MULTIFRACTAL PROPERTIES OF THE INDIAN FINANCIAL MARKET

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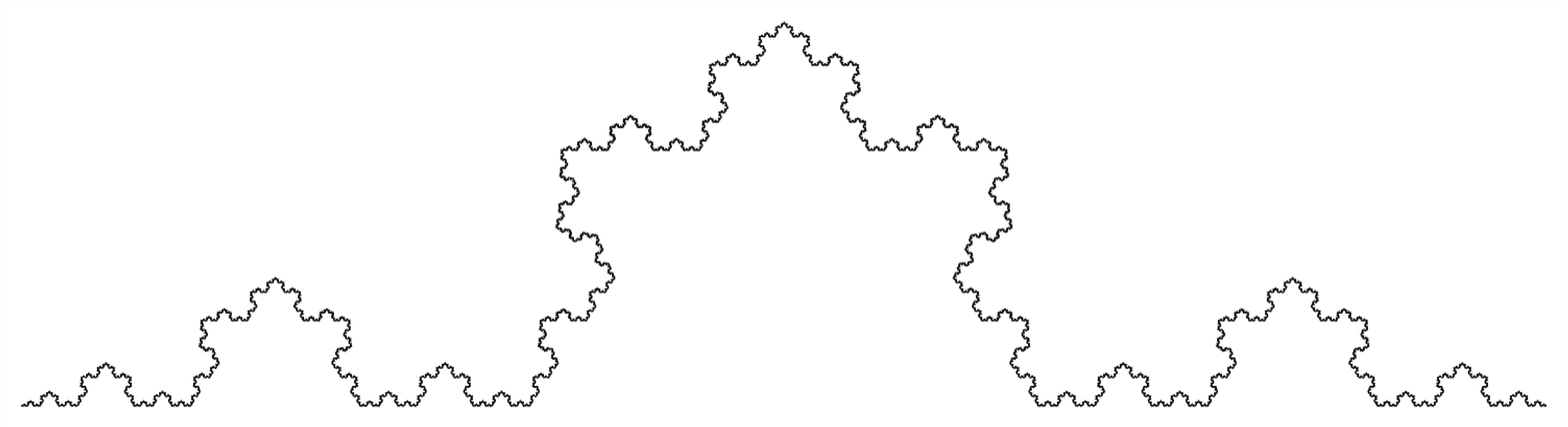
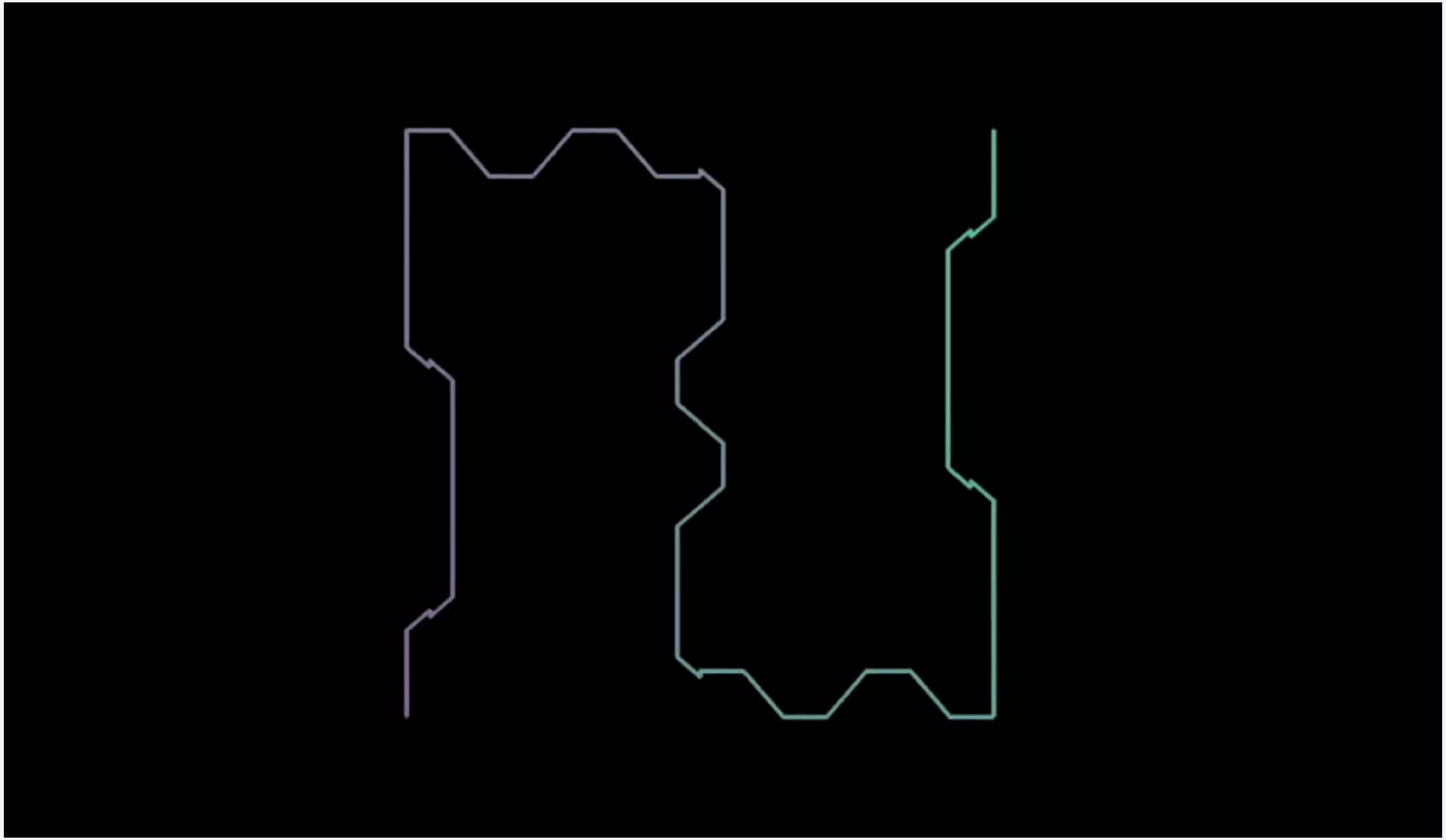
CONTENT

INTRODUCTION

- What is the objective of the paper?
- What is multifractality?





INTRODUCTION

- **Multifractality Due to a Broad Probability Density Function**
  - This type of multifractality occurs when the time series has a wide range of values that are not equally probable. Specifically, the probability distribution of the values of the time series has heavy tails
- **Multifractality due to different long-range correlations for small and large fluctuations**
  - This type of multifractality arises from different long-range correlations in the time series for small and large fluctuations. It means that the time series exhibits different correlation structures depending on the size of the fluctuations.

INTRODUCTION

Statistical Mechanics and its relation to Economics

Complex Systems

- Both fields deal with complex systems composed of many interacting components.
- In statistical mechanics, these components are particles or molecules, while in economics, they are agents such as individuals, firms, or financial entities.

