Birla Institute of Technology and Science, Pilani

Work Integrated Learning Programmes Division

Cluster Programme - M.Tech. in Data Science and Engg.

II Semester 2018-19

Course Number	DSECF ZC416
Course Name	Mathematical Foundation for Data Science
Nature of Exam	Open Book
Weightage for grading	40%
Duration	2 hours and 30 minutes
Date of Exam	15/09/2019 (10:00 - 12:30)

# Pages	2
# Questions	5

Instructions

1. All questions are compulsory
2. Questions are to be answered in the order in which they appear in this paper and in the page numbers mentioned before each of them.

Pages 2-5

Q1a Prove or disprove: If A is a 4×3 matrix and the equation $Ax = 0$ has no non-zero solutions, then $\text{rank}(A) = 3$. (2)

b) Prove that the union of two subspaces is a subspace if and only if one is contained in the other. (2)

c) What kind of conic section or a pair of straight lines is represented by the quadratic form $4x_1^2 + 12x_1x_2 + 13x_2^2 = 16$? (2)

d) Apply four steps of the power method with scaling using the initial guess as [11] to $\begin{pmatrix} 7 & -3 \\ -3 & 1 \end{pmatrix}$ (2)

Pages 5-8

Q2a) Prove that if graphs G and H are isomorphic, then their complements $\bar{G}$ and $\bar{H}$ are also isomorphic. (2)

b) Prove or disprove: There exists a connected simple graph with 6 vertices and 5 edges such that G has Euler circuit? (2)

c) Draw two non-isomorphic 5 vertex, 5 edge simple graphs with the same degree sequence (2)

d) Write a pseudocode to decide whether a graph is bipartite based on the coloring theorem. (2)

Pages 8-11

Q3a) Prove that $\left\lfloor \frac{n}{2} \right\rfloor \left\lceil \frac{n}{2} \right\rceil = \left\lfloor \frac{n^2}{4} \right\rfloor$ for all $n \in \mathbb{N}$. (2)

b) Determine whether the symmetric difference operation is associative. (2)

c) If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer. (2)

d) Draw the graph of the function $f(x) = \left\lceil \left\lfloor x - \frac{1}{2} \right\rfloor + \frac{1}{2} \right\rceil$ in the domain $[-3, 3]$. (2)

Pages 12-15

Q4a) Find the transitive closure using Warshall's algorithm of the relation given by $\{(a,e),(b,a),(b,d),(c,d),(d,a),(d,c),(e,a),(e,b),(e,c),(e,e)\}$ on $\{a,b,c,d,e\}$. Do not use any other method. (2)

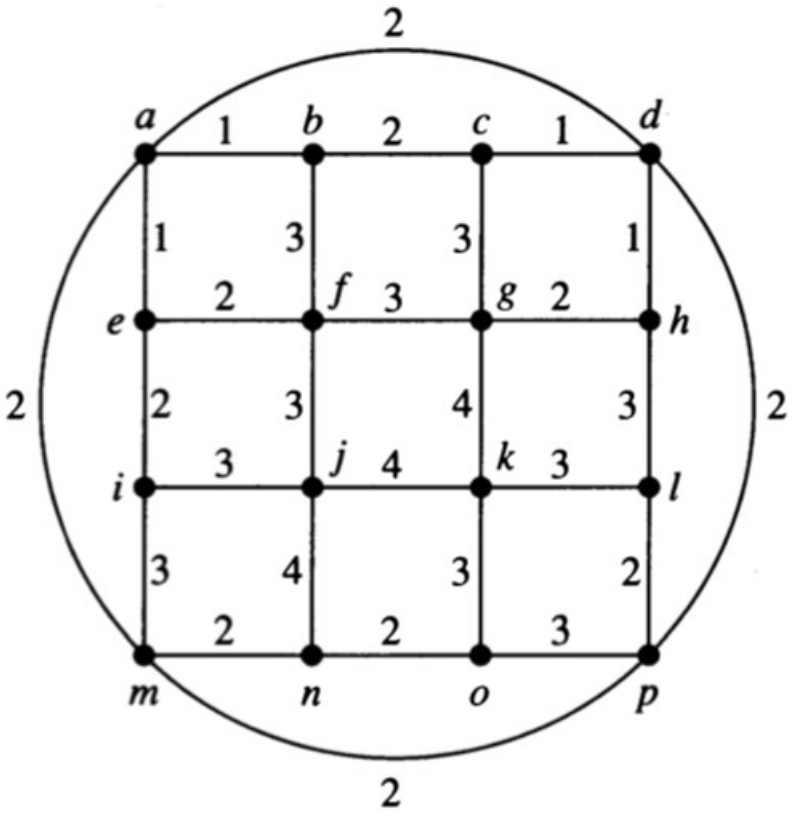
b) Suppose that $R1$ and $R2$ are reflexive relations on a set A, then show that $R1 \oplus R2$ is irreflexive. (2)

c) Determine whether the poset defined by $(P(S), \supseteq)$ is a lattice, where $P(S)$ is the power set of S. (2)

d) Draw the Hasse diagram for inclusion on the power set of the set S, $P(S)$, where $S = \{a,b,c,d\}$. (2)

Pages 16-19

Q5a) Use Kruskal's algorithm to find the minimum spanning tree for the weighted graph given below. (4)



b) Represent the expressions $(x + xy) + (x/y)$ and $x + ((xy + x)/y)$ using binary trees. Write the expressions in prefix and postfix notations. (2)

c) Show that the NOR operator represented by $\downarrow$ is functionally complete. Explicitly derive all the operations. (2)