Design and Analysis of Algorithms

TAE - 1

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(A - 58)
AI - Sem 3

Set 1 :

Q. A.

1.
```
for i = 1 to n/2 do :
    for j = i to n-i do :
        for k = 1 to j do :
            print "foobar"
        end
    end
end
```

Soln. Formula for cost of loop: $n(3+b)+2$

Outer most loop : $n = n/2$, $b = ?$

Inner most loop : $n = n$, $b = 1$

Cost $= n(3+1)+2 = 4n+2$

Middle loop: $n = n$, $b = 4n+2$

$$\begin{aligned} \text{cost} &= n(3+4n+2)+2 \\ &= n(4n+5)+2 \\ &= 4n^2+5n+2 \end{aligned}$$

Whole cost : $n = n/2$, $b = 4n^2 + 5n + 2$

$$= \frac{n}{2}(3 + 4n^2 + 5n + 2) + 2$$

$$= \frac{4n^3 + 5n^2 + 5n}{2} + 2$$

$$= O(n^3)$$

2\. Soln. Inner loop : $n = n$, $b = 14$

Cost $= n(3+14) + 2$

$= 17n + 2$

Outer loop : $n = m$, $b = 17n + 2 + 2$

Cost $= m(3 + 17n + 4) + 2$

$= 17mn + 7m + 2$

Add fn. : $17mn + 7m + 2 + 1$

$= 17mn + 7m + 3$

$= O(mn)$

Q. B.

1\. $T(n) = 3T(n/4) + n \log n$

By Master method,

Case 3 : $a < b^k$: $3 < 4^1$

(a) $p \geq 0 \Rightarrow T(n) = \Theta(n^k \log^p n)$

$a = 3, b = 4, k = 1, p = 1$

$T(n) = \Theta(n \log n)$

2. $T(n) = 4T(n/2) + n / \log n$

Here $a = 4, b = 2, k = 1, p = -1$

$a > b^k$

$\therefore$ By master method case 1:

$T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 4}) = \Theta(n^2)$

Set - 2:

Q. A.

1.

Soln. Inner loop: $n = n, b = 1$

Formula $= n(3+b) + 2$

Cost $= n(3+1) + 2$

$= 4n + 2$

Outer loop: $n = m, b = 4n+2+1$

Cost $= m(3+4n+3) + 2$

$= 4mn + 6m + 2$.

Add f^n:

$$\text{Cost} = 4mn + 6m + 2 + 1$$
$$= \Theta(mn)$$

2.

Soln. Innermost : $n = n$, $b = 2$

$$\text{Cost} = n(3+2) + 2$$
$$= 5n + 2$$

Outer : $n = n$, $b = 1 + 5n + 2 + 2$

$$\text{Cost} = n(3 + 5n + 5) + 2$$
$$= 5n^2 + 8n + 2$$
$$= \Theta(n^2)$$

Q. B.

1. $T(n) = 2T(n/2) + n/\log n$

Soln. $a = 2$, $b = 2$, $k = 1$, $p = -1$

$a = b^k$

& $p = -1$

By Case 2(b), $T(n) = \Theta(n^{\log_b a} \log \log n)$

$\Rightarrow T(n) = \Theta(n^{\log_2 2} \log \log n)$

$= \Theta(n \log \log n)$

2. $T(n) = 4T(n/2) + n^2$

Soln.

$a = 4, \; b = 2, \; k = 2, \; p = 0$

Here, $a = b^k$

& $p > -1$

Therefore,

By master's theorem case 2 (a),

$T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$\Rightarrow T(n) = \Theta(n^{\log_2 4} \log^{0+1} n)$

$= \Theta(n^2 \log n)$

Set 3:

Q. A.

1.

Soln. Inner loop : $n = n$, $b = 1+1+1 = 3$

Cost $= n(3+b)+2$

$= n(3+3)+2$

$= 6n+2$

Fn. : worst case : else,

Cost $= 1+1+6n+2+1$

$= 6n+5$

$= \Theta(n)$

Best case : if $n = 1$ or $n = 2$

Cost $= 1$

$= \Omega(1)$

2.

Solⁿ. Innermost : $n = z$, $b = 12$

Cost $= z(3 + 12) + 2$

$= 15z + 2$

Mid loop : $n = y$, $b = 15z + 2 + 4$

Cost $= y(3 + 15z + 6) + 2$

$= 15yz + 9y + 2$

Outer loop : $n = x$, $b = 15yz + 9y + 2$

cost $= x(3 + 15yz + 9y + 2) + 2$

$= 15xyz + 9xy + 5x + 2$

$= O(xyz)$

Q. B.

1. $T(n) = 4T(n/2) + \log n$

Solⁿ.

Here, $a = 4$, $b = 2$, $k = 0$, $p = 1$

$a > b^k$

Case 1: $T(n) = \Theta(n^{\log_b a} n)$

$T(n) = \Theta(n^{\log_2 4}) = \Theta(n^2)$

2. $T(n) = 2T(n/2) + n / (\log n)$

Soln.

Here, $a = 2$, $b = 2$, $k = 1$, $p = -1$

Case 2(b) $\Rightarrow$ $T(n) = \Theta(n^{\log_b a} \log\log n)$.

$T(n) = \Theta(n^{\log_2 2} \log\log n)$

$= \Theta(n \log\log n)$.

Set - 4

Q.A.

1.

Soln. Inner loop: $n = n$, $b = 1$

Cost $= n(3+1) + 2$

$= 4n + 2$

Mid loop: $n = n$, $b = 4n+2$

Cost $= n(3 + 4n + 2) + 2$

$= 4n^2 + 5n + 2$

Outer loop: $n = n$, $b = 4n^2 + 5n + 2$

$$\text{Cost} = n(3 + 4n^2 + 5n + 2) + 2$$
$$= 4n^3 + 5n^2 + 5n + 2$$

Condundrum fn :

$$\text{Cost} = 1 + 4n^3 + 5n^2 + 5n + 2 + 1$$
$$= 4n^3 + 5n^2 + 5n + 4$$
$$= O(n^3)$$

2.

Soln. Innermost : $n = n$, $b = 1$

$$\text{Cost} = n(3 + 1) + 2$$
$$= 4n + 2$$

Mid : $n = n$, $b = 4n + 2$

$$\text{Cost} = n(3 + 4n + 2) + 2$$
$$= 4n^2 + 5n + 2$$

Outer : $n = n$, $b = 4n^2 + 5n + 2$

$$\text{Cost} = n(3 + 4n^2 + 5n + 2) + 2$$
$$= 4n^3 + 5n^2 + 5n + 2.$$

Mystery fn. :

$$\text{Cost} = 1 + 4n^3 + 5n^2 + 5n + 2 + 1$$
$$= 4n^3 + 5n^2 + 5n + 4$$
$$= O(n^3)$$

Q. B.

1. $T(n) = 4T(n/2) + \log n$

Soln

$a = 4, b = 2, k = 0, p = 1$

$a > b^k$

Case 1: $T(n) = \Theta(n^{\log_b a})$

$\Rightarrow T(n) = \Theta(n^{\log_2 4}) = \Theta(n^2)$

2. $T(n) = 3T(n/4) + n\log n$

Soln.

$a = 3, b = 4, k = 1, p = 1$

$a < b^k, p \geq 0$

Case 3(a) $T(n) = \Theta(n^k \log^p n)$

$\Rightarrow T(n) = \Theta(n^1 \log^1 n) = \Theta(n\log n)$

Set - 5.

Q. A.

1.

Soln.

Innermost: $n = n+n, b = 1$

Cost $= 2n(3+1) + 2$

$= 8n + 2$

Mid: $n = n, b = 8n+2$

$$\text{Cost} = n(3 + 8n + 2) + 2$$
$$= 8n^2 + 5n + 2$$

Outer : $n = n$, $b = 8n^2 + 5n + 2$

$$\text{Cost} = n(3 + 8n^2 + 5n + 2) + 2$$
$$= 8n^3 + 5n^2 + 5n + 2$$

Pesky fn:

$$\text{Cost} = 1 + 8n^3 + 5n^2 + 5n + 2 + 1$$
$$= 8n^3 + 5n^2 + 5n + 4$$
$$= O(n^3).$$

2

soln. Inner most : $n = 100$, $b = 1$

$$\text{Cost} = 100(3+1) + 2$$
$$= 402$$

Mid : $n = n$, $b = 402$

$$\text{Cost} = n(3 + 402) + 2$$
$$= 405n + 2$$

Outer : $n = n$, $b = 405n + 2$

$$\text{Cost} = n(3 + 405n + 2) + 2$$
$$= 405n^2 + 5n + 2$$
$$= O(n^2)$$

Q. B.

1. $T(n) = 6T(n/3) + n^2 \log n$.

Soln.

Here, $a = 6$, $b = 3$, $k = 2$, $p = 1$

$a = b^k$ & ~~[illegible]~~ $p > -1$

Case 2 (a)

$T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$\Rightarrow T(n) = \Theta(n^{\log_3 6} \log^2 n)$

$\Rightarrow T(n) = \Theta(n^2 \log^2 n)$

2. $T(n) = 4T(n/2) + n^2$

Soln.

$a = 4$, $b = 2$, $k = 2$, $p = 0$

$a = b^k$ & $p \geq -1$.

Case 2 (a)

$T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$\Rightarrow T(n) = \Theta(n^{\log_2 4} \log^{0+1} n)$

$= \Theta(n^2 \log n)$