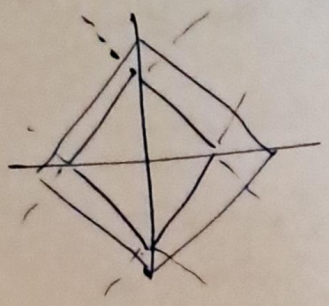
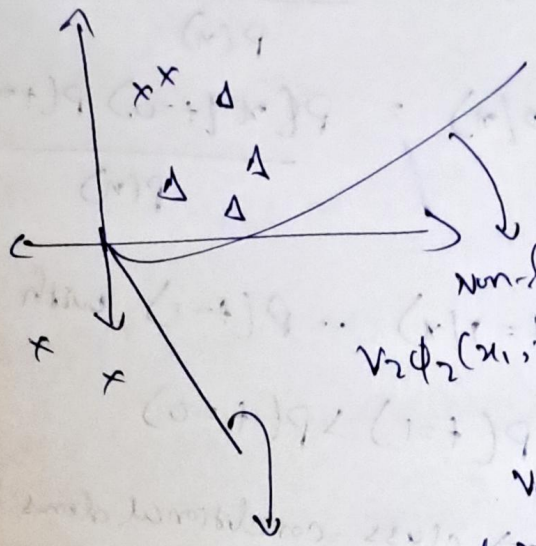
05/09/2024

LASO



classification function



Non-linear boundary

$v_2\phi_2(x_1, x_2) + v_1\phi_1$

(x_1, x_2)

$v_0 = 0$

Boundary separate x and Δ

$= v_2 x_2 + w_1 x_1 + w_0 = 0$

$f(x) = w_2 x_2 + w_1 x_1 + w_0$

$f(x) > 0 \Rightarrow$ class 1

$f(x) < 0 \Rightarrow$ class 0 / class -1

class = $\text{sign}[f(x)] \in \{-1, 1\}$

Bycastom classifcation

$t \in \{0, 1\}$ → binary classification

$P(t_i = 1 / x_i) > P[t_i = 0 / x_i]$

Decision Rule: then we assign x_i to class 1 ; x_i → ith sample

Assuming $p(x_i|t_i=1) \sim \mathcal{N}(\mu_0, \sigma_1)$

Bayes Rule

$$P(t/x) = \frac{P(x|t=1)\,P(t=1)}{P(x)}$$

$$P(t=0/x) = \frac{P(x|t=0)\,P(t=0)}{P(x)}$$

compare

$P(t=1/x) \cdot\cdot P(t=1)$ with $P(x|t=0)\,P(t=0)$

if $P(t=1) > P(t=0)$

$P(x/t)$ → class conditional density

$P(x)$ → prior probability of a class

Algo to build a bayesian classifier

1) Do EDA

2) a) $P(t=c) \leftarrow \frac{n_c}{\sum n_c}$

b) Estimate

θ_c = parameter for $P(x|t=c)$

3) Set decision criterion as y_i = argmax

y_i = argmax $\{ x_i|\theta_c) P(t=c) \}$

Special Case

Assumptions

1) All class conditional densities are gaussian

2) Shared cov are same $\Rightarrow \Sigma_0 = \Sigma_1 = \Sigma$

$$\frac{P(t=1/x)}{P(t=0/x)} > 1 \Rightarrow C \text{ is } 1 \text{ not } 0$$

$$\log P(t=1/x) - \log P(t=0/x) > 0$$

$$\log P(x/t=1) + \log P(t=0) - \log P(x/t=0) - \log (t=0)$$

$$P(x/t=1) = \frac{1}{\sqrt{(2\pi)^d \det(\Sigma)}} e^{-\frac{1}{2}(x-\mu_1)^T \Sigma^{-1} (x-\mu_1)}$$

$$\therefore x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \quad \mu_i = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \quad \Sigma_1 = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_{22}^2 \end{bmatrix}$$