TRANSFORM DOMAIN PATTERN ANALYSIS

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CONTENT

- **Signal Transformation**
- **Frequency Domain Representation of Signal**
- **Feature Extraction and Analysis**
- **Multiresolution Representation**
- **Wavelet Transform**
- **Discrete Cosine Transform**

Frequency Domain Representation of a Signal

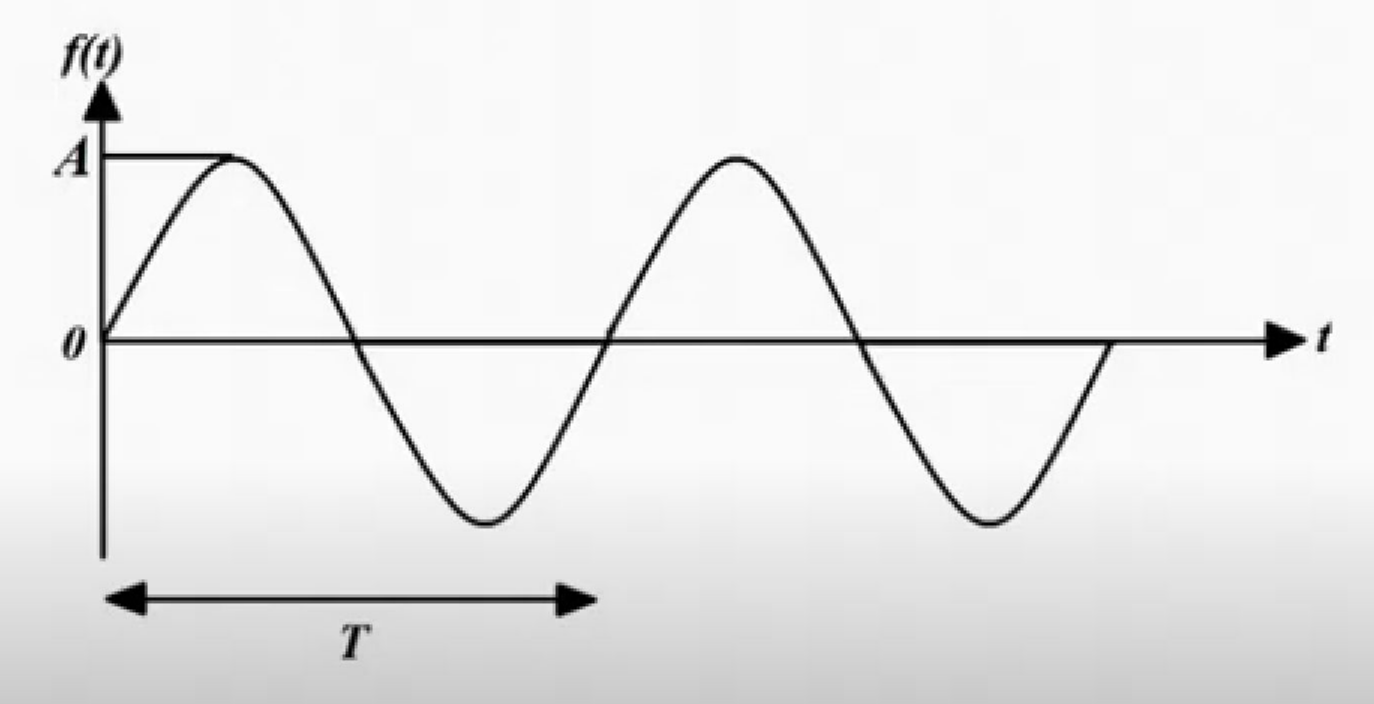
Representation Of Signals

There are two ways in which signals are represented :

Time Domain Representation

Frequency Domain Representation

Time Domain Representation

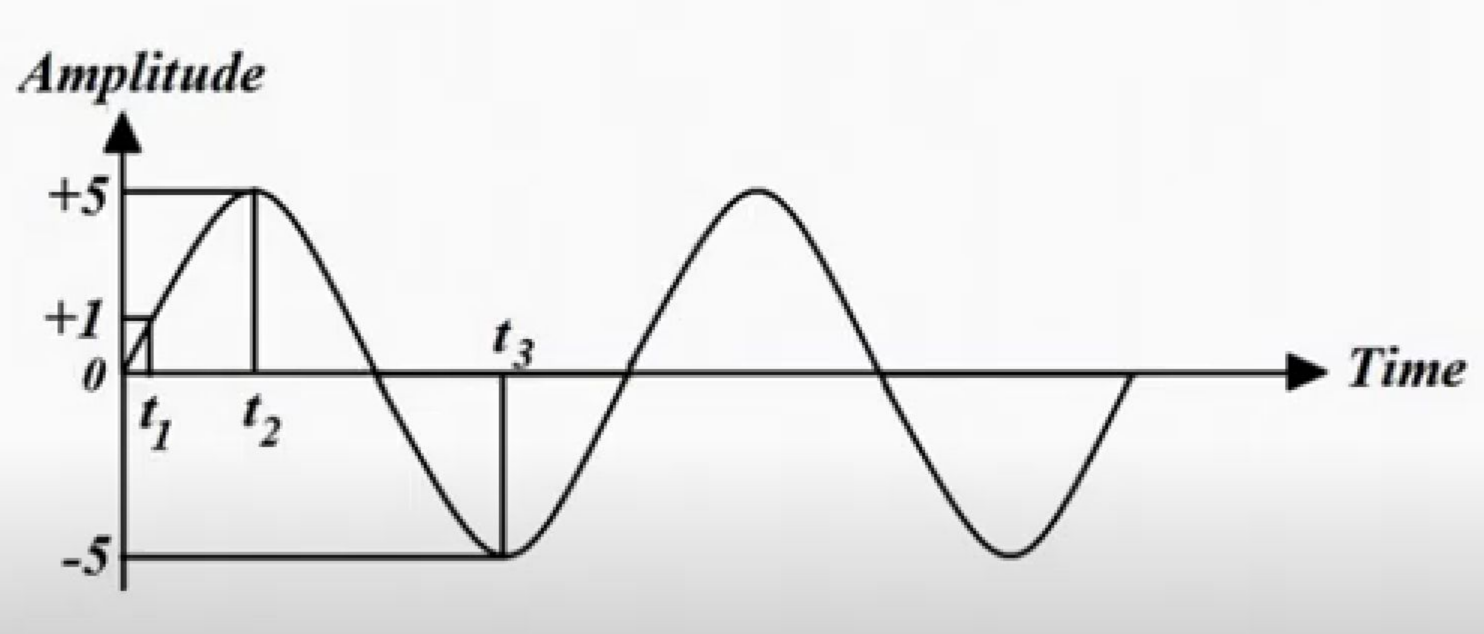


Time Domain Representation

Basic Signal Characteristics :

- **Peak Amplitude**
- **Time Period**
- **Frequency**
- **Phase**
- **Wavelength**

Time Domain Representation

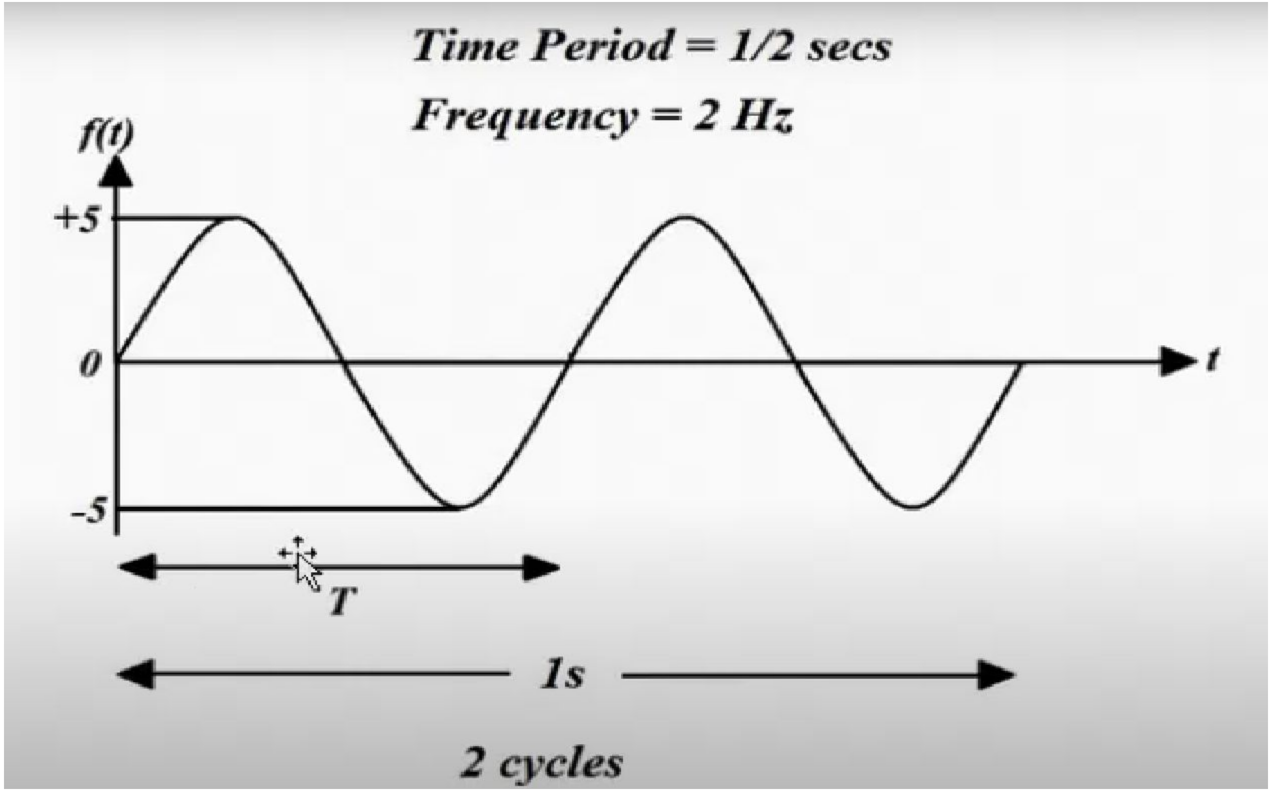


Time Domain Representation

- The time domain representation shows ***changes in amplitude w.r.t time.***

- So it is basically an ***amplitude vs time plot.***

Frequency Domain Representation

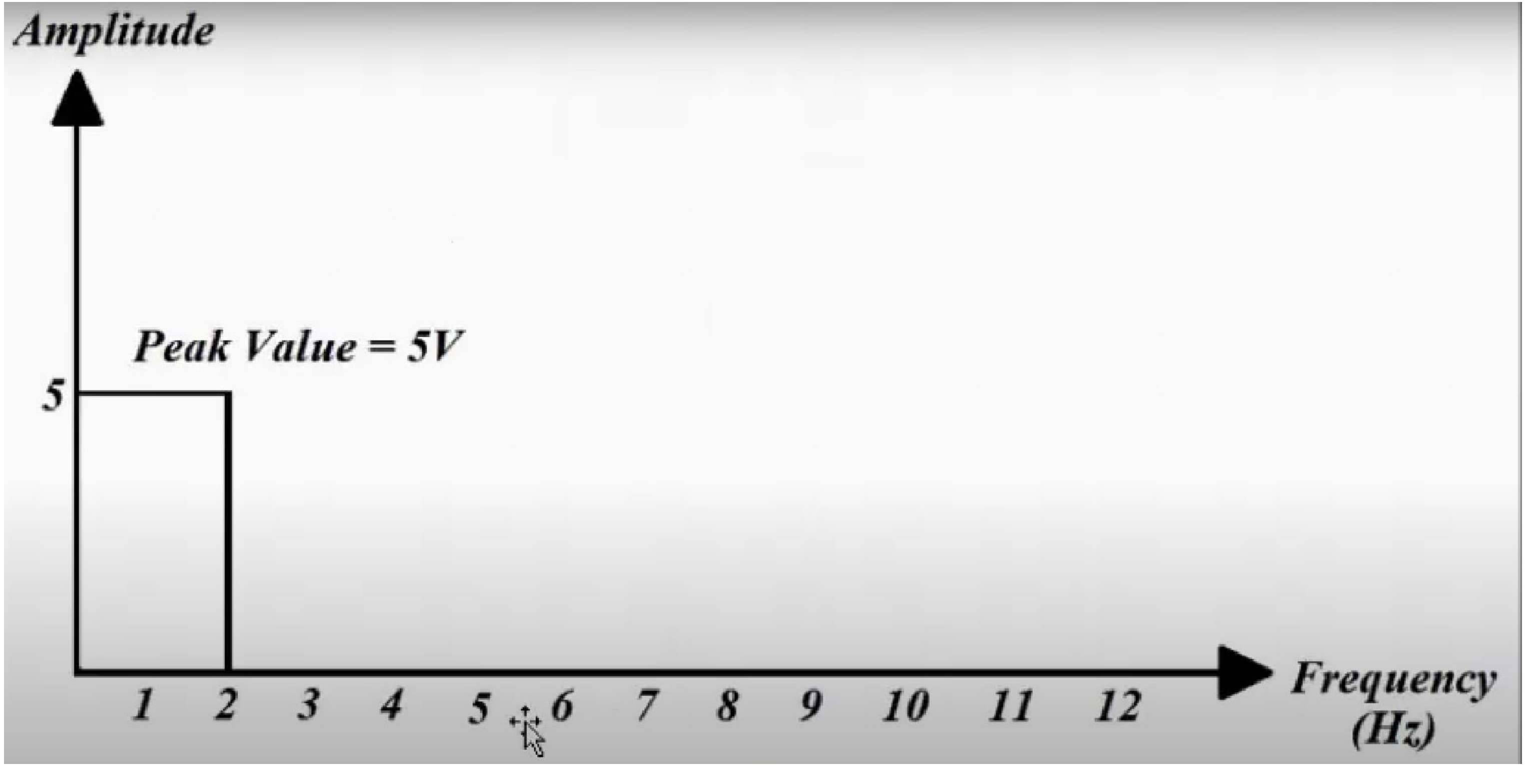


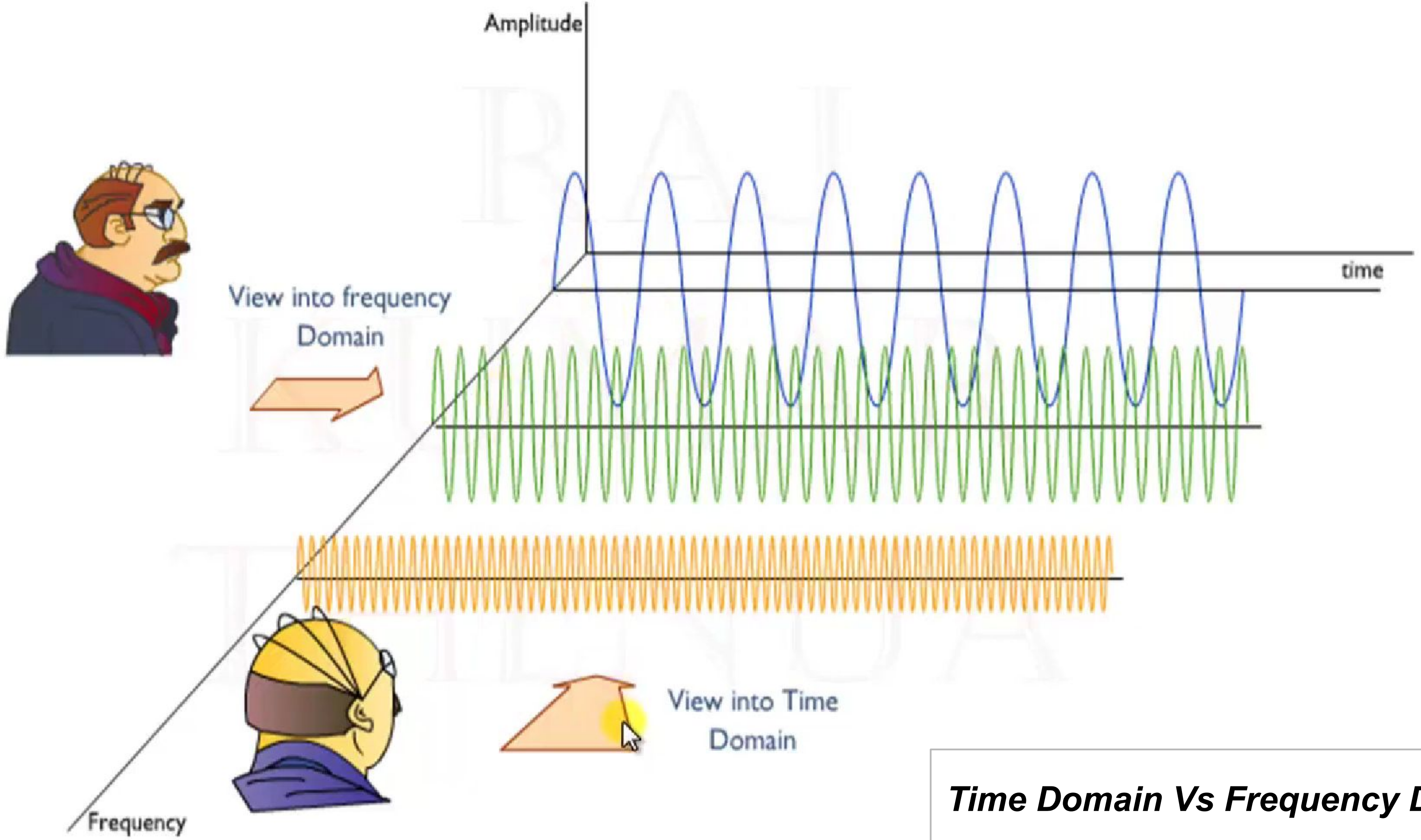
Frequency Domain Representation

- In frequency domain representation, only two things are important: ***Peak Amplitude*** and ***Frequency.***