Non-linearity and Feedback:

- Both systems exhibit non-linear interactions and feedback mechanisms.
- In physics, these can be forces between particles, while in economics, they can be market forces, supply-demand interactions, or network effects

METHOD

1. Calculate Logarithmic Returns and Normalize it:

$$g_t = \frac{\log(P(t+1)) - \log(P(t))}{\sigma}$$

where *P(t)* is the closing price at time *t* and *σ* is standard deviation

2. Profile Calculation

$$Y(i) = \sum_{k=1}^{i} [g_k - \langle g \rangle]$$

where ⟨g⟩ is the mean of the normalized returns.

METHOD

3. Segmentation

- The profile $Y(i)$ is divided into N_s**=int(**N/s**)** non-overlapping segments of equal length s.
- Since the length of the series may not be an exact multiple of s, the same procedure is repeated starting from the opposite end, resulting in **2**N_s segments.

METHOD

4. Detrending and Variance Calculations

- We Calculate the local trend for each of the ***$2N_S$*** segments by a polynomial fit of the time-series. Then determine the variance.

$$F^2(s,\nu) = \frac{1}{s}\sum_{i=1}^{s}\left[Y((\nu-1)s+i) - y_\nu(i)\right]^2$$

For each segment ν from **1 to N_S**

$$F^2(s,\nu) = \frac{1}{s}\sum_{i=1}^{s}\left[Y(N-(\nu-N_s)s+i) - y_\nu(i)\right]^2$$

For each segment ν from N_S **+1 to 2N_S**

y_ν**(i)** is the fitting polynomial in segment ν, representing the local trend

METHOD

5. qth-Order Fluctuation Calculation

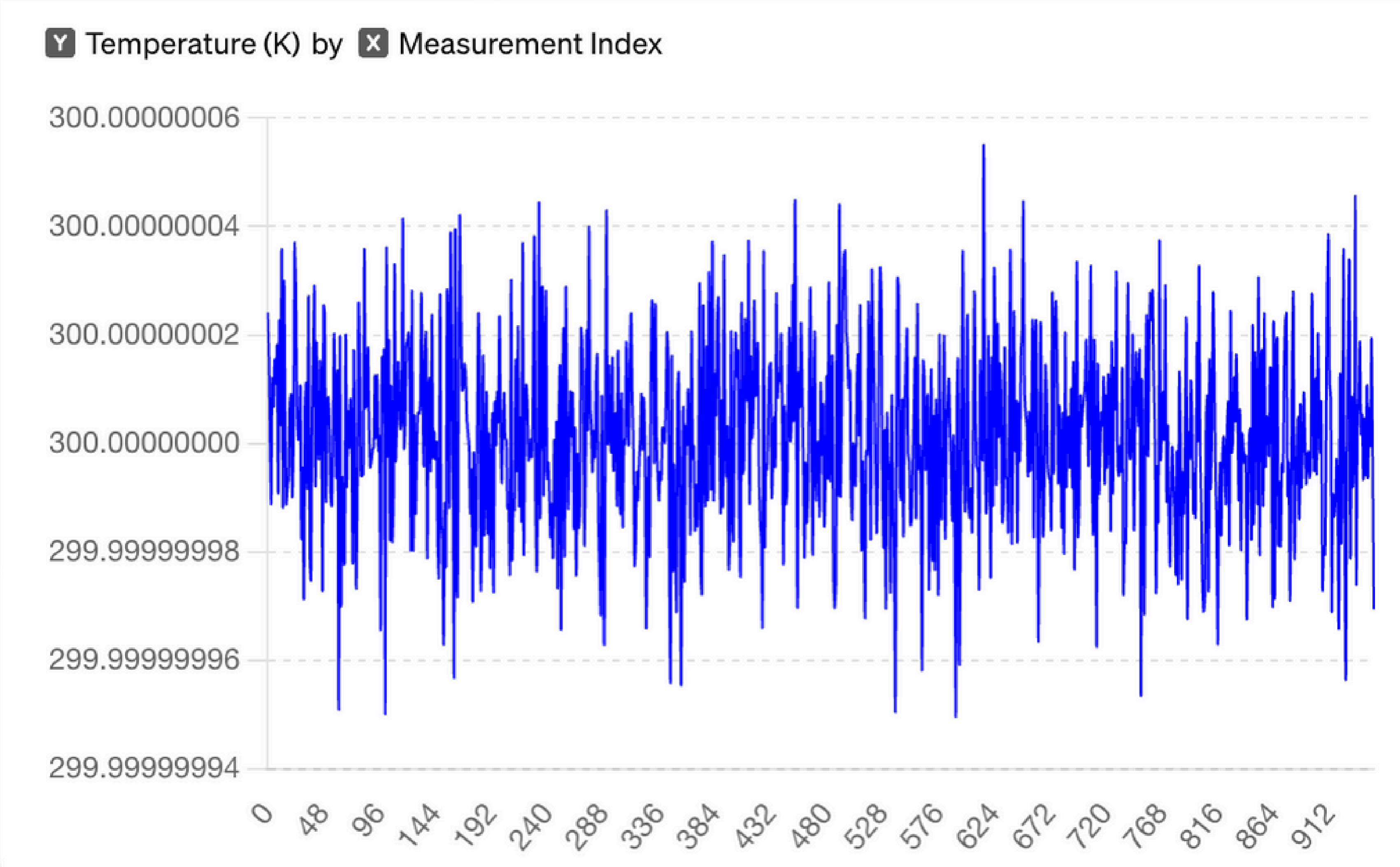
- Next we obtain the qth-Order Fluctuation Function

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} [F^2(s,\nu)]^{q/2} \right\}^{1/q}$$

