3D Projections

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3D Viewing

- The 3D viewing process is inherently more complex than the 2 D viewing process.
- In two dimensional viewing we have 2D window and 2D viewport and objects in the world coordinates are clipped against the window and are then transformed into the viewport for display.
- The complexity added in the three dimensional viewing is because of the added dimension and the fact that even though objects are three dimensional the display devices are only 2D.
- The mismatch between 3D objects and 2D displays is compensated by introducing projections. The projections transform 3D objects into a 2D projection plane.

3D Viewing and projection

- Viewing in 3D involves the following considerations: - We can view an object from any spatial position,
- eg. In front of an object

  Behind the object

  In the middle of a group of objects

  Inside an object, etc.
- We have already seen how to display a 2 dimensional picture on a view window, we have seen line clipping and polygon clipping that can be used to clip those regions of a 2 dimensional picture that are outside the window boundaries. We have seen window to view port transformation for mapping a 2D scene to view port coordinates.

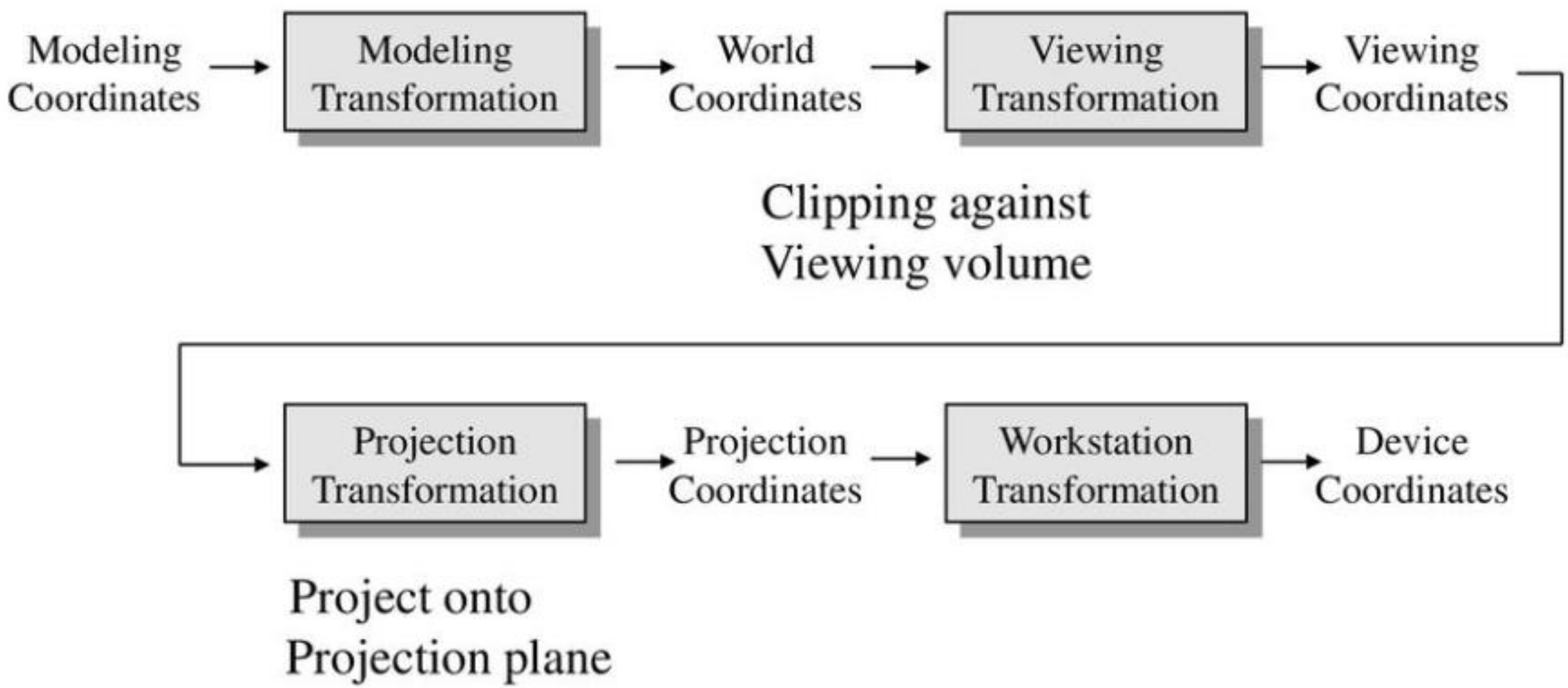
3D Viewing and projection

- For 3 dimensional applications, display of 3 dimensional objects involves a number of steps.
- Generating a view of a 3D scene is similar to taking a photograph of a scene using a camera.
- To take a photo, first we place the camera at a particular position. Then we change the direction of the camera.
- We can point the camera to a particular scene and we can rotate the camera around the 3D object.
- Finally, when we put the switch, the scene will be adjusted to the size of the window of the camera and light form the scene will be projected on to the camera film.