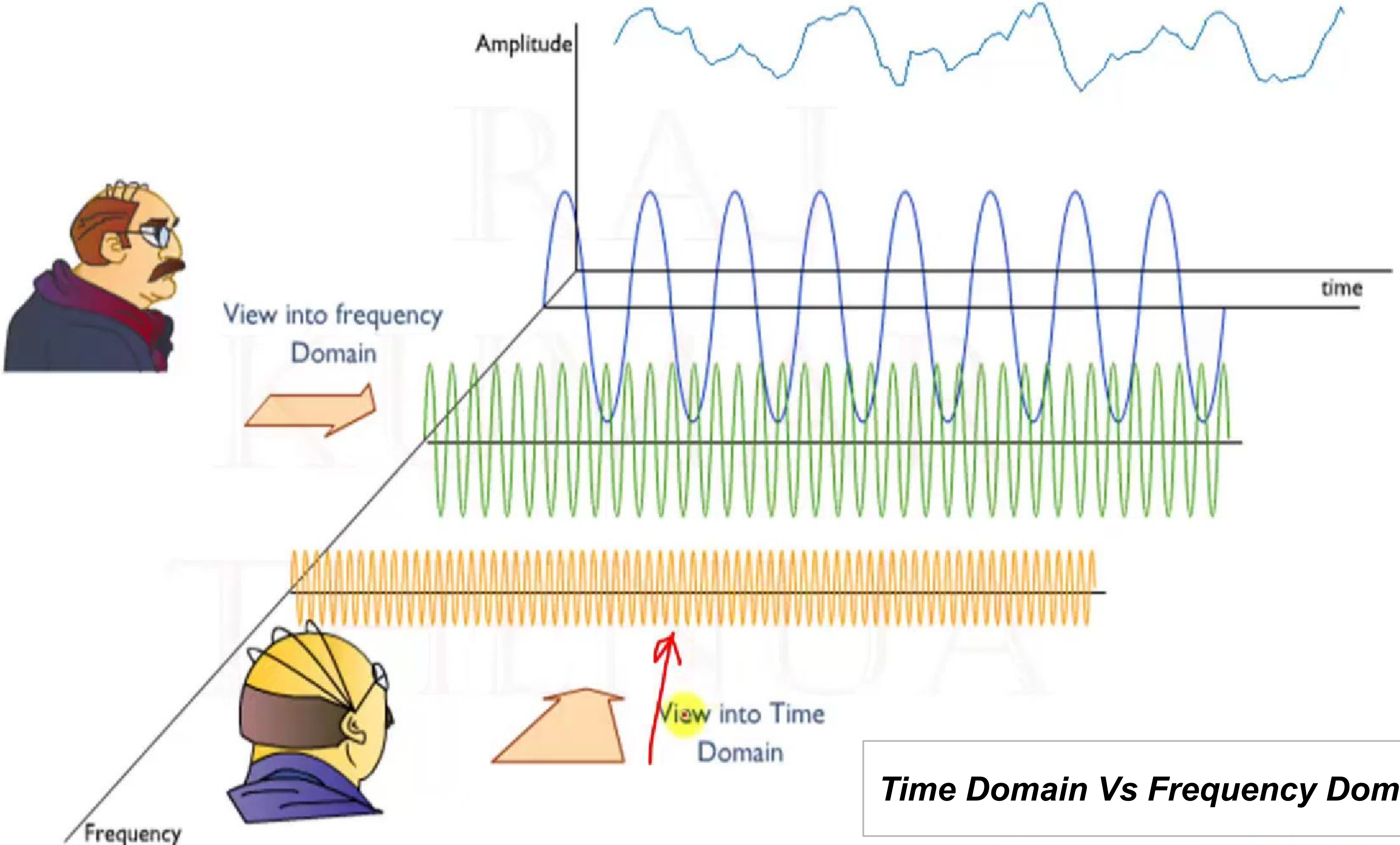
- So it is an ***amplitude vs frequency plot.***

Frequency Domain Representation

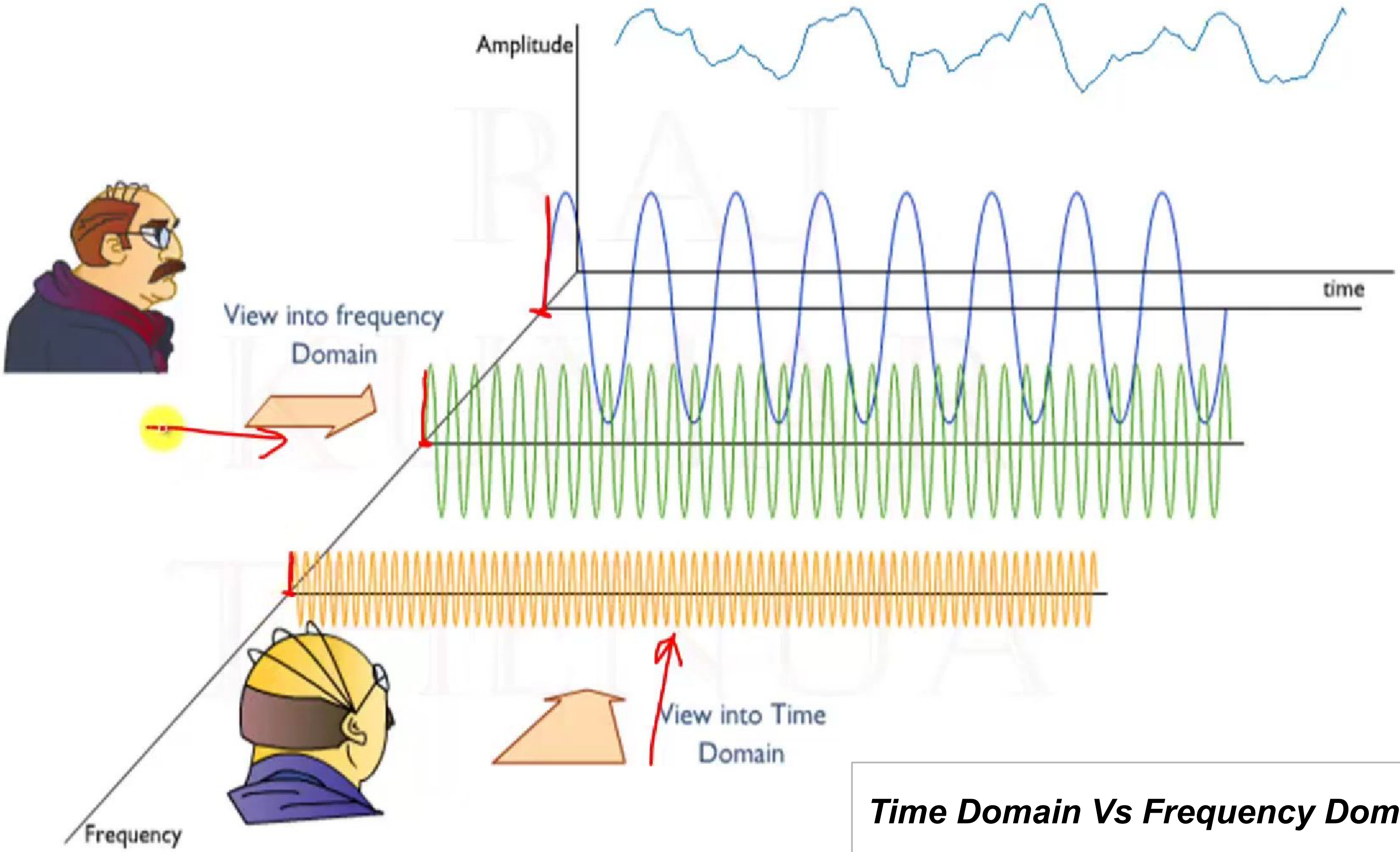




Time Domain Vs Frequency Domain



Time Domain Vs Frequency Domain



Time Domain Vs Frequency Domain

What is Frequency Domain?

- ***The Frequency Domain refers to the analytic space in which mathematical functions or signals are conveyed in terms of frequency, rather than time.***

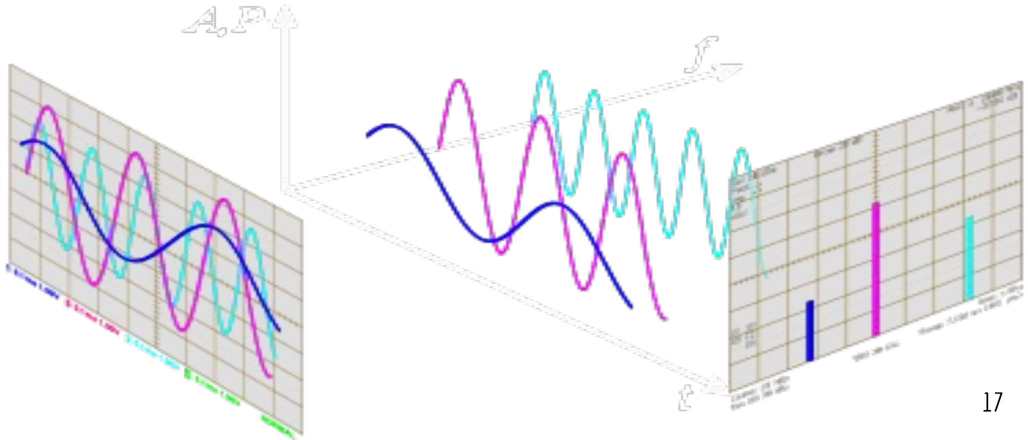
- *For example,* where a time-domain graph may display changes over time, a frequency-domain graph displays how much of the signal is present among each given frequency band.

Continue...

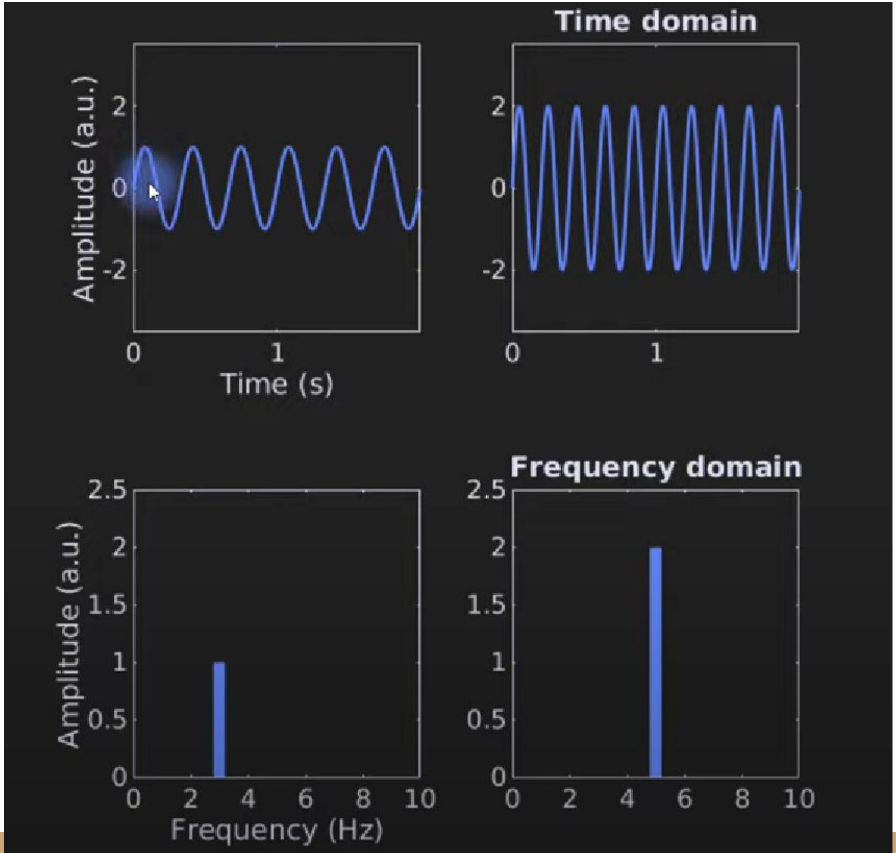
- It is possible, however, to convert the information from a time-domain to a frequency-domain. An example of such transformation is a ***Fourier transform.***
- The Fourier transform converts the time function into a set of sine waves that represent different frequencies. The frequency-domain representation of a signal is known as the ***"spectrum"*** of frequency components.

Continue...

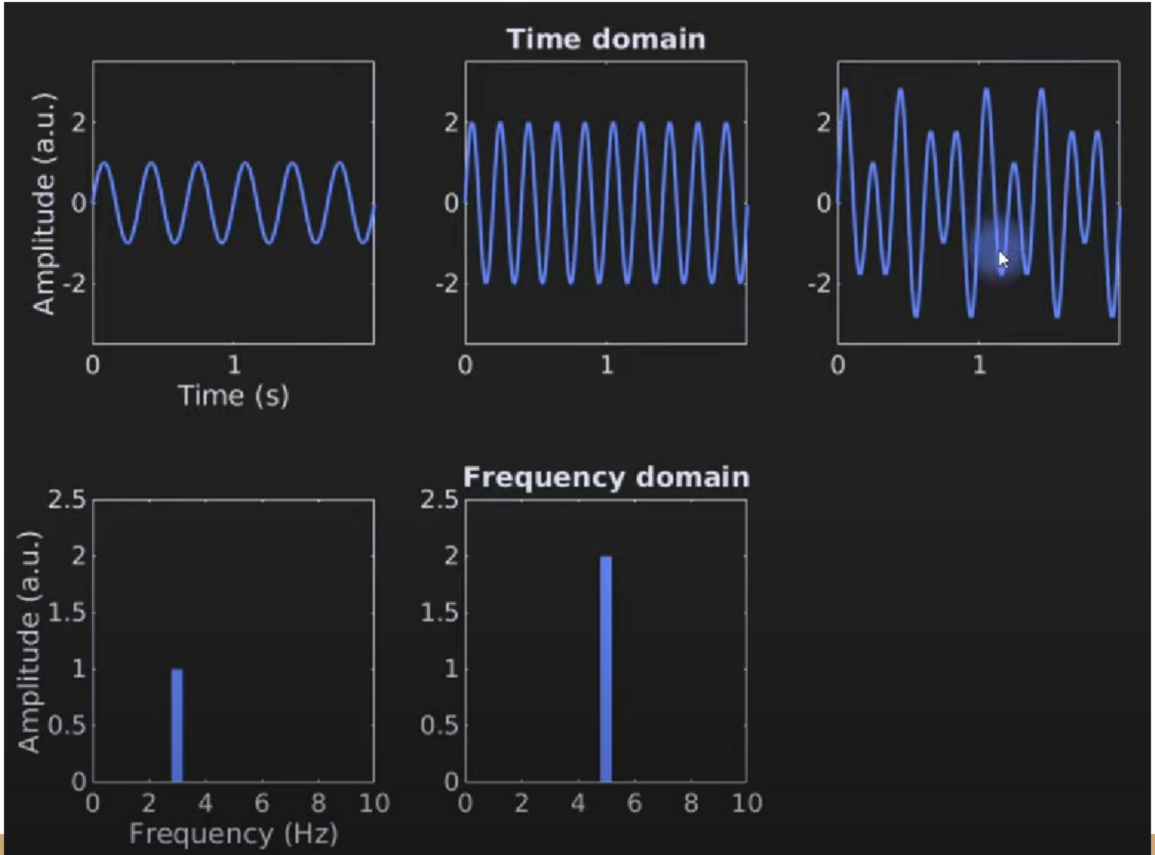
A discipline in which the frequency domain is used for graphical representation is in music. Often audio producers and engineers display an audio signal within a frequency domain in order to better understand the shape and character of an audio signal.



