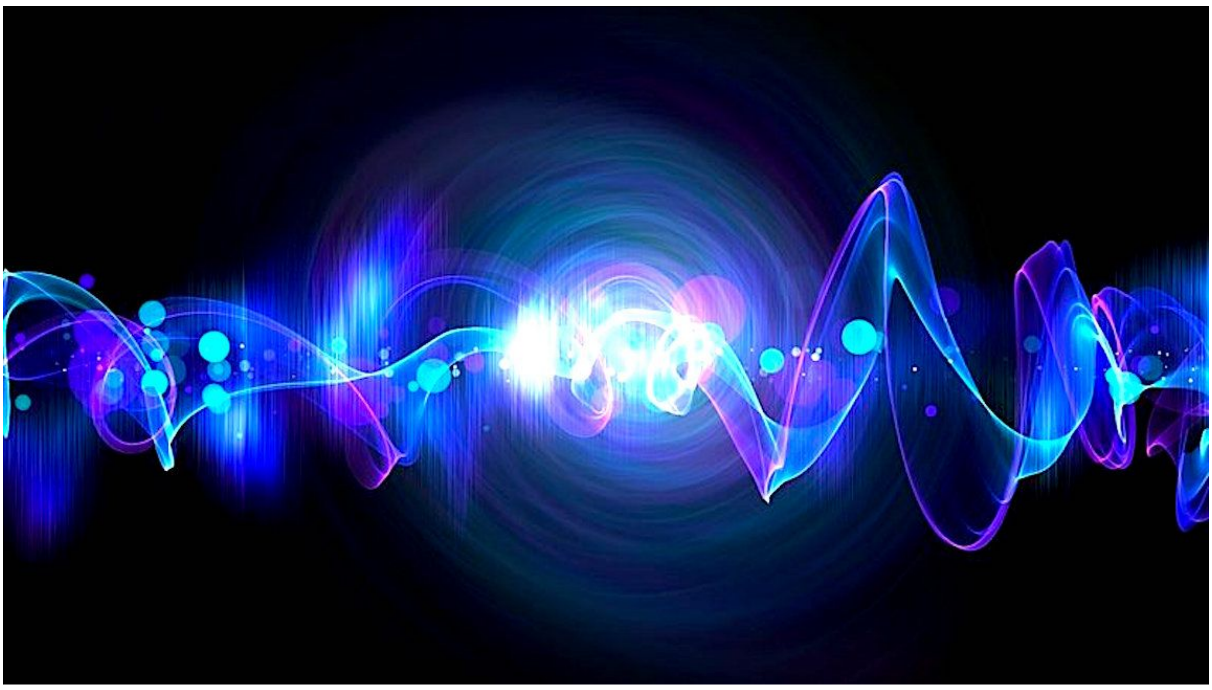
SIGNALS AND SYSTEMS

CONVOLUTION AND CORRELATION



DATE - *13^{TH} September 2019*

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ROLL NO. *- 18EC01044*

THEORY

PERIODIC SIGNALS:

A signal x(t) is called periodic if there is a positive constant T such that

$x(t)= x(t+nT)$, $n= \pm 1, \pm 2, \pm 3,\ldots$, for all t. Such a T is called a period of the signal. The smallest value of T for which $x(t)= x(t+nT)$ for all t, is called the fundamental period of the signal, and is often denoted as T_0.

FOURIER SERIES :

A Fourier series is an expansion of a [periodic function] in terms of an infinite sum of [sines] and [cosines]. Fourier series make use of the [orthogonality] relationships of the [sine] and [cosine] functions. The computation and study of Fourier series is known as [harmonic analysis] .
Fourier series is used for periodic signals and Fourier Transform is used for aperiodic signals.

The functions are integral harmonics of the fundamental frequency 'f0' of the composite signal. Using the series, we can decompose any periodic signal into its harmonics. The expression of a Fourier series is

$x(t) = A_o + {}_{n=1}^{Inf}\sum An \cos(2\pi nt) + {}_{n=1}^{Inf}\sum Bn \sin(2\pi nf0t)$

Where A_o, An and Bn are real and called Fourier Trigonometric Coefficient, given as

$A_o = (1/T) \int x(t)dx$

$An = (2/T)_o^T\int x(t) \cos(2\pi nf_o t)\, dt$

$Bn = (2/T) \int_{o}^{T} x(t) \sin(2\pi n f_o t)\, dt$

Fourier series can also represented as complex (exponential) function as shown below

$x(t) = \sum_{n=-Inf}^{Inf} C_n \exp(j2\pi n f_o t)$

Where Co, Cn, and C-n are complex coefficient, given as

$C_o = A_o$
$C_n = (1/2)(A_n - jB_n)$
$C_{-n} = (1/2)(A_n + jB_n)$

GIBBS EFFECT:

The Gibbs phenomenon is an overshoot (or "ringing") of Fourier series and other eigenfunction series occurring at simple discontinuities.

As more frequencies are added to the reconstruction, the signal becomes closer to the final solution. The interesting thing is how the final solution is approached at the edges in the signal. When only some of the frequencies are used in the reconstruction, each edge shows overshoot and ringing (decaying oscillations). This overshoot and ringing is known as the Gibb's effect.

