CS 357 / MATH 357
Fall 2013

Numerical Methods – Homework 4
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Due: 10/15/2013

Note: Homework is due **5pm** on the due date. Please submit your homework through the dropbox in the Siebel Center basement. Make sure to include your name and **netid** in your homework.

Problem 1 [10pt] Consider the following sparse matrix:

$$A = \begin{bmatrix} 11 & 0 & 0 & 0 & 0 & 0 & 14 \\ 0 & 0 & 0 & 0 & 23 & 0 & 0 \\ 0 & 0 & 0 & 0 & 33 & 34 & 0 \\ 0 & 15 & 0 & 0 & 43 & 44 & 0 \\ 0 & 0 & 0 & 0 & 0 & 54 & 0 \\ 0 & 0 & 62 & 0 & 0 & 0 & 0 \\ 0 & 0 & 72 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Write a python program to find the vectors AA, JA and IA for the Compressed Sparse Row structure.

(b) Convert the CSR format to LIL format using `scipy.sparse.csr_matrix` and then print the LIL format data and rows using attributes associated with the LIL matrix data type.

Print a copy of the code along with all the output.

Problem 2 [10pt] In this problem we will be using Secant and Newton's method to find a root of a function and make a comparison between the two methods.

(a) Use `matplotlib` to plot the function $f(x) = (5 - x)\, exp(x) - 5$, for x between 0 and 5. (This function is associated with the *Wien radiation law*, which gives a method to estimate the surface temperature of a star).

(b) Use the algorithm for the secant method on slide 6 of lecture 12 as a guide to write a python program to find a root of $f(x)$ by secant method, using initial guess $x_0 = 5$. Print out your approximate solution x_k and run until $|f(x_k)| \leq 10^{-8}$.

(c) Use the algorithm for Newton's method on slide 31 of lecture 11 as a guide to write a python program to find a root of $f(x)$ by newton method, using initial guess as $x_0 = 5$. Print out your approximate solution x_k and run until $|f(x_k)| \leq 10^{-8}$.

(d) Use matplotlib to plot $log(x_k)$ vs k for both methods on a single plot. This plot shows the relative rate of convergence.

Problem 3 [3pt] Find the root or roots of $ln[\frac{1+x}{1-x^2}] = 0$ using the bisection method and initial interval as $[\frac{-3}{4}, \frac{1}{4}]$.

Problem 4 [2pt] True/False questions

(a) **(Truel/False)** The Bisection method converges faster than the Newton method for root finding.

(b) **(Truel/False)** The Newton method guarantees convergence.