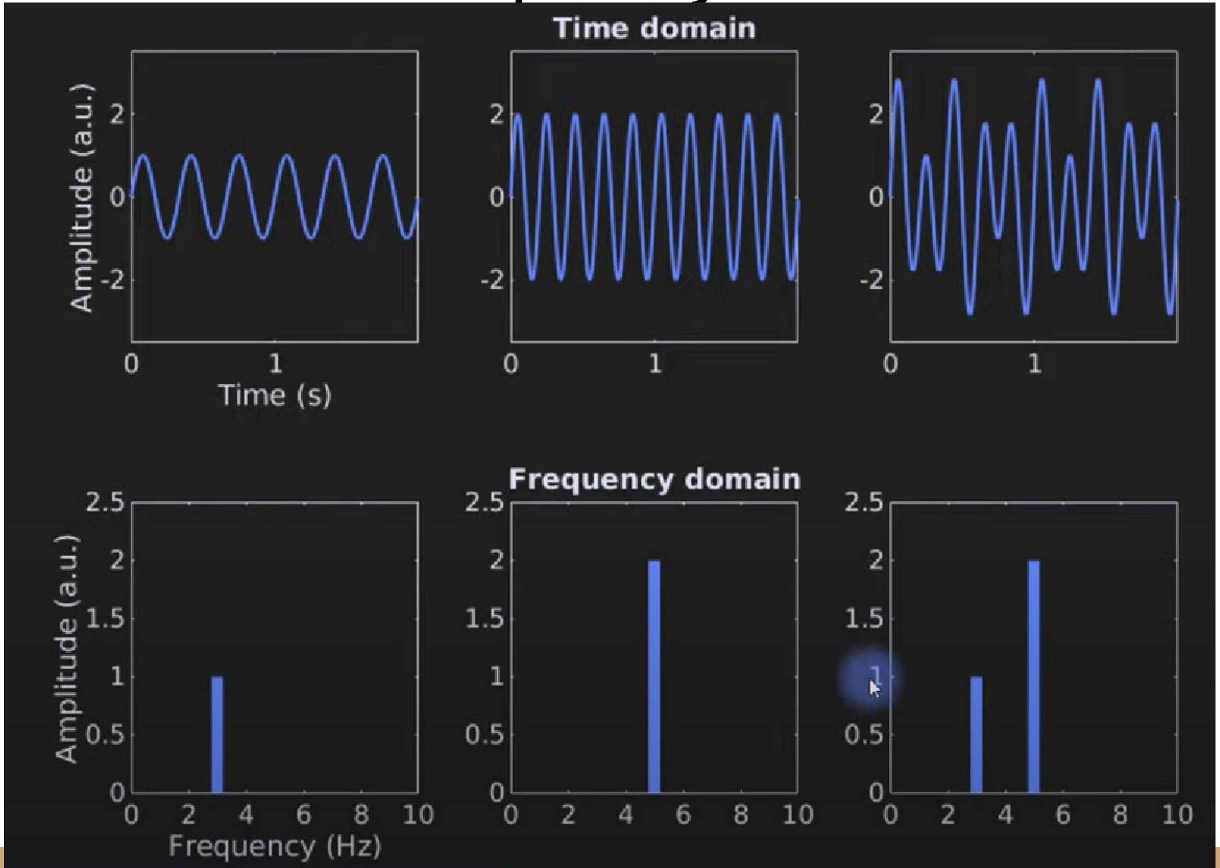
Time Domain Vs Frequency Domain



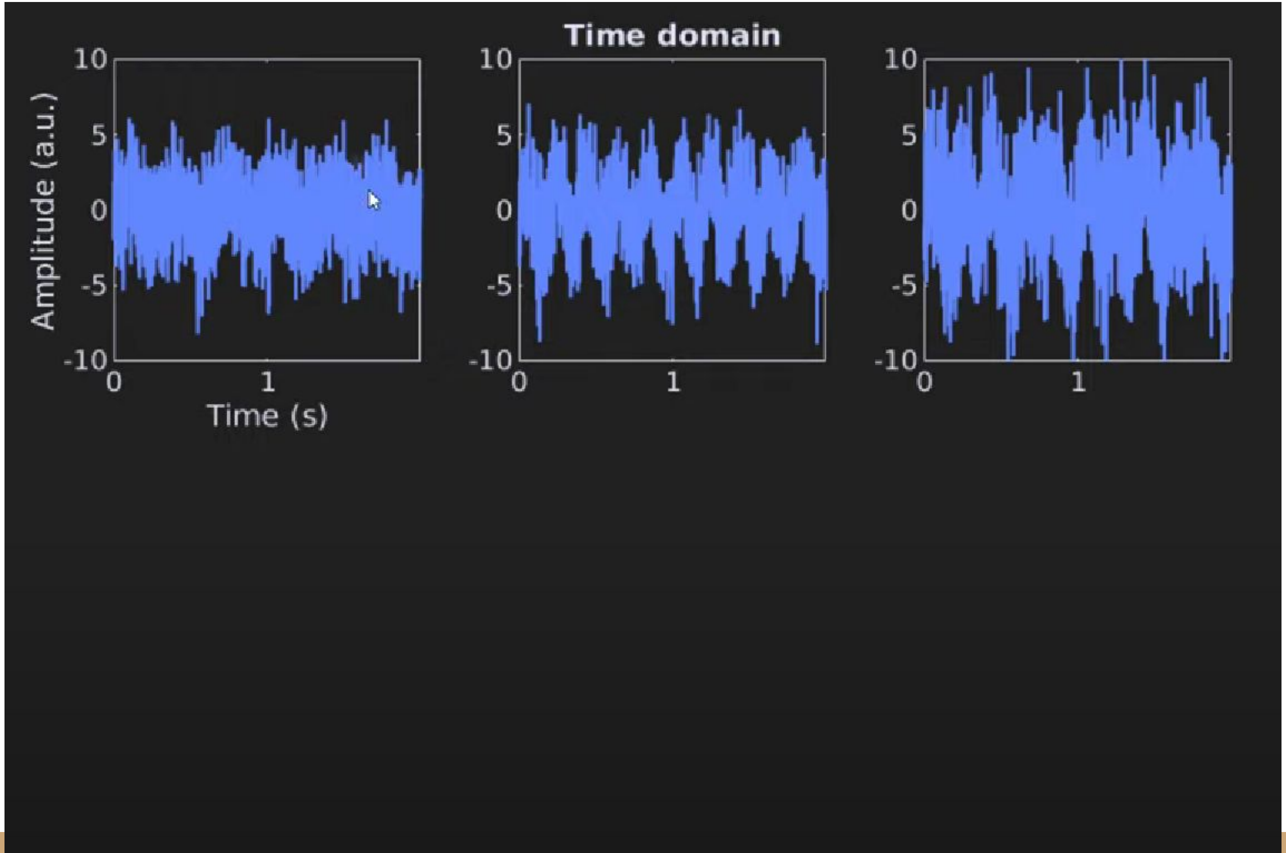
Time Domain Vs Frequency Domain



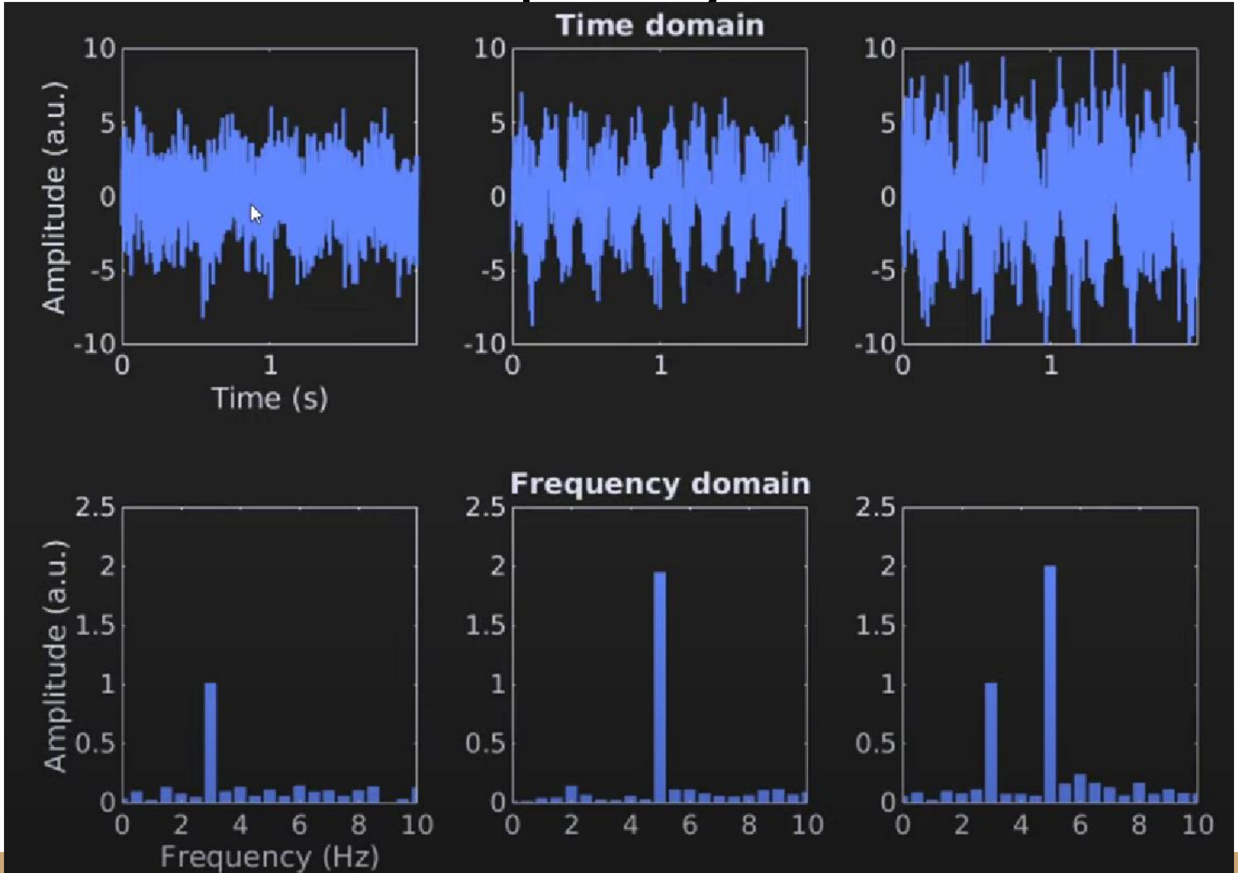
Time Domain Vs Frequency Domain



Time Domain Vs Frequency Domain



Time Domain Vs Frequency Domain



Transformation from Time domain to Frequency domain

Fourier Transform

Laplace Transform

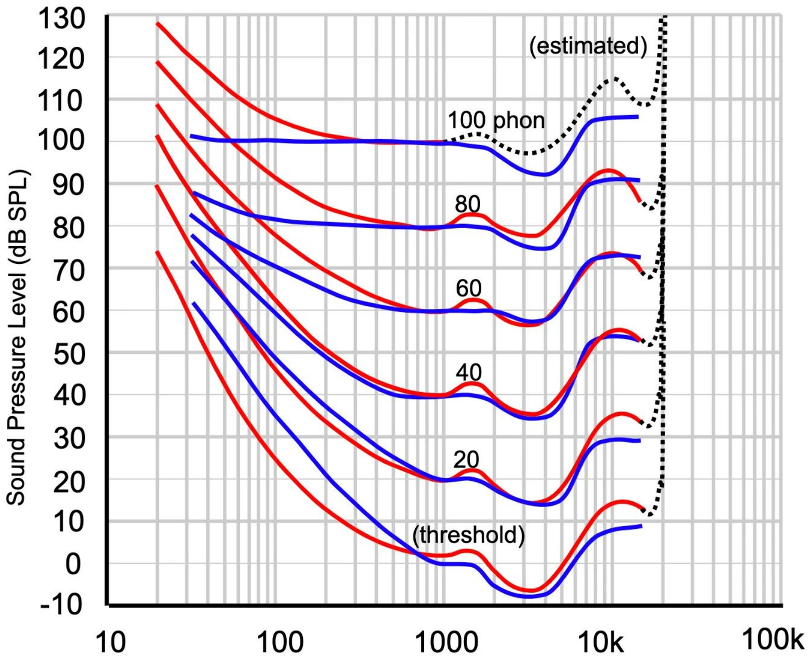
Z- Transform

Application of Frequency Domain

- For example, auditory sounds exist between a range of 20-20,000Hz, and some frequencies are harder for the human ear to withstand.
- The frequency ***3,400Hz*** is a harsh frequency, and the human ear is specifically tuned to respond viscerally to that sound.
- An audio engineer may reduce the strength of that frequency in the frequency domain using ***an audio equalizer***. By displaying the audio signal in the frequency domain, an engineer can boost and reduce signals to make the sounds more pleasant for the human ear.

Application of Frequency Domain

- ***The Fletcher-Munson curve*** is a widely used function that lays atop the frequency domain that audio engineers often reference when mixing various frequencies. The function's curve selectively boosts and reduces frequencies to allow the audio engineer to raise the gain of the signal while mitigating the unpleasant sounds.



Equal-loudness contours (red) (from ISO 226:2003 revision)
Fletcher–Munson curves shown (blue) for comparison

Feature Extraction and Analysis

Feature Extraction and Analysis

- Feature extraction is a process of dimensionality reduction by which an initial set of raw data is reduced to more manageable groups for processing.
- Feature extraction is the name for methods that select and /or combine variables into features, effectively reducing the amount of data that must be processed, while still accurately and completely describing the original data set.

Feature Extraction and Analysis

- In machine learning, pattern recognition, and image processing, feature extraction starts from an initial set of measured data and builds derived values (features) intended to be informative and non-redundant, facilitating the subsequent learning and generalization steps, and in some cases leading to better human interpretations.

Feature Extraction and Analysis

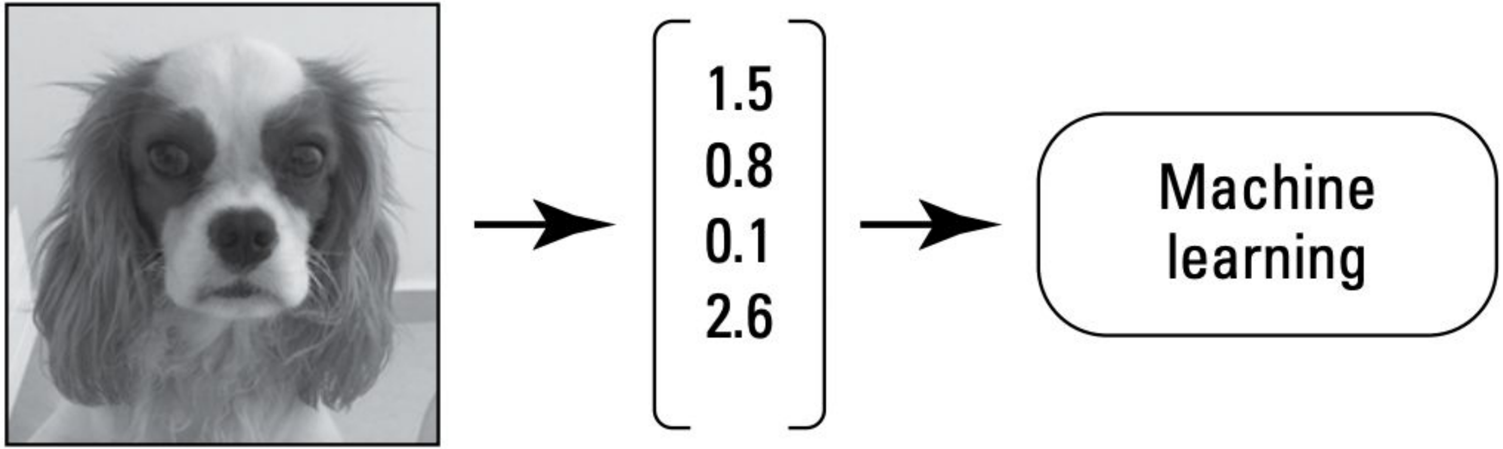


FIGURE 2-1: Feature extraction for the dog detector.

