Turing Machine

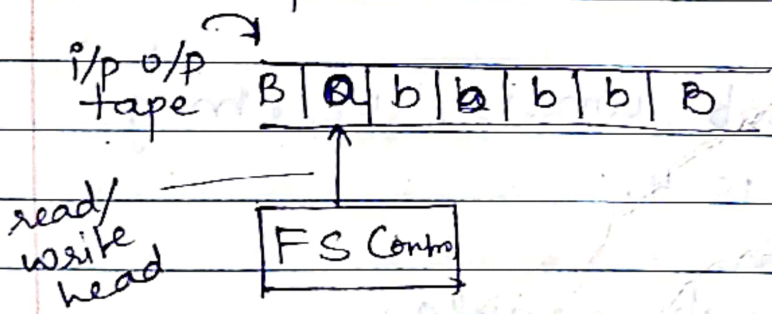
→ FSM do not have memory, hence cannot solve p...
to store data to be used for later computation
→ FSM, External memory, store (write), ability to consume ab as i/p (multiplication)
(L→R)

Elements
① A head (R/W, L/R/S)
② Infinite tape (alphabet: ab)
③ finite set of symbols
④ finite set of states

Turing machine

→ Conceived by Eng mathematician Alan Turing as model of human "computation"
→ Turing Machine is considered to be a simple model of a computer and is most powerful m/c.
→ T.M can be used to perform
(i) Language Recognition
(ii) Computation of some mathematical functions

* Model of T.M



* Components of TM :-

TM consists of finite set of states, i/p or o/p tape and read/write head

Depending on the state and the tape symbol
(i) TM can change the state or remain in same state
(ii) TM can change the tape symbol / keep it the same
(iii) TM can move the head {L, R, S}

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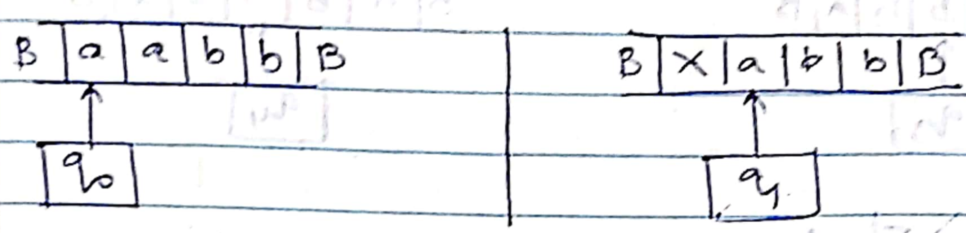
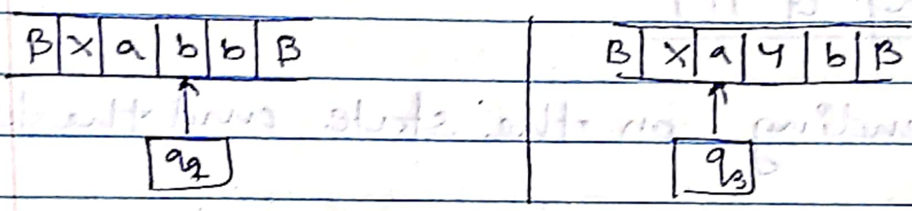
Examples of Transition function



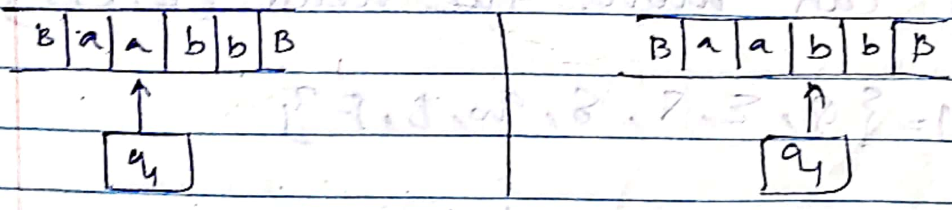
δ :-

Q \ Γ	a
q_0	(q_1, X, R)



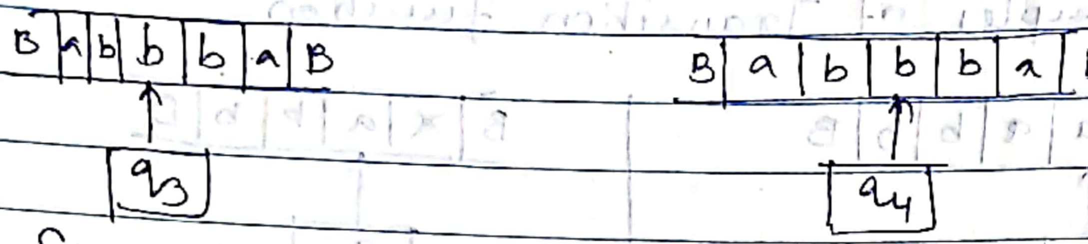
δ :-

Q \ Γ	b
q_2	(q_3, Y, L)