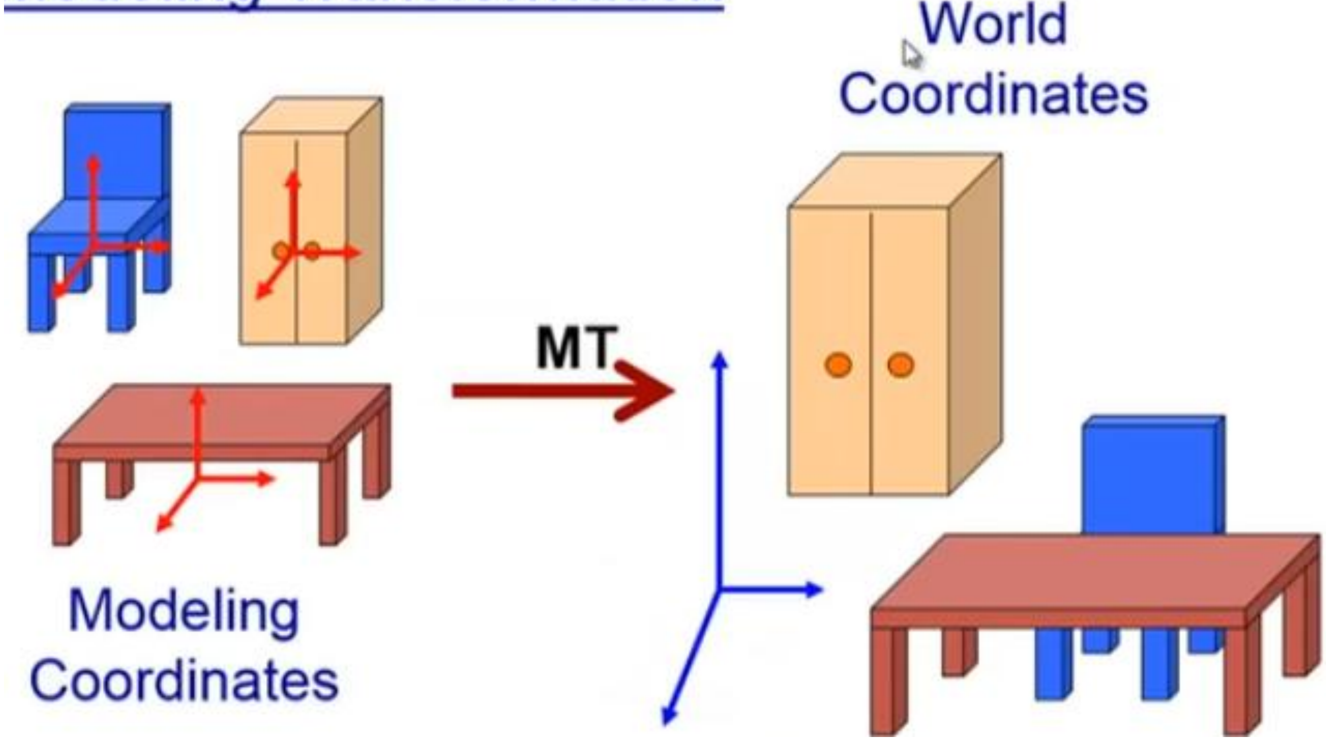
Steps involved in 3d viewing (conceptual)

- In 3 dimensional viewing, we specify a view volume. Then a projection on to a projection plane and a mapping of this projected scene on to the view surface.
- Objects in 3D are clipped against the view volume and are then projected on to a plane.
- The contents in the projection plane are mapped (transformed) in to the view port for display.

3D Viewing Pipeline



Modeling Transformation

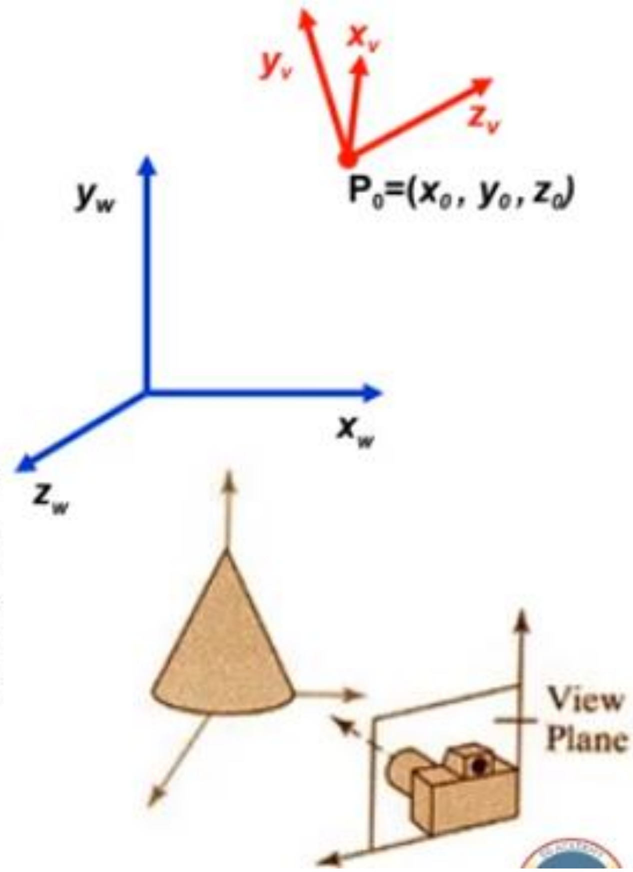


Viewing Coordinates

- Generating a view of an object in 3D is similar to photographing the object.
- Whatever appears in the viewfinder is projected onto the flat film surface.
- Depending on the position, orientation and aperture size of the camera corresponding views of the scene is obtained.

