# HOMEWORK-4

In [1]:
```
#sympy library used to express matrices in terms of symbols
# matplot lib used to plot the graph

from sympy import *
from IPython.display import Image,display,HTML
import matplotlib.pyplot as plt
from matplotlib import cm
import numpy as np

from mpl_toolkits.mplot3d.axes3d import get_test_data
init_printing()

thetadot,t=symbols('thetadot,t')
theta,d,a,alpha=symbols('theta,d,a,alpha')
d_1,d_3,d_5,d_7,a_3=symbols('d_1,d_3,d_5,d_7,a_3')
theta_1,theta_2,theta_3,theta_4,theta_5,theta_6,theta_7=symbols('theta_1,theta_2,theta_3
Rot_z_theta=Matrix([[cos(theta),-sin(theta),0,0],
                    [sin(theta),cos(theta),0,0],
                    [0,0,1,0],
                    [0,0,0,1]])
Trans_z_d=Matrix([[1,0,0,0],
                  [0,1,0,0],
                  [0,0,1,d],
                  [0,0,0,1]])
Trans_x_a=Matrix([[1,0,0,a],
                  [0,1,0,0],
                  [0,0,1,0],
                  [0,0,0,1]])
Rot_x_alpha=Matrix([[1,0,0,0],
                   [0,cos(alpha),-sin(alpha),0],
                   [0,sin(alpha),cos(alpha),0],
                   [0,0,0,1]])
```

In [2]:
```
# Transfromation matrix for each frame
A=Rot_z_theta*Trans_z_d*Trans_x_a*Rot_x_alpha
```

In [3]:
```
# Substituting values form DH table
A_1=A.subs({theta:theta_1,d:d_1,a:0,alpha:pi/2})
A_2=A.subs({theta:theta_2,d:0,a:0,alpha:-pi/2})
A_3=A.subs({theta:0,d:d_3,a:a_3,alpha:-pi/2})
A_4=A.subs({theta:theta_4,d:0,a:-a_3,alpha:pi/2})
A_5=A.subs({theta:theta_5,d:d_5,a:0,alpha:pi/2})
A_6=A.subs({theta:theta_6,d:0,a:a_3,alpha:pi/2})
A_7=A.subs({theta:theta_7,d:d_7,a:0,alpha:0})
```

In [4]:
```
# Transformation matrix for each frame with respect to origin

Transformation=A_1*A_2*A_3*A_4*A_5*A_6*A_7
Transformation
T_1=A_1
T_2=A_1*A_2
T_4=A_1*A_2*A_3*A_4
T_5=A_1*A_2*A_3*A_4*A_5
T_6=A_1*A_2*A_3*A_4*A_5*A_6
T_7=A_1*A_2*A_3*A_4*A_5*A_6*A_7
```

In [5]:
```
# 1st frame wrt origin
T_1
```

Out[5]:

$$\begin{bmatrix}\cos(\theta_1) & 0 & \sin(\theta_1) & 0\\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0\\ 0 & 1 & 0 & d_1\\ 0 & 0 & 0 & 1\end{bmatrix}$$

In [6]:
```
# 2nd frame wrt origin
T_2
```

Out[6]:

$$\begin{bmatrix}\cos(\theta_1)\cos(\theta_2) & -\sin(\theta_1) & -\sin(\theta_2)\cos(\theta_1) & 0\\ \sin(\theta_1)\cos(\theta_2) & \cos(\theta_1) & -\sin(\theta_1)\sin(\theta_2) & 0\\ \sin(\theta_2) & 0 & \cos(\theta_2) & d_1\\ 0 & 0 & 0 & 1\end{bmatrix}$$

In [7]:
```
# 4th framw frame wrt origin
T_4
```

Out[7]:

$$\begin{bmatrix}\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)+\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) & -\sin(\theta_1) & -\sin(\theta_2)\cos(\theta_1)\cos(\theta_4)+\sin(\theta_{\cdot}\\ \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)+\sin(\theta_1)\cos(\theta_2)\cos(\theta_4) & \cos(\theta_1) & -\sin(\theta_1)\sin(\theta_2)\cos(\theta_4)+\sin(\theta_{\cdot}\\ \sin(\theta_2)\cos(\theta_4)-\sin(\theta_4)\cos(\theta_2) & 0 & \sin(\theta_2)\sin(\theta_4)+\cos(\theta_2\\ 0 & 0 & 0\end{bmatrix}$$

