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SetOptions[EvaluationNotebook[], CellContext → Notebook]

(*********************************************)
(*********************************************)
(******* Notebook for Bennett Mechanism *******)
(*********************************************)
(*********************************************)

(*Applying Specific Bennnett Conditions*)
a = c = b = d;
θ1 = θ3 = θ; θ2 = θ4 = -θ;

(*********************************************)
(* Absolute Coordinates for each joint axis *)
(*********************************************)

r0 = {0, 0, 0};
r1 = {x1, y1, z1};
r2 = {x2, y2, z2};
r3 = {x3, y3, z3};
ψ0 = {α0, β0, γ0};
ψ1 = {α1, β1, γ1};
ψ2 = {α2, β2, γ2};
ψ3 = {α3, β3, γ3};

Q = Join[r1, ψ1, r2, ψ2, r3, ψ3];
(*Complete Vector Containing all the system variables*)

(***************************************************)
(* Rotational Matrices based on Z-X-Z Euler Angles *)
(***************************************************)

R0 = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}};
R1 = {{Cos[α1] Cos[γ1] - Sin[α1] Cos[β1] Sin[γ1],
     Sin[α1] Cos[β1] (-Cos[γ1]) + Cos[α1] (-Sin[γ1]), Sin[α1] Sin[β1]},
    {Cos[α1] Cos[β1] Sin[γ1] + Sin[α1] Cos[γ1], Cos[α1] Cos[β1] Cos[γ1] - Sin[α1] Sin[γ1],
     Cos[α1] (-Sin[β1])}, {Sin[β1] Sin[γ1], Sin[β1] Cos[γ1], Cos[β1]}};
R2 = {{Cos[α2] Cos[γ2] - Sin[α2] Cos[β2] Sin[γ2],
     Sin[α2] Cos[β2] (-Cos[γ2]) + Cos[α2] (-Sin[γ2]), Sin[α2] Sin[β2]},
    {Cos[α2] Cos[β2] Sin[γ2] + Sin[α2] Cos[γ2], Cos[α2] Cos[β2] Cos[γ2] - Sin[α2] Sin[γ2],
     Cos[α2] (-Sin[β2])}, {Sin[β2] Sin[γ2], Sin[β2] Cos[γ2], Cos[β2]}};
R3 = {{Cos[α3] Cos[γ3] - Sin[α3] Cos[β3] Sin[γ3],
     Sin[α3] Cos[β3] (-Cos[γ3]) + Cos[α3] (-Sin[γ3]), Sin[α3] Sin[β3]},
    {Cos[α3] Cos[β3] Sin[γ3] + Sin[α3] Cos[γ3], Cos[α3] Cos[β3] Cos[γ3] - Sin[α3] Sin[γ3],
     Cos[α3] (-Sin[β3])}, {Sin[β3] Sin[γ3], Sin[β3] Cos[γ3], Cos[β3]}};
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(***************************************************************************)
(* Positions of various points from different local coordinate systems *)
(***************************************************************************)
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s^0_A = {a, -l Cos[θ_1], -l Cos[θ_1]};
s^1_A = {0, 0, -l};
s^0_B = {a, l Cos[θ_1], l Cos[θ_1]};
s^1_B = {0, 0, l};
s^1_C = {b, -l Cos[θ_2], -l Cos[θ_2]};
s^2_C = {0, 0, -l};
s^1_D = {b, l Cos[θ_2], l Cos[θ_2]};
s^2_D = {0, 0, l};
s^2_e = {c, -l Cos[θ_3], -l Cos[θ_3]};
s^3_e = {0, 0, -l};
s^2_F = {c, l Cos[θ_3], l Cos[θ_3]};
s^3_F = {0, 0, l};
s^3_G = {d, -l Cos[θ_4], -l Cos[θ_4]};
s^0_G = {0, 0, -l};
s^3_H = {d, l Cos[θ_4], l Cos[θ_4]};
s^0_H = {0, 0, l};

```
(*************************)
(* Constraint Equations *)
(*************************)
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Φ^1 = Join[$r_1 + R_1.s^1_A - r_0 - R_0.s^0_A$, $r_1 + R_1.s^1_B - r_0 - R_0.s^0_B$];
Φ^2 = Join[$r_2 + R_2.s^2_C - r_1 - R_1.s^1_C$, $r_2 + R_2.s^2_D - r_1 - R_1.s^1_D$];
Φ^3 = Join[$r_3 + R_3.s^3_e - r_2 - R_2.s^2_e$, $r_3 + R_3.s^3_F - r_2 - R_2.s^2_F$];
Φ^4 = Join[$r_0 + R_0.s^0_G - r_3 - R_3.s^3_G$, $r_0 + R_0.s^0_H - r_3 - R_3.s^3_H$];

