MA 323 - Monte Carlo Simulation Assignment - 2

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1 QUESTION - 1:

a) The Linear Congruence Generator used to generate first 17 values of U_i is of the form:

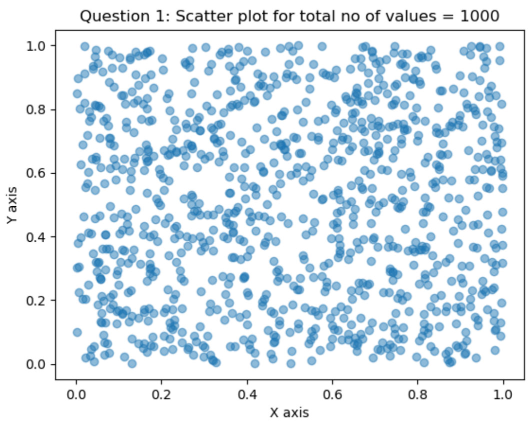
$$x_{i+1} = (ax_i + b) \bmod m$$
$$u_{i+1} = x_{i+1}/m$$

with a = 1229, b = 1, m = 2048.

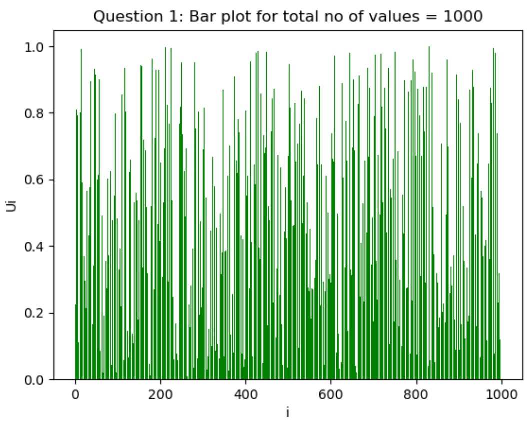
c)

- **Plots for 1000 values:**

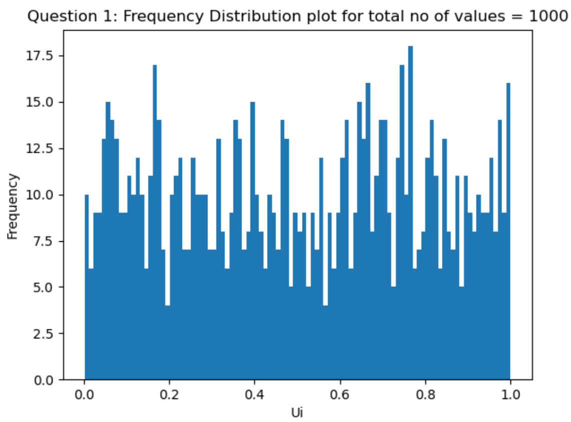
i) Scatter Plot (U_i, U_{i+1})



ii) Bar Plot (U_i vs i)