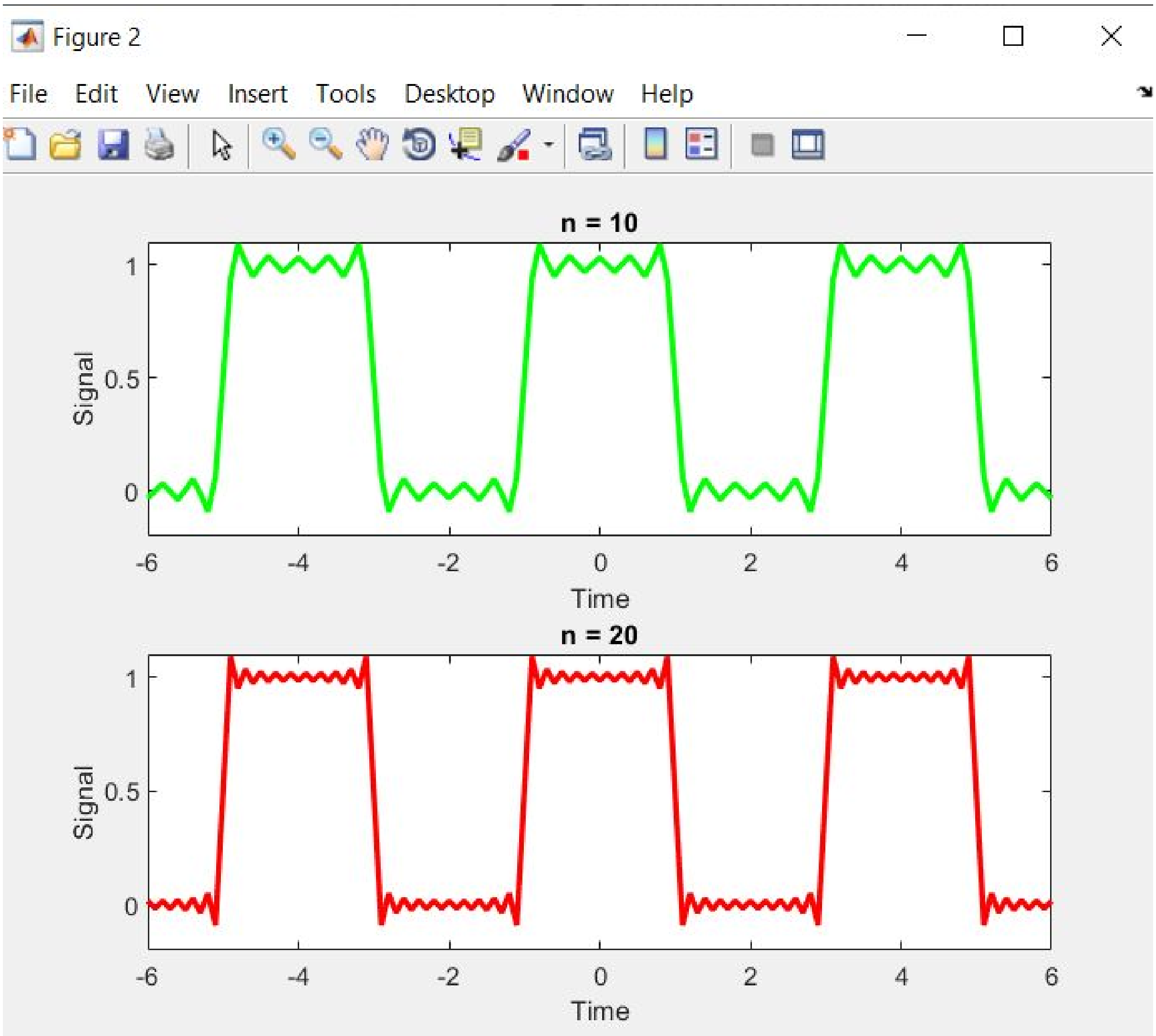
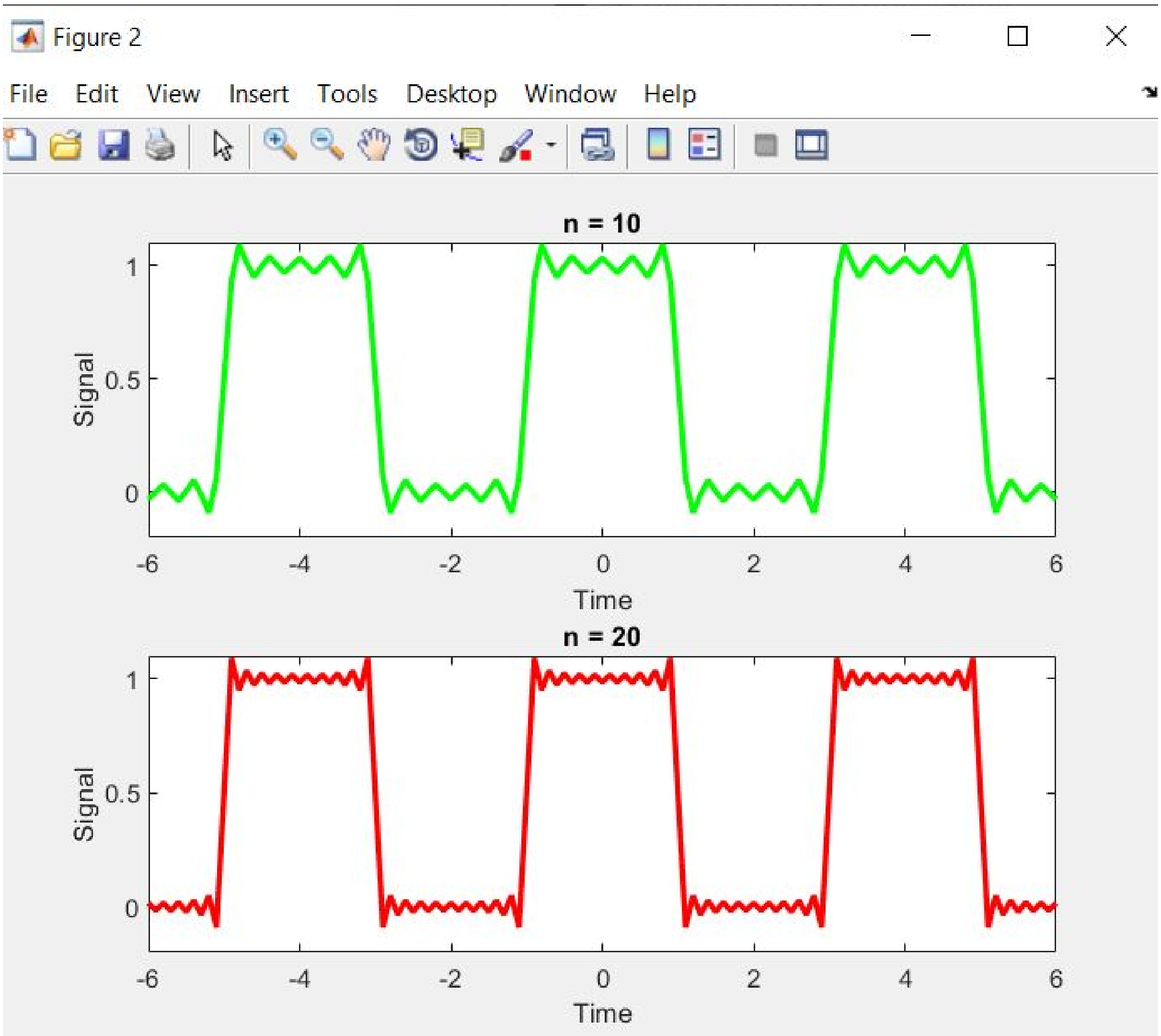


Figure 2
File Edit View Insert Tools Desktop Window Help
n = 10
Signal
Time
n = 20
Signal
Time