In [8]:
```
# 5th frame wrt origin
T_5
```

Out[8]:

$$\begin{bmatrix}(\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)+\cos(\theta_1)\cos(\theta_2)\cos(\theta_4))\cos(\theta_5)-\sin(\theta_1)\sin(\theta_5) & -\sin(\theta_2)\cos(\theta_1)\,\mathrm{c}\\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)+\sin(\theta_1)\cos(\theta_2)\cos(\theta_4))\cos(\theta_5)+\sin(\theta_5)\cos(\theta_1) & -\sin(\theta_1)\sin(\theta_2)\,\mathrm{c}\\ (\sin(\theta_2)\cos(\theta_4)-\sin(\theta_4)\cos(\theta_2))\cos(\theta_5) & \sin(\theta_2)\,\mathrm{si}\\ 0 & \end{bmatrix}$$

In [9]:
```
# 6th frame wrt origin
T_6
```

Out[9]:

$$\left[\begin{matrix}\left(\left(\sin{\left(\theta_{2} \right)} \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{1} \right)} + \cos{\left(\theta_{1} \right)} \cos{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)}\right) \cos{\left(\theta_{5} \right)} - \sin{\left(\theta_{1} \right)} \sin{\left(\theta_{5} \right)}\right) \cos{\left(\theta_{6} \right)} + \left(- \sin{\left(\theta_{2} \right)} \cos{\left(\theta_{1} \right)} \cos{\left(\theta_{4} \right)} + \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{1} \right)} \cos{\left(\theta_{2} \right)}\right) \sin{\left(\theta_{6} \right)}\\\left(\left(\sin{\left(\theta_{1} \right)} \sin{\left(\theta_{2} \right)} \sin{\left(\theta_{4} \right)} + \sin{\left(\theta_{1} \right)} \cos{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)}\right) \cos{\left(\theta_{5} \right)} + \sin{\left(\theta_{5} \right)} \cos{\left(\theta_{1} \right)}\right) \cos{\left(\theta_{6} \right)} + \left(- \sin{\left(\theta_{1} \right)} \sin{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)} + \sin{\left(\theta_{1} \right)} \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{2} \right)}\right) \sin{\left(\theta_{6} \right)}\\\left(\sin{\left(\theta_{2} \right)} \sin{\left(\theta_{4} \right)} + \cos{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)}\right) \sin{\left(\theta_{6} \right)} + \left(\sin{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)} - \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{2} \right)}\right) \cos{\left(\theta_{5} \right)} \cos{\left(\theta_{6} \right)}\\0\end{matrix}\right.$$

In [10]:

```python
# 7th frame wrt orign
T_7
```

Out[10]:

$$\left[\begin{matrix}\left(\left(\left(\sin{\left(\theta_{2} \right)} \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{1} \right)} + \cos{\left(\theta_{1} \right)} \cos{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)}\right) \cos{\left(\theta_{5} \right)} - \sin{\left(\theta_{1} \right)} \sin{\left(\theta_{5} \right)}\right) \cos{\left(\theta_{6} \right)} + \left(- \sin{\left(\theta_{2} \right)} \cos{\left(\theta_{1} \right)} \cos{\left(\theta_{4} \right)} + \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{1} \right)} \cos{\left(\theta_{2} \right)}\right) \sin{\left(\theta_{6} \right)}\right) \cos{\left(\theta_{7} \right)} + \left(\left(\sin{\left(\theta_{2} \right)} \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{1} \right)} + \cos{\left(\theta_{1} \right)} \cos{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)}\right) \sin{\left(\theta_{5} \right)} + \sin{\left(\theta_{1} \right)} \cos{\left(\theta_{5} \right)}\right) \sin{\left(\theta_{7} \right)}\\\left(\left(\left(\sin{\left(\theta_{1} \right)} \sin{\left(\theta_{2} \right)} \sin{\left(\theta_{4} \right)} + \sin{\left(\theta_{1} \right)} \cos{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)}\right) \cos{\left(\theta_{5} \right)} + \sin{\left(\theta_{5} \right)} \cos{\left(\theta_{1} \right)}\right) \cos{\left(\theta_{6} \right)} + \left(- \sin{\left(\theta_{1} \right)} \sin{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)} + \sin{\left(\theta_{1} \right)} \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{2} \right)}\right) \sin{\left(\theta_{6} \right)}\right) \cos{\left(\theta_{7} \right)} + \left(\left(\sin{\left(\theta_{1} \right)} \sin{\left(\theta_{2} \right)} \sin{\left(\theta_{4} \right)} + \sin{\left(\theta_{1} \right)} \cos{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)}\right) \sin{\left(\theta_{5} \right)} - \cos{\left(\theta_{1} \right)} \cos{\left(\theta_{5} \right)}\right) \sin{\left(\theta_{7} \right)}\\\left(\left(\sin{\left(\theta_{2} \right)} \sin{\left(\theta_{4} \right)} + \cos{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)}\right) \sin{\left(\theta_{6} \right)} + \left(\sin{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)} - \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{2} \right)}\right) \cos{\left(\theta_{5} \right)} \cos{\left(\theta_{6} \right)}\right) \cos{\left(\theta_{7} \right)} + \left(\sin{\left(\theta_{2} \right)} \cos{\left(\theta_{4} \right)} - \sin{\left(\theta_{4} \right)} \cos{\left(\theta_{2} \right)}\right) \sin{\left(\theta_{5} \right)} \sin{\left(\theta_{7} \right)}\\0\end{matrix}\right.$$

In [11]:

```python
# End effector position
Xp=Transformation.col(3)
Xp.row_del(3)
```

In [12]:

```python
# substituting values to end effector positions
Xp_copy=Xp.subs({d_1:33.30,
    d_3:31.60,
    d_5:38.40,
    a_3:8.80,
    d_7:20.7})
```

In [13]:

```python
# Taking individual x,y,z for the end effector position
```

```
x=Xp[0]
y=Xp[1]
z=Xp[2]
```

In [14]:
```
# Setting up the Jacobian Matrix

J=Matrix(([diff(x,theta_1),diff(x,theta_2),diff(x,theta_4),diff(x,theta_5),diff(x,theta_
        [diff(y,theta_1),diff(y,theta_2),diff(y,theta_4),diff(y,theta_5),diff(y,theta_6)
        [diff(z,theta_1),diff(z,theta_2),diff(z,theta_4),diff(z,theta_5),diff(z,theta_6)
        [T_1[0,2],T_2[0,2],T_4[0,2],T_5[0,2],T_6[0,2],T_7[0,2]],
        [T_1[1,2],T_2[1,2],T_4[1,2],T_5[1,2],T_6[1,2],T_7[1,2]],
        [T_1[2,2],T_2[2,2],T_4[2,2],T_5[2,2],T_6[2,2],T_7[2,2]]))
```

In [15]:
```
# Substituting values to Jacobian Matrix
J=J.subs({d_1:33.30,
    d_3:31.60,
    d_5:38.40,
    a_3:8.80,
    d_7:20.7})
```

In [16]:
```
J
```

Out[16]:

