AOD DA-2

15BEC0338
Shivam Shukla

1) $\begin{bmatrix} 6 & 2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -3 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)\left(\left((3-\lambda)^2 - 1\right) + 2(-9+3\lambda+2)\right) + 2(3-6+2\lambda) = 0$$

$$(6-\lambda)(9+\lambda^2-6\lambda-1) + 2(3\lambda-7) + 2(2\lambda-3) = 0$$

$$\Rightarrow -\lambda^3 - 9\lambda + 6\lambda^2 + \lambda + 54 + 6\lambda^2 - 36\lambda - 6 + 6\lambda$$
$$- 14 + 4\lambda - 6 = 0$$

$$-\lambda^3 + 12\lambda^2 - 34\lambda + 28 = 0$$

$$\lambda^3 - 12\lambda^2 + 34\lambda - 28 = 0$$

Product of roots = d/a

$abc = +28$ (a, b, c are roots)

$16 \times c = 28$

$c = 7/4$

$= 1.75$

~~Bez~~ $a = 8.3$ $b = 2$

$\lambda_1 = 8.3$ $\lambda_2 = 1.75$ $\lambda_3 = 2$

are the eigen values

Third eigen value is 1.75

for n^2

If eigenvalues of $A \Rightarrow \lambda$, $A^n = \lambda^n$

$\lambda_1' = (\lambda_1)^2$ $\lambda_2' = (\lambda_2)^2$ $\lambda_3' = (\lambda_3)^2$

$\lambda_1' = 8.3^2$ $\lambda_2' = 1.75^2$ $\lambda_3' = 2^2$

$\lambda_1' = 68.89 \quad \lambda_2' = 3.0625 \quad \lambda_3' = 4$

For A^{-1},

$\lambda_1'' = \frac{1}{\lambda_1} \quad \lambda_2'' = \frac{1}{\lambda_2} \quad \lambda_3'' = \frac{1}{\lambda_3}$

$\lambda_1'' = 0.120 \quad \lambda_2'' = 0.5714 \quad \lambda_3'' = 0.5$

Q 1(b) Given $f(x) = x - x^2$ in $(0, l)$

$$F(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

Here range $(0, l)$

$f(x)$ was given and asked for sin series,

So we know $a_0 = 0 \quad a_n = 0$

For half range sine series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^l (lx - x^2)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^l lx\sin\frac{n\pi x}{l}\,dx - \frac{2}{l}\int_0^l x^2\sin\frac{n\pi x}{l}\,dx$$

Solving

$u = x \qquad dv = \sin\frac{n\pi x}{l}\,dx$

$u' = 1 \qquad v = \dfrac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}} = -\cos\left(\frac{n\pi x}{l}\right)\times\frac{l}{n\pi}$

$u'' = 0$

$$v_1 = -\left(\frac{l}{n\pi}\right)^2 \sin\left(\frac{n\pi x}{l}\right)$$

$$= -\sin\left(\frac{n\pi x}{l}\right)\times\frac{l^2}{n^2\pi^2}$$

$$= \frac{2l}{l}\left[x \times \left(-\cos\left(\frac{n\pi x}{l}\right) \times \frac{l}{n\pi}\right) + \sin\frac{n\pi x}{l} \times \frac{l^2}{n^2\pi^2}\right]_0^l$$

$$= \frac{2l}{l}\left(-\cos(n\pi) \times \frac{l^2}{n\pi} - 0\right)$$

Solving (2)

$$dv = \sin\left(\frac{n\pi x}{l}\right) dx$$

$$u = x^2$$

$$u' = 2x$$

$$u'' = 2$$

$$u''' = 0$$

$$v = -\cos\frac{n\pi x}{l} \times \frac{l}{n\pi}$$

$$v_1 = -\sin\frac{n\pi x}{l} \times \frac{l^2}{n^2\pi^2}$$

$$v_{11} = \cos\left(\frac{n\pi x}{l}\right) \frac{l^2}{n^3 \cancel{l}\pi^3}$$

$$\frac{-2}{l}\left[x^2 \times -\cos\left(\frac{n\pi x}{l}\right) \times \frac{l}{n\pi} + 2x \times \frac{l^2}{n^2\pi^2}\sin\left(\frac{n\pi x}{l}\right) + \right.$$

$$\left. 2\cos\frac{n\pi x}{l} \times \frac{l^3}{n^3\pi^3}\right]_0^l$$

$$= \frac{2l^2}{n\pi}\left[\frac{2}{n^2\pi^2}(1-\cos n\pi) + \cos n\pi\right.$$

Adding

$$\Rightarrow \frac{4l^2}{n^3\pi^3}(1-\cos n\pi)$$

$\cos n\pi \Rightarrow (-1)^n$

$n \Rightarrow$ even

$$\frac{4l^2}{n^3\pi^3}(1-1) = 0$$

when $n \Rightarrow$ odd

$$\frac{4l^2}{n^3\pi^3}(1+1) = \frac{8l^2}{n^3\pi^3}$$

2. (a)

$$y'' + 4y = x^4 + \cos^2 x$$

Complementary function

$$m^2 + 4 = 0$$

$$m = \pm 2i$$

$$y_c = (A\cos 2x + B\sin 2x)e^{0x}$$

$$y_c = A\cos 2x + B\sin 2x$$

Now particular integral

$$x^4 + \cos^2 x = x^4 + \frac{1+\cos 2x}{2}$$

$$= x^4 + \frac{1}{2} + \frac{\cos 2x}{2}$$

$$y_p = P y_1 + Q y_2$$

$$= -2\sec 2x \cdot \cos 2x + 2\log\left|\tan\left(\frac{\pi}{4}+x\right)\right| \sin 2x$$

$$y_p = Cx^4 + Dx^3 + Ex^2 + Fx + G + H\sin 2x + K\cos 2x$$

Not satisfy
Multiply with x

$$\Rightarrow y_p = Cx^5 + Dx^4 + Ex^3 + Fx^2 + Gx + Hx\sin 2x + Ix\cos 2x$$

$$y_p' = 5Cx^4 + 4Dx^3 + 3Ex^2 + 2Fx + G + H\sin 2x$$
$$+ Hx\cos 2x(2) + I\cos 2x - I(2)x\sin 2x$$

$$y_p'' = 20Cx^3 + 12Dx^2 + 6Ex + 2F + H\cos 2x(2) + 2H\cos 2x$$
$$- 4Hx\sin 2x - 2I\sin 2x - 2I\sin 2x - 4Ix\cos 2x$$

$4C = 0$ $\quad$ $4D = 1$ $\quad$ $2F = 1/2$

$\boxed{C = 0}$ $\quad$ $\boxed{D = 1/4}$ $\quad$ $\boxed{F = \frac{1}{4}}$

$4E = 0$

$\boxed{E = 0}$

$4G + 6E = 0$

$\boxed{G = 0}$

$4H = 1/2$

$\boxed{H = 1/8}$

$\boxed{I = 0}$

$$y_p = \frac{1}{4} x^4 + \frac{1}{4} x^2 + \frac{1}{8} x \sin 2x$$

$$y = y_c + y_p$$

$$y = A \cos 2x + B \sin 2x + \frac{1}{4} x^4 + \frac{1}{4} x^2 + \frac{1}{8} x \sin 2x$$

2(b) $y'' + 4y = 4\sec^2 2x$

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m \pm 2i$$

$$y_c = e^{0x}(A\cos 2x + B\sin 2x)$$

$$= A\cos 2x + B\sin 2x$$

$$= A\cos 2x + B\sin 2x$$

The particular integral

$$y_p = Py_1 + Qy_2$$

$$y_c = A\cos 2x + B\sin 2x$$

$$y_1 = \cos 2x \qquad y_2 = \sin 2x$$

$$P = -\frac{1}{2}\int \sin 2x \cdot 4\sec^2 2x \, dx$$

$$= -\frac{1}{2}\int 4\tan 2x \sec 2x \, dx$$

$$P = -2\sec 2x$$

$$Q = \int \frac{\cos 2x \cdot 4\sec^2 2x}{2} dx$$

$$= \frac{1}{2}\int 4\sec 2x \, dx = 2\log\left|\tan\left(\frac{\pi}{4}+x\right)\right|$$

$$y_p = Py_1 + Qy_2$$

$$= -2 + 2\sin 2x \cdot \log\left|\tan\left(\frac{\pi}{4}+x\right)\right|$$

$$y = y_c + y_p$$

$$y = A\cos 2x + B\sin 2x + 2\sin 2x \log\left(\tan\left(\frac{\pi}{4}+x\right)\right) - 2$$