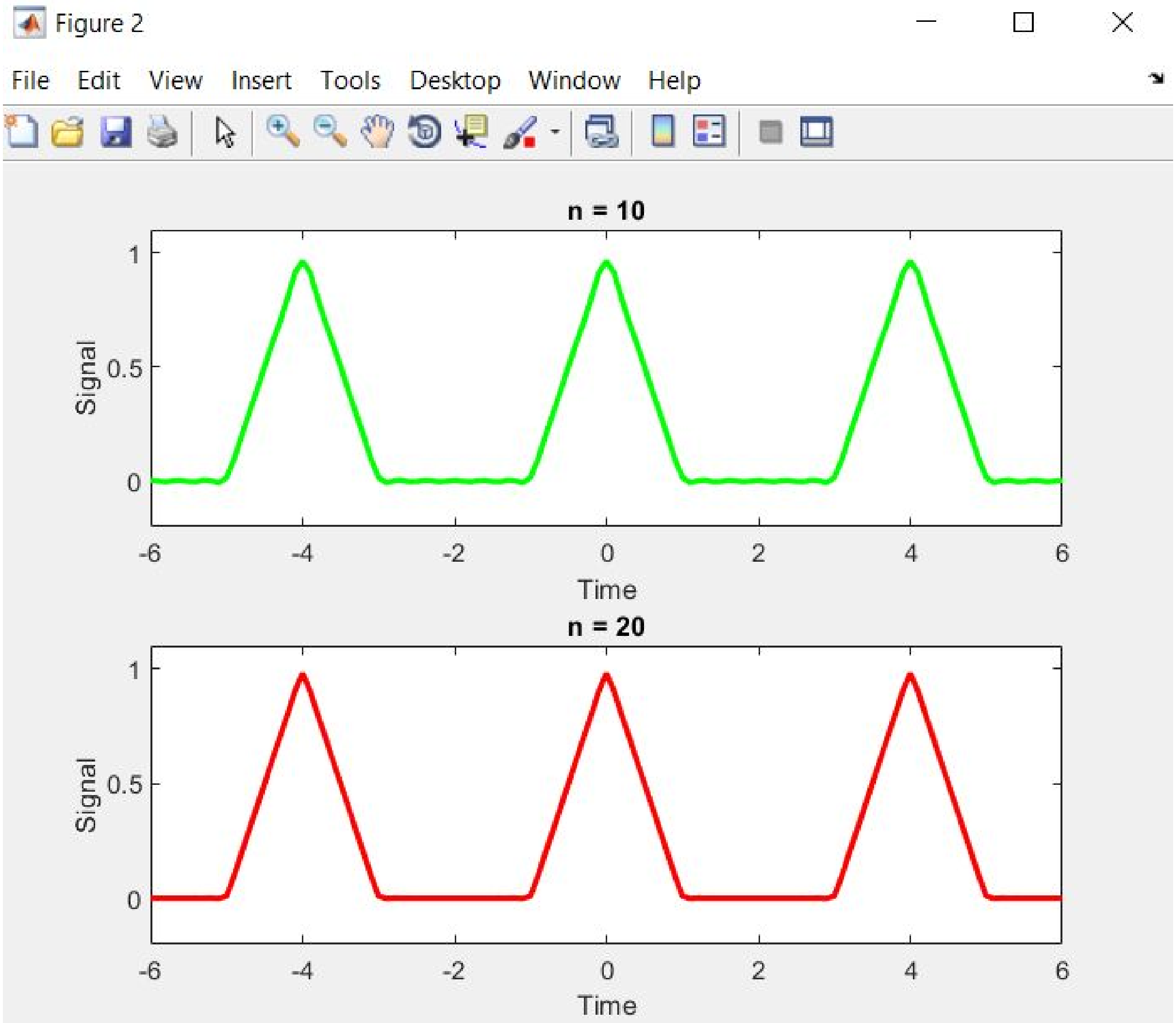
RESULTS

Part1

Rectangular Input



Triangular Input



PART 2

Ques 1

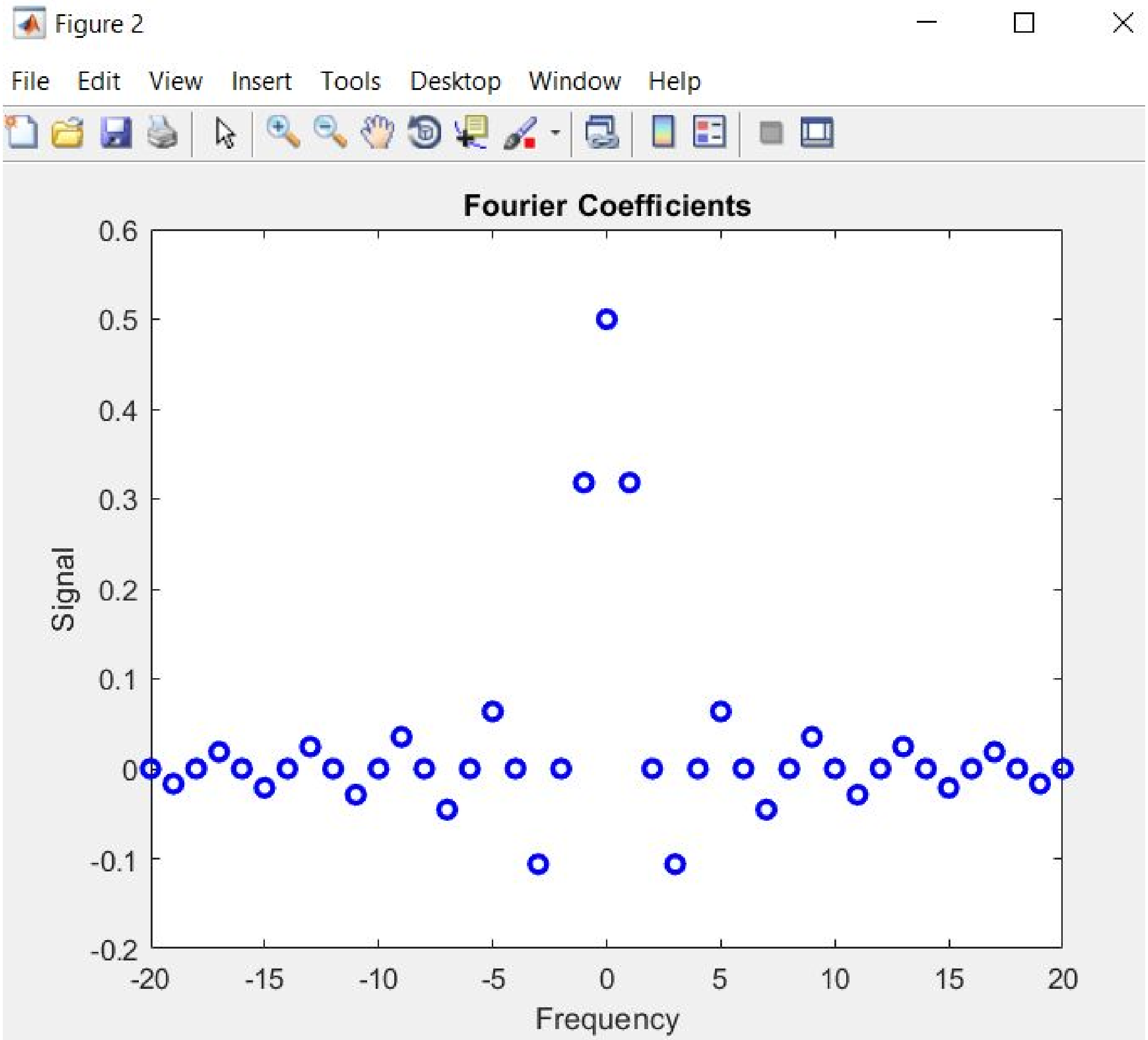


Figure 2
File Edit View Insert Tools Desktop Window Help
Fourier Coefficients
Signal
Frequency