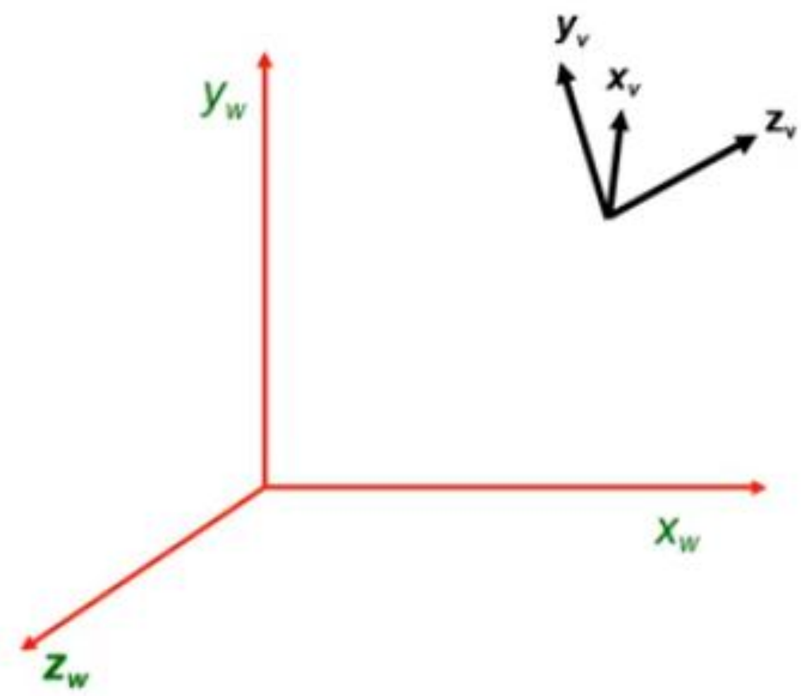
Specifying the viewing coordinates

- For a particular view of a scene first we establish viewing-coordinate system.
- A view-plane (or projection plane) is set up perpendicular to the viewing z-axis.
- World coordinates are transformed to viewing coordinates, then viewing coordinates are projected onto the view plane.



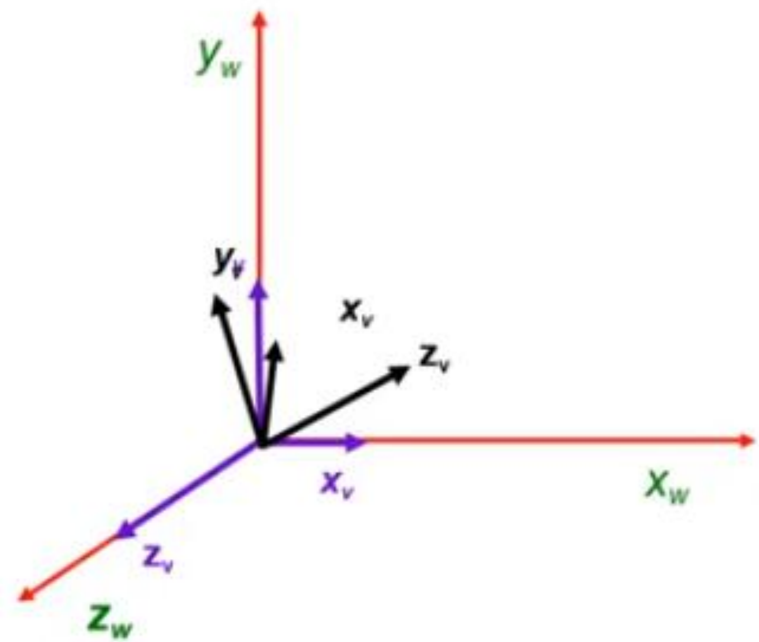
Transformation from world to viewing coordinates

First, we translate the view reference point to the origin of the world coordinate system



Transformation from world to viewing coordinates

Second, we apply rotations to align the x_v, y_v and z_v axis with the world x_w, y_w and z_w axis, respectively.



Transformation
from world to viewing coordinates

If the view reference point is specified at word position (x_0, y_0, z_0), this point is translated to the world origin with the translation matrix **T.**

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_o \\ 0 & 1 & 0 & -y_o \\ 0 & 0 & 1 & -z_o \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation from world to viewing coordinates

- The rotation sequence requires 3 coordinate-axis transformation depending on the direction of **N.**
- First we rotate around x_w-*axis* to bring z_v into the x_w -z_w plane.
- Then, we rotate around the world y_w axis to align the z_w and z_v *axes.*
- The final rotation is about the world z_w axis to align the y_w and y_v *axes.*

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\beta & -\sin\beta & 0 & 0 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

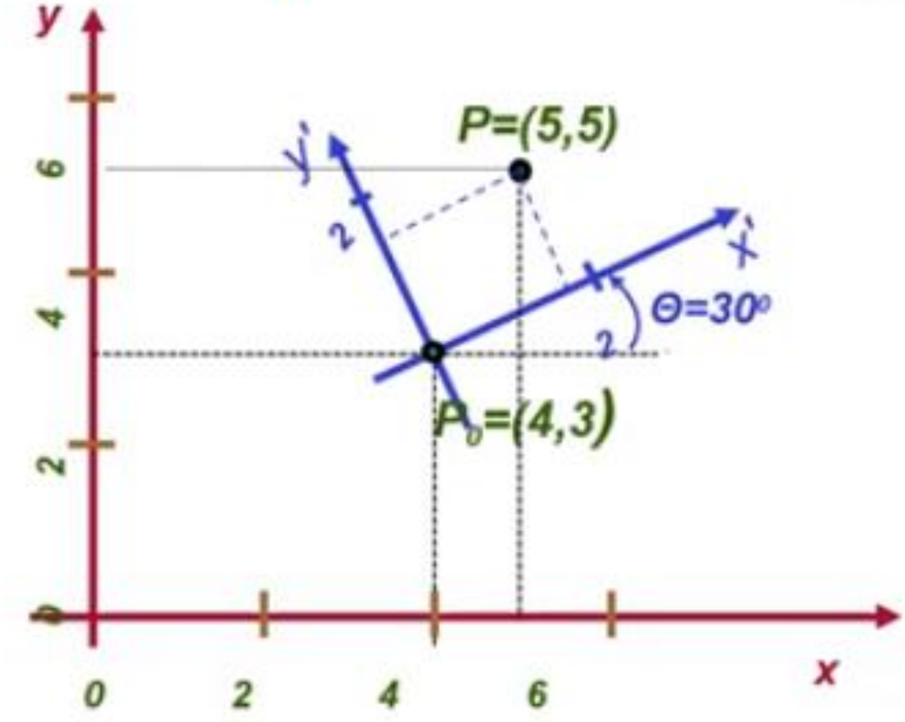
Transformation from world to viewing coordinates

