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INDEX :

DISCRETE MATHS

1) MATHEMATICAL LOGIC

(i) Propositional logic

imp

- declarative stat. which is either T/F but not both : Propositions.
- to connect propositions : connectives

($\neg, \wedge, \vee, \rightarrow, \leftrightarrow$)

eg: $2+4=7 \rightarrow F$ (propositⁿ)
What is the time? $\rightarrow$ (Not prop.)

1) Negation ($\neg$) : unary connective.

2) conjunction ($\wedge$) : p and q

eg $\rightarrow$ Pg 3 [example & exception]

3) Disjunction ($\vee$) : p or q

eg $\rightarrow$ Pg 4 [eg, exception] [Mysore]

4) conditional ($\longrightarrow$) :

- if p then q.

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

if, $p \equiv F$, dont check q, $\therefore p \rightarrow q = T$
if $p = T$, check q, if $q = T$: $p \rightarrow q = T$
$q = F$: $p \rightarrow q = F$

* Other rep of $p \to q$ are:
imp (1) p implies q.
(2) p only if q.
(3) p is sufficient for q.
(4) q if p.
(5) q is necessary for p.

eg → Pg 5 (eg 1)

5) Biconditional ($\longleftrightarrow$)

- "p if and only if q". [other forms]

p	q	$p \longleftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

(behaves like exNOR).

* Other rep of $p \leftrightarrow q$:
imp → (i) p iff q.
(ii) if p then q, & conversely.
(iii) p is necessary & sufficient for q.

* Propositional functions

- functⁿ whose var. are propositions.
eg: $f(p,q,r) = [(p \land q) \to r \land p \land q] \leftrightarrow p$

→ we have 4 PF of single V.
→ we have 16 PF of 2 variables.
$2^2 = 4$ rows
∴ 16 functions

6) EXOR (∇, $\oplus$)

- I will go either by bus or train but not both.

7) NAND ($\uparrow$) : $p \uparrow q \equiv \neg(p \land q)$
8) NOR ($\downarrow$) : $p \downarrow q = \neg(p \lor q)$

NOTE: No. of Propositional functⁿs of 'n variab' is: ⟶ 2^{2^n} [Vimp → like BF]
imp

* Well Formed Formulas

- PF which gives a value with no ambiguity in order.
eg: $p \to q \to r$: not WFF. [Associated]
But, $(p \to q) \to r$: WFF.

* Types of Function:

(i) Tautology : WFF always true
(ii) Contradiction: WFF always false
(iii) Contingency : WFF neither taut / contra.
(iv) Satisfiable : WFF which has atleast one truth V → true.

imp
Satisfiable : not contradiction
unsatisfiable : contradiction

* Converse / Inverse / Contrapositive

imp
(i) Converse of $p \to q$ is $q \to p$
(ii) Inverse of $p \to q$ is $\neg p \to \neg q$.
(iii) Contrapositive of $p \to q$ is $\neg q \to \neg p$

$\boxed{p \to q \equiv \neg q \to \neg p}$ [log. equivalent]

exp ≡ its contrapositive.

* Logical Equivalence

- Two WFF are logically equivalent if P & Q : same truth value.

diag → Pg 14–15 (Law of equivalences)

* Logical Implications

imp

A WFF, P is said to logically imply a WFF, Q if $P \to Q$ is a tautology

diag → Pg 17 (Rules of inference)

eg → Pg 9 (Rev Copy) (Imp) (Used for CS)

* Fallacies

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array} \quad \times \qquad \begin{array}{l} p \to q \\ \neg p \\ \hline \therefore \neg q \end{array} \quad \times$$

[not Modulus Pollen] [not mod Tollen]

* PROBLEMS

imp

- During Simplification, always convert : $\boxed{p \to q \text{ to } \neg p \vee q}$

eg → Pg 20 (Gate CS 2000)

eg → Pg 21 (2004) (Coll-Grp)

imp

$$\boxed{\begin{array}{l} (A \wedge B) \to C \equiv (A \to C) \vee (B \to C) \\ (A \vee B) \to C \equiv (A \to C) \wedge (B \to C) \end{array}}$$

eg → Pg 22 (2014) [opt]. [Wrong]

$$* \boxed{p \oplus q = \overline{p \odot q} = \overline{p} \odot q = p \odot \overline{q} = \overline{\overline{p} \oplus \overline{q}}}$$

eg → CS-2015 (Pg 25) [Substitute values]

* $p \wedge (p \to Q)$ implies:

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

I: false ×
II: Q ✓
III: true ✓
IV: P∨Q. ✓ ∵ Q is true.
V: ¬Q∨P. ✓ ∵ P is true.

eg → Pg 27 (q→p & p→q) imp

eg → Pg 27/28 (all 3 eqs on Fallacy)

(ii) PREDICATIVE LOGIC

- Predicate: open proposition which doesn't give either true / false.

eg: $P(x)$ | x is Prime no.

* Quantifiers

- converts Predicates to propositions

universal quant: $\forall x P(x)$: $P(x)$ is true for every value of x

existential quant: $\exists x P(x)$: $P(x)$ is true for some value of x [Imp]

[eg] → Pg 14 (Rev Copy) Q2

[eg] → Pg 32 (All men are Good)

imp →
$$\boxed{\begin{array}{l}\forall x[M(x) \longrightarrow G(x)] \\ \exists x[M(x) \wedge G(x)]\end{array}}$$

$\forall x \rightarrow$ conditional $(\rightarrow)$ Rule of thumb

$\exists x \rightarrow$ and $(\wedge)$

* Equivalences between Quantifiers

$$\boxed{\sim \forall x \sim P(x) \equiv \exists x P(x)}$$

↳ when ~ comes $\forall x$: became $\exists x$.

1. $\forall x[P(x) \wedge Q(x)] = (\forall x P(x)) \wedge (\forall x Q(x))$
2. $\exists x[P(x) \vee Q(x)] = (\exists x P(x)) \vee (\exists x Q(x))$

imp → $\forall$ distributive over $\wedge$, $\exists$ distributive over $\vee$ } Equivalence

* Implications b/w Quantifiers

1. $[\forall x P(x)] \vee [\forall x Q(x)] \Rightarrow \forall x (P(x) \vee Q(x))$

$\boxed{\forall x \text{ with } \vee : \text{take common}}$

2. $\exists x[P(x) \wedge Q(x)] \Rightarrow \exists x P(x) \wedge \exists x Q(x)$ (imp)

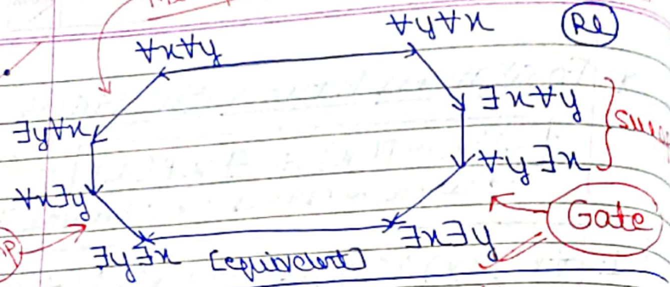
$\boxed{\exists x \text{ with } \wedge : \text{can distribute}}$

Converse not true.