Feature Extraction and Analysis

- When the input data to an algorithm is too large to be processed and it is suspected to be redundant (e.g. the same measurement in both feet and meters), then it can be transformed into a reduced set of features (also named a feature vector).
- The selected features are expected to contain the relevant information from the input data, so that the desired task can be performed by using this reduced representation instead of the complete initial data.

Feature Extraction and Analysis

Image

Feature Extraction and Analysis

0	2	15	0	0	11	10	0	0	0	0	9	9	0	0	0
0	0	0	4	60	157	236	255	255	177	95	61	32	0	0	29
0	10	16	119	238	255	244	245	243	250	249	255	222	103	10	0
0	14	170	255	255	244	254	255	253	245	255	249	253	251	124	1
2	98	255	228	255	251	254	211	141	116	122	215	251	238	255	49
13	217	243	255	155	33	226	52	2	0	10	13	232	255	255	36
16	229	252	254	49	12	0	0	7	7	0	70	237	252	235	62
6	141	245	255	212	25	11	9	3	0	115	236	243	255	137	0
0	87	252	250	248	215	60	0	1	121	252	255	248	144	6	0
0	13	113	255	255	245	255	182	181	248	252	242	208	36	0	19
1	0	5	117	251	255	241	255	247	255	241	162	17	0	7	0
0	0	0	4	58	251	255	246	254	253	255	120	11	0	1	0
0	0	4	97	255	255	255	248	252	255	244	255	182	10	0	4
0	22	206	252	246	251	241	100	24	113	255	245	255	194	9	0
0	111	255	242	255	158	24	0	0	6	39	255	232	230	56	0
0	218	251	250	137	7	11	0	0	0	2	62	255	250	125	3
0	173	255	255	101	9	20	0	13	3	13	182	251	245	61	0
0	107	251	241	255	230	98	55	19	118	217	248	253	255	52	4
0	18	146	250	255	247	255	255	255	249	255	240	255	129	0	5
0	0	23	113	215	255	250	248	255	255	248	248	118	14	12	0
0	0	6	1	0	52	153	233	255	252	147	37	0	0	4	1
0	0	5	5	0	0	0	0	0	14	1	0	6	6	0	0

Image

Machine store images in the form of a matrix of numbers.

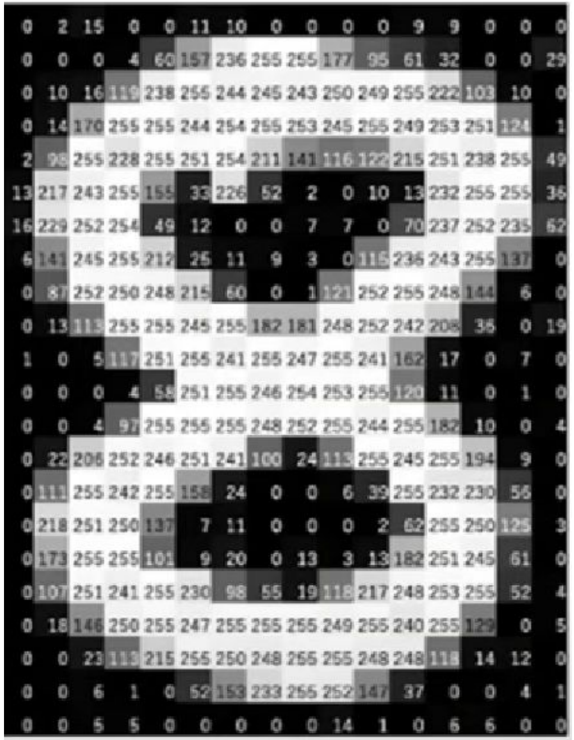
Feature Extraction and Analysis

These numbers, or a pixel values denotes the intensity or brightness of the pixel. Smaller numbers represent black (closer to 0) and larger numbers denotes white (closer to 255).

Image

0	2	15	0	0	11	10	0	0	0	0	9	9	0	0	0
0	0	0	4	60	157	236	255	255	177	95	61	32	0	0	29
0	10	16	119	238	255	244	245	243	250	249	255	222	103	10	0
0	14	170	255	255	244	254	255	253	245	255	249	253	251	124	1
2	98	255	228	255	251	254	211	141	116	122	215	251	238	255	49
13	217	243	255	155	33	226	52	2	0	10	13	232	255	255	36
16	229	252	254	49	12	0	0	7	7	0	70	237	252	235	62
6	141	245	255	212	25	11	9	3	0	115	236	243	255	137	0
0	87	252	250	248	215	60	0	1	121	252	255	248	144	6	0
0	13	113	255	255	245	255	182	181	248	252	242	208	36	0	19
1	0	5	117	251	255	241	255	247	255	241	162	17	0	7	0
0	0	0	4	58	251	255	246	254	253	255	120	11	0	1	0
0	0	4	97	255	255	255	248	252	255	244	255	182	10	0	4
0	22	206	252	246	251	241	100	24	113	255	245	255	194	9	0
0	111	255	242	255	158	24	0	0	6	39	255	232	230	56	0
0	218	251	250	137	7	11	0	0	0	2	62	255	250	125	3
0	173	255	255	101	9	20	0	13	3	13	182	251	245	61	0
0	107	251	241	255	230	98	55	19	118	217	248	253	255	52	4
0	18	146	250	255	247	255	255	255	249	255	240	255	129	0	5
0	0	23	113	215	255	250	248	255	255	248	248	118	14	12	0
0	0	6	1	0	52	153	233	255	252	147	37	0	0	4	1
0	0	5	5	0	0	0	0	0	14	1	0	6	6	0	0

Machine store images in the form of a matrix of numbers.



Why Feature Extraction is Useful?

- The process of feature extraction is useful when you need to reduce the number of resources needed for processing without losing important or relevant information.
- Feature extraction can also reduce the amount of redundant data for a given analysis. Also, the reduction of the data and the machine's efforts in building variable combinations (features) facilitate the speed of learning and generalization steps in the machine learning process.

Dimensionality Reduction

- ***Dimensionality reduction***, or ***dimension reduction***, is the transformation of data from a high-dimensional space into a low-dimensional space so that the low-dimensional representation retains some meaningful properties of the original data.

- Working in high-dimensional spaces can be undesirable for many reasons; raw data are often sparse and analyzing the data is usually computationally intractable (hard to control or deal with).

Dimensionality Reduction…

- ***Dimensionality reduction*** is common in fields that deal with large numbers of observations and/or large numbers of variables, such as signal processing, speech recognition, neuroinformatics, and bioinformatics.

- ***Dimensionality reduction*** can be used for noise reduction, data visualization, cluster analysis, or as an intermediate step to facilitate other analyses.

Multiresolution Representation

Multiresolution Representation

- Multiresolution theory is concerned with the representation and analysis of signals (or image) at more than one resolution.

- The appeal of such an approach is obvious features that might go undetected at one resolution may be easy to spot at another.

- **Representation of a signal (e.g., an images) in more than one resolution/scale.**

Multiresolution Representation

- Many signals or image contain information at different scales or levels of details(e.g., people vs Building)
- Analyzing information at the same scale will not be effective.

Use windows of different size (i.e., vary scale)

Multiresolution Representation

Alternatively, use the same window size but analyze information at different resolutions.

High resolution

Small size objects should be examined at a high resolution

Low resolution

Large size objects should be examined at a low resolution

Multiresolution Representation

- Multiresolution theory incorporates:

    - **Sub-band coding from signal Processing**
    - **Quadrature mirror filtering from digital speech recognition**
    - **Pyramidal image processing**

Sub-band coding from signal Processing

- In Sub-band coding, a signal is decomposed into a set of band-limited components, called sub-bands, which can be reassembled to reconstruct the original signal without error.

- Originally developed for speech and image compression.

- Each sub-band is generated by bandpass filtering the input.

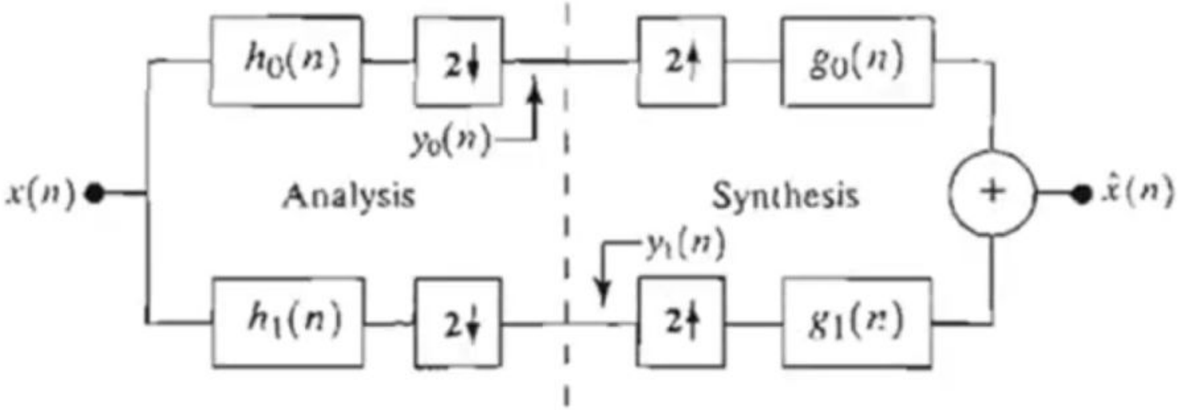
Sub-band coding from signal Processing

- Since the bandwidth of the resulting subbands is smaller than that of the original signal, the subbands can be downsampled without loss of information.
- Reconstruction of the original signal is accomplished by upsampling, filtering and summing the individual subbands.

Note: Up-sampler - Used to increase the sampling rate by an integer factor.

• Down-sampler - Used to decrease the the sampling rate by an integer factor.

Sub-band coding from signal Processing



a) A two-band filter bank for one-dimensional subband coding and decoding

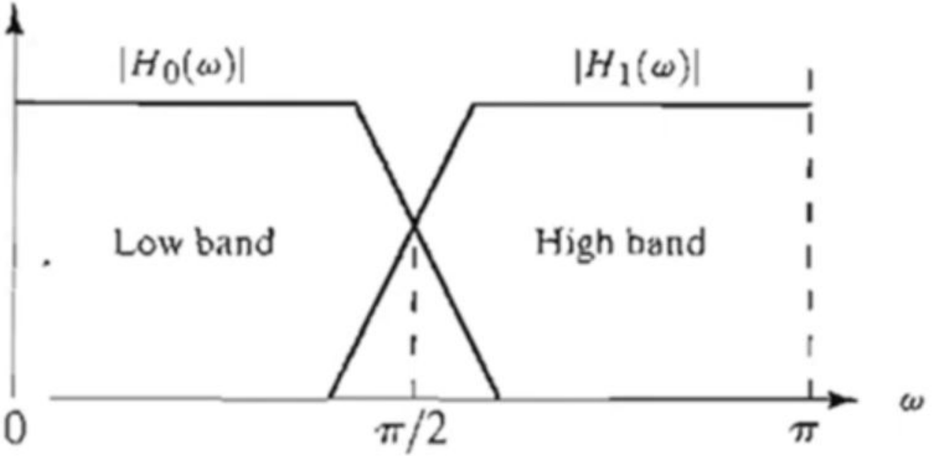
Sub-band coding from signal Processing

- The input of the system is a one-dimensional, band-limited discrete-time signal x (n) for n = 0,1,2,...

- The output sequence, $\hat{x}$(n) is, formed through the decomposition of x(n) into $y_0(n)$ and $y_1(n)$via analysis filters $h_0(n)$ and $h_1(n)$ and subsequent recombination via synthesis filters $g_0(n)$ and $g_1(n)$.

- Filters $h_0(n)$ and $h_1(n)$ are half-band digital filters whose idealized transfer characteristics,$H_0$and H_1 are shown as:

Sub-band coding from signal Processing



(b) its spectrum splitting properties.

Sub-band coding from signal Processing

- Filter H_0 is a lowpass filter whose output is an approximation of x(n) ;
- Filter H_1 is a highpass filter whose output is the high frequency or detail part of x (n).
- All filtering is performed in the time domain by convolving each filter's input with its impulse response-its response to a unit amplitude impulse function, δ(n).

Sub-band coding from signal Processing

- We wish to select $h_0(n)$, $h_1(n)$, $g_0(n)$ and $g_1(n)$ so that the input can be reconstructed perfectly.

- The Z-transform, a generalization of the discrete Fourier transform, is the ideal tool for studying discrete-time, sampled-data systems.

Sub-band coding from signal Processing

The Z-transform of sequence x(n) for n=0,1,2 is:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Down-sampling by a factor of 2:

$$x_{down}(n) = x(2n) \Leftrightarrow X_{down}(z) = \frac{1}{2}\left[X(z^{1/2}) + X(-z^{1/2})\right]$$

Up-sampling by a factor of 2:

$$x^{up}(n) = \begin{cases} x(n/2), n = 0,2,4\ldots \\ 0 \qquad \text{otherwise} \end{cases} \Leftrightarrow X^{up}(z) = X(z^2)$$

Sub-band coding from signal Processing

If the sequence x(n) is down-sampled and subsequently up-sampled to yield x^(n), then:

$$\hat{X}(z) = \frac{1}{2}\left[X(z) + X(-z)\right]$$

Thus, we can express the system's output as:

$$\hat{X}(z) = \frac{1}{2}G_0(z)\left[H_0(z)X(z) + H_0(-z)X(-z)\right]$$
$$+\frac{1}{2}G_1(z)\left[H_1(z)X(z) + H_1(-z)X(-z)\right]$$

Sub-band coding from signal Processing

Rearranging the terms, we get,

$$\hat{X}(z) = \frac{1}{2}\left[H_0(z)G_0(z) + H_1(z)G_1(z)\right]X(z)$$

$$+\frac{1}{2}\left[H_0(-z)G_0(z) + H_1(-z)G_1(z)\right]X(-z)$$

Now, for error-free reconstruction of input, we impose the conditions:

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$$

$$H_0(z)G_0(z) + H_1(z)G_1(z) = 2$$

Sub-band coding from signal Processing

If

$$\begin{cases} H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \\ H_0(z)G_0(z) + H_1(z)G_1(z) = 2 \end{cases}$$

$$\Downarrow$$

Then

$$X(z) = \hat{X}(z)$$

Sub-band coding from signal Processing

Filter	QMF	CQF	Orthonormal
$H_0(z)$	$H_0^2(z) - H_0^2(-z) = 2$	$H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = 2$	$G_0(z^{-1})$
$H_1(z)$	$H_0(-z)$	$z^{-1}H_0(-z^{-1})$	$G_1(z^{-1})$
$G_0(z)$	$H_0(z)$	$H_0(z^{-1})$	$G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1}) = 2$
$G_1(z)$	$-H_0(-z)$	$zH_0(-z)$	$-z^{-2K+1}G_0(-z^{-1})$

Perfect reconstruction filter families.

