Easy Memorization of Integration Formulae from differentiation:

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ $\qquad\qquad$ $\frac{d}{dx}(x)^n = n \cdot x^{n-1}$

e.g. $\int x^2 \, dx = \frac{x^{2+1}}{2+1} + c$ $\qquad\qquad$ $\frac{d}{dx}(x)^2 = 2 \cdot x^{2-1}$

$= \frac{x^3}{3} + c$ $\qquad\qquad$ $= 2 \cdot x$

$\int x \, dx = \frac{x^2}{2} + c \longleftrightarrow \frac{d}{dx}(x^2) = 2x$

$\int 1 \, dx = x + c \longleftrightarrow \frac{d}{dx}(x) = 1$

$\int \frac{1}{\sqrt{x}} \, dx = 2\sqrt{x} + c \longleftrightarrow \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$\int \frac{1}{x} \, dx = \log|x| + c \longleftrightarrow \frac{d}{dx}(log x) = \frac{1}{x}$

$\int a^x \, dx = \frac{a^x}{\log a} + c \longleftrightarrow \frac{d}{dx}(a^x) = a^x \log_e a$

$\int e^x \, dx = e^x + c \longleftrightarrow \frac{d}{dx}(e^x) = e^x$

Integration and it's Application:

Standard formulae of integration:

1) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ $\Longrightarrow$ $\frac{d}{dx}(x)^n = n \cdot x^{n-1}$

2) $\int \frac{1}{x} dx = log|x| + c$ $\Longrightarrow$ $\frac{d}{dx}(logx) = \frac{1}{x}$

3) $\int 1 \, dx = x + c$ $\Longrightarrow$ $\frac{d}{dx}(x) = 1$

4) $\int e^x \, dx = e^x + c$ $\Longrightarrow$ $\frac{d}{dx}(e^x) = e^x$

5) $\int a^x \, dx = \frac{a^x}{\log a} + c$ $\Longrightarrow$ $\frac{d}{dx}(a^x) = a^x \log_e a$

6) $\int sinx \, dx = -cosx + c$ $\Longrightarrow$ $\frac{d}{dx}(cosx) = -sinx$

7) $\int cosx \, dx = sinx + c$ $\Longrightarrow$ $\frac{d}{dx}(sinx) = cosx$

8) $\int sec^2x \, dx = tanx + c$ $\Longrightarrow$ $\frac{d}{dx}(tanx) = sec^2x$

9) $\int cosec^2x \, dx = -cotx + c$ $\Longrightarrow$ $\frac{d}{dx}(cotx) = -cosec^2x$

10) $\int secx \, tanx \, dx = secx + c$ $\Longrightarrow$ $\frac{d}{dx}(secx) = secx \, tanx$

11) $\int cosecx \, cotx \, dx = -cosecx + c$ $\Longrightarrow$ $\frac{d}{dx}(cosecx) = -cosecx \, cotx$

12) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

13) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

14) $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$

15) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + c$

16) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right| + c$

17) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$

18) $\int \frac{dx}{|x|\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1}\frac{x}{a} + c$

Working Rules of integration:

1. $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

2. $\int k\, f(x)\, dx = k \int f(x)\, dx$, where k is a non-zero constant.

e.g. $I = 3x^2 + 5x - 7$

Sol. $I = \int (3x^2 + 5x - 7)\, dx$

$$= 3 \int x^2\, dx + 5 \int x\, dx - 7 \int 1\, dx$$

$$= 3\, \frac{x^3}{3} + 5\, \frac{x^2}{2} - 7x + c \quad = x^3 + \frac{5}{2}\, x^2 - 7x + c$$

Integration by the method of Substitution:

a) $\int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + c$

e.g. $I = \frac{cosx}{sinx}$

Sol. $I = \int \frac{cosx}{sinx}\, dx$

$= \log|sinx| + c$

b) $\int [f(x)]^n\ f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$

e.g. $I = \int \frac{logx}{x}\, dx$

Sol. let $u = logx\ \therefore \frac{du}{dx} = \frac{1}{x}\ \therefore\ \frac{1}{x}\, dx = du$

$\therefore I = \int u\, du = \frac{u^2}{2} + c$

$\therefore I = \frac{1}{2}\, (logx)^2 + c$

e.g. $I = \int e^{sinx}\ cosx\, dx$

Sol. let $u = sinx\ \therefore\ \frac{du}{dx} = cosx\ \therefore cosx\, dx = du$

$\therefore I = \int e^u\ du = e^u + c\ \therefore I = e^{sinx} + c$

Integration by parts:

$$\boldsymbol{\int u\, v\ dx = u\ \int v\ dx\ -\ \int \left[\frac{du}{dx}\ \int v\ dx\right]\ dx}$$

❖ **Evaluate:** $\boldsymbol{\int x \sin x dx}$

Sol.: I=$\int x \sin x dx$

By **LIATE** Rule**:** x is **Algebraic function** and $\sin x$ is **Trigonometric function**

Let $u = x$ and $v = \sin x$

Using integration by parts, we get

$$\boldsymbol{\int uv\ dx = u \int v\, dx - \int \left[\frac{du}{dx} \int v\ dx\right]\ dx}$$

$$\int x \sin x\ \ dx$$

$$= x\ \int \sin x\ \ dx\ -\ \int \left[\frac{d}{dx}(x)\ \int \sin x\ \ dx\right]\ dx$$

$$= x\ (-\cos x)\ -\ \int [1\ (-\cos x)]\, dx$$

$$= -x \cos x + \int \cos x\ dx$$

$$= -x \cos x + \sin x + c$$

Definite Integral:

$$\boldsymbol{\int_a^b f(x)\ dx = [F(x)]_a^b = F(b) - F(a)}$$

e.g. $\int_2^5 x^2\ dx$

Sol. $I = \int_2^5 x^2\ dx$

$$= \left[\frac{x^3}{3}\right]_2^5$$

$$= \frac{1}{3}\,[5^3 - 2^3]$$

$$= \frac{1}{3}\,[125 - 8] = \frac{117}{3} = 39$$