



# Additive Manufacturing Research

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# Objectives

## **1. NDI and NDE**

What methods of collecting and evaluating data which can be used for determining Probability of Detection?

## **2. Probability of detection**

What statistical methods can be used to examine data for Probability of Detection?



# **Summary of Pulsed Thermography Study**



# Informational Articles

## Notes

- **Ultrasonic testing (UT):** type of NDT that uses ultrasonic waves to detect to identify flaws within an object.

### Two methods for Ultrasonic Testing

#### 1. Pulse echo testing

A Piezoelectric Transducer releases sound waves into an object. These waves are measured by looking at the depth of each sound wave sent through the material.

#### 2. Through-transmission

An emitter is used to send sound waves, and a receiver is used to receive sound energy. The emitter and receiver are placed on opposite sides of the object.

( For both methods, imperfections in the material will cause a decrease in the amount of sound received, doing so flags the location of the flaw)



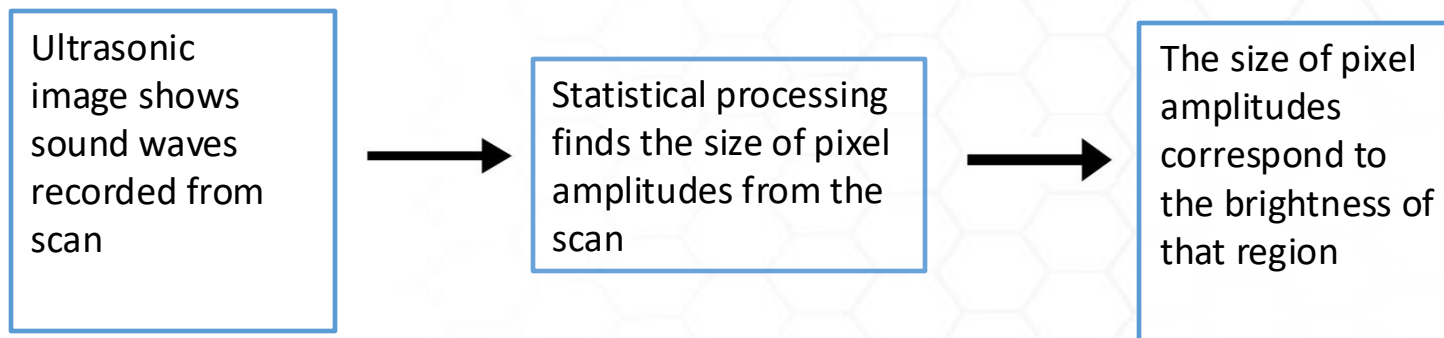


# Research Papers/Studies



## Notes

- This article discusses the difficulties that come with capturing ultrasonic images of concrete structures.
- This research aims to enhance the analysis of ultrasonic images of concrete structures by associating statistical values with the brightness in different regions. Doing so would make drawing conclusions about identifying defects more straightforward and reliable.
- The statistical values associated with the brightness are found by using statistical processing to estimate the size of pixel amplitudes found in the ultrasonic image.





## Dealing with irregularities found in data

**Grain Noise:** (irregularities in soundwaves that occur due to very small imperfections on the surface of the object being tested)

- In ultrasonic testing, grain noise are random irregularities that occur in sound waves they mainly occur due to irregular surface conditions. Grain noise negatively affects data we collect. Grain noise cannot be filtered out

### **Spatial Averaging**

- We use spatial averaging to deal with grain noise, spatial averaging ignores echoes from sound waves that are less than 6 Decibel (dB) from the data we collect.

### **Multiple Scattering**

- A particle or wave scatters multiple times because of the particles that are surrounding it  
Ex. when light passes through something like dust it scatters multiple times before reaching the end

## Preferred methods for Data Analysis

### **Gaussian distribution/Normal distribution (best used for low resolution systems)**

- The multiple scattering that occurs in low-resolution systems looks similar to thermal noise. Gaussian distribution is used to characterize thermal noise. Hence it is also recommended to use Gaussian distribution to characterize low-resolution systems.

### **Trimmed Mean vs. Weibull Distribution**

- If a scan includes multiple instances of high amplitudes due to grain noise do not use **Weibull distribution** to characterize this data. Instead use statistical methods like **Trimmed Mean** to exclude a certain percentage of extreme values.

Overall this research shows how a combination of ultrasound and statistical image processing can be used as an effective method for evaluating concrete structures through non destructive inspection.

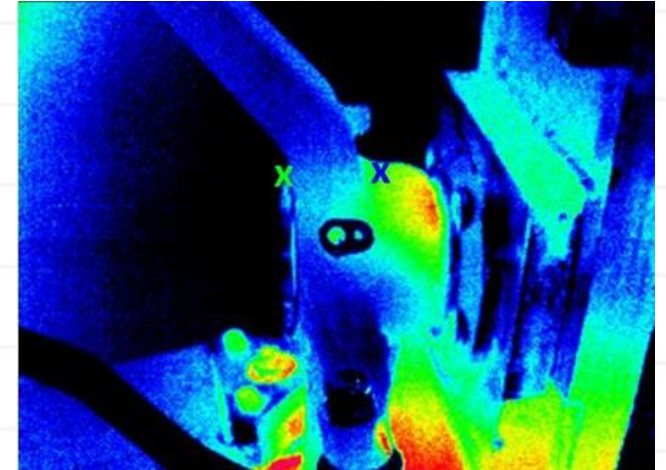
## Notes

- The purpose of this study is to examine how Pulsed Thermography (PT) can be used to identify shallow spherical defects in metal objects produced by laser powder bed fusion (L-PBF).

## Thermography

Thermography is a non-destructive testing method that utilizes the heat radiated from an object to identify any cracks and irregularities it may have. This process employs a specialized camera to capture a detailed image of the object, illustrating its various regions with distinct colors. These colors represent the invisible heat emitted by specific areas of the object, detectable solely by the thermal camera.

Once the thermal image is captured, it undergoes analysis using imaging software. This software plays a crucial role in determining and locating any cracks within the object. Cracks are often shown by lines or patterns of different colors in the thermal image.

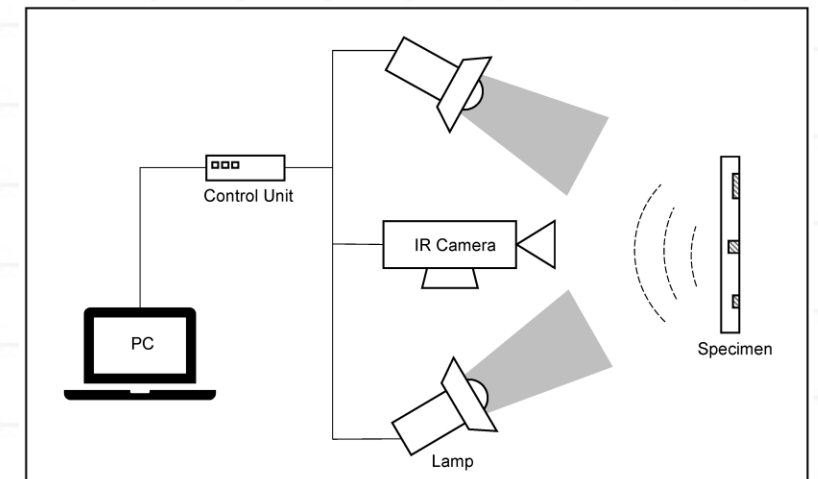


## What is Pulsed Thermography (PT)

Pulsed Thermography is a type of active thermography that uses an external energy source in short pulses, to heat the surface of the object under examination.

A thermal camera is then used to examine the temperature changes on the object's surface after the external energy source is applied.

Irregularities within the surface of the object alter the way that specific region releases heat. The thermal camera is able to capture and highlight these irregularities, doing so provides a detailed and accurate depiction of the object's structural characteristics.





