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In[1]:= SetOptions[EvaluationNotebook[], CellContext -> Notebook]

In[2]:= (*****
(*****
(***** Notebook for Bennett Mechanism *****)
(*****
(*****

In[3]:= (*****
(* Absolute Coordinates for each joint axis *)
(*****

In[4]:= r0 = {0, 0, 0};
r1 = {x1, y1, z1};
r2 = {x2, y2, z2};
r3 = {x3, y3, z3};
psi0 = {alpha0, beta0, gamma0};
psi1 = {alpha1, beta1, gamma1};
psi2 = {alpha2, beta2, gamma2};
psi3 = {alpha3, beta3, gamma3};

In[12]:= q = Join[r1, psi1, r2, psi2, r3, psi3];
(*Complete Vector Containing all the system variables*)

In[13]:= (*****
(* Rotational Matrices based on Z-X-Z Euler Angles *)
(*****

In[14]:= R0 = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}};
R1 = {{Cos[alpha1] Cos[gamma1] - Sin[alpha1] Cos[beta1] Sin[gamma1],
Sin[alpha1] Cos[beta1] (-Cos[gamma1]) + Cos[alpha1] (-Sin[gamma1]), Sin[alpha1] Sin[beta1]},
{Cos[alpha1] Cos[beta1] Sin[gamma1] + Sin[alpha1] Cos[gamma1], Cos[alpha1] Cos[beta1] Cos[gamma1] - Sin[alpha1] Sin[gamma1],
Cos[alpha1] (-Sin[beta1])}, {Sin[beta1] Sin[gamma1], Sin[beta1] Cos[gamma1], Cos[beta1]}};
R2 = {{Cos[alpha2] Cos[gamma2] - Sin[alpha2] Cos[beta2] Sin[gamma2],
Sin[alpha2] Cos[beta2] (-Cos[gamma2]) + Cos[alpha2] (-Sin[gamma2]), Sin[alpha2] Sin[beta2]},
{Cos[alpha2] Cos[beta2] Sin[gamma2] + Sin[alpha2] Cos[gamma2], Cos[alpha2] Cos[beta2] Cos[gamma2] - Sin[alpha2] Sin[gamma2],
Cos[alpha2] (-Sin[beta2])}, {Sin[beta2] Sin[gamma2], Sin[beta2] Cos[gamma2], Cos[beta2]}};
R3 = {{Cos[alpha3] Cos[gamma3] - Sin[alpha3] Cos[beta3] Sin[gamma3],
Sin[alpha3] Cos[beta3] (-Cos[gamma3]) + Cos[alpha3] (-Sin[gamma3]), Sin[alpha3] Sin[beta3]},
{Cos[alpha3] Cos[beta3] Sin[gamma3] + Sin[alpha3] Cos[gamma3], Cos[alpha3] Cos[beta3] Cos[gamma3] - Sin[alpha3] Sin[gamma3],
Cos[alpha3] (-Sin[beta3])}, {Sin[beta3] Sin[gamma3], Sin[beta3] Cos[gamma3], Cos[beta3]}};

In[18]:= (*****
(* Positions of various points from different local coordinate systems *)
(*****

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In[19]:= s_A^0 = {a, -1 Cos[θ1], -1 Cos[θ1]};
          s_A^1 = {0, 0, -1};
          s_B^0 = {a, 1 Cos[θ1], 1 Cos[θ1]};
          s_B^1 = {0, 0, 1};
          s_C^1 = {b, -1 Cos[θ2], -1 Cos[θ2]};
          s_C^2 = {0, 0, -1};
          s_D^1 = {b, 1 Cos[θ2], 1 Cos[θ2]};
          s_D^2 = {0, 0, 1};
          s_e^2 = {c, -1 Cos[θ3], -1 Cos[θ3]};
          s_e^3 = {0, 0, -1};
          s_F^2 = {c, 1 Cos[θ3], 1 Cos[θ3]};
          s_F^3 = {0, 0, 1};
          s_G^3 = {d, -1 Cos[θ4], -1 Cos[θ4]};
          s_G^0 = {0, 0, -1};
          s_H^3 = {d, 1 Cos[θ4], 1 Cos[θ4]};
          s_H^0 = {0, 0, 1};

In[35]:= (*****
(* Constraint Equations *)
(*****)