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-20
-15
-10
-5
0
5
10
15
20

Ques 2

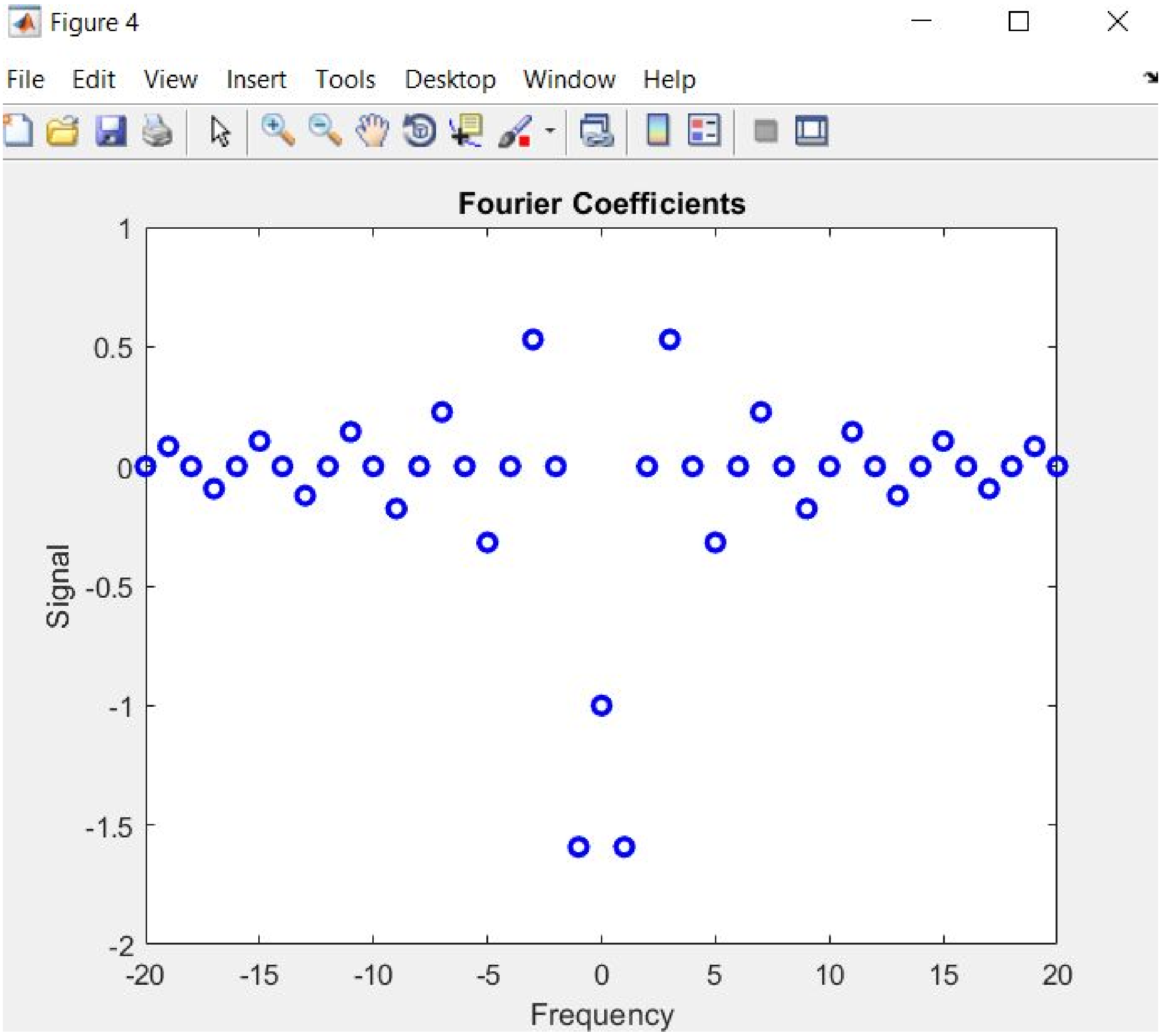


Figure 4
File Edit View Insert Tools Desktop Window Help
Fourier Coefficients
Signal
Frequency
1
0.5
0
-0.5
-1
-1.5
-2
-20
-15
-10
-5
0
5
10
15
20

