G.H. RAISONI COLLEGE OF ENGINEERING

2020-2021 EVEN TERM

CAE-1 EXAMINATION SUMMER-2021

(ONLINE TERM)

DEPARTMENT: ARTIFICIAL INTELLIGENCE

SEM/SEC: 4th / A

SUBJECT: MLA DATE:

ROLL NO: A-58

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Q3. Bayes Theorem is a method to determine conditional probabilities, the probability of one event occuring given that another event has already occured. Thus, conditional probabilities are a must in determining accurate predictions and probabilities in Machine learning.

$$P(A|B) = \frac{P(B|A)\ P(A)}{P(B)}$$

Probablity that man speaks the truth $= \frac{2}{3}$

Probablity that man lies $= \frac{1}{3}$

Probablity of getting a 4 $= \frac{1}{6}$

Probablity of not getting a 4 $= \frac{5}{6}$

Applying Bayes Theorem,

Formula

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$\Rightarrow \frac{\frac{1}{6} \times \frac{2}{3}}{\frac{1}{6} \times \frac{2}{3} + \frac{5}{6} \times \frac{1}{3}}$$

$$\Rightarrow \frac{\frac{2}{18}}{\frac{2}{18} + \frac{5}{18}} \Rightarrow \frac{\frac{2}{18}}{\frac{7}{18}} \Rightarrow \frac{2}{18} \times \frac{18}{7}$$

$$\Rightarrow \frac{2}{7} = 0.28$$

Pg.no. (2)

b. Principal Component Analysis is an unsupervised, non-parametric statistical technique primarily used for dimensionality reduction in machine learning.

Given data,

x	y
6.7	8.3
4	5.5
8.3	3.2
5.2	6.0
7.4	3.4
9.1	5.1
7	4.8

First step is to calculate mean of each column.

$$\bar{x} = \frac{6.7 + 4 + 8.3 + 5.2 + 7.4 + 9.1 + 7}{7}$$

$$= 6.81$$

$$\bar{y} = \frac{8.3 + 5.5 + 3.2 + 6.0 + 3.4 + 5.1 + 4.8}{7}$$

$$= 5.18$$

Center values in each column

$$\Rightarrow C = A - M$$

-0.114	3.114
-2.814	0.314
1.485	-1.985
-1.61	0.814
0.8585	-1.785
2.285	-0.085
0.185	-0.385

Next step is to calculate covariance matrix of centered matrix C.

$$\text{Cov}(x, y) = \frac{\sum (x_i - \bar{x}) \times (y_i - \bar{y})}{N}$$

$$= \frac{\sum_{1}^{7} (x_i - 6.81) \times (y_i - 5.18)}{7}$$

$= -1.1364$

$$Cov(x,x) = \frac{\sum (x_i - \bar{x})^2}{N}$$

$$= \sum_{i=1}^{7} \frac{(x_i - 6.81)^2}{7}$$

$$= 3.0580$$

$$Cov(y,y) = \frac{\sum (y_i - \bar{y})^2}{N}$$

$$= \sum_{i=1}^{7} \frac{(y_i - 5.18)^2}{7}$$

$$= 2.9580$$

Covariance matrix

$$= \begin{bmatrix} Cov(x,x) & Cov(x,y) \\ Cov(y,x) & Cov(y,y) \end{bmatrix}$$

$$= \begin{bmatrix} 3.058 & -1.136 \\ -1.136 & 2.958 \end{bmatrix}$$

Finally, we calculate the eigen-decomposition of the covariance matrix.

$$(A - \lambda I)v = 0$$

or

$$|A - \lambda I| = 0$$

$$A - \lambda I = \begin{bmatrix} 3.058 & -1.136 \\ -1.136 & 2.958 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3.058 - \lambda & -1.136 \\ -1.136 & 2.958 - \lambda \end{bmatrix}$$

$$|A - \lambda I| = (3.058 - \lambda)(-2.958 - \lambda) - (-1.136)(-1.136) = 0$$

Solving above eqn. we get (Eigen values):

$\lambda_1 = 4.145$ & $\lambda_2 = 1.870$

Putting λ in eigendecomposition eqn.

$$\begin{bmatrix} 3.058-\lambda & -1.136 \\ -1.136 & 2.958-\lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

For $\lambda_1 = 4.145$: $v_1 = 0.7224, v_2 = -0.691$

For $\lambda_2 = 1.870$: $v_1 = 0.6914, v_2 = 0.7224$

∴ Eigen vectors & eigen values

values = $[4.145, \ 1.870]$

vectors = $\begin{bmatrix} 0.722 & -0.691 \\ -0.691 & 0.722 \end{bmatrix}$

105.

a. Suppose, $K = 3$. Three objects are random

Centroid	Objects	
	A1	A2
c_1	3.8	9.9
c_2	7.8	12.2
c_3	6.2	18.5

Let d_1, d_2, d_3 from c_1, c_2, c_3.

Pg no. 7

Distance Calculation:

A_1	A_2	d_1	d_2	d_3	cluster
6.8	12.6	4.0	1.1	5.9	2
0.8	9.8	3.0	7.4	10.1	1
1.2	11.6	3.1	6.6	8.5	1
2.8	9.6	1.0	5.6	9.5	1
3.8	9.9	0.0	4.6	8.9	1
4.4	6.5	3.5	4.4	12.1	1
4.8	1.1	8.4	4.8	17.5	1
6.0	19.9	1.1	6.0	1.4	3
6.2	18.5	19.9	6.2	0.0	3
7.6	17.4	8.9	7.6	1.8	3
7.8	12.2	6.4	7.8	6.5	2
6.6	7.7	4.6	6.6	10.8	1
8.2	4.5	7.0	8.1	14.1	1
8.4	6.9	5.5	8.4	11.8	2
9.0	3.4	8.3	9.9	15.4	1
9.6	11.1	5.9	2.1	18.1	2

Calculation of new centroid :

New Centroid	Objects	
	A1	A2
C_1	4.6	7.1
C_2	8.2	10.7
C3	6.6	18.6

Cluster after second iteration

Centroid	Revised Centroids	
	A1	A2
C_1	5.6	7.1
C_2	8.1	12.0
C3	6.6	18.6

Ans no. (9)