** PROBABILITY

→ Random exp: where outcome not certain.

→ For each random exp, Sample Space is defined: (Set of all possible outcomes)

→ Event: Any Subset of SS.

$E_1 = \{1, 3, 5\}$ // getting odd no

$E_3 = \{HH, HT, TH\}$ // atleast 1 H.

→ if, $E_1 \cap E_2 = \phi \Rightarrow$ (mutually exclusive events)

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* Probability: For each event E in S, we assig a no. P(E)

$P(E_1) = 1/2$, $P(E_3) = 3/4$ imp

→ $P(S) = 1$

→ imp if E_1, E_2 are mutually exclusive events.

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$

if not mutually exclusive:

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

→ E & E^c are also mutually exclusive.

$\therefore P(E \cup E^c) = P(S) = 1$ $\quad \because \boxed{S = E \cup E^c}$

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* Random Variables → • take SS as i/p, & o/p a number.
• its a function. imp

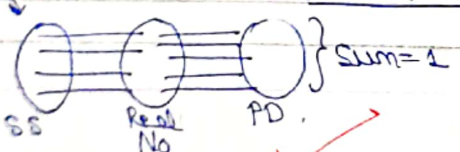
eg $S = \{HH, HT, TH, TT\}$

X = count #heads or count # tails →

X :	0	1	2
P(X) :	1/4	1/2	1/4

$\boxed{\sum P(X) = 1}$ ↳ $P(X=0)$ ↳ $P(X=2)$

X:



eg → Pg 5 [Flip a coin, until we get 1st head]

* Random Variable → discrete RV [RV are points, finite, countable set]
→ continuous RV [RV gives range
X: [lifetime of car, [0, 80]]
(∞ points in range)]

imp

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* Discrete Random Variable

① Bernoulli Random Variable : Experiment whose outcomes can be classified as either a success or a failure. 'X=1': success / 'X=0': failure.

$\left\{ \begin{array}{l} P(X=0) = 1-p \\ P(X=1) = p \end{array} \right.$ where p: prob of success

$\boxed{\text{prob mass funct}^n}$

eg → Pg 7 [Q1 - Toss 2 coins]

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② Binomial Random Variable: If n indep. experiment, each of which result in success with prob 'p' are to be performed.

X: represents #success in n trials.

pmf → $\boxed{P(X=r) = {}^nC_r\, p^r q^{n-r}}$ $\quad q = 1-p$

- Each is a Bernoulli expriment

eg → Pg 8 [Binomial RV] imp

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③ Poisson Random Variable

$$\boxed{P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}}$$

→ Poisson RV is used to approximate Binomial RV. when n↑, p↓ (prob of success ↓)

$\boxed{\lambda = np}$ // Poisson used in place of Binomial RV.

imp

* Continuous Random Variable

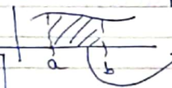
→ X = years of material : [1850 – 1965]
= lifetime of a car

X → discrete RV. [countable set / Prob is calculated for each point] [Pmf]
X → Continuous RV [uncountable set / Prob is calculated for a range] [Pdf]

Prob(point) = 0.

- $\boxed{P(x \in B) = \int_B f(x)\,dx}$

f(x) : Pdf
B : range [a, b]
$\int f(x)$: AUC



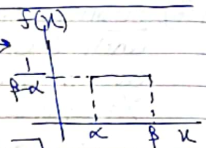
→ $P(x \in [a,b])$.

$\Rightarrow \boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$ → Prob(S) = 1

① Uniform Random Variable

(imp)

$$\boxed{f(x) = \begin{cases} \frac{1}{\beta-\alpha}, & \text{if } x \in [\alpha, \beta] \\ 0, & \text{otherwise} \end{cases}}$$



$$\Rightarrow \boxed{\int_{-\infty}^{\infty} f(x)\,dx = 0 + \left[\frac{1}{\beta-\alpha} \cdot \beta-\alpha\right] + 0 = 1}$$

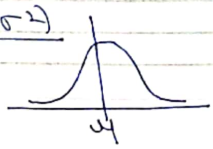
eg → Pg 13 [Solve Geometrically]

② Exponential Random Variable (λ)

$$\boxed{f(x) = \begin{cases} \lambda \cdot e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}}$$

③ Normal Random Variable (μ, σ²)

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$



* Expectation : To guess which outcome in the SS has the most probability to come out.