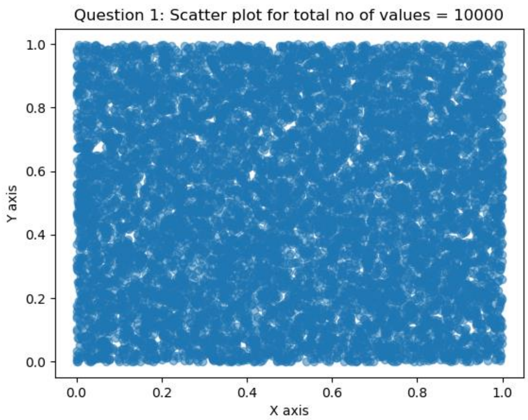


iii) Frequency Distribution of U_i

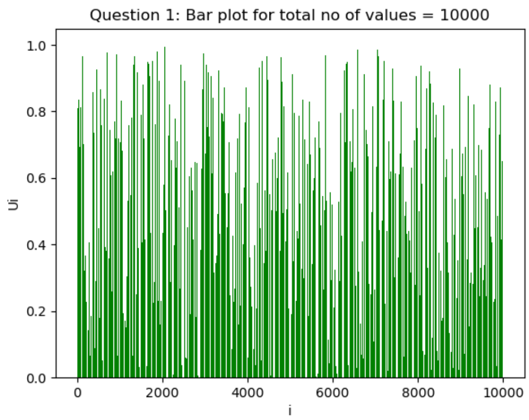


- **Plots for 10000 values:**

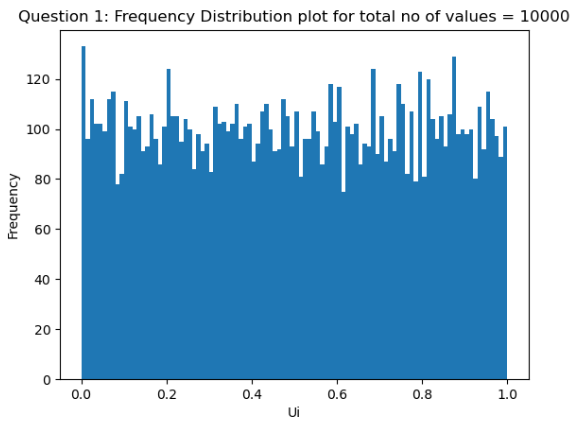
i) **Scatter Plot** (U_i, U_{i+1})



ii) **Bar Plot** (U_i vs i)

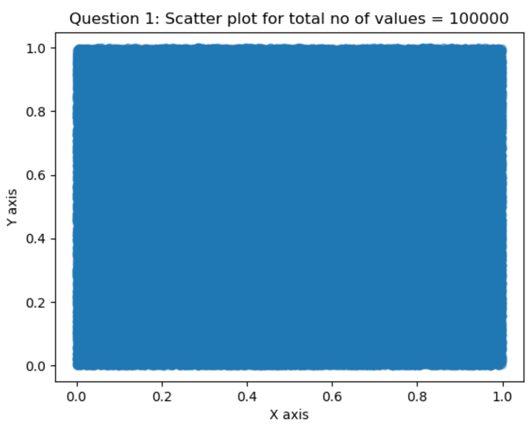


iii) **Frequency Distribution of U_i**

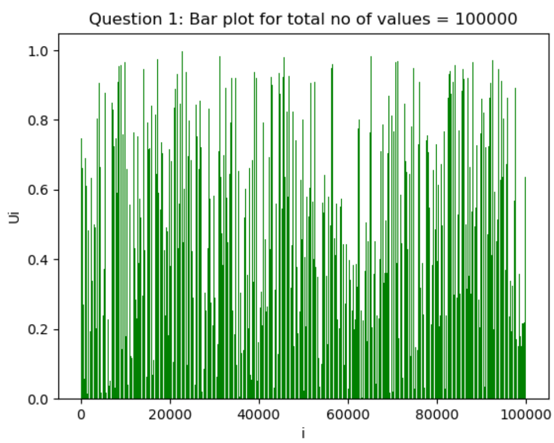


- **Plots for 100000 values:**

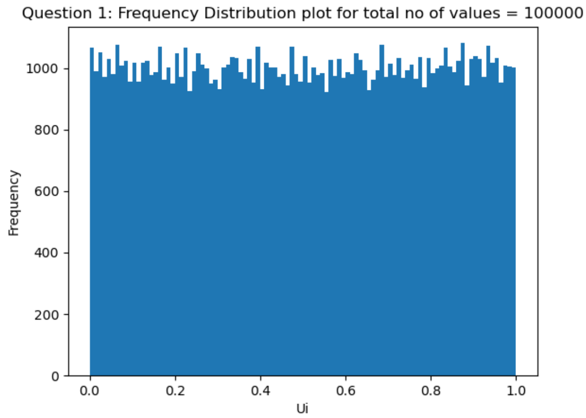
i) **Scatter Plot** (U_i, U_{i+1})



ii) **Bar Plot** (U_i vs i)



iii) Frequency Distribution of U_i



- **Observations:**
  1. The scatter plot suggests that the U_i 's do not follow any particular pattern, so, they are almost completely random.
  2. The frequency distribution plots suggest that the random generator follows the 2 properties of the ideal random generator:
     a) Each U_i is uniformly distributed between 0 and 1.
     b) The U_i are mutually independent.

