Assignment 1: CS 215

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Contents

Question 1

(a) The situation is equivalent to distributing n books to n people. The total number of ways of doing that is $n!$. There is only 1 way in which everyone gets his book back. So the probability of it happening is $\boxed{1/n!}$

(b) There is only 1 way of distributing m books to their respective m owners. And for this way there are $(n-m)!$ ways of distributing the left $n-m$ books among left $n-m$ people for a total of $1 \,\text{x}\, (n-m)! = (n-m)!$ ways. So the probability of it happening is $\boxed{(n-m)!/n!}$

(c) There are $m!$ way of distributing the m books belonging to the last m people to the first m people. And for each such way there are $(n-m)!$ ways of distributing the left $n-m$ books among left $n-m$ people for a total of $m! \,\text{x}\, (n-m)! = m!(n-m)!$ ways. So the probability of it happening is $\boxed{m!(n-m)!/n!}$

(d)

Question 2

Question 3

Question 4

Question 5

(a).

$$P(C_1|Z_1) = \tfrac{1}{3}$$
$$P(C_2|Z_1) = \tfrac{1}{3}$$
$$P(C_3|Z_1) = \tfrac{1}{3}$$

(b) $P(H_3|C_1, Z_1) = \frac{1}{2}$ (if car is present behind first door so rest two doors have equal probability)

$P(H_3|C_2, Z_1) = 1$ (if car is present behind door 2, so the host will always pick door 3)

$P(H_3|C_3, Z_1) = 0$ (if car is present behind door 3 , them host will never select it)

(c)

$$P(H_3|C_2, Z_1) = 1$$
$$P(C_2, Z_1) = P(C_2|Z_1) \times P(Z_1) = \tfrac{1}{3} \cdot \tfrac{1}{3} = \tfrac{1}{9}$$
$$P(H_3|Z_1) = \tfrac{1}{3} \cdot \tfrac{1}{2} + \tfrac{1}{3} \cdot 1 + \tfrac{1}{3} \cdot 0 = \tfrac{1}{2}$$

$$\begin{aligned} P(H_3, Z_1) &= P(H_3|Z_1) \times P(Z_1) \\ &= \frac{1}{2} \cdot \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

$$P(C_2|H_3, Z_1) = \frac{P(H_3|C_2, Z_1)P(C_2, Z_1)}{P(H_3, Z_1)}$$
$$= \frac{1 \times \frac{1}{9}}{\frac{1}{6}}$$
$$= \frac{2}{3}$$

(d)

$$P(C_1|H_3, Z_1) = \frac{P(H_3|C_1, Z_1)P(C_1, Z_1)}{P(H_3, Z_1)}$$
$$= \frac{\frac{1}{2} \times \frac{1}{9}}{\frac{1}{6}}$$
$$= \frac{1}{3}$$

(e) $P(C_2|H_3, Z_1) > P(C_1|H_3, Z_1)$

So, Switching will be beneficial

(f)

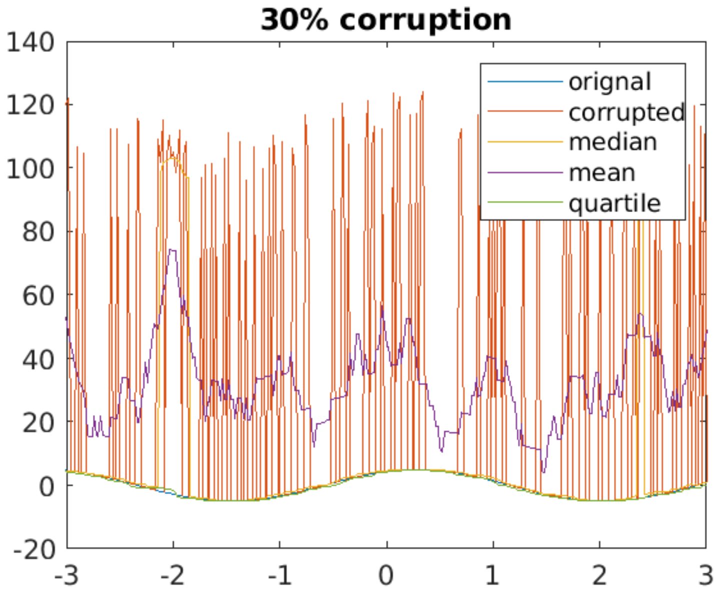
$$\forall_i P(H_3|C_i, Z_1) = \frac{1}{2}$$
$$\forall_i P(C_i, Z_1) = \frac{1}{3}$$
$$P(H_3|Z_1) = \frac{1}{2}$$
$$P(H_3, Z_1) = P(H_3|Z_1) \times P(Z_1) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$P(C_2|H_3, Z_1) = \frac{P(H_3|C_2, Z_1)P(C_2, Z_1)}{P(H_3, Z_1)}$$
$$= \frac{\frac{1}{2} \times \frac{1}{9}}{\frac{1}{6}}$$
$$= \frac{1}{3}$$

$$P(C_1|H_3, Z_1) = \frac{P(H_3|C_1, Z_1)P(C_1, Z_1)}{P(H_3, Z_1)}$$
$$= \frac{\frac{1}{2} \times \frac{1}{9}}{\frac{1}{6}}$$
$$= \frac{1}{3}$$

$P(C_2|H_3, Z_1) = P(C_1|H_3, Z_1)$. So switching won't create any difference.

Question 6


30% corruption
orignal
corrupted
median
mean
quartile
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120
100
80
60
40
20
0
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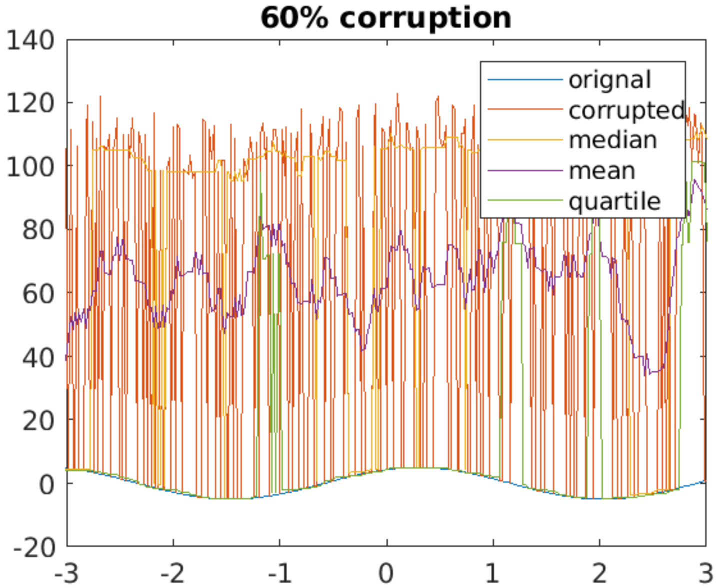
-3
-2
-1
0
1
2
3


60% corruption
orignal
corrupted
median
mean
quartile
140
120
100
80
60
40
20
0
-20
-3
-2
-1
0
1
2
3