δ :-

Q \ Γ	a
q_1	(q_1, a, R)



δ:-

$Q \backslash \Gamma$	b
q_3	(q_4, b, S)

Working of TM

Depending on the state and the tape symbol,

1. TM can change the state or remain in the same state
2. TM can change the tape symbol / keep it the same
3. TM can move the head $\{L, R, S\}$

$$M = \{Q, \Sigma, \Gamma, \delta, q_0, B, F\}$$

where,

Q = Finite set of states

Σ = i/p alphabets

Γ = tape alphabets

δ = transition fn

q_0 = start state

B = Blank

F = finite set of final states

Type I

① Design TM to recognise $L = \{a^n b^n \mid n \geq 1\}$

⟹ Theory: B ~~first a~~ ~~first b~~ B

X Y

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

$Q = \{q_0, q_1, q_2, q_3, q_f\}$

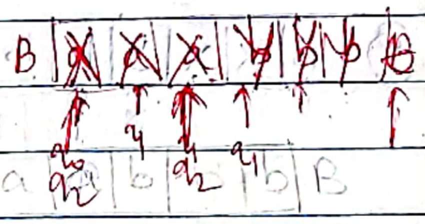
$\Sigma = \{a, b\}$

$\Gamma = \{a, b, X, Y, B\}$

$q_0 = q_0$

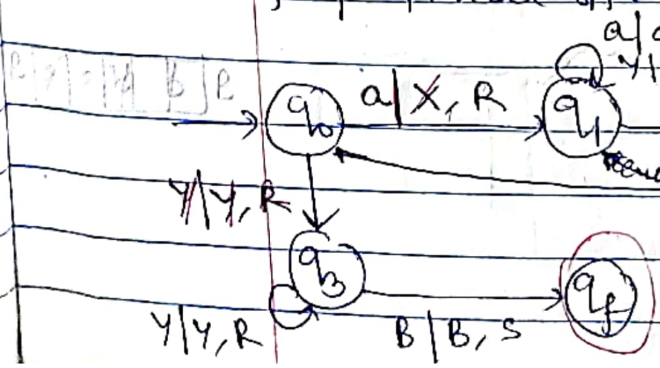
$B = B$

$F = \{q_f\}$



δ :-

Q \ Γ	a	b	X	Y	B
→ q_0	(q_1, X, R)			(q_3, Y, R)	
a→X q_1	(q_1, a, R)	(q_2, Y, L)		(q_1, Y, R)	
b→Y q_2	(q_2, a, L)		(q_0, X, R)	(q_2, Y, L)	
q_3				(q_3, Y, R)	(q_f, B, S)
q_f	final state				



9)