QMF: quadrature mirror filters

CQF: conjugate mirror filters

Sub-band coding from signal Processing

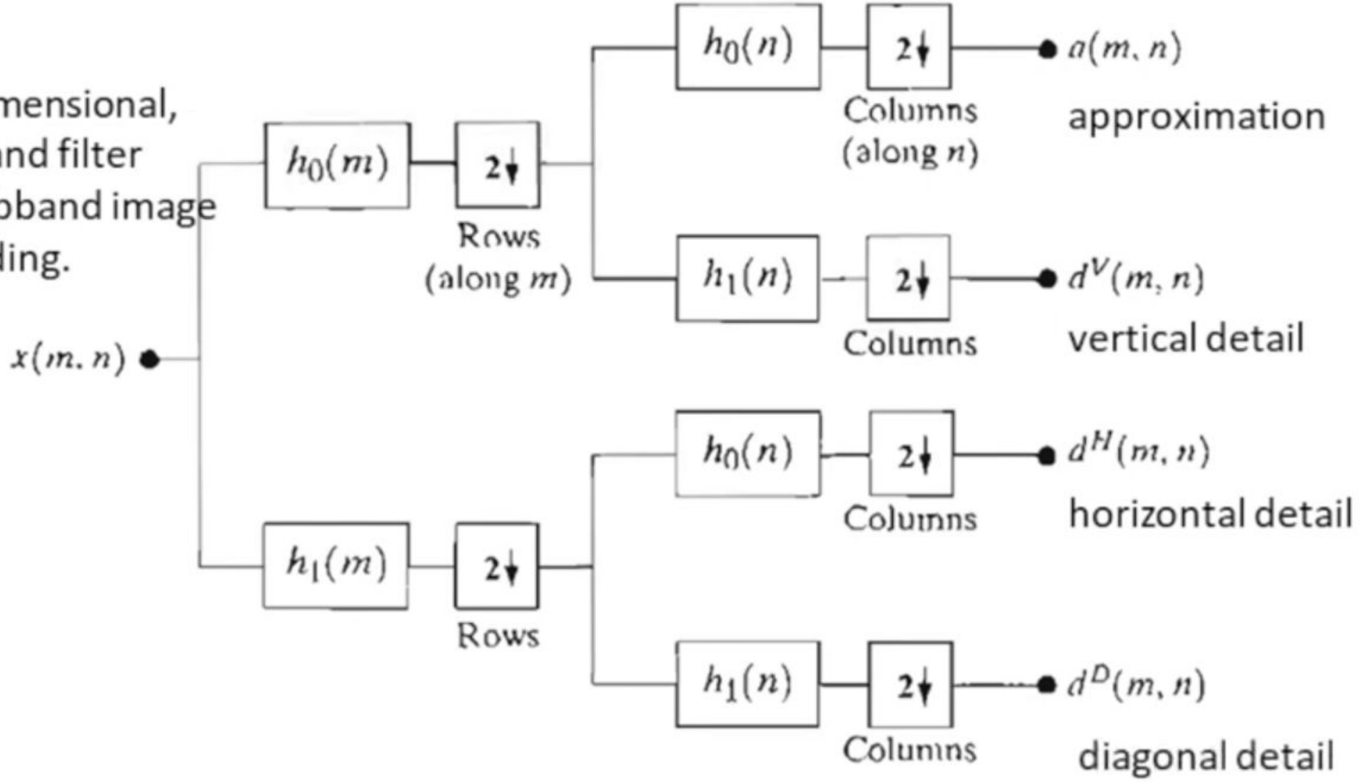
For each class, a "prototype" filter is designed to a particular specification and the remaining filters are computed from the prototype.

Columns 1 and 2 of Table are classic results from the filter bank literature- namely, quadrature mirror filters (QMFs) and conjugate quadrature filters (CQFs).

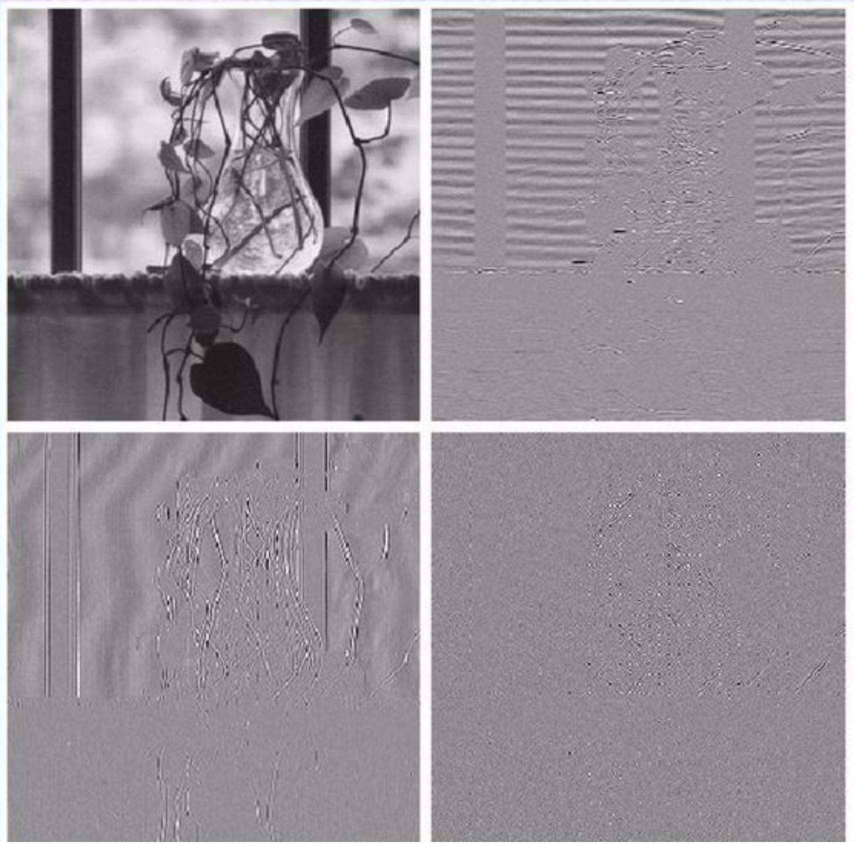
The filters in column three, which are later used in the development of the fast wavelet transform are called orthonormal.

Sub-band coding from signal Processing

A two-dimensional, four-band filter bank for subband image coding.



Sub-band coding from signal Processing

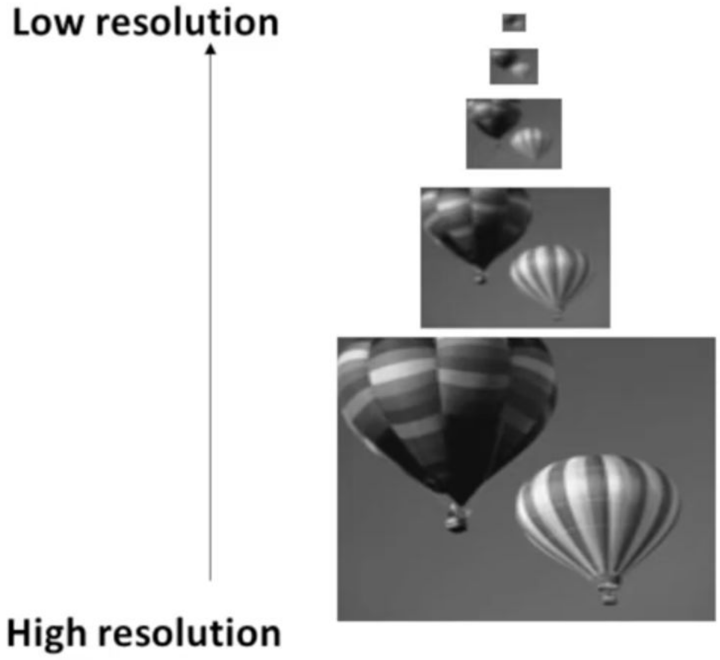


four band split the image using subband coding system

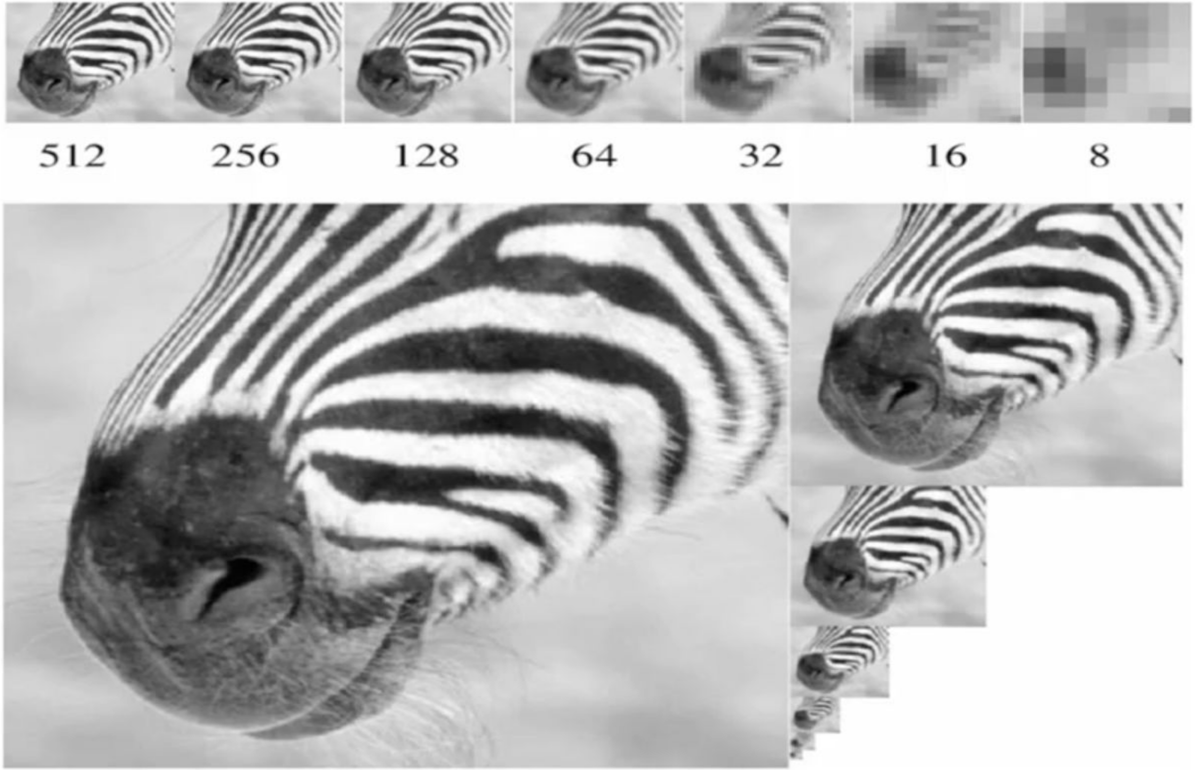
Pyramidal image processing

- Image pyramid is a powerful, but conceptually simple structure for representing images at more than one resolution.
- Originally it was devised for machine vision and image compression applications.
- An image pyramid is a collection of decreasing resolution images arranged in the shape of a pyramid.

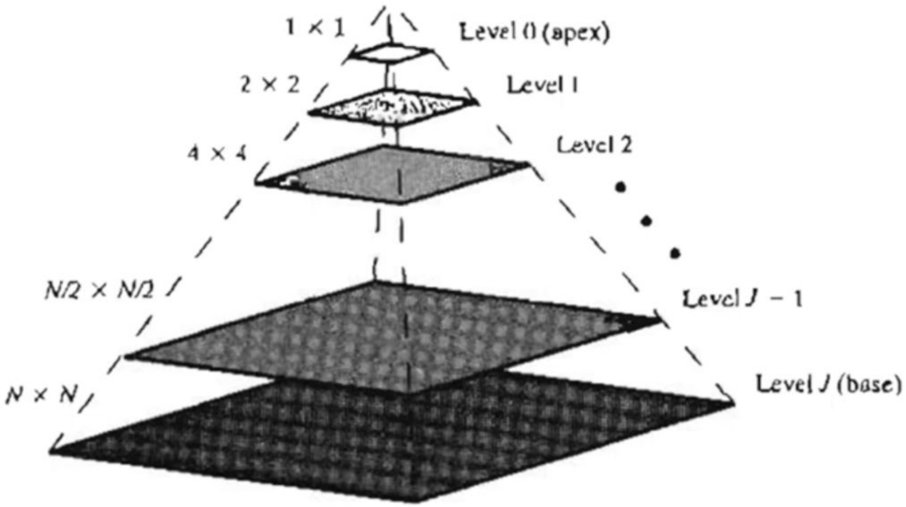
Pyramidal image processing



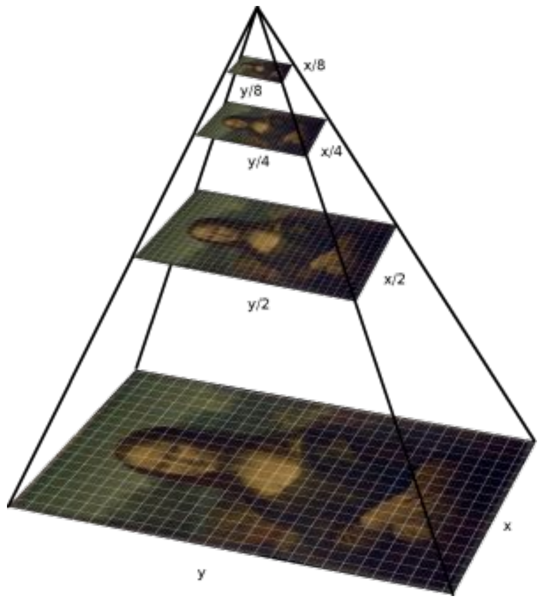
Pyramidal image processing



Pyramidal image processing



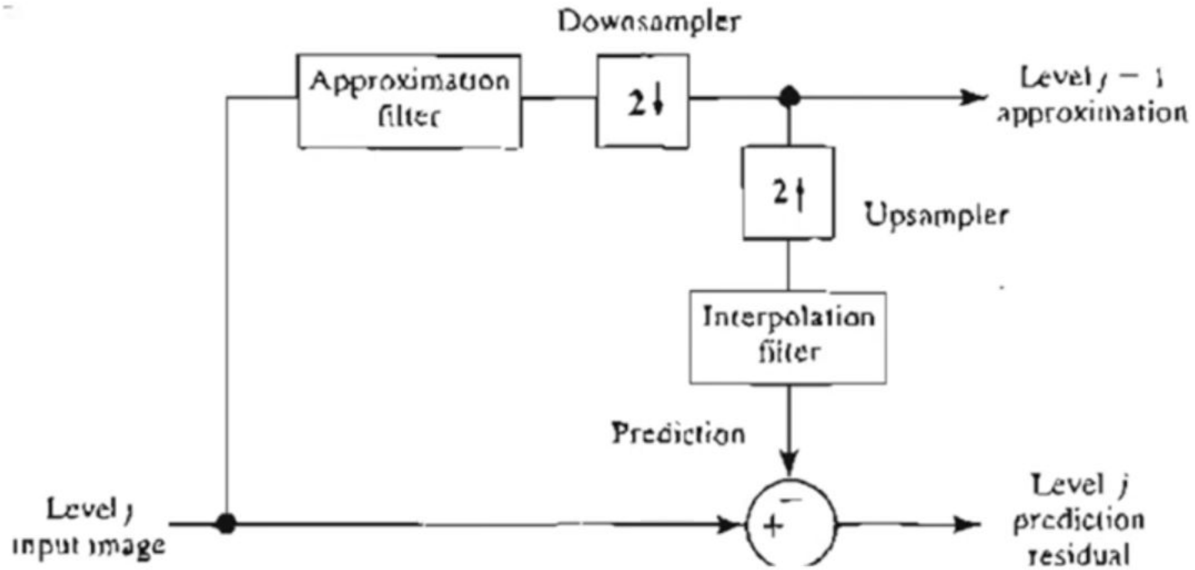
A pyramidal image structure



Pyramidal image processing

- The base of the pyramid contains a high-resolution representation of the image being processed; the apex contains a low-resolution approximation.
- As we move up in the pyramid, both size and resolution decrease.
- Since base level J is size 2^J X 2^J or N X N, where $J = log_2 N$, intermediate level j is size 2^j X 2^j , where $0 \leq j \leq J$.

Pyramidal image processing



System block diagram for creating pyramidal image structure

Pyramidal image processing

- Both the original image, which is at the base of the pyramid, and its P reduced resolution approximations can be accessed and manipulated directly.
- The information at level j is the difference between the level j approximation of the corresponding approximation pyramid and an estimate of that approximation based on the level j - 1 prediction residual.
- This difference can be coded-and therefore stored and transmitted-more efficiently than the approximation.

Pyramidal image processing

- Approximation and prediction residual pyramids are computed in an iterative fashion.
- Each pass is composed of three sequential steps:
- 1) Compute a reduced-resolution approximation of the input image.
- This is done by filtering the input and downsampling (i.e., subsampling) the filtered result by a factor of 2.

Pyramidal image processing

- A variety of filtering operations can be used:
- Neighborhood averaging, which produces a mean pyramid,
- Lowpass Gaussian filtering, which produces a Gaussian pyramid, or
- No filtering, which results in a subsampling pyramid.
- Without the filter, aliasing can become pronounced in the upper levels of the pyramid

Pyramidal image processing

2) Upsample the output of the previous step-again by a factor of 2-and filter the result.

- This creates a prediction image with the same resolution as the input.
- The interpolation filter determines how accurately the prediction approximates the input to Step 1.
- If the interpolation filter is omitted, the prediction is an upsampled version of the Step I output and the blocking effects of pixel replication may become visible.

Pyramidal image processing

3) Compute the difference between the prediction of Step 2 and the input to Step 1.

- This difference, labeled the level j prediction residual, can be later used to reconstruct progressively the original image.
- In the absence of quantization error, a prediction residual pyramid can be used to generate the corresponding approximation pyramid, including the original image, without error.

