MAE263B - Final Exam

Name: ___________________________

	Criterion	Score	Maximum
	Correctness of Analysis (Errors in work)		
1	Kinematic Analysis		22
2	Dynamics		33
3	Trajectory Generation		33
	Thoroughness of Work (Could someone else figure out what you did?)		
1	Kinematic Analysis		6
2	Dynamics		3
3	Trajectory Generation		3
	Total		100

General Instructions

- **Due Date submission window:** March 13 ~ 25 (Midnight)
- **Submission Method:** Bruinlearn
- **Submission Content:**
  - Written documents and graphs
  - Matlab Code (embedded in the written doc)
  - Video Clip of the moving arm (Matlab Simulation)
- **Individual work and submission**: This is a final take-home EXAM. Collaboration is not allowed. The submission should reflect your own understanding of the material
- **Open books open Notes**

Robotic Arm Definition and Configuration

The following gripper and tool (Fig. 1) is added to the Puma 560 (Fig.2). The circle is attached to the last link of the robotic arm. The gripper includes two fingers holding a laser cutter tool (Fig. 1). Given the coordinate system (in red) the position of the tip of the tool where the origin of the tool frame is attached expressed in the gripper coordinate system is ${}^{G}P_{T} = [-0.1, 0, 0.08]m$. The position of the origin of the gripper coordinate system expressed in the last coordinate system of the robot (frame 6) is ${}^{6}P_{G} = [0, 0, 0.05625]m$. The position of the origin of the tool frame with respect to the wrist frame is the sum of these frames.

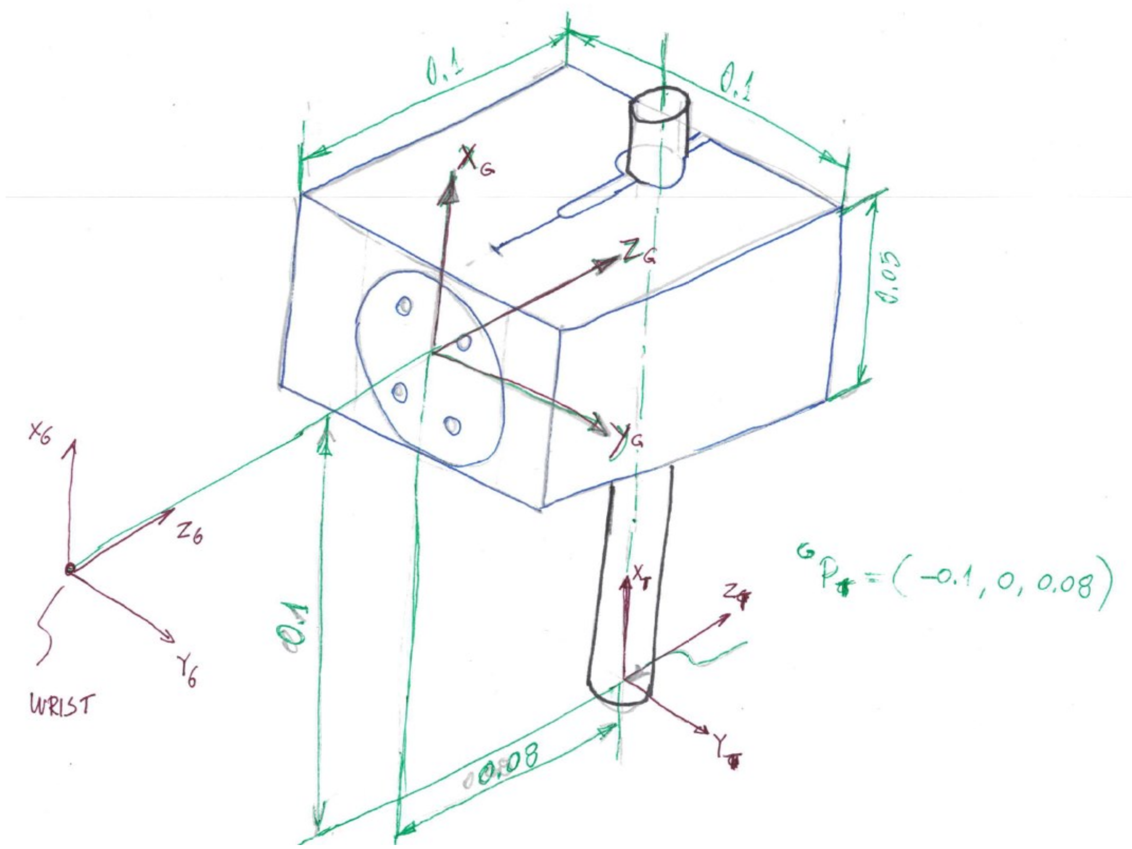


Fig.1 End Effector (gripper) Holding a plasma cutting tool. (Note: Frame G here is defined the attachment of the gripper. Do not confuse it with the Goal frame defined later on).