① B | a | a | b | b | B

q_0

② B | X | a | b | b | B

q_1

③ B | ~~X~~ | a | Y | b | B

$\boxed{q_2}$

④ B | X | a | ~~Y~~ | b | B

q_0

⑤ B | X | X | Y | b | B

q_1

⑥ B | X | X | Y | Y | B

q_2

⑦ B | X | X | Y | Y | B

q_0

⑧ B | X | X | Y | Y | B

q_f

VQ

2) Design TM to recognize

$L = \{ a^n b^n c^n \mid n \geq 1 \}$

	X	X	Y	Y	Z	Z	
B	a	a	b	b	c	c	B

$M = \{ Q, \Sigma, \Gamma, \delta, q_0, B, F \}$

$Q = \{ q_0, q_1, q_2, q_3, q_4, q_f \}$ $q_0 = \{ q_0 \}$

$\Sigma = \{ a, b, c \}$ $B = \{ B \}$ Blank

$\Gamma = \{ a, b, c, X, Y, Z, B \}$ $F = \{ q_f \}$

δ :-

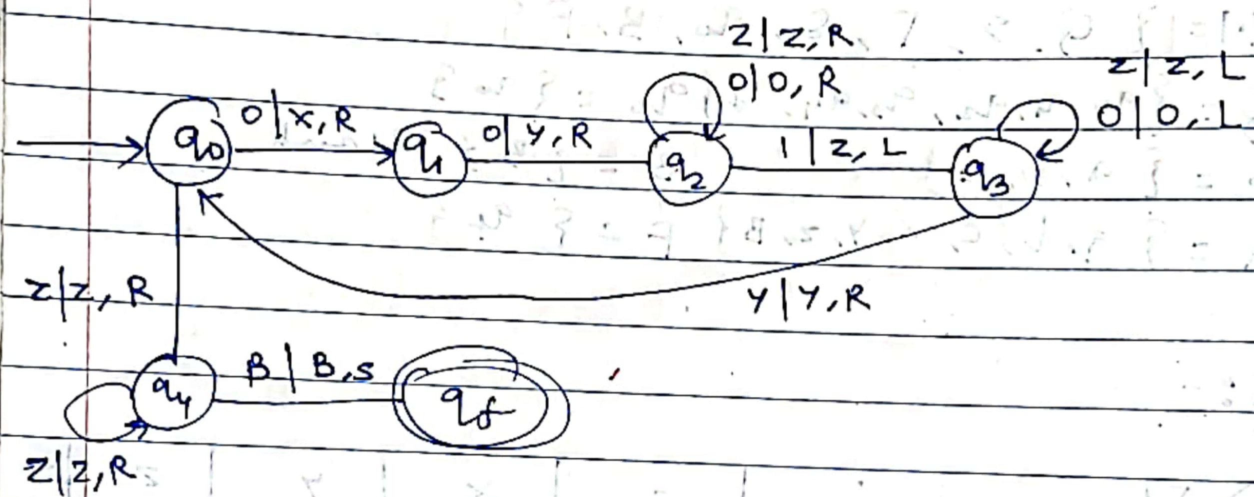
Q \ Γ	a	b	c	X	Y	Z	B
→ q_0	(q_1, X, R)			$\cancel{q_4}$,	q_4, Y, R		
q_1	q_1, a, R	q_2, Y, R			q_1, Y, R		
q_2		q_2, b, R	q_3, Z, $\cancel{R}$ L			q_2, Z, R	
q_3	q_3, a, L	q_3, b, L		q_0, X, R	q_3, Y, L	q_3, Z, L	
q_4					q_4, Y, R	q_4, Z, R	q_f, B, S
q_f	final state						

transition diagram :

3) Design TM to recognize

$L = \{ 0^{2n} 1^{n} \mid n \geq 1 \}$

	x	y	x	y	z	z	
B	0	0	0	0	1	1	B



④ Design TM to recognize

$L = \{ a^{n} b^{2n} c^{m} \mid n, m \geq 1 \}$

	x	x	y	z	y	z				
B	a	a	b	b	b	b	c	c	c	B

Q \ Σ	0	1	X	Y	Z	B
→ q0	q1, X, R				q4, Z, R	
q1	q2, Y, R					
q2	q2, 0, R	q3, Z, L			Z|Z,R	
q3	q3, 0, L			q0, Y, R	Z|Z,L	
q4					q4, Z, R	B|B,S

δ/Γ	a	b	X	Y	B

Q.5] Design TM to recognize

$L = \{\underline{x} \mid n_a(x) = n_b(x)\}$

B b b a a b a B

* *

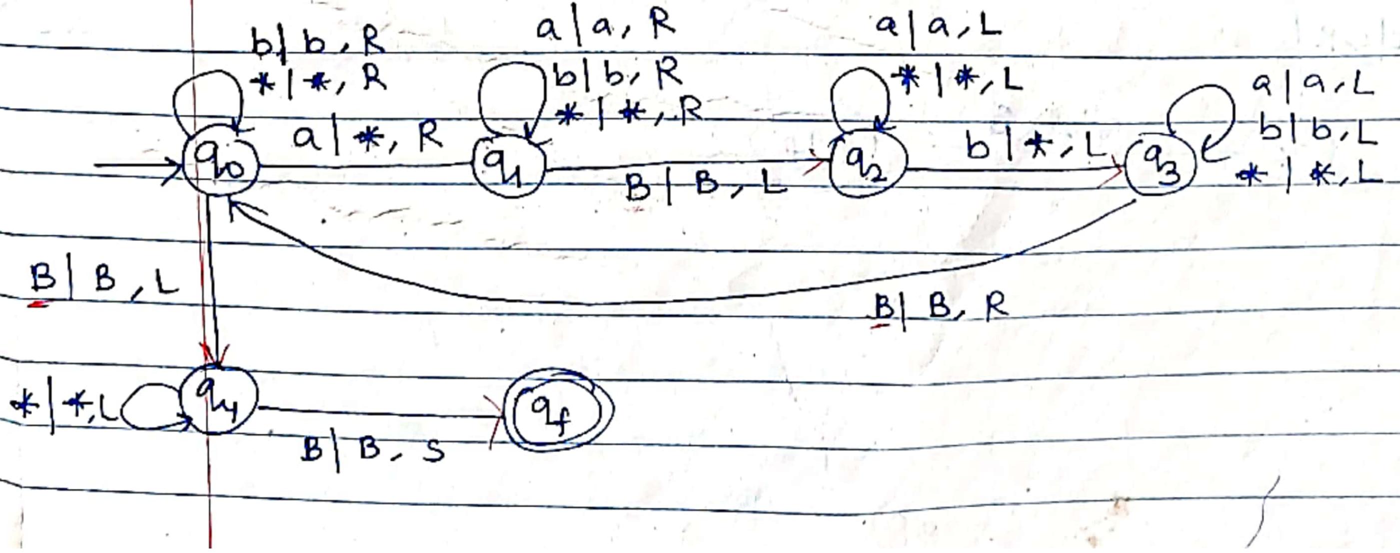
B b b * a * a B   a, b a a bb

* *   b

B b * * * * a B

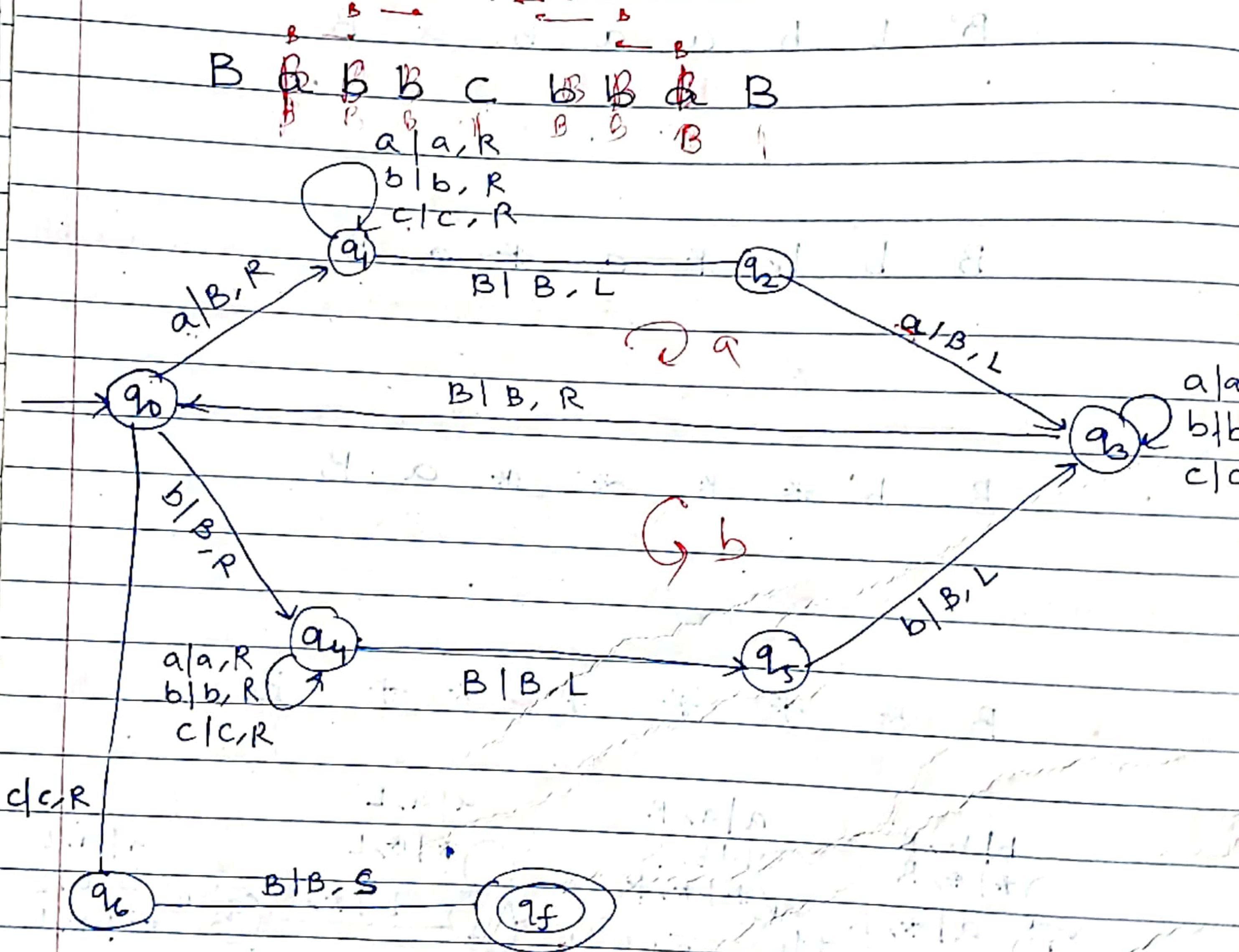
* *

B * * * * * * B



6) Design TM to recognize

$L = \{ wcw^R \mid w \in \{a,b\}^*, \; w^R - \text{Reverse of } w \}$

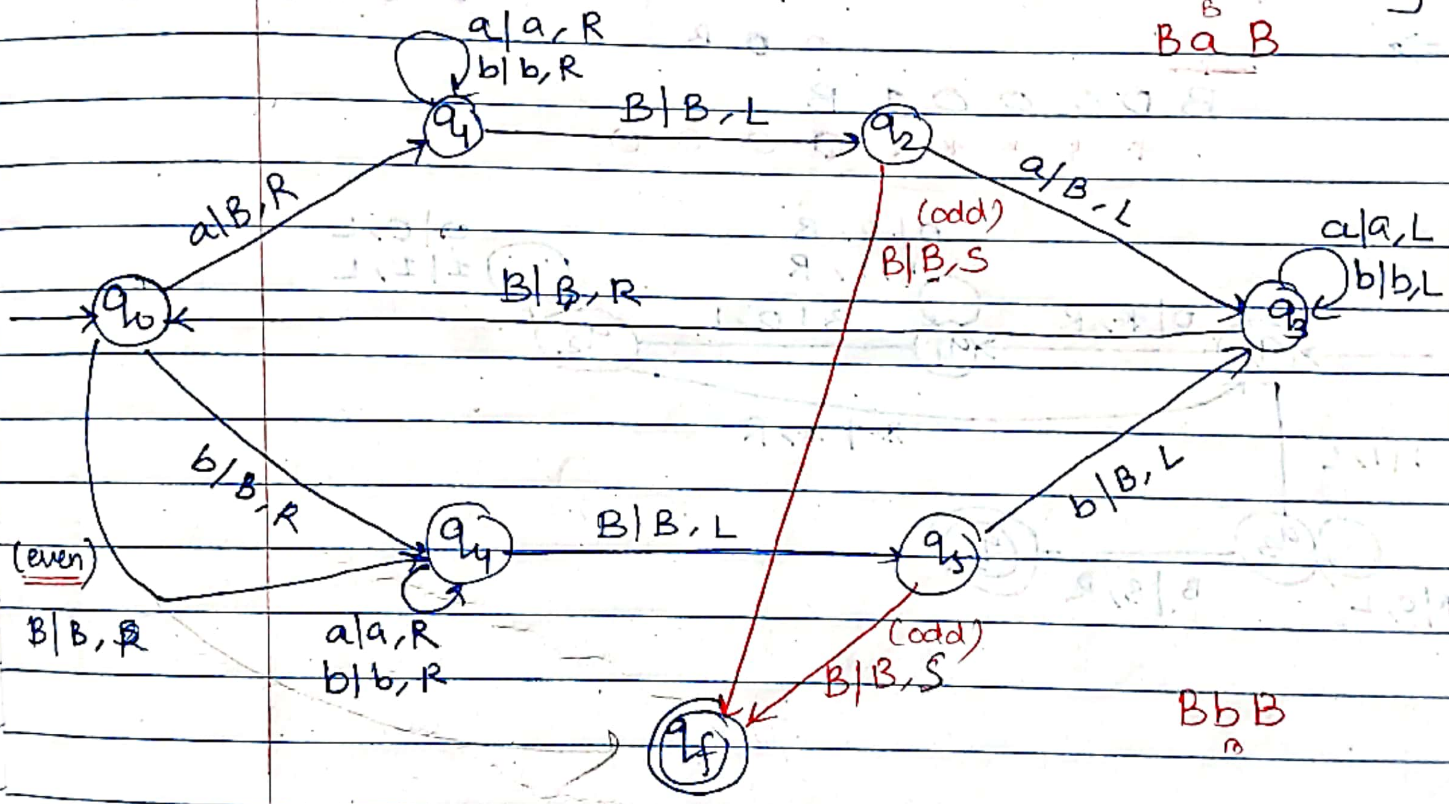


δ/Γ	a	b	X	Y	B

w

7) Design TM to recognize

L = {w | w is a palindrome over Σ={a,b}}



aaabb

B B|B B B B

BaaB
BB

BabaB
BBB

Type II

i/p | ~~0~~ | ~~0~~ | ~~0~~ | 1 | B0B

o/p | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

① Design TM to copy the given number 'n'

i/p $B0^n1B$

o/p $B0^n10^nB$

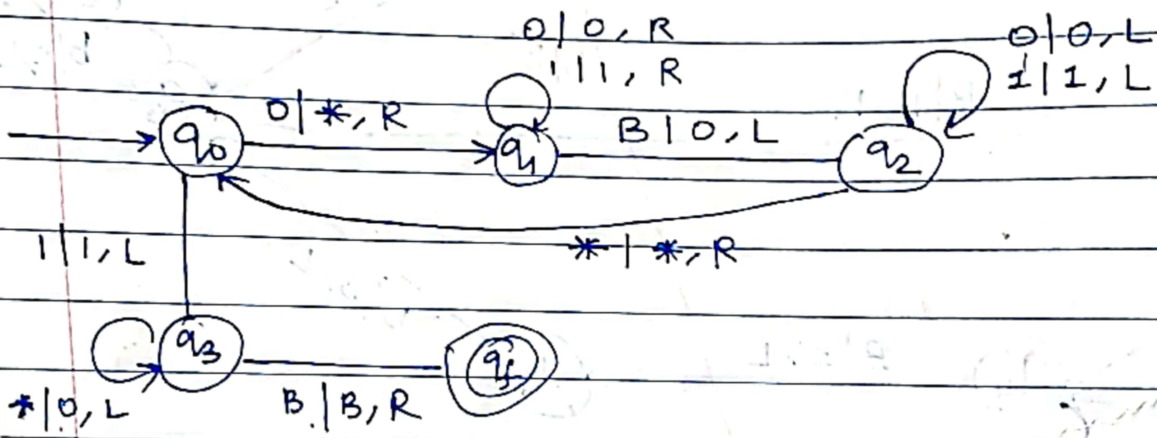
⇒

B 0 0 0 0 1 0 I

0 0 B

B 0 0 0 0 1 B

* * * * 0 0 0 0



② Design TM for recognizing language
$L = \{ x \mid n_a(x) > n_b(x) \}$

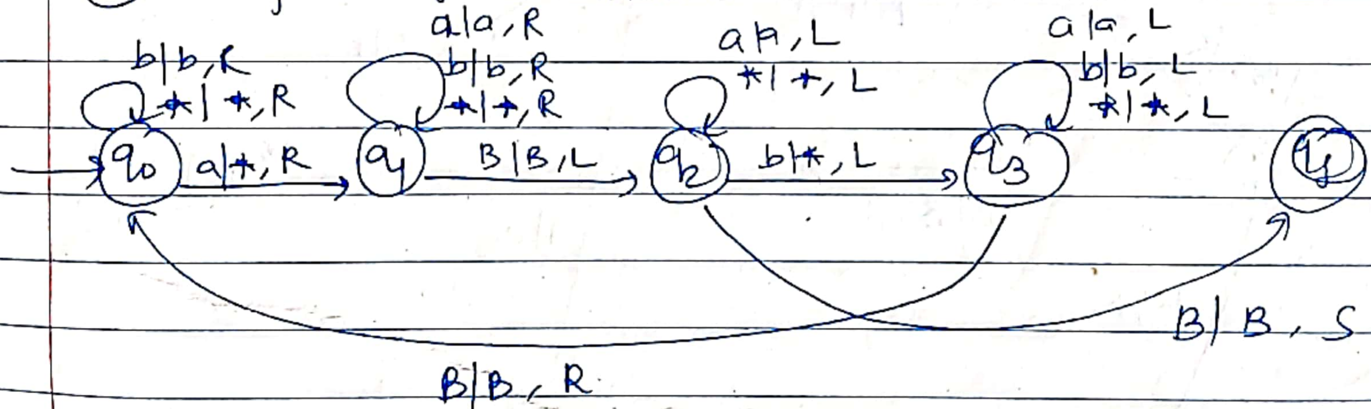
sol^n:

B a a b a b b a B

* ⟶

⟵ *

1. q_0 - a make it *
2. q_1 - goto Rights
3. q_2 - b make it *
4. q_3 - goto lefts
5. q_f - final state