* Sentences with Multiple Quantifiers

- order of Quantifiers important in a statement.

Multiple Quantifiers



eg → Pg 17 (Rev Copy) (Q1)

~~eg~~ → Pg 40 (CS-2012) } Gate Qs

eg → Pg 40 (CS-2013) } imp

imp → 'n' direction problem

Q "Some real no.s are rational".

$\exists x[\text{real}(x) \wedge \text{rational}(x)]$

Q "There are Graphs which are not connected".

$\exists x[G(x) \wedge \neg C(x)]$

* $\boxed{\forall x[P(x) \rightarrow Q(x)] \Rightarrow \forall x P(x) \rightarrow \forall x Q(x)}$ [Imp]

VI [Gate Q]

2] SET THEORY AND ALGEBRA

* Set: well defined collection of objects.
* Empty Set / Null Set: no elements.
* Universal Set: all elements.
* Subset ($A \subseteq B$): every element of A is in B. imp

eg → Pg 19 (Rev Copy) [$A \subseteq B$ & $A \not\subseteq B$]

* Power Set $P(A)$ or 2^A. [Set of all Subsets of A].

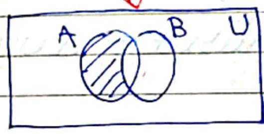
$A = \{1, 2, 3\}$

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

$\boxed{\text{if } |A| = n, \text{ then } |P(A)| = 2^n}$ [Imp]

* Operations on Sets:

(i) Complement: $A^c \equiv U - A$
(ii) Union: $A \cup B$
(iii) Intersection: $A \cap B$
(iv) Difference of Sets: $A - B$: [present in A but not in B]



[Present in A or B but Not both.]

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(v) Symmetric diff of Sets: EXOR

$\boxed{A \oplus B = (A - B) \cup (B - A)}$

$\boxed{(A \cup B) - (A \cap B)}$

* PROBLEMS

eg → Pg 21 (Rev Copy) (Numbering Math)

$S = \{1\}$ $P(S) = \{\phi, \{1\}\}$

eg → Pg 22 (Rev Copy)

Power Set

$\boxed{P(A \cap B) = P(A) \cap P(B)}$
$\boxed{P(A \cup B) \neq P(A) \cup P(B)}$ → don't work with union.

eg → Pg 50

imp

* RELATIONS

eg: $A = \{x, y\}$, $B = \{1, 2, 3\}$

- Cartesian Product
  $A \times B = \{(x,1), (x,2), (x,3), (y,1), (y,2), (y,3)\}$

$\boxed{\phi \notin A \times B \text{ but, } \phi \subseteq A \times B}$

imp

ϕ is subset of every Set.

- $A \times B \neq B \times A$ imp
  $\boxed{A \times (B \cup C) = (A \times B) \cup (A \times C)}$
  $\boxed{A \times (B \cap C) = (A \times B) \cap (A \times C)}$

* Relations: A binary relation is a subset of $A \times B$ imp
  $R = \{(x,1), (x,3), (y,3)\}$
  y related to 3 / $yR3$

- Total no. of relations from A to B is: $|P(A \times B)| = 2^{|A||B|}$ binary relation [P/A on ordered pairs]

- $A = \{1, 2, 3\}$
  $\Delta = \{(1,1), (2,2), (3,3)\}$ [Diagonal R]

* Types of Binary Relation:

Let R be relation on Set A
i.e. $R \subseteq A \times A$

(1) Reflexive – for $\forall x$, xRx

(2) Irreflexive – If even one (x,x) found, not irreflexive.

(3) Symmetric – If (x,y) found; (y,x) must be present.

(4) Asymmetric – If (x,y) found; (y,x) found even for 1 such pair, not asymmetric.
$xRy \Rightarrow y \not R x$

(5) Transitive — $\forall x, y, z$;
$xRy \land yRz \rightarrow xRz$
must be present

* Compatible relatⁿ : reflexive & symmetric
Equivalence relatⁿ : reflexive, symmetric and transitive
imp → Partial order relⁿ [Poset] : reflexive, anti-symmetric, transitive.

1. $R = \{(a,a), (b,b), (c,c), (a,b), (b,c)\}$

$a \rightarrow b \rightarrow c$

if all loops → reflexive
no loop → irreflexive
some loop → nothing

eg → Pg 57 (Q 4) [also irreflexive]

(6) AntiSymmetric — If (x, y) found; (y, x) also present & x, y are distinct no's, not antiSymmetric.
$xRy \land yRx \rightarrow x = y$ (imp)

~~eg~~ → Pg 27 (Rev Copy) (Q 6)

eg → Pg 58 (Q5) [antisymmetric]

eg → Pg 61 (Q9) (transitive) [not transitive]

* Reflexive, Symmetric, Transitive Closure (Floyd Warshall) (imp) — use Digraph [4 way]

→ Transitive closure of R is $O(n^3)$
$R \cup R^2 \cup R^3 \ldots$

$R^2 = R \cdot R$
$R^3 = R \cdot R^2$

eg → Pg 61 – 63

* No. of Relations :

Let $S = \{1, 2, 3 \ldots n\}$

① No. of relations on S is $2^{n \cdot n} = \boxed{2^{(n^2)}}$

② No. of reflexive relations on S is :

$\boxed{2^{(n^2 - n)}}$ [n^2 → total ordered pairs]

③ No. of irreflexive relations on S is :

$\boxed{2^{(n^2 - n)}}$ $\{(1,1), (2,2) \ldots\}$ (a, a)

④ No. of Symmetric relations on S is : (imp)

$\boxed{2^n * 2^{\left(\frac{n^2 - n}{2}\right)}}$ $\{(1,1), (2,2) \ldots$ (p/a, p/a) $|(1,2), (2,1)| \ldots$ (d, d)

⑤ No. of Asymmetric relations on S is :

$\boxed{3^{\left[\frac{n^2 - n}{2}\right]}}$ $\{|(1,2), (2,1)| \ldots$ (a, a / a, p / p, a)

union → Power set
equivalence Reln [Vimp]

(6) No. of AntiSymmetric relⁿ on S is:

$\boxed{2^n * 3^{\left[\frac{n^2-n}{2}\right]}}$

$\{(1,1), (2,2), \cdots (n,n)\}$ — p/a p/a

$(1,2), (2,1)$
a a
a p
p a

NOTES:

(1) Asymmetric ⇒ irreflexive
(2) Asymmetric ⇒ AntiSymmetric [imp]

$\underbrace{\overset{T}{xRy} \wedge \overset{F}{yRx}}_{F} \longrightarrow x=y$ ∴ always T.

(3) Irreflexive & transitive ⇒ AntiSymmetric [Asymmetric also]

$xRy \wedge \overset{F}{yRx} \longrightarrow x=y$

∵ if, yRx existed, due to trans: xRx also ∴ irreflexive X.

[imp]

* If R, S are equivalence relations:
(i) $R \cap S$ ⇒ equivalence relation
(ii) $R \cup S$ ⇒ need not be equivalence relⁿ.

