

```

(*****)
(*Dynamics of A Planar Mechanism*)
(*****)
SetOptions[EvaluationNotebook[], CellContext -> Notebook]

(*Defining Absolute Coordinates*)
Do[{rj = {xj[t], yj[t]}}, {j, 4}]
q = Join[r1, {θ1[t]}, r2, {θ2[t]}, r3, {θ3[t]}, r4, {θ4[t]}]
{x1[t], y1[t], θ1[t], x2[t], y2[t], θ2[t], x3[t], y3[t], θ3[t], x4[t], y4[t], θ4[t]}

(*Defining Constraints*)
Φ = {y1[t], θ1[t], x2[t], θ2[t], x2[t] - x1[t] + y2[t] - y1[t],
    θ2[t] - θ1[t], x1[t] - x3[t], y1[t] - y3[t], x2[t] - x4[t], y2[t] - y4[t],
    x3[t] - Sin[θ3[t]] - x4[t] - Cos[θ4[t]], y3[t] + Cos[θ3[t]] - y4[t] - Sin[θ4[t]]}
{y1[t], θ1[t], x2[t], θ2[t], -x1[t] + x2[t] - y1[t] + y2[t],
    -θ1[t] + θ2[t], x1[t] - x3[t], y1[t] - y3[t], x2[t] - x4[t], y2[t] - y4[t],
    -Cos[θ4[t]] - Sin[θ3[t]] + x3[t] - x4[t], Cos[θ3[t]] - Sin[θ4[t]] + y3[t] - y4[t]}

```

```

MatrixForm[%]

```

$$\begin{pmatrix} y_1[t] \\ \theta_1[t] \\ x_2[t] \\ \theta_2[t] \\ -x_1[t] + x_2[t] - y_1[t] + y_2[t] \\ -\theta_1[t] + \theta_2[t] \\ x_1[t] - x_3[t] \\ y_1[t] - y_3[t] \\ x_2[t] - x_4[t] \\ y_2[t] - y_4[t] \\ -\cos[\theta_4[t]] - \sin[\theta_3[t]] + x_3[t] - x_4[t] \\ \cos[\theta_3[t]] - \sin[\theta_4[t]] + y_3[t] - y_4[t] \end{pmatrix}$$

```

J = D[Φ, {q}] (* Jacobian Matrix *)

```

```

{{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
 {-1, -1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
 {1, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, -Cos[θ3[t]], -1, 0, Sin[θ4[t]]},
 {0, 0, 0, 0, 0, 0, 0, 1, -Sin[θ3[t]], 0, -1, -Cos[θ4[t]]}}

```

```

q' = D[q, t]

```

```

{x1'[t], y1'[t], θ1'[t], x2'[t], y2'[t], θ2'[t], x3'[t], y3'[t], θ3'[t], x4'[t], y4'[t], θ4'[t]}

```

$$l_1 = l_2 = 1 / 2;$$

$$l_3 = l_4 = 1;$$

$$m = 1;$$

$$(*\theta_3=\theta_4=0;*)$$

$$M_1 = \left\{ \{m, 0, 0\}, \{0, m, 0\}, \left\{0, 0, \frac{m l_1^2}{12}\right\} \right\}$$

$$M_2 = \left\{ \{m, 0, 0\}, \{0, m, 0\}, \left\{0, 0, \frac{m l_2^2}{12}\right\} \right\}$$

$$M_3 = \left\{ \left\{m, 0, \frac{1}{2} (-m) l_3 \sin[\theta_3[t]]\right\}, \left\{0, m, \frac{1}{2} m l_3 \cos[\theta_3[t]]\right\}, \right. \\ \left. \left\{\frac{1}{2} (-m) l_3 \sin[\theta_3[t]], \frac{1}{2} m l_3 \cos[\theta_3[t]], \frac{m l_3^2}{3}\right\} \right\}$$

$$M_4 = \left\{ \left\{m, 0, \frac{1}{2} (-m) l_4 \sin[\theta_4[t]]\right\}, \left\{0, m, \frac{1}{2} m l_4 \cos[\theta_4[t]]\right\}, \right. \\ \left. \left\{\frac{1}{2} (-m) l_4 \sin[\theta_4[t]], \frac{1}{2} m l_4 \cos[\theta_4[t]], \frac{m l_4^2}{3}\right\} \right\}$$

$$\left\{ \{1, 0, 0\}, \{0, 1, 0\}, \left\{0, 0, \frac{1}{48}\right\} \right\}$$

$$\left\{ \{1, 0, 0\}, \{0, 1, 0\}, \left\{0, 0, \frac{1}{48}\right\} \right\}$$

$$\left\{ \left\{1, 0, -\frac{1}{2} \sin[\theta_3[t]]\right\}, \left\{0, 1, \frac{1}{2} \cos[\theta_3[t]]\right\}, \left\{-\frac{1}{2} \sin[\theta_3[t]], \frac{1}{2} \cos[\theta_3[t]], \frac{1}{3}\right\} \right\}$$

$$\left\{ \left\{1, 0, -\frac{1}{2} \sin[\theta_4[t]]\right\}, \left\{0, 1, \frac{1}{2} \cos[\theta_4[t]]\right\}, \left\{-\frac{1}{2} \sin[\theta_4[t]], \frac{1}{2} \cos[\theta_4[t]], \frac{1}{3}\right\} \right\}$$

$$M = \text{ArrayFlatten}[\{\{M_1, 0, 0, 0\}, \{0, M_2, 0, 0\}, \{0, 0, M_3, 0\}, \{0, 0, 0, M_4\}\}]$$

$$\left\{ \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \right.$$

$$\left\{0, 0, \frac{1}{48}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{0, 0, 0, 0, 0, \frac{1}{48}, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 1, 0, -\frac{1}{2} \sin[\theta_3[t]], 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 1, \frac{1}{2} \cos[\theta_3[t]], 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, -\frac{1}{2} \sin[\theta_3[t]], \frac{1}{2} \cos[\theta_3[t]], \frac{1}{3}, 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, -\frac{1}{2} \sin[\theta_4[t]], 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, \frac{1}{2} \cos[\theta_4[t]], 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \sin[\theta_4[t]], \frac{1}{2} \cos[\theta_4[t]], \frac{1}{3}\right\} \right\}$$

```
MatrixRank[J]
```

```
11
```

```
(*RowReduce[J] //MatrixForm*)
```

```
J = Drop[J, {2}, 0] (*Eliminating a dependent constraint, arbitrarily*)
```

```
{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
{-1, -1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 1, 0, 0, 0, 0, 0, 0},
{1, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, -1, 0},
{0, 0, 0, 0, 0, 0, 1, 0, -Cos[θ3[t]], -1, 0, Sin[θ4[t]]},
{0, 0, 0, 0, 0, 0, 0, 1, -Sin[θ3[t]], 0, -1, -Cos[θ4[t]]}
```

```
JIn = PseudoInverse[J]
```

A very large output was generated. Here is a sample of it:

```
{ <<1>> }
```

Show Less

Show More

Show Full Output

Set Size Limit...

```
Qe = {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
q' = D[q, t]
```

```
{x1'[t], y1'[t], θ1'[t], x2'[t], y2'[t], θ2'[t], x3'[t], y3'[t], θ3'[t], x4'[t], y4'[t], θ4'[t]}
```

```
Qd = - (∂{q} (J . q')) . q'
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, -Sin[θ3[t]] θ3'[t]2 - Cos[θ4[t]] θ4'[t]2,
Cos[θ3[t]] θ3'[t]2 - Sin[θ4[t]] θ4'[t]2}}
```

MIn = Inverse[M]

$$\begin{aligned}
& \left\{ \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \right. \\
& \{0, 0, 48, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 48, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, \frac{\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2}{\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2}, \right. \\
& - \frac{\cos[\theta_3[t]] \sin[\theta_3[t]]}{4 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2 \right)}, \frac{\sin[\theta_3[t]]}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2 \right)}, 0, 0, 0 \Big\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, - \frac{\cos[\theta_3[t]] \sin[\theta_3[t]]}{4 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2 \right)}, \right. \\
& \frac{\frac{1}{3} - \frac{1}{4} \sin[\theta_3[t]]^2}{\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2}, - \frac{\cos[\theta_3[t]]}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2 \right)}, 0, 0, 0 \Big\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, \frac{\sin[\theta_3[t]]}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2 \right)}, \right. \\
& - \frac{\cos[\theta_3[t]]}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2 \right)}, \frac{1}{\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2}, 0, 0, 0 \Big\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2}{\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2}, \right. \\
& - \frac{\cos[\theta_4[t]] \sin[\theta_4[t]]}{4 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)}, \frac{\sin[\theta_4[t]]}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)} \Big\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, - \frac{\cos[\theta_4[t]] \sin[\theta_4[t]]}{4 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)}, \right. \\
& \frac{\frac{1}{3} - \frac{1}{4} \sin[\theta_4[t]]^2}{\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2}, - \frac{\cos[\theta_4[t]]}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)} \Big\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\sin[\theta_4[t]]}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)}, \right. \\
& - \frac{\cos[\theta_4[t]]}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)}, \frac{1}{\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2} \Big\} \Big\}
\end{aligned}$$

