G.H. RAISONI COLLEGE OF ENGG., NAGPUR

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CAE - 2 EXAMINATION FOR SPLIT - 2

DEPARTMENT: AI

SEM/SEC : 3 / A DATE: 28/08/2020

SUBJECT : DM

ROLL NO : 58

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CO2.

Q.3.

i) Let e be the identity element for $*$ in G.

Then we have $a * a^{-1} = e$, where $a^{-1} \in G$

Also $(a^{-1})^{-1} * a^{-1} = e$

Therefore, $(a^{-1})^{-1} = a * a^{-1}$.

Thus, by right cancellation law, we have $(a^{-1})^{-1} = a$.

ii)

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ii.) Let a and $b \in G$ and G is a group for $*$, then
$a * b \in G$ (closure)

Therefore, $(a*b)^{-1} * (a*b) = e$

Let a^{-1} and b^{-1} be the inverses of a and b respectively, then
$a^{-1}, b^{-1} \in G$.
——— (1)

Therefore, $(b^{-1} * a^{-1}) * (a * b)$
$= b^{-1} * (a^{-1} * a) * b$
(associativity)
$= b^{-1} * e * b = b^{-1} * b$
$= e$
——— (2)

From (1) and (2) we have
$(a * b)^{-1} * (a * b) = (b^{-1} * a^{-1}) * (a * b)$
$(a * b)^{-1} = b^{-1} * a^{-1}$
by right cancellation law.

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Q CO2.

2. Let $M = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$

given M is non singular.

Now
Multiplication ~~of~~ of a 2×2 matrix with any other 2×2 matrix would yeild a 2×2 matrix.

$\therefore$ Closure property satisfied ——— ①

Let N be any other 2×2 matrix & P be a 2×2 matrix.

$(MN)P = M(NP)$

$\therefore$ Associativity satisfied ——— ②

Multiplying

$M \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$

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$\therefore$ Identity element is present. —— 3

Given M is non-singular & every non-singular matrix has a inverse which is unique.

$\because$ Inverse present —— 4

from 1, 2, 3 & 4,

Set of all matrices is Group.

CO2.

1. $(G, *)$ Group $a, b \in G$

~~For every there existance inverse.~~

~~$a * a^{-1} = e = a^{-1} * a^{-1}$~~

~~$a * e = e * a^{-1}$~~

Given: $a * b = a * b^{-1}$ & $b * a = a * b^{-1}$

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Now,

$a * b = b * a^{-1}$

$a * b * b^{-1} = b * a^{-1} * b^{-1}$ (Multiply $b*$ both sides)

$a * (e) = b * (ab)^{-1}$ $(b * b^{-1} = e)$

$a = b * (ab)^{-1}$ ($a * e = a$ identity law)

$a * (ab) = b * (ab)^{-1} * (ab)$

$a^2 b = b * e$ $(a * a^{-1} = e)$

$a^2 b * b^{-1} = b * b^{-1}$ (b^{-1} multiply both sides)

$a^2 = e$

$\therefore a * a = e$

$\therefore a = a^{-1}$ —— ①

$\therefore a * a^{-1} = e = a^{-1} * a^{-1}$

$a * e = e * a^{-1}$

$e * a = a * a^{-1} = a * e$

$\therefore a^{-1} = a$ —— ②

So, $a^4 = e$

$\therefore o(a) = a^4$

So, 4 is order of a.

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