The complete transformation from world to viewing coordinate transformation matrix is obtained as the matrix product

$$\mathbf{M}_{wc,vc} = \mathbf{R}_z \cdot \mathbf{R}_y \cdot \mathbf{R}_x \cdot \mathbf{T}$$

Transformation
from world to viewing coordinates

Example For 2d System

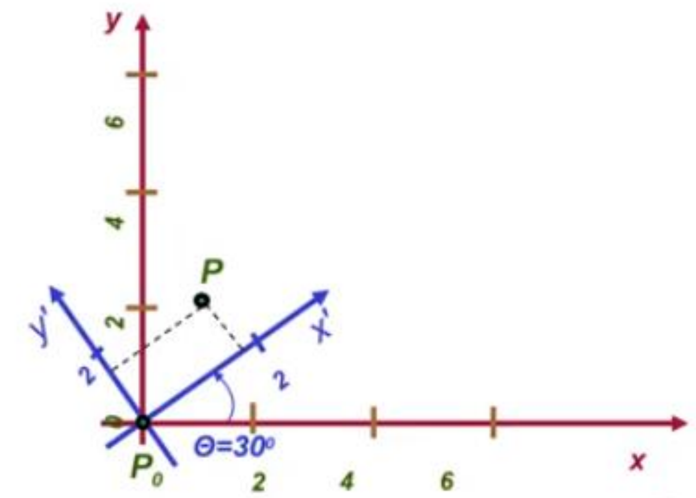


Transformation from world to viewing coordinates

Example For 2d System

Translation:

$$T = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

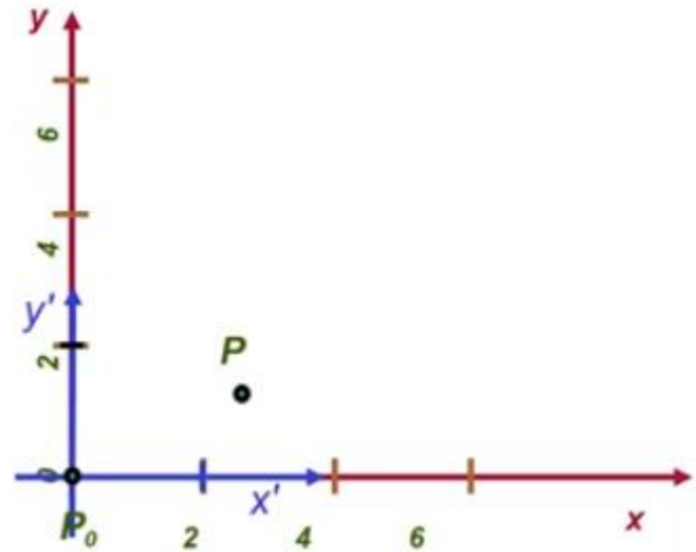


Transformation from world to viewing coordinates

Example For 2d System

Rotation

$$R = \begin{bmatrix} .866 & 0.5 & 0 \\ -0.5 & .866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Transformation from world to viewing coordinates

Example For 2d System

New coordinate

$$\mathbf{M}_{wc.vc} = \begin{bmatrix} .866 & 0.5 & 0 \\ -0.5 & .866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{X} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} .866 & 0.5 & -4.964 \\ -0.5 & .866 & -.598 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{X} \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.866 \\ 1.232 \\ 1 \end{bmatrix}$$

Projection Transformations

The projection transform defines the *viewing volume*

The viewing volume determines
• How an object is projected onto the screen (i.e., orthographic projection or perspective projection)
• Which objects or portions of objects are clipped out of the final image

Projection

- For centuries, artists, engineers, designers, drafters, and architects have been facing difficulties and constraints imposed by the problem of representing a three-dimensional object or scene in a two-dimensional medium - the problem of projection.
- The implementers of a computer graphics system face the same challenge. We have seen the method to convert world coordinate description of objects to viewing coordinates. Next we have to project 3 dimensional objects on to the 2 dimensional view planes.
- Projection is the process of converting 3D object into 2D object.

