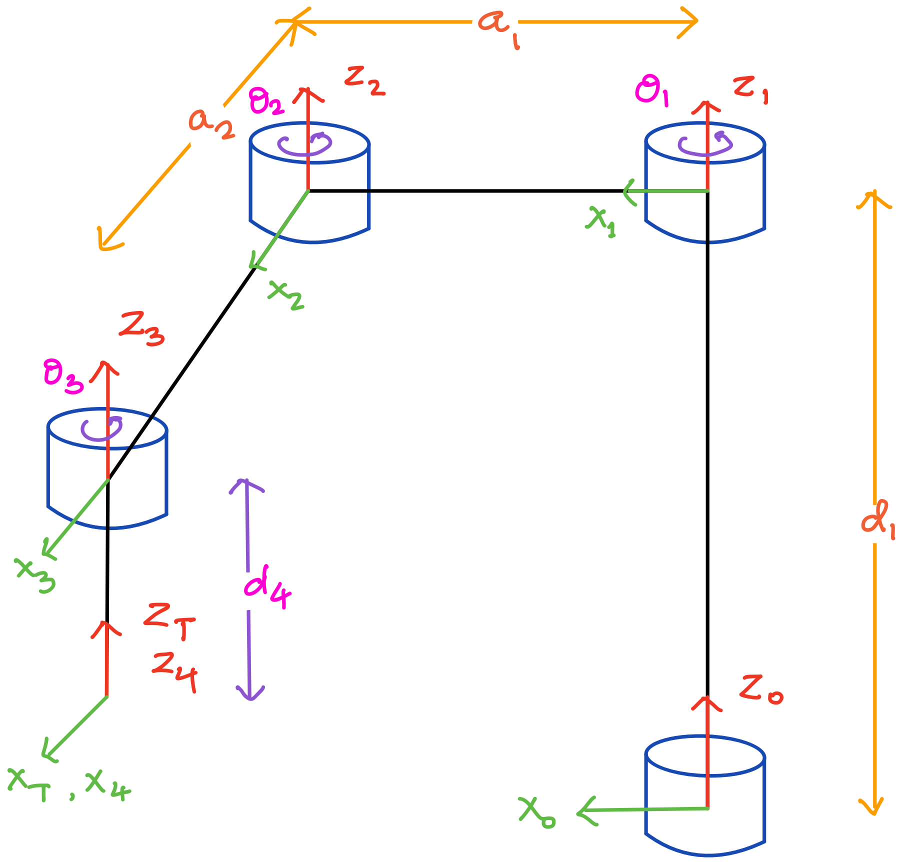
SHIVAM KUMAR PANDA (105730045)



DH Parameters

$i-1$	i	α_{i-1}	a_{i-1}	d_i	θ_i
0	1	0	0	d_1	θ_1
1	2	0	a_1	0	θ_2
2	3	0	a_2	0	θ_3
3	4	0	0	$-d_4$	0

$i-1$	i	α_{i-1}	a_{i-1}	d_i	θ_i
0	1	0	0	0.4	θ_1
1	2	0	0.325	0	θ_2
2	3	0	0.225	0	θ_3
3	4	0	0	$-d_4$	0

Here the frame 4 and frame T (tool) is coincided to make things simpler.

So $d_4 \in [0.17, 0.32]$

Forward Kinematics

$${}^{0}T_4 = {}^{0}T_1 \cdot {}^{1}T_2 \cdot {}^{2}T_3 \cdot {}^{3}T_4$$

$$= \begin{bmatrix} c_{123} & -s_{123} & 0 & a_2 c_{12} + a_1 c_1 \\ s_{123} & c_{123} & 0 & a_2 s_{12} + a_1 s_1 \\ 0 & 0 & 1 & d_1 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{123} & -s_{123} & 0 & 0.225\, c_{12} + 0.325\, c_1 \\ s_{123} & c_{123} & 0 & 0.225\, s_{12} + 0.325\, s_1 \\ 0 & 0 & 1 & 0.4 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Kinematics

Step 1:

$$T_E = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_E \cdot {}^0T_1^{-1} =$$

$$\begin{bmatrix} r_{11}c_1 - r_{21}s_1 & r_{12}c_1 - r_{22}s_1 & r_{13}c_1 + r_{23}s_1 & p_x c_1 + p_y s_1 \\ r_{21}c_1 - r_{11}s_1 & r_{22}c_1 - r_{12}s_1 & r_{23}c_1 - r_{13}s_1 & p_y c_1 - p_x s_1 \\ r_{31} & r_{32} & r_{33} & p_z - d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{12} \cdot T_{23} \cdot T_{34} =$$

$$\begin{bmatrix} c_{23} & -s_{23} & 0 & a_1 + a_2 c_2 \\ s_{23} & c_{23} & 0 & a_2 s_2 \\ 0 & 0 & 1 & -d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Comparing both the sides we get,

$$p_z - d_1 = -d_4 \quad \text{—— (1)}$$

$$\Rightarrow \quad d_4 = d_1 - p_z$$

$$p_x c_1 + p_y s_1 = a_1 + a_2 c_2 \quad \text{—— (2)}$$
$$p_y c_1 - p_x s_1 = a_2 s_2 \quad \text{—— (3)}$$

Squaring and adding the equations we get.

$$p_x^2 + p_y^2 = a_1^2 + a_2^2 + 2a_1a_2c_2$$

$$c_2 = \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$s_2 = \pm\sqrt{\frac{4a_1^2a_2^2 + a_1^2 + a_2^2 - p_x^2 - p_y^2}{2a_1a_2}}$$

$$\theta_2 = \operatorname{atan2}\left(\pm\sqrt{\frac{4a_1^2a_2^2 + a_1^2 + a_2^2 - p_x^2 - p_y^2}{2a_1a_2}}, \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2a_1a_2}\right)$$

The two solutions for θ_2 represents the elbow-up and elbow-down positions.

$$\theta_1 = \boxed{\beta + \psi} \qquad \theta_2 < 0$$

$$\boxed{\beta - \psi} \qquad \theta_2 > 0$$

where $\beta = \operatorname{atan2}(p_y, p_x)$

$$c\psi = \frac{p_x^2 + p_y^2 + a_1^2 - a_2^2}{2 a_1 \sqrt{p_x^2 + p_y^2}}$$

$0 \leq \psi \leq 180°$

Once we know θ_1 and θ_2, θ_3 can be determined before doing ${}^0T_1^{-1}$,

$$c_{123} = r_{11}$$

$$s_{123} = r_{21}$$

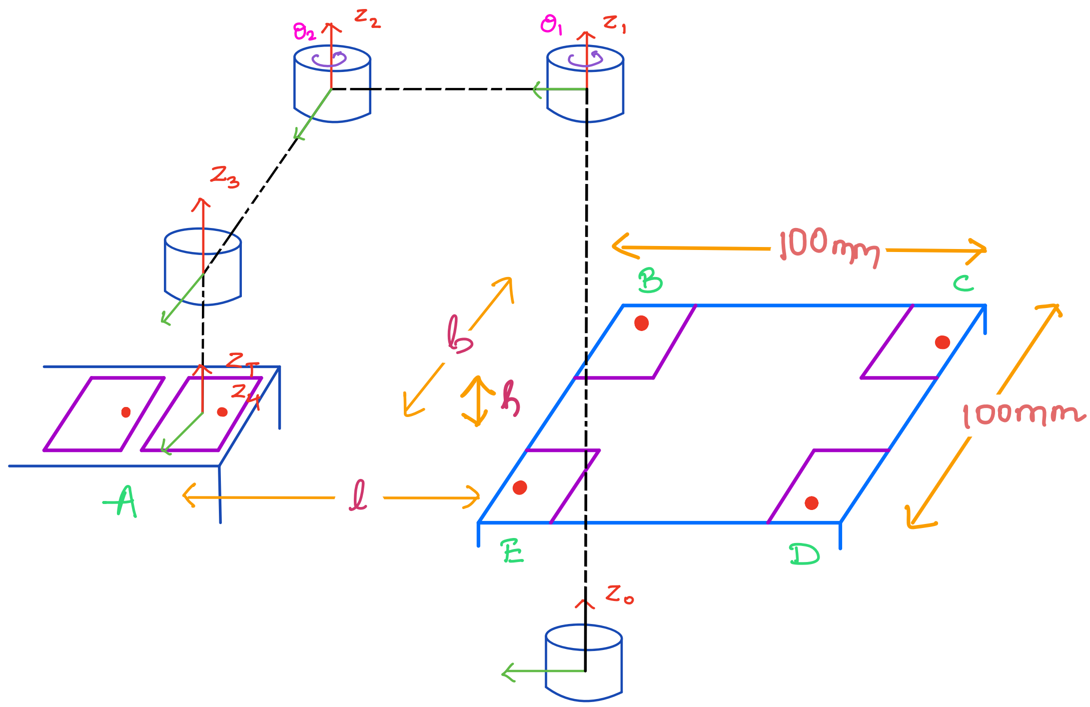
$$\Rightarrow \quad \theta_{123} = \operatorname{atan2}(s_{123}, c_{123})$$

$$\theta_{123} = \operatorname{atan2}(r_{21}, r_{11})$$

$$\Rightarrow \quad \theta_3 = \theta_{123} - \theta_1 - \theta_2$$

$$\boxed{\theta_3 = \operatorname{atan2}(r_{21}, r_{11}) - \theta_1 - \theta_2}$$

Locations of Feeder & PCB



l = distance between the centre of the chip on the feeder to the left edge of the PCB board.

b = distance between the centre of the chip on the feeder to the upper edge of the PCB board.

h = distance between the bottom surface of chip on the feeder to the top surface of the PCB on which it will be placed.

Let's take $l = 0.320\,m$

$b = 0.05\,m$

$h = 0$

$$^{0}T_A = \begin{bmatrix} 0 & -1 & 0 & 0.325 \\ 1 & 0 & 0 & 0.225 \\ 0 & 0 & 1 & 0.100 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0.18 \\ 0 & 0 & 1 & 0.100 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_C = \begin{bmatrix} 0 & -1 & 0 & -0.09 \\ 1 & 0 & 0 & 0.18 \\ 0 & 0 & 1 & 0.100 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

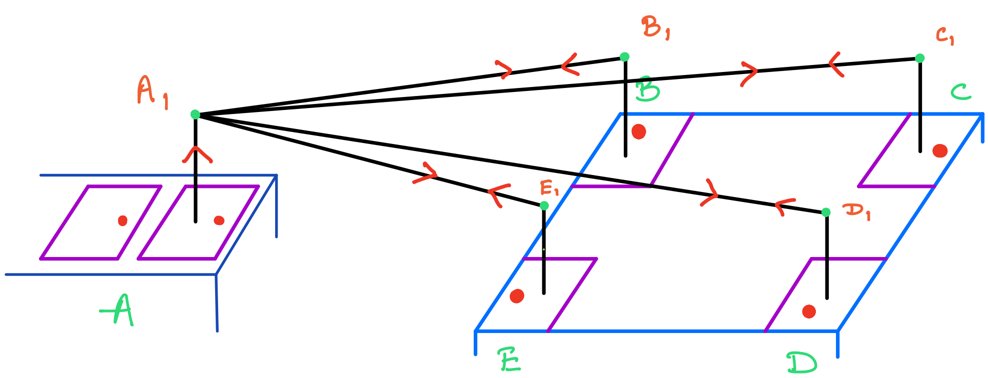
$$
{}^{0}T_{D} = \begin{bmatrix} 1 & 0 & 0 & -0.09 \\ 0 & 1 & 0 & 0.27 \\ 0 & 0 & 1 & 0.100 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_{E} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0.27 \\ 0 & 0 & 1 & 0.100 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Locations of the via points

To avoid collision we can lift up the chip to a safe height after grasping it up. Let's take this safety clearance to be $c = 10mm$.

We can reach from the feeder location to the PCB locations using a straight line path.



We choose to have same paths for the forward journey from feeder to PCB and the respective return journey from PCB to feeder.

Based on the figure above we have 5 via points - A_1, B_1, C_1, D_1, E_1

$$^{0}T_{A_1} = \begin{bmatrix} 0 & -1 & 0 & 0.325 \\ 1 & 0 & 0 & 0.225 \\ 0 & 0 & 1 & 0.110 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_{B_1} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0.18 \\ 0 & 0 & 1 & 0.110 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_{C_1} = \begin{bmatrix} 0 & -1 & 0 & -0.09 \\ 1 & 0 & 0 & 0.18 \\ 0 & 0 & 1 & 0.110 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^{0}T_{D_1} = \begin{bmatrix} 1 & 0 & 0 & -0.09 \\ 0 & 1 & 0 & 0.27 \\ 0 & 0 & 1 & 0.110 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_{E_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0.27 \\ 0 & 0 & 1 & 0.110 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

MATLAB code

In the MATLAB code the Z_4 or the Z_T axis is rotated $180°$, hence $\alpha_3 = 180°$ and Z_4 or Z_T points downwards. This was done since the prismatic joint limits was not accepting negative values.