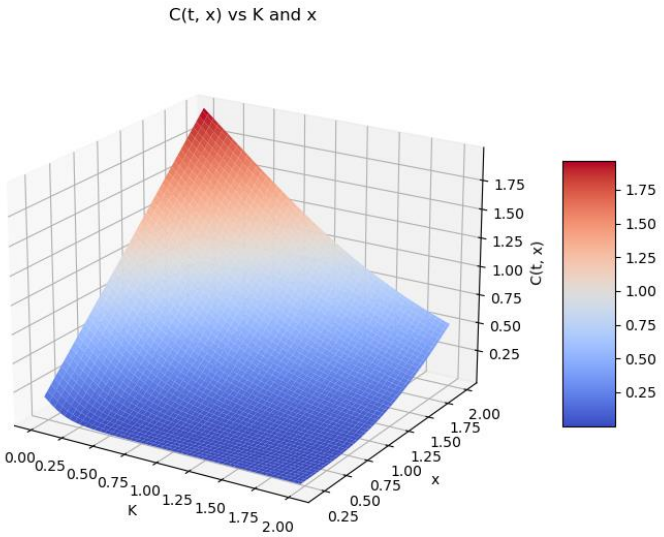


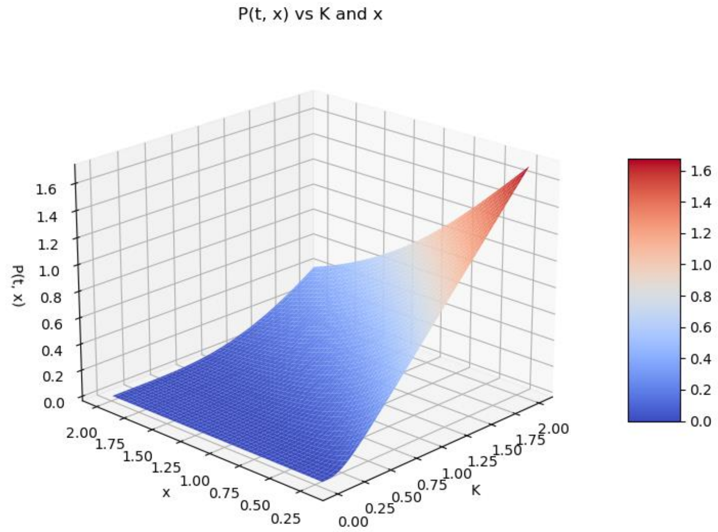
xi. Variation of C and P with T & σ:



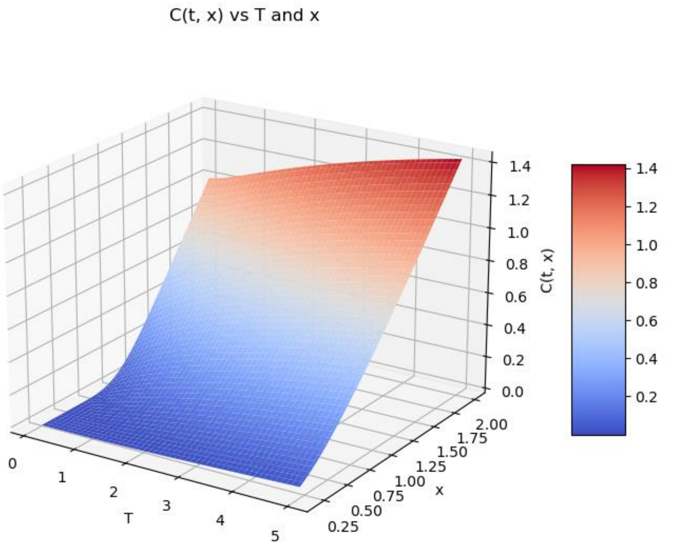


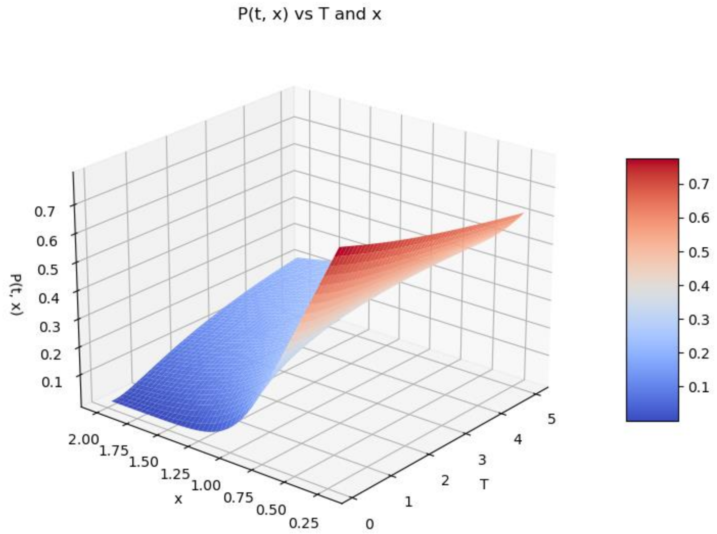
xii. Variation of C and P with K & x:



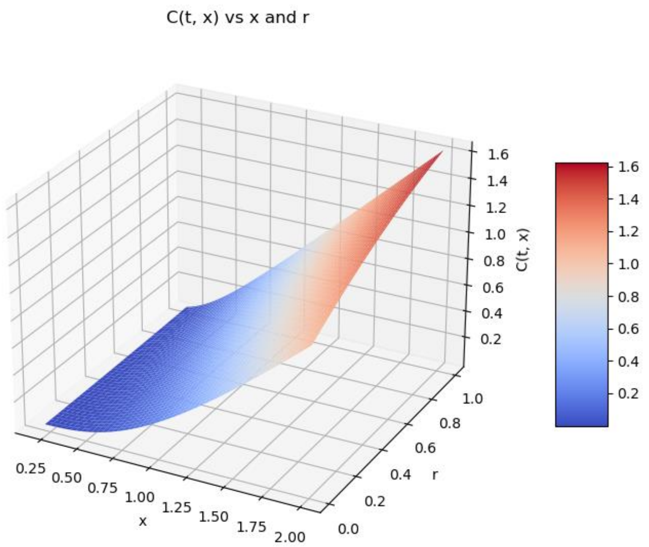


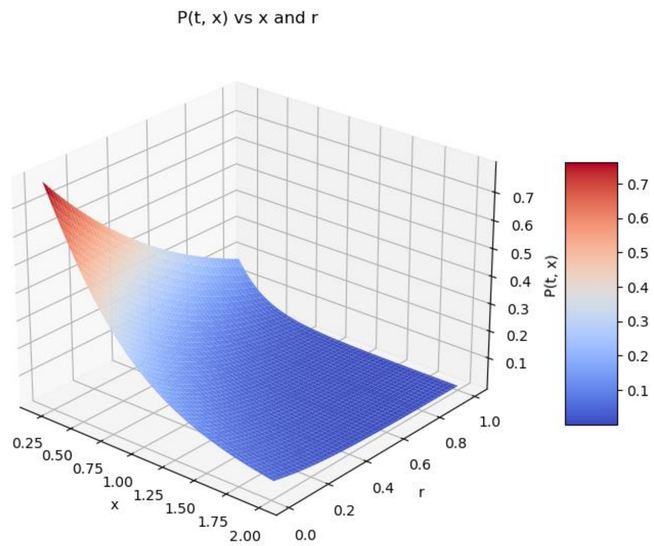
xiii. Variation of C and P with T & x:





xiv. Variation of C and P with x & r:





xv. Variation of C and P with x & σ: