Trigonometry

- **Units of Angles (Degree and Radian)**
- Degree: $1^{\circ} = \frac{\pi}{180}$ $\quad \rightarrow$ for e.g. $135^{\circ} = 135 \times \frac{\pi}{180} = \frac{3\pi}{4}$
- Radian: $1^{R} = \frac{180^{\circ}}{\pi}$ $\quad \rightarrow$ for e.g. $\frac{11\pi}{3} = \frac{11\pi}{3} \times \frac{180^{\circ}}{\pi} = 660^{\circ}$

Degree $\left(\text{Multiply by } \frac{\pi}{180}\right)$	**Radian** $\left(\text{Multiply by } \frac{180^{\circ}}{\pi}\right)$
360°	2π
180°	π
90°	$\frac{\pi}{2}$
45°	$\frac{\pi}{4}$
60°	$\frac{\pi}{3}$
30°	$\frac{\pi}{6}$

- **Compound Angles (Formulas):**

1. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
3. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - sin\,\alpha\sin\beta$
4. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + sin\,\alpha\sin\beta$
5. $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
6. $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

- **Allied Angles:**
- For X-axis ($(\pi \pm \theta)$ or $(2\pi \pm \theta)$), function does not change

i.e. Sin → Sin

Cos → Cos

Tan → Tan …

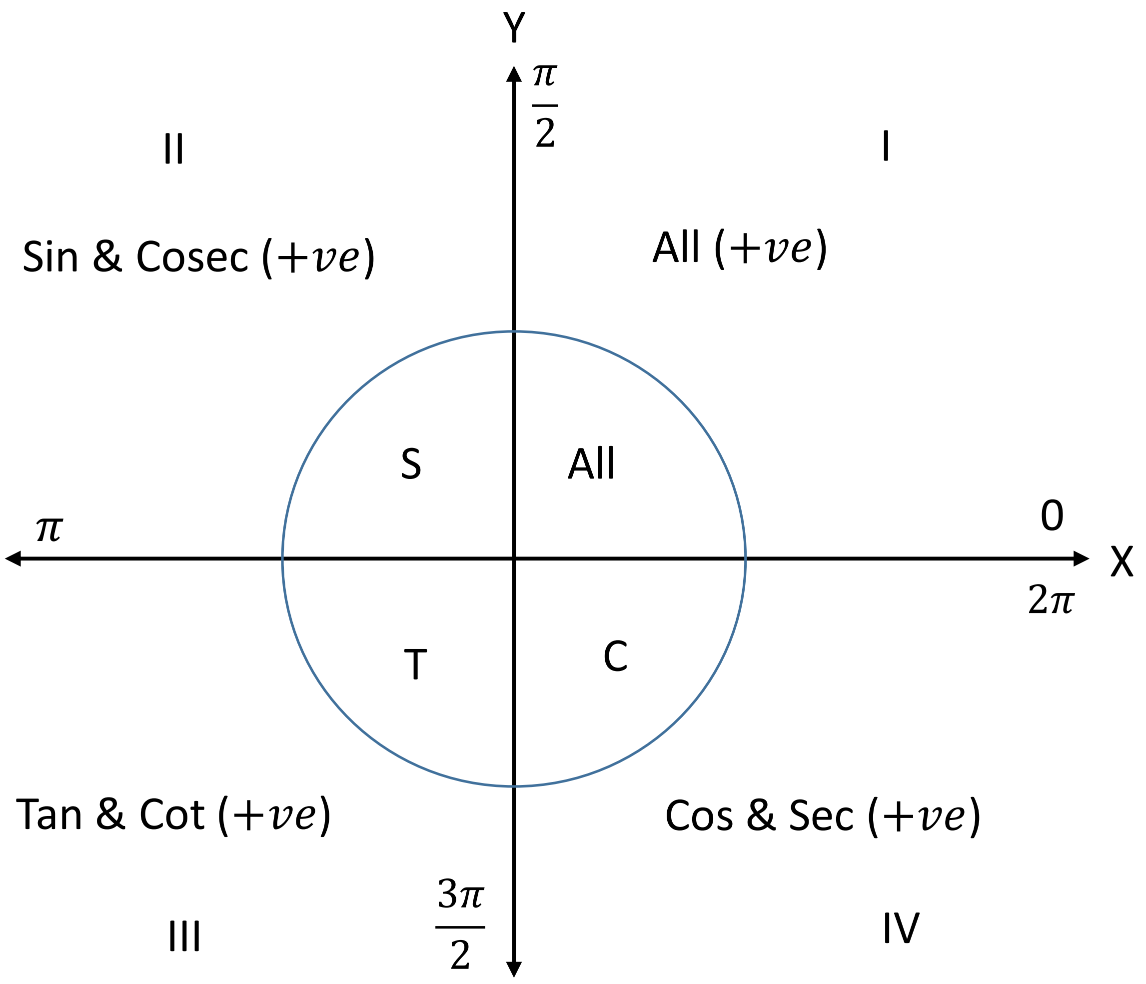
- For Y-axis $\left(\left(\frac{\pi}{2} \pm \theta\right) \; or \; \left(\frac{3\pi}{2} \pm \theta\right)\right)$, function changes