$$\left[\begin{matrix} 20.7\left(\left(-\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_5)-\sin(\theta_5)\cos(\theta_1)\right)\sin(\theta_6) \\ +\,8.8\left(\left(-\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_5)-\sin(\theta_5)\cos(\theta_1)\right)\cos(\theta_6) \\ +\,8.8\left(\sin(\theta_1)\sin(\theta_2)\cos(\theta_4)-\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\right)\sin(\theta_6) \\ -\,20.7\left(\sin(\theta_1)\sin(\theta_2)\cos(\theta_4)-\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\right)\cos(\theta_6)+8.8\sin(\theta_1)\sin(\theta_2)\sin(\theta_4) \\ +\,38.4\sin(\theta_1)\sin(\theta_2)\cos(\theta_4)+31.6\sin(\theta_1)\sin(\theta_2)-38.4\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)+8.8\sin \\ (\theta_1)\cos(\theta_2)\cos(\theta_4)-8.8\sin(\theta_1)\cos(\theta_2) \\ 20.7\left(\left(\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)+\cos(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_5)-\sin(\theta_1)\sin(\theta_5)\right)\sin(\theta_6) \\ +\,8.8\left(\left(\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)+\cos(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_5)-\sin(\theta_1)\sin(\theta_5)\right)\cos(\theta_6) \\ +\,8.8\left(-\sin(\theta_2)\cos(\theta_1)\cos(\theta_4)+\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\right)\sin(\theta_6) \\ -\,20.7\left(-\sin(\theta_2)\cos(\theta_1)\cos(\theta_4)+\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\right)\cos(\theta_6)-8.8\sin(\theta_2)\sin(\theta_4)\cos \\ (\theta_1)-38.4\sin(\theta_2)\cos(\theta_1)\cos(\theta_4)-31.6\sin(\theta_2)\cos(\theta_1)+38.4\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) \\ -\,8.8\cos(\theta_1)\cos(\theta_2)\cos(\theta_4)+8.8\cos(\theta_1)\cos(\theta_2) \\ 0 \\ \\ \\ \\ \\ \sin(\theta_1) \\ \\ -\cos(\theta_1) \\ \\ 0 \end{matrix}\right.$$

In [67]:
```
q_current=Matrix(([0],
          [0],
          [pi/2],
          [0],
```

```
           [pi],
           [0]))
J_initial=J.subs({theta_1:q_current[0],
    theta_2:q_current[1],
    theta_4:q_current[2],
    theta_5:q_current[3],
    theta_6:q_current[4],
    theta_7:q_current[5]})
J_initial
```

Out[67]:

$$\begin{bmatrix} 0 & -49.2 & 17.6 & 0 & -8.8 & 0 \\ 67.9 & 0 & 0 & -8.8 & 0 & 0 \\ 0 & 67.9 & -59.1 & 0 & 20.7 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In [17]:

```
# The velocity matrix for end effector velocity
X_dot=Matrix(([0],
             [-10*sin(pi/2+ thetadot*t)*thetadot],
             [10*cos(pi/2 + thetadot*t)*thetadot],
             [0],
             [0],
             [0]))
X_dot=X_dot.subs({thetadot:(2*pi)/5})
X_dot
```

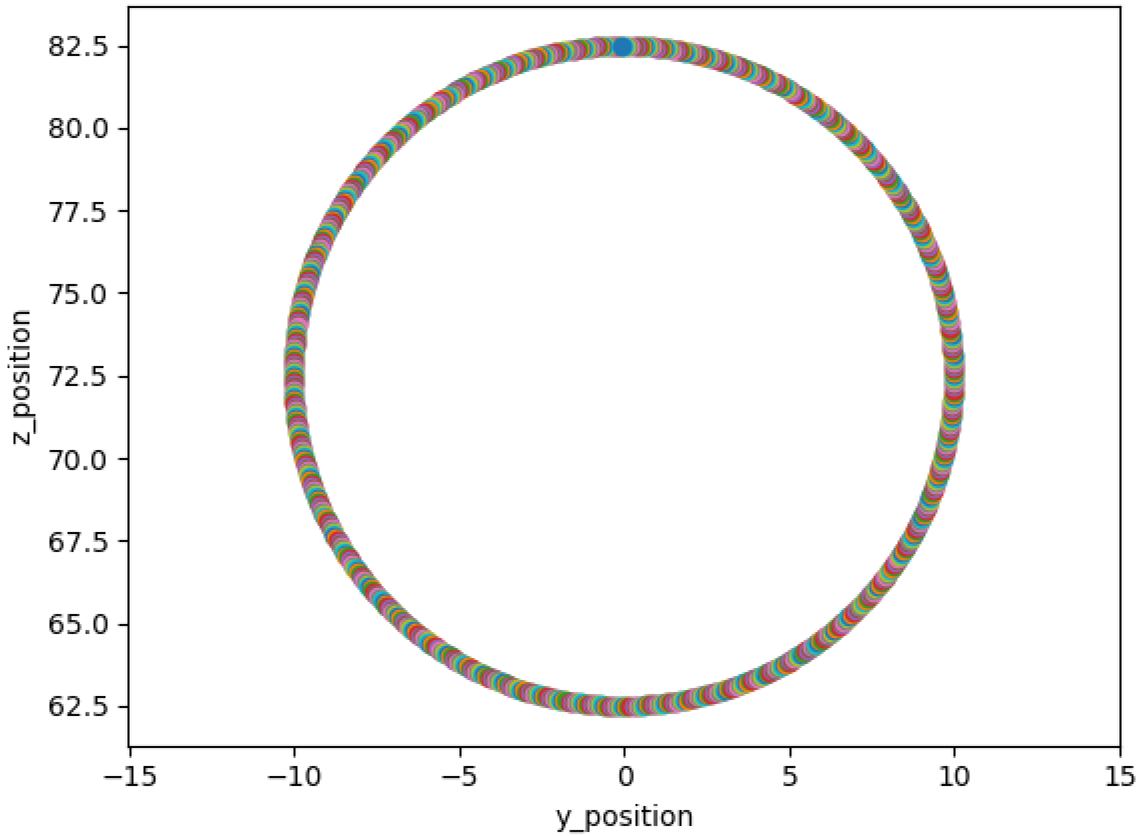
Out[17]:

$$\begin{bmatrix} 0 \\ -4\pi\cos\left(\frac{2\pi t}{5}\right) \\ -4\pi\sin\left(\frac{2\pi t}{5}\right) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In [63]:

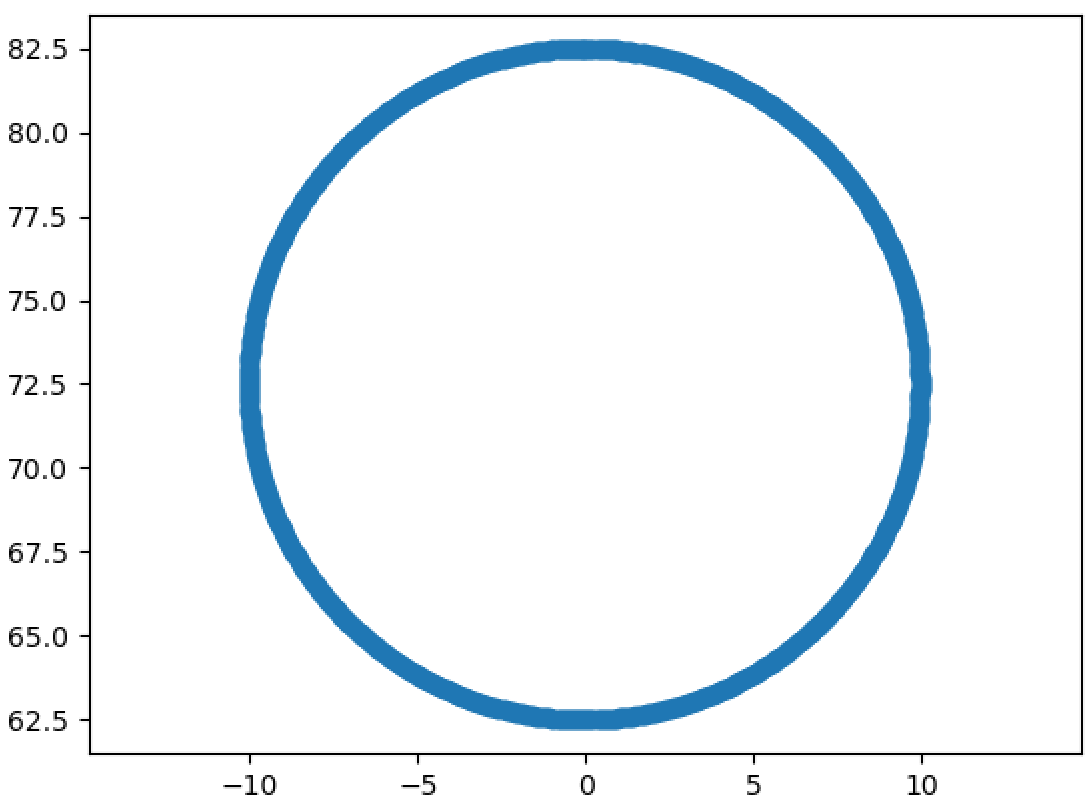
```
# List1 to store the q_current values
# y_plt to store the y values of the end effector
# z_plt to store the z values of the end effector

x=67.9
list1=[]
x_plt=[]
y_plt=[]
z_plt=[]
i=0
j=0
x=[]
# The initial q_current values are given to us
q_current=Matrix(([0],
          [0],
          [pi/2],
          [0],
          [pi],
          [0]))

# Using while loop for
while(i<=5):
```

```
Orientation=J


Orientation=Orientation.subs({theta_1:q_current[0],
theta_2:q_current[1],
theta_4:q_current[2],
theta_5:q_current[3],
theta_6:q_current[4],
theta_7:q_current[5]
})

Orientation_inverse=Orientation.evalf().inv()

q_dot=Orientation_inverse*X_dot
Z=q_dot
Z=Z.subs({t:i}).evalf()
q_current=q_current+Z*0.005

list1.append(q_current)

x_plt.append(Xp_copy[0].subs({theta_1:q_current[0],
theta_2:q_current[1],
theta_4:q_current[2],
theta_5:q_current[3],
theta_6:q_current[4],
theta_7:q_current[5]}).evalf())

y_plt.append(Xp_copy[1].subs({theta_1:q_current[0],
theta_2:q_current[1],
theta_4:q_current[2],
theta_5:q_current[3],
theta_6:q_current[4],
theta_7:q_current[5]}).evalf())

z_plt.append(Xp_copy[2].subs({theta_1:q_current[0],
theta_2:q_current[1],
theta_4:q_current[2],
theta_5:q_current[3],
theta_6:q_current[4],
theta_7:q_current[5]}).evalf())

i=i+0.005
plt.axis("equal")
plt.scatter(y_plt[j],z_plt[j])
j+=1
plt.xlabel("y_position")
plt.ylabel("z_position")
x.append(67.9)
```



