Fibonacci morphism.

→ Fibonacci words.

f : letters → strings substitutions.

substitute each letter in a string by the corresponding string

$f(a) = ab$

$f(b) = b$

a is a prefix of $f(a)$.

a
ab
aba
abaab
abaababa

Fibonacci infinite sequence → $\lim_{n\to\infty} f^n(a)$ word.

$$f^n(a) = f^{n-1}(f(a))$$

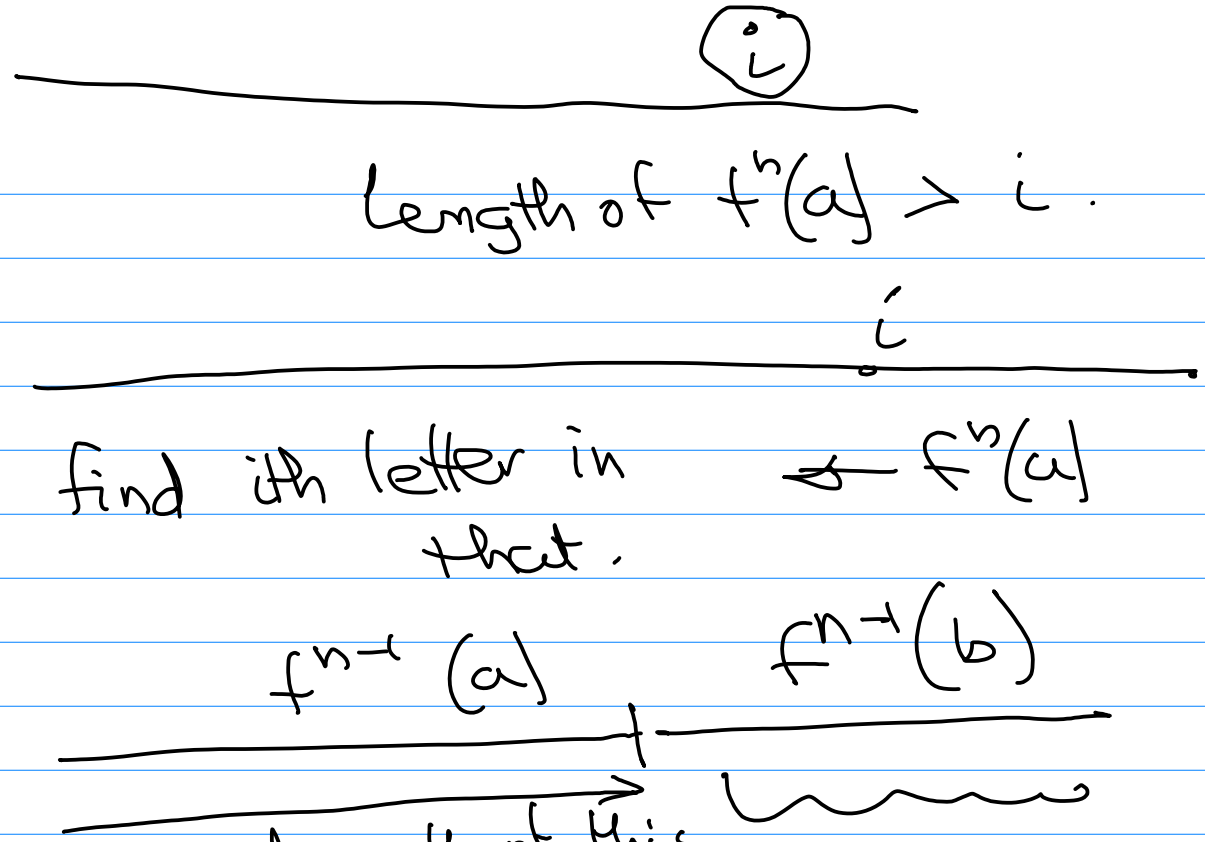
length of $f^n(a)$ → Fibonacci number!

$$= f^{n-1}(ab)$$

$$= f^{n-1}(a) f^{n-1}(b)$$

$$= f^{n-1}(a) f^{n-2}(f(b))$$

$$= f^{n-1}(a) \cdot f^{n-2}(a)$$



$f(a) = abc$ (i) otherwise in left part!

$f^n(a) = f^{n-1}(a) \cdot f^{n-1}(b) \cdot f^{n-1}(c)$.

ith character.

$f(a) = ab$
$f(b) = b$ → different behaviour.

→ lengths grow exponentially in n!

10^{18} character.

→ unsigned long long

→ $2^{64} - 1$

substring $f(a) = ab$

and subsequence

$f(a) = abc$
$f(b) = bc$
$f(c) = bc.$

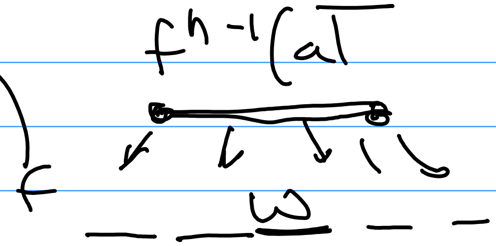
↳ whether a given string is a substring of $f^n(a)$ or not!

lots of search. www.ams.org/– popular articles.

$w.$

$f^n(a)$ for some $\underline{a}$.

substring of $= f(f^{n-1}(a))$



can we do reverse!

from which string could w have been obtained as a substring by applying f!

x aaa
x bb.

$w = \underline{baaba}\,\underline{baaba}\,\underline{baaba}$ valid substring

$f^{-1}(w) = a\,b\,a\,a\,b\,a\,a\,b\,a\,\frac{a}{b}$ or

w $\quad f^n$.

f^{n-1} a b a a b a a b a $\frac{a}{b}$ or

f^{n-2} { a b a b a b a
a b a b a a

a a a

aaba

We these to limit possible inverses.

{ bb is not possible not valid!

aaa is not valid ✗ }

Show that there will be at most two choices in the Fibonacci word!

ICPC World Finals. 2012.

$\ell + \frac{\ell}{\gamma} + \frac{\ell}{\gamma^2} + \cdots$

10^5 length.

$n \leq 100$

$\log_c \rightarrow$ redu

↳ compute number of occurrences of this in $f^n(a)$! Fibonacci word!

↰ length reduces by γ $(1+\sqrt{5})/2$

subsequence is easier!

Fibonacci → every string over $\{a, b\}$ is a subsequence!

infinitely many occurrences of a and b.

abaab

$\underline{a} \rightarrow b \rightarrow a \rightarrow$

<u>shortest prefix.</u>

find longest prefix of this

longest prefix of w which is a subsequence of $f^n(a)$!

$f^{n-1}(a) \cdot f^{n-1}(b)$ subsequence of

required for only two pairs.

finding longest prefix of w in $f^{n-1}(a)$.

a required → char within some small distance

↳ do it recursively for suffixes and characters for which it is required!

assume chars infinitely often!

next character if it exists can be found within a $O(1)$ short distance.

Need to take care of special cases.

growth rate may be polynomial.

Sturmian. $a \to ab$, $b \to bc$, $c \to d$, $d \to c$ } length of $f^n(a)$?

↓

no. of distinct substrings of length n = $n+1$

number of distinct substrings of length n in the infinite word

$\mathcal{O}(1)$ $\mathcal{O}(n)$ $\mathcal{O}(n \log\log n)$ bb

Prove this $\mathcal{O}(n \log n)$ $\mathcal{O}(n^2)$! aaa

for Fibonacci. only a small no. actually occur! $\to 2^n$ not do

use small forbidden substrings to reduce no. of possible inverses!

aba

aa aab
ab aba
ba baa
bab

exactly one of these in both ways

xbb

Recursive definitions & sequences!

No quiz next week!

Midsem
written assignment → on the day
→ describing solution. the exam is scheduled!
open book / ↓
except consultation with 6 hrs.
each other! working ← deadline
no copying on paper! for submission!

won't be needed but can use if you want!