Pyramidal image processing

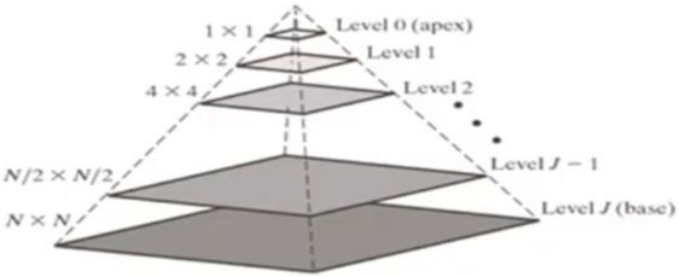
- Executing this procedure P times produces two intimately related P + 1 level approximation and prediction residual pyramids.
- The level j - 1 approximation outputs are used to populate the approximation pyramid; the level j prediction residual outputs are placed in the prediction residual pyramid.
- If a prediction residual pyramid is not needed, Steps 2 and 3, together with the upsampler, interpolation filter, and summer, can be omitted.

Pyramidal image processing

Approximation pyramid:

- At each reduced resolution level we have a filtered and downsampled image.

$$f_{\downarrow 2}(n) = f(2n)$$

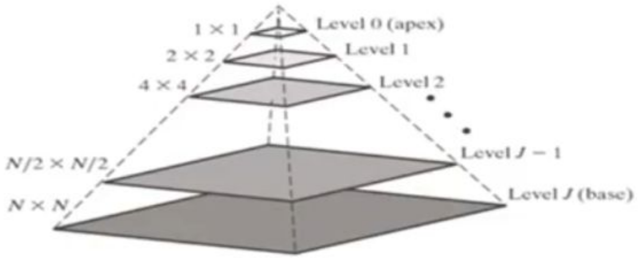


Pyramidal image processing

Prediction pyramid:

- A prediction of each high resolution level is obtained by upsampling (inserting zeros) the previous low resolution level (prediction pyramid) and interpolation (filtering).

$$f_{2\uparrow}(n) = \begin{cases} f(n/2) & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$



Pyramidal image processing

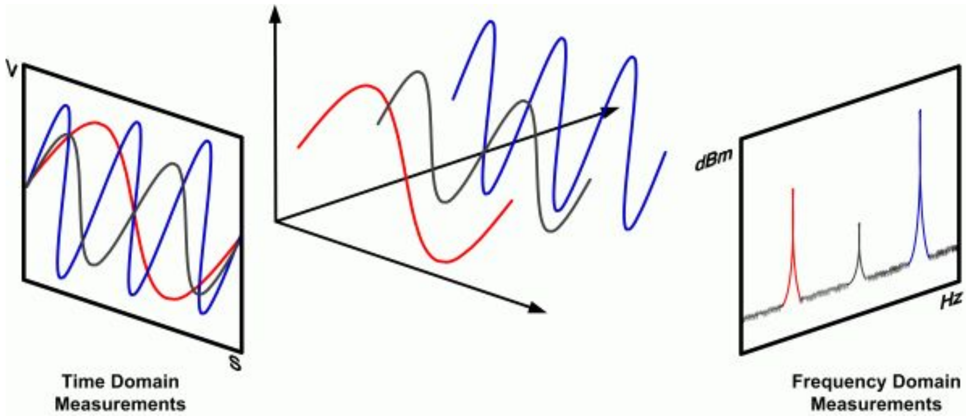
Prediction residual pyramid:

- At each resolution level, the prediction error is retained along with the lowest resolution level image.
- The original image may be reconstructed from this information.

Wavelet Transform

Fourier Transform

Fourier Transform is a mathematical model which helps to transform the signals between two different domains, such as transforming signal from frequency domain to time domain or vice versa. Fourier transform has many applications in Engineering and Physics, such as signal processing, RADAR, and so on.



Fourier Transform

The Mathematical expression for Fourier transform is

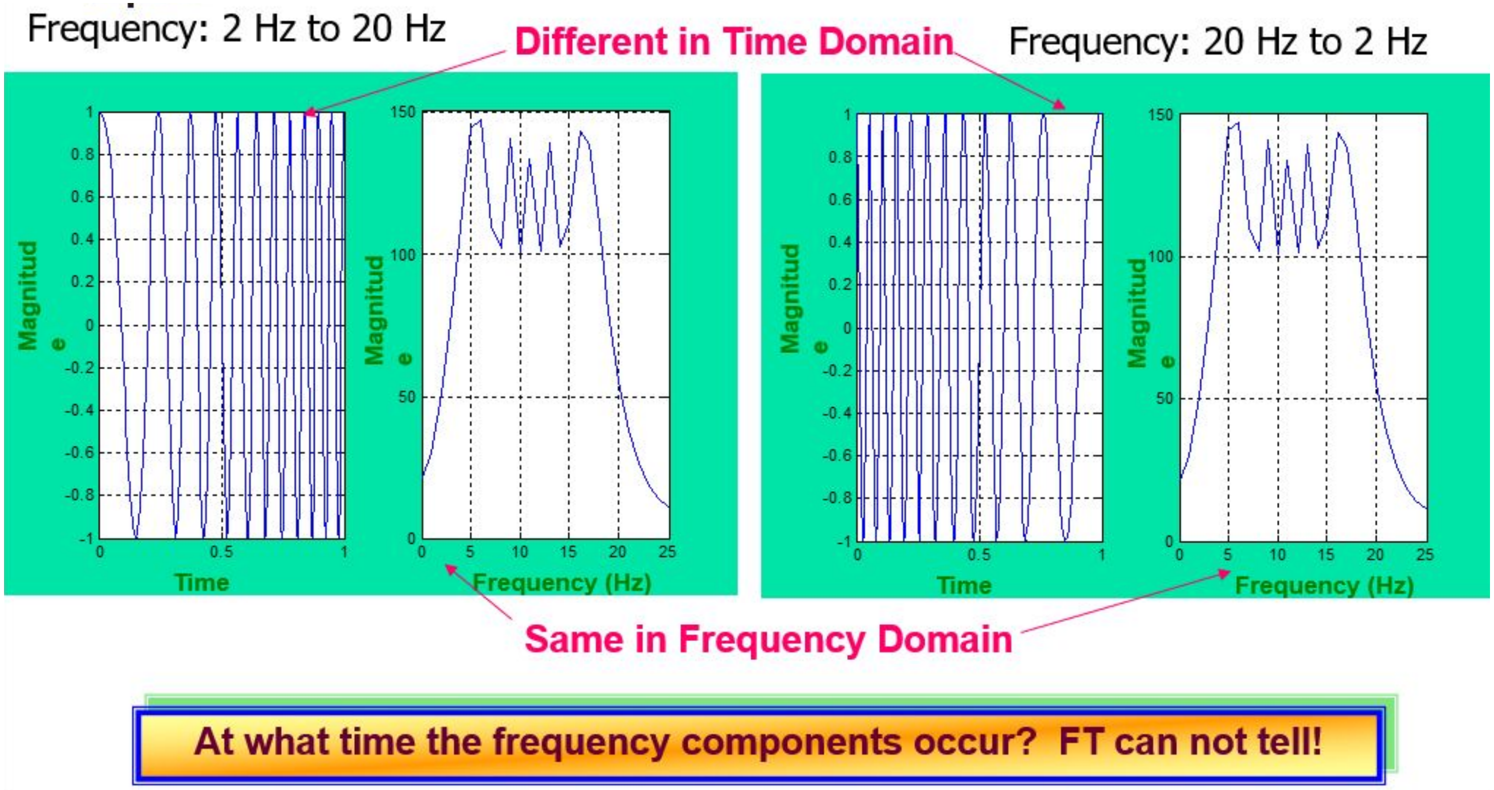
$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

And the inverse of this function can be considered as the function of time which we use for converting the frequency domain function to the time domain function.

Inverse Fourier Transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$$

Fourier Transform

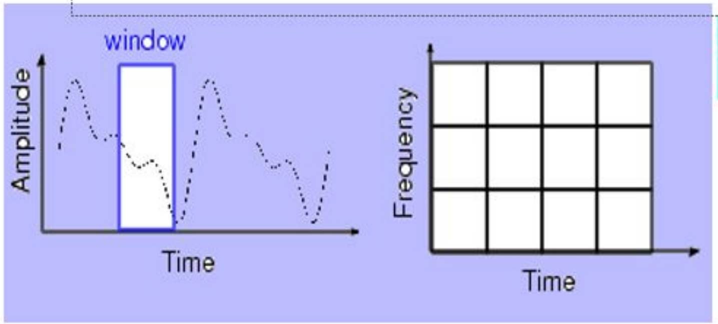


Fourier Transform

- FT Only Gives what Frequency Components Exist in the Signal
- The Time and Frequency Information can not be Seen at the Same Time
- Time-frequency Representation of the Signal is Needed

SORT TIME FOURIER TRANSFORM (STFT)

- Dennis Gabor (1946) Used STFT
  - To analyze only a small section of the signal at a time -- a technique called *Windowing the Signal*.
- The Segment of Signal is Assumed *Stationary*
- A 3D transform



$$\mathrm{STFT}_X^{(\omega)}(t', f) = \int_t \left[x(t) \bullet \omega^*(t - t') \right] \bullet e^{-j2\pi f t} dt$$

$\omega(t)$: the window function

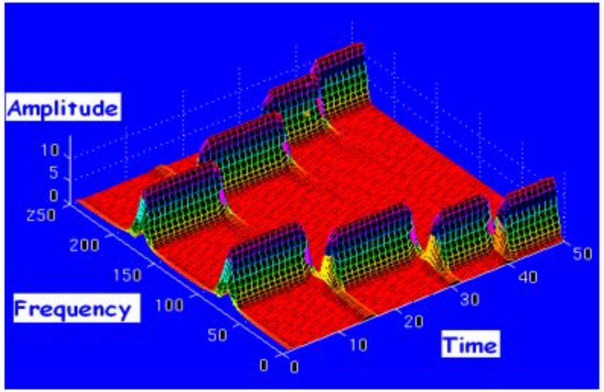
A function of time and frequency

PARITA PATEL

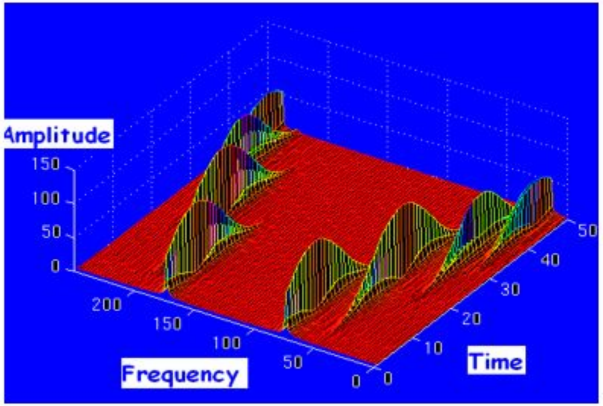
SORT TIME FOURIER TRANSFORM (STFT)

- Unchanged Window
- Dilemma of Resolution
  - Narrow window -> poor frequency resolution
  - Wide window -> poor time resolution
- Heisenberg Uncertainty Principle
  - Cannot know what frequency exists at what time intervals

Via Narrow Window

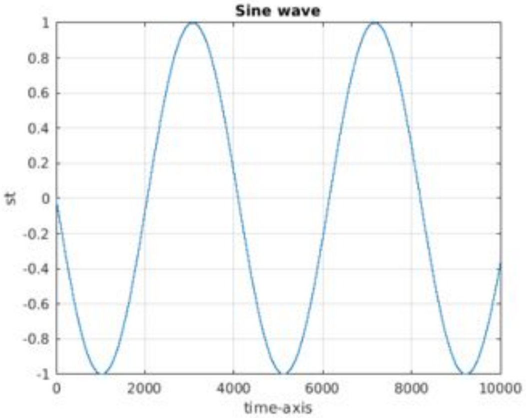


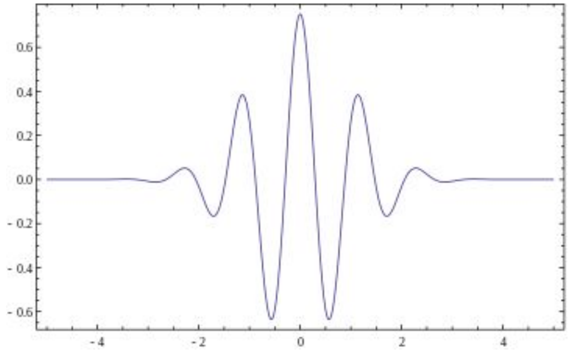
Via Wide Window



Wavelet Transform

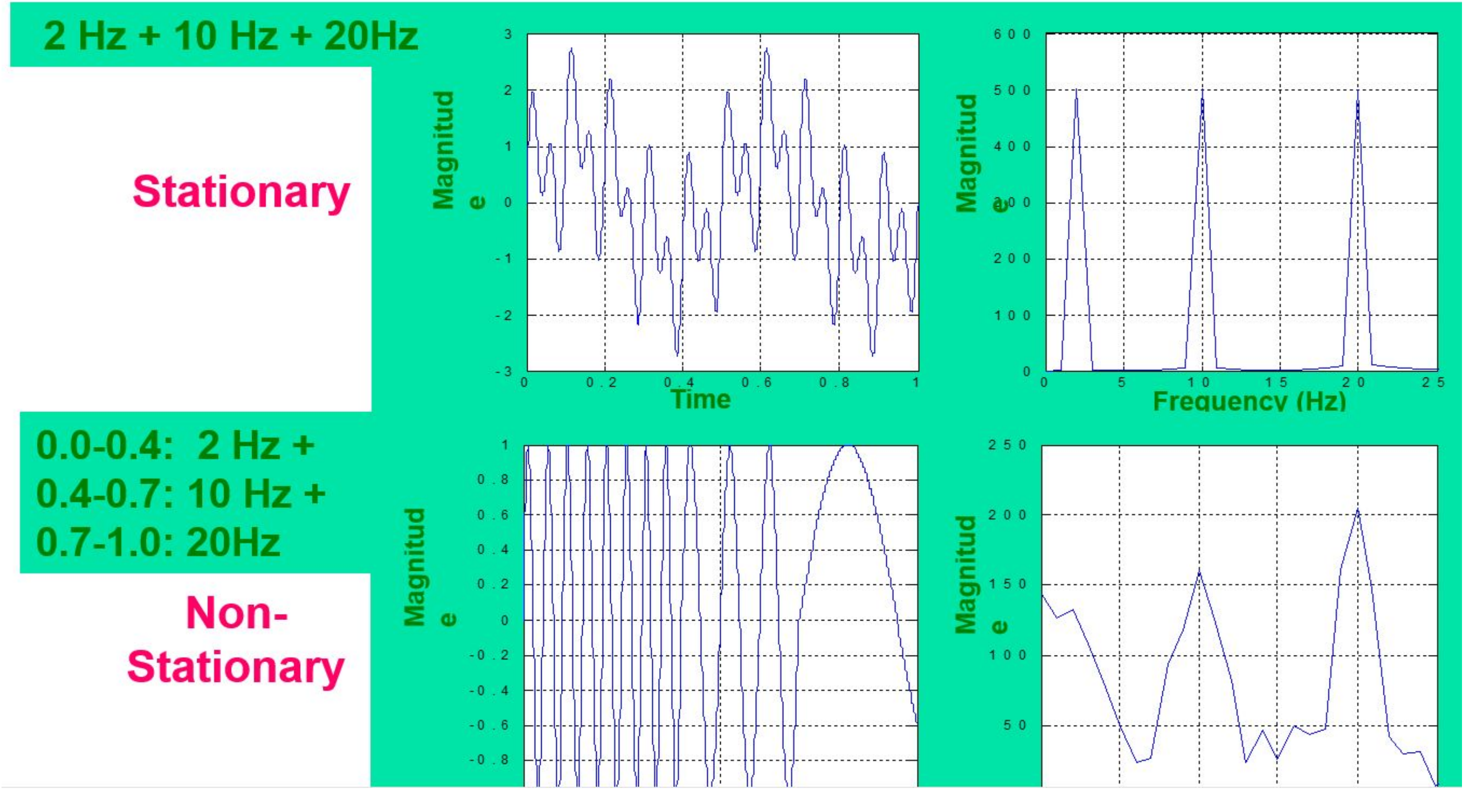
A wavelet is a small wave which has its energy concentrated in time to give a tool for analysis at transient, Non-stationary.