③ Design TM to perform m-n which is defined as
m-n if m>n , 0 if m≤n

i/p $B0^m10^nB$

o/p $B0^pB$ $p = m-n$

eg: 5-3

B00000 1000B

B ⟶ B

⟵ B

9) B B 0 0 0 0 1 0 0 B B

B ⟶

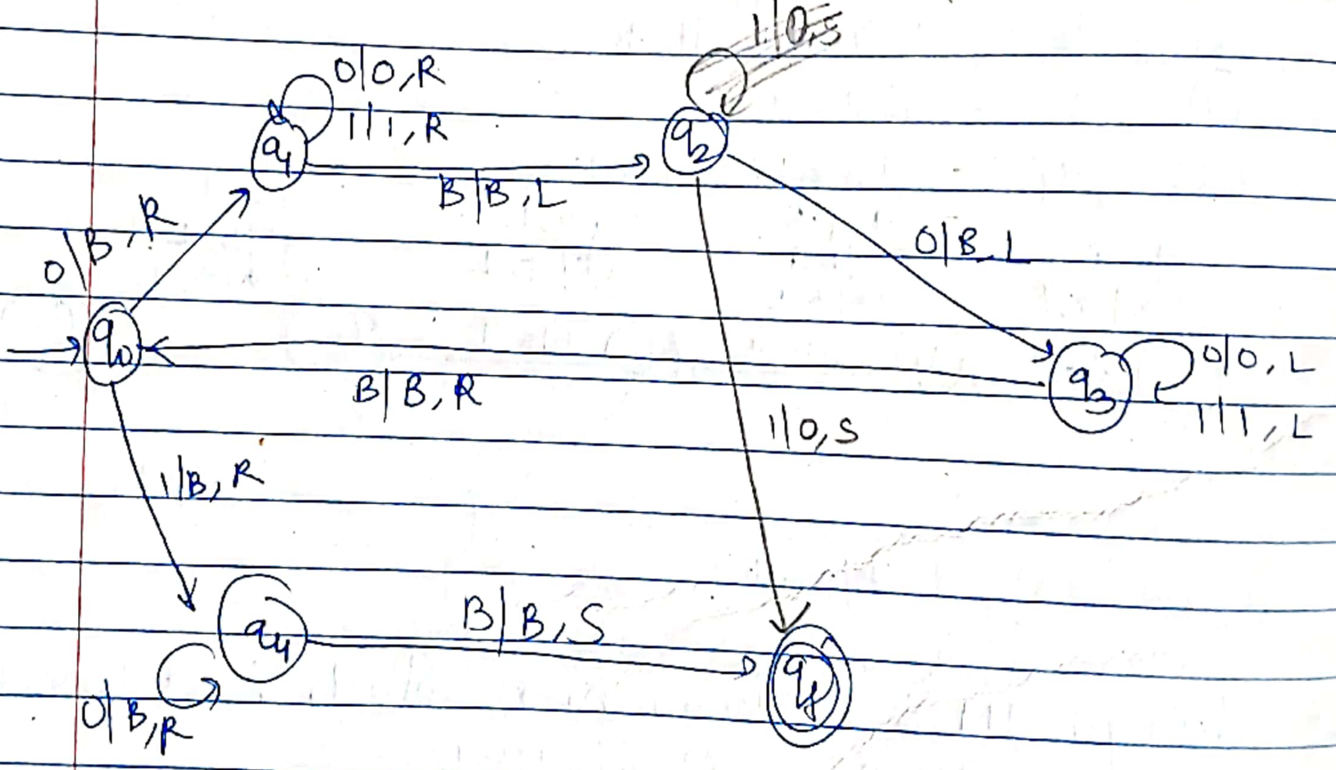
⟵ B

B B B 0 0 0 1 0 B B B

B ⟶

⟵ B

B B B B 0 0 1 B B B B



B B B B B B B   B B B B B B B

Γ | a | b | X | Y | B |

(4) Design TM to compare m & n and leave on the tape 'G' if $m > n$

'E' if $m = n$

'L' if $m < n$

if $B0^m 1 0^n B$

if $B\,p\,B$ $\quad p = G|E|L$

Sol^n 5 & 2

B 0 0 0 0 0 1 00 B $\quad$ B BB B B B B $\quad$ B B B B B B B B B

B

B

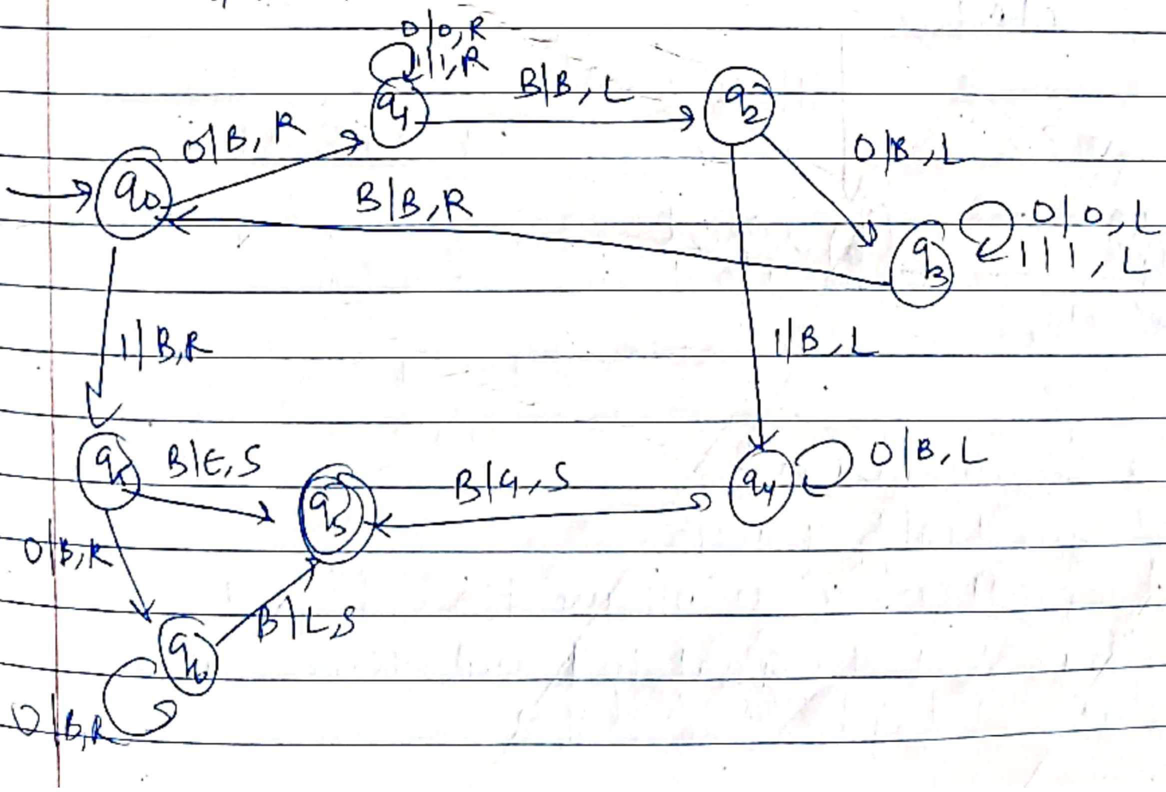
B 0 0 0 0 1 0 B

B

B

B 0 0 0 1 B

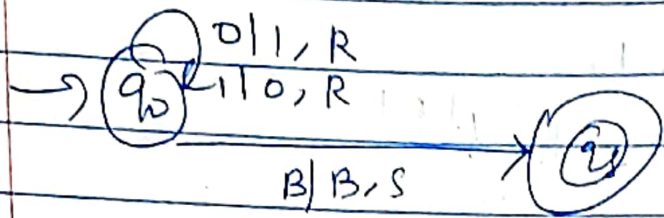
G B B B



5) Design TM to find 1's complement of Binary number

Sol^n Eg: B 10011100 B

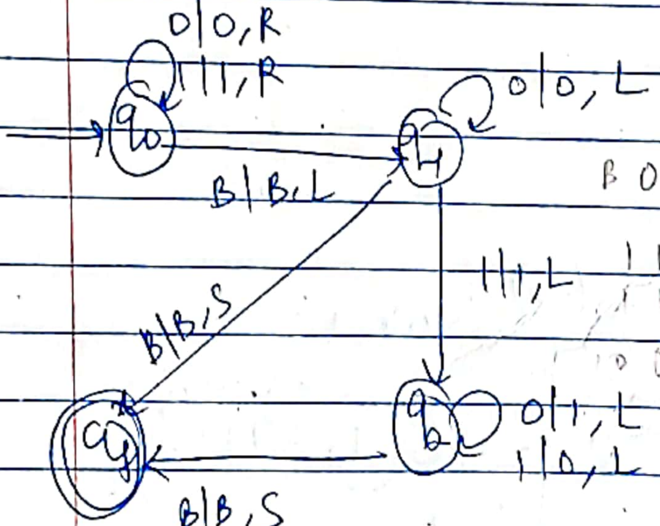
01100011.



6) Design TM to find 2's complement of binary number

eg. 1 : BB01011 0000BB

10101 0000



10100 1111

1 1 1 1 1

101010.000

B 0000 B

1 1 1 1

1 0 1 0 0 1 1 1 1

0 0 0 0

b 0 b 0 1 0 0 0 0

1001011 0110100

0110101 0110101

For 2's complement:

Begin Right to left

Keep all 0's as 0 till we find first 1

keep that 1 equal to 1 and then replace all 0's by 1 & 1's by 0

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δ/Γ	a	b	X	Y	B

⑦ Design TM to increment Binary by 1

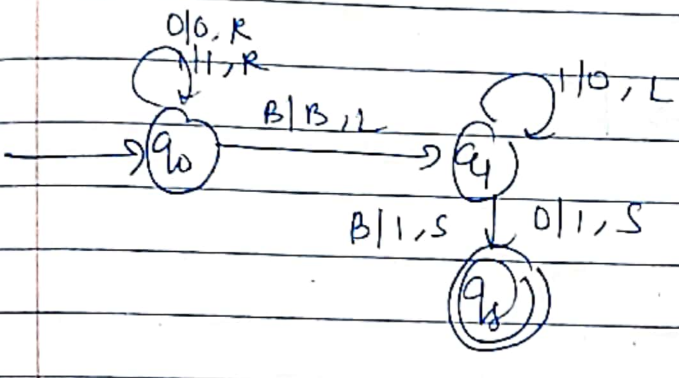
Sol^n

0011   0101

100   10

B0011B   B100B

100   11



B1111B

⑧ Design TM to check whether the given unary number is divisible by 3

Sol^n:

	S \ I	0
	$\rightarrow q_s$	q_1
0	q_0^*	q_1
1	q_1	q_2
2	q_2	q_0

— q_0

111

1111111

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

$Q = \{q_0, q_1, q_2, q_f\}$

$q_0 = q_0$

$\Sigma = \{0\}$

$\Gamma = \{0, B\}$

$B = B$

$F = \{q_f\}$

$Q \backslash \Sigma$	ϕ	B
$\rightarrow q_0$	(q_1, ϕ, R)	(q_f, B, S)
q_1	(q_2, ϕ, R)	
q_2	(q_0, ϕ, R)	
q_f^*	final state	