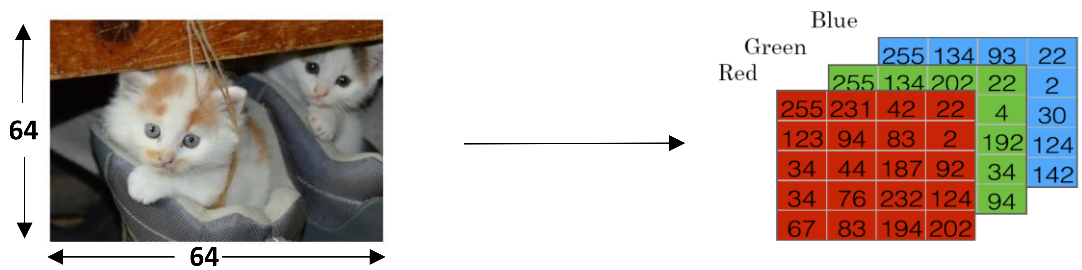
Binary Classification

In a binary classification problem, the result is a discrete value output.

For example - account hacked (1) or not hacked (0)
- a tumor malign (1)or benign(0)

Example: Cat vs Non-Cat

The goal is to train a classifier for which the input is an image represented by a feature vector, x, and predicts whether thecorresponding label y is 1 or 0. In this case, whether this is a cat image(1)or a non-cat image (0).



An image is stored in the computer in three separate matrices corresponding to the Red, Green, and Blue color channels of the image. The three matrices have the same size as the image, for example, the resolutionof the cat image is 64 pixels X 64 pixels, the three matrices (RGB) are 64 X 64 each.

The value in a cell represents the pixel intensity which will be used to create a feature vector of n-dimension. In pattern recognition and machine learning, a feature vector represents an image, Then the classifier's job is to determine whether it contain a picture of a cat or not.

To create a feature vector, x, the pixel intensity values will be "unrolled" or "reshaped" for each color. The dimension of the input feature vector x is $n = 64x\,64x\,3 = 12288$.

$$x = \begin{bmatrix} 255 \\ 231 \\ 42 \\ \vdots \\ 255 \\ 134 \\ 202 \\ \vdots \\ 255 \\ 134 \\ 93 \\ \vdots \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}1\\1\\1\\1\end{matrix}}\right\} \text{red} \\ \left.\vphantom{\begin{matrix}1\\1\\1\\1\end{matrix}}\right\} \text{green} \\ \left.\vphantom{\begin{matrix}1\\1\\1\\1\end{matrix}}\right\} \text{blue} \end{matrix}$$

Logistic Regression

Logistic regression is a learning algorithm used in a supervised learning problem when the output y are all either zero or one. The goal of logistic regression is to minimize the error between its predictions and training data.

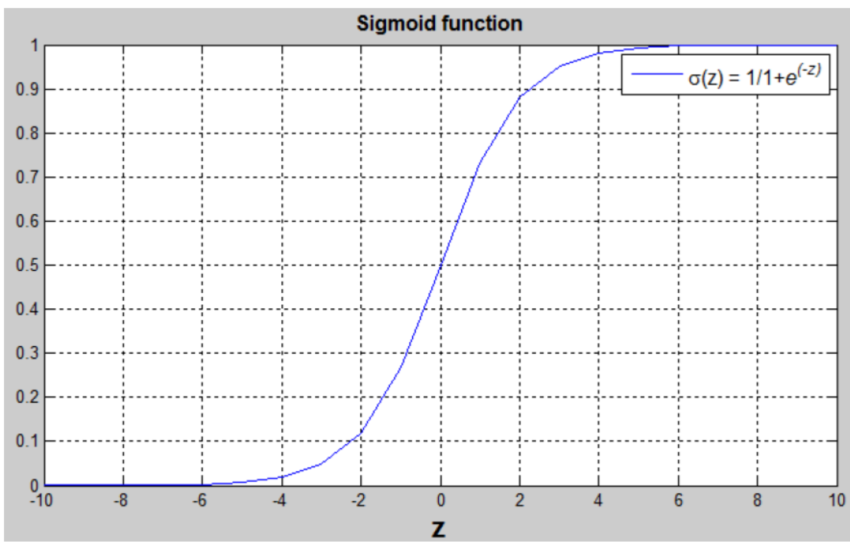
Example: Cat vs No - cat

Given an image represented by a feature vector x, the algorithm will evaluate the probability of a cat being in that image.

$$Given\ x\ ,\ \hat{y} = P(y = 1|x), \text{where } 0\ \leq \hat{y} \leq 1$$

The parameters used in Logistic regression are:

- The input features vector: $x\ \in \mathbb{R}^{n_x}$, where n_x is the number of features
- The training label: $y \in 0,1$
- The weights: $w\ \in \mathbb{R}^{n_x}$, where n_x is the number of features
- The threshold: $b\ \in \mathbb{R}$
- The output: $\hat{y} = \sigma(w^T x + b)$
- Sigmoid function: s = $\sigma(w^T x + b)$ = $\sigma(z)$= $\frac{1}{1+\ e^{-z}}$



$(w^T x + b)$ is a linear function $(ax + b)$, but since we are looking for a probability constraint between [0,1], the sigmoid function is used. The function is bounded between [0,1] as shown in the graph above.

Some observations from the graph:

- If z is a large positive number, then $\sigma(z)$ = 1
- If z is small or large negative number, then $\sigma(z)$ = 0
- If $z = 0$, then $\sigma(z)$ = 0.5

Logistic Regression: Cost Function

To train the parameters w and b, we need to define a cost function.

Recap:

$\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1 + e^{-z^{(i)}}}$

$x^{(i)}$ the i-th training example

$Given \left\{ \left(x^{(1)}, y^{(1)}\right), \cdots, \left(x^{(m)}, y^{(m)}\right) \right\},\ we\ want\ \hat{y}^{(i)} \approx y^{(i)}$

Loss (error) function:

The loss function measures the discrepancy between the prediction ($\hat{y}^{(i)}$) and the desired output ($y^{(i)}$). In other words, the loss function computes the error for a single training example.

$L\left(\hat{y}^{(i)}, y^{(i)}\right) = \frac{1}{2}(\hat{y}^{(i)} - y^{(i)})^2$

$L\left(\hat{y}^{(i)}, y^{(i)}\right) = -(y^{(i)} \log\left(\hat{y}^{(i)}\right) + \left(1 - y^{(i)}\right)\log(1 - \hat{y}^{(i)})$

- If $y^{(i)} = 1$: $L\left(\hat{y}^{(i)}, y^{(i)}\right) = -\log\left(\hat{y}^{(i)}\right)$ where $\log\left(\hat{y}^{(i)}\right)$ and $\hat{y}^{(i)}$ should be close to 1
- If $y^{(i)} = 0$: $L\left(\hat{y}^{(i)}, y^{(i)}\right) = -\log\left(1 - \hat{y}^{(i)}\right)$ where $\log\left(1 - \hat{y}^{(i)}\right)$ and $\hat{y}^{(i)}$ should be close to 0

Cost function

The cost function is the average of the loss function of the entire training set. We are going to find the parameters $w\ and\ b$ that minimize the overall cost function.

$$J(w, b) = \frac{1}{m}\sum_{i=1}^{m} L\left(\hat{y}^{(i)}, y^{(i)}\right) = -\frac{1}{m}\sum_{i=1}^{m}\left[\left(y^{(i)} \log\left(\hat{y}^{(i)}\right) + \left(1 - y^{(i)}\right)\log(1 - \hat{y}^{(i)}\right)\right]$$