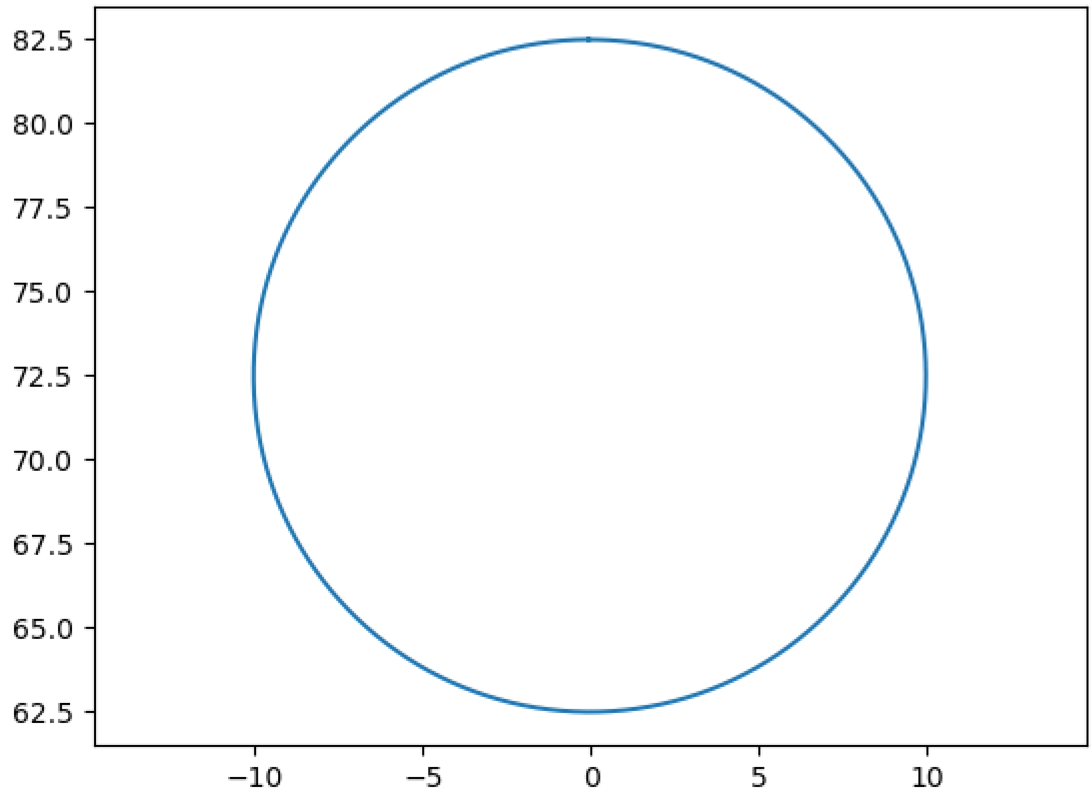

In [64]:
```
# Plot of the y and z scatter
plt.axis("equal")
plt.scatter(y_plt, z_plt)
```

Out[64]: <matplotlib.collections.PathCollection at 0x7f18fd5e79d0>



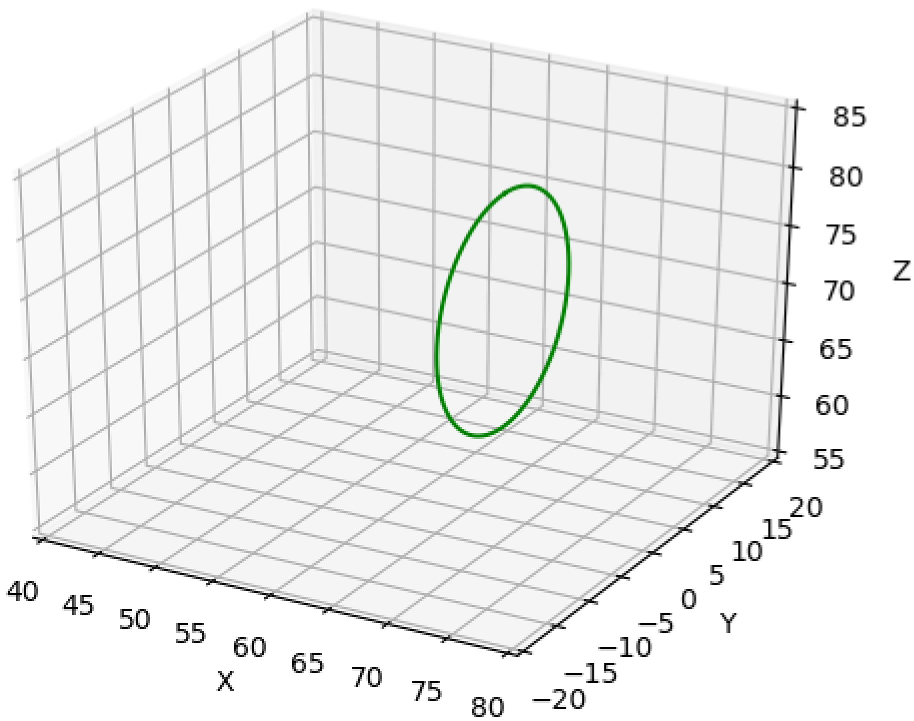

In [65]:
```
# normal plot for y and z
plt.axis("equal")
plt.plot(y_plt,z_plt)
```

Out[65]: [<matplotlib.lines.Line2D at 0x7f18fd81b1f0>]




In [66]:

```
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import pyplot as plt

fig = plt.figure()
ax = fig.add_subplot(111,projection='3d')
ax.axes.set_xlim3d(left=40, right=80)
ax.axes.set_ylim3d(bottom=-20, top=20)
ax.axes.set_zlim3d(bottom=55, top=85)
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.plot3D(x_plt,y_plt,z_plt, color="green")
plt.show()
```



85
80
75
70
65
60
55
Z
20
15
10
5
0
−5
−10
−15
−20
Y
40
45
50
55
60
65
70
75
80
X


In [ ]: