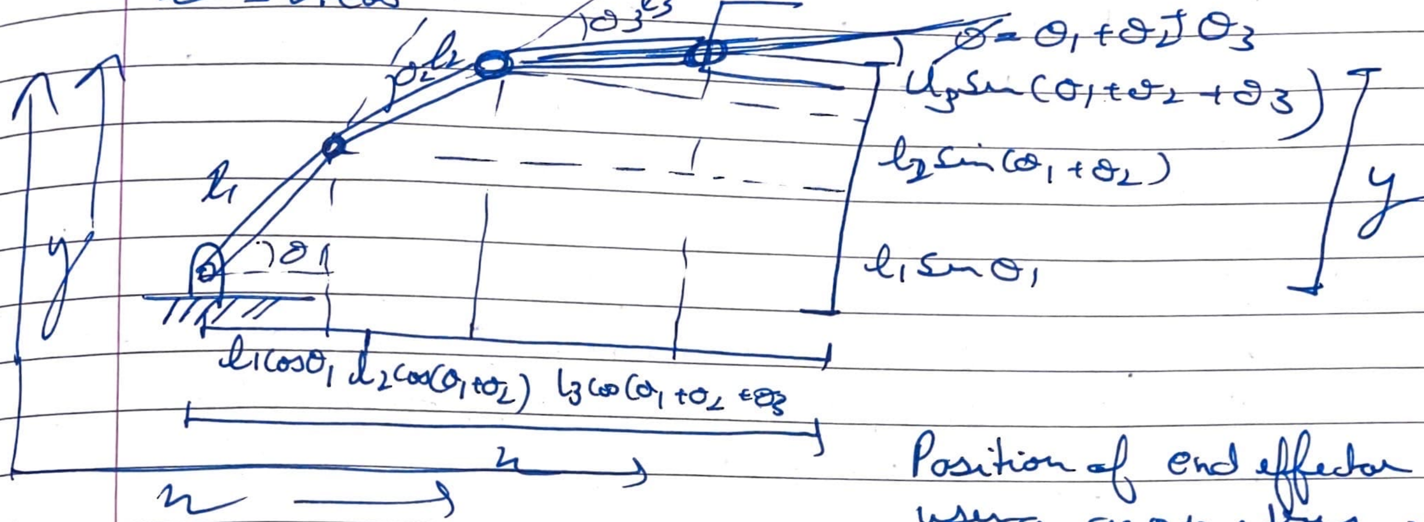
Ans 2

~~x = l1 cos~~



Position of end effector using geometry

$$\boxed{\begin{aligned} x &= l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + l_3\cos(\theta_1+\theta_2+\theta_3) \\ y &= l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) + l_3\sin(\theta_1+\theta_2+\theta_3) \end{aligned}}$$

$\phi = \theta_1 + \theta_2 + \theta_3$

differentiating x & y to get velocity of end effector

$$\dot{x} = -l_1\sin\theta_1\dot{\theta}_1 - l_2\sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) - l_3\sin(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)$$

$$\dot{y} = \cancel{l_1\cos\theta\dot{\theta}_1 + l_2\cos(\theta_1+\theta_2) + l_3\cos}$$
$$= l_1\cos\theta_1\dot{\theta}_1 + l_2\cos(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) + l_3(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)$$

differentiating ϕ to get ~~end effector~~ joint velocities

$$\dot{\phi} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3$$

for ~~we~~ part one we are given
$\theta_1, \theta_2, \theta_3$ joint angles &
$\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$, joint velocities

Part-1 :

∴ Forward kinematic Position can be represented as following in matrix form

$$\cancel{\begin{bmatrix} x \\ y \\ \phi \end{bmatrix} = \begin{bmatrix} l_1c_1 + l_2c_2 + l_3 \\ \\ \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \\ \phi \end{bmatrix} = \begin{bmatrix} l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + l_3\cos(\theta_1+\theta_2+\theta_3) \\ l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) + l_3\sin(\theta_1+\theta_2+\theta_3) \\ \theta_1 + \theta_2 + \theta_3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ \phi \end{bmatrix} = \underbrace{\begin{bmatrix} l_1\cos\theta_1 & l_2\cos(\theta_1+\theta_2) & l_3\cos(\theta_1+\theta_2+\theta_3) \\ l_1\sin\theta_1 & l_2\sin(\theta_1+\theta_2) & l_3\sin(\theta_1+\theta_2+\theta_3) \\ \theta_1 & \theta_2 & \theta_3 \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{3\times1}$$