In[36]:= Φ1 = Join[r1 + R1.s_A^1 - r0 - R0.s_A^0, r1 + R1.s_B^1 - r0 - R0.s_B^0];
          Φ2 = Join[r2 + R2.s_C^2 - r1 - R1.s_C^1, r2 + R2.s_D^2 - r1 - R1.s_D^1];
          Φ3 = Join[r3 + R3.s_e^3 - r2 - R2.s_e^2, r3 + R3.s_F^3 - r2 - R2.s_F^2];
          Φ4 = Join[r0 + R0.s_G^0 - r3 - R3.s_G^3, r0 + R0.s_H^0 - r3 - R3.s_H^3];

In[40]:= (*****
(* Complete Constraint Equation *)
(*****)

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In[41]:=  $\Phi = \text{Join}[\Phi^1, \Phi^2, \Phi^3, \Phi^4]; \text{MatrixForm}[\Phi]$ 
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Out[41]//MatrixForm=
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$$\begin{pmatrix} -a - 1 \sin[\alpha_1] \sin[\beta_1] + x_1 & 1 \cos[\theta_1] + 1 \cos[\alpha_1] \sin[\beta_1] \cdot & -1 \cos[\beta_1] + 1 \cos[\theta_1] + z_1 & -a + 1 \sin[\alpha_1] \sin[\beta_1] + x_1 & -1 \cos[\theta_1] - 1 \cos[\alpha_1] \sin[\beta_1] & 1 \cos[\beta_1] - 1 \cos[\theta_1] + z_1 \\ 1 \cos[\theta_2] \sin[\alpha_1] \sin[\beta_1] - 1 \sin[\alpha_2] \sin[\beta_2] + 1 \cos[\theta_2] (-\cos[\beta_1] \cos[\gamma_1] \sin[\alpha_1] - \cos[\alpha_1] & -1 \cos[\alpha_1] \cos[\theta_2] \sin[\beta_1] + 1 \cos[\alpha_2] \sin[\beta_2] - b (\cos[\gamma_1] \sin[\alpha_1] + \cos[\alpha_1] \cos[\beta_1] \sin[\gamma_1] & -1 \cos[\beta_2] + 1 \cos[\beta_1] \cos[\theta_2] + 1 \cos[\gamma_1] \cos[\theta_2] \sin[\beta_1] & -1 \cos[\theta_2] \sin[\alpha_1] \sin[\beta_1] + 1 \sin[\alpha_2] \sin[\beta_2] - 1 \cos[\theta_2] (-\cos[\beta_1] \cos[\gamma_1] \sin[\alpha_1] - \cos[\alpha_1] & 1 \cos[\alpha_1] \cos[\theta_2] \sin[\beta_1] - 1 \cos[\alpha_2] \sin[\beta_2] - b (\cos[\gamma_1] \sin[\alpha_1] + \cos[\alpha_1] \cos[\beta_1] \sin[\gamma_1] & 1 \cos[\beta_2] - 1 \cos[\beta_1] \cos[\theta_2] - 1 \cos[\gamma_1] \cos[\theta_2] \sin[\beta_1] \\ 1 \cos[\theta_3] \sin[\alpha_2] \sin[\beta_2] - 1 \sin[\alpha_3] \sin[\beta_3] + 1 \cos[\theta_3] (-\cos[\beta_2] \cos[\gamma_2] \sin[\alpha_2] - \cos[\alpha_2] & -1 \cos[\alpha_2] \cos[\theta_3] \sin[\beta_2] + 1 \cos[\alpha_3] \sin[\beta_3] - c (\cos[\gamma_2] \sin[\alpha_2] + \cos[\alpha_2] \cos[\beta_2] \sin[\gamma_2] & -1 \cos[\beta_3] + 1 \cos[\beta_2] \cos[\theta_3] + 1 \cos[\gamma_2] \cos[\theta_3] \sin[\beta_2] & -1 \cos[\theta_3] \sin[\alpha_2] \sin[\beta_2] + 1 \sin[\alpha_3] \sin[\beta_3] - 1 \cos[\theta_3] (-\cos[\beta_2] \cos[\gamma_2] \sin[\alpha_2] - \cos[\alpha_2] & 1 \cos[\alpha_2] \cos[\theta_3] \sin[\beta_2] - 1 \cos[\alpha_3] \sin[\beta_3] - c (\cos[\gamma_2] \sin[\alpha_2] + \cos[\alpha_2] \cos[\beta_2] \sin[\gamma_2] & 1 \cos[\beta_3] - 1 \cos[\beta_2] \cos[\theta_3] - 1 \cos[\gamma_2] \cos[\theta_3] \sin[\beta_2] \\ 1 \cos[\theta_4] \sin[\alpha_3] \sin[\beta_3] + 1 \cos[\theta_4] (-\cos[\beta_3] \cos[\gamma_3] \sin[\alpha_3] - \cos[\alpha_3] \sin[\gamma_3] & -1 \cos[\alpha_3] \cos[\theta_4] \sin[\beta_3] - d (\cos[\gamma_3] \sin[\alpha_3] + \cos[\alpha_3] \cos[\beta_3] \sin[\gamma_3]) + 1 \cos[\theta_4] & -1 + 1 \cos[\beta_3] \cos[\theta_4] + 1 \cos[\gamma_3] \cos[\theta_4] \sin[\beta_3] & -1 \cos[\theta_4] \sin[\alpha_3] \sin[\beta_3] - 1 \cos[\theta_4] (-\cos[\beta_3] \cos[\gamma_3] \sin[\alpha_3] - \cos[\alpha_3] \sin[\gamma_3] & 1 \cos[\alpha_3] \cos[\theta_4] \sin[\beta_3] - d (\cos[\gamma_3] \sin[\alpha_3] + \cos[\alpha_3] \cos[\beta_3] \sin[\gamma_3]) - 1 \cos[\theta_4] & 1 - 1 \cos[\beta_3] \cos[\theta_4] - 1 \cos[\gamma_3] \cos[\theta_4] \sin[\beta_3] \cdot \end{pmatrix}$$

