MA 323 - Monte Carlo Simulation Assignment - 12

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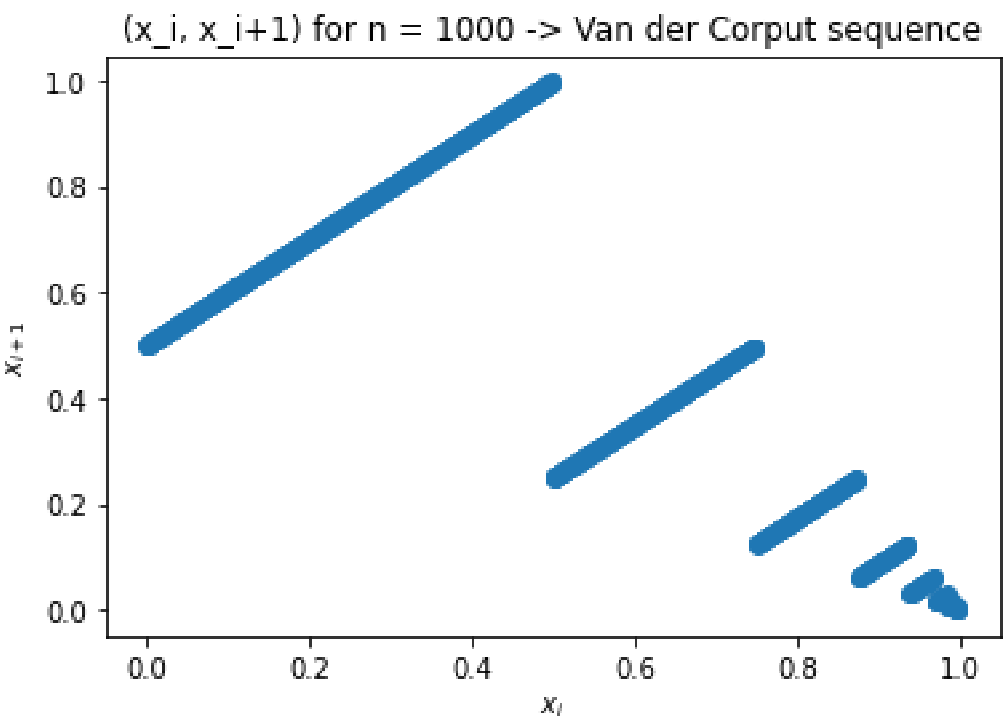
1 QUESTION - 1:

The first 25 values of the Van der Corput sequences using the radical function $x_i = \phi_2(i)$ are:

SI No.	Van der Corput Sequence	SI No	Van der Corput Sequence
1	0	**14**	0.6875
2	0.5	**15**	0.4375
3	0.25	**16**	0.9375
4	0.75	**17**	0.03125
5	0.125	**18**	0.53125
6	0.625	**19**	0.28125
7	0.375	**20**	0.78125
8	0.875	**21**	0.15625
9	0.0625	**22**	0.65625
10	0.5625	**23**	0.40625
11	0.3125	**24**	0.90625
12	0.8125	**25**	0.09375
13	0.1875		

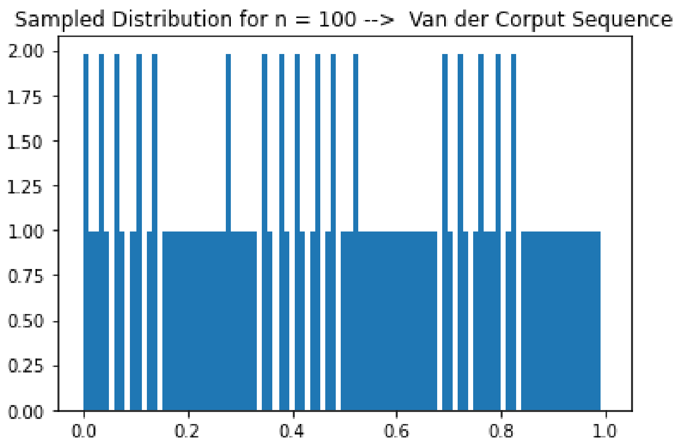
The plot for the first 1000 values in the form of (x_i, x_{i+1}) is:

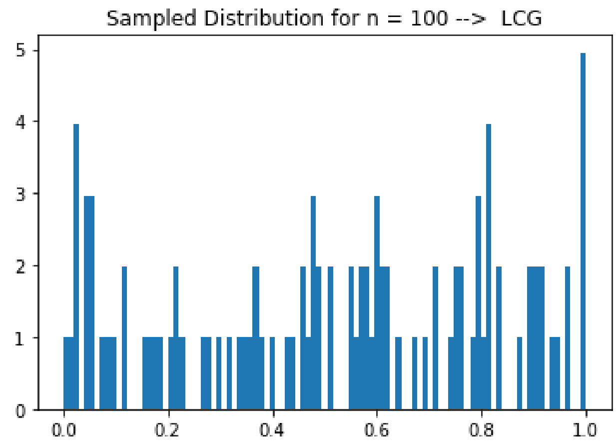


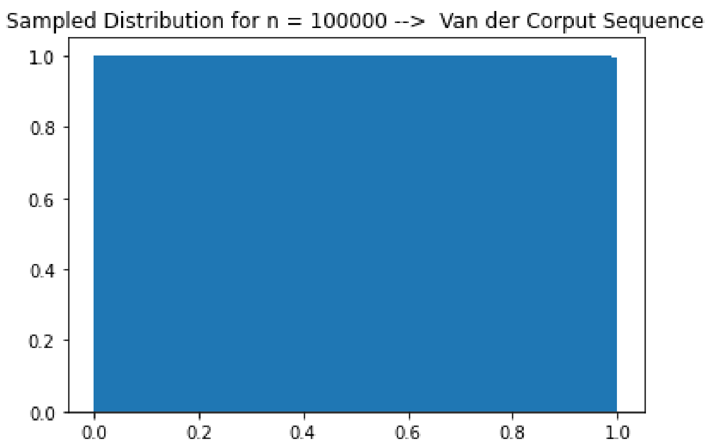
Observations:

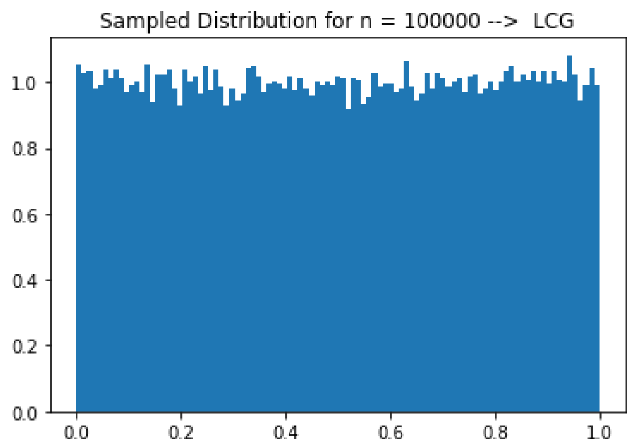
1. As we can observe the points are forming a set of lines parallel to the line y = x, it shows that the points are not uniformally distributed. Similar plots in previous labs was shown to cover the whole area (which is not the case here).
2. I found that this plot helps in what is known as “Spectral Test”. LCGs have a property that when plotted in 2 dimensions, lines will form, on which all possible outputs can be found. The spectral test compares the distance between these planes; the further apart they are, the worse the generator is.

Sampled Distributions:









LCG Used: Lagged Fibonacci Generator

$$U_i = (U_{i-17} - U_{i-5})$$

If $U_i < 0$, set $U_i = U_i + 1$

The first 17 values were generated using following General Linear Congruence Generator:

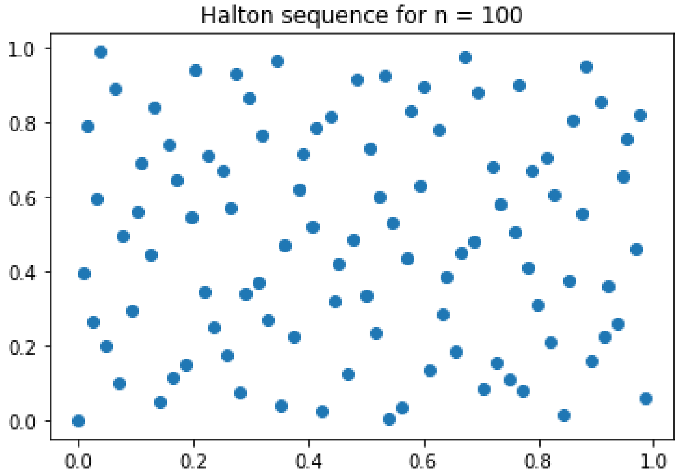
$x_{i+1} = (a.x_i + b) \bmod m$

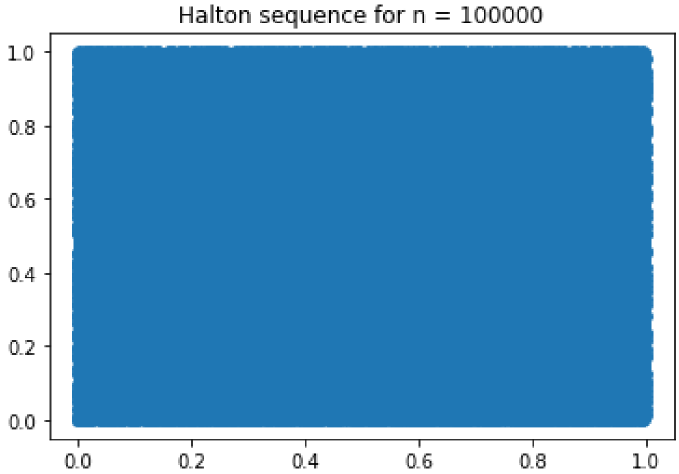
$u_{i+1} = x_{i+1} / m$, with a = 1229, b = 1, m = 2048 and seed (x_0) = 1

Observations:

1. We can observe that the plot for the Van der Corput sequence is more uniformally distributed, than the LCG used – Lagged Fibonacci Generator.
2. In both the plots of Van der Corput sequence, the proportional of points belonging to a fixed interval is almost proportional to the length of the interval – a very important property of the random numbers. But this property is not fully satisfied by the Lagged Fibonacci Generator.

2 QUESTION - 2:





Observations:

1. We can observe that the points are more uniformally and equi-distantly located in the R^2 plane. They completely cover the whole region.

2. So, this sequence generates the required set of points which are used in the Quasi-Monte Carlo simulation, and achieves our end goal.