(7) No of compatible relations on S is:

$\boxed{2^{\left[\frac{n^2-n}{2}\right]}}$

$\{(1,1), (2,2) \cdots$ — p p
$(1,2), (2,1)$ — p/a p/a

* If $S = \{1,2,3 \cdots n\}$
Smallest equivalence relⁿ = n
Largest equivalence relⁿ = n^2.

n^2 → all ordered Pairs

eg → Pg 69 (CS-2002) ∴ φ not equivalence

NOTE: φ is everything except reflexive. [depends upon US] [Test]

eg → Pg 70 (2016-Set 2) [VI]

* FUNCTIONS

A relation f from X to Y is a function:
(i) for every $x \in X$,
(ii) there is a unique image $y \in Y$, [imp]

$\boxed{y = f(x)}$

eg → Pg 72 [Every $x \in X$, should take 1 arrow exactly]

* TYPES OF FUNCTIONS:

(1) One to one function / Injective [imp]

$f(x_1) = f(x_2) \implies x_1 = x_2$
if, $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$ — contrapos.

→ diff element should have diff images.

(2) Onto function / Surjective

– every element in Y should have pre-img.

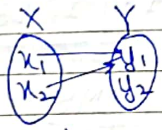
range = codomain

[eg] → Pg 32 (Rev Copy)

(3) Bijective:

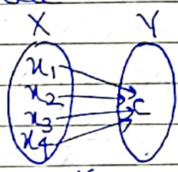
$\boxed{\text{Injective + Surjective : Bijective}}$

(4) Many to one:



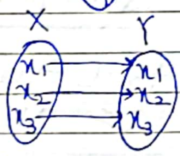
(5) Into funct^n: not onto.

(6) Constant funct^n:



(7) Identity funct^n:

imp → $\boxed{f(x_1) = x_1}$



(8) Inverse of function:

$f^{-1} = \{(b,a) \mid (a,b) \in f\}$

– f should be a bijective function.

(9) Composition of function:

If $f: A \to B$ and $g: B \to C$ then,

imp → $g \circ f: A \to C$ is:

$\boxed{g\circ f(x) = g[f(x)]}$

(i) $g\circ f \neq f\circ g$ but if, $f: A \to A$, $g: A \to A$
[Not commutative] $f\circ g = g\circ f$.

(ii) $(f\circ g)\circ h = f\circ(g\circ h)$ // Associativ.

[eg] → Pg 34 (Proof) [Rev Copy]

(iii) If f, g are one-one ⇒ gof is one-one
If f, g are onto ⇒ gof is onto.
If f, g are bijections ⇒ gof is bijection.

(iv) $\boxed{(g\circ f)^{-1} = f^{-1} \circ g^{-1}}$ imp

(v) If gof is one-one ⇒ f is one-one
If gof is onto ⇒ g is onto
[gof]

VI

* No. of Functions:

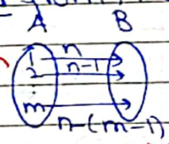
$|A| = m, |B| = n$.

(1) No. of functions from A to B is:

$\boxed{n^m}$ [each elem in A have n choices]

(2) No. of one-one functions from A to B:

imp $\boxed{\dfrac{n!}{(n-m)!} = {}^nP_m}$



(3) No. of onto functions from A to B:

$$\boxed{{}^nC_0\, n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m - {}^nC_3 (n-3)^m}$$ [Vimp]

Scanned with CamScanner

④ No. of bijective functⁿˢ from A to B :

$\Rightarrow \boxed{n!}$ imp

⑤ No. of many to one function :

$\boxed{n^m - {}^nP_m}$ imp

⑥ No. of constant functions = n
⑦ No. of identity functions = 1

$\boxed{eg} \rightarrow$ Pg 79 (2006)

[TOC Q]

$\boxed{eg} \rightarrow$ Pg 79 (2014) imp

$\{0,1\}^4 \Rightarrow (0,1) \times (0,1) \times (0,1) \times (0,1)$

$\Rightarrow$ 16 elements / ordered Quadple.

∴ (P/A) per quadple $\Rightarrow 2^{16}$ relations.

* Groups [GROUPS]

$\boxed{eg} \rightarrow$ Description of no.s [8.1]

* Binary operation ↪ closed

- (2,3) : element of N×N
  5 : element of N. imp
- '+' is binary operation from N×N to N.
- '+', '*', are BO on N, but '−' is not.
- 'Union' & 'Intersection' are BO on Set P(A)

* Algebraic System

- A set together with 1 or more BO is called an Algebraic System

eg: (Z, +) → '+' is BO on Set Z

∴ '+' is closed on Z.

* General Properties: of Algebraic System

let * be a Binary operation on Set A.

(1) Associative: $(a*b)*c = a*(b*c)$
$\forall a,b,c \in A$

(2) Identity:
There exists $e \in A$, such that
$a*e = a = e*a$

$\boxed{\begin{array}{l} e=0, \text{ for addition} \\ e=1, \text{ for multiplicⁿ} \end{array}}$

(3) Inverse:
There exists $b \in A$, such that:
$a*b = e = b*a$

$\boxed{\begin{array}{l} b=-a,\ e=0 \text{ ; for addition} \\ b=1/a,\ e=1 \text{ ; for multiplicⁿ} \end{array}}$

(4) Commutative:
$\forall a,b \in A$; $a*b = b*a$.

* DEFINITIONS

[VI]

SemiGroup	:	associative
Monoid	:	associative, identity element
Group	:	associative, identity elem, inverse element
Abelian Group	:	associative, identity elem, inverse elem, commutative

* Examples :

eg → Pg 40 (Rev Copy) (Q 1)

(i) (N,+) : ~~monoid~~ ✗, semigroup ✓.
(ii) (Z,+) : abelian Group.
(iii) (N,X), (W,X), (Z,X) : Monoid.
(iv) (Q,X), (R,X) : Monoids
↳ due to 0, inverse of 0 dne.
(v) Set of all nxn non-singular matrices is a Group, but not Abelian, wrt Multiplication.
∵ non-Singular ⇒ inverse ✓
but, AB ≠ BA (matrix mul)

eg → Pg 87 (9) O(G) = 4

* PROPERTIES

imp

① Once you have checked its a Group, check order; if <= 6 → Abelian Grp

imp

② Every finite group of Prime Order must be abelian.
O(G) = 19 → Abelian
O(G) = 4 → Abelian (<= 6).

③ In a group, inverse of identity element is itself. (Obviously)

* SUB-GROUP → (H,*) : non-empty subset of (G,*), which is also a Group.

eg → Pg 89

imp

Identity element & Group is always a SubGroup, but improper SubGroup.

eg → Pg 90 [all n multiples of Z → SG]

* LAGRANGE'S THEOREM

If H is any SG of a finite group G, then O(H) divides O(G), but converse not true ← imp

eg → Pg 92 (2006)

eg → Pg 93 (2006) [Vimp] (3xb)%15=e (2xe)%15=2

eg → Pg 93 (Based on ~~AG~~ Lagrange)

* POSETS & HASSE DIAGRAM

* Set → Poset by relation '$\leq$', if reflexive, antisymmetric & transitive.