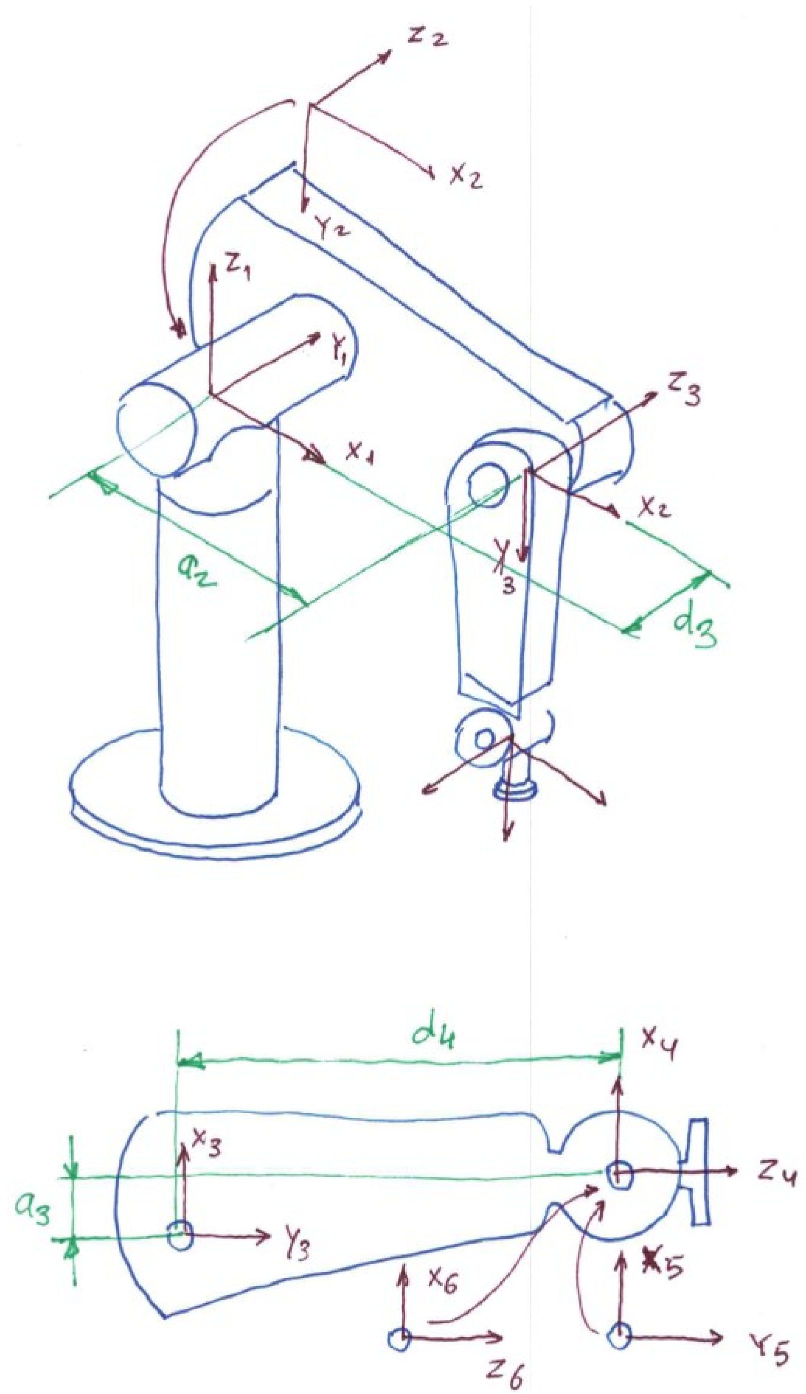


Fig. 2 - The PUMA 560 Coordinate system and geometrical dimensions -
$a_2 = 0.4318m; a_3 = 0.0191m \; d_3 = 0.1254m \; d_2 = 0.4318m$

Definitions of the Leading Frames

- **Frame {0} (Frame {B})** - The base frame is aligned with Frame {1}
- **Frame {6} (Frame {W})** – The wrist frame is last frame of the manipulator located at the wrist
- **Frame {S} –** The “station” frame is located at the base of the robotic arm with coordinates aligned with the base frame (Frame {0}) ${}^{0}P_{SORG} = [0,0,-0.672]m$.
- **Frame {G}** – The Goal frame (trajectory frame) depends on where you place the cube. You are free to position the cube anywhere within the workspace of the manipulator.
- **Frame {T}** – The tool frame is aligned with respect to the wrist frame and defined by two vectors with respect to the wrist (see Fig.1)

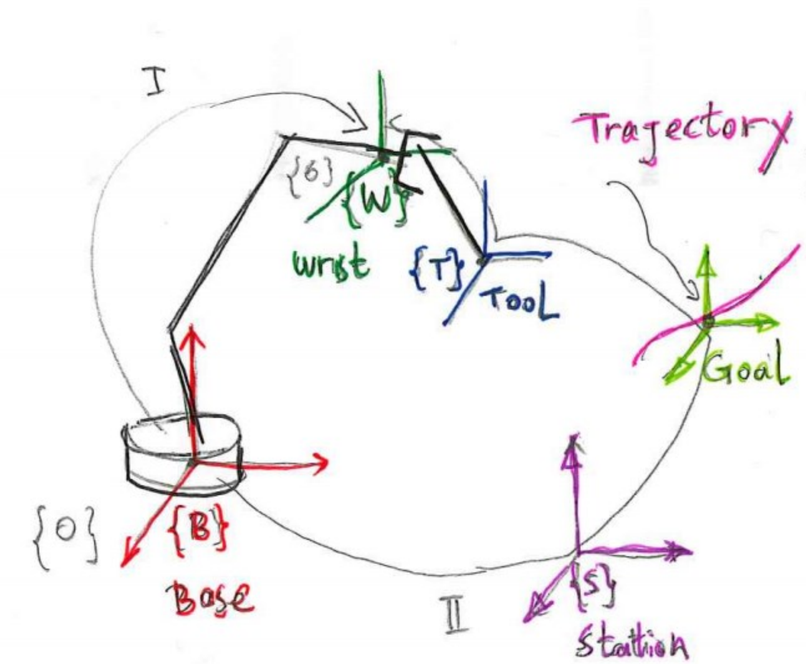


Figure 3: Frame defenition

Use the following equation for solving the inverse kinematics

$${}^{B}_{S}T\,{}^{S}_{G}T\,{}^{G}_{T}T\,{}^{T}_{W}T\,{}^{B}_{S}T = {}^{0}_{1}T(\theta_1)\,{}^{1}_{2}T(\theta_2)\,{}^{2}_{3}T(\theta_3)\,{}^{3}_{4}T(\theta_4)\,{}^{4}_{5}T(\theta_5)\,{}^{5}_{6}T(\theta_6)$$

Notes:

- The left hand side of the previous equation is well defined given the geometry of the manipulator and the trajectories
- For the purpose of solving the inverse kinematics while following the trajectory you may assume that the tool frame {T} and the goal frame {G} are perfectly align. However in between the via point the tool position / ordination using the direct kinematics may deviate from the goal which constitutes the trajectory errors.

$$
{}^{G}_{T}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Part 1 – Kinematic Analysis

MAE 163B - Undergraduate Students: Required

MAE 263B - Graduate Students: Required

1) **Forward Kinematics** – Solve the forward kinematics by expressing all the homogeneous transformation matrixes ${}^{0}_{1}T, {}^{1}_{2}T, {}^{2}_{3}T, {}^{3}_{4}T, {}^{4}_{5}T, {}^{5}_{6}T, {}^{6}_{T}T$ as well as the homogeneous transformation matrix ${}^{0}_{6}T, {}^{0}_{T}T, ({}^{6}_{T}T)^{-1}$. Note that frame 6 is the last frame of the wrist joint and frame T is located at the tip of the tool.

2) **Invers Kinematics** - Solve the inverse kinematics using one of the two methods
   a) Geometric / Hybrid solution - Note that the last three joint are a wrist with three intersecting joints. Use the geometric approach to solve for the position of the wrist and use the Euler formulation to defined the last three angles of the wrist.
   b) Analytical solutions.

3) **Jacobian Matrix** - Calculate the Jacobian Matrix in the tool's frame (${}^{T}J$) or the base frame (${}^{0}J$) based on the method used. Note that end result of each method will express the Jacobian in either frame 0 or frame 6. Use the special transformation matrix or the velocity mapping to provide the expression of the Jacobian in the other fame (0 or 1). For expressing the Jacobian use one of the following methods:
   a) Velocity propagation (assume that the velocity vector is defined as $\begin{bmatrix} v_x & v_y & v_z & \omega_x & \omega_y & \omega_z \end{bmatrix}^T$)
   b) Static force propagation (assume that the force/torque vector is defined as $\begin{bmatrix} f_x & f_y & f_z & n_x & n_y & n_z \end{bmatrix}^T$) and applied on the tool tip
   c) Explicit derivation of the Jacobian.
   d) Regardless of the method you have used transform the Jacobian to the based frame ${}^{0}J$ of it is not already expressed in frame 0
4) **Singularities** – Use the Jacobian matrix and find analytically the singular configurations of the arm and wrist. Sketch by hand or via the toolbox the arm configurations in the singular configurations.