3)

$$A = \begin{pmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{pmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{pmatrix} 2-\lambda & 2 & -7 \\ 2 & 1-\lambda & 2 \\ 0 & 1 & -3-\lambda \end{pmatrix} = 0$$

$$(2-\lambda)(-3+3\lambda-\lambda+\lambda^2-2) - 2(-6-2\lambda) - 7(2) = 0$$

$$-\lambda^3 + 13\lambda - 12 = 0$$

$$\lambda^3 - 13\lambda + 12 = 0$$

$$\begin{array}{c|cccc} 3 & 1 & 0 & -13 & 12 \\ & 0 & 3 & 9 & -12 \\ \hline & & & & \boxed{0} \end{array}$$

$\therefore \lambda = 1, 3, -4$

$$\lambda^2 + 3\lambda - 4 = 0$$

$$\lambda = 1, -4$$

at $\lambda = 1$

$$x + 2y - 7z = 0$$

$$2x + 0y + 2z = 0$$

$$\frac{x}{-4} = \frac{-y}{2+14} = \frac{z}{-4}$$

$$\frac{x}{1} = \frac{y}{-4} = \frac{z}{-1}$$

$$\begin{pmatrix} 1 \\ -4 \\ -4 \end{pmatrix}$$

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$$-x+2y-7z=0$$
$$2x-2y+2z=0$$

$$\frac{x}{4+(-14)}=\frac{y}{-2+14}=\frac{z}{2-4}$$

$$\begin{pmatrix}5\\6\\1\end{pmatrix}$$

$$\frac{x}{-10}=\frac{y}{-12}=\frac{z}{-2}$$

$$y=-4$$

$$6x+2y-7z=0$$
$$2x+5y+2z=0$$

$$\frac{x}{4+35}=\frac{-y}{12+14}=\frac{z}{30-4}$$

$$\frac{x}{3}=\frac{-y}{2}=\frac{z}{2}$$

$$\begin{pmatrix}3\\-2\\2\end{pmatrix}$$

$$m=\begin{pmatrix}1&5&3\\-4&6&-2\\-1&1&2\end{pmatrix}$$

The Matrix M is not symmetric

$$D = M^{-1} A M$$

$$M^{-1} \quad \frac{\text{Adj } M}{|M|}$$

$$[\text{Adj } M] = \begin{pmatrix} 14 & -7 & -28 \\ 16 & 5 & -16 \\ 2 & -6 & 26 \end{pmatrix}$$

$$= (7)(5)\begin{pmatrix} 2 & -1 & -4 \\ 2 & 1 & -2 \\ 2 & -6 & 26 \end{pmatrix}$$

$$\frac{1}{|M|} = \frac{1}{70}$$

$$M^{-1} = \text{Adj} \frac{1}{|M|} = \frac{1}{70}\begin{pmatrix} 14 & -7 & -28 \\ 16 & 5 & -16 \\ 2 & -6 & 26 \end{pmatrix}$$

Thus

$$D = M^{-1} A M$$

$$D = \frac{1}{70}\begin{pmatrix} 14 & -7 & -28 \\ 10 & 5 & -16 \\ 2 & -6 & 26 \end{pmatrix}\begin{pmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{pmatrix}\begin{pmatrix} 1 & 5 & 3 \\ -4 & 6 & -2 \\ -1 & 1 & 2 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ & 0 & -4 \end{pmatrix}$$

$$A^n = M D^n M^{-1}$$

$$A^4 = \frac{1}{70}\begin{pmatrix} 1 & 5 & 3 \\ -4 & 6 & -2 \\ -1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1^4 & 0 & 0 \\ 0 & 3^4 & 0 \\ 0 & 0 & (-4)^4 \end{pmatrix}\begin{pmatrix} 14 & -7 & -28 \\ 10 & 5 & -10 \\ 2 & -6 & 21 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 80 & -37 & 227 \\ 54 & 79 & -258 \\ 26 & -38 & 179 \end{pmatrix}$$

4) Given equation

$$2x_1x_2 + 2x_2x_3 + 2x_3x_1$$

$$\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\begin{bmatrix} 0-\lambda & 1 & 1 \\ 1 & 0-\lambda & 1 \\ 1 & 1 & 0-\lambda \end{bmatrix}$$

$$(-\lambda)(\lambda^2-1) - 1(-\lambda-1) + 1\ (1+\lambda) = 0$$

$$\lambda^3 - 3\lambda - 2 = 0$$

$$\lambda = -1, -1, 2$$

$$D = M^{-1} A M$$

$$M^{-1} = \frac{\text{Adj } M}{|M|}$$

$$[\text{Adj } M] = (7)(5) \begin{bmatrix} 2 & -1 & -4 \\ 2 & 1 & -2 \\ 2 & -6 & 26 \end{bmatrix}$$

$$\frac{1}{|M|} = \frac{1}{70}$$

$$M^{-1} = \frac{1}{70} \begin{pmatrix} 14 & -7 & -28 \\ 16 & 5 & -16 \\ 2 & -6 & 26 \end{pmatrix}$$

$$D = M^{-1} A M$$

$$D = \frac{1}{70}\begin{pmatrix} 14 & -7 & -28 \\ 10 & 5 & -10 \\ 2 & -6 & 26 \end{pmatrix}\begin{pmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{pmatrix}\begin{pmatrix} 1 & 5 & 3 \\ -4 & 6 & -2 \\ -1 & 1 & 2 \end{pmatrix}$$

when $\lambda = 2$

$$-2x + y + z = 0$$

$$x - 2y + z = 0$$

$$\frac{x}{1+2} = \frac{-y}{-2-1} = \frac{z}{4-1}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\lambda = -1$$

$$x + y + z = 0 \qquad x + y + z = 0$$

$$x = -y - z$$

$$M = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$N = \begin{pmatrix} -1/\sqrt{2} & \frac{-1}{\sqrt{2}} & 1/\sqrt{3} \\ \frac{1}{\sqrt{2}} & 0 & 1/\sqrt{3} \\ 0 & 1/\sqrt{2} & 1/\sqrt{3} \end{pmatrix}$$

$$N = \frac{1}{2\sqrt{3}} \begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$D = N^{-1} A N$$

$$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Canonical form of is $Y^{-1} DY$

$$= -1\, y_1^2 - (-1) y_2^2 + 2y_3^2$$

$$= -1\, y_1^2 - y_2^2 + 2y_3^2$$

DA - 2

MAT 2002 - F2 + TF2

Shivam Shukla

15BEC0338

1. $r^2 \frac{d^2u}{dr^2} + r\frac{du}{dr} - u = -r^3$

$r = e^t \Rightarrow t = \log r$

$r\frac{du}{dr} = Du$

$r^2 \frac{d^2u}{dr^2} = D(D-1)u$

$D(D-1)u + Du - u = -e^{3t}$

$(D^2-1)u = -e^{3t}$

$CF = (D^2-1)u_c = 0$

$u_c = e^{\lambda t}$

$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

$CF = y_c = c_1 e^t + c_2 e^{-t}$

$PI = (D^2-1)u_p = -e^{3t}$

$$8Ae^{3t} = -e^{3t} \Rightarrow A = -\frac{1}{8}$$

$$PI = -\frac{1}{8}e^{3t}$$

$$y = c_1e^{t} + c_2e^{-t} - \frac{1}{8}e^{3t} = c_1 x + \frac{c_2}{x} - \frac{1}{8}x^3$$

2. $$9s^2\frac{d^2x}{ds^2} + 9s\frac{dx}{ds} + (9s^2-1)x = 0$$

$$x = \sum a_n s^{n+r}$$

$$\frac{dx}{ds} = \sum a_n (n+r) s^{n+r-1}$$

$$\frac{d^2x}{ds^2} = \sum a_n (r+n)(n+r-1) s^{n+r-2}$$

$$\Rightarrow 9\sum a_n(r+n)(n+r-1)s^{n+r} + 9\sum a_n(n+r)s^{n+r}$$
$$\sum a_n s^{r+n} = 0$$

Equing least power term 9 s to 0

comparing coefficients

$$9r^2 - 1 = 0$$

$$r = \pm\frac{1}{3}$$

x^{r+1}

$$9a_2(r+2)(r+1) + 9a_2(r+2) + 9a_0 - a_2 = 0$$

$$\Rightarrow \quad a_2 = \frac{-9a_0}{9(r+2)^2 - 1} = \frac{9a_0}{1 - 9(r+2)^2}$$

$$a_3 = \frac{-a_1}{\left(r+1-\frac{5}{3}\right)\left(r+1+\frac{7}{3}\right)}, \quad a_4 = \frac{a_0}{\left(r+\frac{1}{3}\right)\left(r+\frac{5}{3}\right)\left(r+\frac{13}{3}\right)\left(r+\frac{7}{3}\right)}$$

