FA to RE

formula | Arden's method

(1) FA to RE using formula

$$r_{ij}^{k} = (r_{ik}^{k-1})(r_{kk}^{k-1})^{*}(r_{kj}^{k-1}) + r_{ij}^{k-1}$$

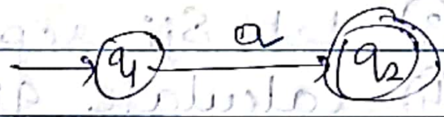
r_{ij} = transition of states q_i to q_j (i.e source state to target state)

$i=1, j=1, \boxed{k=1}$

$r_{11}^{1} = r_{11}^{0}(r_{11}^{0})^{*}(r_{11}^{0}) + r_{11}^{0}$

$= \epsilon(\epsilon)^{*}\epsilon + \epsilon$

$= \epsilon$



$r_{12}^{1} = a$

$r_{21}^{1} = \phi$

$r_{22}^{1} = \epsilon$

$r_{12}^{2} = (r_{12}^{1})(r_{22}^{1})^{*}(r_{22}^{1}) + (r_{12}^{1})$

$= a(\epsilon)^{*}\epsilon + a$

$= a+a$ Idempotent Law

$= a$

$\boxed{r = a}$

$$R = Q + RP$$
$$\Downarrow$$
$$R = QP^*$$

(2) Arden's theorem

(i) Arden's theorem is useful for checking the equivalence of two reg exp as well as in conversion from DFA to reg exp.

Algorithm:-

(1) Let q_1 be the initial state.

(2) There are $q_2, q_3, \ldots q_n$ be the number of states. The final state may be some q_j where $j \leq n$.

(3) Let δ_{ji} represents the transition from q_j to q_i.

(4) Calculate q_i such that

$$q_i = q_j \times \delta_{ji}$$

If q_i is start state,

$$q_i = q_j \times \delta_{ji} + \epsilon$$

(5) Illy compute the final state which ultimately gives the reg exp r.

Let P & Q be reg expression.

Reg exp R is given as

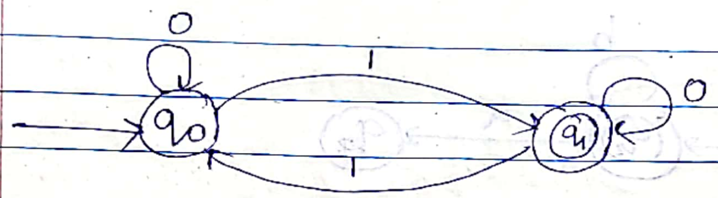
$$R = Q + RP$$

which has unique solution as $\boxed{R = QP^*}$

$$R = Q + \underline{R}P = Q + \underline{Q}P^*P = Q(\epsilon + P^*P)$$

$$= QP^* \quad [\because \epsilon + R^*R = R^*]$$

1



$q_0 = q_0 0 + q_1 1 + \epsilon$

$q_1 = q_0 1 + q_1 0$

$q_1 = q_0 1 + q_1 0$

R Q R P

$q_1 = q_0 1 0^*$

R Q P*

$q_0 = q_0 0 + \underline{q_1} 1 + \epsilon$

2

$q_0 = q_0 0 + \underline{q_0 1 0^*} 1 + \epsilon$

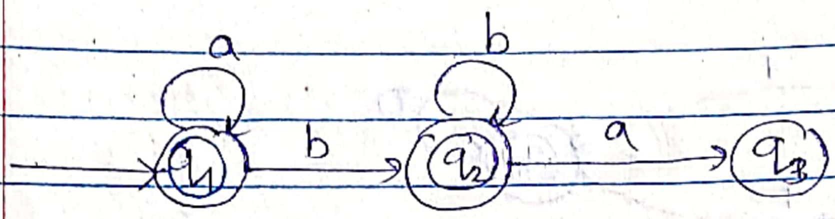
$q_0 = \epsilon + q_0 (0 + 1 0^* 1)$

R Q R P

$\boxed{q_0 = \epsilon (0 + 1 0^* 1)^*}$

$\boxed{q_1 = (0 + 1 0^* 1)^* 1 0^*}$ // eq^n ~~for~~ final state.

②



$q_1 = q_1 a + \epsilon$

$q_2 = q_1 b + q_2 b$

$\underset{R}{q_1} = \underset{Q}{\epsilon} + \underset{R}{q_1} \underset{P}{a}$

$q_1 = \epsilon a^*$

$\boxed{q_1 = a^*}$ $\qquad \because \epsilon R^* = R^*$

Substituting q_1 in q_2,

$\underset{R}{q_2} = \underset{Q}{a^* b} + \underset{R}{q_2} \underset{P}{b}$

$\boxed{q_2 = a^* b b^*}$

or

$q_2 = a^* b^+ \qquad \because LL^* = L^+$

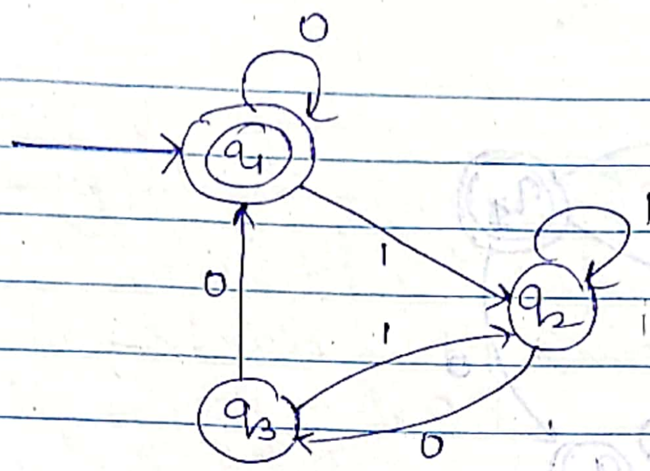
$r = q_1 + q_2$

$= a^* + a^* b b^*$

$= a^*(\epsilon + bb^*)$

$= a^* b^*$

(3)



incoming transitions.

$q_1 = q_1 0 + q_3 0 + \epsilon$

$q_2 = q_1 1 + q_2 1 + q_3 1$

$q_3 = q_2 0$

$q_2 = q_1 1 + q_2 1 + q_2 01$

$q_2 = q_1 1 + q_2 (1 + 01)$

$R \quad Q + R \, P$

$q_2 = q_1 1 (1+01)^*$

Substituting q_2 to q_1,

$q_1 = q_1 0 + q_3 0 + \epsilon$

$= q_1 0 + q_2 00 + \epsilon$

$= q_1 0 + q_1 1 (1+01)^* 00 + \epsilon$

$q_1 = q_1 [0 + 1(1+01)^* 00] + \epsilon$

$q_1 = \epsilon + q_1 [0 + 1(1+0)^* 00]$

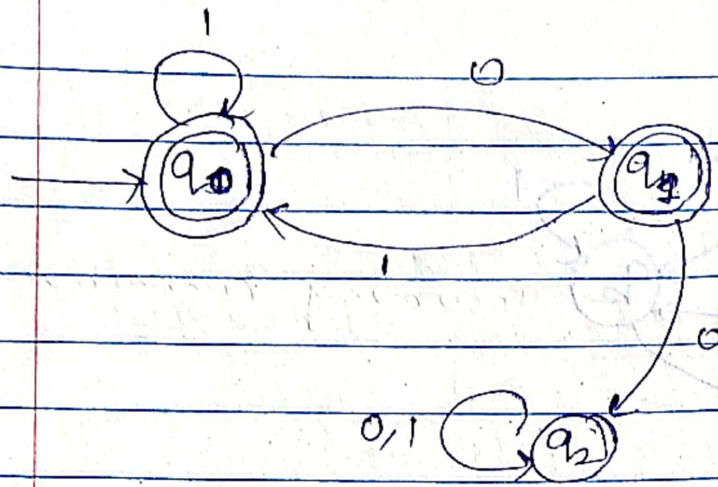
$R \quad Q + R P$

$q_1 = \epsilon [0 + 1(1+0)^* 00]^*$

$\boxed{q_1 = [0 + 1(1+0)^* 00]^*}$ $\boxed{\because \epsilon \cdot R = R}$

//

(4)



$q_0 = q_0 1 + q_1 1 + \epsilon$

$q_1 = q_0 0$

$q_0 = q_0 1 + q_0 01 + \epsilon$

$q_0 = q_0 (1+01) + \epsilon$

$q_0 = \epsilon + q_0 (1+011)$

R   Q + RP

$q_0 = \epsilon (1+01)^*$

$\boxed{q_0 = (1+01)^*}$

$q_1 = q_0 0$

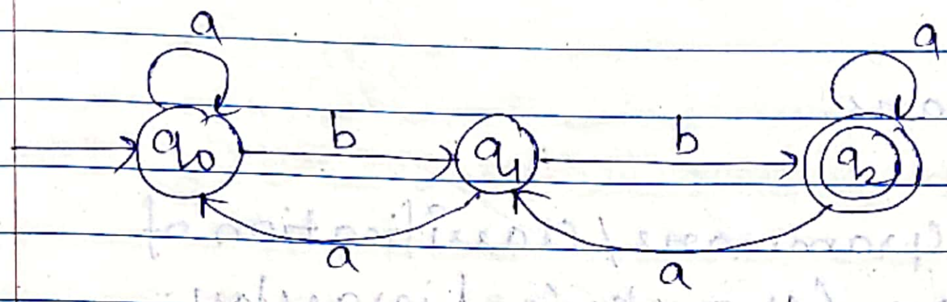
$\boxed{q_1 = (1+01)^* 0}$

$r = q_0 + q_1$

$= (1+01)^* + (1+01)^* 0$

$\boxed{r = (1+01)^* (\epsilon + 0)}$ //

(5)



$$q_0 = q_0 a + q_1 a + \epsilon$$

$$q_1 = q_0 b + q_2 a$$

$$\underbrace{q_2}_{R} = \underbrace{q_1 b}_{Q} + \underbrace{q_2}_{R}\underbrace{a}_{P}$$

$$q_2 = q_1 b a^*$$

Substituting q_2 in q_1

$$\underbrace{q_1}_{R} = \underbrace{q_0 b}_{Q} + \underbrace{q_1}_{R}\underbrace{b a^* a}_{P}$$

$$q_1 = q_0 b(ba^*a)^*$$

Substituting q_1 in q_0

$$q_0 = q_0 a + q_0 b(ba^*a)^* a + \epsilon$$

$$\underbrace{q_0}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_0}_{R}\underbrace{(a+b(ba^*a)^*a)}_{P}$$

$$\boxed{q_0 = (a+b(ba^*a)^*a)^*}$$

$$\boxed{q_1 = (a+bba^*aa)^* bba^*a}$$

$$\boxed{q_2 = (a+bba^*aa)^* bba^*a\, ba^*}$$

$$q_2 = (a+bba^+a)^* bba^+ ba^*$$

⇒ If on input w, $\delta(p, w) = NA(a)$ & $\hat{\delta}(q, w) = NA(b)$ decision cannot be taken evaluate further for all i/ps on a & b

⇒ If on input w, $\hat{\delta}(p, w) = NA$ & $\hat{\delta}(q, w) = A$ then decision is ~~j~~ it is Distinguishable

★★ DFA minimization (table-filling method)

- Our goal is to understand when two distinct states p and q can be replaced by a single state that behaves like both p & q. We say that states p and q are equivalent if:

• For all input strings w, $\hat{\delta}(p, w)$ is an accepting state if and only if $\hat{\delta}(q, w)$ is an accepting state.

• We do not require that $\hat{\delta}(p, w)$ and $\hat{\delta}(q, w)$ are the same state, only that either both are accepting or both are non-accepting.