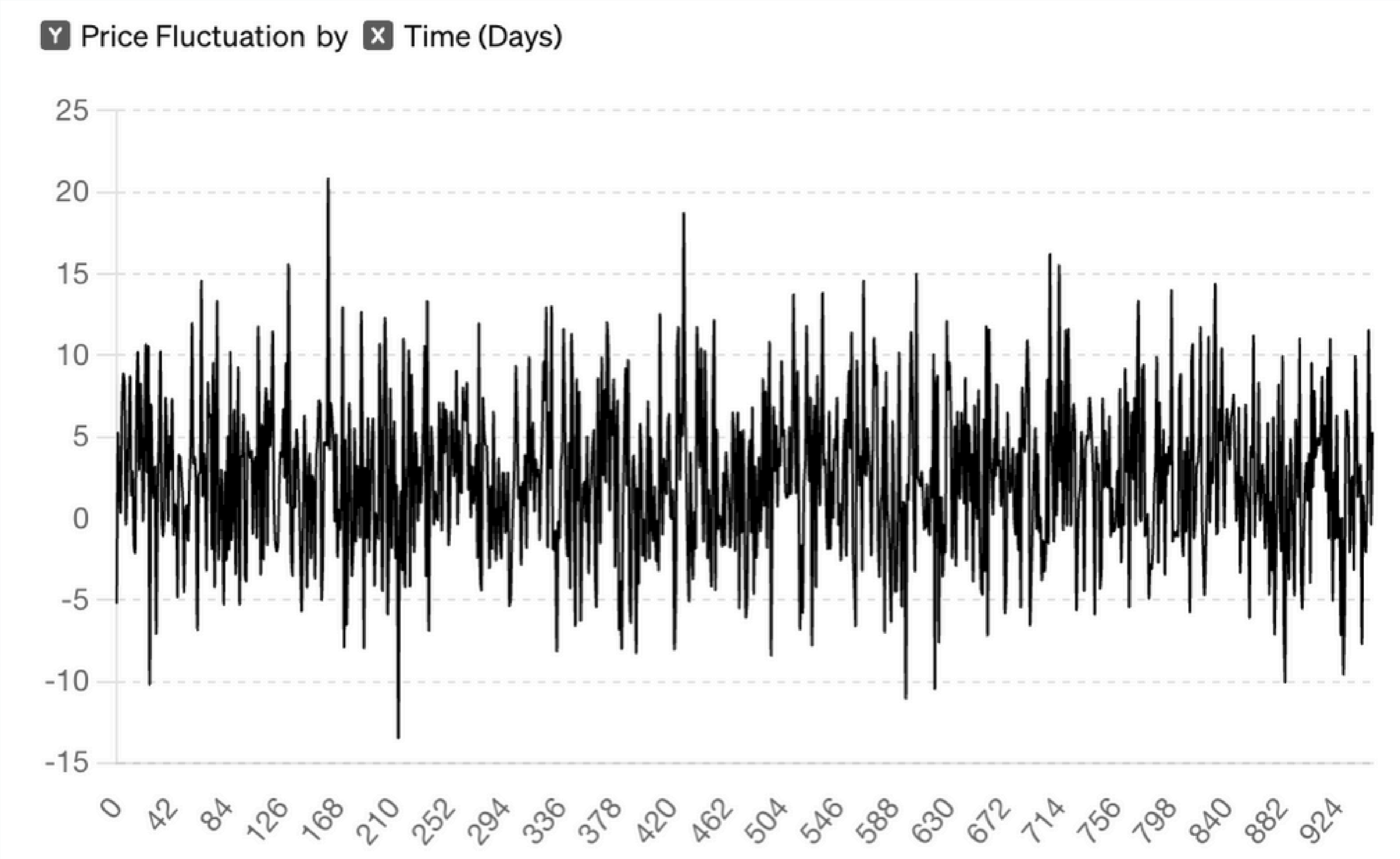
- When ***q=2***, the fluctuation function corresponds to the root mean square fluctuation, which is commonly used to assess the overall variability of the data.
- For ***q>2***, the fluctuation function gives more weight to larger deviations, highlighting the impact of extreme events.
- For ***q<2***, the fluctuation function is more sensitive to smaller fluctuations.

INTRODUCTION

Statistical Mechanics and its relation to Economics



Temperature Fluctuations



Stock Price Fluctuations

METHOD

6. Scaling Behaviour of Fluctuation Function

- If the time series exhibits multifractal behavior, the fluctuation functions $F_q(s)$ will scale as a power-law with respect to s

$$F_q(s) \sim s^{h(q)}$$

h(q) is the generalized Hurst exponent for the moment ***q***.

- The family of scaling exponents ***h(q)*** can be obtained by observing the slope of the log–log plot of ***Fq(s)*** versus ***s***

- The relationship between the generalized Hurst exponent ***h(q)*** and the classical multifractal scaling exponent ***τ(q)*** is given by

$$\tau(q) = qh(q) - 1$$

METHOD

6. Scaling Behaviour of Fluctuation Function

- Using the spectrum of generalized Hurst exponents ***h(q)***, we can calculate the singularity strength ***α*** and the singularity spectrum ***f(α)***