Wavelet

STATIONARITY OF SIGNAL



Wavelet Transform

- An important aspect of biomedical signals is that the information of interest is often a combination of phenomena that are transient (e.g., spike and action potentials) and diffuse (e.g., small oscillations and semi-periodic activations).

- Such phenomena are characterized by the local information that exist in the time-domain, frequency-domain, or both. To analyze such signals, there is a need for methods that are sufficiently versatile to capture events that present these extremes by localizing information from a time-frequency point of view.

- One of the heavily utilized time-frequency analysis methods is the wavelet transform and its generalization of wavelet-packet transform.

- Wavelets divide the signal of interest into different frequency components, whereby each component may be studied at a resolution matched to its scale.

Wavelet

normalization

shift in time

$$\Psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\,\Psi\!\left(\frac{t-\tau}{s}\right)$$

change in scale:
big s means long
wavelength

wavelet with
scale, s and time, τ

Mother wavelet

The “Wavelet” Family

- The wavelet functions or simply “**Wavelets**" have a gender: these are father wavelets (φ), also known as the scaling function, and mother wavelets (ψ).

- The father wavelet itself is constructed from the mother wavelet by dilating the mother wavelet. Dilating the mother wavelet produces a low frequency wavelet "father wavelet" that maps onto a low frequency region of the signal.

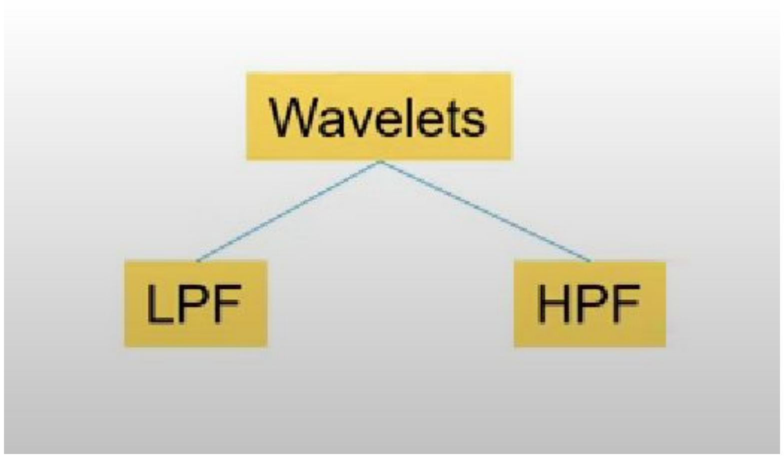
- On the other hand, compressing or squeezing the mother wavelet produces a high frequency wavelet that maps onto a higher frequency range of the signal.

$$\int \phi(t)dt = 1 \qquad \int \psi(t)dt = 0$$

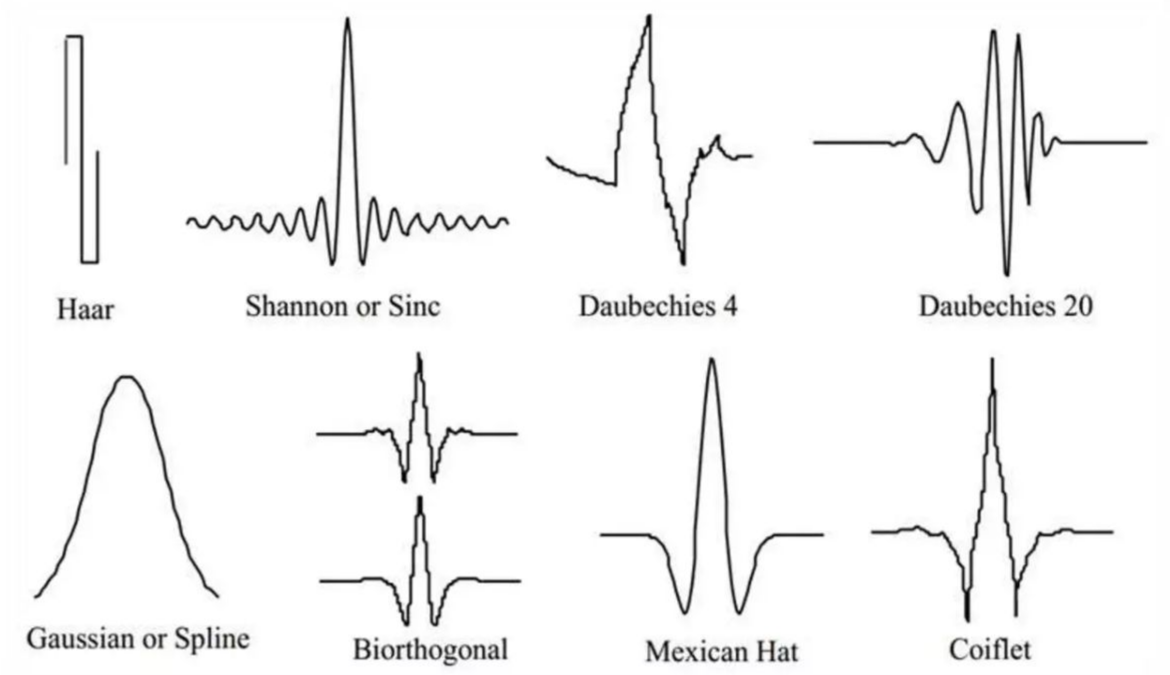
7

Wavelets

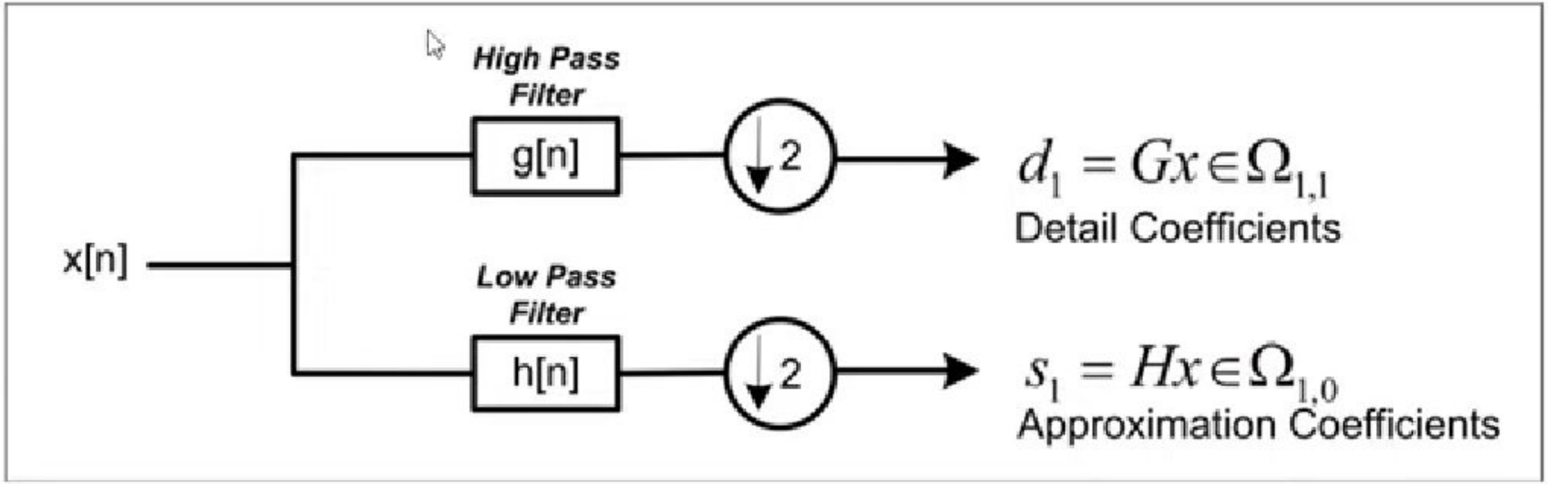
- A **Father wavelet** is good at representing the **smooth and low-frequency** parts of a signal
- A **Mother wavelet** is good at representing the detail and high-frequency parts of a signal



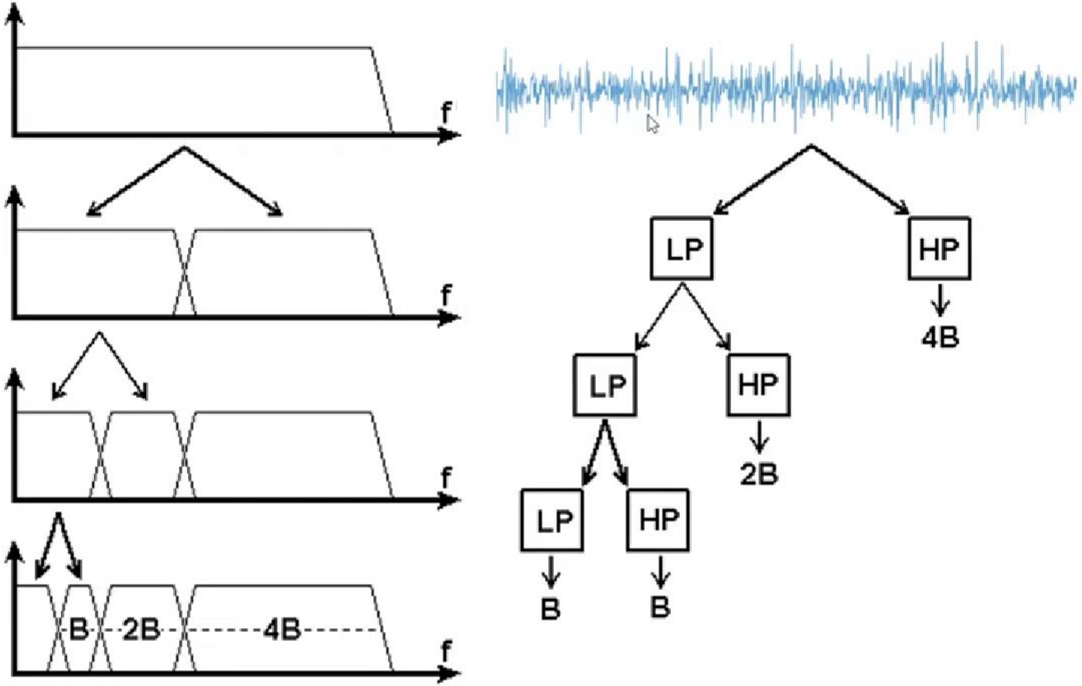
Here are some of the most popular mother wavelets :