Forward kinematics velocity eqn are given by deriving the position eqns

∴ we get

$$\cancel{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} l_1\cos\theta_1\dot{\theta}_1 & l_2\cos(\theta_1+\theta_2)[\dot{\theta}_1+\dot{\theta}_2] & l_3\cos(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3) \\ l_1\sin\theta_1\dot{\theta}_1 & l_2\sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2] & l_3\sin \\ \end{bmatrix}}$$

Forward kinematic Eqns for ~~Posi~~ velocity →

$\dot{x}$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -l_1 \sin\theta_1 \dot{\theta}_1 & -l_2 \sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) & l_3 \sin(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3) \\ +l_1 \cos\theta_1 \dot{\theta}_1 & l_2 \cos(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) & l_3 \cos(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3) \\ \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}$$

∴ We can represent the above eqn as $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = A \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$A^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = A^{-1} A \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$A^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = I \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

Part ② We got the velocity equations from forward kinematics.
Here we are simplifying the Matrix to convert it to a form where joint velocities are in a separate matrix.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -l_1 \sin\theta_1 \dot{\theta}_1 & -l_2 \sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) & -l_3 \sin(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3) \\ +l_1 \cos\theta_1 \dot{\theta}_1 & l_2 \cos(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) & l_3 \cos(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3) \\ \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}_{3\times 3}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -l_1 \sin\theta_1 \dot{\theta}_1 & -l_2 \sin(\theta_1+\theta_2)\dot{\theta}_1 - l_2 \sin(\theta_1+\theta_2)\dot{\theta}_2 & -l_3 \sin(\theta_1+\theta_2+\theta_3)\dot{\theta}_1 - l_3 \sin(\theta_1+\theta_2+\theta_3)\dot{\theta}_2 - l_3 \sin(\theta_1+\theta_2+\theta_3)\dot{\theta}_3 \\ +l_1 \cos\theta_1 \dot{\theta}_1 & l_2 \cos(\theta_1+\theta_2)\dot{\theta}_1 + l_2 \cos(\theta_1+\theta_2)\dot{\theta}_2 & l_3 \cos(\theta_1+\theta_2+\theta_3)\dot{\theta}_1 + l_3 \cos(\theta_1+\theta_2+\theta_3)\dot{\theta}_2 + l_3 \cos(\theta_1+\theta_2+\theta_3)\dot{\theta}_3 \\ \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}_{3\times 3}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} (-l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) - l_3 \sin(\theta_1+\theta_2+\theta_3))\dot{\theta}_1 & (-l_2 \sin(\theta_1+\theta_2) - l_3 \sin(\theta_1+\theta_2+\theta_3))\dot{\theta}_2 & -l_3 \sin(\theta_1+\theta_2+\theta_3)\dot{\theta}_3 \\ (l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3))\dot{\theta}_1 & (l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3))\dot{\theta}_2 & l_3 \cos(\theta_1+\theta_2+\theta_3)\dot{\theta}_3 \\ \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} (-l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) - l_3 \sin(\theta_1+\theta_2+\theta_3)) & (-l_2 \sin(\theta_1+\theta_2) - l_3 \sin(\theta_1+\theta_2+\theta_3)) & (-l_3 \sin(\theta_1+\theta_2+\theta_3)) \\ (l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)) & (l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)) & l_3 \cos(\theta_1+\theta_2+\theta_3) \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

The ~~Eqn~~ we Matrix Eqn is of form

$A = BC$ $\quad A \rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix}$ $\quad$ B is the large matrix

$\boxed{\text{Then to } C}$ $\quad C \rightarrow \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$

multiply by B^{-1} $\rightarrow B^{-1}A = B^{-1}BC$

$B^{-1}A = IC$

$\boxed{B^{-1}A = C}$

Using this we can find $\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$

The solution is done using Python Sympy the code as well as screenshot of the $\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$ matrix is shared in the PDF.