→ X = getting # Heads. (imp)

X	0	1	2
P(X)	1/4	1/2	1/4

$\sum X P(X) = 0 + \frac{1}{2} + \frac{1}{2} = 1$ [Before conducting exp, there is more chance to get HT/TH to come out]

① E(X) of Bernoulli RV

X	0	1
P(X)	1-p	p

∴ $E(X) = \sum X \cdot P(X) = p$.
(Exp # success in 1 trial)

∴ $\boxed{1 \text{ success in } 1/p \text{ trials}}$ ← (imp)

Q if p = 0.2, how many expected # trials to get 1 success → E = 5

② E(X) of Binomial RV

np Success in n Trials

$P(X=r) = {}^nC_r\, p^r q^{n-r}$;

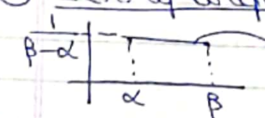
$\boxed{E(X) = \sum_{r=0}^{n} X \cdot P(X) \Rightarrow np}$ (if prob of single success is p, then expected # success in n Trials is np)

Q Expected # success in 10 trials, p = 0.6 ⇒ 6.

③ E(X) of Poisson RV = λ.

④ Continuous RV ⇒ $\boxed{E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx}$

④ E(X) of uniform RV



$\therefore \boxed{E(X) = \frac{\alpha+\beta}{2}}$

⑤ E(X) [Exponential RV] = $1/\lambda$.

⑥ E(X) [Normal RV] = μ (mean)

* **Independent Event**

- if an event has already occured, it won't change prob of next event.

$P(H_1 \cap H_2) = P(H_1) \cdot \underbrace{P(H_2 / H_1)}_{P(H_2)} = P(H_1) \cdot P(H_2)$.

↳ Head in 1st Flip

imp

$\therefore \boxed{P(A \cap B) = P(A) \cdot P(B)}$ $\boxed{P(A/B) = P(A)}$

* If any 2 events are independent, $P(A \cap B) = P(A)P(B)$ & converse ✓

* Pick ball from bag (with replacement) ⟹ independent events.

* if A, B are independent events

(i) $A, \bar{B}$
(ii) $B, \bar{A}$
(iii) $\bar{A}, \bar{B}$

} all are independent. ← imp

* if $E_1, E_2, E_3 \cdots E_n$ are independent: $\bar{E_1}, E_4, E_5$ are also independent.

eg → Pg 23 [Q 4 – Interview] ✓

* Derive if events are independent based upon the formula.

eg → Pg 25 [eg 7] (2 dice). ← imp

* To check if E, F, G events are independent.

$P(E \cap F) = P(E) \cdot P(F)$
$P(E \cap G) = P(E) \cdot P(G)$
$P(F \cap G) = P(F) \cdot P(G)$
$P(E \cap F \cap G) = P(E) P(F) P(G)$

} (all should satisfy)

imp

$\therefore \sum_{i=2}^{n} {}^{n}C_i$ is the # of eq^n's to verify.

* If A, B, C are independent events. ← imp

A & $B \cup C$
B & $A \cup C$
C & $A \cup B$

} are also independent.

(all valid comb like: $(A \cup B) \cap C$, $(\overline{A \cup B}) \cap C$ ....

* if Mutually exclusive:

$\underbrace{P(A \cap B)}_{\phi} = P(A) \cdot P(B) = \underline{\underline{0}}$.

* if $P(A) = 1$ (A = SS)

$\underbrace{P(A \cap B)}_{B} = \underbrace{P(A)}_{1} \cdot P(B) = P(B)$ ✓

* **Total Probability**

⟹

2G 3R	1G 4R	3G 2R
A	B	C

∵ Choose a bag & then pick a ball. $P(G) = ?$

$P(G) = P(A \cap G) + P(B \cap G) + P(C \cap G)$

depend events → $= P(A) \cdot P(G/A) + P(B) \cdot P(G/B) + P(C) P(G/C)$.

$= \frac{1}{3} \cdot \frac{2}{5} + \frac{1}{3} \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{3}{5}$ (Total Probab.)

∵ A & G (Choosing G B depends on the bag chosen).

eg → Pg 31 [SS, & disjoint events] →

imp

E_1 E_2 E_3

eg → Pg 33 [Q2] ✓ (1 ball definitely red, pick a ball, What is the probability that it is red.)

eg → Pg 35 [Q3] [Prob of getting 2 Heads out of 4F & 3U coins] ✓ imp

* Bayes' Theorem

– Reverse Probability : Predict the Src.

A	B	C
2R 3G	1R 4G	3R 2G

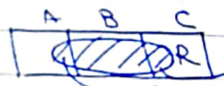
$P(A/R)$: Prob that bag A is chosen, given the ball chosen is red.

$\Rightarrow P(A/R) = \dfrac{P(A \cap R)}{P(R)}$ ⟶ Total Prob imp

$$\boxed{P(A/R) = \frac{P(A)\cdot P(R/A)}{P(A)\cdot P(R/A) + P(B)\cdot P(R/B) + P(C)\cdot P(R/C)}}$$

eg → Pg 37 [All Qs] ✓ imp

Geometric Visualisation of BT



→ TP = P(R).

$P(A/R) = \dfrac{P(A \cap R)}{P(R)}$ ✓

eg → Pg 41 [i) Green, odd K Green Balls] ✓