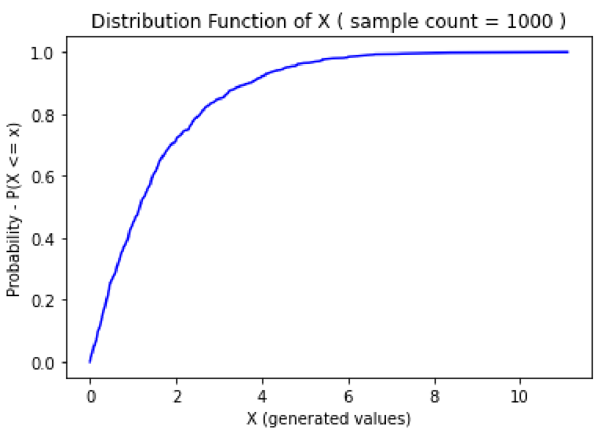
The frequency of different numbers lying in same length intervals are almost same. So, the given random generator behaves like a good random generator.

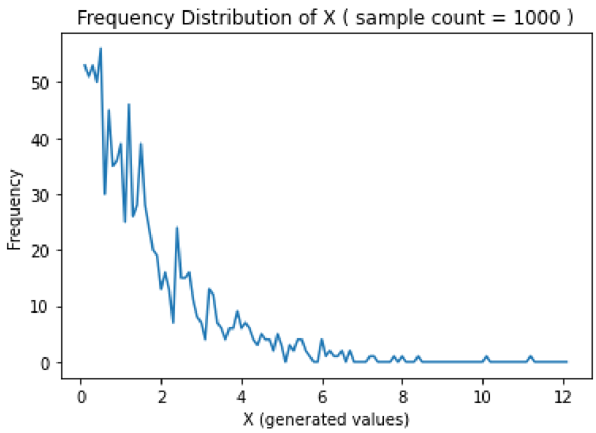
2 QUESTION - 2:

Mean (ɵ) = π / 2 *(assumed)*

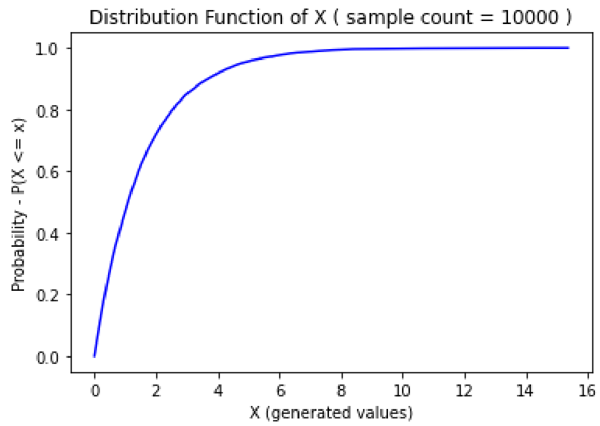
I have plotted the graphs for 5 different cases in which the number of values generated are 1000, 10000, 100000, 1000000, 10000000.

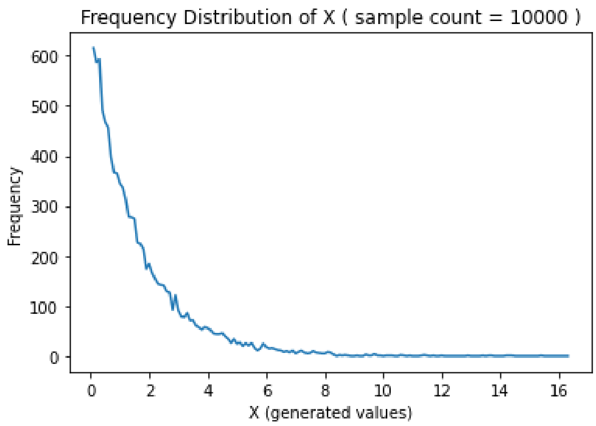
Case i): Total number of values generated = 1000