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In[42]:= (*****  
(* Jacobian Matrix *)  
(*****)
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In[43]:=  $\Phi_q = D[\Phi, \{q\}]; \text{MatrixForm}[\Phi_q] (* \text{Jacobian Matrix} *)$ 
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Out[43]//MatrixForm=
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$$\begin{pmatrix} 1 & 0 & 0 & -1 \cos[\alpha_1] \sin[\beta_1] \\ 0 & 1 & 0 & -1 \sin[\alpha_1] \sin[\beta_1] \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \cos[\alpha_1] \sin[\beta_1] \\ 0 & 1 & 0 & 1 \sin[\alpha_1] \sin[\beta_1] \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \cos[\alpha_1] \cos[\theta_2] \sin[\beta_1] - b (-\cos[\gamma_1] \sin[\alpha_1] - \cos[\alpha_1] \cos[\beta_1] \sin[\gamma_1]) + 1 C c \\ 0 & -1 & 0 & 1 \cos[\theta_2] \sin[\alpha_1] \sin[\beta_1] + 1 \cos[\theta_2] (-\cos[\beta_1] \cos[\gamma_1] \sin[\alpha_1] - \cos[\alpha_1] \sin[\gamma_1]) \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 \cos[\alpha_1] \cos[\theta_2] \sin[\beta_1] - b (-\cos[\gamma_1] \sin[\alpha_1] - \cos[\alpha_1] \cos[\beta_1] \sin[\gamma_1]) - 1 C c \\ 0 & -1 & 0 & -1 \cos[\theta_2] \sin[\alpha_1] \sin[\beta_1] - 1 \cos[\theta_2] (-\cos[\beta_1] \cos[\gamma_1] \sin[\alpha_1] - \cos[\alpha_1] \sin[\gamma_1]) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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In[44]:= (*****
(* Here Starts the Method-A+ *)
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In[45]:=  $\Phi_q^1 = \Phi_q[\{1, 2, 3, 4, 5, 6\}, \text{All}];$ 
 $\Phi_q^2 = \Phi_q[\{7, 8, 9, 10, 11, 12\}, \text{All}];$ 
 $\Phi_q^3 = \Phi_q[\{13, 14, 15, 16, 17, 18\}, \text{All}];$ 
 $\Phi_q^4 = \Phi_q[\{19, 20, 21, 22, 23, 24\}, \text{All}];$ 
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In[49]:=  $\Phi_q^{-1} = \text{Drop}[\Phi_q, \{1, 6\}, 0];$ 
 $\Phi_q^{-2} = \text{Drop}[\Phi_q, \{7, 12\}, 0];$ 
 $\Phi_q^{-3} = \text{Drop}[\Phi_q, \{13, 18\}, 0];$ 
 $\Phi_q^{-4} = \text{Drop}[\Phi_q, \{19, 24\}, 0];$ 
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In[53]:=  $\text{MatrixRank}[\Phi_q^{-1}]$ 
 $\text{MatrixRank}[\Phi_q]$ 
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Out[53]= 17
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$Aborted (*Aborted Due to RAM overshoot*)
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