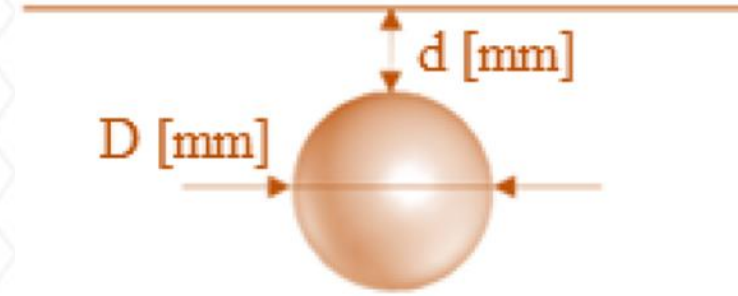


## Summary

This study examined specimens manufactured in 316L Stainless Steel through L-PBF, all of which were produced to have shallow spherical defects of various sizes.

Pulsed Thermography identified 50% of defects with a diameter and depth ( $D/d$ ) of 4mm or less. And identified 90% of defects with a  $D/d$  of 6.5mm or less.

When the aspect ratio of the  $D/d$  is greater than 4mm or 6.5mm, the defects have "specific shapes" which are easier for Pulsed Thermography to detect .



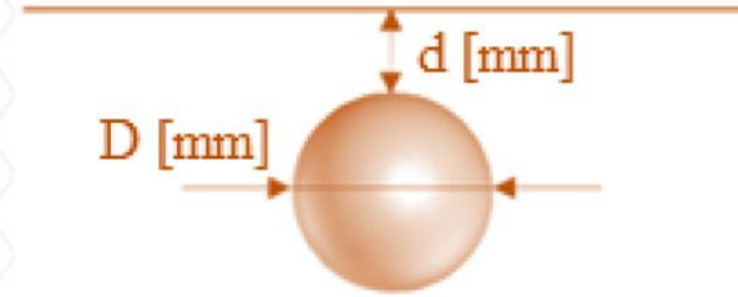


## Summary

"Different shapes" are defects with  $D/d$  between 3mm and 4mm. These defects were significantly harder to detect.

Applying post processing algorithms like TSR and PCT were found to be a more effective way of detecting these shapes. These methods achieved around 70% detection.

The researchers in this study believe that PT has potential and that with the help of better equipment PT can be a useful way to non destructively examine 3D-printed parts.







# **Educational Videos/Resources**

# Weibull distribution using the fatigue test as an example (survival/failure/reliability analysis)

## Notes

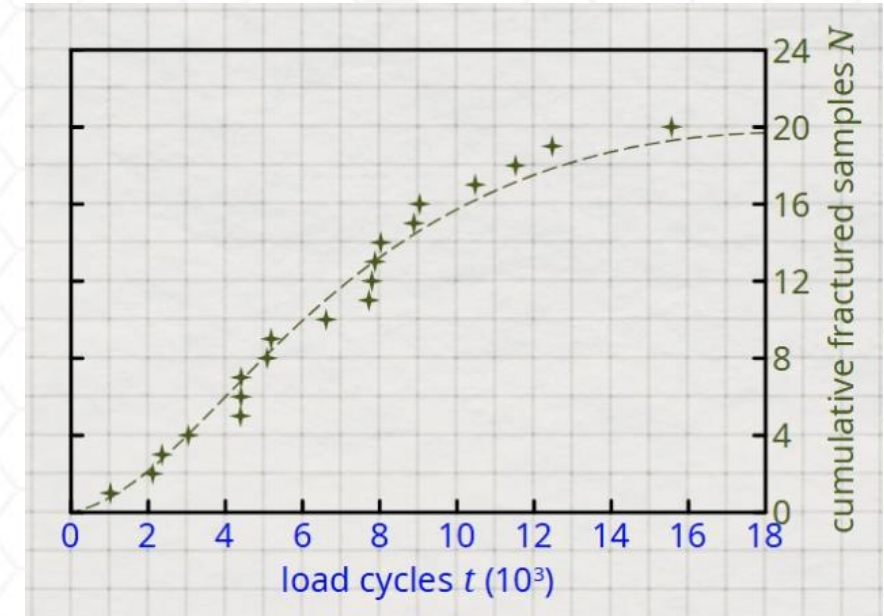
- This video demonstrates how Weibull distribution can be used to examine data from fatigue testing.

### What is a Wohler curve?

Indicates the total number of loads an object can withstand until it is fractured.

**Cumulative frequency:** the total number of samples that have fractured up until a certain number of load cycles

**Ex.** The graph to the right shows that at 5,000 load cycles the cumulative frequency is 8 (since 8 samples have fractured after enduring 5,083 load cycles)



## Weibull Distribution

**Shape Parameter (K)** : The likeliness of an object breaking under stress testing.

### If $K < 1$

Objects have a higher chance of breaking ( objects are less likely to withstand a high number of load cycles)

### If $K > 1$

Objects have a smaller chance of breaking (objects are more likely to withstand a high number of load cycles)

### If $K = 1$

All objects have an equal chance of breaking at any point (failure rate is constant over time)

#### Weibull distribution

$$F(t) = 1 - e^{-\left(\frac{t}{T}\right)^k}$$

$k$  : shape parameter  
 $T$  : scale parameter  
 $t$  : variable

**Scale parameter ( $\lambda$ ):** Examines the distribution of the number of load cycles an object can withstand

- **Larger  $\lambda$ ,** tells us object can withstand higher number of load cycles before breaking

**Time to failure (t) :** Represents the number of load cycles an object can endure before failing.

- We can solve directly for time of failure using

$$t = T \left[ \ln \left( \frac{1}{1-F(t)} \right) \right]^{\left( \frac{1}{k} \right)}$$

**F(t)-** represents the probability of failure

For example, F(100) represents the probability of an object breaking after a 100 load cycles. If F(100) = .29, that means there is a 29% chance that the object will not be able to withstand 100 load cycles



# Technical Notes



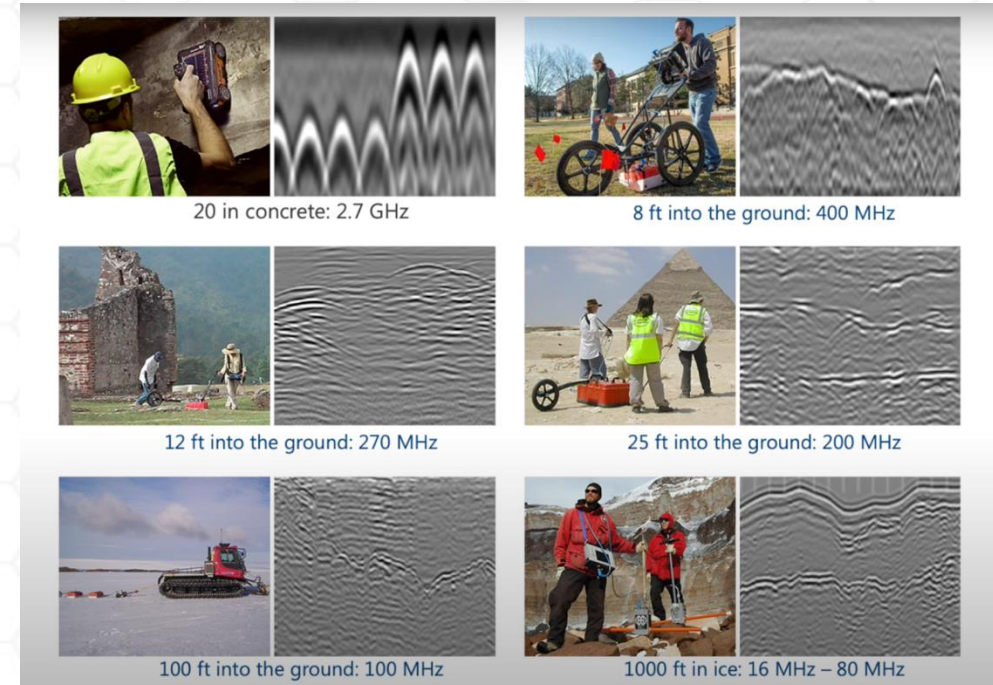
## Notes

- The purpose of this study is to examine how Ground-penetrating radar (GPR) can be used to identify defects in concrete structures.

