

CSE 5311 – Programming Project

Disjoint sets

Union-Find refers to a class of algorithms used to maintain a data structure for disjoint sets, many of which have been analyzed by Tarjan (For references, see [Tarjan75], [wiki]) These algorithms are often used in connected components algorithms, which motivates the following problem:

Consider a graph that initially contains n vertices and no edges. Randomly include one edge at a time until the graph is connected; that is, until there is only one connected component. Plot the expected number of edges that must be included before the graph is connected against n . Can you find a function $f(n)$ such that $f(n)$ is close to the expected number of edges that must be included? For a few values of n , use your samples to estimate $g_n(m) = \Pr\{\text{after } m \text{ edges, the } n \text{ vertex graph is connected}\}$. Plot your estimates of g_n ; do you notice anything surprising? Can you find any threshold beyond which the graph gets connected with high probability? Use graph plots to show your results.

[Tarjan75] Tarjan, Robert Endre (1975). "Efficiency of a Good But Not Linear Set Union Algorithm".
Journal of the ACM 22 (2): 215–225. doi:10.1145/321879.321884

[wiki] http://en.wikipedia.org/wiki/Disjoint-set_data_structure.