Jt = Transpose[J]

$$\begin{aligned}
& \{ \{0, 0, 0, -1, 0, 1, 0, 0, 0, 0, 0\}, \{1, 0, 0, -1, 0, 0, 1, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0\}, \\
& \{0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, -1, 0, 0, 0, 1, 0\}, \{0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 1\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, -\cos[\theta_3[t]], -\sin[\theta_3[t]]\}, \\
& \{0, 0, 0, 0, 0, 0, 0, -1, 0, -1, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, -1, 0, -1\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, \sin[\theta_4[t]], -\cos[\theta_4[t]]\} \}
\end{aligned}$$

JMinJt = Inverse[J.Min.Jt]

A very large output was generated. Here is a sample of it:

$$\left\{ \left\{ \frac{1}{\langle\langle 1 \rangle\rangle} \left(- \left(- \frac{\cos[\theta_3[t]] \sin[\theta_3[t]]}{4 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_3[t]]^2 - \frac{1}{4} \sin[\theta_3[t]]^2 \right)} - \frac{\sin[\theta_3[t]]^2}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\langle\langle 1 \rangle\rangle]^2 - \frac{1}{4} \sin[\theta_3[t]]^2 \right)} - \right. \right. \right. \\ \left. \cos[\theta_3[t]] \left(- \frac{\cos[\theta_3[t]]}{2 \left(\frac{1}{3} - \langle\langle 1 \rangle\rangle - \frac{1}{4} \langle\langle 1 \rangle\rangle^2 \right)} - \frac{\sin[\langle\langle 1 \rangle\rangle]}{\frac{1}{3} - \langle\langle 1 \rangle\rangle - \langle\langle 1 \rangle\rangle} \right) - \right. \\ \left. \left. \frac{\cos[\theta_4[t]] \sin[\theta_4[t]]}{4 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)} \right) (\langle\langle 1 \rangle\rangle) + \langle\langle 43 \rangle\rangle + \langle\langle 1 \rangle\rangle \right\}, \\ \left. - \frac{\cos[\theta_4[t]] \sin[\theta_4[t]] (\langle\langle 1 \rangle\rangle)}{4 \left(\frac{1}{3} - \frac{1}{4} \cos[\langle\langle 1 \rangle\rangle]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)} + \langle\langle 24 \rangle\rangle + \left(- \frac{\cos[\langle\langle 1 \rangle\rangle]^2}{2 \left(\frac{1}{3} - \langle\langle 1 \rangle\rangle - \frac{1}{4} \langle\langle 1 \rangle\rangle} \right)} + \frac{\langle\langle 1 \rangle\rangle - \langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} \right) (\langle\langle 1 \rangle\rangle) \right\}, \\ 0, \\ \left. - \left(- \frac{\langle\langle 1 \rangle\rangle^2}{2 (\langle\langle 1 \rangle\rangle)} + \frac{\frac{1}{3} - \langle\langle 1 \rangle\rangle}{\frac{1}{3} - \langle\langle 1 \rangle\rangle - \langle\langle 1 \rangle\rangle} \right) (\langle\langle 1 \rangle\rangle) + \langle\langle 23 \rangle\rangle \right\}, \\ 0, \\ \left. \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, \right. \\ \left. \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, \right. \\ \left. \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, \right. \\ \left. \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, \right. \\ \left. \frac{\langle\langle 53 \rangle\rangle + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}}{\langle\langle 1 \rangle\rangle}, \right. \\ \left. \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, \right. \\ \left. \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} \right\}, \\ \langle\langle 9 \rangle\rangle, \left\{ \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, \langle\langle 8 \rangle\rangle, \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} \right\} \}$$

Show Less Show More Show Full Output Set Size Limit...

(*q''=Min.Qe+Min.Jt.(JMinJt.(Qd-J.Min.Qe))*)


```

s = NDSolve[{eqs, x1[0] == 1, y1[0] == 0,  $\theta_1$ [0] == 0, x1'[0] == 0, y1'[0] == 0,
   $\theta_1'$ [0] == 0, x2[0] == 0, y2[0] == 1,  $\theta_2$ [0] == 0, x2'[0] == 0, y2'[0] == 0,  $\theta_2'$ [0] == 0,
  x3[0] == 1, y3[0] == 0,  $\theta_3$ [0] == 0, x3'[0] == 0, y3'[0] == 0,  $\theta_3'$ [0] == 0,
  x4[0] == 0, y4[0] == 1,  $\theta_4$ [0] == 0, x4'[0] == 0, y4'[0] == 0,  $\theta_4'$ [0] == 0},
  {x1, y1,  $\theta_1$ , x2, y2,  $\theta_2$ , x3, y3,  $\theta_3$ , x4, y4,  $\theta_4$ , x1', y1',  $\theta_1'$ , x2', y2',  $\theta_2'$ , x3', y3',
   $\theta_3'$ , x4', y4',  $\theta_4'$ }, {t, 0, 5}, Method -> {"EquationSimplification" -> "Solve"}]

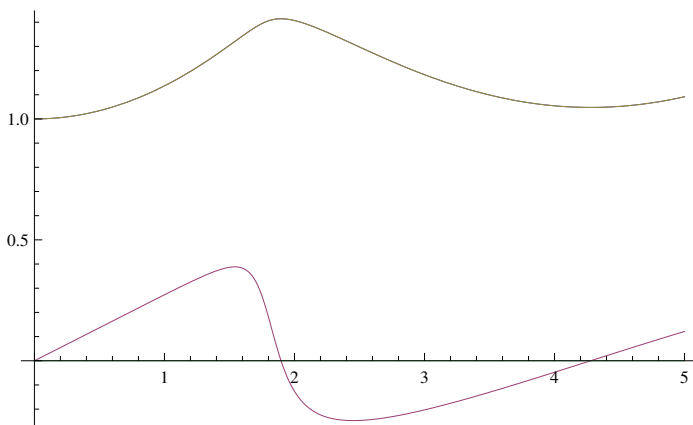
{{x1 -> InterpolatingFunction[{{0., 5.}}, <>],
  y1 -> InterpolatingFunction[{{0., 5.}}, <>],
   $\theta_1$  -> InterpolatingFunction[{{0., 5.}}, <>],
  x2 -> InterpolatingFunction[{{0., 5.}}, <>],
  y2 -> InterpolatingFunction[{{0., 5.}}, <>],
   $\theta_2$  -> InterpolatingFunction[{{0., 5.}}, <>],
  x3 -> InterpolatingFunction[{{0., 5.}}, <>],
  y3 -> InterpolatingFunction[{{0., 5.}}, <>],
   $\theta_3$  -> InterpolatingFunction[{{0., 5.}}, <>],
  x4 -> InterpolatingFunction[{{0., 5.}}, <>],
  y4 -> InterpolatingFunction[{{0., 5.}}, <>],
   $\theta_4$  -> InterpolatingFunction[{{0., 5.}}, <>],
  x1' -> InterpolatingFunction[{{0., 5.}}, <>],
  y1' -> InterpolatingFunction[{{0., 5.}}, <>],
   $\theta_1'$  -> InterpolatingFunction[{{0., 5.}}, <>],
  x2' -> InterpolatingFunction[{{0., 5.}}, <>],
  y2' -> InterpolatingFunction[{{0., 5.}}, <>],
   $\theta_2'$  -> InterpolatingFunction[{{0., 5.}}, <>],
  x3' -> InterpolatingFunction[{{0., 5.}}, <>],
  y3' -> InterpolatingFunction[{{0., 5.}}, <>],
   $\theta_3'$  -> InterpolatingFunction[{{0., 5.}}, <>],
  x4' -> InterpolatingFunction[{{0., 5.}}, <>],
  y4' -> InterpolatingFunction[{{0., 5.}}, <>],
   $\theta_4'$  -> InterpolatingFunction[{{0., 5.}}, <>]}}

