Here is the corrected version with the proper formula for vector projection:

Linear Algebra Formula Guide

1. Vectors

- **Definition**: A vector is an ordered set of numbers representing magnitude and direction in space.

$$\vec{v} = (\,v\,)_1\, v_2 \vdots v_n$$

- **Scalars**: A scalar is a single number that can scale a vector (e.g., k).

- **Vector Addition**: The sum of two vectors is done element-wise.

$$\vec{v} + \vec{u} = (\,v\,)_1 + u_1 v_2 + u_2 \vdots v_n + u_n$$

- **Scalar Multiplication**: Multiply each component of the vector by a scalar k.

$$k\vec{v} = (\,k\,)\, v_1 k v_2 \vdots k v_n$$

- **Dot Product (Inner Product)**: The dot product of two vectors $\vec{v}$ and $\vec{u}$ is:

$$\vec{v} \cdot \vec{u} = v_1 u_1 + v_2 u_2 + \ldots + v_n u_n$$

This yields a scalar.

- **Vector Projection**: The projection of vector $\vec{v}$ onto vector $\vec{u}$ is:

$$Proj_{\vec{u}} \vec{v} = \frac{\vec{v} \cdot \vec{u}}{\|\vec{u}\|^2} \vec{u}$$

- **Cosine Similarity**: The cosine of the angle between two vectors is given by:

$$\cos(\theta) = \frac{\vec{v} \cdot \vec{u}}{\|\vec{v}\| \|\vec{u}\|}$$

where $\|\vec{v}\|$ is the norm (magnitude) of the vector.

- **Orthogonal Vectors**: Two vectors are orthogonal if their dot product is zero:

$$\vec{v} \cdot \vec{u} = 0$$

- **Normal and Ortho-normal Vectors**: - A **normal vector** has a magnitude (norm) of 1.

$$\|\vec{v}\| = 1$$

- **Ortho-normal vectors** are orthogonal and have a unit norm.

- **Vector Norm**: The length (or magnitude) of a vector is:

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$$

- **Vector Space**: A set of vectors that satisfies closure under addition and scalar multiplication.

- **Linear Combination**: A vector $\vec{v}$ is a linear combination of vectors $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_k$ if:

$$\vec{v} = c_1 \vec{u}_1 + c_2 \vec{u}_2 + \ldots + c_k \vec{u}_k$$

where $c_1, c_2, \ldots, c_k$ are scalars.

- **Linear Span**: The set of all linear combinations of a set of vectors is called the span of those vectors.

- **Linear Independence**: A set of vectors is linearly independent if none of the vectors can be expressed as a linear combination of the others.

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2. Matrices

- **Matrix Definition**: A matrix is a rectangular array of numbers arranged in rows and columns.

$$A = (a)_{11} \, a_{12} \ldots a_{1n} a_{21} a_{22} \ldots a_{2n} \vdots \vdots \ddots \vdots a_{m1} a_{m2} \ldots a_{mn}$$

- **Matrix Addition**: Add corresponding elements of two matrices.

$$A + B = (a)_{11} + b_{11} a_{12} + b_{12} \ldots a_{1n} + b_{1n} \ldots \ldots \ddots \ldots$$

- **Transpose of a Matrix**: Flip a matrix over its diagonal.

$$A^T = (a)_{11} \, a_{21} \ldots a_{12} a_{22} \ldots \vdots \vdots \ddots$$

- **Scalar Multiplication**: Multiply every element by a scalar k.

$$kA = (k) \, a_{11} k a_{12} \ldots k a_{21} k a_{22} \ldots$$

- **Matrix Multiplication**: Multiply two matrices by summing the product of rows and columns.

$$(AB)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

- **Matrix Multiplication Properties**: 1. **Associativity**: $A(BC) = (AB)C$ 2. **Distributivity**: $A(B + C) = AB + AC$ 3. **Non-Commutativity**: $AB \neq BA$ in general.

- **Hadamard Product**: Element-wise multiplication of two matrices.

$$(A \circ B)_{ij} = a_{ij} b_{ij}$$

- **Determinant**: For a 2×2 matrix A, the determinant is:

$$det(A) = a_{11} a_{22} - a_{12} a_{21}$$

For larger matrices, use cofactor expansion.

- **Identity Matrix**: A matrix with ones on the diagonal and zeros elsewhere.

$$I = (1) \, 0 \ldots 0 1 \ldots \vdots \vdots \ddots$$

- **Invertible Matrix and Inverse**: A matrix A is invertible if there exists a matrix A^{-1} such that:

$$AA^{-1} = A^{-1} A = I$$

- **Rank**: The rank of a matrix is the number of linearly independent rows or columns.

- **Trace**: The trace of a matrix is the sum of its diagonal elements.

$$tr(A) = a_{11} + a_{22} + \ldots + a_{nn}$$

- **Symmetric Matrix**: A matrix is symmetric if $A = A^T$.

- **Diagonal Matrix**: A matrix is diagonal if all off-diagonal elements are zero.

- **Orthogonal Matrix**: A matrix is orthogonal if $A^T A = I$.

- **Ortho-Normal Matrix**: A matrix whose rows and columns are orthogonal unit vectors.

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3. Eigenvalues and Eigenvectors

- **Eigenvalue Equation**: For a matrix A, a scalar λ is an eigenvalue if there exists a non-zero vector $\vec{v}$ such that:

$$A\vec{v} = \lambda\vec{v}$$

$\vec{v}$ is the eigenvector corresponding to λ.

- **Characteristic Equation**: To find eigenvalues, solve the characteristic equation:

$$det(A - \lambda I) = 0$$

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4. Principal Components Analysis (PCA)

- **Principal Components**: In PCA, the principal components are the eigenvectors of the covariance matrix of the data, and the eigenvalues represent the variance along those components.

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5. Singular Value Decomposition (SVD)

- **Definition**: For an $m \times n$ matrix A, the singular value decomposition is:

$$A = U\Sigma V^T$$

where: - U is an $m \times m$ orthogonal matrix. - Σ is an $m \times n$ diagonal matrix with singular values. - V^T is the transpose of an $n \times n$ orthogonal matrix V.