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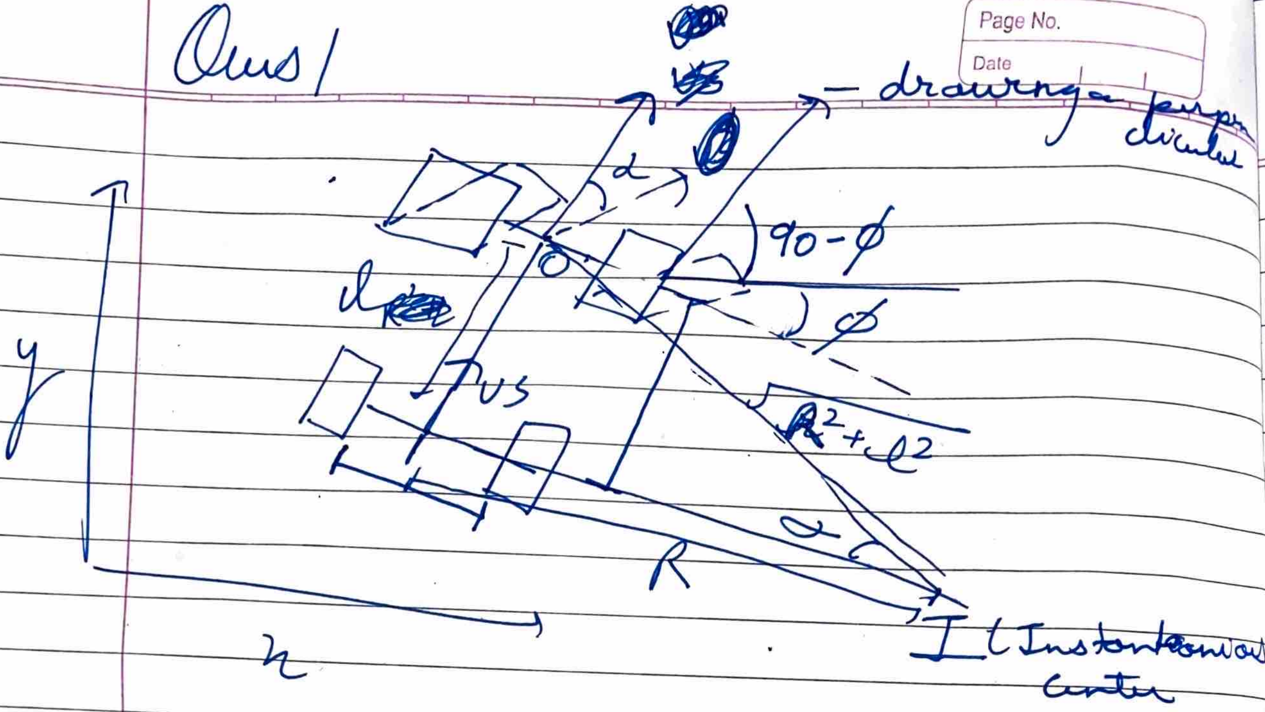
Ans 1



∴ We are given the following

$90-\phi$ Robot orientation
steering angle → α
Diameter of wheel = 0.5 $\boxed{\text{radius } r = 0.25}$
Initial position of point O (x_i, y_i, ϕ_i)

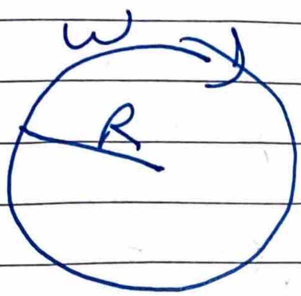
drive speed → ω
distance b/w wheel = ~~4m~~ 1.5 m
Chassis length = 4m

Time duration T

No slipping

For rear wheels as it's a rear wheel drive
$\omega_L + \omega_R = 2\omega$

As the vehicle is rotating we know that in a circular motion



ω of the object remains same.

now a car is not a point object
∴ when the car rotates about the instantaneous center it has different velocity in the front & the rear wheels.

Let V_s be the velocity of the car at the rear wheel mid point & V_{so} be velocity of point O

∴ $\frac{V_s}{R} = \frac{V_{so}}{\sqrt{R^2+l^2}}$

$$V_{so} = \frac{V_s\sqrt{R^2+l^2}}{R}$$

$$V_{so} = \frac{V_s\sqrt{R^2+l^2}}{R}$$

$\frac{l}{R} = \tan\alpha$

$l = R\tan\alpha$

$$V_{so} = \frac{V_s\sqrt{R^2+R^2\tan^2\alpha}}{R}$$

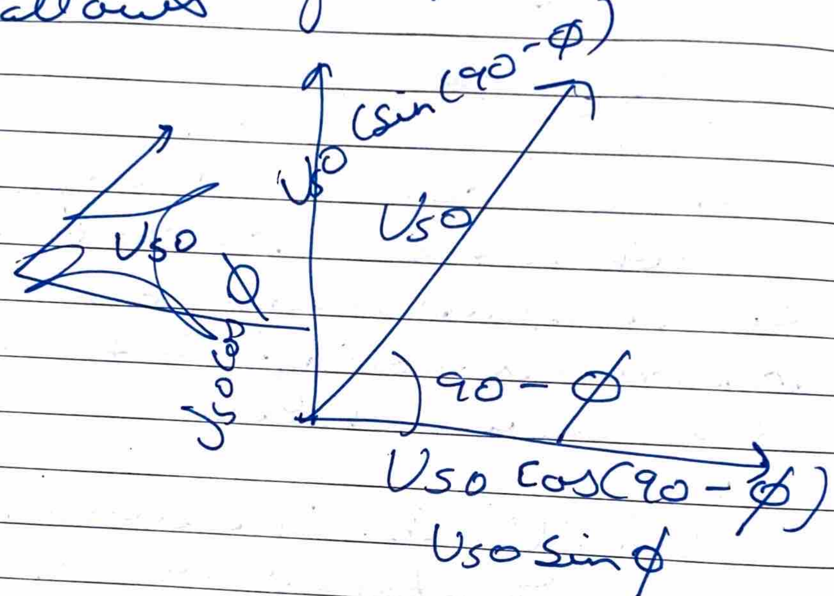
$$V_{so} = \frac{V_s\cancel{R}\sqrt{1+\tan^2\alpha}}{\cancel{R}}$$

$$V_{so}\ V_s\sqrt{\sec^2\alpha}$$

$$V_{so} = V_s\sec\alpha$$

∴ The velocity of front ~~wheels~~ point O is
$V_{sO} = V_s \sec\alpha$

Now the equation for State space ~~etc~~ for point O are as follows



$$\therefore\ \dot{x} = V_{sO}\sin\phi$$
$$\dot{y} = V_{sO}\cos\phi$$
$$\dot{\phi} = \frac{V_{sO}\tan\alpha}{L}$$

$$V_{sO} = V_s \sec\alpha$$

$$\therefore\ \dot{x} = V_s \sec\alpha \sin\phi$$
$$\dot{y} = V_s \sec\alpha \cos\phi$$
$$\dot{\phi} = \frac{V_s \sec\alpha \tan\alpha}{L}$$

~~V_s~~

$$V_s = \frac{\omega_R r + \omega_L r}{2} = \frac{r(\Sigma\omega)}{2}$$

$$\boxed{\theta = 90-\phi}$$

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$d\theta = \rho\, d\theta$$

$$d\theta = \frac{\tan\alpha}{L}\, dD$$

$$D = V_{sO}$$

$$\dot{\theta} = \frac{V_{sO}\tan\alpha}{L}$$

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$V_s = rw$
$V_s = 0.25w$

$\therefore \dot{x} = 0.25 w \sec\alpha \sin\phi$
$\dot{y} = 0.25 w \sec\alpha \sin\phi$
$\dot{\theta} = 0.25 w \sec\alpha \tan\phi$

For the trajector points

~~xcurrent~~

We know that

$\dot{\phi} = \frac{V_s \sec\alpha \tan\alpha}{L}$ $\therefore$ ~~with increase~~ ~~cons with the~~

ϕ is changing with time

as ϕ changes $\dot{x}$ & $\dot{y}$ will also change

to get the current value of ~~ϕ~~ x & y we use the fallowing eqns

$x_{current} = x_{current} + \dot{x} \times \Delta t$

$y_{current} = y_{current} + \dot{y} \times \Delta t$

for $t = 0$ $x_{current} = x_{initial} = x_i$
$y_{current} = y_{initial} = y_i$

$\phi_{current} = \phi_{current} + \dot{\phi} \times \Delta t$

at $t = 0$ $\phi_{current} = \phi_{initial} = \phi_i$

∴ Using a for loop &

appending the x current values
& y current values in

xlist & y list & plotting
these points we get the
trajectory of the ~~veh~~ vehicle

All this is coded in python