

## PRT - Ex-3

### Bayes Theorem Basics

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

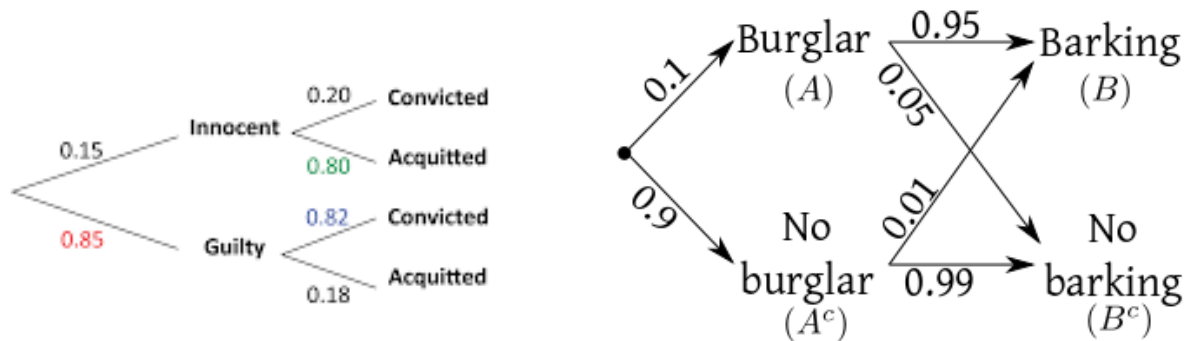
THE PROBABILITY OF "B" BEING TRUE GIVEN THAT "A" IS TRUE

THE PROBABILITY OF "A" BEING TRUE

THE PROBABILITY OF "A" BEING TRUE GIVEN THAT "B" IS TRUE

THE PROBABILITY OF "B" BEING TRUE

### Examples



### • Medical Diagnosis

- suppose we know from statistical data that flu causes fever in 80% of cases, approximately 1 in 10,000 people have flu at a given time, and approximately 1 out of every 50 people is suffering from fever:

$$\Pr(\text{fever} | \text{flu}) = 0.8 \quad \Pr(\text{flu}) = 0.0001 \quad \Pr(\text{fever}) = 0.02$$

- Given a patient with fever, does she have flu? Answer by applying Bayes' rule:

$$\begin{aligned} \Pr(\text{flu} | \text{fever}) &= [ \Pr(\text{fever} | \text{flu}) \cdot \Pr(\text{flu}) ] / \Pr(\text{fever}) \\ &= 0.8 \times 0.0001 / 0.02 = 0.004 \end{aligned}$$

## Application of Bayes Theorem - Exercises

### **1. Chocolate Problem:**

Suppose a Chocolate manufacturing company is sending 40% of the Chocolates to Stores-A and 60% of the Chocolates to Stores-B. After a day Stores-A reports that 7% of the Chocolates delivered are defective and Stores-B reports that 5% of Chocolates delivered are defective.

- a) Find the probability that Chocolates sent to Stores-A and it is defective.

**Ans)**  $P(\text{Chocolate to A and it is defective})$

$$P(A)P(D|A) = 0.4(0.07) = 0.028$$

- b) Find the probability that a product is sent to company A and it is not defective.

**Ans)**  $P(\text{Chocolate to A and not defective})$

$$P(A)P(D'|A) = 0.4(0.93) = 0.372$$

- c) Find the probability that a product is sent to company B and it is defective.

**Ans)**  $P(\text{Chocolate to B and defective})$

$$P(B)P(D|B) = 0.6(0.05) = 0.03$$

- d) Find the probability that a product is sent to company B and it is not defective.

**Ans)**  $P(\text{Chocolate to B and not defective})$

$$P(B)P(D'|B) = 0.6(0.95) = 0.57$$

### **2. Faculty Problem:**

A Department in an university has rated 80% of its faculty as satisfactory (S) and 20% as unsatisfactory (S'). On seeing their resume, it showed that 75% of the satisfactory faculty had previous work experience (E) in teaching, while 20% of the unsatisfactory faculty had no teaching experience (E') in the profession.

If a faculty who has had previous work experience is hired, what is the approximate empirical probability that this faculty will be an unsatisfactory employee?

$$\text{Ans) } P(S'|E) = \frac{P(S') * P(E|S')}{P(S') * P(E|S') + P(S) * P(E|S)} = \frac{0.2 * 0.8}{0.2 * 0.8 + 0.8 * 0.75} = 0.2105$$

**OR**

	<b>S</b>	<b>S'</b>	<b>TOTAL</b>
<b>E</b>	600	160	760
<b>E'</b>	200	40	240
<b>TOTAL</b>	800	200	<b>1000</b>

$$\text{Ans) } P(S'|E) = \frac{160}{760} = 0.2105$$

### 3. Driver Problem:

Suppose a survey of 1000 drivers in a metropolitan area during a 3-year period was taken. The following results were found.

Age Group	18-25	26-39	40-55	55+	
0-1 Accidents	100	150	250	75	575
2-3 Accidents	150	25	125	25	325
3+ accidents	50	25	25	0	100
Totals	300	200	400	100	1000

Suppose we randomly select a driver from the group. What is the probability that the driver,

- a) Is in the 26-39 age group, given they have more than 3 accidents.

**Ans)**  $25/100=0.25$

- b) Had 2-3 accidents, given they are in the 18-25 age group.

**Ans)**  $150/300=0.5$

- c) Had 0-1 accidents, given they are in the 40-55 age group.

**Ans)**  $250/400=0.625$

- d) Is in the 40-55 age group, given they had 0-1 accidents

**Ans)**  $250/575=0.435$