Homework 2

ENPM662: Introduction to Robot Modeling

Deadline : October 9, 2022

Instructions

1. **Submit your assignment as your_directoryID_hw2.zip**
2. **Your submission must contain your code, instructions to run it, and the report as a PDF only**

1 Homogeneous Transformations

1.1 Composition of transforms

The world axes are fixed. Consider the following sequence of rotations and translations:

1. Rotate by ϕ about the world x-axis.
2. Translate by y along the current y-axis.
3. Rotate by θ about the world z-axis.
4. Rotate by ψ about the current x-axis.

Consider 4×4 homogeneous transformation matrices R_{angle} (with zero translation), $T_{distance}$ (with identity rotation). Write the matrix production equation using rotation matrices, R_{angle}, or translation matrices, $T_{distance}$, that will give the resulting pose of the frame and explain why you chose that order.

1.2 Modeling beyond rigid transformations

Consider a camera rigidly mounted on a drone hovering over a plane. If the camera's view is modelled as a cone with an apex angle $\alpha = 45°$, derive an expression for coverage area A (when defined) in terms of 3 consecutive rotations (ψ, θ, ϕ) and 3D location (d_x, d_y, d_z) relative to the world frame on the ground plane as shown in Fig. 1. Refer to the Sec. 3 for hints.

Write a python function that takes the above six inputs (define values of your choice) and outputs the area.

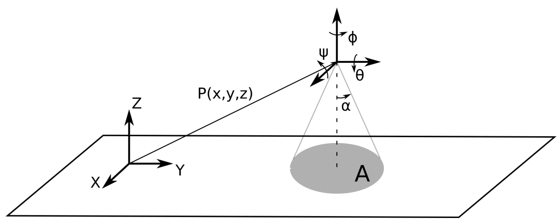

Figure 1: Representation of drone and camera model

1.3 Transform Estimation

Consider the location of a point P relative to frames A, A' be (x, y, z) and (x', y', z') respectively. Assuming that the frame A is only rotated about Z axis with an angle ϕ and translated freely in 3D space to the frame A'. Derive an expression for the transformation matrix H from the frame A to A' Fig. 2.

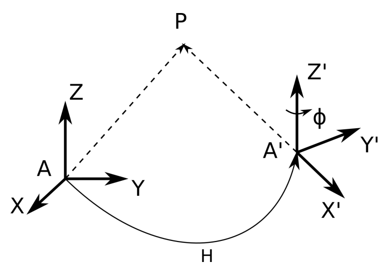

Figure 2: Illustration of reference frames

2 Kinematics

2.1 Trajectory Optimisation

A drone is in constant motion and it moves from a position ***X, Y, Z*** to ***X', Y', Z'*** as shown in figure 3. If the final orientation is $\psi_g = 35°$, $\theta_g = 15°$, $\phi_g = 20°$ with $\omega_{max} = 1deg/s$, Plot the trajectory of the drone such that it reaches the final orientation in the shortest amount of time, you can represent the trajectory as plots of six quantities $\psi, \theta, \phi, \omega_x, \omega_y, \omega_z$ w.r.t time. Please describe your computations in the report.
Assumptions:

- The position ***X, Y, Z*** is aligned with the global frame of reference.

- ψ, θ, ϕ are consecutive rotations about the global X, Y, Z axes to obtain the drone's orientation.
- $\omega_x, \omega_y, \omega_z$ are angular velocities of the drone about the drone's local X, Y, Z axes.
- The angular velocities of the drone can be changed arbitrarily i.e., you can decide any profile for $\omega_x, \omega_y, \omega_z$ (including initial and final values) as long as all $|\omega_x|$, $|\omega_y|$, $|\omega_z| \leq \omega_{max}$

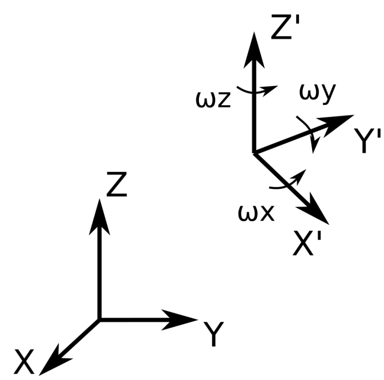

Figure 3: Illustration of angular speeds expressed in the drone's local frame

3 Appendix

1. Re-write equations in the homogeneous matrix multiplication form.
2. Area of an ellipse of form $ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0$ is given by

$$A = \frac{-\pi}{(ac - b^2)^{3/2}} \begin{vmatrix} a & b & d \\ b & c & e \\ d & e & f \end{vmatrix}$$