Sub

$r = 1/3$,

$$y_1 = a_0 x^{1/3}\left(1 - \frac{1}{4\cdot\frac{4}{3}}x^2 + \frac{1}{4^2 2!\cdot\frac{4}{3}\cdot\frac{7}{3}}x^4 + \cdots\right)$$

$r = -1/3$

$$y_2 = a_0 x^{-1/3}\left(1 - \frac{1}{4\cdot\frac{2}{3}}x^2 + \frac{1}{4^2 + 2!\cdot\frac{2}{3}\cdot\frac{5}{3}}x^4 \cdots\right)$$

$$y = c_1 y_1 + c_2 y_2$$

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SLP form

$$(Ry')' + (\lambda p + q)y = 0 \quad -①$$

$$9s^2 \frac{d^2 r}{ds^2} + 9s \frac{dr}{ds} + (9s^2 - 1)r = 0$$

$$\Rightarrow \frac{d^2 r}{ds^2} + \frac{1}{s}\frac{dr}{ds} + \left(s - \frac{1}{9s}\right) r = 0$$

$$\left(s\frac{dr}{ds}\right)' + \left(s - \frac{1}{9s}\right) r = 0$$

$$R = s \quad p = s \quad \lambda = 1 \quad q = \frac{1}{9s}$$

3. $$\frac{d^2 y}{dx^2} + \mu y = 0$$

$$\lambda^2 + 1 = 0 \Rightarrow \lambda \pm i$$

$$y = c_1 \cos \lambda x + c_2 \sin \lambda x$$

Applying boundary conditions

$$c_2 \sin \lambda\pi = 0 \qquad y'(\pi) = y'(-\pi)$$

$$c_1 s_1 \quad -\lambda\pi = 0 \Rightarrow c_1 = c_2 = 0 \rightarrow \text{trivial}$$

$$\lambda = \frac{n\pi}{\pi} = n \rightarrow +\text{ve int.} \quad \lambda = n \Rightarrow \mu = n^2$$

Eigen fns $\cos nx$, $\sin nx$

$n = 0, 1, 2, 3 \ldots$

let $I = \int_{-\pi}^{\pi} y_n y_m \, dx$

$$= \int_{-\pi}^{\pi} \sin(m+n)x + \cos(m-n)x \, dx$$

$m \neq n$

$\Rightarrow I = 0$

$\int_{-\pi}^{\pi} y_n y_m \, dx = 0$ $\Rightarrow$ orthogonal in nature

4. $T = 100 \text{ gal} = T_2$

Fertilizer $= 150$

Rate of fertil^n $= 2$

$$y_1' = \frac{2}{100} y_2 - y_1 \frac{2}{100}$$

$$y_2' = \frac{2}{100} y_1 - \frac{2}{100} y_2$$

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -0.02 & 0.02 \\ 0.02 & -0.02 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -0.02-\lambda & 0.02 \\ 0.02 & -0.02-\lambda \end{vmatrix} = 0$$

$$\underline{\underline{\lambda = 0, -0.04}}$$

$$AX = \lambda X$$

$\lambda = 0.04$

$$X_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

For $\lambda = 0$

$$X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = [X_1] e^{0t} + [X_2] e^{-0.04t}$$

$$y_1 = 75 - 75 e^{-0.04t}$$

$$y_2 = 75 + 75 e^{-0.04t}$$

5. Series LRC circuit

$$E = V_L + V_R + V_C$$

$$e^{-t} \sin t = L \frac{di}{dt} + IR + \frac{Q}{C}$$

$$\frac{d^2 q}{dt^2} + 2\frac{dq}{dt} + 5q = e^{-t} \sin t$$

Laplace :

$$s^2 q - sq(0) - q'(0) + 2(sq - q(0)) + 5q = \frac{1}{(s+1)^2+1}$$

$q(0) = 0 \quad q'(0) = 1$ on substitution,

$$Q(s^2 + 2s + 5) = \frac{1}{(s+1)^2+1} + 1$$

$$Q = \frac{1}{((s+1)^2+1)((s+1)^2+4)} + \frac{1}{(s+1)^2+4}$$

$$= \frac{1}{3} \times \frac{2}{(s+1)^2+2^2} + \frac{1}{3} \times \frac{1}{(s+1)^2+1^2}$$

$$q(t) = \frac{1}{3} e^{-t} \sin 2t + \frac{1}{3} e^{-t} \sin t$$

$$i(t) = \frac{1}{3} e^{-t} (\cos t + 2\cos 2t - \sin t - \sin 2t)$$