Determinant & Matrices

- Second Order Determinant:
- $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

- Example: $\begin{vmatrix} 4 & -3 \\ 1 & 2 \end{vmatrix} = (4)(2) - (-3)(1)$

  $= 8 + 3$

  $= 11$

- Third Order Determinant:

- $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

- Example: $\begin{vmatrix} 2 & 4 & 3 \\ 1 & 0 & -1 \\ 2 & 3 & 5 \end{vmatrix} = 2\begin{vmatrix} 0 & -1 \\ 3 & 5 \end{vmatrix} - 4\begin{vmatrix} 1 & -1 \\ 2 & 5 \end{vmatrix} + 3\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix}$

  $= 2(0 + 3) - 4(5 + 2) + 3(3 - 0)$

  $= 6 - 40 + 9$

  $= -25$

- Matrix: A rectangular array of m rows and n columns enclosed by [ ] is called a matrix of order $m \times n$

- Example: Order of $\begin{bmatrix} 2 & -4 & 1 \\ 6 & 8 & 5 \end{bmatrix}$ is 2×3
- Example: Order of $\begin{bmatrix} 8 & 9 \\ -1 & -6 \end{bmatrix}$ is 2×2

- Example: If $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ and B $= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ then $2A + 3B = \cdots$
- Solution: $2A + 3B = 2\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} + 3\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 9 \\ -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 15 \\ -5 & 0 \end{bmatrix}$$

- Matrix Multiplication:
- If order of matrix A is 4×3 and order of matrix B is 3×5 then order of matrix AB is 4×5
- Example: If $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and B $= \begin{bmatrix} 5 & 1 \\ -2 & 1 \end{bmatrix}$,then AB=…
- Solution: $AB = \begin{bmatrix} (1)(5) + (3)(-2) & (1)(1) + (3)(1) \\ (-2)(5) + (5)(-2) & (-2)(1) + (5)(1) \end{bmatrix} = \begin{bmatrix} 5 - 6 & 1 + 3 \\ -10 - 10 & -2 + 5 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -20 & 3 \end{bmatrix}$

- Adjoint of a matrix:

  If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $adjA = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- Example: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $adjA = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ of a matrix:

- Inverse of a matrix:
- $A^{-1} = \frac{1}{|A|} \cdot \ adjA$
- Example: If $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$, then A^{-1}=….
- Solution: $|A| = 5 + 8 = 13$

$$adjA = \begin{bmatrix} 5 & 2 \\ -4 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \ adjA = \frac{1}{13} \cdot \begin{bmatrix} 5 & 2 \\ -4 & 1 \end{bmatrix}$$