## What is GPR?

GPR is a non-destructive detection and imaging method that uses electromagnetic waves to identify defects. It is commonly applied to concrete structures.

GPR transmits frequencies between 10MHz - 2.7 GHz, depending on the material being examined.





## **What is GPR?**

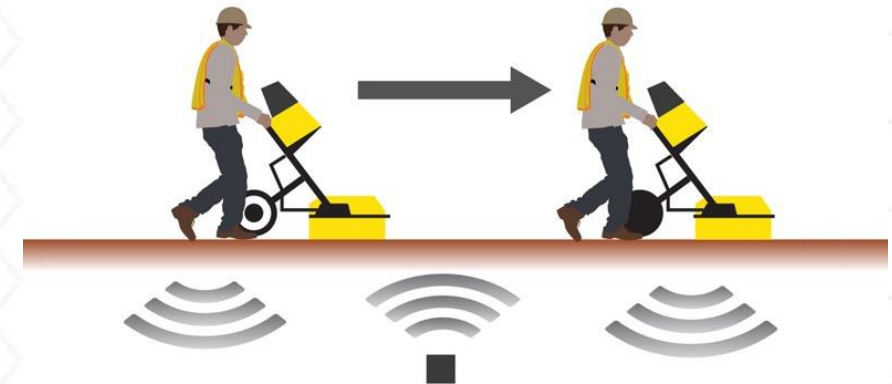
How does GPR work?

A transmitter and receiver are used. The transmitter releases energy at roughly 4000 pulses/sec at the object being examined.

The object under examination reflects this energy to the receiver in the form of a high-frequency electromagnetic reflection.

The GPR scans record these reflections. Color is added to these scans they are then analyzed to determine the amplitude of each reflection along with its two-way travel time (the time it takes for the energy to travel through the object under examination and reflect to the receiver).

### GROUND PENETRATING RADAR



## **What is the Dielectric Constant ?**

Dielectric constant, a measure of how fast radar travels through a material. The dielectric constant is needed to properly calibrate the GPR machine.

The more moisture in a material the the larger the dielectric constant will be. The larger the dielectric constant the faster radar travels.

There are multiple ways to calculate the dielectrics of concrete.



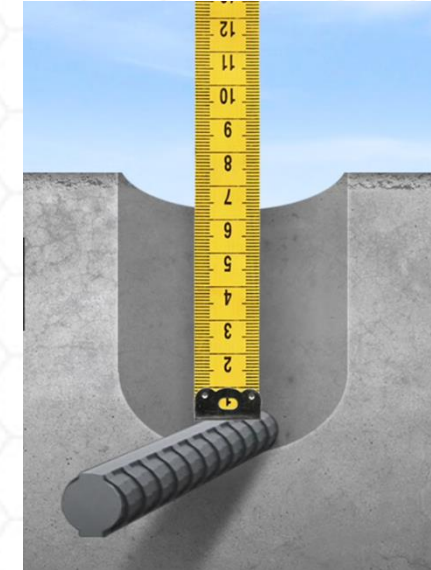
### **Method 1**

Determine the dielectric constant based on when the concrete was poured. Dry concrete has a dielectric constant between 4-6. While wet concrete has a dielectric between 10-15.

### **Method 2 (Ground Truth with a known depth)**

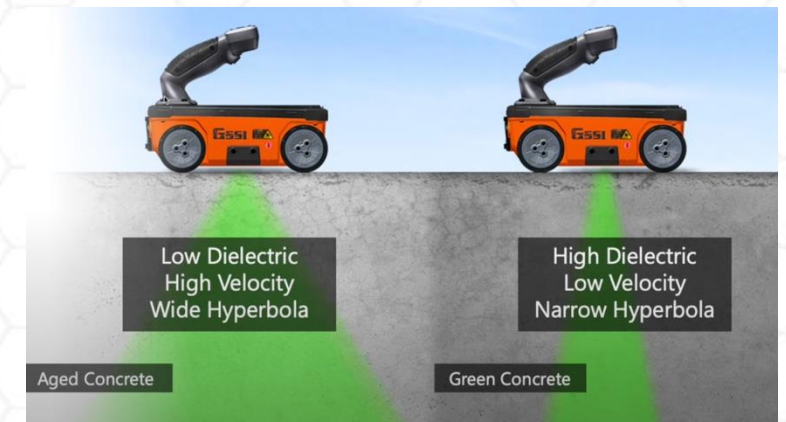
Comparing depth from the GPR of the object to its actual depth and then calibrating the GPR machine to its actual depth.

Ex. A rebar lies seven inches in the ground, the GPR records its depth as nine inches. Calibrate the GPR machine to its actual depth instead of the depth recorded, this gives you the dielectric constant.



### **Method 3 (Hyperbola fitting)**

When the hyperbola formed by the scan from the GPR is wider the concrete is dry and as a result, the dielectric constant will be smaller.





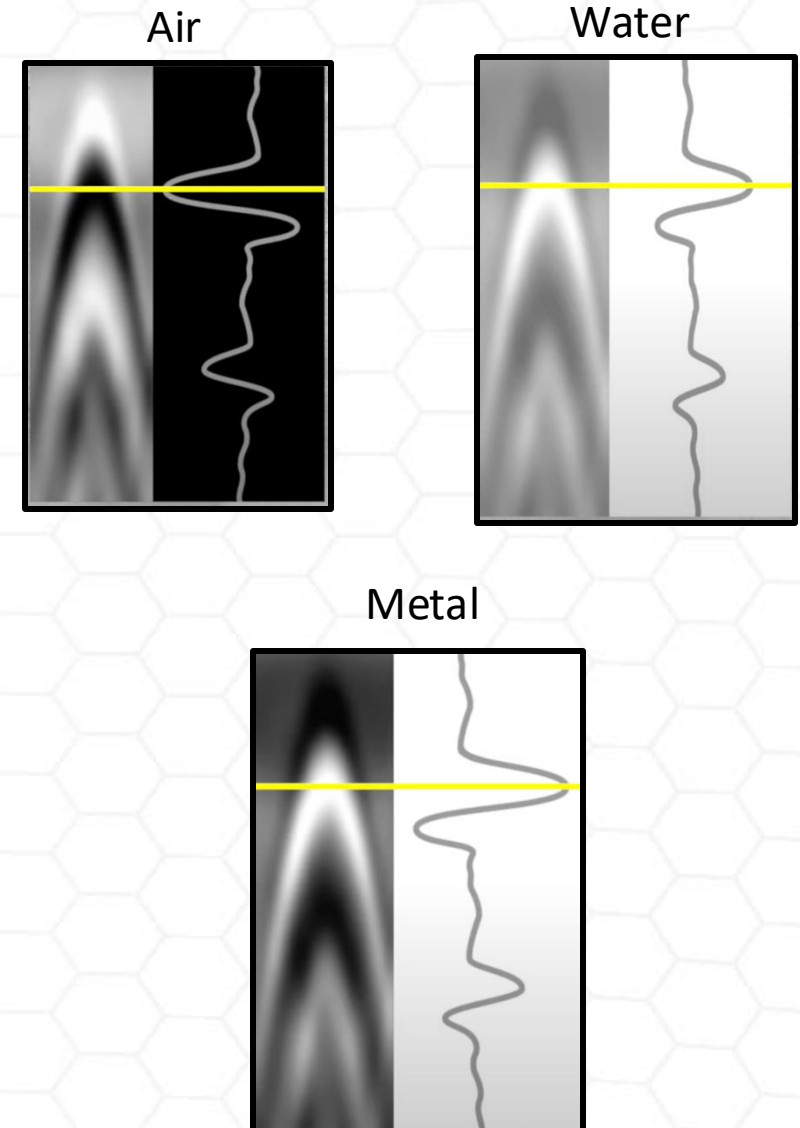
## Determining material types:

An **oscilloscope** is used to analyze scans from GPR machine. Scans with amplitude to the right are positive and to the left are negative.

