Function and Limit

Definition of a Function

Let A and B be two non – empty sets. IF every element of set A is related to a unique element of set B, then such a relation is called a function from set A to the set B, and is denoted by the symbol $f: A \to B$.

Example 1,

If $f(x) = e^x$, then prove that:

$(1) f(x+y) = f(x) \cdot f(y)$

$(2) f(x-y) = \frac{f(x)}{f(y)}$

Solution:

Here, $f(x) = e^x$, $f(y) = e^y$

$f(x+y) = e^{x+y}$

$f(x-y) = e^{x-y}$

$(1) f(x+y) = f(x) \cdot f(y)$

L.H.S.

$f(x+y)$

$= e^{x+y}$

$= e^x \cdot e^y$

$= f(x) \cdot f(y)$

R.H.S.

$(2) f(x-y) = \frac{f(x)}{f(y)}$

L.H.S.

$f(x-y)$

$= e^{x-y}$

$= e^x \cdot e^{-y}$

$= \frac{e^x}{e^y}$

$= \frac{f(x)}{f(y)}$

R.H.S.

Limit of a Function

Let $f(x)$ be a function of a real variable x. If the difference between $f(x)$ and a fix value l can be made as small as possible by taking the values of x sufficiently near to a (but not equal to a), then we say that x tends to a , $f(x)$ tend to l . Symbolically, we write $\lim_{x \to a} f(x) = l$.

Properties of Limit

Let f and g be to functions of x , then

(1) $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

(2) $\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

(3) $\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

(4) $\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\left[\lim_{x \to a} f(x)\right]}{\left[\lim_{x \to a} g(x)\right]}; g(x) \neq 0$

(5) $\lim_{x \to a} k = k, k$ is a constant.

(6) $\lim_{x \to a}[kf(x)] = k \lim_{x \to a} f(x), k$ is a constant.

(7) $\lim_{x \to a} \log[f(x)] = \log\left[\lim_{x \to a} f(x)\right]$

Standard forms of Limit

(1) $\lim_{n\to\infty} \frac{1}{n} = 0$

(2) $\lim_{n\to\infty} r^n = 0\ ; where\ \ |\ r\ | < 1, r \in R$

(3) $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}\ , n \in N; x, a \in R (x \neq a)$

(4) $\lim_{x\to 0} \frac{a^x - 1}{x} = \log_e a\ ; a > 0$

(5) $\lim_{x\to 0} \frac{e^x - 1}{x} = \log_e e\ = 1$

(6) $\lim_{n\to 0} (1 + x)^{\frac{1}{x}} = e$

(7) $\lim_{n\to\infty} \left(1 + \frac{1}{x}\right)^x = e$

(8) $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

(9) $\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$

(10) $\lim_{x\to 0} \frac{\sin^{-1} x}{x} = 1$

(11) $\lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1$

Remember

- $\sum 1 = n$
- $\sum n = \frac{n(n+1)}{2}$
- $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum n^3 = \frac{n^2(n+1)^2}{4}$