(1) $[N, \leq]$ (less than/eq to)
(2) $[P(A), \subseteq]$ → Poset.

eg → Pg 43 (Rev Copy)

$\boxed{[Z, |] \text{ is not a Poset}}$; not reflexive due to 0

But, $[N, |]$ is [illegible]

imp

- 2 elements in Poset are comparable, if either $a \leq b$ or $b \leq a$.
- if $a \leq b$; $\boxed{b \text{ covers } a}$, such that, there is no element x : $a < x$ & $x < b$.

eg → Pg 98 $[P(A), \subseteq]$ → Hasse Diag

eg → Pg 45 (Rev Copy) $[D_{12}]$

$[D_n, |]$ is a chain/totally ordered PoSet (all pairs comparable); if $n = p^m$, p → prime

imp

* HASSE DIAGRAM

- glb → greatest lower bound
- lub → lowest upper bound.

eg → Pg 102 (All 5 Qs)

imp

glb → $\cap$ (meet)
lub → $\cup$ (join)

* LATTICES

1) Meet Semilattice - glb - Intersectn ↓

imp

- Poset in which each pair of elem. has a glb.
  $a \wedge b$ or $a * b = glb(a,b)$

2) Join-Semilattice - lub - Union ↑

- Poset in which each pair of element has a lub.
  $a \vee b$ or $a \oplus b = lub(a,b)$

3) Lattice - both glb & lub. $(D_n, |)$ → $\boxed{n = p^m}$

NOTE: Every TOSet is a lattice

eg → Pg 105 (2)

eg → Pg ~~107~~ (5) Vimp (lub/glb dne).

* $[P, |]$ (P → Set of all +ve integers)
  ↳ Poset
  $x \wedge y$ = meet = gcd(x, y) ↓
  $x \vee y$ = join = lcm(x, y) ↑

Vimp

3 GRAPH THEORY

1. $G = (V, E)$; [V → Vertices, E → edges]

$$\boxed{\begin{array}{l}\text{No. of Vertices} = O(G) \\ \text{No. of edges} = \text{Size}(G)\end{array}}$$

2. Parallel edges / loops : MultiGraph
without ∥ edge / self loops : Simple G.
3. No. of edges incident on node = degree
$\boxed{deg(loop) = 2}$
4. Graph in which each vertex is of same degree 'K' : K-regular. imp

eg → Pg 111 (degree seq.)

eg → Pg 112(2) [Indegree/outdegree]

* 1st Theorem of Graph Theory

· Sum of degrees of vertices of G is twice the no. of edges of G.

imp → $\boxed{\sum_{V \in V(G)} deg(V) = 2|E|}$ (HandShaking theorem)

(i) In any Graph, no. of odd degree vertices must be even.
(ii) [Sum of indegrees = sum of outdegrees = No. of edges]. → Directed Graph
imp

//

[Handshaking theorem]

⇒ $\boxed{\text{If sum of all vertices is odd} \Rightarrow \text{Invalid Graph.}}$

$O(n^2 \log n)$

eg → Pg 115 [2010 & 2014]

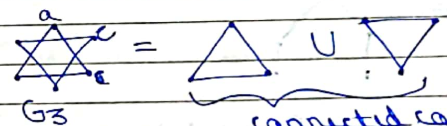
eg → Pg 117 [2016] // 23 diff compound
imp

eg → Pg 120 (2003) // min degree = K

* Max no. of edges in a n node [K_n] undirected Graph without self loop / Parallel edges = $\boxed{\frac{n(n-1)}{2}}$

* No. of undirected Graphs constructed from n vertices ⇒ $\boxed{2^{\frac{n(n-1)}{2}}}$ (P/A)

* CONNECTEDNESS



connected component.

(not connected)

– There is no direct / indirect path b/w a & c.

eg → Pg 123 (2018) [Vimp]
→ Tough

//

* TREE

- connected Graph with no cycles.
- $\boxed{\text{Tree with n Vertex} \Rightarrow (n-1) \text{ edges}}$

- Forest: Group of trees.
- Binary tree: Each node atmost 2 children.

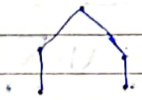
* SPECIAL GRAPHS

1. Null Graph (N_n) : no edges.

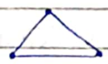
   N_2 . .

2. Path Graph (P_n)

   - All vertices connected by 1 Path

   $P_5 \Rightarrow$ 

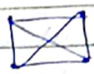
3. Cycle Graph (C_n) [from a cycle (all vertex)].

   $C_3 \Rightarrow$ 

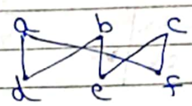
4. Wheel Graph (W_{1N})

   $W_{1,4} \rightarrow$  - connected Graph - cycles.

5. Complete Graph (K_N)

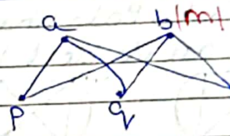
   K_4:  $\rightarrow$ $\boxed{{}^nC_2 \text{ edges.}}$

6. BiPartite Graph ($K_{3,3}$)

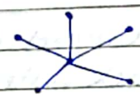
   {a, b, c} & {p, q, r} are not connected (with each other)

7. Complete BiPartite Graph ($K_{m,n}$)

   $K_{2,3}$ 

   $\Rightarrow 2 * 3 = 6$ edges (m, n)

   $\therefore$ $\boxed{\text{No. of edges} = mn}$ → imp

8. Star Graph : $K_{1,5}$ 

* Complement of Graph :

If G has 'n' vertices, 'p' edges

$\bar{G}$ has 'n' vertices, $\left[\frac{n(n-1)}{2} - p\right]$ edges.

eg → Pg 131 (CS-2015) [isomorphic]

* Max no. of edges in a complete BiPartite Graph of 12 vertices.

(1,11); (2,10); (3,9); (4,8); (5,7), (6,6); ✓ ⇒ 36 edges

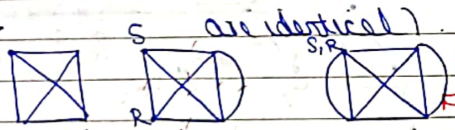
* MULTI GRAPH & EULER CIRCUITS

1. Euler Path: Path that includes each edge of the Graph exactly once & each vertex at least once.

edge → exactly once
vertex → atleast once

[Vimp]

2. [if Euler Path : Graph is traversable
   if Euler Circuit : Eulerian Multi Graph]
   ↳ (Euler Path whose end points are identical).



not traversable — not traversable — eulerian Graph

imp

(i) A connected Graph G is an eulerian if each vertex of G has an even degree.
(ii) A connected Graph G is traversable if G has 0 or exactly 2 vertices of odd degree.

imp

* HAMILTONIAN GRAPH

every vertex → exactly once : Hamilton Path.

- if closed: Hamiltonian cycle
- If G contains HC : Hamiltonian Graph.

If order(G) = n, HC has n edges. Imp

eg → Pg 59 (Rev Copy) } Eulerian X
eg → Pg 136 (1) } Hamiltonian ✓/X

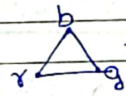
* No. of Hamiltonian cycles in a complete Graph K_n :
$$\Rightarrow \frac{(n-1)!}{2}$$
imp

⇒ In Undirected, CW/ACW in Cycle : Same

* GRAPH COLOURING & CHROMATIC NO (χ)

- $\chi(G)$ = min no. of colours req to colour the vertices, such that adjacent vertex dont receive same colour. [Chromatic No.]

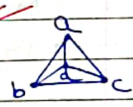
(i) χ(isolated vertex) = 1
(ii) χ(Path) = 2
(iii) Cycle Graph



⇒ 3

$\chi(C_n) = 2 ; n = \text{even}$
$\quad\quad\quad\; 3 ; n = \text{odd}$

(iv) Wheel Graph



$\chi(W_{1,N}) = 3, n = \text{even}$
$\quad\quad\quad\quad\; 4, n = \text{odd}$

(v) χ(tree) = 2 (binary tree).
(vi) χ(biPartite Graph) = 2

imp

(vii) ~~Regular Graph~~/Complete Graph (imp)

$\boxed{\chi(K_n)=n}$; where K_n = CG

* PLANAR GRAPH

- Graph is Planar, if edges dont cross over.
- K_4 is Planar
- $K_{3,3}$ or 3H,3U Problem not Planar. → complete bipartite Graph

* FOUR COLOUR THEOREM (imp)

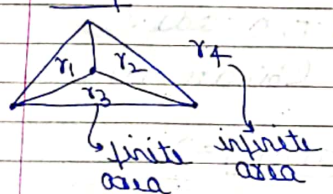
For any Planar Graph G;
$\boxed{\chi(G)\le 4}$; max it can be 4.

* COLOURING MAPS, DUALITY

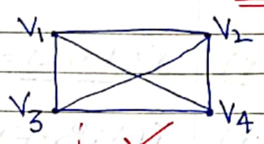
- Map: Graphs without cross overs.
- Map divides plane into regions.
  [Map ⟶ dual Graph]

[eg] ⟶ Pg 140 (2)

Map



Dual Graph $\chi=4$



→ K4 [planar Graph].

Maxim[um]

* WELCH POWEL ALGORITHM

→ gives max colours.

- Arrange degree seq in decreasing order.
- gives $\chi \le n$ (imp)

[eg] ⟶ Pg 63 (Rev Copy)

[eg] ⟶ Pg 143 (CS-2004) (imp)

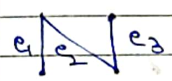
* MATCHING

- Matching in a Graph is a subset of edges in which no two edges are adjacent. (imp)
- Maximal matching: to which no edge can be added
- No of edges in largest maximal matching of G is called: Matching number of G ($\alpha'(G)$).

[eg] ⟶ Pg 145 ($\alpha'(G)=3$)

* PERFECT MATCHING

- A perfect matching is a matching in which every vertex is matched.



$\{e_1, e_3\}$: perfect matching.
$\{e_2\}$: maximal but not perfect.

Vimp

⇒ $\boxed{\text{If no. of vertices is odd} \Rightarrow \text{No perfect match exists}}$ [imp]

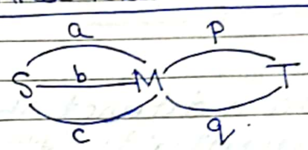
(1) K_{mn} has perfect matching if $m = n$.
(2) $K_{nn} \Rightarrow$ no. of perfect matching is $\boxed{n!}$

(3) K_n has perfect matching if 'n' is even.
like K_4 : $\{e_1, e_3\}$

(4) No. of perfect matchings in K_{2n} is
$$\boxed{\frac{(2n)!}{2^n\, n!}}$$
imp

4] COMBINATORICS

$\boxed{\begin{array}{l}\text{Sum Rule} \longrightarrow \text{OR} \\ \text{And Rule} \longrightarrow \text{and}\end{array}}$



S to M & M to T
$\Rightarrow 3 \cdot 2 = 6$ Ways

[eg] ⟶ Pg 150 [Computer Password]

* PERMUTATION & COMBINATION

^nC_r : unordered selection of r objects from n objects.

^nP_r : arrangement / ordered selection of r from n objects

[eg] ⟶ Pg 151 (eg)

1] THEOREM 1 :

- Select r from n distinct objects & arrange [without repetitions].

$$\boxed{^nP_r = \frac{n!}{(n-r)!}}$$ [ordered]

(i) There are 'n!' permutations of n distinct objects in a row.

imp

(ii) There are $(n-1)!$ permutations of n distinct objects in a circle.

(iii) Garland : CW = ACW

$\therefore$ no. of permutations $= \frac{(n-1)!}{2}$

[No. of HC in K_n]

2] THEOREM 2

- Select r from n distinct objects (without repetition) imp

$$\boxed{C(n,r) = {}^nC_r = \frac{n!}{r!(n-r)!}} \quad \begin{bmatrix} n \\ r \end{bmatrix}$$

(i) ${}^nC_0 = 1 = {}^nC_n$

(ii) ${}^nC_1 = n$

(iii) ${}^nC_r = {}^nC_{n-r}$ imp

3] THEOREM (3) imp

No. of r-permutations of n objects with unlimited repetition

$\therefore \underbrace{n \times n \times n \cdots n}_{r} = \boxed{n^r}$

4] THEOREM (4) imp

- No. of r combinations of n objects with unlimited repetition
- $V(n,r) = C(n+r-1, r)$

(i) no. of non-negative integral soln. to $[x_1 + x_2 + x_3 + \cdots x_n = r]$

(ii) No. of ways of distributing 'r' similar balls into 'n' boxes.

(iii) No. of binary numbers with 'n-1' ones & 'r' zeros.

eg $\rightarrow$ Pg 156 (r balls $\rightarrow$ n boxes)

* PROBLEMS

V. imp

- 3 similar apples thrown on 5 children $\Rightarrow {}^7C_3$ $V(5,3)$
- 3 different apples among 5 children $\Rightarrow$ [repetition + ordered] $\Rightarrow 5^3$
- 3 similar apples among 5 children if each child recvs at most 1 apple $C(5,3)$ $\Rightarrow$ Select 3 among 5 children & give apples $\Rightarrow {}^5C_3$
- 3 diff apples among 5 children if each child recvs at most 1 apple $\Rightarrow {}^5P_3$

Imp

eg $\rightarrow$ Pg 160 [20 diff flavours]

[$V(20,4)$] : 4 scoops thrown to 20 diff flavours. imp

eg $\rightarrow$ Pg 161 [Coins / dice similar]

[Stand. Procedure]

eg $\rightarrow$ Pg 162 [Non-negative integral soln]

* 16 similar apples among 4 people, so that each of them gets atleast 1?

$\Rightarrow V(4,12) \Rightarrow {}^{15}C_{12}$

⇒ Binary Sequences with exactly 10 ones, ↳ length 15.
- Choose the 10 places
⇒ $^{15}C_{10}$.

eg ⟶ Pg 163 (MCQ Test)

⇒ Pick a man & a woman who are not married from 30 married couples? [30H, 30W]
$[^{30}C_1 * ^{29}C_1]$
imp
Indirect counting

eg ⟶ Pg 165 [10 people]

⇒ No. of 4 digit telephone nos having at least one of their digits repeated ⇒
$10^4 - 10.9.8.7$ } distinct

* No. of Squares on chess board = $1^2 + 2^2 + 3^2 + \cdots 64^2$.
* No. of rectangles on chess board $^9C_2 \times {^9C_2}$.
[Select any 2 Horizontal lines & any 2 Vertical lines]

* PRINCIPLE OF INCUSION & EXCLUSION

eg ⟶ Pg 170 [upto 500, 3/5/7]
imp
[ ]

* DEARRANGEMENTS

Permutation of objects that leaves no object in it's original position.

* No. of dearrangements of {1, 2, 3 ⋯ n}
$$\boxed{D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \cdots\cdots\right]}$$
Gate

eg ⟶ Pg 172 [IT-2004]

* GENERATING FUNCTIONS

imp

1. $(1-x)^{-1} = \sum_{r=0}^{\infty} x^r = 1 + x + x^2 + \cdots$
2. $(1-x)^{-n} = \sum_{r=0}^{\infty} V(n,r)\, x^r$ ← Vimp

* Power Series
- $A(x) = \sum_{r=0}^{\infty} a_r x^r$ is called the GF of the sequence $A = \langle a_r \rangle$.

* $A(x) = 1 + 2x + 3x^2 + 4x^3 + \cdots$
⇒ AGP.
∴ $\boxed{\text{Sum of } \infty \text{ AGP} = \frac{a}{1-r} + \frac{dr}{(1-r)^2}}$

eg → Pg 175. [Coeff of x^{12}] (imp)

eg → Pg 176 [CS-2017]

* RECURRENCE RELATIONS

- In Gate: option Substitution is the best technique.

$n \to 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

- Sequence = 0, 2, 4, 6, 8, 10 ....
- Stopping/initial condition : $T(0)=0$ / $T(1)=2$ / $T(2)=4$.
- Recurrence relation : $T(n)=T(n-1)+2$
- Solution of RR [imp] : $\boxed{T(n)=2n}$ (we want to express in terms of n)

~~eg~~ → Pg 3 [Q1]

eg → Pg 4

(imp)

> If initial conditions are not given, a RR can give 2 solutions. ∵ Sequences are different

eg → Pg 6 (2002 Q) [Vimp]

* Towers of Hanoi Problem (PL)

$O(2^n) \to \boxed{T(n)=2T(n-1)+1}$

eg → Pg 10 [Gate 2015]

* Euler phi-function (ϕ) (Gate)

- $\phi(n)$ = The no of positive integers not exceeding n & relatively prime to n. [$\gcd(m,n)=1$].
- $\phi(90) = 2 \times 3^2 \times 5$

$\Rightarrow 2 \times 3^2 \times 5 \times \left[1-\frac{1}{2}\right]\left[1-\frac{1}{3}\right]\left[1-\frac{1}{5}\right]$

$\Rightarrow 24$.

eg → Pg 181 [IT-2005]

* No OF POSITIVE FACTORS

No. of divisors of 2100 is:

$\Rightarrow 3 \times 7 \times 2^2 \times 5^2$.

$\Rightarrow \{1,3\}, \{1,7\}, \{1,2,2^2\}, \{1,5,5^2\}$ (Gate)

$\Rightarrow 2 \cdot 2 \cdot 3 \cdot 3 \Rightarrow \underline{36}$

(Positive distinct integral factors)

* MODULAR ARITHMETIC

If $\boxed{a \equiv b \pmod m}$ — imp

⇒ m divides a − b

eg: $18 \equiv 3 \pmod 5$

∵ 5 divides 18 − 3 i.e. 15

If $a \equiv b \pmod m$ & $c \equiv d \pmod m$

(ii) $a + c \equiv b + d \pmod m$

(iii) $ac \equiv bd \pmod m$

* Fermat's theorem

If p is prime & a, p are coprime, then $\boxed{a^{p-1} \equiv 1 \pmod p}$

Q $3^{51} \bmod 5$?

⇒ $3^4 \equiv 1 \pmod 5$

⇒ $(3^4)^{12} \Rightarrow 1^{12} \pmod 5$

⇒ $(3^4)^{12} \cdot 3^3 \Rightarrow 27 \bmod 5$

⇒ 2

$\boxed{a^{p-1} \bmod p = 1}$ — imp

[p divides $a^{p-1} - 1$]

PREVIOUS YEAR GATE

* Complement of an element in Hasse Diagram. — imp

eg → Q2.3 → Refer GFG.

$f(1,2) = (3,-1)$

* If $f(x,y) = (x+y, x-y)$. $z_1 = x+y$, $z_2 = x-y$

$f^{-1}(x,y) = ?$

[Solve by subst. from option]

* No of equivalence Relations on a set of n elements is given by nth bell number.

if n = 5: — imp — 52

1
1 2
2 3 5
5 7 10 15
15 20 27 37 52

* Set of all strings [Σ*], with the concat operator: [monoid]

doesn't form a group. [identity = ε exists]

∵ [inverse of a string dne]

* $f(E \cup F) = f(E) \cup f(F)$ — Imp

$f(E \cap F) \neq f(E) \cap f(F)$.

eg → Concept → Q.2.31

[P is Prime]

* Set {1, 2, ... p−1} under mult. mod p ⟶ Group

imp: Set of all bijective functions on finite Set ⟶ Group. (n!)

* A → Set of integers
  B → odd integers
  C → Even integers
  $A \cap (B \cap C) = \phi$ (finite).

eg → * Equivalence class concept ⟶ Q2.27

* Distributive lattice (Gate)

if $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
if $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.
where, $\wedge$ = meet ↓
$\vee$ = join ↑

imp → if compliment exists, is always unique [can't have 2 compliment]

eg → Q2.43

* if, $R_1 = \{(1,2), (1,8), (3,6), (5,4), (7,2), (7,8)\}$
$R_2 = \{(2,2), (4,4), (6,2) (6,4), (8,2)\}$
→ $R_1 ; R_2 = \{(1,2), (3,2), (3,4), (5,4), (7,2)\}$

composite relation [used in classes].

each element either 0/1.

* In Symmetric matrix: fill the upper Δ matrix only

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{bmatrix}$$

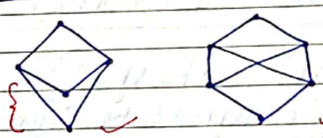
imp → $\frac{n(n+1)}{2}$ elements ∴ $2^{\frac{n^2+n}{2}}$ diff symmetric matrix

* Generator of a cyclic Group:

if, b*b*b ... *b can generate all other members from group {a b c d}
then, b = Generator. [Vimp]

eg → Q2.65 [ME]

not compar {



} Not a Lattice

* To be a ring/field/integral domain, we need to specify 2 binary operations.

→ Set of functions from N to {0,1} = 2^N = uncountab

→ Set of Rational No = countable.

Cantor's theorem

eg → Q2.75 [Good Q]

$X \to Y \vee Z$

imp

* If X then Y, unless Z
  $X \to Y \vee Z$
  $\Rightarrow (\neg X \vee Y) \vee Z$
  $\Rightarrow \bar{X} + Y + Z$

To prove it's valid / not: 3 procedure [Vimp]
$P \to Q$

(i) consider the case
(ii) disprove, consider [P=T, Q=F], & if satisfies, not Void.

P	Q
F	T
F	F
T	T

V. imp

(iii) $\neg P \vee Q = \bar{P} + Q$ & open it to get the result == 1 (valid)

$\forall x \forall y (R(x,y) \to R(y,x))$
∴ All sets may/maynot be Symmetric
∴ Satisfiable but not Valid

* In CNF [P(S)], there is a truth assignment; for at which at least half the clauses evaluate to true.

$P(x) = \neg(x=1) \wedge \forall y (\exists z (x = y * z)$
$\Rightarrow (y=x) \vee (y=1))$.

∴ x is not eq to 1 & if there exists same z for all y, such that product of y & z is x, then y is either the no. itself or 1
⇒ Prime no.

* Negation of Quantifiers – Q.
  CS → Q1.31 imp [Pg 53] (soln)

* None of my friends are perfect
  $\to \forall x [F(x) \to \neg P(x)]$.
  $\to \neg \exists x (F(x) \wedge P(x))$

Use disjunction $p \vee \neg p$ = cancel.
CS → Q1.49 [Use contrapositive]

* $\neg \exists x (\forall y \neg R(x,y)) \Leftrightarrow \forall x \exists y R(x,y)$

GRAPH THEORY.

* If no. of vertices = n.
  No. of edges in MST = n − 1.

* A bridge can't be a part of a Simple cycle.
  Bridge: edge, whose removal disconnects a graph.

* No. of distinct minimum Spanning trees → Q4.47
  [data Structures]

Vimp

* C_5 is the only Graph which is isomorphic to its complement.

* Which of the following graph is isomorphic to? (imp)

(i) no. of vertices same

(ii) no. of edges same.

(iii) degree sequence same

(iv) search if same cycle graph exists.

[defn of isomorphic]

eg → Q4.38

K_{33} → non-planar Graph with min no. of edges. [biPartite]

[6 vertices / 9 edges]

* Let G be a graph with n vertices and if every vertex has a degree of at least $\frac{n-1}{2}$, then G is connected. (imp) [connected]

→ A connected Graph has euler circuit ⟷ all its Vert have even degree

eg → Q.4.27 [degree of complement. imp] (imp)

* MST is the subset of edges that connects all Vertex & has minimum

total weight

Famous Non-Planar Graph

K_{33} : Non-Planar Graph with min no. of edges
K_5 : Non-Planar Graph with min no. of vertices

* No. of distinct simple graphs with upto 3 nodes. [unlabelled]

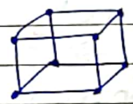
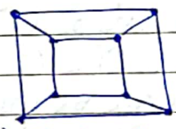
eg → Q4.6 (7) labelled → 11 (imp)

Vertex removed from G, no. of components lie b/w : [K−1 and n−1] → Star

K → no. of component earlier

n → no. of nodes

isolated vertex

 →  [imp]

[Non-Planar Graph]

* A Graph is non-Planar if & only if it contains a subGraph which is homeomorphic to K_5 or K_{33} [Kuratowski Theorem] (imp)

* Max no. of edges in a connected planar simple Graph $= 3n-6$

* Max no. of edges in undirected Graph with $\left[\begin{array}{l} N \text{ vertex} \\ K \text{ components} \end{array}\right.$

→ each component: 1 vertex.
last component: $N-(K-1)$ vertices.
make it complete: $^{n-(K-1)}C_2$
imp

$\Rightarrow \dfrac{(n-K)(n-K+1)}{2}$

* Combinatorics

→ No. of Binary strings, n 0s, K 1s, that no two ones are adjacent.
_0_0_ ... 0_ ⇒
imp → K can be placed anywhere in these (n+1) gaps, ∴ $^{n+1}C_K$.

eg → Q3.8 [V(n,r) Q on flowers] (Stand. Q).

* 4 digit even nos having all 4 digits distinct → Case 1: 0 $\;9\times 8\times 7\times 1$
Case 2: ≠0 $\;8\times 8\times 7\;\; 4$
imp

eg → Q3.9 (Use pigeon hole principle)
∴ $\left\lfloor \dfrac{n-1}{h} \right\rfloor + 1 = 3$; h = 4 (holes)
n = pigeons.
or, GO (Asim - Last Ans)

* If $a_n = 2n+3$ (imp)
Generating funct^n → $\sum_{n=0}^{\infty} a_n x^n$.

eg → Q3.39 [AGP formula used — Vimp]. [GO]

* $\sum_{x=1}^{99} \dfrac{1}{x(x+1)} \longrightarrow \sum_{x=1}^{99} \left[\dfrac{1}{x} - \dfrac{1}{x+1}\right]$

* If, $P = \sum_{\substack{1\le i\le 2K \\ i \text{ odd}}} i$ ⇒ Sum of all odd nos from [1, 2K]
if K=5
∴ $P = 1+3+5+7+9 \longrightarrow 25$

* Exponent of 11 in the prime fact of 300!
$300\times 299\times 298\times \cdots 242\times \cdots 121\times \cdots 11\cdots$
⇒ $\left\lfloor \dfrac{300}{11} \right\rfloor = 27$, $\left\lfloor \dfrac{27}{11} \right\rfloor = 2$, $\left\lfloor \dfrac{2}{11} \right\rfloor = 0$.
∴ $27+2 = 29$ (imp)

eg → Q3.34 [No. of 4 digit nos having their digits in non-dec. order] {1,2,3}, use tree method [ME].

* 10 percent
[Pair no. of 1s, 2s]
[take each order & find all possible permutations]

1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 2
1 1 1 1 1 1 2 2
1 1 1 1 2 2 2
1 1 2 2 2 2
2 2 2 2 2

imp

eg → Q3.18 [GO – Digvijay Ans] imp
(Pigeon hole)

eg → Q.3.22 [Vimp – Robot movement]
(ME Ans)

TEST

* If any 2 vertices of tree are chosen, 1 cycle created [joined by edge]. (if n = 10)
$^{10}C_2 = 45$ cycles [cyclic cardinality]

* If in a degree sequence, all degrees is distinct → Graph not possible. imp

* 3 digit no. divisible by 6 with (12345) [div by 2 & 3]
eg → Q6 [8 nos] 2 :, 4 : imp

* Matrix representation of a relation with 4 elements {1,2,3,4} is given:
$R = \{(1,1),(1,2),(1,4),(2,1),(3,2),(3,3)\}$
$R^4 = ?$ [Do it by Graph]
eg → Q10

[Total SS = 32]

→ If M is a set of cardinality 5
$\{x, y, z, a, b\}$
Subsets not containing $x, y, z = 4$

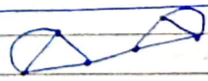
ϕ → only relation which is symmetric and asymmetric. [transitive also]

* No. of edge disjoint hamiltonian cycles in $K_n = \left\lfloor \frac{n-1}{2} \right\rfloor$ ∴ $K_{101} = 50$

* If H → cyclic Group [order = 20]
if a = generator, n = 8
$\boxed{Order(a^x) = \frac{n}{gcd(x,n)}} = \frac{20}{gcd(8,20)} = 5$ imp

* Degree of each vertex in $K_n = n-1$
∴ By removing $(n-1)$ edges ∴ Graph can be disconnected. imp

* If a funct^n has multiple roots ⇒ many to one. 1A
if funct^n is strictly increasing / strictly decreasing: $f(x) = \ln x + x$ ∴ one to one [injective]
S↑ S↓

*  3-regular connected Graph.