**Metal:** The radar sent from GPR is reflected perfectly back to it. The scan shows a positive amplitude.

**Air:** Scan shows a negative amplitude, through a pattern which becomes white, then black, then white.

**Water:** The scan shows a positive amplitude. The pattern of the scan matches the pattern seen in the scan of a metal. Except the colors in the scan are less bold.





## EXTRA FEATURES

Focus mode is an option seen in certain GPR machines it allows you to turn the tails of scans into dots. Doing so helps reveal details obscured by the tails of the scans.

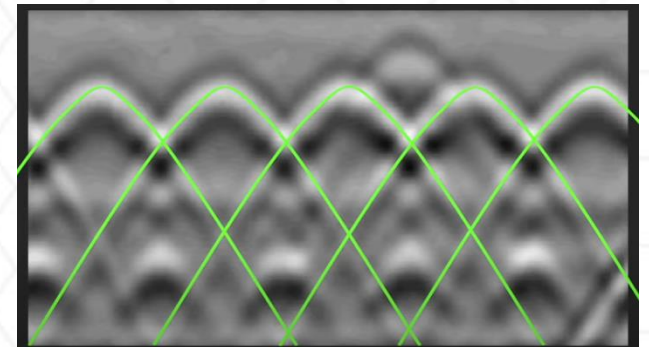
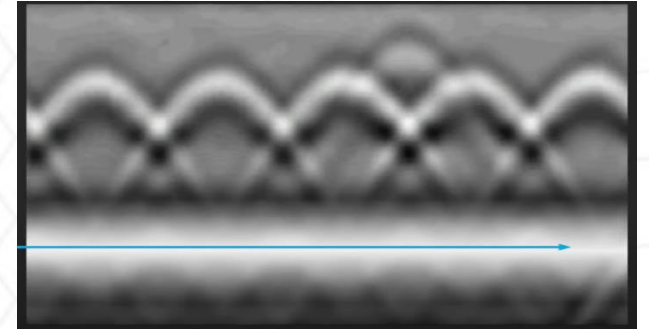
### **Auto Gain**

Often a significant portion of the GPR scan ends up being absorbed and is not recorded. Auto Gain makes our scans easier to visually interpret by making them bolder.

### **Manual Gain (similar to Auto Gain)**

Allows you to physically select and amplify amplitudes from scans. Doing so increases the readability of the scan.

If we add too much gain, truncation occurs and details from the scan are lost.



# What kind of machine learning algorithms can be used for finding the probability of detection of flaws in 3D printed parts?

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## How can ML be used to find flaws within a 3D-printed object?

1. **Acquire a data set of 3D-printed objects with known flaws.**
2. **Use a CT scan** to capture 2D slices of the surface details of the object with no flaws.
3. **Manipulate** the images into a usable format (the slices could need to be resized or have their brightness/contrast adjusted)
4. **Build your model**, identify what mathematical and statistical techniques can be used as its basis
  - Bayes' Theorem
  - Reliability Analysis
  - Monte Carlo Simulation
5. **Train your model**, teach your computer how to spot flaws.
  - If needed split data into two categories. Use most of your data for training (roughly  $\frac{3}{4}$ ), and use the rest for testing.
6. **Evaluate Model:** Evaluate your model's performance using the test data. Find the probability of detection to assess how well the model detected flaws.



## Bayes' Theorem

Bayes' theorem helps us determine the likelihood of something based on what we already know.

1. Identify your prior probability distribution (an initial guess or assumption about what our probability of detection is).
2. Gather new evidence, such as inspection results or characteristics of the printed parts.
3. Combine your prior belief with the new evidence to get a likelihood function (this is an estimate of the probability of detecting flaws in 3D-printed objects based on prior knowledge)

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Diagram illustrating Bayes' Theorem with labels:

- $P(A|B)$ : Probability of A occurring given evidence B has already occurred
- $P(B|A)$ : Probability of B occurring given evidence A has already occurred
- $P(A)$ : Probability of A occurring
- $P(B)$ : Probability of B occurring

## **Monte Carlo Simulation**

- 1. Model the printing process** (develop a model that simulates the 3D printing process)
- 2. Specify criteria for the flaw**, this criteria will work as the basis for finding the quality of the 3D-printed object  
Examples of criteria for the flaw
  - Surface Roughness (how smooth or rough the surface of the 3D-printed object is)
  - Dimensional accuracy is about whether the 3D-printed object has the right size and shape as the original object from the control group (the object with no flaws)
  - Structural Integrity (a measure of how strong the 3D-printed object is)
  - Defects such as Voids (empty spaces or holes inside the object) or Cracks (breaks or fractures in the object)
- 3. Create a virtual model of each sample**, from the data set that we are testing
- 4. After creating a model for each sample, compare it against the predefined flaw criteria.**
- 5. Calculate the frequency of flaws** across all regions of the object
- 6. Use the frequency of flaws to assess the quality of each object**
  - If the number of flaws detected in the sample is close to the actual number of flaws in the object, it suggests a high POD.
  - If the number of flaws detected in the sample is far from the actual number of flaws in the object, it suggests a low POD.

## **Reliability Analysis**

(estimate the number of flaws in a 3D-printed object by assessing the failure characteristics)

1. **Analyze old data**, identify patterns in regions with defects, find trends and common failure points
2. **If needed categorize failures by types** (surface defects, dimensional inaccuracies, structural failures) and determine how frequent and severe each type of flaw is.
3. **Calculate reliability metrics** like the failure rate (the average amount of time between failures)
4. **Determine if certain types of flaws are associated with specific types of failures**  
Types of failures
  - Layer Adhesion Failure (layers do not properly stick to one another during the printing process)
  - Overcuring or Undercuring (object begins to deform due to uneven cooling, overcuring makes object brittle, undercuring makes object soft)
5. **Predictive modeling**, based on this data create a model to estimate the number of flaws a 3D-printed object will have.

(Statistical techniques such as regression analysis or Bayes' Theorem can also be used to help build this model.)



# **Using Statistical Likelihood Techniques for Flaw Detection**

## **Bayesian inference**

### **Step 1: Establish your Prior Distribution**

The prior distribution is your best guess about the average flaw size based on scans of similar parts (what you already know), it is found before you look at the CT scan of the actual part .

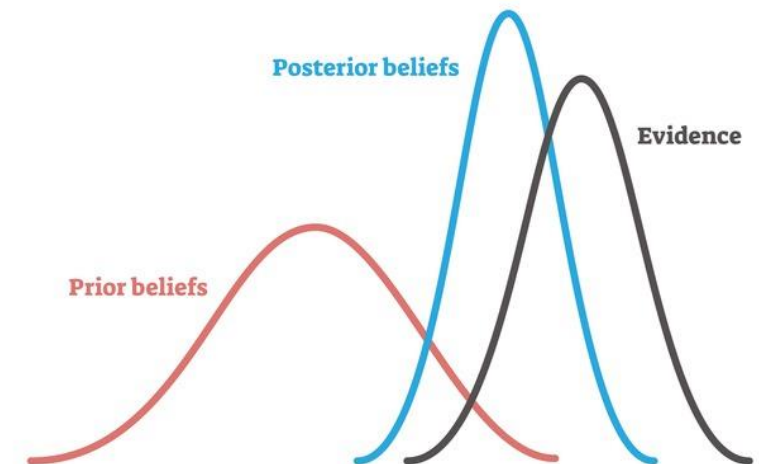
- **Example:** If you know that flaws in previous scans of the similar objects are about 2 mm in size, this is your **Prior Mean**.

