```
In[1]:= SetOptions[EvaluationNotebook[], CellContext → Notebook]

In[2]:= (**********************************************)
        (**********************************************)
        (******* Notebook for Bennett Mechanism *******)
        (**********************************************)
        (**********************************************)

In[3]:= (**********************************************)
        (* Absolute Coordinates for each joint axis *)
        (**********************************************)

In[4]:= r0 = {0, 0, 0};
        r1 = {x1, y1, z1};
        r2 = {x2, y2, z2};
        r3 = {x3, y3, z3};
        ψ0 = {α0, β0, γ0};
        ψ1 = {α1, β1, γ1};
        ψ2 = {α2, β2, γ2};
        ψ3 = {α3, β3, γ3};

In[12]:= q = Join[r1, ψ1, r2, ψ2, r3, ψ3];
         (*Complete Vector Containing all the system variables*)

In[13]:= (***************************************************)
         (* Rotational Matrices based on Z-X-Z Euler Angles *)
         (***************************************************)

In[14]:= R0 = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}};
         R1 = {{Cos[α1] Cos[γ1] - Sin[α1] Cos[β1] Sin[γ1],
              Sin[α1] Cos[β1] (-Cos[γ1]) + Cos[α1] (-Sin[γ1]), Sin[α1] Sin[β1]},
             {Cos[α1] Cos[β1] Sin[γ1] + Sin[α1] Cos[γ1], Cos[α1] Cos[β1] Cos[γ1] - Sin[α1] Sin[γ1],
              Cos[α1] (-Sin[β1])}, {Sin[β1] Sin[γ1], Sin[β1] Cos[γ1], Cos[β1]}};
         R2 = {{Cos[α2] Cos[γ2] - Sin[α2] Cos[β2] Sin[γ2],
              Sin[α2] Cos[β2] (-Cos[γ2]) + Cos[α2] (-Sin[γ2]), Sin[α2] Sin[β2]},
             {Cos[α2] Cos[β2] Sin[γ2] + Sin[α2] Cos[γ2], Cos[α2] Cos[β2] Cos[γ2] - Sin[α2] Sin[γ2],
              Cos[α2] (-Sin[β2])}, {Sin[β2] Sin[γ2], Sin[β2] Cos[γ2], Cos[β2]}};
         R3 = {{Cos[α3] Cos[γ3] - Sin[α3] Cos[β3] Sin[γ3],
              Sin[α3] Cos[β3] (-Cos[γ3]) + Cos[α3] (-Sin[γ3]), Sin[α3] Sin[β3]},
             {Cos[α3] Cos[β3] Sin[γ3] + Sin[α3] Cos[γ3], Cos[α3] Cos[β3] Cos[γ3] - Sin[α3] Sin[γ3],
              Cos[α3] (-Sin[β3])}, {Sin[β3] Sin[γ3], Sin[β3] Cos[γ3], Cos[β3]}};

In[18]:= (**********************************************************************)
         (* Positions of various points from different local coordinate systems *)
         (**********************************************************************)
```

In[19]:=
$$s^0_A = \{a, -l\, Cos[\theta_1], -l\, Cos[\theta_1]\};$$
$$s^1_A = \{0, 0, -l\};$$
$$s^0_B = \{a, l\, Cos[\theta_1], l\, Cos[\theta_1]\};$$
$$s^1_B = \{0, 0, l\};$$
$$s^1_C = \{b, -l\, Cos[\theta_2], -l\, Cos[\theta_2]\};$$
$$s^2_C = \{0, 0, -l\};$$
$$s^1_D = \{b, l\, Cos[\theta_2], l\, Cos[\theta_2]\};$$
$$s^2_D = \{0, 0, l\};$$
$$s^2_e = \{c, -l\, Cos[\theta_3], -l\, Cos[\theta_3]\};$$
$$s^3_e = \{0, 0, -l\};$$
$$s^2_F = \{c, l\, Cos[\theta_3], l\, Cos[\theta_3]\};$$
$$s^3_F = \{0, 0, l\};$$
$$s^3_G = \{d, -l\, Cos[\theta_4], -l\, Cos[\theta_4]\};$$
$$s^0_G = \{0, 0, -l\};$$
$$s^3_H = \{d, l\, Cos[\theta_4], l\, Cos[\theta_4]\};$$
$$s^0_H = \{0, 0, l\};$$

In[35]:=
```
(*************************)
(* Constraint Equations *)
(*************************)
```

In[36]:=
$$\Phi^1 = Join\left[r_1 + R_1.s^1_A - r_0 - R_0.s^0_A,\ r_1 + R_1.s^1_B - r_0 - R_0.s^0_B\right];$$
$$\Phi^2 = Join\left[r_2 + R_2.s^2_C - r_1 - R_1.s^1_C,\ r_2 + R_2.s^2_D - r_1 - R_1.s^1_D\right];$$
$$\Phi^3 = Join\left[r_3 + R_3.s^3_e - r_2 - R_2.s^2_e,\ r_3 + R_3.s^3_F - r_2 - R_2.s^2_F\right];$$
$$\Phi^4 = Join\left[r_0 + R_0.s^0_G - r_3 - R_3.s^3_G,\ r_0 + R_0.s^0_H - r_3 - R_3.s^3_H\right];$$

In[40]:=
```
(**********************************)
(* Complete Constraint Equation *)
(**********************************)
```

In[41]:=
```
Φ = Join[Φ^1, Φ^2, Φ^3, Φ^4]; MatrixForm[Φ]
```

Out[41]//MatrixForm=

$$\begin{pmatrix}
-a - l\,Sin[\alpha_1]\,Sin[\beta_1] + x_1 \\
l\,Cos[\theta_1] + l\,Cos[\alpha_1]\,Sin[\beta_1]\ \cdot \\
-l\,Cos[\beta_1] + l\,Cos[\theta_1] + z_1 \\
-a + l\,Sin[\alpha_1]\,Sin[\beta_1] + x_1 \\
-l\,Cos[\theta_1] - l\,Cos[\alpha_1]\,Sin[\beta_1] \\
l\,Cos[\beta_1] - l\,Cos[\theta_1] + z_1 \\
l\,Cos[\theta_2]\,Sin[\alpha_1]\,Sin[\beta_1] - l\,Sin[\alpha_2]\,Sin[\beta_2] + l\,Cos[\theta_2]\,(-Cos[\beta_1]\,Cos[\gamma_1]\,Sin[\alpha_1] - Cos[\alpha_1 \\
-l\,Cos[\alpha_1]\,Cos[\theta_2]\,Sin[\beta_1] + l\,Cos[\alpha_2]\,Sin[\beta_2] - b\,(Cos[\gamma_1]\,Sin[\alpha_1] + Cos[\alpha_1]\,Cos[\beta_1]\,Sin[\gamma_1 \\
-l\,Cos[\beta_2] + l\,Cos[\beta_1]\,Cos[\theta_2] + l\,Cos[\gamma_1]\,Cos[\theta_2]\,Sin[\beta \\
-l\,Cos[\theta_2]\,Sin[\alpha_1]\,Sin[\beta_1] + l\,Sin[\alpha_2]\,Sin[\beta_2] - l\,Cos[\theta_2]\,(-Cos[\beta_1]\,Cos[\gamma_1]\,Sin[\alpha_1] - Cos[\alpha_1 \\
l\,Cos[\alpha_1]\,Cos[\theta_2]\,Sin[\beta_1] - l\,Cos[\alpha_2]\,Sin[\beta_2] - b\,(Cos[\gamma_1]\,Sin[\alpha_1] + Cos[\alpha_1]\,Cos[\beta_1]\,Sin[\gamma_1] \\
l\,Cos[\beta_2] - l\,Cos[\beta_1]\,Cos[\theta_2] - l\,Cos[\gamma_1]\,Cos[\theta_2]\,Sin[\beta_1 \\
l\,Cos[\theta_3]\,Sin[\alpha_2]\,Sin[\beta_2] - l\,Sin[\alpha_3]\,Sin[\beta_3] + l\,Cos[\theta_3]\,(-Cos[\beta_2]\,Cos[\gamma_2]\,Sin[\alpha_2] - Cos[\alpha_2 \\
-l\,Cos[\alpha_2]\,Cos[\theta_3]\,Sin[\beta_2] + l\,Cos[\alpha_3]\,Sin[\beta_3] - c\,(Cos[\gamma_2]\,Sin[\alpha_2] + Cos[\alpha_2]\,Cos[\beta_2]\,Sin[\gamma_2 \\
-l\,Cos[\beta_3] + l\,Cos[\beta_2]\,Cos[\theta_3] + l\,Cos[\gamma_2]\,Cos[\theta_3]\,Sin[\beta \\
-l\,Cos[\theta_3]\,Sin[\alpha_2]\,Sin[\beta_2] + l\,Sin[\alpha_3]\,Sin[\beta_3] - l\,Cos[\theta_3]\,(-Cos[\beta_2]\,Cos[\gamma_2]\,Sin[\alpha_2] - Cos[\alpha_2 \\
l\,Cos[\alpha_2]\,Cos[\theta_3]\,Sin[\beta_2] - l\,Cos[\alpha_3]\,Sin[\beta_3] - c\,(Cos[\gamma_2]\,Sin[\alpha_2] + Cos[\alpha_2]\,Cos[\beta_2]\,Sin[\gamma_2] \\
l\,Cos[\beta_3] - l\,Cos[\beta_2]\,Cos[\theta_3] - l\,Cos[\gamma_2]\,Cos[\theta_3]\,Sin[\beta_2 \\
l\,Cos[\theta_4]\,Sin[\alpha_3]\,Sin[\beta_3] + l\,Cos[\theta_4]\,(-Cos[\beta_3]\,Cos[\gamma_3]\,Sin[\alpha_3] - Cos[\alpha_3]\,Sin[\gamma_3 \\
-l\,Cos[\alpha_3]\,Cos[\theta_4]\,Sin[\beta_3] - d\,(Cos[\gamma_3]\,Sin[\alpha_3] + Cos[\alpha_3]\,Cos[\beta_3]\,Sin[\gamma_3]) + l\,Co \\
-l + l\,Cos[\beta_3]\,Cos[\theta_4] + l\,Cos[\gamma_3]\,Cos[\theta_4]\,Sin[\beta_3] \\
-l\,Cos[\theta_4]\,Sin[\alpha_3]\,Sin[\beta_3] - l\,Cos[\theta_4]\,(-Cos[\beta_3]\,Cos[\gamma_3]\,Sin[\alpha_3] - Cos[\alpha_3]\,Sin[\gamma \\
l\,Cos[\alpha_3]\,Cos[\theta_4]\,Sin[\beta_3] - d\,(Cos[\gamma_3]\,Sin[\alpha_3] + Cos[\alpha_3]\,Cos[\beta_3]\,Sin[\gamma_3]) - l\,Cos \\
l - l\,Cos[\beta_3]\,Cos[\theta_4] - l\,Cos[\gamma_3]\,Cos[\theta_4]\,Sin[\beta_3]\ \cdot
\end{pmatrix}$$

