Summer Semester, Carry Over Examinations, 2018-19

B. Tech. I Year II Semester

Sub: Engineering Mathematics II (AHM 2201/BMAS 0102)

Time: 3 Hrs. **Max. Marks: 80**

SECTION A **(2 x 10 = 20 Marks)**

Note: Attempt ALL questions of this section.

Q.1. Test the following infinite series for convergence or divergence:

$$\log\frac{1}{2} - \log\frac{2}{3} + \log\frac{3}{4} - \log\frac{4}{5} + \cdots$$

Q.2. Test the convergence of the series:

$$1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \cdots$$

for all values of x.

Q.3. Prove that,

$$\beta(m,n) = \beta(n,m) \text{ if } m > 0, n > 0.$$

Q.4. Evaluate using Beta and Gamma functions:

$$\int_0^\infty \frac{dx}{1+x^4}$$

Q.5. Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of the circle

$$x^2 + y^2 = 9.$$

Q.6. Determine the area of region bounded by the curves

$$xy = 2, 4y = x^2 \text{ and } y = 4.$$

Q.7. If $\vec{A} = yx^2\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$ then find curl curl $\vec{A}$.

Q.8. Find div $(r^3\vec{r})$ if $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

Q.9. Using Green's theorem, evaluate the integral $\oint_c (xy\,dy - y^2 dx)$,

where c is the square cut from the first quadrant by the lines x = 1,

y = 1.

Q.10. Solve the partial differential equation:

$$DD'(D - 2D' - 3)z = 0$$

The terms have their usual meanings.

SECTION B (5x3x2 = 30 Marks)

Note: Attempt ANY TWO parts of each question of this section.

Q.1. (a) Test the convergence or divergence of the series:

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin\frac{1}{n}$$

(b) Test the series for convergence and divergence:

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots + \frac{x^n}{n^2+1} + \cdots$$

(c) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{sin\theta}} \times \int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\, d\theta$

using Beta and Gamma functions.

Q.2. (a) Change the order of integration in $\int_0^a \int_y^a \frac{x\, dx\, dy}{x^2+y^2}$ and hence evaluate the same.

(b) Evaluate the following by changing in to polar coordinates:

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\ dx.dy.$$

(c) Find the directional derivative of f (x, y, z) = 2 x y + z^2 at the point (1, −1, 3) in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

Q.3. (a) Prove the vector identity:

$$curl\ (u\vec{a}) = u\ curl\ \vec{a} + (grad\ u) \times \vec{a}$$

(b) Find the work done when a force

$$\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$$

moves a particle in the xy- plane from (0, 0) to (1, 1) along the parabola $y^2 = x$.

(c) Solve the partial differential equation:

$$(mz - ny)p + (nx - lz)q - ly + mx = 0$$

The terms have their usual meanings.

SECTION C **(10x3 = 30 Marks)**

Note: Attempt ALL questions of this section.

Q.1. Test the following series for convergence and divergence:

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \cdots, x > 0.$$

Q.2. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes in A, B and C. Apply Dirichlet's integral to find the volume of the tetrahedron OABC. Also find its mass if the density at any point is $k\,x^2y^2z^2$, if k is a constant.

Q.3. Solve the partial differential equation:

$$(D^2 - 3DD' + 2D'^2)z = e^{2x-y} + e^{x+y} + \cos(x + 2y)$$