i.e. Sin ↔ Cos

Tan ↔ Cot

Cosec ↔ Sec

- **For e.g.:** For cosec $\left(\frac{3\pi}{2} + \theta\right)$
- $\left(\frac{3\pi}{2} + \theta\right)$ $lies\ in\ IV\ quadrant$
- $In\ IV\ quadrant\ cosec\ is\ -ve$
- As $\frac{3\pi}{2}$ $is\ on\ Y-axis, so$ function changes, i.e. Cosec → Sec
- $\therefore$ cosec $\left(\frac{3\pi}{2} + \theta\right) = -\sec\theta$



- Allied Angles:

Angle in Degree	$-\theta$	$90^0 - \theta$	$90^0 + \theta$	$180^0 - \theta$	$180^0 + \theta$
Angle in radian	$-\theta$	$\frac{\pi}{2} - \theta$	$\frac{\pi}{2} + \theta$	$\pi - \theta$	$\pi + \theta$
Sinθ	-Sinθ	Cosθ	Cosθ	Sinθ	-Sinθ
Cosθ	Cosθ	Sinθ	-Sinθ	-Cosθ	-Cosθ
Tanθ	-Tanθ	Cotθ	-Cotθ	-Tanθ	Tanθ
Cosecθ	-Cosecθ	Secθ	Secθ	Cosecθ	-Cosecθ
Secθ	Secθ	Cosecθ	-Cosecθ	-Secθ	-Secθ
Cotθ	-Cotθ	Tanθ	-Tanθ	-Cotθ	Cotθ

Principal Periods of Trigonometric Functions:

- $\sin(ax+b)$: $\frac{2\pi}{|a|}$
- $\cos(ax+b)$: $\frac{2\pi}{|a|}$
- $\text{cosec}(ax+b)$: $\frac{2\pi}{|a|}$
- $\sec(ax+b)$: $\frac{2\pi}{|a|}$

- $\tan(ax+b)$: $\frac{\pi}{|a|}$
- $\cot(ax+b)$: $\frac{\pi}{|a|}$

- Examples:

1. Principal Period of $\sin\left(\frac{7x}{3}\right) = \frac{2\pi}{\left|\frac{7}{3}\right|} = \frac{3\times 2\pi}{7} = \frac{6\pi}{7}$
2. Principal Period of $\cos(-6x+2) = \frac{2\pi}{|-6|} = \frac{2\pi}{6} = \frac{\pi}{3}$
3. Principal Period of $\tan(5x+4) = \frac{\pi}{|5|} = \frac{\pi}{5}$
4. Principal Period of $\sin\left(\frac{x}{3}\right) + \tan\left(\frac{x}{7}\right)$

$$= LCM\ \left(PP\ of\ \sin\left(\frac{x}{3}\right),PP\ of\ tan\left(\frac{x}{7}\right)\right)$$

$$= LCM\left(\frac{2\pi}{\left|\frac{1}{3}\right|},\frac{\pi}{\left|\frac{1}{7}\right|}\right)$$

$$= LCM\left(\frac{3\times 2\pi}{1},\frac{7\times\pi}{1}\right) = LCM\ (6\pi, 7\pi) = 42\pi$$

- **Multiple and Submultiple Angles:**

1. $\sin 2\theta = 2\sin\theta\cos\theta$
2. $\sin 2\theta = \frac{2\tan\theta}{1+tan^2\theta}$
3. $\cos 2\theta = cos^2\theta - sin^2\theta$
4. $\cos 2\theta = 2cos^2\theta - 1$
5. $\cos 2\theta = 1 - 2sin^2\theta$
6. $\cos 2\theta = \frac{1-tan^2\theta}{1+tan^2\theta}$
7. $\tan 2\theta = \frac{2\tan\theta}{1-tan^2\theta}$
8. $\sin 3\theta = 3\sin\theta - 4sin^3\theta$
9. $\cos 3\theta = 4cos^3\theta - 3\cos\theta$
10. $\tan 3\theta = \frac{3\tan\theta - tan^3\theta}{1-3tan^2\theta}$

- **Sums into Product (Factor Form)**

1. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \rightarrow S + S = 2SC$

2. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) \rightarrow S - S = 2CS$

3. $\cos C + cos\ D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \rightarrow C + C = 2CC$

4. $-\cos C + cos\ D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) \rightarrow -C + C = 2SS$

Or

$\cos C - cos\ D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) \rightarrow C - C = -2SS$