### **Step 2: Establish your Likelihood Distribution (What the Data Tells You)**

- The likelihood function is how likely the data you get from the CT scan is.
- **Example:** If the CT scan shows something that looks like a 2.5 mm flaw, this is your **Likelihood Mean**

**Variance:** The variance in the prior and likelihood distribution affects how confidently you can estimate the flaw size. A more precise variance will increase the reliability of your estimate.

## **BAYESIAN ANALYSIS**





### Step 3: Compute the Updated Guess (Posterior Distribution)

- **Posterior Flaw Size ( $\mu_{\text{Posterior}}$ ):**

- $\mu_{\text{Posterior}} = ((\sigma^2_{\text{Likelihood}} * \mu_{\text{Prior}}) + (\sigma^2_{\text{Prior}} * \mu_{\text{Likelihood}})) / (\sigma^2_{\text{Likelihood}} + \sigma^2_{\text{Prior}})$

- **Posterior Variance ( $\sigma_{\text{Posterior}}$ ):**

- $\mu_{\text{Posterior}} = ((\sigma^2_{\text{Prior}} * \sigma^2_{\text{Likelihood}}) / (\sigma^2_{\text{Prior}} + \sigma^2_{\text{Likelihood}}))$

#### Plugging in the Values:

- Prior Mean = 2 mm
- Prior Variance = 0.5 mm
- Likelihood Mean = 2.5 mm
- Likelihood Variance = 0.3 mm

#### Posterior Mean:

Posterior Mean =  $(0.3^2 * 2 + 0.5^2 * 2.5) / (0.3^2 + 0.5^2) = \underline{\underline{2.37 \text{ mm}}}$

#### Posterior Variance:

Posterior Variance =  $(0.5^2 * 0.3^2) / (0.5^2 + 0.3^2) = \underline{\underline{.066 \text{ mm}}}$



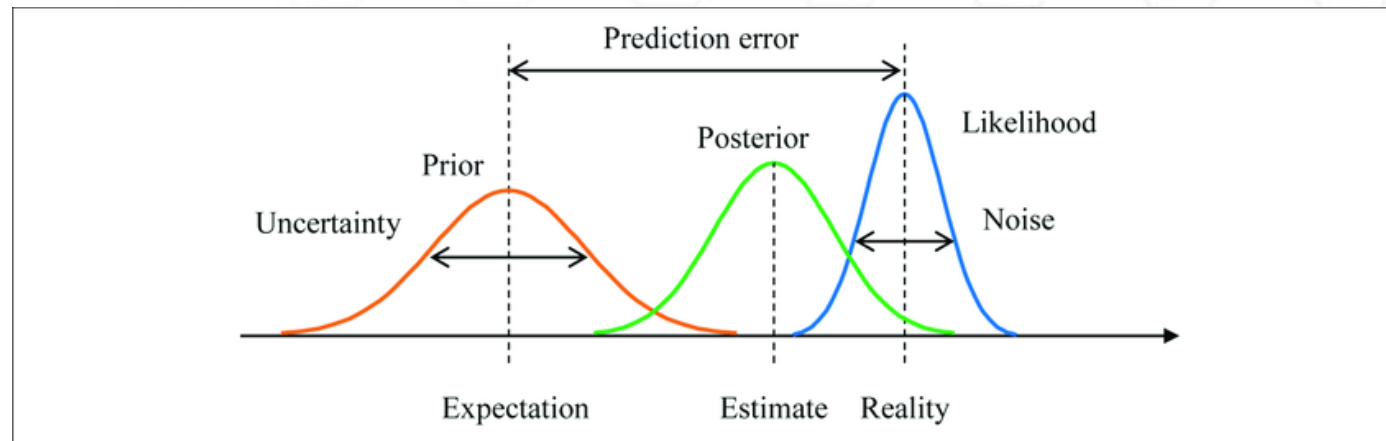
## Step 4: Use the Updated Guess (Posterior Distribution)

- **What It Is:**

The posterior distribution is your new estimate of the flaw size after looking at both your prior knowledge and the new data.

- **Why This Matters:**

Bayesian inference helps you make more accurate predictions about flaws by continually improving your estimates as you gather more data.



## Maximum Likelihood Estimation (MLE)

### 1. Model Selection:

- **Choose a Distribution:** Based on prior CT scans of similar shapes choose a probability distribution that fits the flaw sizes and shapes. A Normal or Weibull distribution works best
- **Parameterization:** Decide what parameters you want to use for your model.
  - For a normal distribution use mean and standard deviation as your parameters.
  - For a Weibull distribution use shape and scale as parameters

### 2. Create a Likelihood Function:

If you are using a normal distribution use the following formula as your likelihood function

$$L(\mu, \sigma^2) = \prod_{i=1}^n (1 / \sqrt{2\pi\sigma^2}) \exp(-(x_i - \mu)^2 / 2\sigma^2)$$

$\prod_{i=1}^n$  : This represents the product from  $i = 1$  to  $n$ , where  $n$  is total number of observations (from of similar shape)

$\sqrt{\phantom{x}}$  : square root.

**exp()** : exponential function (e raised to the power of the value in the parentheses)

$\mu$  : **Mean**

$\sigma$  : **Variance**

$x_i$  : each observed flaws size (from scan of similar shape)

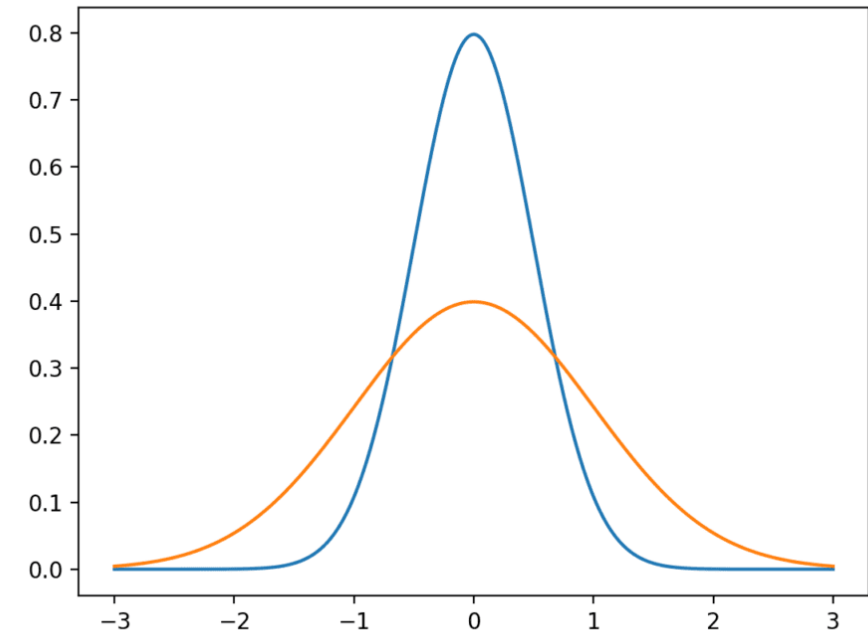


### Step 3: Maximize the Likelihood Function:

- Take the log of your likelihood function and then find the values of the parameters  $\mu$  and  $\sigma$  which maximize the log-likelihood function (this is usually using libraries found in statistical software).

### Step 4: Estimate Parameters:

- The values that maximize the log-likelihood function are your **Maximum Likelihood Estimation** estimates. These are considered the most likely values for the average flaw size and variance given your observed data from the flaws of a similar shape.



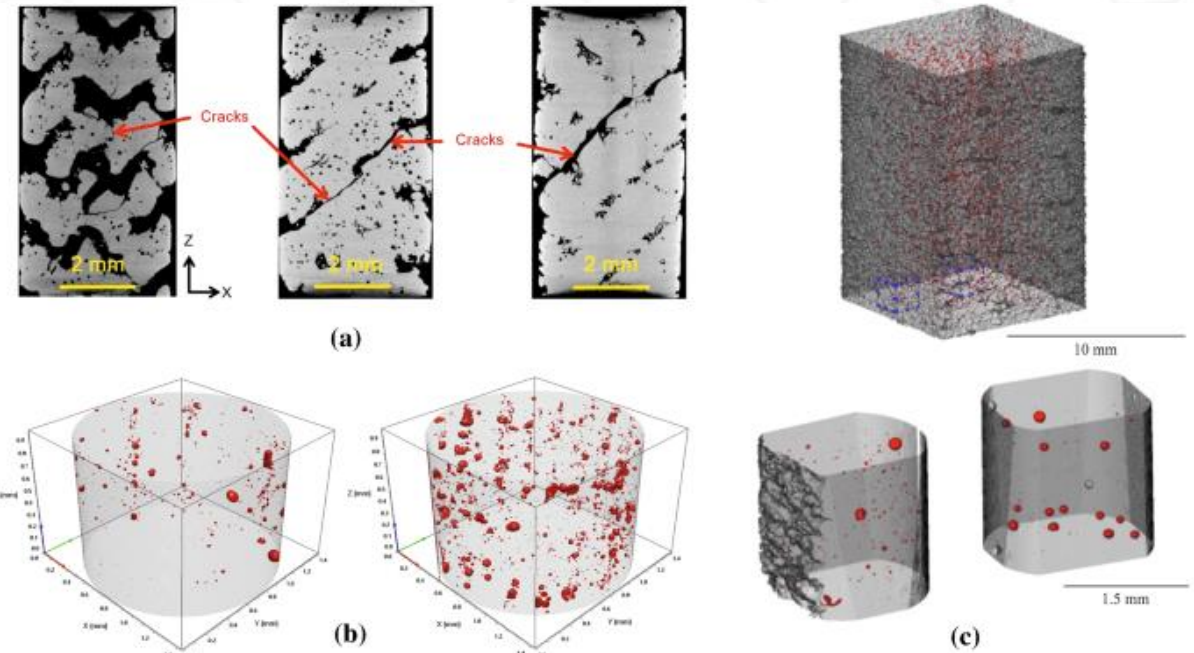


# **Using Statistical Distributions Techniques for Flaw Detection**

## Modeling Flaw Characteristics to Identify Size and Shape Distribution Types

### 1. Collect Data

- Gather data from CT scan results of similar shapes that shows the sizes and shapes of flaws found in these shapes.
- Record this data carefully, note the dimensions (length, width, depth) and shapes (spherical, elliptical) of each flaw.







## 2. Choose the Appropriate Distribution for Flaw Size and Shape

### Size Distribution:

- **Normal Distribution:** Apply this if most of the flaws are roughly the same size
- **Log-Normal Distribution:** Use this if the flaw sizes are positively skewed, meaning that there are lots of small flaws and few large flaws.
- **Weibull Distribution:** This is often used when looking at materials that tend to fail because of small defects.

### Shape Distribution:

- **Spherical Distribution:** Use this for spherical shaped flaws (these flaws are common in materials that break into even fragments)
- **Elliptical Distribution:** Apply this when you have elongated or flattened flaws (these flaws are common in materials that crack in specific directions.)

## 3. Use Software to Fit the Data to the Distribution

### For Flaw Size:

- **Fit the Data:** Check if your flaw sizes follow a pattern, like a normal distribution (bell curve), log-normal distribution, or Weibull distribution.
- **Software:** Python library 'scipy.stats' can be used to match your data to different statistical distributions it might fall under for size

### For Flaw Shape:

- **Analyze Shapes:** Look at whether the flaws are round or elliptical.
- **Fit the Data:** Use software to match the shape data to the right distribution.

#### 4. Validate the Fit:

Use goodness-of-fit tests, like the Chi-square test, to check how well the chosen distribution fits your actual data.

- Chi-square test:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$\chi^2$  = chi squared

$O_i$  = Actual number of flaws

$E_i$  = Expected number of flaws (based on scan of similar part)

- A low chi squared value means that your model fits well. A high value suggests a poor fit.

#### 5. Use the Distributions in Modeling:

Once you've identified the distribution that best describes your flaw size and shape, you can use this information in two ways:

1. **In Bayesian Inference:** The identified distribution becomes your prior distribution.
1. **Maximum Likelihood Estimation (MLE):** The same distribution can also be used to determine your likelihood equation which is then used to solve parameters (like average flaw size and shape).

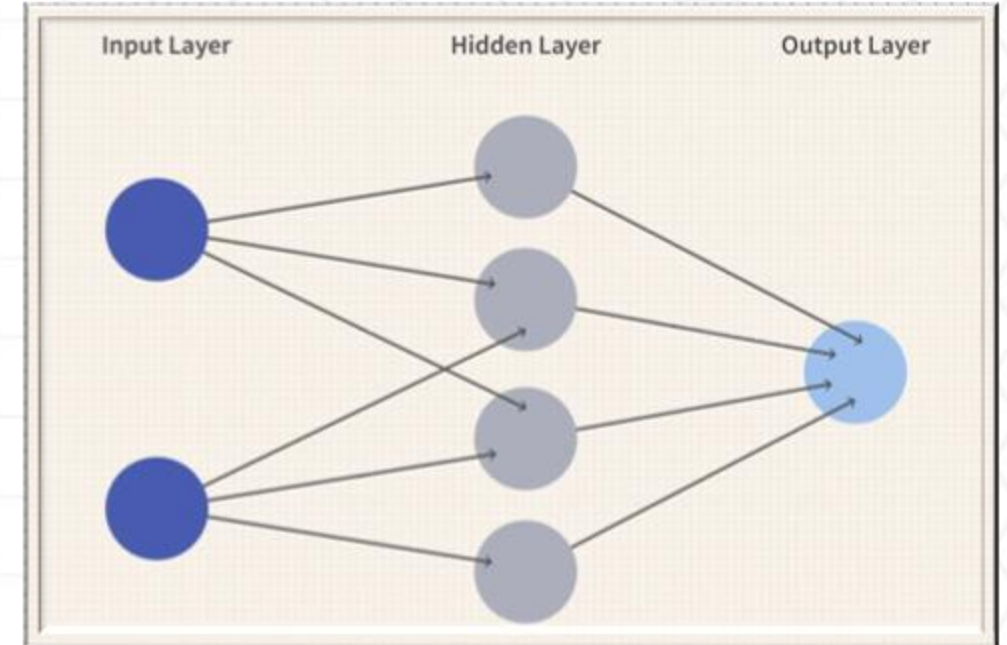


# Using Machine Learning for Flaw Detection

## What is a neural network?

A neural network is a computer model that mimics how our brain works

- **Input Layer:** Starting point, this is where data (from a CT scan or image) is inputted.
- **Hidden Layers:** Occur in the middle, most of the learning is done here. The hidden layers work to transform the input data.
- **Output Layer:** The final layer gives a result. This is where we can identify what's in a CT scan or image.



## Using Machine Learning to estimate flaw sizes in an object

### 1. Collect Data

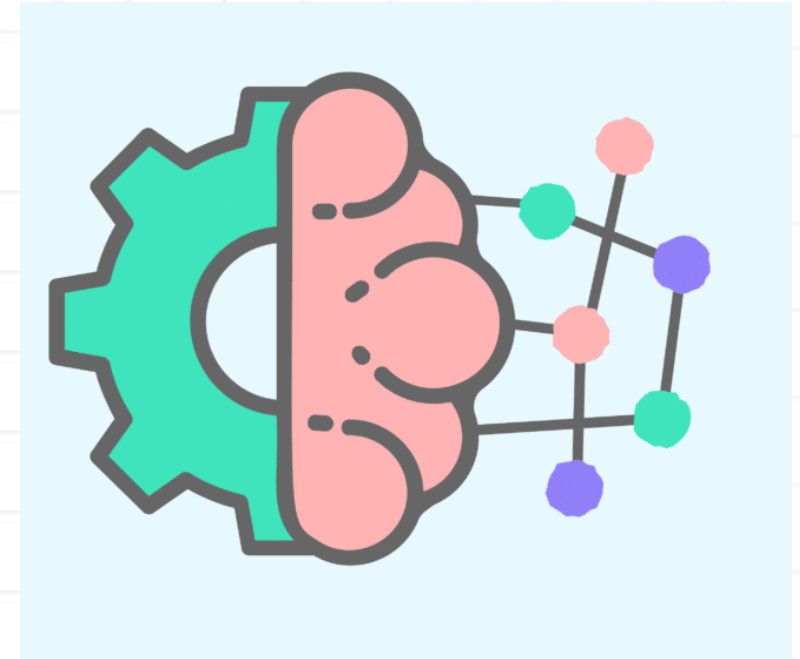
- Collect CT Scans of similar objects then identify features like the location, size and the shape of each flaw

### 2. Train the Model

- **Choose a Model:** Pick a ML model
- **Neural Network** works best, its used for directly looking at images to find flaws.
- **Train the Model:** Use the data from your CT scans of similar objects to teach the model how to predict flaws and estimate their size.

### 3. Check Accuracy

Test your model with new scans to see how well it finds flaw sizes and types.





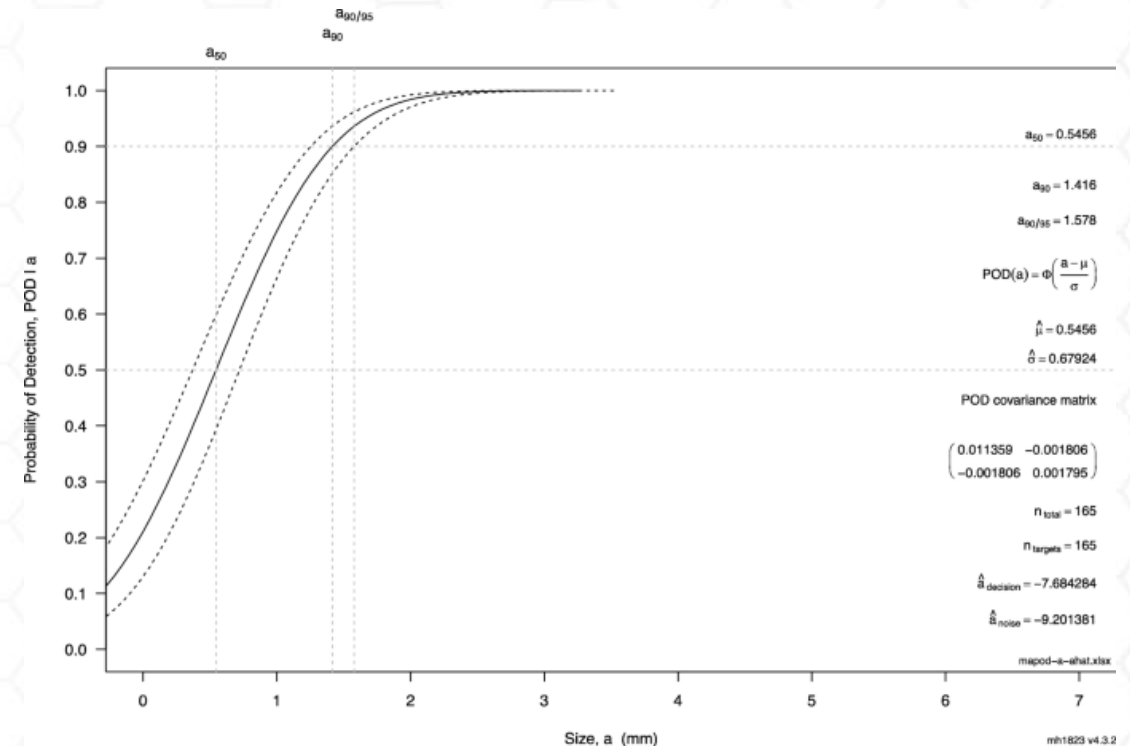


# **ASTM standards related to Probability of Detection (POD) for Non-Destructive Evaluation (NDE)**

## ASTM-E2862-12 Standard Practice for Probability of Detection Analysis for Hit/Miss Data

This ASTM standard gives a structured approach for creating a Probability of Detection (POD) analysis using hit/miss data. A "hit" is when a flaw is detected, while a "miss" occurs when a flaw is not caught.

This standard addresses how hit/miss data can be used to develop POD curves. A POD curve is a graph that shows the probability of detecting a flaw based on its size. The purpose of this study was to determine how effective different non-destructive testing methods are.



## Key Points:

### **1.High-Quality Data:**

The standard highlights the importance of using high-quality data. The method you use for data collection should be consistent. It's important to look at variables like environmental conditions and equipment calibration (ex. printer calibration) to ensure your data is accurate.

### **2.Large Sample Size:**

The standard also discusses the importance of using a large sample size to create a POD curve. A small sample size can cause the POD confidence intervals to be too wide, making them unreliable.

### **3.Documentation and Summarization:**

This standard underscores the importance of proper documentation. Make sure that your procedures for test setup and data collection are well-documented. Proper documentation allows you to be able to reproduce your data in the same manner it was collected.

### **4.Confidence Intervals in POD Analysis:**

This standard discusses the importance of using confidence intervals in your POD analysis. Confidence intervals give a range of what your actual POD is likely to be. Doing so helps with understanding how reliable different methods of detection are.

## Steps for creating a POD curve

### 1. Data Collection

- We will be using existing data from scans of 3D printed objects where an algorithm has already attempted to identify potential errors

### 2. Classify Data

#### -True Positive:

The algorithm correctly identifies a flaw.

#### -False Positive:

The algorithm incorrectly identifies a flaw where no flaw is present.

#### -True Negative:

The algorithm correctly identifies that there is no flaw.