$$\alpha = h(q) + q\frac{dh(q)}{dq}$$

$$f(\alpha) = q[\alpha - h(q)] + 1$$

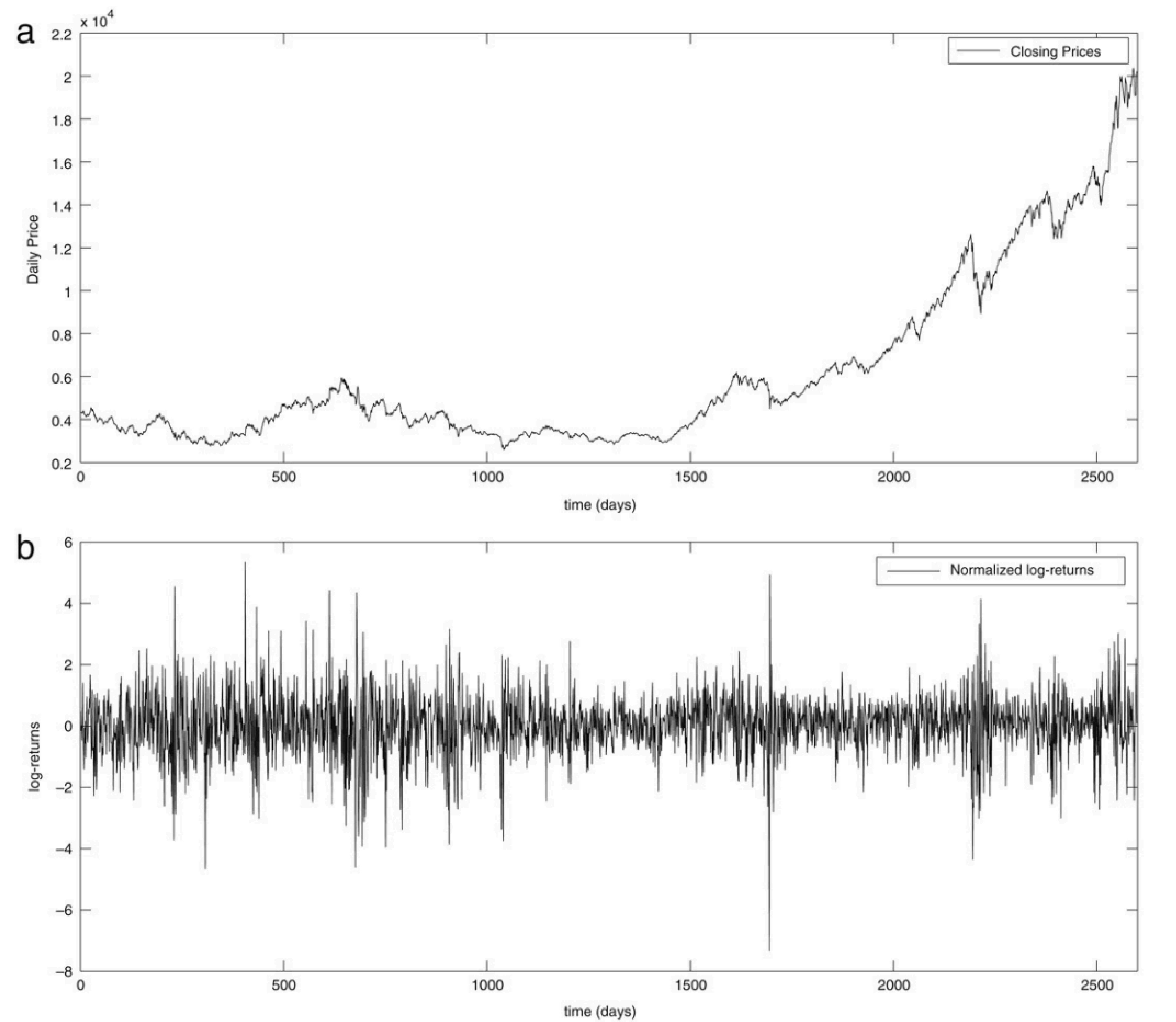
MFDFA ANALYSIS ON MARKETS

Data Processing

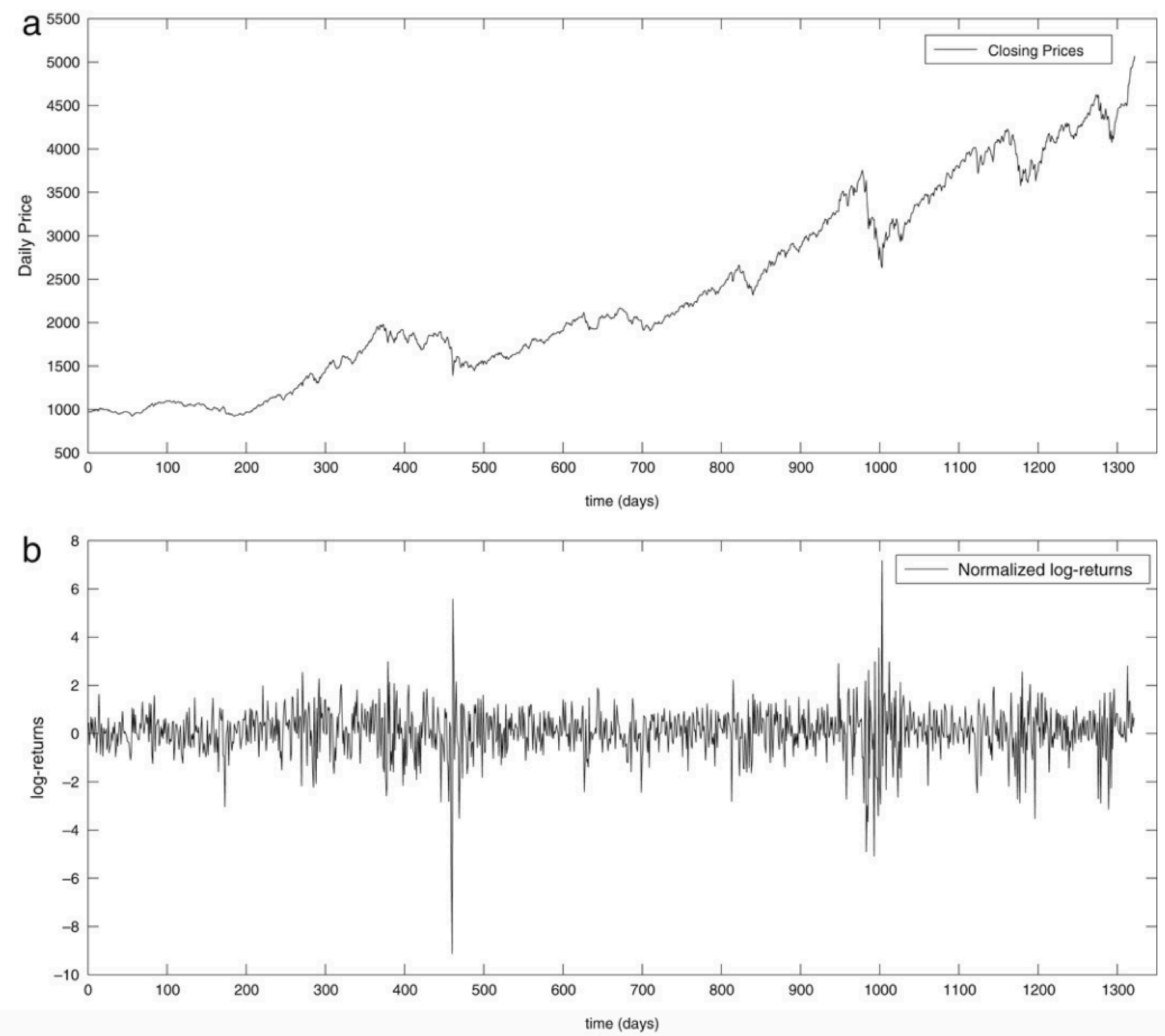
- BSE data from July 1997 to December 2007 was taken.
- NSE data from August 2002 to December 2007 was taken.
- S&P 500 data from July 1997 to December 2007 was taken.

- The normalized log returns were then calculated and plotted

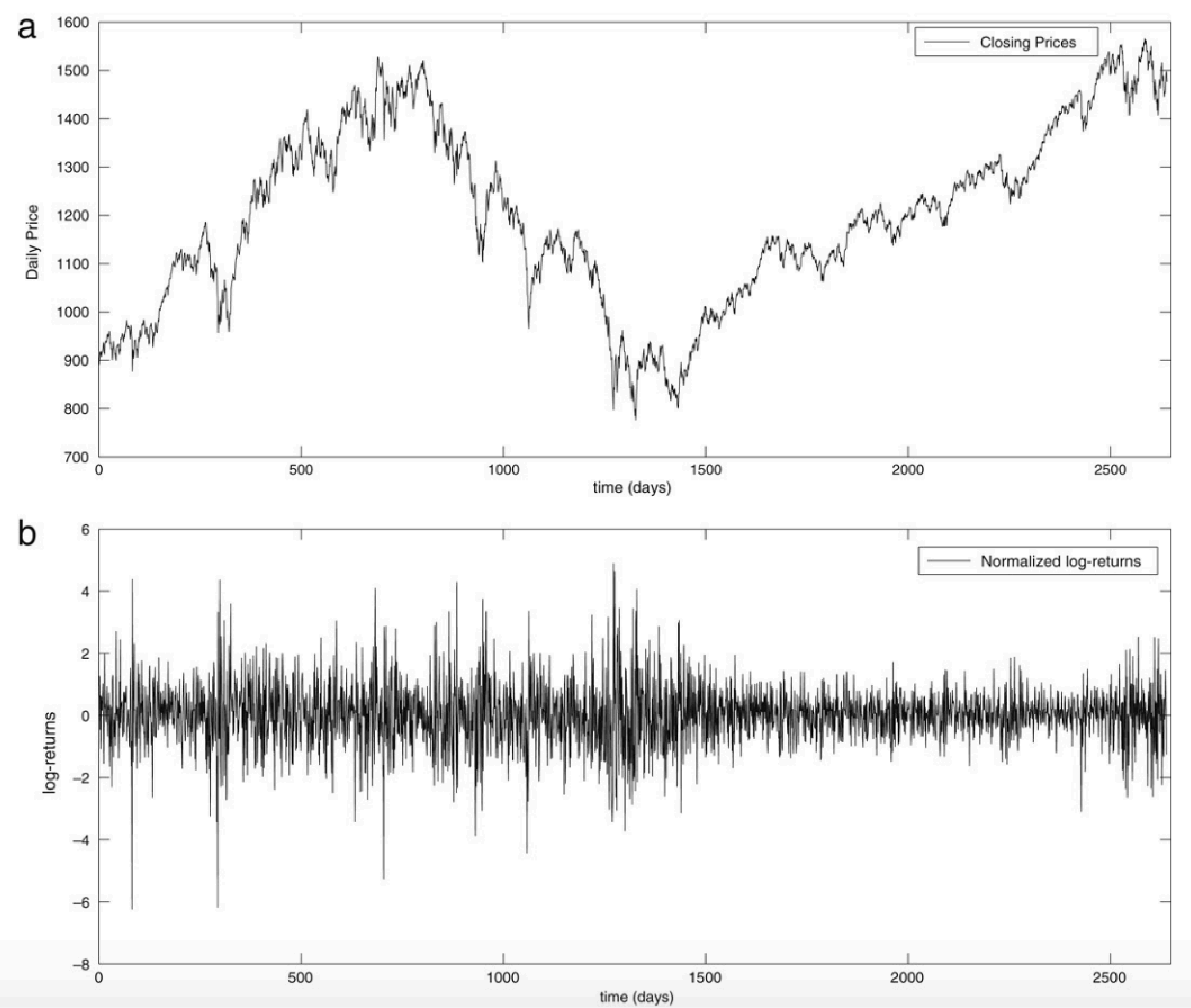
BSE


a
x 10^4
Closing Prices
Daily Price
time (days)
b
Normalized log-returns
log-returns
time (days)

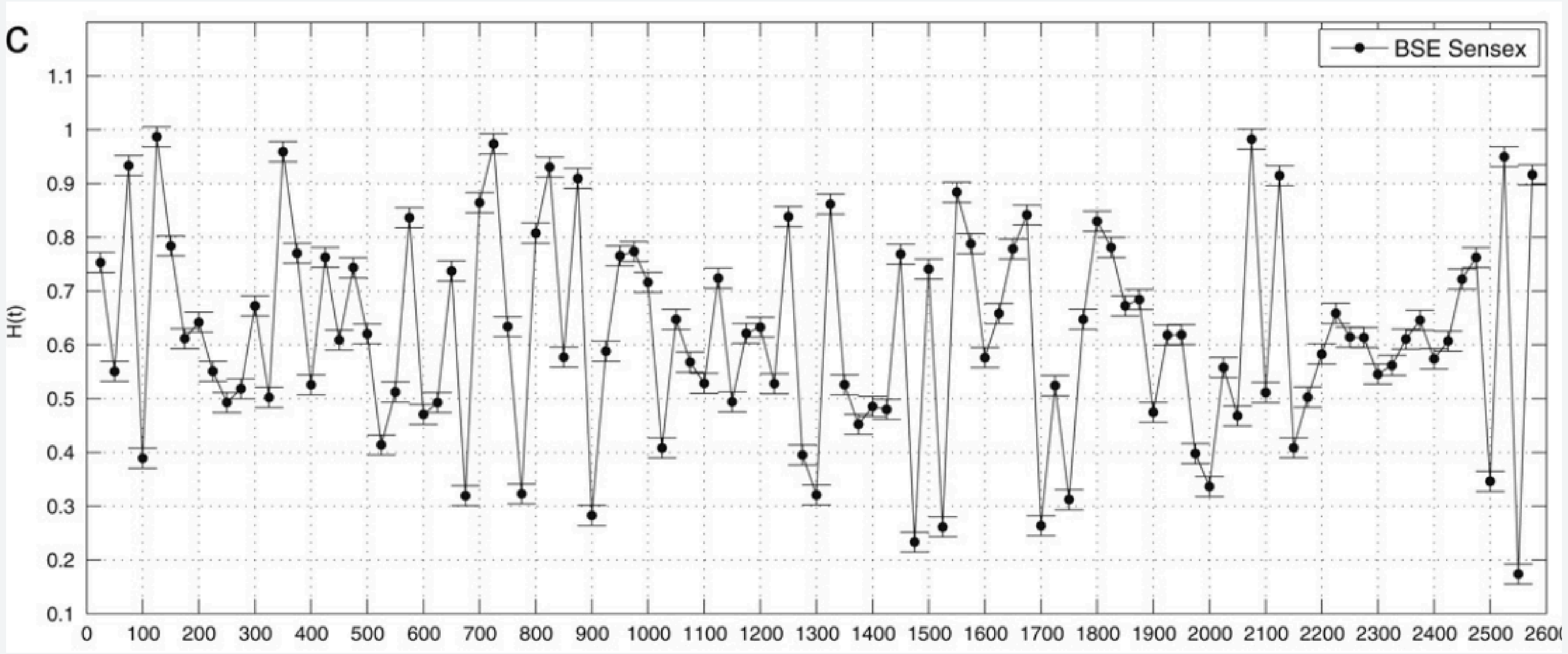
NSE

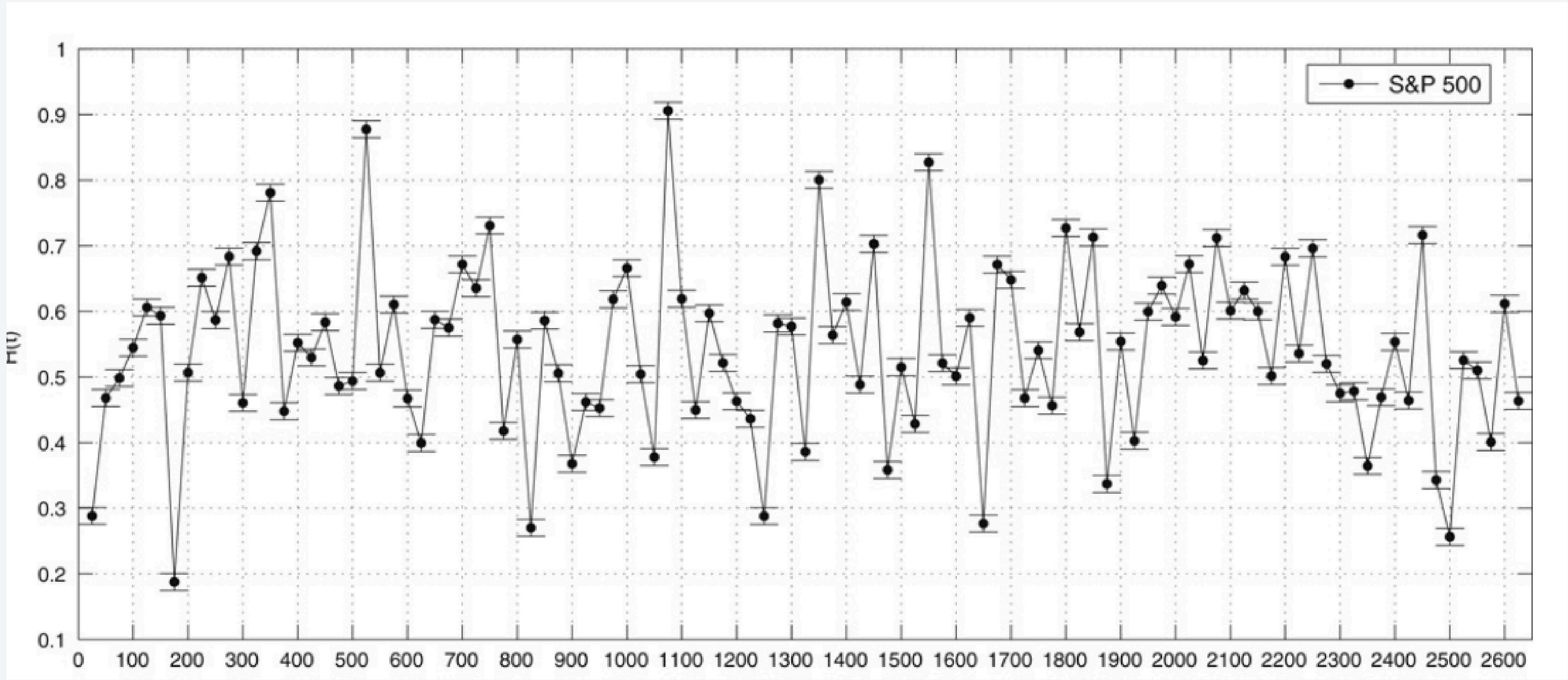

a
Closing Prices
Daily Price
time (days)
b
Normalized log-returns
log-returns
time (days)