5) **Symbolic Matlab Script** – This section is not required but if you develop a symbolic Matlab script add it as a separate *.m file. Name the file <MSR6>_<initials>_<student number>.m (e.g. MSR6_JR_123456.m)

Part 2 – Dynamics

MAE 163B - Undergraduate Students: Required

MAE 263B - Graduate Students: Required

6) **Dynamics Equation** - Derive the dynamic equations of the robotic arm using only the **<u>first 3 DOF</u>** and formulate the three equations of motion using the Newton Euler method. Use the standard form to express the final result i.e. collect all the terms into matrixes. Note that the two methods should generate the same sets of three differential equations. Assume that The external forces and torques are acting on the tool in three orthogonal directions

$$\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) + F(\theta,\dot{\theta})$$

Submission requirements

Define the equations in a parametric way. Do not use specific numerical values of the Puma. Arrange the equations in a matrix form.

Mass and Inertia - Parameter

(1) **Mass Links 1,2,3** - Assume that every link has a mass and it center of mass is expressed with respect to its coordinate system

(2) **Inertia Links 1,2,3** - Assume that every link has an inertia matrix expressed with respect to the center of mass

$$ {}^{i}I_{c} = \begin{bmatrix} {}^{i}I_{c_{xx}} & 0 & 0 \\ 0 & {}^{i}I_{c_{yy}} & 0 \\ 0 & 0 & {}^{i}I_{c_{zz}} \end{bmatrix} $$

(3) Mass / Inertia Links 4,5,6, gripper - Lump the mass of the last 3 DOF with the end effector and the tool and assume that the

manipulator carries it at the origin of frame 4. Assume no inertia for the lumped mass

Part 3 – Trajectory Generation

MAE 163B - Undergraduate Students: Required (Tracing 1 letter – "B")

MAE 263B - Graduate Students: Required (Tracing 3 letters)

7) **Trajectory Generation –** The tool tip need to follow a trajectory defined by the 3 letters placed on the three faces of a cube with a 0.005m ± 0.0005m (5 ± 0.5 mm) away from the surface of the cube. You will need to add as many via points as necessary to meet the ± 0.5 mm accuracy. The trajectory includes the following phases (note that if is you plan a trajectory for only one letter phase 1 2 7 are only applicable):

   Phase 1: Move from the reference arm configuration depicted in fig.2 to the starting point of the first letter. Make sure that the tool does not hit the block (10 s)

   Phase 2: Trace the first letter (20 s)

   Phase 3: Transition from the end of the first letter to the beginning of the second letter (10 s)

   Phase 4: Trace the second letter (20 s)

   Phase 5: Transition from the end of the second letter to the beginning of the third letter (10s)

   Phase 6: Trace the third letter (20 s)

   Phase 7: Transition from the end of the third letter to the reference arm configuration (10)

   Definitions of the letters

   - First Letter (Substitute for the letter A): The first letter of your first name
   - Second letter: The letter B

- Third letter (Substitute for the letter C): The first letter of your family name

The cube's dimensions are 0.1x0.1x0.1 m (10x10x10 cm).