Ques 3

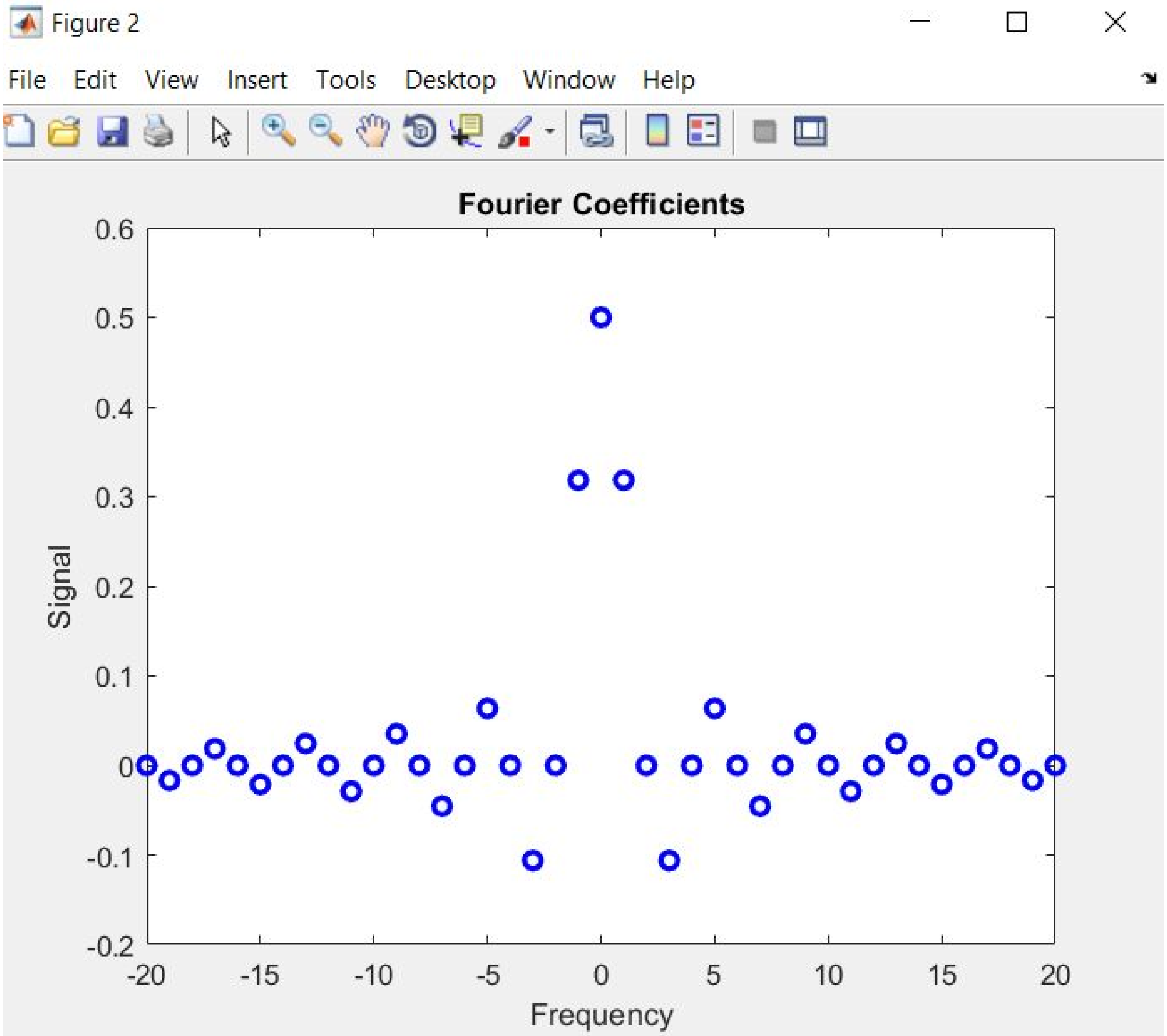


Figure 2
File Edit View Insert Tools Desktop Window Help
Fourier Coefficients
Signal
Frequency
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-20
-15
-10
-5
0
5
10
15
20

Ques 4

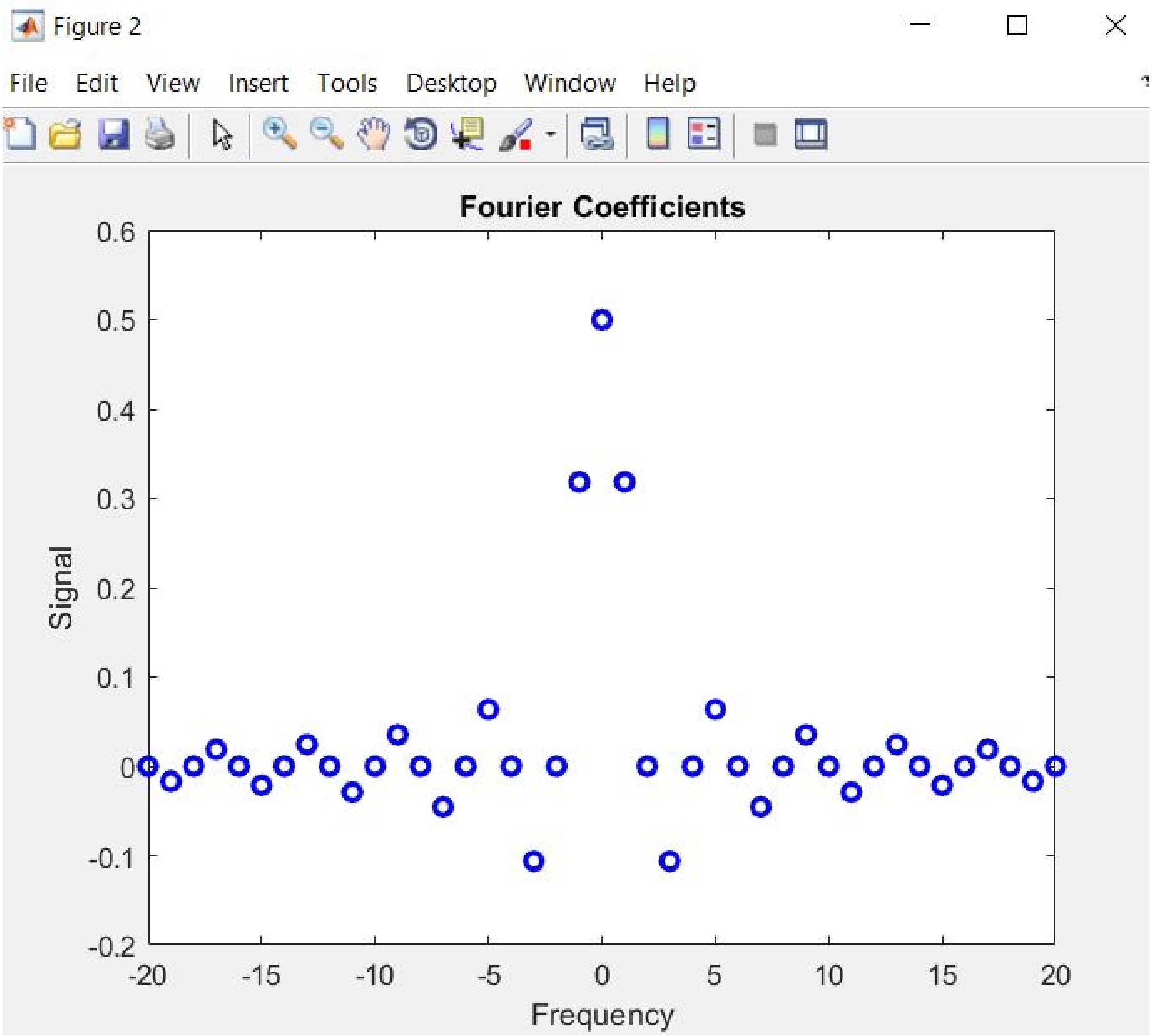


Figure 2
File Edit View Insert Tools Desktop Window Help
Fourier Coefficients
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
Signal
-20
-15
-10
-5
0
5
10
15
20
Frequency