In[42]:=
```
(********************)
(* Jacobian Matrix *)
(********************)
```

In[43]:= Φ_q = D[Φ, {q}]; MatrixForm[Φ_q] (* Jacobian Matrix *)

Out[43]//MatrixForm=

$$\begin{pmatrix}
1 & 0 & 0 & -l\,\mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\beta_1] \\
0 & 1 & 0 & -l\,\mathrm{Sin}[\alpha_1]\,\mathrm{Sin}[\beta_1] \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & l\,\mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\beta_1] \\
0 & 1 & 0 & l\,\mathrm{Sin}[\alpha_1]\,\mathrm{Sin}[\beta_1] \\
0 & 0 & 1 & 0 \\
-1 & 0 & 0 & l\,\mathrm{Cos}[\alpha_1]\,\mathrm{Cos}[\theta_2]\,\mathrm{Sin}[\beta_1] - b\,(-\mathrm{Cos}[\gamma_1]\,\mathrm{Sin}[\alpha_1] - \mathrm{Cos}[\alpha_1]\,\mathrm{Cos}[\beta_1]\,\mathrm{Sin}[\gamma_1]) + l\,\mathrm{Co} \\
0 & -1 & 0 & l\,\mathrm{Cos}[\theta_2]\,\mathrm{Sin}[\alpha_1]\,\mathrm{Sin}[\beta_1] + l\,\mathrm{Cos}[\theta_2]\,(-\mathrm{Cos}[\beta_1]\,\mathrm{Cos}[\gamma_1]\,\mathrm{Sin}[\alpha_1] - \mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\gamma \\
0 & 0 & -1 & 0 \\
-1 & 0 & 0 & -l\,\mathrm{Cos}[\alpha_1]\,\mathrm{Cos}[\theta_2]\,\mathrm{Sin}[\beta_1] - b\,(-\mathrm{Cos}[\gamma_1]\,\mathrm{Sin}[\alpha_1] - \mathrm{Cos}[\alpha_1]\,\mathrm{Cos}[\beta_1]\,\mathrm{Sin}[\gamma_1]) - l\,\mathrm{C} \\
0 & -1 & 0 & -l\,\mathrm{Cos}[\theta_2]\,\mathrm{Sin}[\alpha_1]\,\mathrm{Sin}[\beta_1] - l\,\mathrm{Cos}[\theta_2]\,(-\mathrm{Cos}[\beta_1]\,\mathrm{Cos}[\gamma_1]\,\mathrm{Sin}[\alpha_1] - \mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

In[44]:=
```
(******************************)
(* Here Starts the Method-A+ *)
(******************************)
```

In[45]:=
Φ_q^1 = Φ_q[[{1, 2, 3, 4, 5, 6}, All]];
Φ_q^2 = Φ_q[[{7, 8, 9, 10, 11, 12}, All]];
Φ_q^3 = Φ_q[[{13, 14, 15, 16, 17, 18}, All]];
Φ_q^4 = Φ_q[[{19, 20, 21, 22, 23, 24}, All]];

In[49]:=
Φ_q^{-1} = Drop[Φ_q, {1, 6}, 0];
Φ_q^{-2} = Drop[Φ_q, {7, 12}, 0];
Φ_q^{-3} = Drop[Φ_q, {13, 18}, 0];
Φ_q^{-4} = Drop[Φ_q, {19, 24}, 0];

In[53]:=
MatrixRank[Φ_q^{-1}]
MatrixRank[Φ_q]

Out[53]= 17

$Aborted (*Aborted Due to RAM overshoot*)