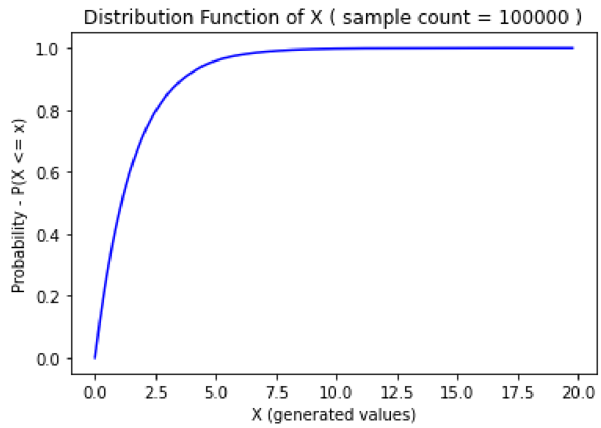


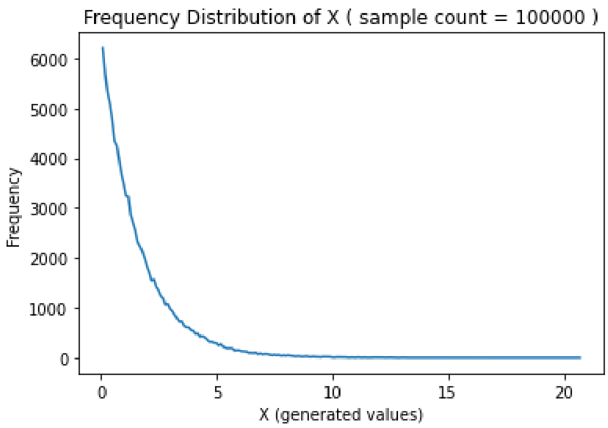
Case ii): Total number of values generated = 10000



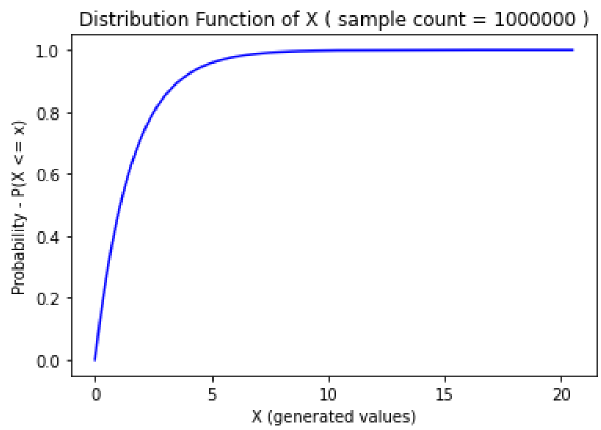


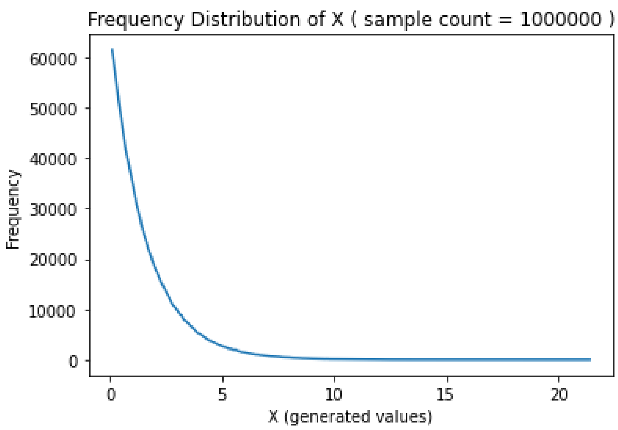
Case iii): Total number of values generated = 100000