Ques *5*

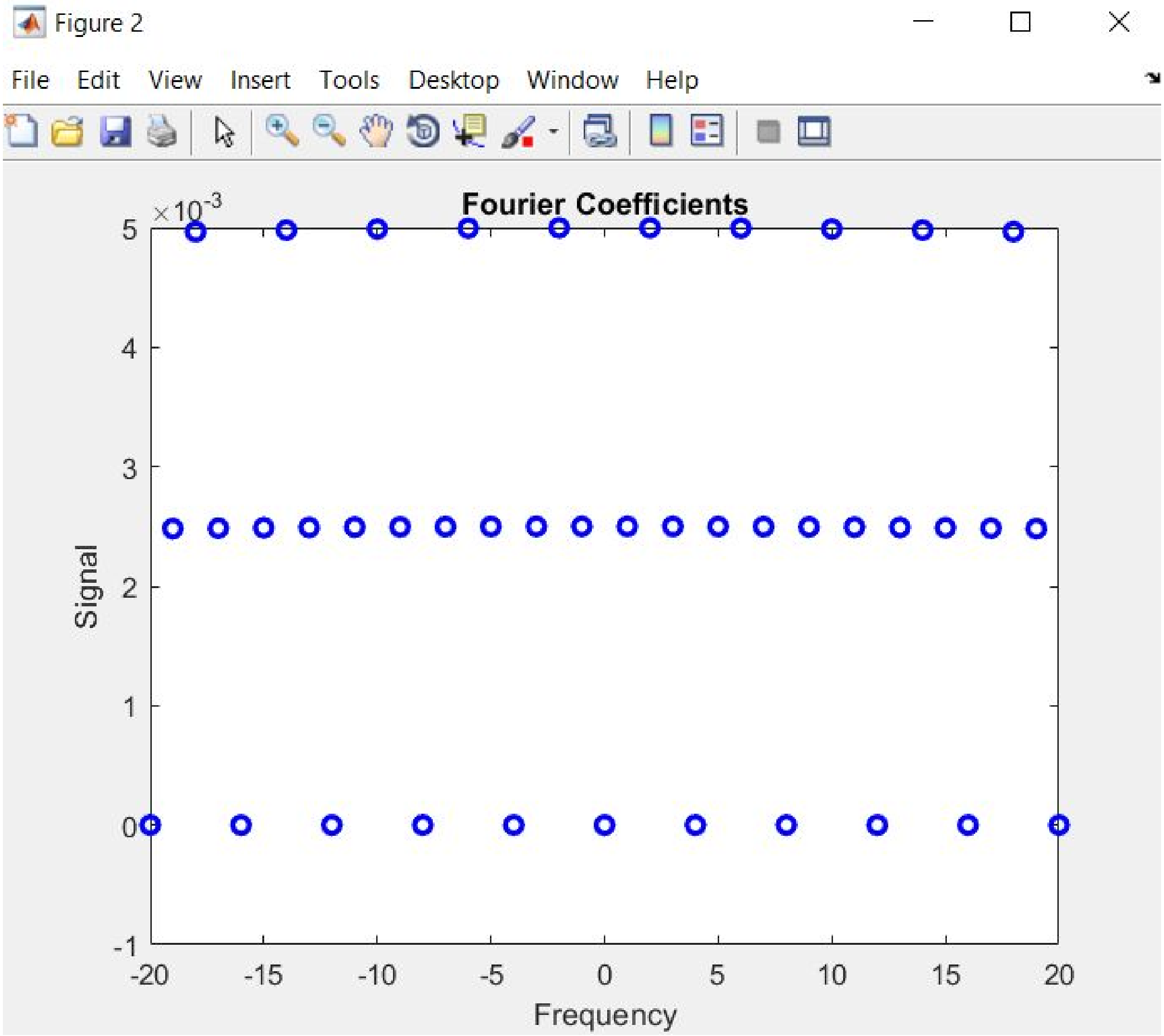


Figure 2
File Edit View Insert Tools Desktop Window Help
Fourier Coefficients
×10^-3
5
4
3
2
1
0
-1
Signal
-20
-15
-10
-5
0
5
10
15
20
Frequency