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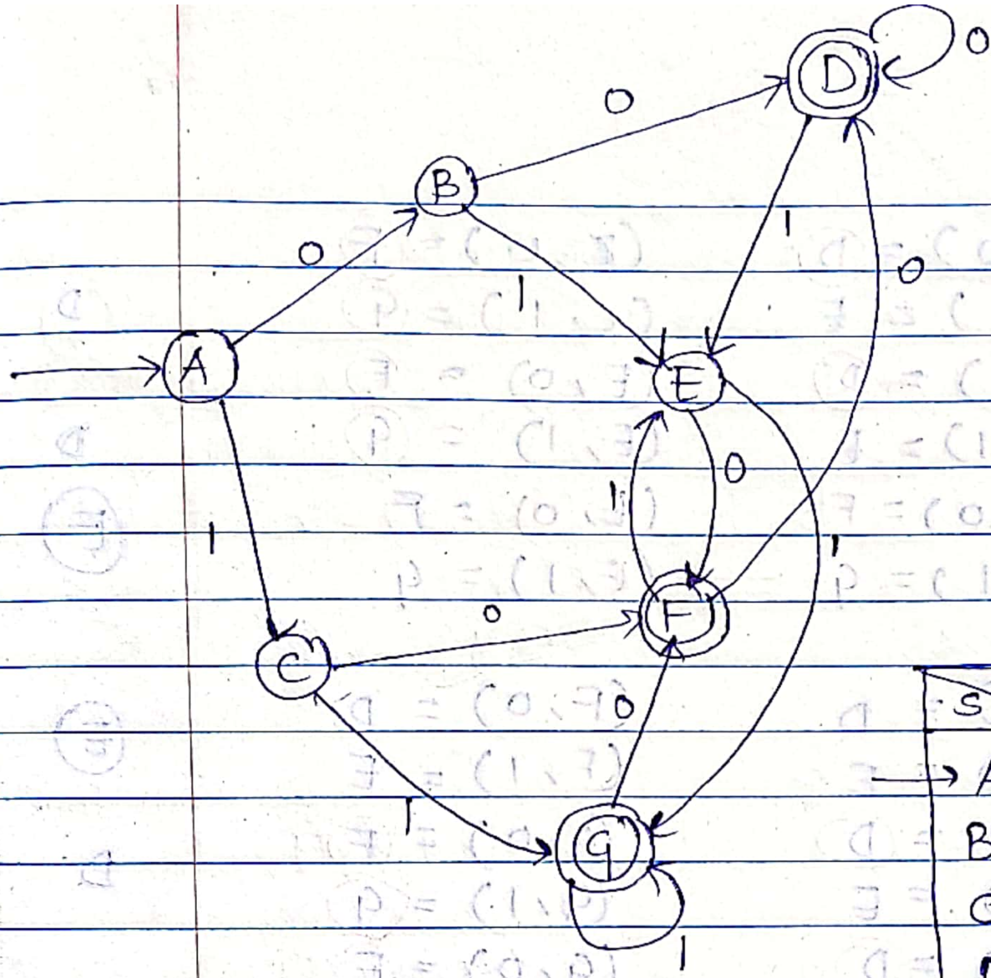
• If two states are not equivalent, then we say they are distinguishable. That is, state p is distinguishable from state q if there is at least one string w such that one of $\hat{\delta}(p, w)$ and $\hat{\delta}(q, w)$ is accepting, and the other is not accepting.

Note: ★★ (1) eliminate any state that cannot be reached from the start state.



$S \backslash \Sigma$	0	1
→ A	B	C
B	D	E
C	F	G
D*	D	E
E	F	G
F*	D	E
G*	F	G

	A	B	C	D*	E	F*
B	x					
C	x	x				
→ D*	1	1	1			
E	x	x	O	1		
→ F*	1	1	1	O	1	
→ G*	1	1	1	x	1	x