```
(*********************************)
(* Complete Constraint Equation *)
(*********************************)
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phi = Join[Φ^1, Φ^2, Φ^3, Φ^4]; MatrixForm[Φ]

Φ

```
(*******************)
(* Jacobian Matrix *)
(*******************)
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Φ_q = D[phi, {Q}]; MatrixForm[Φ_q] (* Jacobian Matrix *)
```

$$
\begin{pmatrix}
1 & 0 & 0 & -l\,\mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\beta_1] \\
0 & 1 & 0 & -l\,\mathrm{Sin}[\alpha_1]\,\mathrm{Sin}[\beta_1] \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & l\,\mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\beta_1] \\
0 & 1 & 0 & l\,\mathrm{Sin}[\alpha_1]\,\mathrm{Sin}[\beta_1] \\
0 & 0 & 1 & 0 \\
-1 & 0 & 0 & l\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\beta_1] - d\,(-\mathrm{Cos}[\gamma_1]\,\mathrm{Sin}[\alpha_1] - \mathrm{Cos}[\alpha_1]\,\mathrm{Cos}[\beta_1]\,\mathrm{Sin}[\gamma_1]) + l\,\mathrm{Cos} \\
0 & -1 & 0 & l\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\alpha_1]\,\mathrm{Sin}[\beta_1] + l\,\mathrm{Cos}[\theta]\,(-\mathrm{Cos}[\beta_1]\,\mathrm{Cos}[\gamma_1]\,\mathrm{Sin}[\alpha_1] - \mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\gamma_1] \\
0 & 0 & -1 & 0 \\
-1 & 0 & 0 & -l\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\beta_1] - d\,(-\mathrm{Cos}[\gamma_1]\,\mathrm{Sin}[\alpha_1] - \mathrm{Cos}[\alpha_1]\,\mathrm{Cos}[\beta_1]\,\mathrm{Sin}[\gamma_1]) - l\,\mathrm{Co} \\
0 & -1 & 0 & -l\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\alpha_1]\,\mathrm{Sin}[\beta_1] - l\,\mathrm{Cos}[\theta]\,(-\mathrm{Cos}[\beta_1]\,\mathrm{Cos}[\gamma_1]\,\mathrm{Sin}[\alpha_1] - \mathrm{Cos}[\alpha_1]\,\mathrm{Sin}[\gamma_1 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
$$

```
Dimensions[%45]
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{24, 18}

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(******************************)
(* Here Starts the Method-A+ *)
(******************************)

Φ_q^1 = Φ_q[[{1, 2, 3, 4, 5, 6}, All]];
Φ_q^2 = Φ_q[[{7, 8, 9, 10, 11, 12}, All]];
Φ_q^3 = Φ_q[[{13, 14, 15, 16, 17, 18}, All]];
Φ_q^4 = Φ_q[[{19, 20, 21, 22, 23, 24}, All]];

Φ_q^-1 = Drop[Φ_q, {1, 6}, 0];
Φ_q^-2 = Drop[Φ_q, {7, 12}, 0];
Φ_q^-3 = Drop[Φ_q, {13, 18}, 0];
Φ_q^-4 = Drop[Φ_q, {19, 24}, 0];
```

```
MatrixRank[Φq^-1]
MatrixRank[Φq^-2]
MatrixRank[Φq^-3]
(*MatrixRank[Φq^-4]*)
```

17

16

16

```
MatrixRank[Φq^1]
MatrixRank[Φq^2]
MatrixRank[Φq^3]
MatrixRank[Φq^4]
```

5

6

6

5

```
(*Giving specific numerical values to position coordinates*)
{x1, y1, z1} = {10, 0, 0};
{x2, y2, z2} = {5, 8.138, -2.962};
{x3, y3, z3} = {-4.519, 8.921, 0};
{α1, β1, γ1} = {240, 20, 0} * Pi / 180;
{α2, β2, γ2} = {148.4, 34.46, 31.57} * Pi / 180;
{α3, β3, γ3} = {0, 20, 63.13} * Pi / 180;
```

```
MatrixRank[Φq^1]
MatrixRank[Φq^2]
MatrixRank[Φq^3]
MatrixRank[Φq^4]
```

5

6

6

5

```
MatrixRank[Φq^-1]
MatrixRank[Φq^-2]
MatrixRank[Φq^-3]
MatrixRank[Φq^-4]
```

$\mathbf{MatrixRank}[\Phi_q^{-1}]$
$\mathbf{MatrixRank}[\Phi_q^{-2}]$
$\mathbf{MatrixRank}[\Phi_q^{-3}]$
$\mathbf{MatrixRank}[\Phi_q^{-4}]$

17

16

16

15

$\mathbf{MatrixRank}[\Phi_q]$

18