S_1: $\forall x \forall y [spider(x) \wedge spider(y) \Rightarrow x = y]$.
[atmost one spider]
imp $\forall x [\sim spider(x)] \vee \exists x [spider(x) \wedge$

[Cartesian Product] (imp)

* $P \times (Q \times R) \neq P \times Q \times R$. → not compatible

$(x,(y,z)) = (x,y,z)$.

[ordered pair] [ordered triplet]

* No. of subgroups of A ? (finite grp)

if $|A| = 24$

$24 = \underbrace{3^1 * 2^3}_{\text{primes}} \longrightarrow (1+1)*(3+1) = 8$

[Positive integral factors]

* Trivial Tree ⟶ only one vertex (imp)

In non-Trivial Tree ⟶ atleast one vertex of degree 1 [must]. ~~leaf~~

Given: $f(x) + 2f(1-x) = 3x$

$\therefore f(x) + 2f(1-x) = 3x$ —(i)

$f(1-x) + 2f(x) = 3(1-x)$ —(ii)

Solve: $\rightarrow f(x) = 2-3x$

* Set-Subset Problem $S = \alpha, \beta, \gamma, \delta, \varepsilon$ (imp)

eg → Q30 [Imp]

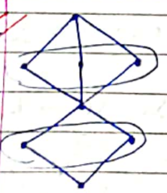
S_1 & S_2 need to be disjoint $S_1 \cap S_2 = \phi$.

ϕ ↳ no. of subset not containing ϕ

ϕ 2^5

$\{\alpha, \beta, \gamma\}$ 2^4 → no. of subsets not containing α.

*



2 comparable with 3.

$\therefore$ No. of Topological Orderings $= 2! \times 3! = 12$

[Doubt] (imp)

Set Problem

* $\boxed{{}^nC_0 + {}^nC_2 + {}^nC_4 + \dots {}^nC_n = 2^{n-1}}$

(No. of Subsets containing even no. of elements. [Imp] (obvious half subsets)

* If degree of each vertex = 4.

vertices = 5!

use handshaking ⟶ $5! \times 4 = 2E$

$\therefore E = 240$ edges

* $\forall x\, \alpha \rightarrow (\exists y\, \beta \rightarrow (\forall u \exists v\, \gamma))$ (Mistake)

$\Rightarrow \forall x\, \alpha \rightarrow (\forall y \neg\beta \vee \forall u \exists v\, \gamma)$

$\Rightarrow \exists x \neg\alpha \vee (\forall y \neg\beta \vee \forall u \exists v\, \gamma)$

* MULTI SUBJ. TEST

* Atleast one of G or $\bar{G}$ must be connected. (at least deg $\frac{n-1}{2}$). (imp)

if $\bar{G} \rightarrow$ connected, $G \rightarrow$ connect / discon.

* $Y = 0011$ (3) → Base 10

9s complement of $Y = 6$. (0110)

* Pigeon Hole Principle:

m Pigeons occupy n Holes $(m>n)$

[Pigeons must be grtr than holes]

(imp) at least: $\left\lfloor \frac{m-1}{n} \right\rfloor + 1$ occupy same hole.

$\left\lceil \frac{m}{n} \right\rceil$ pigeons.

Scanned with CamScanner

Approach

$\Rightarrow (p \to p \vee q) \wedge (q \to \neg p \vee q)$ ← Use SA

$\Rightarrow (\bar{p} + p + q)(\bar{q} + \bar{p} + q)$

$\Rightarrow 1 \cdot 1 = 1$ (True) ∴ Tautology

* Ratio of Chromatic no. to diameter for C_{40}

∴ CN of $C_{40} = 2$ (vimp)

CN of diameter $= \lfloor \frac{n}{2} \rfloor = \lfloor \frac{40}{2} \rfloor = 20$

∴ Ratio = 0.1

* $\boxed{\text{Diameter of } C_n = \lfloor \frac{n}{2} \rfloor}$

* Beautiful Q on $V(n,r)$ (Vimp)

No. of terms in the expansion of $(x+y+z+w)^3$?

Now, if $(x+y)^2 = x^2 + y^2 + 2xy$

Each term deg 2

∴ $\sum x^{n_1} y^{n_2}$

$\Rightarrow n_1 + n_2 = 3$ ∴ Similarly

$\Rightarrow \sum x^{n_1} y^{n_2} z^{n_3} w^{n_4}$

$\Rightarrow n_1 + n_2 + n_3 + n_4 = 3$ [3 similar balls in 4 boxes]

∴ $V(4,3) = {}^6C_3 = 20$

[Read Statements]

* $\forall x(\text{odd}(x) \wedge \text{even}(x) \to \text{integer}(x))$ ✗

For every x, which is odd & even both. [Not possible]

* Another modif. on $V(n,r)$:

How many ways can we dist. at most 10 identical balls to 3 boxes? (imp)

∴ $x_1 + x_2 + x_3 \le 10$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 10$

$\Rightarrow V(4,10) = {}^{13}C_{10} = {}^{13}C_3$

* $T: N \to N$ (Natural no.) [injective, surjective Q]

$T(n) = \begin{cases} n^2, & n \text{ is odd} \\ 2n+1, & n \text{ is even} \end{cases}$

∵ $T(3) = 9$, $T(4) = 9$ — 3, 4 → 9 (many to one)

∴ Not injective

∵ Range covers (Squares of odd nos)

Range ≠ codomain

∴ Not Surjective

[Approach]

* Given: BCD → excess 3 circuit.

ABCD → PQRS

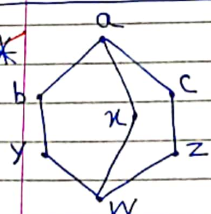
excess 3 code for R (boolean funct)?

Dud

A B C D	P Q R S	dec. val
0 0 0 0	0 0 1 1	1

⇒ & draw K Map of R (A B C D)

*



complem. (y) = {x, c, z}

complem (z) = {x, b, y}.

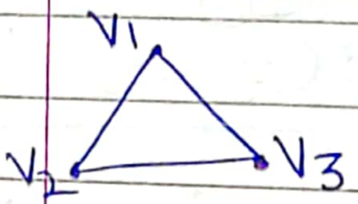
[Complement Problem]

∴ Not a distributive lattice

* No. of ways to assigning colours to a K_3, such that atleast 2 vertices have the same colours.

$C = \{red, Green, Blue, Yellow\}$.

Total − (All V. have diff colours).



Total $= 4^3 = 64$

diff colours $= {}^4C_3 \cdot 3!$

$\therefore\ 64 - 4 \cdot 6 = 40$