Dt : 1 August / Basic Maths for DS/ML

- In py. addition of vector and scalar is done in following way:

$$\bar{x}+a = \begin{bmatrix} x_1+a \\ x_2+a \\ x_3+a \end{bmatrix} = \bar{x} + a\mathbb{1} \quad \left\{ \mathbb{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in R^{n\times 1} \right\}$$

$$\bar{x}\cdot\bar{y} = \bar{x}^T\bar{y} = \langle x, y \rangle = \underbrace{x_1y_1 + x_2y_2}_{\text{scalar}}$$

$$\overset{N\times 1}{\bar{x}} \odot \overset{N\times 1}{\bar{y}} = \begin{bmatrix} x_1y_1 \\ x_2y_2 \\ \vdots \\ x_Ny_N \end{bmatrix}$$

mask

- V is a vector space if for all $V_1, V_2 \in V$
$cV_1 \in V$ and $V_1+V_2 \in V$; $c_1V_1 + c_2V_2 \in V$
for $c, c_1, c_2 \in R$

$$XX^{-1} = I = \begin{bmatrix} 1 & 0 & \cdots & \\ 0 & 1 & & \\ \vdots & & \ddots & \\ \vdots & & & 1 \end{bmatrix}$$

X^{-1} only exist if X is invertible $\Leftrightarrow$ full rank

Pseudo-Inverse :

$$\cancel{X_{M\times N} X^{+}_{N\times M} = I_{M\times M}} \qquad X^{+}_{N\times M} X_{M\times N} = I_{N\times N}$$

$$\left[\underbrace{(X^H X)^{-1}}_{} X^H \right] X$$

↳ this should be invertible

Eigen decomposition :

$$\underset{N\times N}{\bar{A}}\underset{N\times 1}{\bar{v}_1} = \lambda_1 \bar{v}_{1\,N\times 1}$$

λ_1 → eigen value of $\bar{A}$

$\bar{v}_1$ → eigen vector of $\bar{A}$

Implication of $\lambda_1 = 0$ ⇒ Rank deficiency

$$\bar{A} = \bar{Q}\,\bar{\Lambda}\,\bar{Q}^{-1}$$

$$\begin{bmatrix} \bar{v}_1 & \bar{v}_2 & \dots & \bar{v}_N \end{bmatrix} \begin{bmatrix} \lambda_1 & \dots & \\ 0 & \lambda_2 & \\ \vdots & & \lambda_3 \\ & & & \ddots \end{bmatrix} \bar{Q}^{-1}$$

Tensor ↔ Representation
↓
Transpose of tensor

function :

Smooth function ⇒ Derivative is continuous at x.

for Lipshitz Continuity :

$$|f(x + \Delta x) - f(x)| \le K\,\Delta x \Rightarrow \text{Lipshitz continious}$$

$K \in \mathbb{R}^{+}$

$f'(x) = 0$ → maxima
↳ minima

$f''(x) = 0$ → point of inflecti

~~$f'''(x) = 0$ → Saddle point~~

$$\Delta f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \alpha \vec{v}$$

α → mg, $\vec{v}$ → dir

$$f(x_1, x_2) = 5x_1^2 + 3x_2^2$$

$$\nabla f = \begin{bmatrix} 10\,x_1 \\ 6\,x_2 \end{bmatrix}$$

at critical pt.

$x_1 = 0$, $x_2 = 0$

Hessian matrix

$$Hf = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

for maximum all eigen value should be negative

for minima all eigen val should be ~~pos~~ positive

for saddle point one should be positive and other negative.

Constrained Optimization using Lagrange multiplier

maximize $f(x)$, subjected to $g(x) = 0$

$$\mathcal{L}(x) = f(x) + \lambda g(x) \; ; \; \lambda \neq 0$$

$$\cancel{\nabla f(x)} = 0 \; ; \; \boxed{\nabla \mathcal{L}(x) = 0} \Rightarrow \nabla f(x) = -\lambda \nabla g(x)$$