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(*****)
(* Bennett Mechanism Reaction Solvability Analysis (RSA) *)
(* Using Natural Coordinates *)
(*****)
SetOptions[EvaluationNotebook[], CellContext -> Notebook]

MyNorm[a_] := a[[1]]^2 + a[[2]]^2 + a[[3]]^2

Do[{rj = {xj, yj, zj}}, {j, 8}]
Do[{uj = {ej, fj, gj}}, {j, 8}]
Q = Join[r1, r2, r3, r4, r5, r6, r7, r8, u1, u2, u3, u4, u5, u6, u7, u8];

(*Constraint Equations*)
R1 = Join[r2 - r3, u2 - u3];
R2 = Join[r4 - r5, u4 - u5];
R3 = Join[r6 - r7, u6 - u7];
R4 = Join[r8 - r1, u8 - u1];
ϕ = Join[R1, R2, R3, R4];
(*Adding 8 Unit vector constraints*)
Do[{ϕ = Join[ϕ, {MyNorm[uj] - 1}]}, {j, 8}]

(*Adding Rigid link constraints*)
Do[{ϕ = Join[ϕ, {(rj+1 - rj).uj}]}, {j, 1, 7, 2}]
Do[{ϕ = Join[ϕ, {(rj+1 - rj).uj+1}]}, {j, 1, 7, 2}]
Do[{ϕ = Join[ϕ, {MyNorm[rj+1 - rj] - a^2}]}, {j, 1, 7, 2}]
Do[{ϕ = Join[ϕ, {uj.uj+1 - Cos[αj]}]}, {j, 1, 7, 2}]

(*Adding Fixind(Ground) constraints*)
ϕ = Join[ϕ, r1];
ϕ = Join[ϕ, r2 - {p, q, t}];

ϕ // MatrixForm

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$$\begin{pmatrix} x_2 - x_3 \\ y_2 - y_3 \\ z_2 - z_3 \\ e_2 - e_3 \\ f_2 - f_3 \\ g_2 - g_3 \\ x_4 - x_5 \\ y_4 - y_5 \\ z_4 - z_5 \\ e_4 - e_5 \\ f_4 - f_5 \\ g_4 - g_5 \\ x_6 - x_7 \\ y_6 - y_7 \\ z_6 - z_7 \end{pmatrix}$$