Projection

- There are two basic methods of projection

  1) Parallel Projection

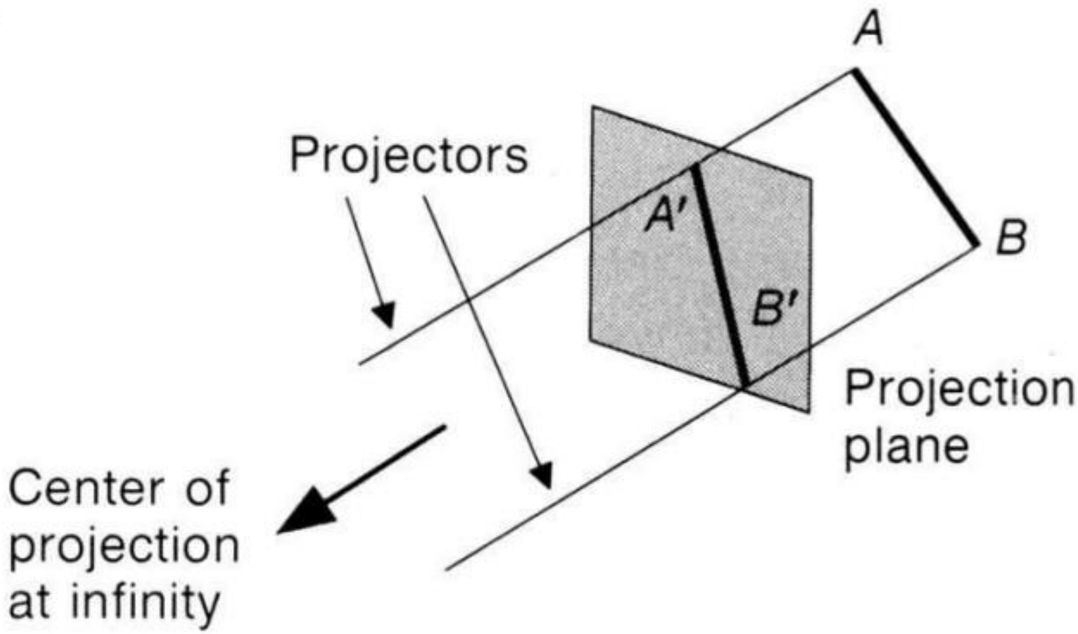
  2) Perspective Projection.
- Key factor is the center of projection. If distance to center of projection is finite : Perspective, If infinite : Parallel

Parallel Projection

- In a parallel projection, coordinate positions are transformed to the view plane along parallel lines. Parallel projection discards z-coordinate and parallel lines from each vertex on the object are extended until they intersect the view plane.
- In parallel projection, we specify a direction of projection instead of center of projection. In parallel projection, the distance from the center of projection to project plane is infinite.
- In this type of projection, we connect the projected vertices by line segments which correspond to connections on the original object.

Parallel Projection

Parallel projections are less realistic, but they are good for exact measurements. In this type of projections, parallel lines remain parallel and angles are not preserved. Various types of parallel projections are shown in the following hierarchy.



Parallel Projection

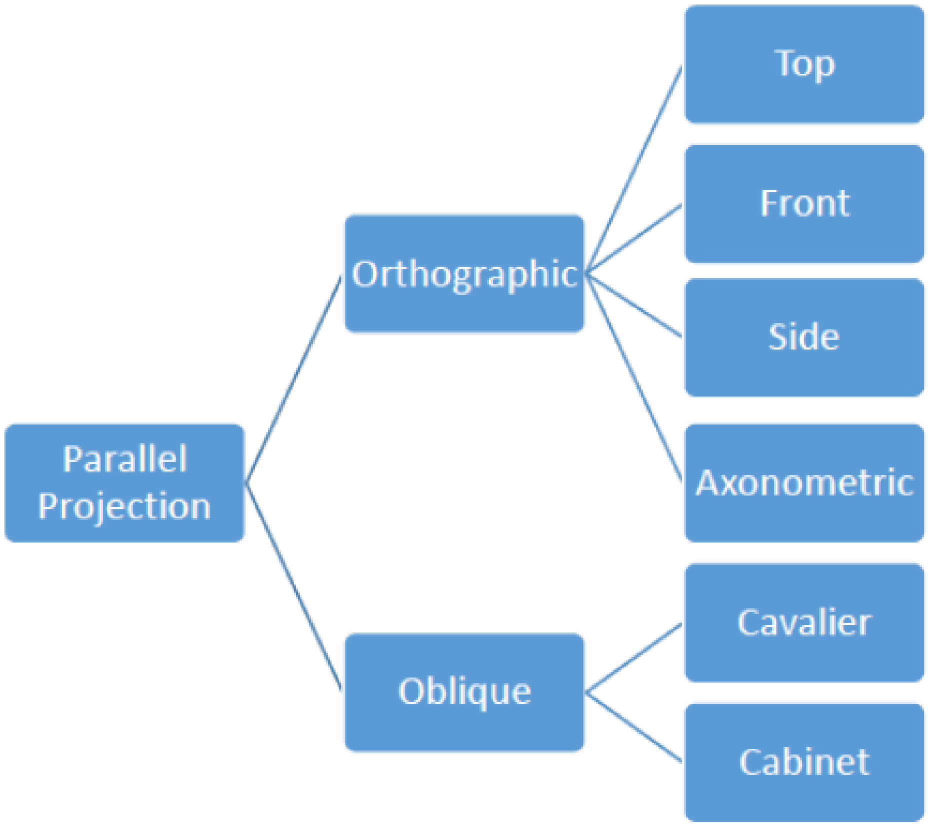
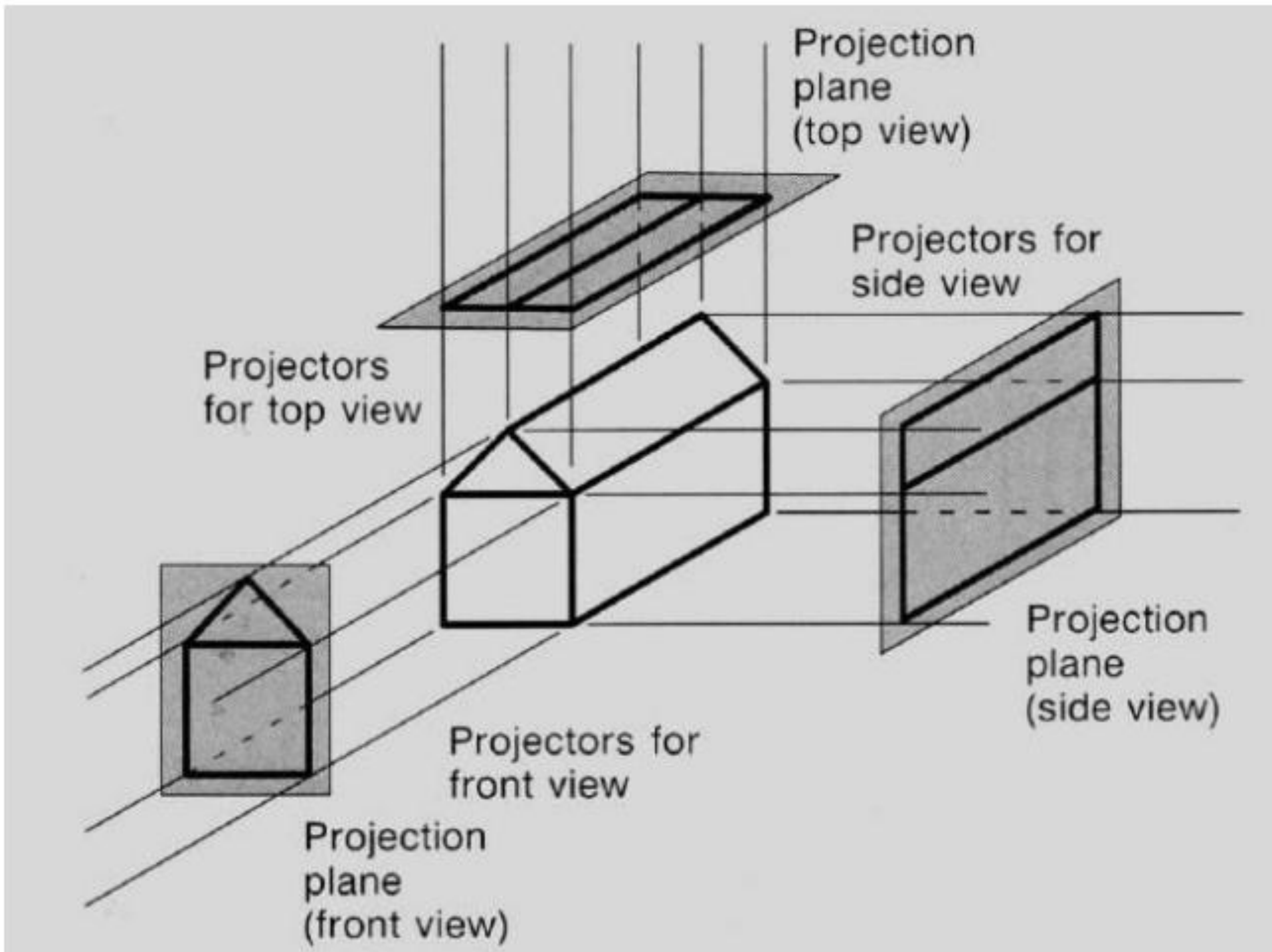