Ques 6

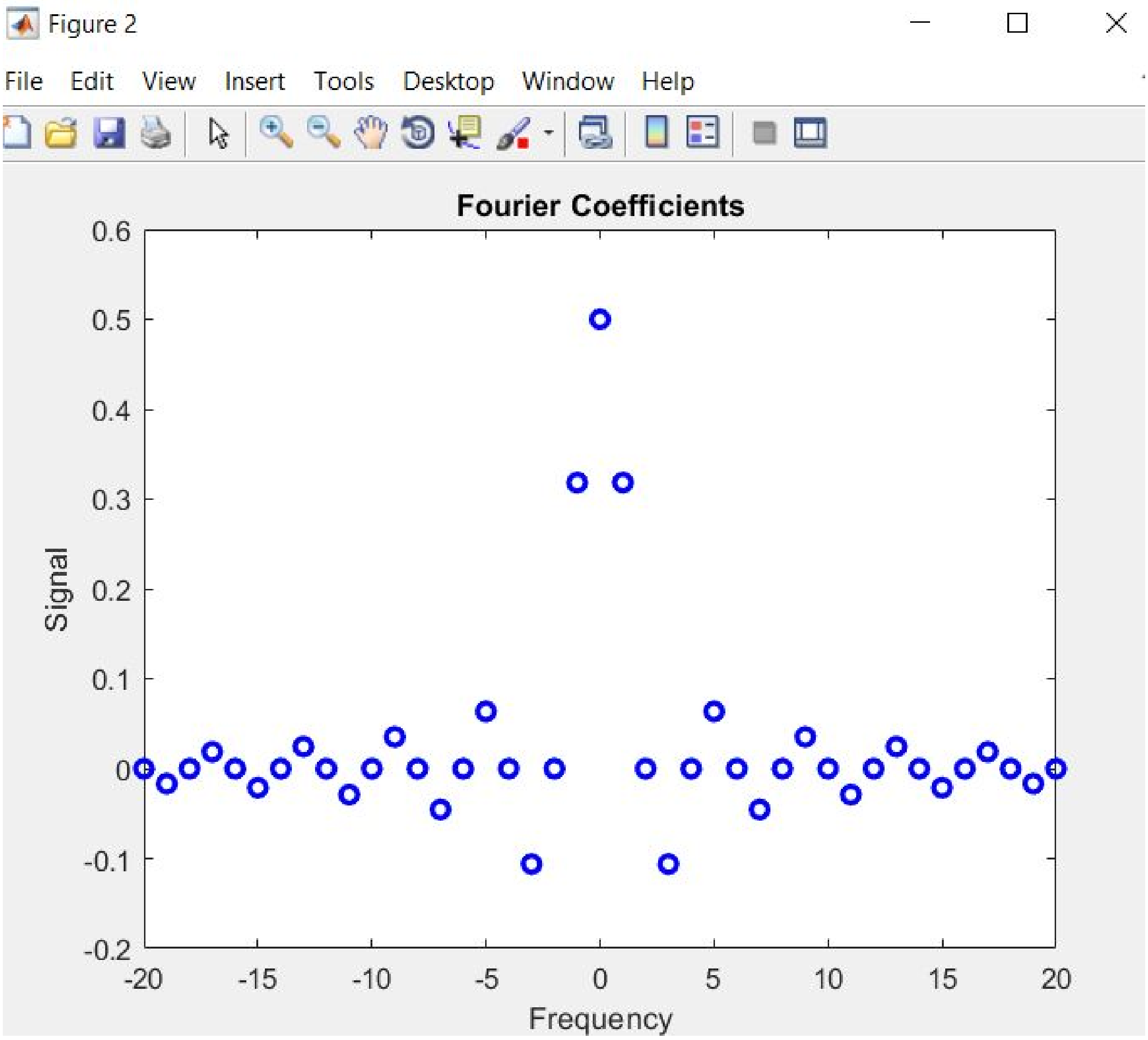


Figure 2
File Edit View Insert Tools Desktop Window Help
Fourier Coefficients
Signal
Frequency
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-20
-15
-10
-5
0
5
10
15
20

Ques 7

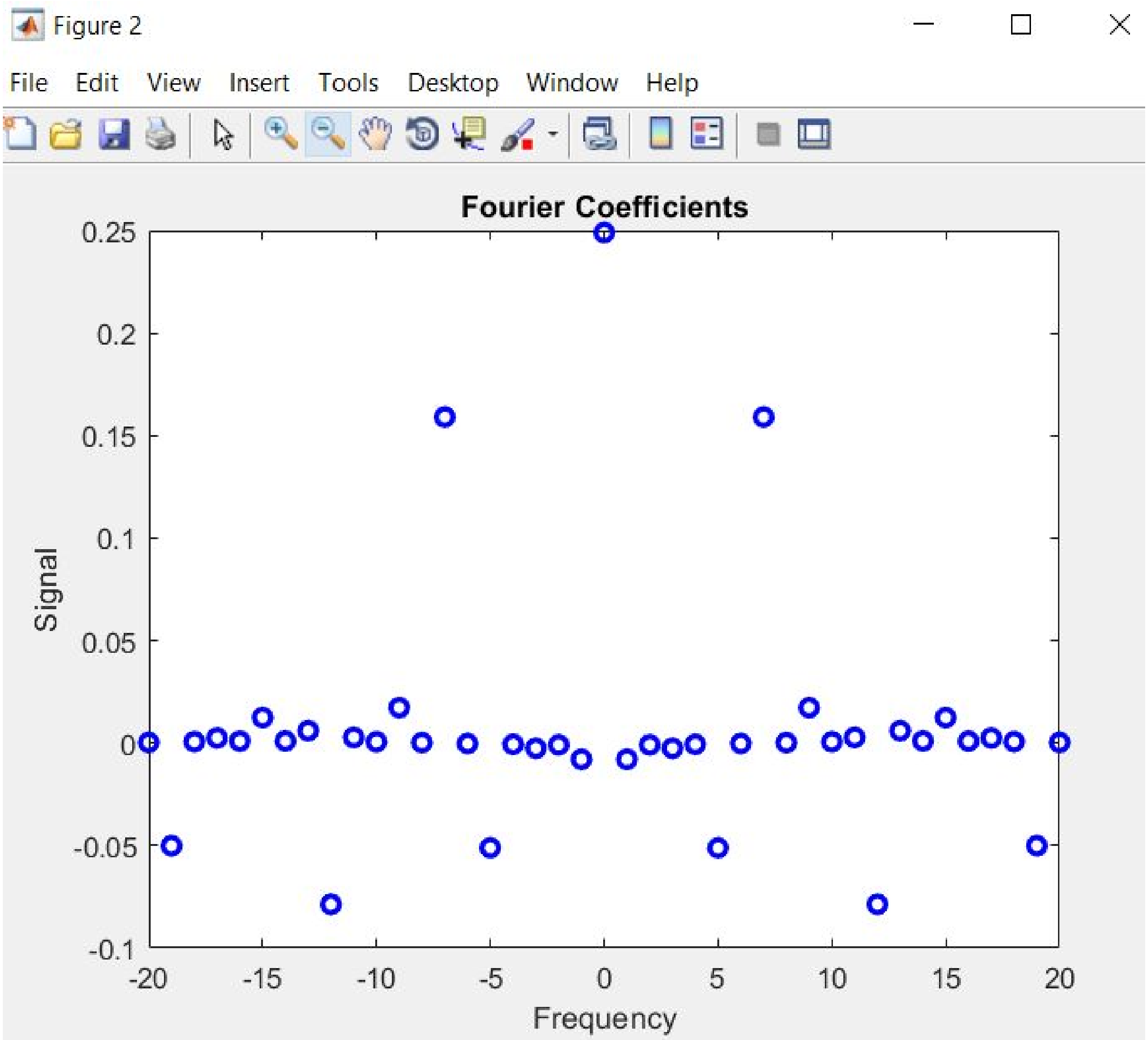


Figure 2
File Edit View Insert Tools Desktop Window Help
Fourier Coefficients
Signal
Frequency
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-20
-15
-10
-5
0
5
10
15
20