(A, B) = (A, 0) = B   (B, 0) = D   D
(A, 1) = C   (B, 1) = E

(A, C) = (A, 0) = B   (C, 0) = F   D
(A, 1) = C   (C, 1) = G

(A, E) = (A, 0) = B   (E, 0) = F   D
(A, 1) = C   (E, 1) = G

(B, C) (B, 0) = D (C, 0) = F
(B, 1) = E (C, 1) = G D

(B, E) (B, 0) = D (E, 0) = F
(B, 1) = E (E, 1) = G D

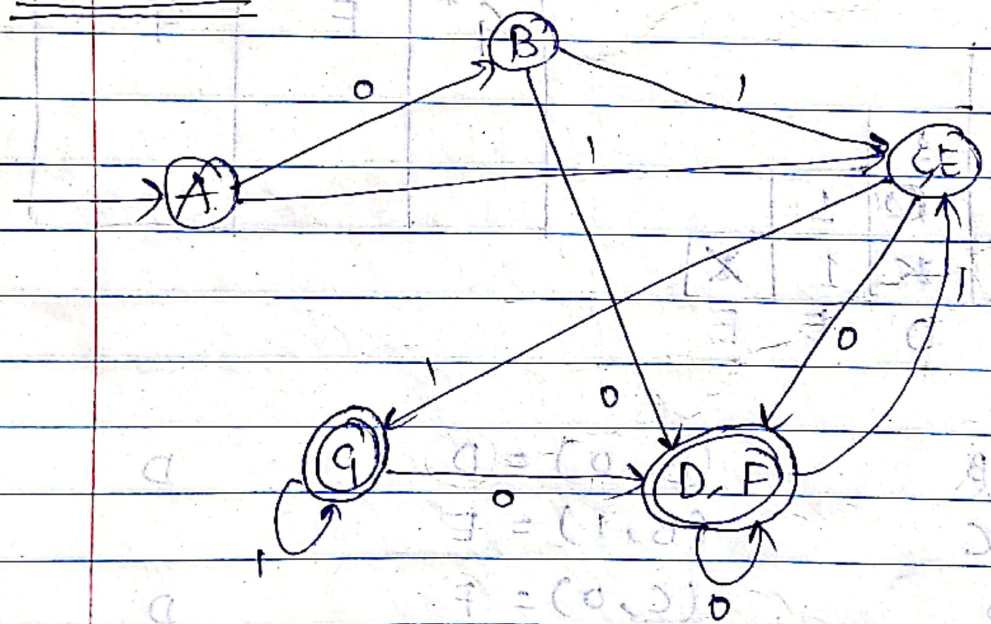
(C, E) (C, 0) = F (E, 0) = F
(C, 1) = G (E, 1) = G E

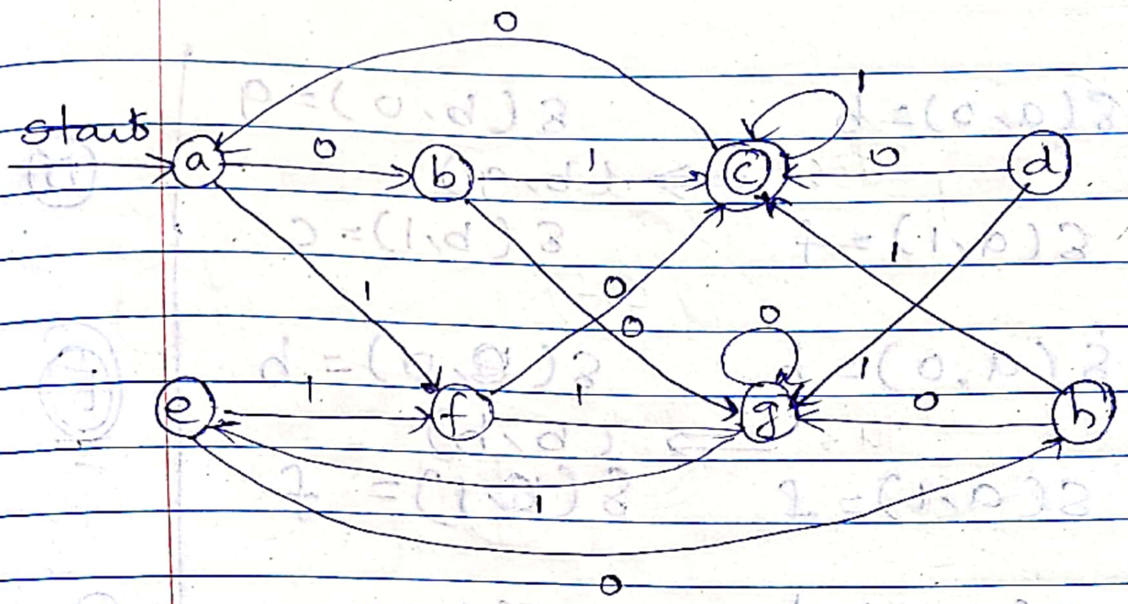
~~(E, G)~~
(D, F) = (D, 0) = D (F, 0) = D
~~(D, G)~~
(D, 1) = E (F, 1) = E E