Figure: Types of Parallel Projection

Orthographic parallel projection

- When the projection is perpendicular to the view plane, we have an orthographic parallel projection.
- Orthographic projections are most often used to produce the front views and top view of the object.
- Front, side, and rear orthographic projections of an object are called elevations; and a top orthographic projection is called a plan view.
- Engineering and architectural drawings commonly employ these orthographic projections, because lengths and angles are accurately depicted, and can be measured from the drawings.

Orthographic parallel projection

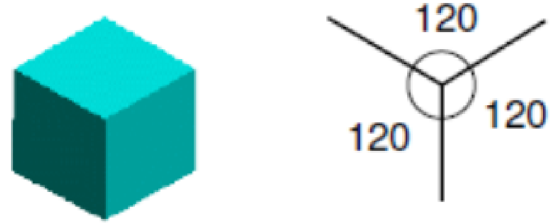


Axonometric Projections

- Orthographic projections that show more than one side of an object are called axonometric orthographic projections.
- The most common axonometric projection is an isometric projection where the projection plane intersects each coordinate axis in the model coordinate system at an equal distance.
- In this projection parallelism of lines are preserved but angles are not preserved.

Axonometric Projections

- Isometric: all angles between principal axes are equal

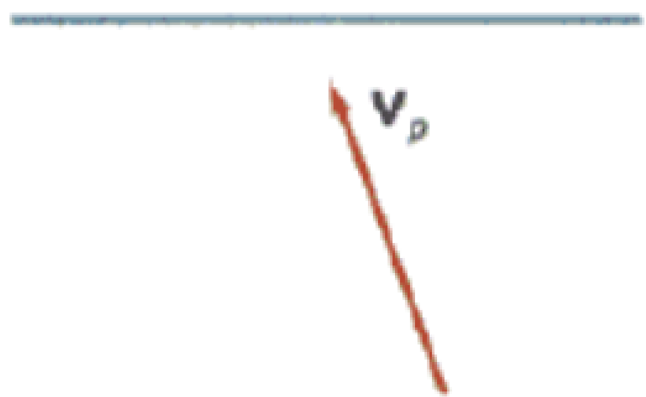


- Dimetric: angles between two principal axes are equal

- Trimetric: all angles different

Oblique parallel projection

- An oblique projection is obtained by projecting points along parallel lines that are not perpendicular to the projection plane. Objects can be visualized better then with orthographic projections. Common oblique parallel projections are Cavalier and Cabinet



Oblique Projection

Oblique parallel projection

- There are two types of oblique projections: Cavalier and Cabinet.
- The Cavalier projection makes 45° angle with the projection plane. The projection of a line perpendicular to the view plane has the same length as the line itself in Cavalier projection.
- The Cabinet projection makes 63.4° angle with the projection plane. In Cabinet projection, lines perpendicular to the viewing surface is projected at ½ their actual length.