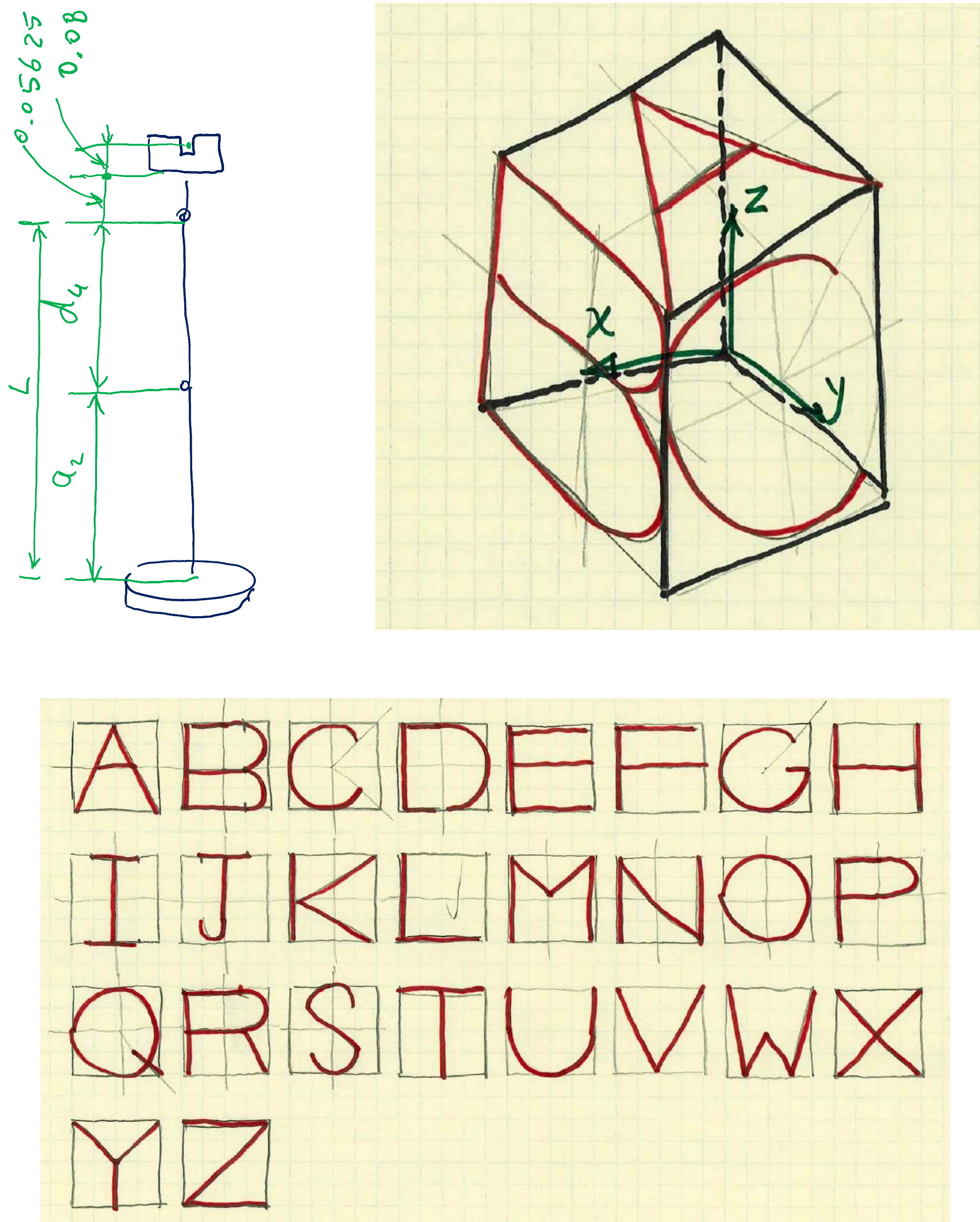


Figure 4 Trajectory definitions

Specifications

- **Trajectory Location** - You may position the cube anywhere within the workspace of the manipulator. It is recommended to aligned the cube coordinate system with the robot coordinate system (frame 0). You may start with the arm configuration in an L shape and check if it is within the workspace of the manipulator if not adjust the location of the cube by translate it with respect to the base frame.
- **Trajectory Definition for a specific letter –** See above phases for the entire trajectory. For each particular letter, the trajectory may start at any point of the letter Design the trajectories such that they will be minimized their length. No analytical analysis is required for minimizing the length of the trajectory just use common sense.
- **Orientation of the Tool Tip** - Note that since the tool is cylindrical the tool can rotate along its long axis without any effect on the trajectory but it should remain perpendicular to the surfaces of the cube.
- **Accuracy and Via Point –** Add as many via points as needed such that the tip of the tool will not deviate from the expected trajectory by more than 0.0005 m (0.5 mm) assuming and form the perpendicular orientation by more than 1 Deg.
- **Completion Time -** The cut must be completed within the time frame listed above. For tracing an individual letter divide the time interval between the various segments and allocate a time based on the length of each segment. In case the joint velocity acceded the maximal angular velocity of each one of the joint add more time to the allocated segment.
- **Velocity –** For straight line segment the tool starts/ends each segment with a zero velocity. For a transition between a straight segment to a

curved segment and vice versa both the velocities and the accelerations should be matched at the transition point.

- **Sampling Frequency –** The sampling frequency is 100Hz meaning that you need to calculate new position and joint angle evert 0.01 sec

Use the invers kinematics to calculate and plot the joint angles as a function of time with the following methods

a) Interpolation at the joint space
b) Interpolation at the end effector tool space

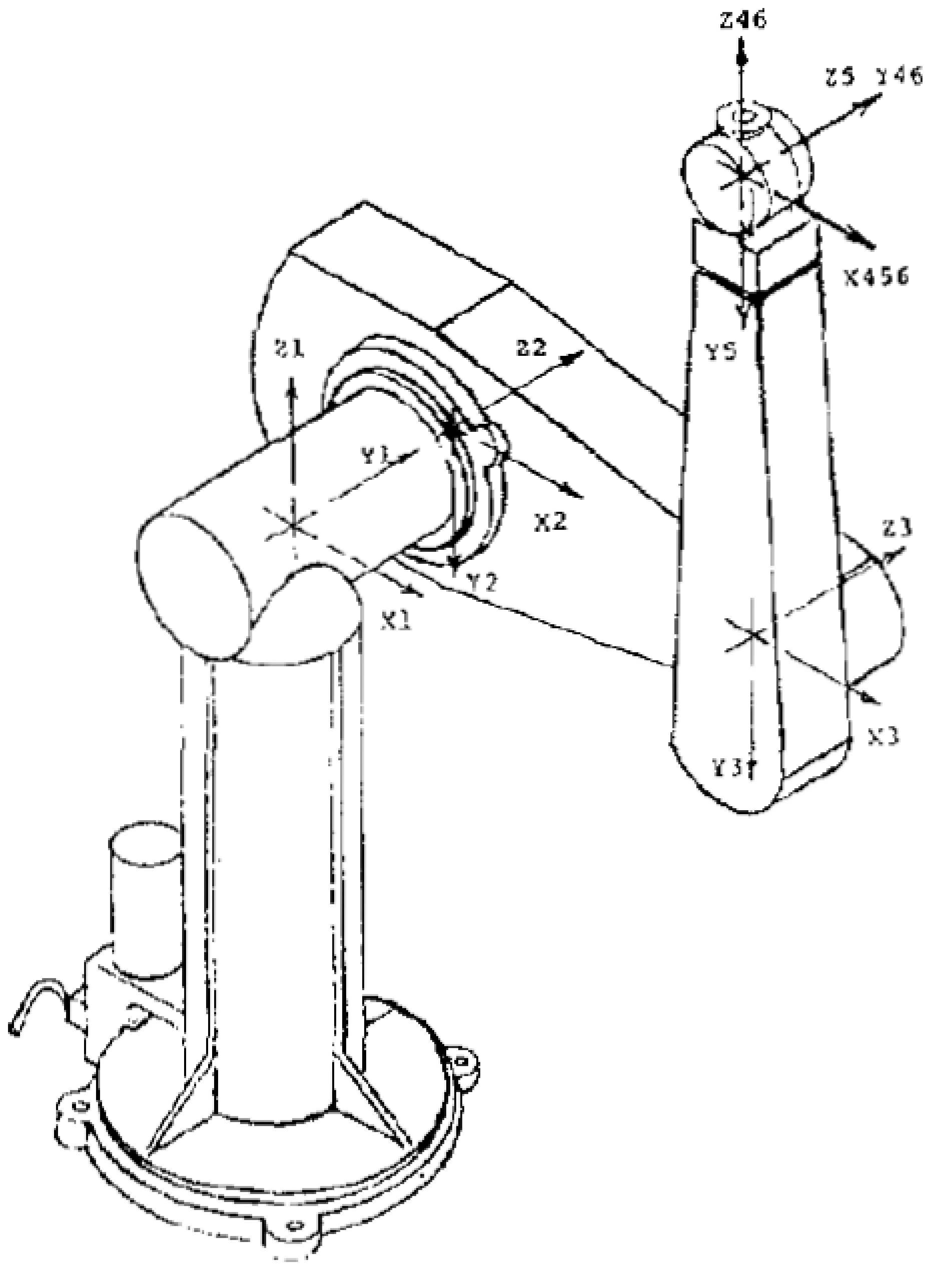
Submission Requirements

Plots Summary – Submit the following plots

- In each one of the plots clearly mark the different phases as defined above.
- Plots the actual end effector trajectory (X.Y,Z) as a 3D graph
- Plot each one of the Euler angles of the tool with respect to the base frame as a function of time.
- Plot the position x,y,z of the tool tip each as a function of time.
- Plot the absolute position error (absolute difference between the required trajectory and the actual trajectory)
- Plot the absolute orientation error of the angles defining the perpendicular direction with respect to the plane (absolute difference between the required trajectory orientation and the actual trajectory orientation) just during the phases
- Plot each of the six joint angles as a function of time
- Verify the following by adding the :
  - The trajectory is within the workspace of the manipulator (use the joint limits)
  - The trajectory is within the tolerances (± 0.1 from the expected trajectory)

  - During the tracing phases the tool needs to be perpendicular to the plane (± 1 deg inclination from the plane)
  - The max velocities are lower than the maximal allowed velocities.
- Summarize in a table how many via points were used for each segment using the two methods.