#### -False Negative:

The algorithm fails to detect a flaw when one is present

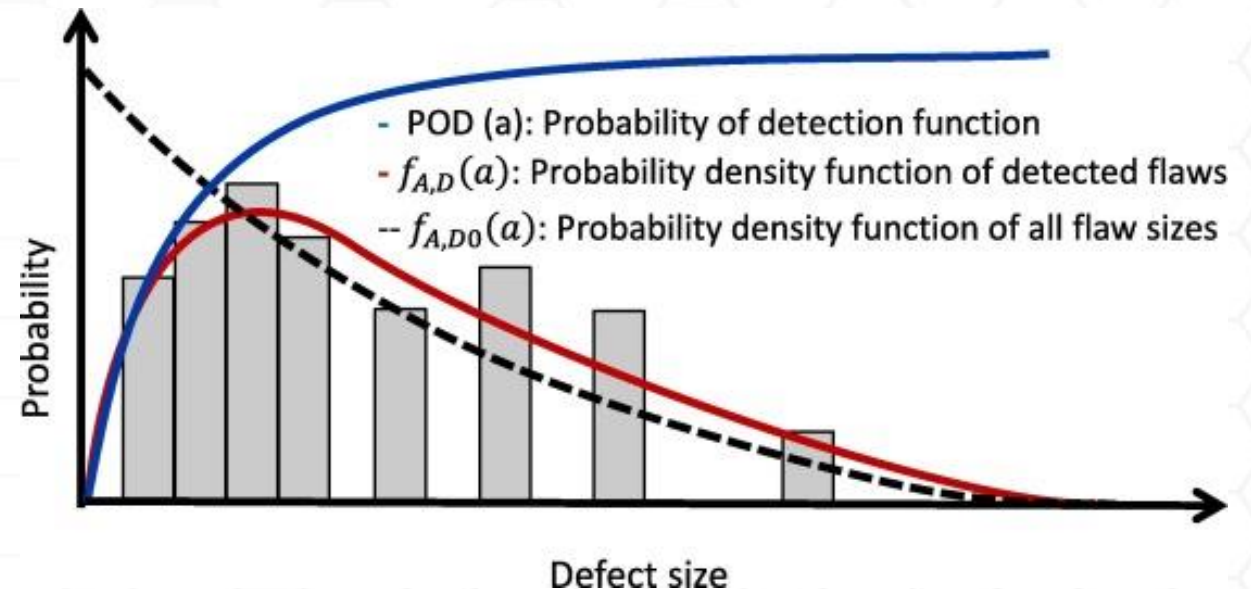
<b>True Positive</b>	<b>False Positive</b>
<b>False Negative</b>	<b>True Negative</b>

### 3. Solve for the POD:

- The probability of detection for each flaw size is the number of **True Positives** / (number of **True Positives** + number of **False Negatives**) for that flaw size.

### 4. Create the POD curve

- On the x-axis, plot flaw size. Use binning to characterize small, medium, and large flaws.
- On the y-axis, plot the POD as a percentage with a range from 0 to 100%.

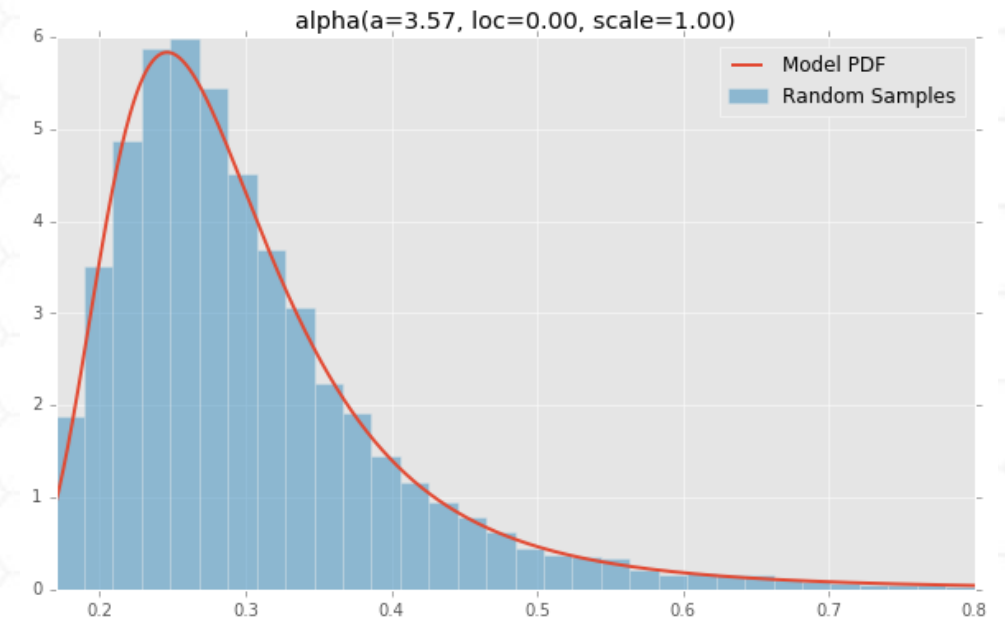




## Finding POD Confidence Intervals

### 1. Fitting the POD Data

- In general we use **logistic regression** to fit the POD data, as it models probabilities (between 0 and 1).
- You can also use software to fit the data to the appropriate distribution. Python's ``scipy.stats`` library can be used to match your data to the different statistical distributions it might fall under.





## 2. Fitting Process

- Logistic Regression:  
This is how you can model detection data using logistic regression.
- Fit the Logistic Regression Model to the Defect Detection Data:

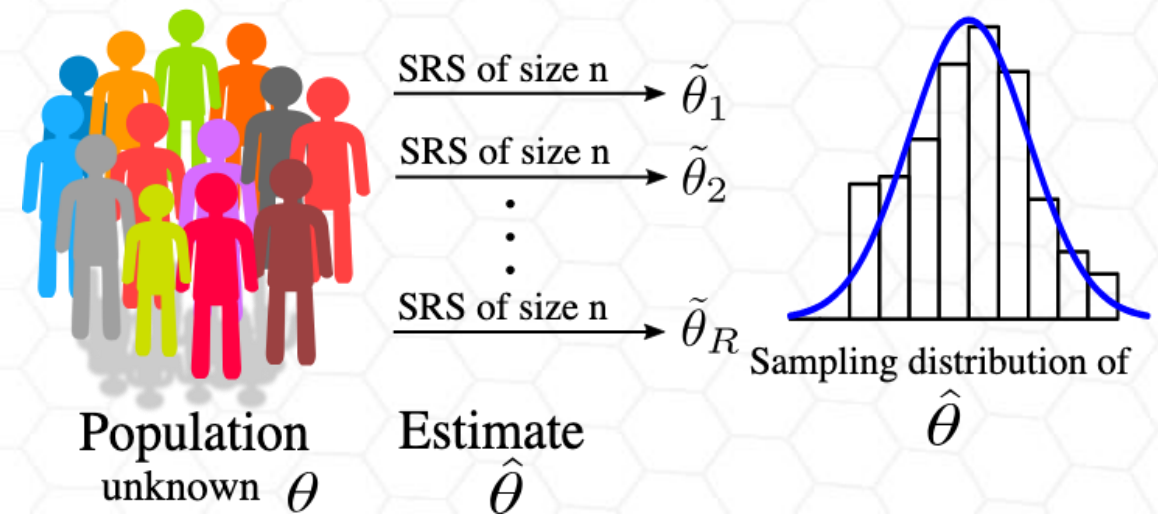
$$p(X) = \frac{e^{a+bX}}{1 + e^{a+bX}}$$

(the logistic regression model equation is the equation for the POD curve)

- $p(X)$  - Probability of detection
- $X$  - Flaw size
- $a$  and  $b$  - Coefficients estimated from the data
- Coefficient Effects:
  - $a$  affects the position of the POD curve along the x-axis
  - $b$  affects the slope of the curve, meaning how quickly the probability changes as the flaw size ( $X$ ) changes.

### 3. Estimate the Confidence Intervals from the Curve

1. Estimate the confidence intervals from the curve from the previous step.
2. To do this, we will use **bootstrapping**, which is a technique where we pull random data points from our original dataset and fit a new logistic regression model to that dataset to create a new POD curve.
3. After repeating this process a few times, we will end up with several POD curves.
4. Plot these POD curves.
  - We use these curves to figure out a range where the true POD curve is likely to lie.
  - A 95% confidence interval means that if we repeated this process 100 times, 95 times out of 100, the POD is likely to fall within this range.



# How do we integrate False Positives and True Negatives into POD Curves?

## 1. False Positive Rate

FPR will tell you often the algorithm incorrectly identifies a flaw when no flaw is present for a given flaw size.

$$\frac{FP}{N} = \frac{FP}{FP + TN}$$

$FP$  = number of false positives

$TN$  = number of true negatives

$N$  = total number of negatives

## 2. True Negative Rate

TNR shows you how well the algorithm, identifies the absence of flaws.

$$TNR = \frac{\text{True Negative}}{\text{True Negative} + \text{False Positive}}$$

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