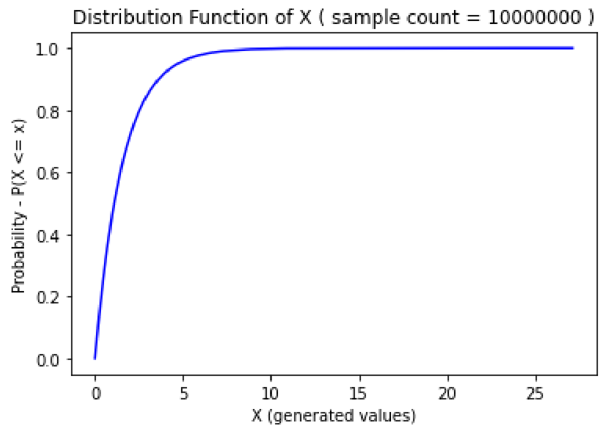


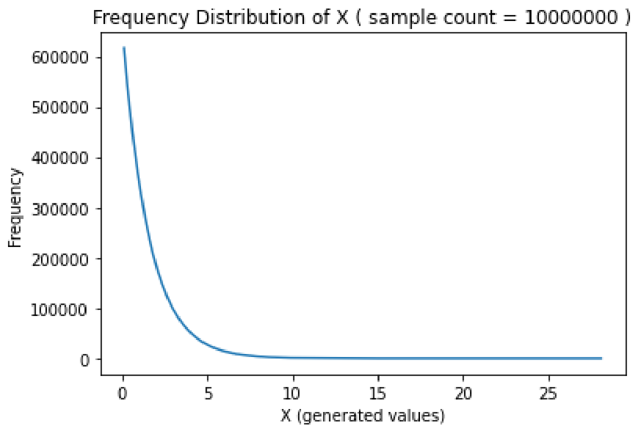
Case iv): Total number of values generated = 1000000



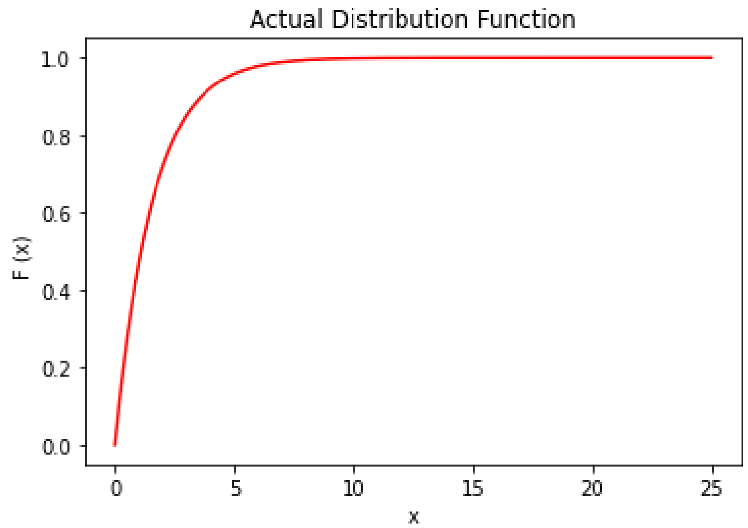


Case v): Total number of values generated = 10000000





Actual Distribution Function:



- Actual Mean = 1.5707963267948966

  Actual Variance = 2.4674011002723395 (here, Variance = ɵ^{2})