Oblique parallel projection

- Cavalier: Angle between projectors and projection plane is 45◦. Depth is projected full scale.
- Cabinet: Angle between projectors and projection plane is 63.4˚. Depth is projected ½ scale. Both the projections are shown in the following figure:

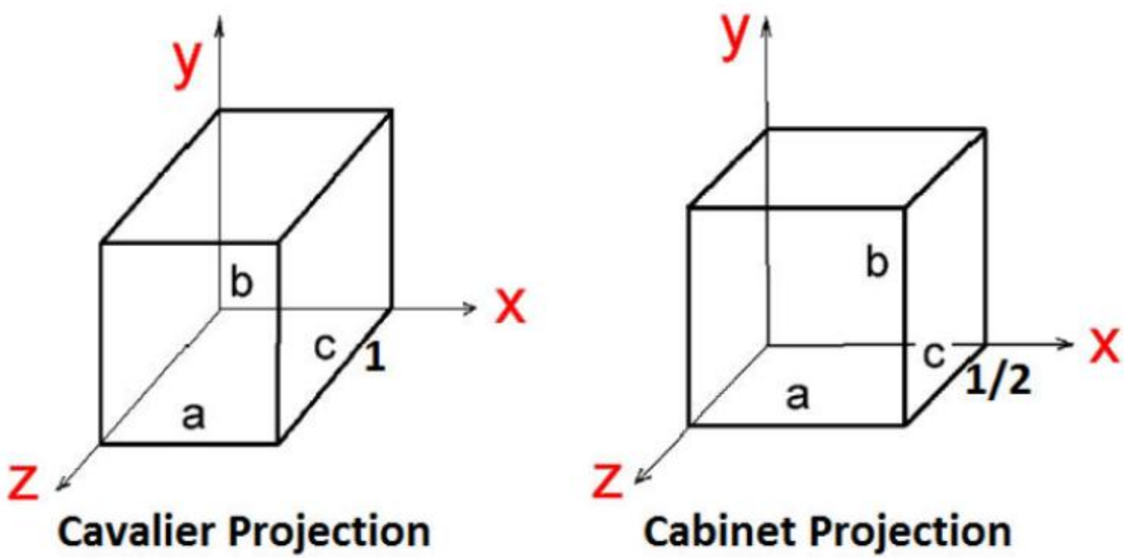
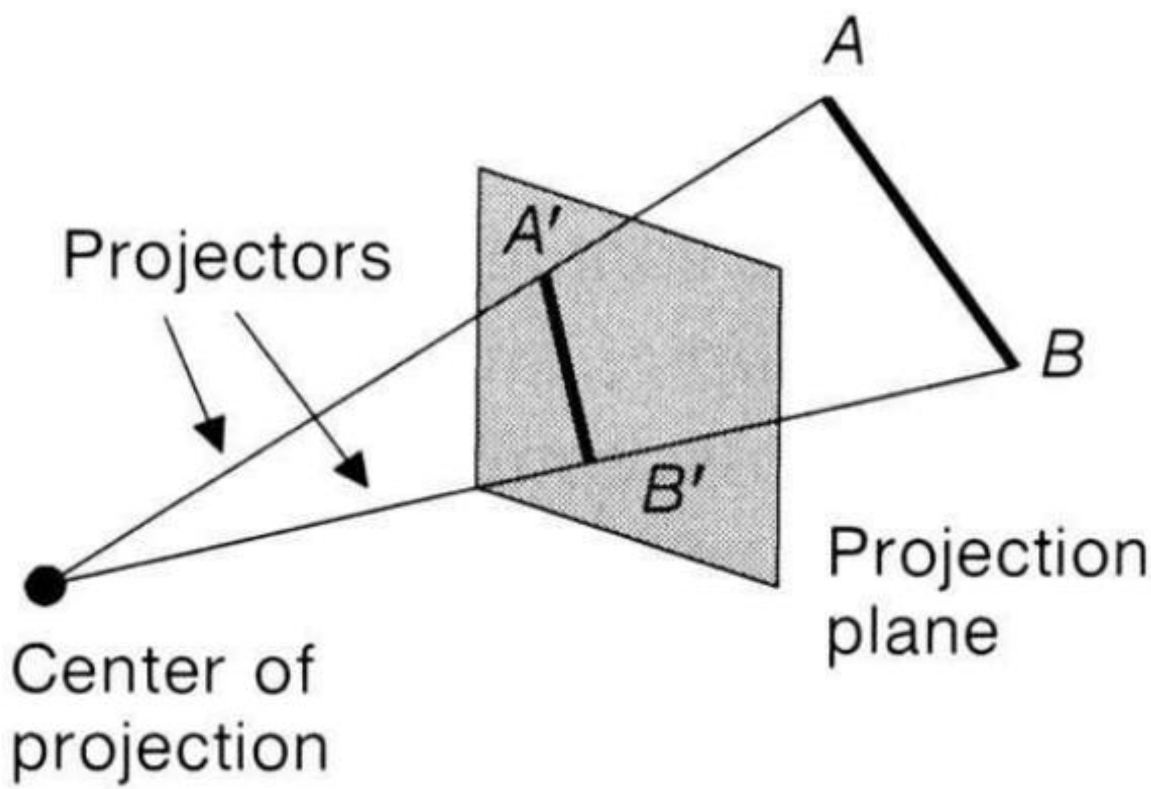


Figure: Cavalier & Cabinet Projection

Perspective Projection

- In perspective projection, the distance from the center of projection to project plane is finite and the size of the object varies inversely with distance which looks more realistic.
- Visual effect is similar to human visual system. In perspective projection, object positions are transformed to the view plane along lines that converge to a point called projection reference point (center of projection).
- Single point centre of projection (i.e. projection lines converge at a point).
- Shapes are projected smaller as their distances to the view plane increase.
- More realistic (human eye is a perspective projector).