Appendix - The Puma 560 Technical Data



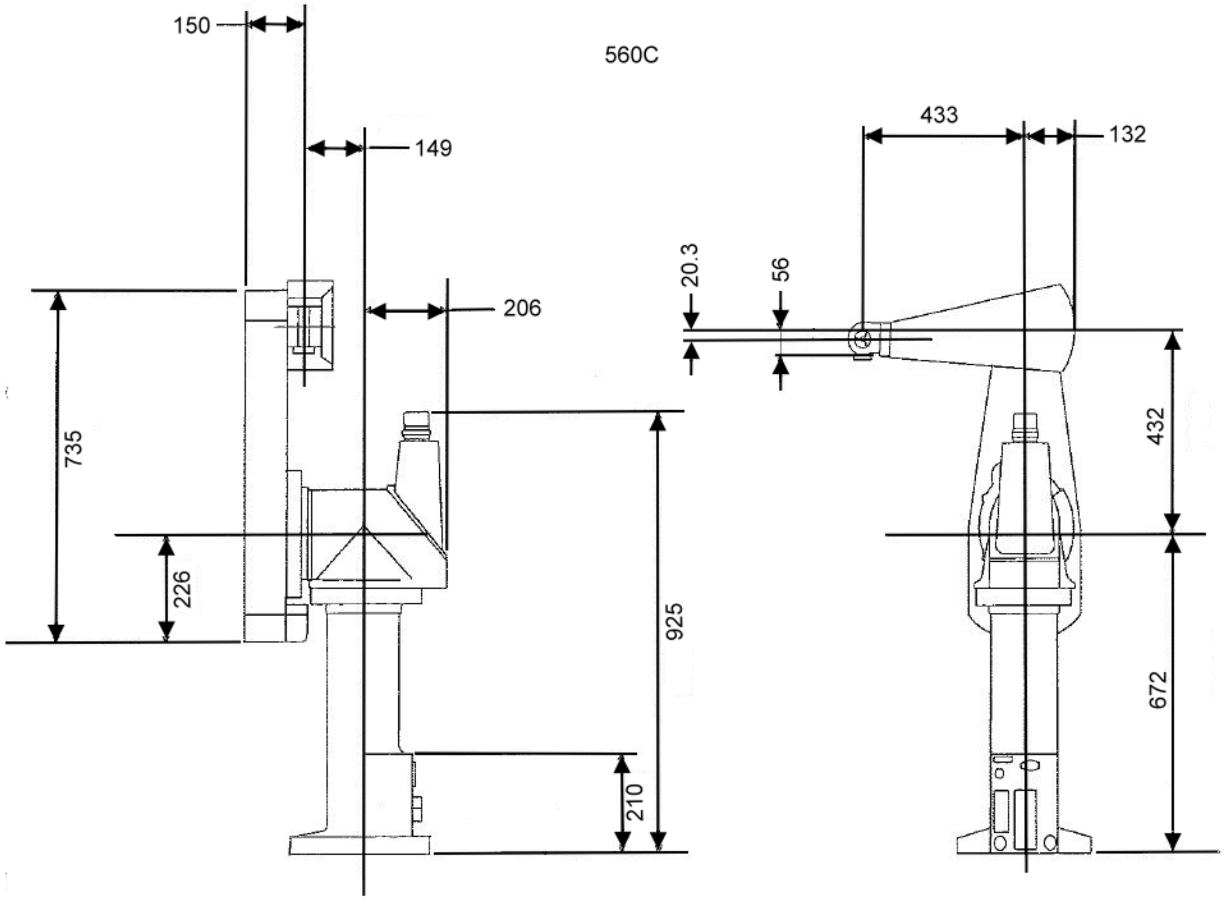


Table A1. Modified Denavit – Hartenberg Parameters

i	α_{i-1} (degrees)	θ_i	a_{i-1} (meters)	d_i (meters)
1	0	q_1	0	0
2	-90	q_2	0	.2435
3	0	q_3	.4318	-.0934
4	90	q_4	-.0203	.4331
5	-90	q_5	0	0
6	90	q_6	0	0

of the center of gravity in the coordinate frame attached to the link. The coordinate frames used are assigned by a modified Denavit-Hartenberg method [Craig 85]. In this variant of the Denavit-Hartenberg method, frame i is attached to link i, and axis Z_i lies along the axis of rotation of joint i. The coordinate frame attachments are shown in Figure 2; they are located as follows:

- Link 1: Z axis along the axis of rotation, +Z up; +Y1 || +Z2.
- Link 2: Z axis along the axis of rotation , +Z away from the base; X-Y plane in the center of the link, with +X toward link 3.
- Link 3: Z3 || Z2; X-Y plane is in the center of link 3; +Y is away from the wrist.
- Link 4: The origin is at the intersection of the axes of joints 4 5 and 6; +Z4 is along the axis of rotation and directed away from link 2; +Y4 || +Z3 when joint 4 is in the zero position.
- Link 5: The origin coincides with that of frame 4; +Z5 is directed away from the base; +Y5 is directed toward link 2 when joint 5 is in the zero position.
- Link 6: The origin coincides with that of frame 4; when joints 5 and 6 are in the zero position frame 6 is aligned with frame 4.
- Wrist: The dimensions are reported in frame 4.

Table 4. Link Masses (kilograms; $\pm 0.01 + 1\%$)

Link	Mass
Link 2	17.40
Link 3	4.80
Link 4*	0.82
Link 5*	0.34
Link 6*	0.09
Link 3 with Complete Wrist	6.04
Detached Wrist	2.24

* Values derived from external dimensions; ±25%.

Table 5. Centers of Gravity. (meters ± 0.003)

Link	r_x	r_y	r_z
Link 2	0.068	0.006	-0.016
Link 3	0	-0.070	0.014
Link 3 With Wrist	0	-0.143	0.014
Link 4*	0	0	-0.019
Link 5*	0	0	0
Link 6*	0	0	0.032
Wrist	0	0	-0.064

* Values derived from external dimensions; ±25%.

Table 6. Diagonal Terms of the Inertia Dyadics and Effective Motor Inertia.

Link	I_{xx}	I_{yy}	I_{zz}	I_{motor}
Link 1	–	–	0.35	1.14(±0.27)
Link 2	0.130 (±3%)	0.524 (±5%)	0.539 (±3%)	4.71 (±0.54)
Link 3	0.066	0.0125	0.086	0.83 (±0.09)
Link 3 With Wrist	0.192 (±4%)	0.0154 (±5%)	0.212 (±4%)	–
Link 4*	1.80×10^{-3}	1.80×10^{-3}	1.30×10^{-3}	0.200 (±0.016)
Link 5*	0.30×10^{-3}	0.30×10^{-3}	0.40×10^{-3}	0.179 (±0.014)
Link 6*	0.15×10^{-3}	0.15×10^{-3}	0.04×10^{-3}	0.193 (±0.015)

* Inertia Diadic terms derived from external dimensions; ±50%.

Table 7. Motor and Drive Parameters

	Joint 1	Joint 2	Joint 3	Joint 4	Joint 5	Joint 6
Gear Ratio	62.61	107.36	53.69	76.01	71.91	76.73
Maximum Torque (N-m)	97.6	186.4	89.4	24.2	20.1	21.3
Break Away Torque (N-m)	6.3	5.5	2.6	1.3	1.0	1.2

Appendix B. limit performances used for PUMA 560

Joint N°	1	2	3	4	5	6
$q_{\max}$ (rad)	π	$3\pi/4$	$3\pi/4$	π	$3\pi/4$	$3\pi/4$
$\dot{q}_{\max}$ (rad/s)	8	10	10	5	5	5
$\ddot{q}_{\max}$ (rad/s^2)	10	12	12	8	8	8
$\dddot{q}_{\max}$ (rad/s^3)	30	40	40	20	20	20
$\tau_{\max}$ (N m)	97.6	186.4	89.4	24.2	20.1	21.3

6 Axis arm with 3 axis making up a spherical wrist.[4]

Maximum reach 878mm from center axis to center of wrist [4]

Software selectable payloads from 4 kg to 2.5 kg [4]

Arm wright: 83 kg (approximate)[5]

Repeatability ±0.1mm[6]

2.5 kg max velocity: 500mm/sec straight line moves [6]

4.0 kg max velocity: 470mm/sec straight line moves [6]

Joint Maximums [7]	Degrees
Waist	320
Shoulder	266
Elbow	284
Wrist Bend	200
Wrist Roll	280
Tool Flange	532

References

https://en.wikipedia.org/wiki/Programmable_Universal_Machine_for_Assembly#cite_note-p1-5-7