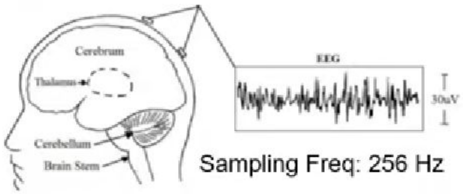
Discrete Wavelet Transform



Wavelet Decomposition

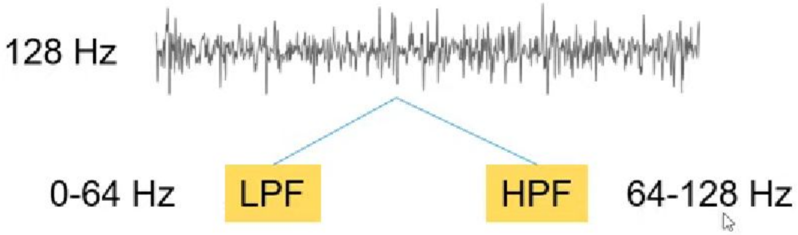


Decomposing Signals

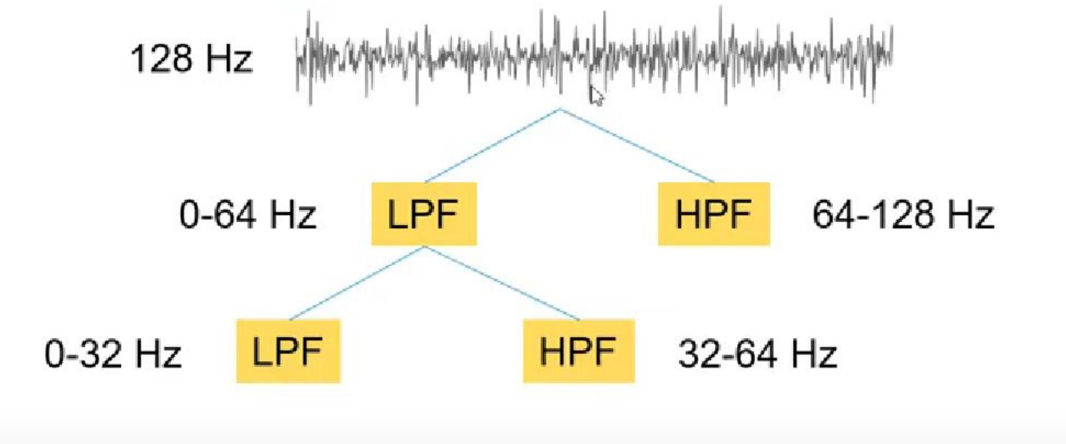


128 Hz

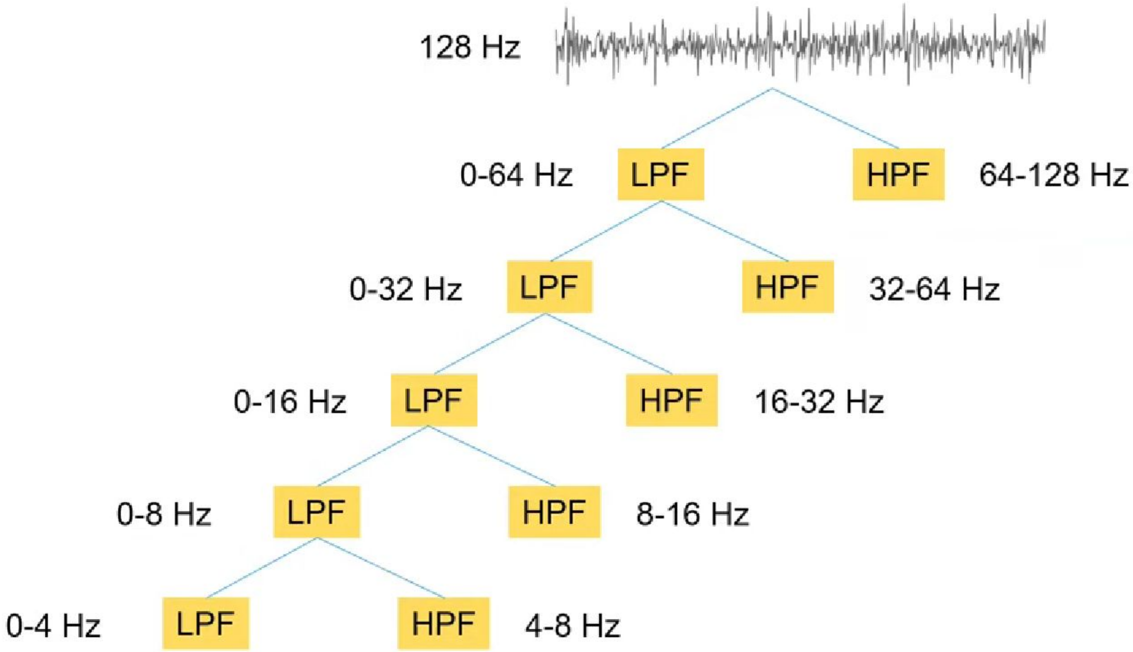
Decomposing Signals



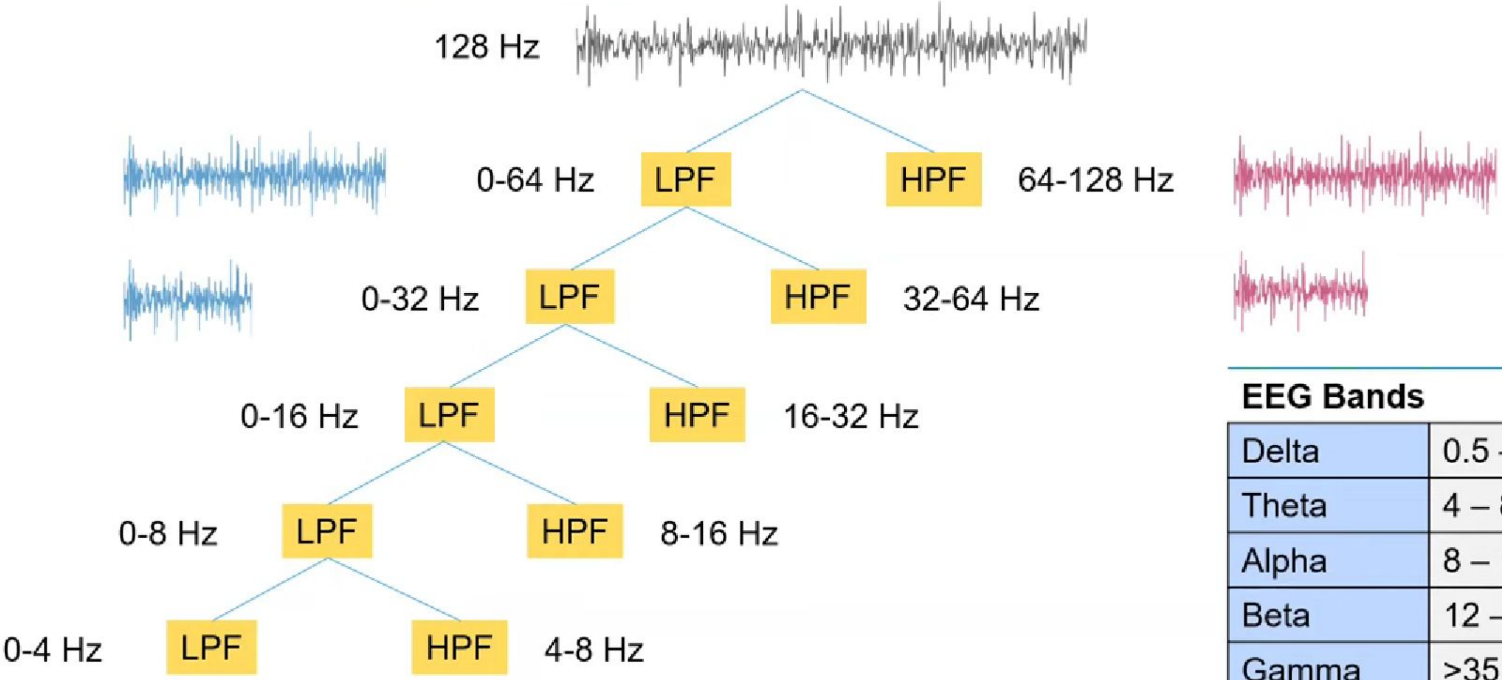
Decomposing Signals



Decomposing Signals



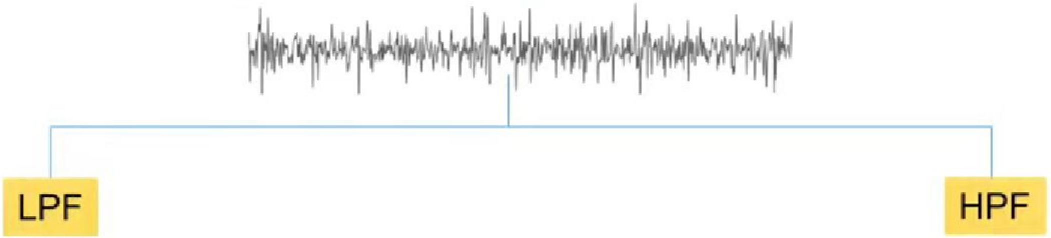
Decomposing Signals



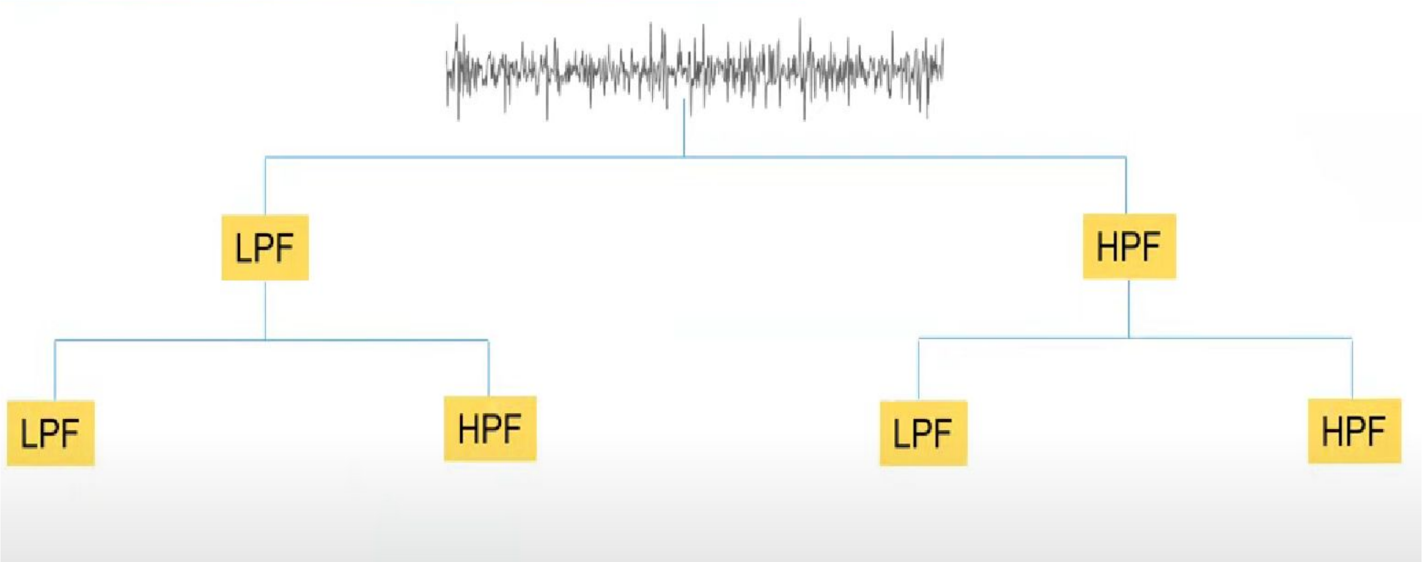
EEG Bands

Delta	0.5 – 4 Hz
Theta	4 – 8 Hz
Alpha	8 – 12 Hz
Beta	12 – 30 Hz
Gamma	>35 Hz

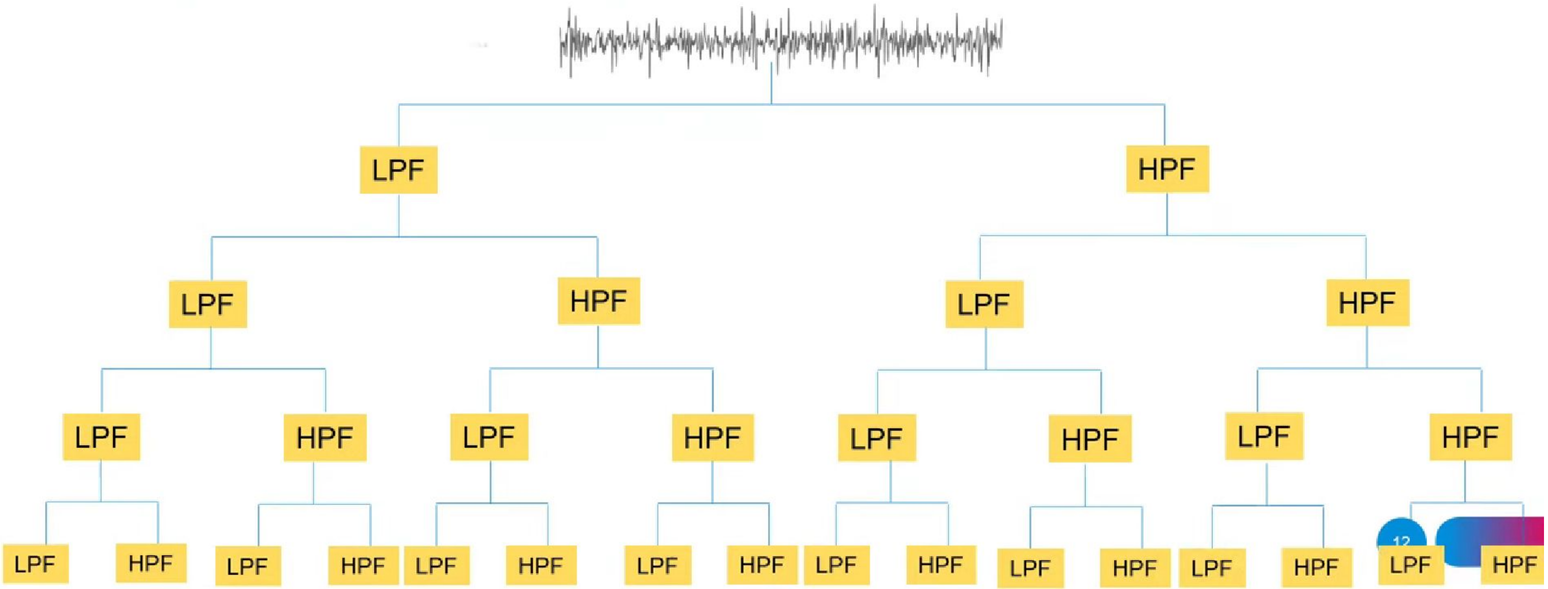
Wavelet Packet Transform



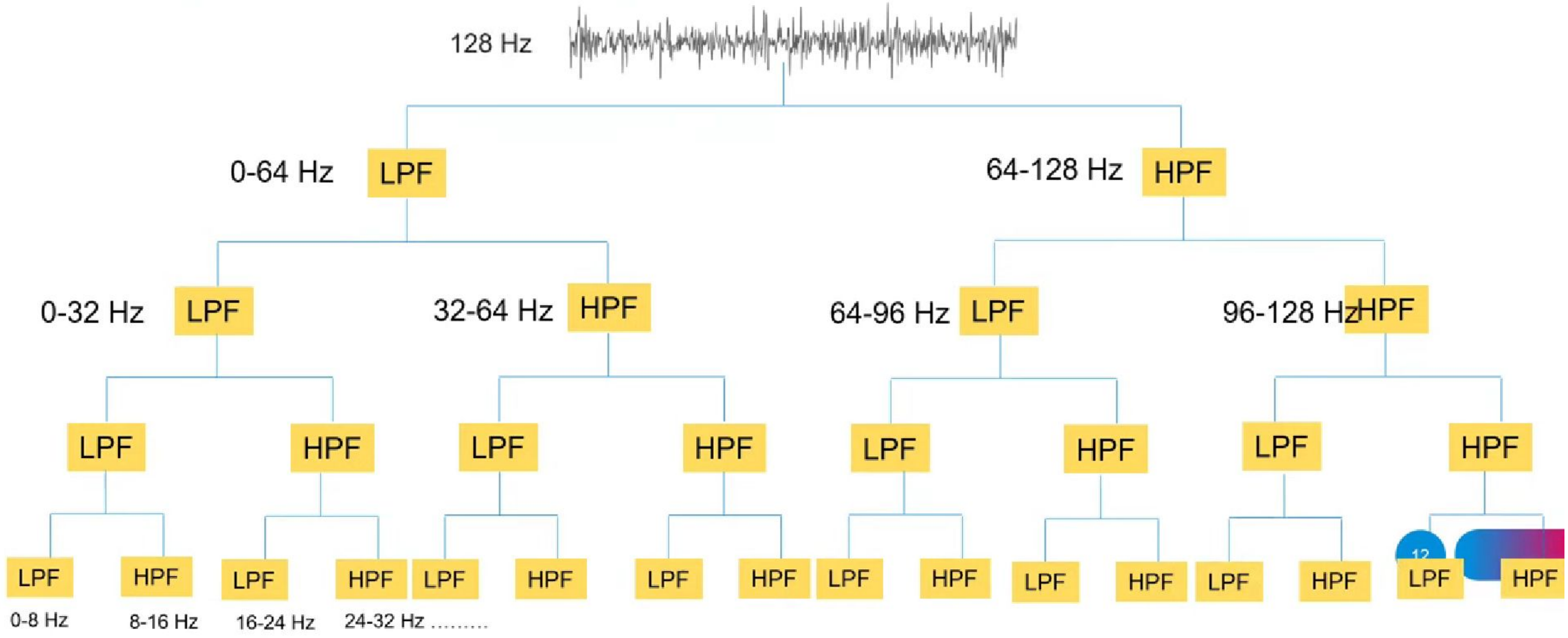
Wavelet Packet Transform



Wavelet Packet Transform

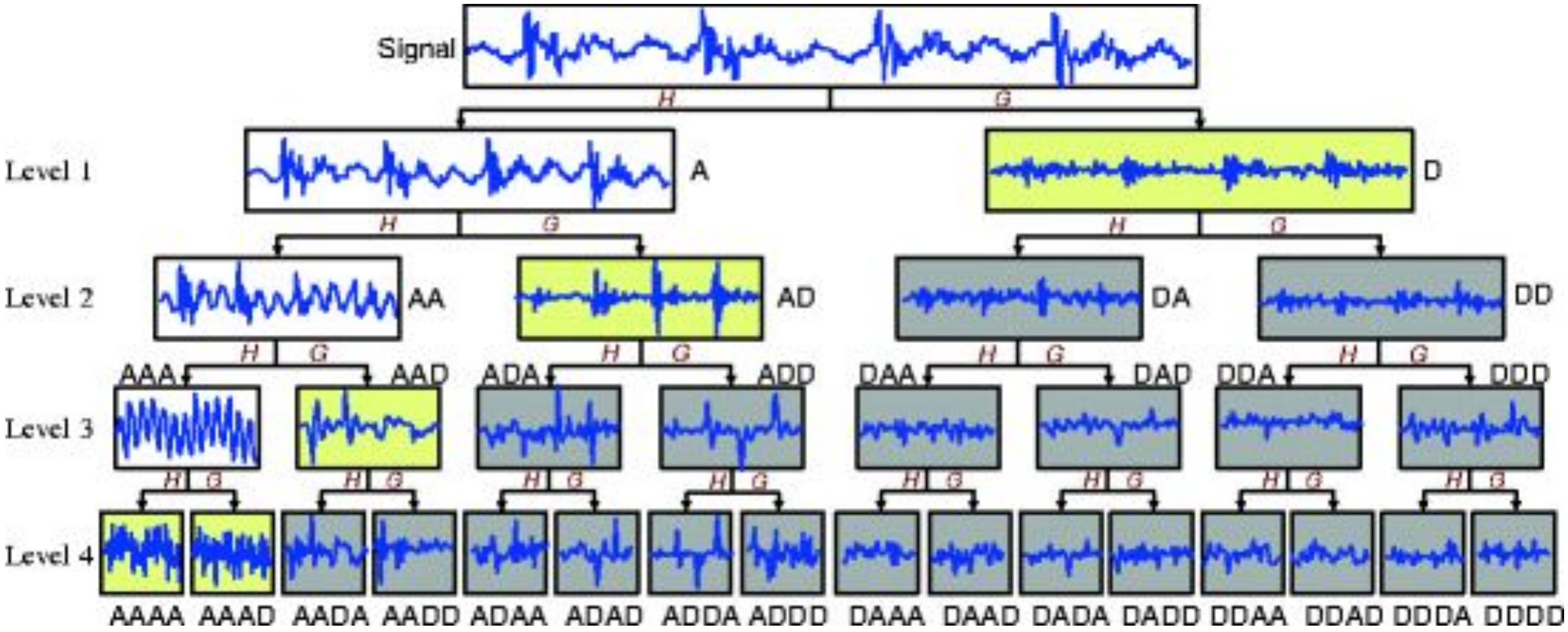


Wavelet Packet Transform



PARITA PATEL

Wavelet Packet Transform



Wavelet Applications

Typical Application Fields:

- Astronomy
- nuclear engineering
- sub-band coding
- signal and image processing
- Music
- magnetic resonance imaging
- speech discrimination
- optics
- Turbulence
- Earthquake-prediction
- human vision, and pure mathematics applications.

Wavelet Applications

Sample Applications:

- Identifying pure frequencies
- De-noising signals
- Detecting discontinuities and breakdown points
- Detecting self-similarity
- Compressing images

Difference between Fourier Transform and Wavelet Transform

Fourier Transform	Wavelet Transform
1. The Mathematical expression for Fourier transform is $$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$	1. The Mathematical expression for Wavelet transform is $$\left\langle f, \psi_{a,b} \right\rangle = \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right)\mathrm{d}t$$
2. It is applicable fo	2. It is appl Non-stationary Signals.
3. It has Zero time Resolution and Very High Frequency Resolution.	3. It has high time Resolution as well as high Frequency Resolution.
4. FT convert signal from time domain to frequency domain signal it provides two dimensional information about any signal ie. Frequency & Amplitude .	4. WT gives a complete three dimensional information about any signal ie. signal time, signal frequency and its amplitude.