Perspective Projection



Perspective Projection

- There are 3 types of perspective projections which are shown in the following chart.
- One point perspective only z axis intersects single vanishing point. One point perspective projection is simple to draw.
- Two point perspective x and z axes intersect two vanishing points. Two point perspective projection gives better impression of depth.
- Three point perspective all axes intersect three vanishing points. Three point perspective projection is most difficult to draw.

The following figure shows all the three types of perspective projection:

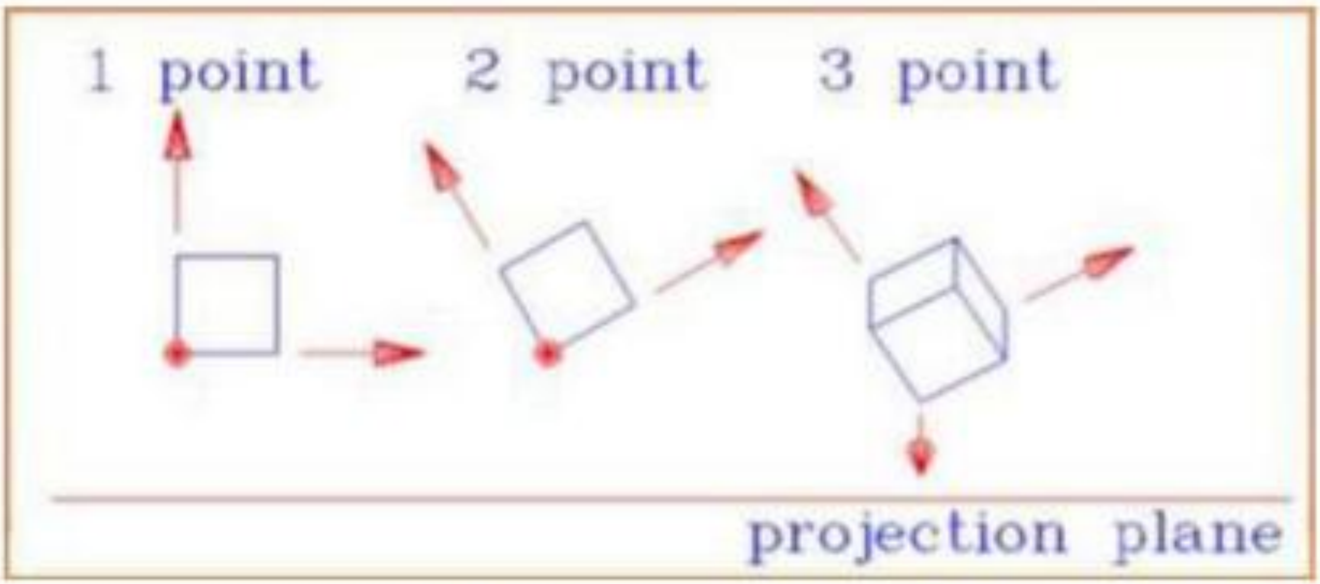
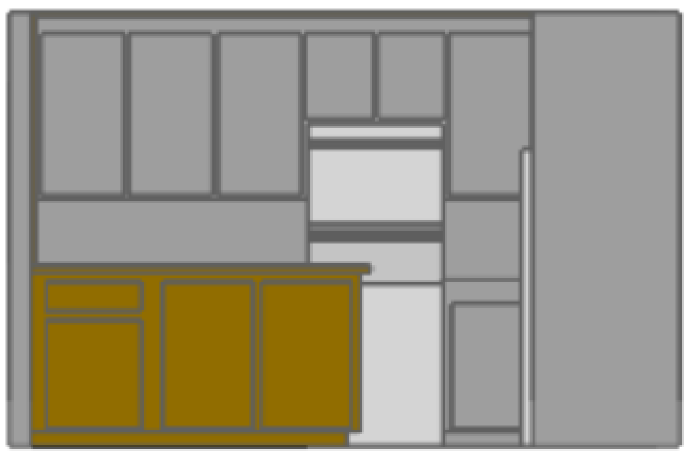
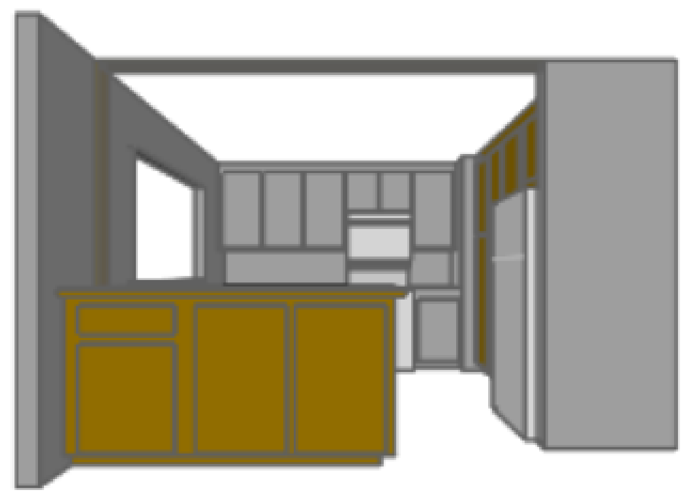


Figure: 1-point, 2-point, 3-point perspective projection

- The following illustration shows the same model in both a parallel projection and perspective projection. Both are based on the same viewing direction.



Parallel projection



Perspective projection