$$\begin{aligned}
& \mathbf{e}_6 - \mathbf{e}_7 \\
& \mathbf{f}_6 - \mathbf{f}_7 \\
& \mathbf{g}_6 - \mathbf{g}_7 \\
& -\mathbf{x}_1 + \mathbf{x}_8 \\
& -\mathbf{y}_1 + \mathbf{y}_8 \\
& -\mathbf{z}_1 + \mathbf{z}_8 \\
& -\mathbf{e}_1 + \mathbf{e}_8 \\
& -\mathbf{f}_1 + \mathbf{f}_8 \\
& -\mathbf{g}_1 + \mathbf{g}_8 \\
& -1 + \mathbf{e}_1^2 + \mathbf{f}_1^2 + \mathbf{g}_1^2 \\
& -1 + \mathbf{e}_2^2 + \mathbf{f}_2^2 + \mathbf{g}_2^2 \\
& -1 + \mathbf{e}_3^2 + \mathbf{f}_3^2 + \mathbf{g}_3^2 \\
& -1 + \mathbf{e}_4^2 + \mathbf{f}_4^2 + \mathbf{g}_4^2 \\
& -1 + \mathbf{e}_5^2 + \mathbf{f}_5^2 + \mathbf{g}_5^2 \\
& -1 + \mathbf{e}_6^2 + \mathbf{f}_6^2 + \mathbf{g}_6^2 \\
& -1 + \mathbf{e}_7^2 + \mathbf{f}_7^2 + \mathbf{g}_7^2 \\
& -1 + \mathbf{e}_8^2 + \mathbf{f}_8^2 + \mathbf{g}_8^2 \\
& \mathbf{e}_1 (-\mathbf{x}_1 + \mathbf{x}_2) + \mathbf{f}_1 (-\mathbf{y}_1 + \mathbf{y}_2) + \mathbf{g}_1 (-\mathbf{z}_1 + \mathbf{z}_2) \\
& \mathbf{e}_3 (-\mathbf{x}_3 + \mathbf{x}_4) + \mathbf{f}_3 (-\mathbf{y}_3 + \mathbf{y}_4) + \mathbf{g}_3 (-\mathbf{z}_3 + \mathbf{z}_4) \\
& \mathbf{e}_5 (-\mathbf{x}_5 + \mathbf{x}_6) + \mathbf{f}_5 (-\mathbf{y}_5 + \mathbf{y}_6) + \mathbf{g}_5 (-\mathbf{z}_5 + \mathbf{z}_6) \\
& \mathbf{e}_7 (-\mathbf{x}_7 + \mathbf{x}_8) + \mathbf{f}_7 (-\mathbf{y}_7 + \mathbf{y}_8) + \mathbf{g}_7 (-\mathbf{z}_7 + \mathbf{z}_8) \\
& \mathbf{e}_2 (-\mathbf{x}_1 + \mathbf{x}_2) + \mathbf{f}_2 (-\mathbf{y}_1 + \mathbf{y}_2) + \mathbf{g}_2 (-\mathbf{z}_1 + \mathbf{z}_2) \\
& \mathbf{e}_4 (-\mathbf{x}_3 + \mathbf{x}_4) + \mathbf{f}_4 (-\mathbf{y}_3 + \mathbf{y}_4) + \mathbf{g}_4 (-\mathbf{z}_3 + \mathbf{z}_4) \\
& \mathbf{e}_6 (-\mathbf{x}_5 + \mathbf{x}_6) + \mathbf{f}_6 (-\mathbf{y}_5 + \mathbf{y}_6) + \mathbf{g}_6 (-\mathbf{z}_5 + \mathbf{z}_6) \\
& \mathbf{e}_8 (-\mathbf{x}_7 + \mathbf{x}_8) + \mathbf{f}_8 (-\mathbf{y}_7 + \mathbf{y}_8) + \mathbf{g}_8 (-\mathbf{z}_7 + \mathbf{z}_8) \\
& -\mathbf{a}^2 + (-\mathbf{x}_1 + \mathbf{x}_2)^2 + (-\mathbf{y}_1 + \mathbf{y}_2)^2 + (-\mathbf{z}_1 + \mathbf{z}_2)^2 \\
& -\mathbf{a}^2 + (-\mathbf{x}_3 + \mathbf{x}_4)^2 + (-\mathbf{y}_3 + \mathbf{y}_4)^2 + (-\mathbf{z}_3 + \mathbf{z}_4)^2 \\
& -\mathbf{a}^2 + (-\mathbf{x}_5 + \mathbf{x}_6)^2 + (-\mathbf{y}_5 + \mathbf{y}_6)^2 + (-\mathbf{z}_5 + \mathbf{z}_6)^2 \\
& -\mathbf{a}^2 + (-\mathbf{x}_7 + \mathbf{x}_8)^2 + (-\mathbf{y}_7 + \mathbf{y}_8)^2 + (-\mathbf{z}_7 + \mathbf{z}_8)^2 \\
& -\text{Cos}[\alpha_1] + \mathbf{e}_1 \mathbf{e}_2 + \mathbf{f}_1 \mathbf{f}_2 + \mathbf{g}_1 \mathbf{g}_2 \\
& -\text{Cos}[\alpha_3] + \mathbf{e}_3 \mathbf{e}_4 + \mathbf{f}_3 \mathbf{f}_4 + \mathbf{g}_3 \mathbf{g}_4 \\
& -\text{Cos}[\alpha_5] + \mathbf{e}_5 \mathbf{e}_6 + \mathbf{f}_5 \mathbf{f}_6 + \mathbf{g}_5 \mathbf{g}_6 \\
& -\text{Cos}[\alpha_7] + \mathbf{e}_7 \mathbf{e}_8 + \mathbf{f}_7 \mathbf{f}_8 + \mathbf{g}_7 \mathbf{g}_8 \\
& \mathbf{x}_1 \\
& \mathbf{y}_1 \\
& \mathbf{z}_1 \\
& -\mathbf{p} + \mathbf{x}_2 \\
& -\mathbf{q} + \mathbf{y}_2 \\
& -\mathbf{t} + \mathbf{z}_2
\end{aligned}$$

$$J = D[\Phi, \{Q\}] \quad (* \text{ Jacobian Matrix } *)$$
$$\begin{aligned} &\{0, 0, 0, 1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \\ &\quad 0, 0\}, \\ &\{0, 0, 0, 0, 1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \\ &\quad 0, 0\}, \\ &\{0, 0, 0, 0, 0, 1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \\ &\quad 0, 0\}, \\ &\{0, \\ &\quad 0, 0, 1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ &\{0, \\ &\quad 0, 0, 0, 1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \end{aligned}$$

[illegible]

[illegible]

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{0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
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MatrixRank[J]
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```
48
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Dimensions[J]
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{54, 48}
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```
RSA[cList_] := (Ji = J[[cList, All]]; J-i = Complement[J, Ji];
  Print["Ji = ", MatrixRank[Ji], " and J-i = ", MatrixRank[J-i]]);
```

```
RSA[{1, 2, 3, 4, 5, 6}]
```

```
Ji = 6 and J-i = 47
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