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Summer Semester, Carry Over Examinations, 2018-19

B. Tech. I Year I Semester

Sub: Engineering Mathematics I (AHM 1201/BMAS 0101)

Time: 3 Hrs. **Max. Marks: 80**

SECTION A **(2 x 10 = 20 Marks)**

Note: Attempt ALL questions of this section.

Q.1. If $W(x,y,z) = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then find

$$xW_x + yW_y + zW_z .$$

Q.2. If $z = e^{ax+by} f(ax - by)$, show that:

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$$

Q.3. If $u = x^{2019} f\left(\frac{y}{x}\right)$, use Euler's theorem to evaluate

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$

Q.4. If $u = f(r, s)$ and $r = x + at$, $s = y + bt$; prove that:

$$a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} = \frac{\partial u}{\partial t}$$

Q.5. If $u = x(1-y)$, $v = xy$, compute $\frac{\partial(u,v)}{\partial(x,y)}$.

Q.6. Find rank of the matrix $A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

Q.7. Define Skew-Hermitian matrix with an example.

Q.8. If $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 9 \\ 0 & 0 & 1 \end{bmatrix}$, find the Eigen values of $4A$ and $2A^{-1}$.

Q.9. Solve the differential equation:

$$\frac{d^2y}{dx^2} - y = e^{-x}$$

Q.10. Find the particular integral of:

$$(D^2 - 2D + 1)\, y = e^x \cos x$$

SECTION B **(5x3x2 = 30 Marks)**

Note: Attempt ANY TWO parts of each question of this section.

Q.1. (a) Expand $x^2y + \sin y + e^x$ in powers of $(x-1)$ and $(y-\pi)$.

(b) If $u = \log\left(\frac{x^2+y^2}{\sqrt{x}+\sqrt{y}}\right)$, Apply Euler's theorem to find

$$xu_x + yu_y.$$

(c) Find the maximum and minimum distances of the point (3, 4, 12) from a sphere whose centre is at (0, 0, 0) and radius 1.

Q.2. (a) Solve the following system of equations by matrix method:

$$x + 2y - z = 3,$$

$$3x - y + 2z = 1,$$

$$2x - 2y + 3z = 2,$$

$$x - y + z = -1.$$

(b) Verify Cayley – Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

(c) Solve the differential equation:

$$(x^2 - 4y)dx = (4x - y^2)dy.$$

Q.3. (a) Solve the linear differential equation:

$$(D^2 + 4)\,y = \cos^2 x.$$

(b) Solve the system of differential equations:

$$\frac{dx}{dt} - y = t,$$

$$\frac{dy}{dt} + x = 1.$$

(c) Solve the differential equation:

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} = n^2 y; \qquad \text{n is a constant.}$$

SECTION C (10x3 = 30 Marks)

Note: Attempt ALL questions of this section.

Q.1. If $u = tan^{-1}(\frac{x^3+y^3}{x-y})$, prove that:

(a) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u,$

(b) $x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} = \sin 4u - \sin 2u.$

Q.2. Solve the differential equation:

$$\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}(1 + y^2)x\, dy = 0.$$

Q.3. Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2} + y = \tan x$$

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