(a) $P(\bar{x}) = \frac{1}{2\pi} \frac{1}{|C|^{0.5}} \exp\left(-\frac{1}{2} (\bar{x}-\bar{\mu})^T C^{-1} (\bar{x}-\bar{\mu})\right)$

We know that as C can be represented as $C = QDQ^T$ (spectral thm.)

$$C^{-1} = Q D^{-1} Q^T$$

$$\Rightarrow P(\bar{x}) = \frac{1}{2\pi} \frac{1}{|C|^{0.5}} \exp\left(-\frac{1}{2} (\bar{x}-\bar{\mu})^T Q D^{-1} Q^T (\bar{x}-\bar{\mu})\right)$$

$$\Rightarrow P(\bar{y}) = \frac{1}{2\pi} \frac{1}{|C|^{0.5}} \exp\left(-\frac{1}{2} (y'-\mu')^T D^{-1} (y'-\mu')\right)$$

$$\{ y' = Q^T x ;\ \mu' = Q^T \mu \}$$

$$\Rightarrow \{ x = Q y' ;\ \mu = Q \mu' \}$$

D^{-1} is a diagonal matrix, with entries : $1/\text{eigenvalues}$ $\Rightarrow D = \begin{pmatrix} 1/e_1 & 0 \\ 0 & 1/e_2 \end{pmatrix}$

$$P(\bar{y}) = 1/2\pi \ 1/|C|^{0.5} \exp\left(-\frac{1}{2} \frac{(y'-\mu')_1^2}{e_1}\right) \exp\left(-\frac{1}{2} \frac{(y'-\mu')_2^2}{e_2}\right)$$

~~(as $|C| = e_1 e_2$)~~

$$P(\bar{y}) = \left[\frac{1}{\sqrt{2\pi}\sqrt{e_1}} \exp\left(\frac{-1}{2}\left(\frac{y_1'-\mu_1'}{\sqrt{e_1}}\right)^2\right)\right]\left[\frac{\exp\left(\frac{-1}{2}\left(\frac{y_2'-\mu_2'}{\sqrt{e_2}}\right)^2\right)}{\sqrt{2\pi}\sqrt{e_2}}\right]$$

[So, we just need to pick up 2 random variables, one from $N(\mu_1', \sqrt{e_1})$ other from $N(\mu_2', \sqrt{e_2})$ and then transform $\bar{y}$ to $\bar{x}$ by $\bar{x} = Q\bar{y}$]