(D, G) = (D, 0) = D (G, 0) = F
(D, 1) = E (G, 1) = G D

(F, G) = (F, 0) = D (G, 0) = F
(F, 1) = E (G, 1) = G D

min DFA :





Q \ Σ	0	1	checklist
1 a	b	f	b, g
2 b	g	c	b, h
3 c*	a	c	f, e
5 e	h	f	g, h
6 f	c	g	
7 g	g	e	
8 h	g	c	

	a	b	c	e	f	g
b						
c	1	1				
e						
f						
g			1			
h			1			

(a, b) E

$\delta(a,0)=b \quad \delta(b,0)=g$

u's b $\Longrightarrow$ (b, g) D

$\delta(a,1)=f \quad \delta(b,1)=c$

u's $\Longrightarrow$ (c, f)

(a, e)

$\hat{\delta}(a,0)=b \quad \delta(e,0)=h$

u's $\Longrightarrow$ (b, h) E

$\delta(a,1)=f \quad \delta(e,1)=f$

(a, f)

$\delta(a,0)=b \quad \delta(f,0)=c$ D

$\delta(a,1)=f \quad \delta(f,1)=g$

(a, g)

$\delta(a,0)=b \quad \delta(g,0)=g$

$\Longrightarrow (b,g)$ D from (b, g)

$\delta(a,1)=f \quad \delta(g,1)=e$

$\Longrightarrow (f,e)$

(a, h)

$\delta(a,0)=b \quad \delta(h,0)=g$

$\Longrightarrow (b,g)$ D

$\delta(a,1)=f \quad \delta(h,1)=c$

(b, e)

$\delta(b,0)=g \quad \delta(e,0)=h$

$\Longrightarrow (g,h)$ D

$\delta(b,1)=c \quad \delta(e,1)=f$

(b, f)

$\delta(b,0)=g \quad \delta(f,0)=c$ D

$\delta(b,1)=c \quad \delta(f,1)=g$

D	(b, g)	$\delta(b,0) = g$ $\delta(b,1) = c$	$\hat{\delta}(g,0) = g$ $\delta(g,1) = e$	D
E	(b, h)	$\delta(b,0) = g$ $\delta(b,1) = c$	$\delta(h,0) = g$ $\delta(h,1) = c$	E
D	(e, f)	$\delta(e,0) = h$ $\delta(e,1) = h$	$\delta(f,0) = c$ $= g$	D
D from (b, g)	(e, g)	$\delta(e,0) = h$ $\delta(e,1) = f$	$\delta(g,0) = g$ $\Rightarrow (h, g)$ $\delta(g,1) = e$ look (e, f)	D
D	(e, h)	$\delta(e,0) = h$ ~~$\delta(e,1)$~~ $\delta(e,1) = f$	$\delta(h,0) = g$ $\Rightarrow (g, h)$ $\delta(h,1) = c$	D
D	(f, g)	$\delta(f,0) = c$	$\delta(g,0) = g$	D
D	(f, h)	$\delta(f,0) = c$	$\delta(h,0) = g$	D
D	(g, h)	$\delta(g,0) = g$ $\delta(g,1) = e$	$\delta(h,0) = g$ $\delta(h,1) = c$	D

min DFA: