Pset 1. Please submit your solution sheets in class on Tuesday Oct.10. 2017

1. Compute $\text{tr}(A)$, $\det(A)$, A^{-1} and AB and CA^{-1}, where

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 2 & 3 & 1 \\ 2 & 7 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 5 & 0 & 10 \\ 2 & 3 & 1 & -1 \\ 2 & 7 & 9 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 5 & 0 \\ 2 & 3 & 1 \\ 2 & 7 & 9 \\ -2 & 3 & 7 \end{bmatrix}.$$

2. Compute

(1)
$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}^{10}$$

(2)
$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{100}$$

(3)
$$\begin{bmatrix} a & 1 & & & \\ & a & 1 & & \\ & & \ddots & \ddots & \\ & & & a & 1 \\ & & & & a \end{bmatrix}^{n}$$

3. For any square matrix $A \in M_n(\mathbb{R})$, show that $\text{rank}(A^n) = \text{rank}(A^{n+1})$

4. Solve the following equations: (If there is no solution, give the reason.)

$$\begin{bmatrix} 1 & 2 & 0 & -4 \\ 1 & -1 & -4 & 9 \\ 2 & -3 & 1 & 5 \\ 3 & -2 & -5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3 \\ 22 \\ -3 \\ 3 \end{bmatrix}$$

5. Find all the eigenvalues and eigenvectors for A. Is A similar to some diagonal matrix?

$$A = \begin{bmatrix} 3 & 3 & -2 \\ 0 & -1 & 0 \\ 4 & 2 & -3 \end{bmatrix}$$