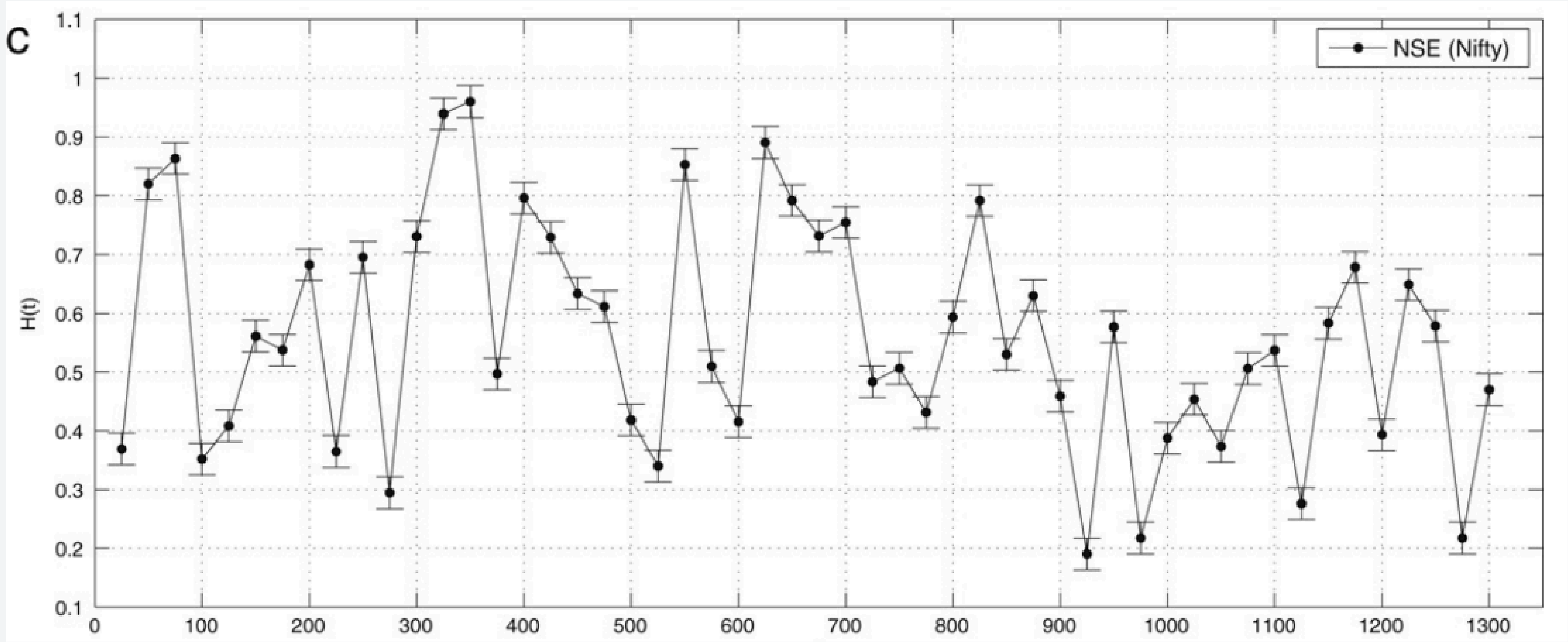
S&P500

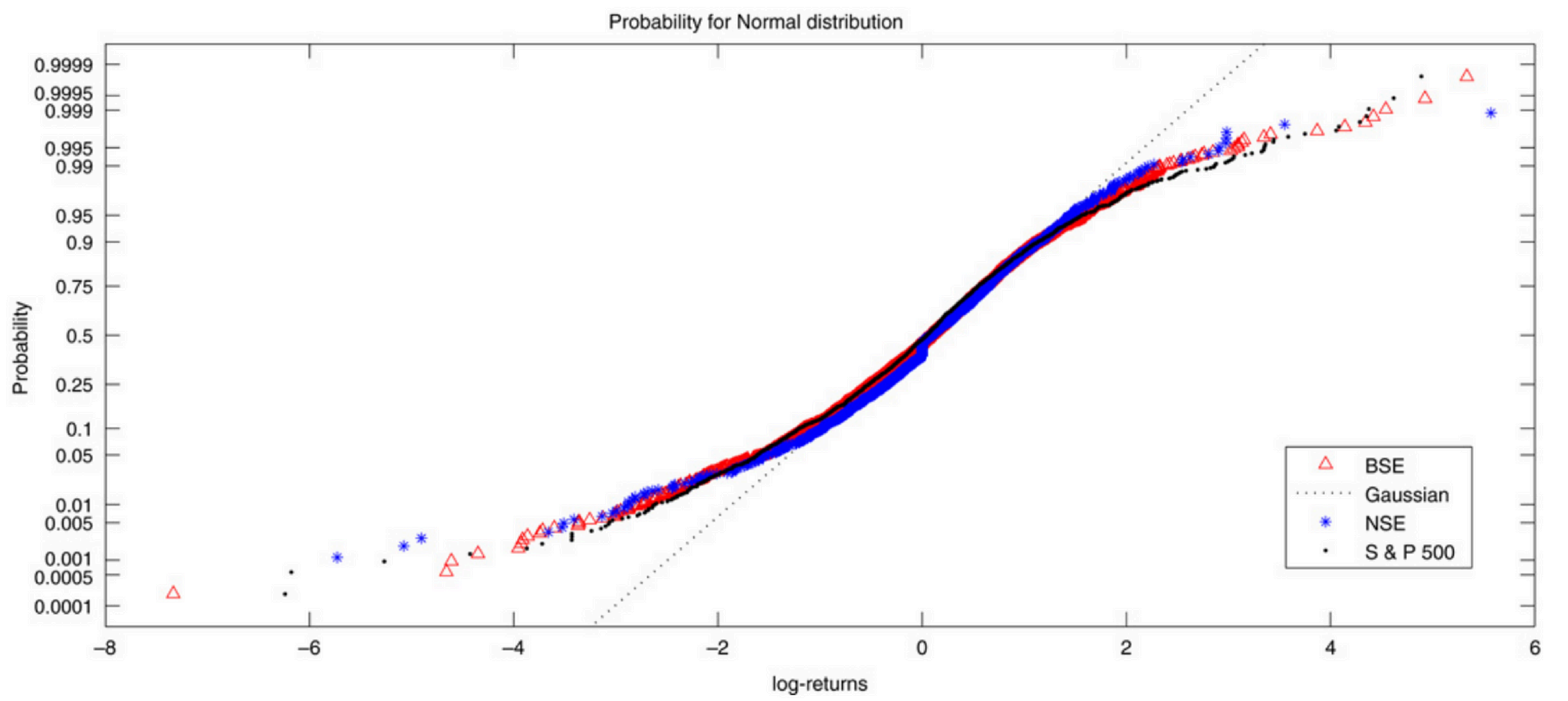


a
Closing Prices
Daily Price
1600
1500
1400
1300
1200
1100
1000
900
800
700
0
500
1000
1500
2000
2500
time (days)
b
Normalized log-returns
log-returns
6
4
2
0
−2
−4
−6
−8
0
500
1000
1500
2000
2500
time (days)

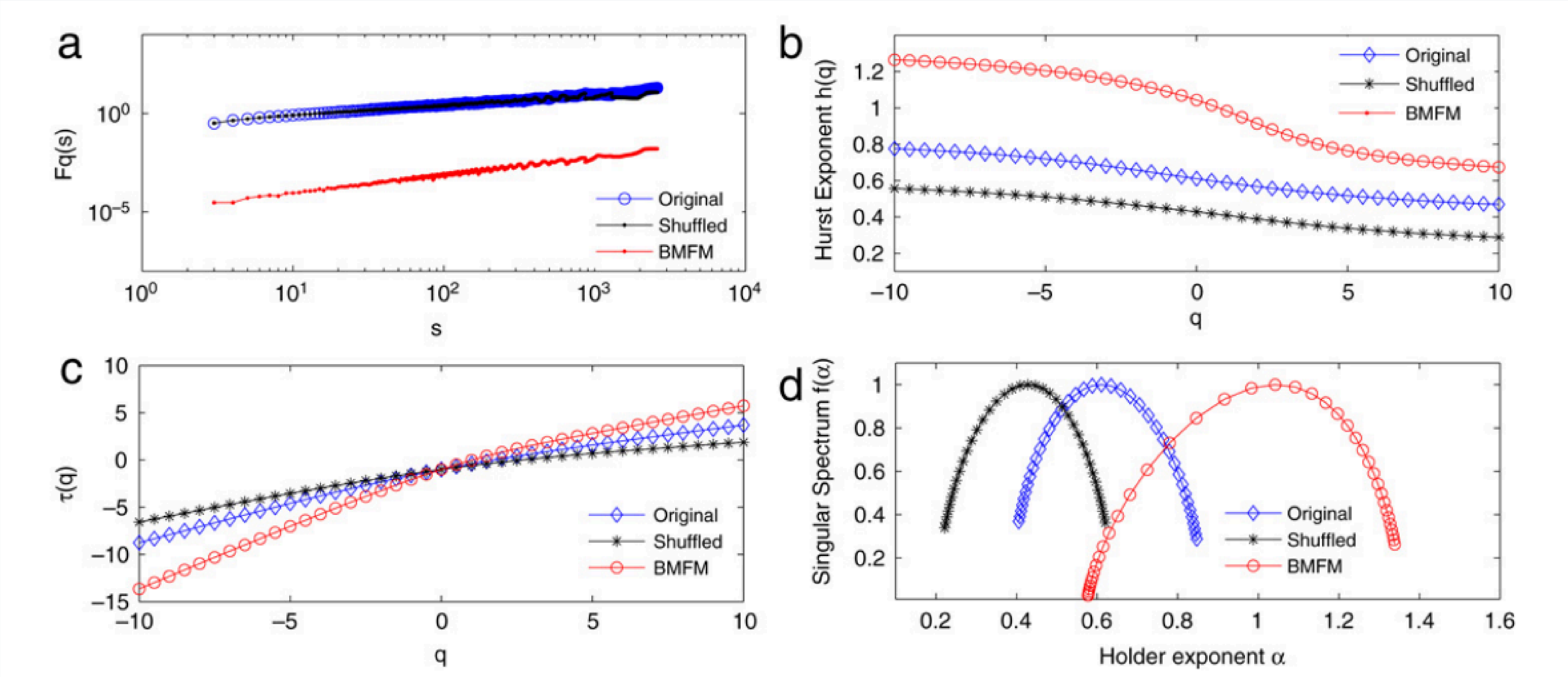

c
BSE Sensex
H(t)


S&P 500
H(t)


c
NSE (Nifty)
H(t)