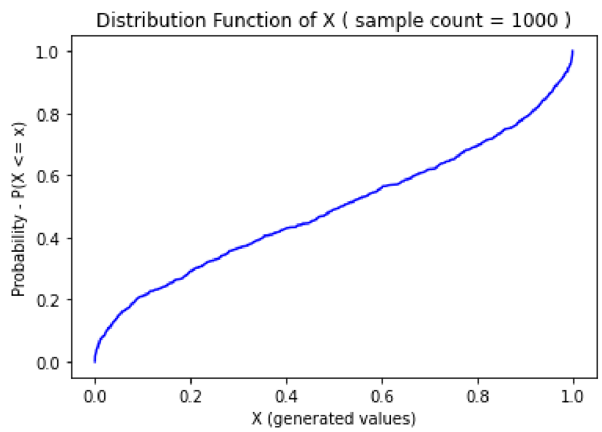
Case	Sample Count	Sample Mean	Sample Variance
i)	1000	1.527608770849963	2.3312738492502976
ii)	10000	1.5354517674732717	2.342475561946338
iii)	100000	1.5644662674427838	2.4328267137230175
iv)	1000000	1.5725458646556747	2.472518537123785
v)	10000000	1.570628987472085	2.465969544567569

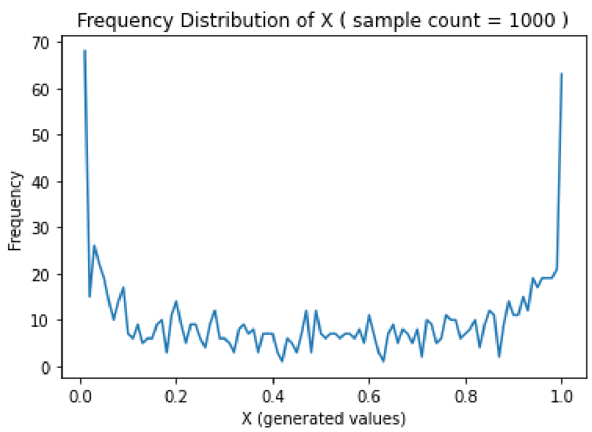
- **Observations:**
    1. As we increase the number of values generated (Sample Count), the mean and the variance of the generated values (X) converge to the actual mean and variance. It is also evident from the distribution function of the X for different values of sample count which approaches to the plot of F(x) as sample count increases. It follows from the Law of Large Numbers.
    2. The distribution function of X is identical to the cdf F(x) from which random variable X was generated. This is because F(x) is a continuous strictly increasing function and U is a uniform distribution function on [0, 1], so, $F^{-1}(U)$ will be a sample from F. This clearly shows the Inverse Transform Method and the following theorem given in Lecture – 2.

3 QUESTION – 3 :

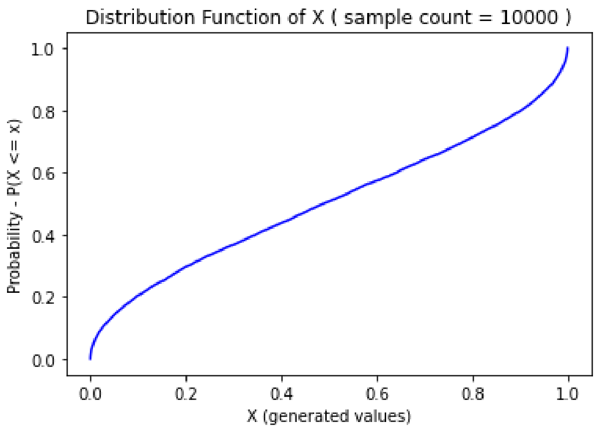
I have plotted the graphs for 5 different cases in which the number of values generated are 1000, 10000, 100000, 1000000, 10000000.

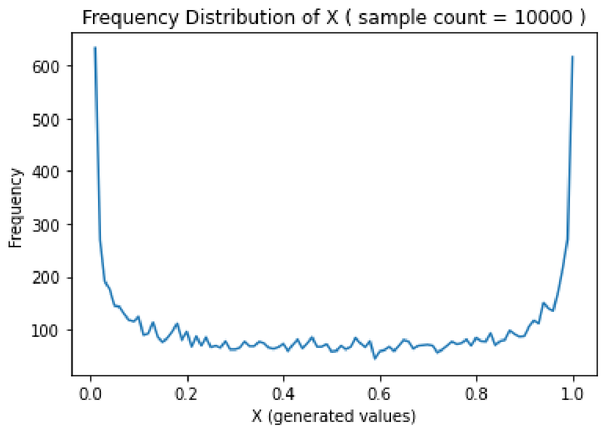
Case i): Total number of values generated = 1000



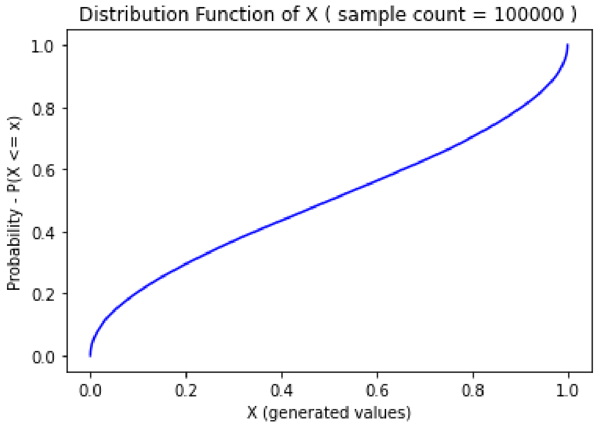


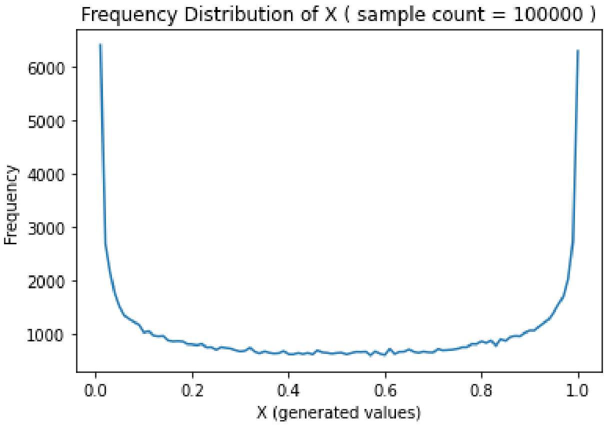
Case ii): Total number of values generated = 10000



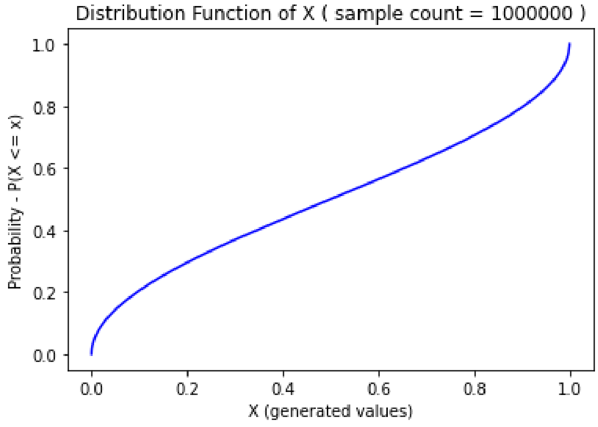


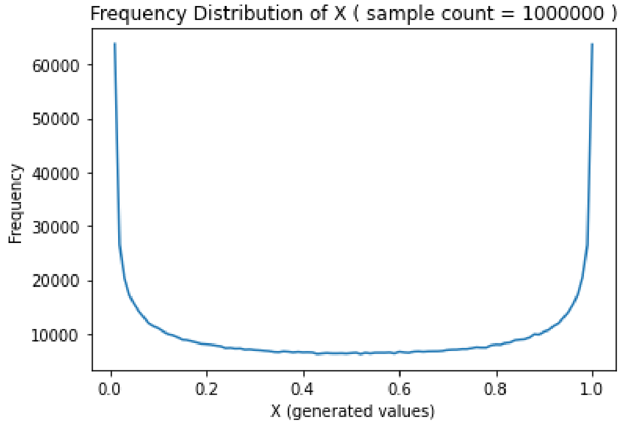
Case iii): Total number of values generated = 100000