```

```

Plot[Evaluate[{x3[t], x3'[t], y2[t],  $\theta_1$ [t]} /. s], {t, 0, 5}, PlotStyle -> Automatic]

```



```

θ3[2] /. s
J /. t → 2 /. s
λ = JMinJt.(Qd - J.Min.Qe)
{0.886271}

{{{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {-1, -1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {1, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, -0.632306, -1, 0, -0.774719},
 {0, 0, 0, 0, 0, 0, 0, 1, -0.774719, 0, -1, -0.632306}}}
```

A very large output was generated. Here is a sample of it:

$$\begin{aligned}
& \left\{ \frac{1}{\langle\langle 1 \rangle\rangle} \left(- \left(- \frac{\cos[\theta_4[t]]^2}{2 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)} + \frac{\frac{1}{3} - \frac{1}{4} \sin[\theta_4[t]]^2}{\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2} \right) \right. \\
& \quad \left(\frac{\cos[\theta_4[t]] \sin[\theta_4[t]] (\langle\langle 32 \rangle\rangle + \langle\langle 1 \rangle\rangle)}{4 \left(\frac{1}{3} - \frac{1}{4} \cos[\theta_4[t]]^2 - \frac{1}{4} \sin[\theta_4[t]]^2 \right)} - \right. \\
& \quad \left. \left(\frac{\cos[\langle\langle 1 \rangle\rangle]^2}{9 (\langle\langle 1 \rangle\rangle)^3} - \frac{\langle\langle 1 \rangle\rangle}{6 \langle\langle 1 \rangle\rangle} + \langle\langle 41 \rangle\rangle + \frac{\langle\langle 1 \rangle\rangle^2 \langle\langle 1 \rangle\rangle^2}{16 (\langle\langle 1 \rangle\rangle) (\langle\langle 1 \rangle\rangle)} \right) (\langle\langle 1 \rangle\rangle) + \langle\langle 23 \rangle\rangle \right) - \\
& \quad - (\langle\langle 1 \rangle\rangle) \left(- \frac{1}{9 (\langle\langle 1 \rangle\rangle)^2} + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} + \frac{\cos[\theta_3[t]] \langle\langle 2 \rangle\rangle \sin[\theta_4[t]]}{16 (\langle\langle 1 \rangle\rangle) \left(\frac{1}{3} - \frac{1}{4} \langle\langle 1 \rangle\rangle^2 - \frac{1}{4} \langle\langle 1 \rangle\rangle^2 \right)} + \langle\langle 60 \rangle\rangle \right) \\
& \quad + \frac{\langle\langle 1 \rangle\rangle}{(\langle\langle 1 \rangle\rangle) (-\sin[\theta_3[t]] \langle\langle 1 \rangle\rangle' [t]^2 - \cos[\langle\langle 1 \rangle\rangle [t]] \langle\langle 1 \rangle\rangle)} + \\
& \quad \frac{(\langle\langle 43 \rangle\rangle + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}) (\cos[\theta_3[t]] \theta_3' [t]^2 - \sin[\theta_4[t]] \theta_4' [t]^2)}{\langle\langle 1 \rangle\rangle}, \\
& \quad \langle\langle 1 \rangle\rangle, 0, \langle\langle 6 \rangle\rangle, \langle\langle 1 \rangle\rangle, \\
& \quad \left. \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} - \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} + \frac{(\langle\langle 1 \rangle\rangle) (\langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle - \langle\langle 1 \rangle\rangle)}{\langle\langle 1 \rangle\rangle} \right\}
\end{aligned}$$

Show Less Show More Show Full Output Set Size Limit...

```

λ /. t → 1 /. s
{{0.539284, -0.528874, 0, 0.352806, 0, -0.377552,
 -0.186478, 0.176067, -0.0831644, -0.141989, 0.0421465}}
```

```

Dimensions[λ]
(*This should come out to be 11, and not 12*)
{11}
```



```
Table[Plot[Evaluate[λ[[j]] /. s], {t, 0, 5}, PlotStyle → Automatic], {j, 1, 11}]
```