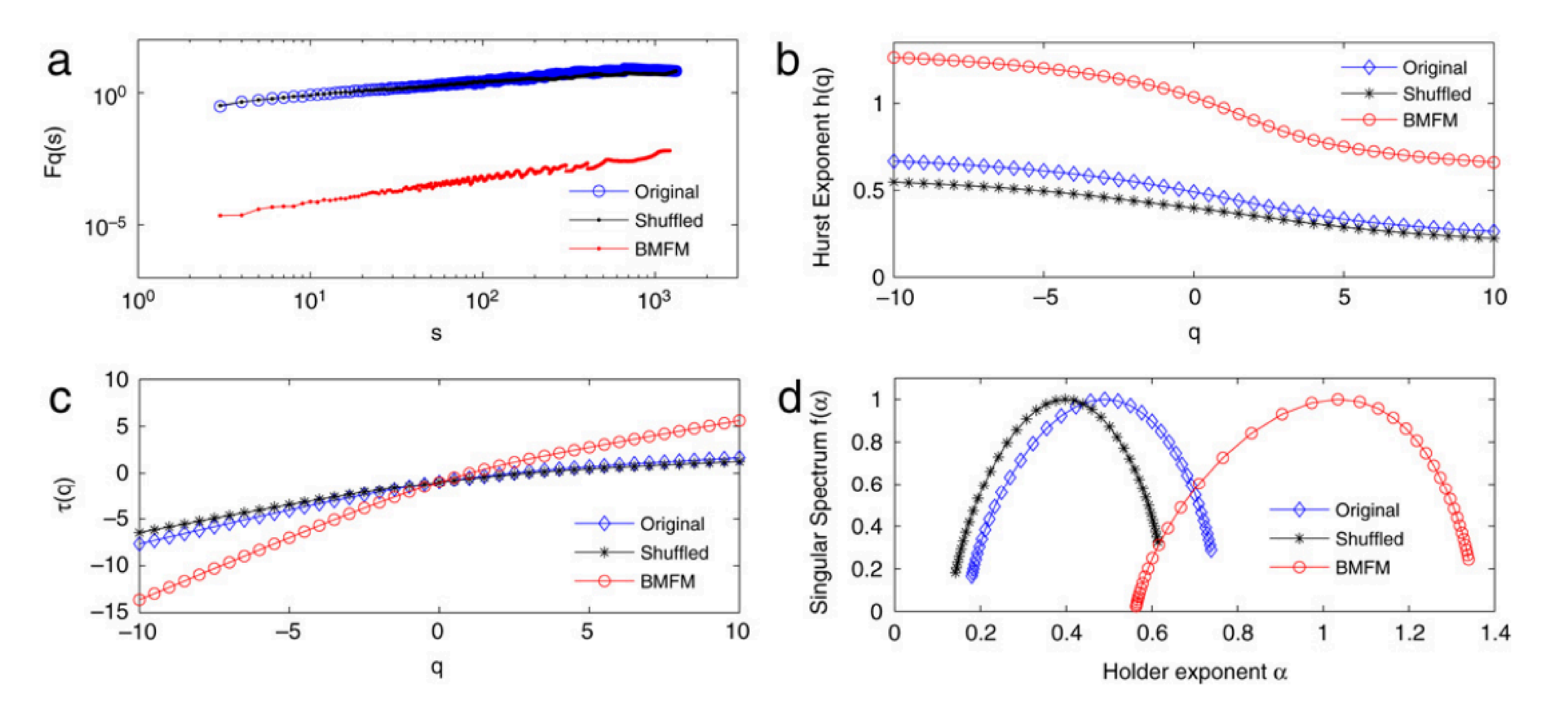


Probability for Normal distribution
0.9999
0.9995
0.999
0.995
0.99
0.95
0.9
0.75
0.5
0.25
0.1
0.05
0.01
0.005
0.001
0.0005
0.0001
Probability
−8
−6
−4
−2
0
2
4
6
log-returns
BSE
Gaussian
NSE
S & P 500



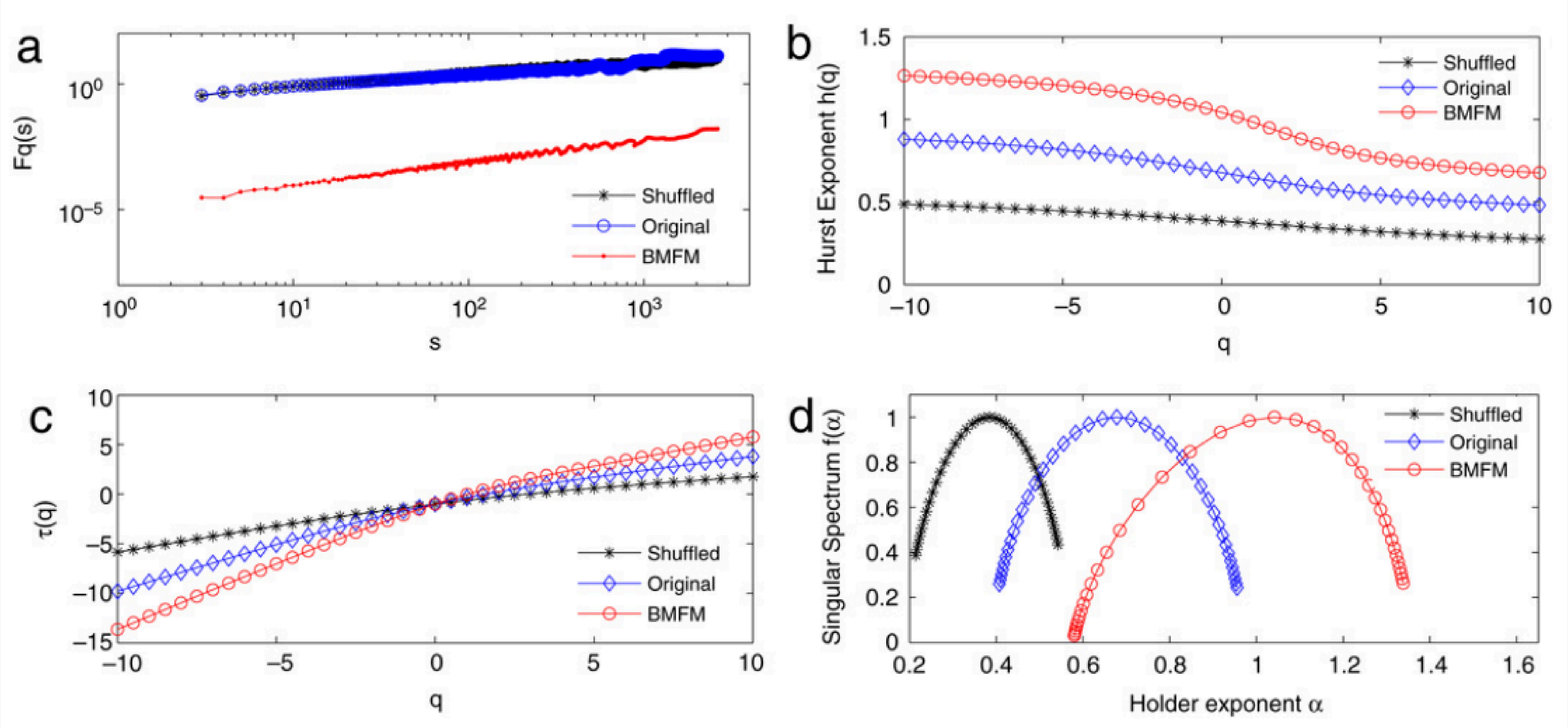
a
Fq(s)
s
Original
Shuffled
BMFM
b
Hurst Exponent h(q)
q
Original
Shuffled
BMFM
c
τ(q)
q
Original
Shuffled
BMFM
d
Singular Spectrum f(α)
Holder exponent α
Original
Shuffled
BMFM

BSE



a
b
c
d
Fq(s)
s
Original
Shuffled
BMFM
Hurst Exponent h(q)
q
τ(q)
Singular Spectrum f(α)
Holder exponent α

NSE


a
Fq(s)
s
Shuffled
Original
BMFM
b
Hurst Exponent h(q)
q
Shuffled
Original
BMFM
c
τ(q)
q
Shuffled
Original
BMFM
d
Singular Spectrum f(α)
Holder exponent α
Shuffled
Original
BMFM

SPX

CONCLUSION

- The MF-DFA method allows a reliable multifractal characterization of the Indian financial markets (BSE and NSE indices) and the US S&P 500 index.
- We find that the multifractal scaling exponents $h(q)$ and $\tau(q)$ have a nonlinear dependence on the moment q. On the basis of the nonlinearity of these scaling exponents and singularity spectrum $f(\alpha)$, we prove that the Indian and US markets both exhibit multifractality.
- By analyzing the results for the original and shuffled time series, we find that the multifractality in these financial markets is due to long-range correlations and broad probability density function.
- The above results are compared with the series generated by the BMFM. We find that the multifractal strengths for the financial markets are smaller than the BMFM.