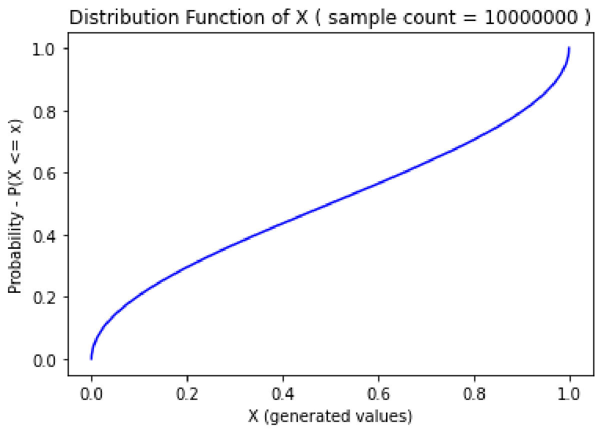


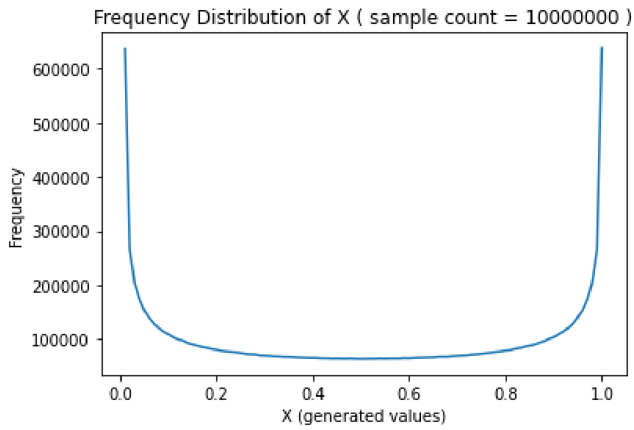
Case iv): Total number of values generated = 1000000



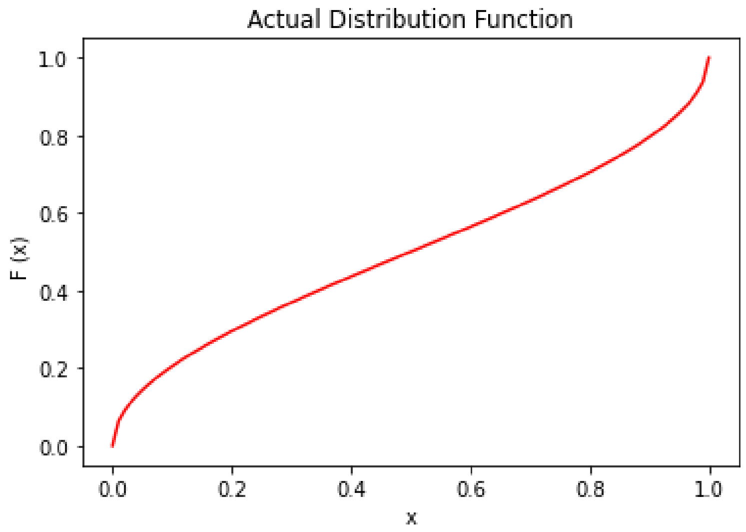


Case v): Total number of values generated = 10000000





Actual Distribution Function:



- **Sample Mean & Variance**

Case	Sample Count	Sample Mean	Sample Variance
i)	1000	0.48833044199671294	0.12727982689751888
ii)	10000	0.49285879887658973	0.12416196616477684
iii)	100000	0.49954063843320945	0.12488154453701807
iv)	1000000	0.5003726470843849	0.12494976854973477
v)	10000000	0.5000121854366314	0.12499836920736974

- **Observations:**
  1. The distribution function of X is identical to the cdf F(x) from which random variable X was generated. This is because F(x) is a continuous strictly increasing function and U is a uniform distribution function on [0, 1], so, $F^{-1}(U)$ will be a sample from F. This clearly shows the Inverse Transform Method and the following theorem given in Lecture – 2.
  2. The distribution functions of X approaches the plot of F(x) as the sample count increases.