

G. H. RAISONI COLLEGE OF ENGG., NAGPUR

2020 - 2021 ODD TERM

CBE-2 EXAMINATION FOR SPLIT-2

DEPARTMENT: AI

SEM/SEC : 3 / A

DATE: 28/08/2020

SUBJECT : DM

ROLL NO : 58

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C02.

Q.3.

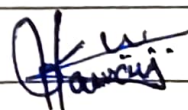
i) Let e be the identity element for $*$ in G .

Then we have $a * a^{-1} = e$, where $a^{-1} \in G$

Also $(a^{-1})^{-1} * a^{-1} = e$

Therefore, $(a^{-1})^{-1} = a * a^{-1}$.

Thus, by right cancellation law, we have $(a^{-1})^{-1} = a$.



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ii.) Let a and $b \in G$ and G is a group for $*$, then
 $a * b \in G$ (closure)

Therefore, $(a * b)^{-1} * (a * b) = e$

Let a^{-1} and b^{-1} be the inverses of a and b respectively, then
 $a^{-1}, b^{-1} \in G$.

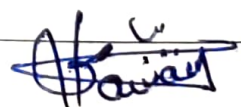
————— ①

Therefore, $(b^{-1} * a^{-1}) * (a * b)$
 $= b^{-1} * (a^{-1} * a) * b$
(associativity)
 $= b^{-1} * e * b = b^{-1} * b$
 $= e$

————— ②

from ① and ② we have
 $(a * b)^{-1} * (a * b) = (b^{-1} * a^{-1}) * (a * b)$
 $(a * b)^{-1} = b^{-1} * a^{-1}$

by right cancellation law.



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Q CO2.

2. Let $M = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$

given M is non singular.

Now

Multiplication of a 2×2 matrix with any other 2×2 matrix would yield a 2×2 matrix.

\therefore Closure property satisfied ——— ①

Let N be any other 2×2 matrix & P be a 2×2 matrix.

$$(MN)P = M(NP)$$

\therefore Associativity satisfied ——— ②

Multiplying

$$M \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

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\therefore Identity element is present.

—— (3)

Given M is non-singular & every non-singular matrix has a inverse which is Unique.

\therefore Inverse present —— (4)

From (1), (2), (3) & (4),

Set of all matrices is Group.

(Q2.

1. $(G, *)$ Group $a, b \in G$

~~For every there existence inverse.~~

$$\del{a * a^{-1} = e = a^{-1} * a}$$

$$\del{a * e = e * a^{-1}}$$

Given: $a * b = a * b^{-1}$ & $b * a = a * b^{-1}$

~~Hanish~~

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Now,

$$a * b = b * a^{-1}$$

$$a * b * b^{-1} = b * a^{-1} * b^{-1} \quad (\text{Multiply both sides})$$

$$a * (e) = b * (ab)^{-1} \quad (b * b^{-1} = e)$$

$$a = b * (ab)^{-1} \quad (a * e = a \text{ identity law})$$

$$a * (ab) = b * (ab)^{-1} * (ab)$$

$$a^2 b = b * e \quad (a * a^{-1} = e)$$

$$a^2 b * b^{-1} = b * b^{-1} \quad (b^{-1} \text{ multiply both sides})$$

$$a^2 = e$$

$$\therefore a * a = e$$

$$\therefore a = a^{-1}$$

$$\therefore a * a^{-1} = e = a^{-1} * a \quad \text{--- (1)}$$

$$a * e = e * a^{-1}$$

$$e * a = a * a^{-1} = a * e$$

$$\therefore a^{-1} = a \quad \text{--- (2)}$$

$$\text{So, } a^4 = e$$

$$\therefore o(a) = 4$$

So, 4 is order of a.

~~Pranav~~

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