Ques 8

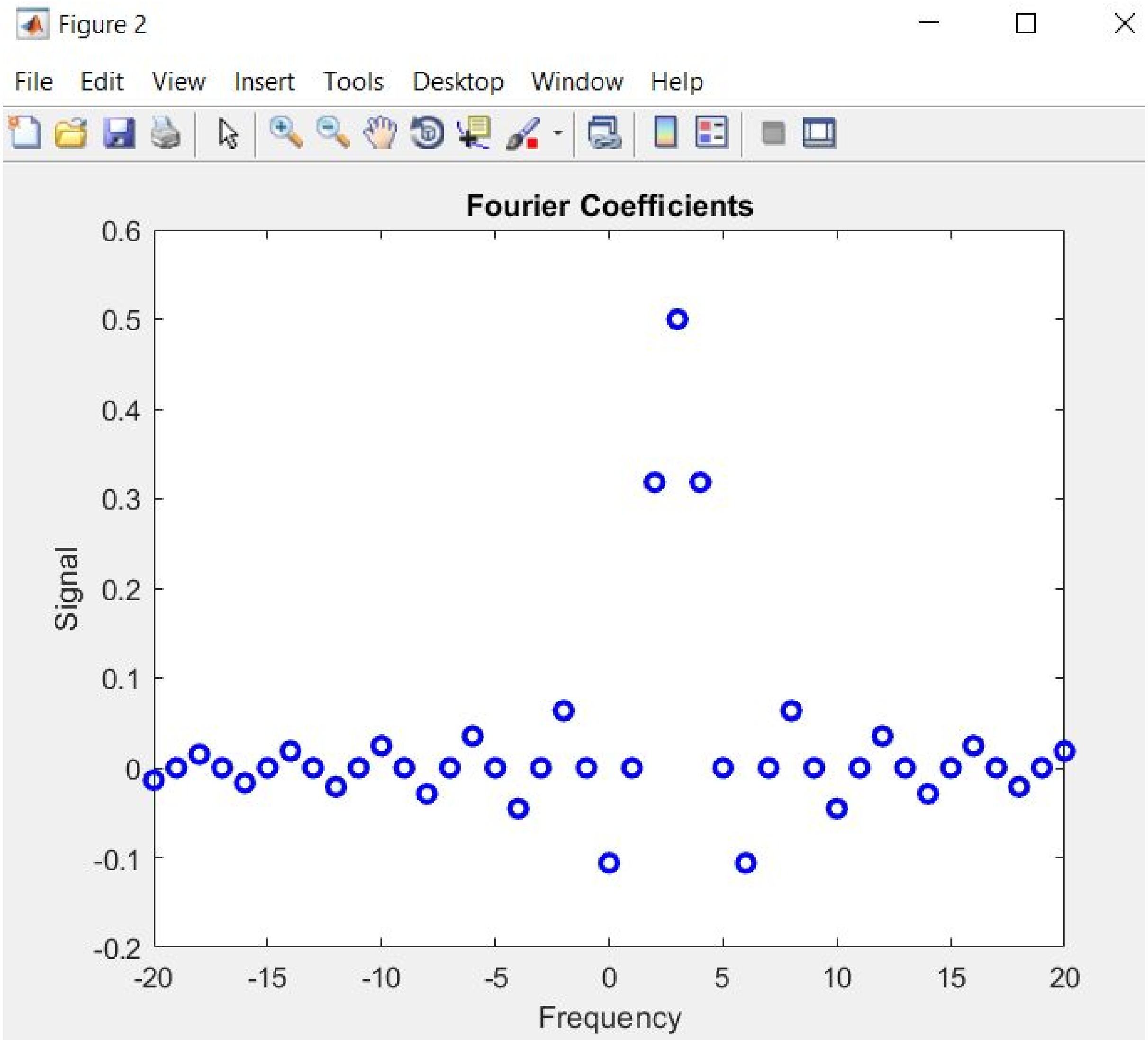


Figure 2
File Edit View Insert Tools Desktop Window Help
Fourier Coefficients
Signal
Frequency
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-20
-15
-10
-5
0
5
10
15
20

CODES

X1

```
function y = x1(tau, T)

    sympref('HeavisideAtOrigin', 1);

    y = @(t)(heaviside(t) - heaviside(t - 2*tau));
    y = @(t) y(mod(t, T));
    y = @(t) y(t+tau);
```

X2

```
function y = x2(tau, T)

    sympref('HeavisideAtOrigin', 1);

        y = @(t)(heaviside(t) - heaviside(t - 2*tau)).*(tau  - 
abs(t-tau));
        y = @(t) y(mod(t, T));
        y = @(t) y(t+tau);
```

FOU_EXP

```
function [y, C] = fou_exp(x, n, T)

    syms t
    y = 0;
    w = 2*pi / T;
    C = sym(zeros(2*n+1, 1));

    for j = -n:n
```

```
    f = @(t) x(t) .* exp(-w*t*j*1i);
    C(j+n+1, 1) = integral(f, -T/2, T/2) / T;
    y = y + C(j+n+1, 1) * exp(w*t*j*1i);

  end
```

FOU_TRIG

```
function y = fou_trig(x, n, T)

  syms t

  w = 2*pi / T;
  f = x;
  a = int(f, t, 0, T) / T;
  y = a;

  for j = 1:n

    f = x * cos(j*w*t);
    a = 2 * int(f, t, -T/2, T/2) / T;
    f = x * sin(j*w*t);
    b = 2 * int(f, t, -T/2, T/2) / T;
    y = y + a * cos(j*w*t) + b * sin(j*w*t);

  end
```

PART2

```
function part2(x, n, T)


    [y, C] = fou_exp(x, n, T);

    %PLOTTING fourier coefficients
    t_p = -n:n;

    figure();
    plot(t_p, C, 'ob', 'LineWidth', 2);
    title('Fourier Coefficients');
    ylabel('Signal');
    xlabel('Frequency');
```

PLOT

```
function Plot(x, yn1, yn2, T)

    t_p_x = -T*1.5:0.01:T*1.5;
    x_p = x(t_p_x);
    t_p_y = -1.5*T:0.1:1.5*T;
    yn1_p = double(subs(yn1, t_p_y));
    yn2_p = double(subs(yn2, t_p_y));

    figure();
    plot(t_p_x, x_p, '-m', 'LineWidth', 2);
    title('Input Signal');
```

```
ylabel('Signal');
xlabel('Time');

figure();

subplot(2, 1, 1);
plot(t_p_y, yn1_p, '-g','LineWidth', 2);
title('n = 10');
ylabel('Signal');
xlabel('Time');
axis([-1.5*T 1.5*T -0.2 1.1]);

subplot(2, 1, 2);
plot(t_p_y, yn2_p, '-r','LineWidth', 2);
title('n = 20');
ylabel('Signal');
xlabel('Time');
axis([-1.5*T 1.5*T -0.2 1.1]);
```

DISCUSSION

- *Applications include complex analysis, signal processing, control systems, speech and language processing, image processing, communications, thermal analysis, vibration analysis, modal decomposition.*
  - *By using FS we can represent signals using exponential or sinusoidal functions.These functions have easy relations with respect to linear transforms such as integration and differentiation. This makes the Fourier Series convenient for solving differential equations*
  - *Fourier series exists when it follows Dirichlet condition*

Given by

1. *Absolutely integrable*
2. *Bounded variation in one time period*
3. *Finite discontinuity in one time period.*

CONCLUSION:

- *If x(t) is any odd function, then its FS consists only of sine terms. If x(t) is an even function, then its FS consists only of cosine terms*
- *As we increase the value of n , the signal will match the original signal.*
- *Thus we can represent a periodic signal using Fourier Series.*