Report on

Application of Set Theory and Group Theory

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Set Theory:

Set theory is the mathematical theory of well-determined collections, called sets, of objects that are called members, or elements, of the set. Pure set theory deals exclusively with sets, so the only sets under consideration are those whose members are also sets. The theory of the hereditarily-finite sets, namely those finite sets whose elements are also finite sets, the elements of which are also finite, and so on, is formally equivalent to arithmetic. So, the essence of set theory is the study of infinite sets, and therefore it can be defined as the mathematical theory of the actual—as opposed to potential—infinite.

The notion of set is so simple that it is usually introduced informally, and regarded as self-evident. In set theory, however, as is usual in mathematics, sets are given axiomatically, so their existence and basic properties are postulated by the appropriate formal axioms. The axioms of set theory imply the existence of a set-theoretic universe so rich that all mathematical objects can be construed as sets. Also, the formal language of pure set theory allows one to formalize all mathematical notions and arguments. Thus, set theory has become the standard foundation for mathematics, as every mathematical object can be viewed as a set, and every theorem of mathematics can be logically deduced in the Predicate Calculus from the axioms of set theory.

Both aspects of set theory, namely, as the mathematical science of the infinite, and as the foundation of mathematics, are of philosophical importance.

Example: A survey asks 200 people “What beverage do you drink in the morning”, and offers choices:

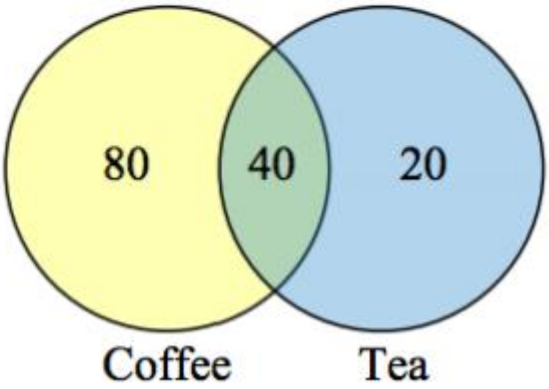
- Tea only
- Coffee only

- Both coffee and tea

Suppose 20 report tea only, 80 report coffee only, 40 report both. How many people drink tea in the morning? How many people drink neither tea or coffee?

⇨ This question can most easily be answered by creating a Venn diagram. We can see that we can find the people who drink tea by adding those who drink only tea to those who drink both: 60 people.

We can also see that those who drink neither are those not contained in the any of the three other groupings, so we can count those by subtracting from the cardinality of the universal set, 200.



200 – 20 – 80 – 40 = 60 people who drink neither.

Applications of Set Theory:

Set Theory is the foundation of many aspects of Computer Systems Engineering and data management.

- ❖ Binary Logic gates used to build microchips in ever digital device.
- ❖ 'Selection' one of the three fundamental constructs of Programming (the other two being sequence & iteration).
- ❖ Sets form the basis of many data structure used in programming, e.g. Set (Java Platform SE 7).
- ❖ Databases, Set Theory determines which data will be included and exclude in searches and selections.

- ❖ Security & Access control lists for users and systems.
- ❖ Distribution lists for messages such as email.
- ❖ Cryptography for secure communications.
- ❖ Bayesian filtering of junk email.
- ❖ Artificial Intelligence decision making.
- ❖ Image Recognition.
- ❖ Linguistics, Translation and proto-language study.

Group Theory:

Group theory studies the algebraic structures known as groups. The concept of a group is central to abstract algebra: other well-known algebraic structures, such as rings, fields, and vector spaces, can all be seen as groups endowed with additional operations and axioms. Groups recur throughout mathematics, and the methods of group theory have influenced many parts of algebra. Linear algebraic groups and Lie groups are two branches of group theory that have experienced advances and have become subject areas in their own right.

Group theory is the study of symmetry, whenever an object or a system's property is invariant under a transformation then we can analyse the object using group theoretic methods.

For example, the mathematical objects like, a circle remains invariant under a rotation, a vector remains invariant under a translation (vector expressed in Cartesian system of coordinates); both of them can be analysed using group theory.

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Mathematicians classify mathematics as a study of patterns and group theory is the study of symmetries, this says enough about the place of group theory at the heart of modern mathematics.

Various physical systems, such as crystals and the hydrogen atom, may be modelled by symmetry groups. Thus, group theory and the closely related representation theory have many important applications in physics, chemistry, and materials science. Group theory is also central to public key cryptography.

Applications of Group Theory:

- In Physics, the Lorentz group expresses the fundamental symmetry of many of the known fundamental laws of nature. The special theory of relativity, Maxwell's field equations in electromagnetism and Dirac equation in theory of electron are all invariant under Lorentz transformations.
- Modern particle physics is based on symmetry principles and by the application of group theory the existence of several particle was predicted before they were experimentally observed.
- In chemistry, the symmetry of a molecule provides us with the information of what energy levels the orbitals will be, what the orbitals symmetries are, what transitions can occur between energy levels, even bond order and all of that is calculated using group theory.
- Group theory is helpful in finding the correct linear combination of wavefunctions that is needed to diagonalize the Hamiltonian. This procedure involves the concept of equivalence which applies to situations where equivalent atoms sit at symmetrically equivalent sites.
- The irreducible representations of the symmetry group of Schrödinger’s equation label the states and specify their degeneracies (e.g., an atom in a crystal field).