

Day-2: Data Structures And Algorithms:

Summary

- Arrays are collections of elements stored continuously in memory
- Each element in an array is indexed starting from 0
- Arrays provide efficient access to elements using their index position
- The continuous memory allocation allows for fast element retrieval

Prefix Sum

Prefix sum is a technique used to efficiently calculate the sum of elements in a subarray. It involves creating an auxiliary array where each element stores the cumulative sum of all elements up to that index.

Key Concepts

- **Prefix sum array:** An array where $\text{prefixSum}[i] = \text{arr}[0] + \text{arr}[1] + \dots + \text{arr}[i]$
- **Range sum query:** Sum of elements from index L to R can be calculated as $\text{prefixSum}[R] - \text{prefixSum}[L-1]$
- **Time complexity:** $O(n)$ for preprocessing, $O(1)$ for each query
- **Space complexity:** $O(n)$ for storing the prefix sum array

Example

Given array: [3, 1, 4, 2, 5]

Prefix sum array: [3, 4, 8, 10, 15]

To find sum from index 1 to 3: $\text{prefixSum}[3] - \text{prefixSum}[0] = 10 - 3 = 7$

Applications

- Efficiently answering multiple range sum queries
- Finding equilibrium index in an array
- Solving subarray sum problems

- 2D matrix sum queries (using 2D prefix sums)

Suffix Sum

Suffix sum is a technique similar to prefix sum, but instead calculates the cumulative sum of elements from a given index to the end of the array. It involves creating an auxiliary array where each element stores the sum of all elements from that index onwards.

Key Concepts

- **Suffix sum array:** An array where $\text{suffixSum}[i] = \text{arr}[i] + \text{arr}[i+1] + \dots + \text{arr}[n-1]$
- **Range sum query:** Sum of elements from index L to R can be calculated as $\text{suffixSum}[L] - \text{suffixSum}[R+1]$
- **Time complexity:** $O(n)$ for preprocessing, $O(1)$ for each query
- **Space complexity:** $O(n)$ for storing the suffix sum array

Example

Given array: [3, 1, 4, 2, 5]

Suffix sum array: [15, 12, 11, 7, 5]

To find sum from index 1 to 3: $\text{suffixSum}[1] - \text{suffixSum}[4] = 12 - 5 = 7$

Applications

- Efficiently answering range sum queries from right to left
- Finding maximum suffix sum in dynamic programming problems
- Solving problems requiring cumulative sums from the end
- Combined with prefix sum for bidirectional range queries

Subarrays

A subarray is a contiguous part of an array. For an array of size n, there are $n * (n+1)/2$ possible subarrays. Understanding subarrays is fundamental for solving many array-based problems.

Key Concepts

- **Subarray definition:** A contiguous sequence of elements within an array
- **Number of subarrays:** For an array of length n , total subarrays = $n*(n+1)/2$
- **Generation:** Can be generated using two nested loops with indices i (start) and j (end)
- **Properties:** Every element is part of multiple subarrays

Common Problems

- Finding maximum/minimum subarray sum
- Counting subarrays with specific properties (sum = k , product < k , etc.)
- Finding longest subarray with certain conditions
- Subarray with given sum

Kadane's Algorithm

Kadane's Algorithm is an efficient algorithm to find the maximum sum of a contiguous subarray. It uses dynamic programming principles and runs in $O(n)$ time complexity.

Key Concepts

- **Algorithm idea:** At each position, decide whether to extend the current subarray or start a new one
- **Current maximum:** $\text{max_current} = \max(\text{arr}[i], \text{max_current} + \text{arr}[i])$
- **Global maximum:** Track the maximum sum seen so far
- **Time complexity:** $O(n)$
- **Space complexity:** $O(1)$

Basic Implementation Logic

```
max_current = max_global = arr[0]
for i in range(1, len(arr)):
    max_current = max(arr[i], max_current + arr[i])
    max_global = max(max_global, max_current)
return max_global
```

Variations of Kadane's Algorithm

- **Maximum subarray sum (basic):** Standard implementation finding the largest sum
- **Minimum subarray sum:** Similar approach but tracking minimum instead of maximum
- **Maximum subarray product:** Track both max and min products (handles negative numbers)
- **Circular array maximum sum:** Consider wrap-around by finding $\max(\text{normal_kadane}, \text{total_sum} - \text{min_subarray_sum})$
- **With indices:** Track start and end positions of the maximum subarray
- **At least K elements:** Modified to ensure subarray has minimum length
- **With deletion allowed:** Allow skipping up to k elements while finding maximum sum
- **2D Kadane's:** Extend to matrices by fixing columns and applying 1D Kadane on rows

Two Pointer Technique

The two pointer technique is an algorithmic pattern that uses two pointers to iterate through data structures, often to optimize time complexity from $O(n^2)$ to $O(n)$. The pointers can move in the same direction or opposite directions.

Types of Two Pointer Approaches

1. Opposite Direction (Left and Right)

- **Use case:** When array is sorted or needs to find pairs
- **Pattern:** Start one pointer at beginning, other at end, move based on condition
- **Applications:** Two sum in sorted array, container with most water, trapping rain water

```
left, right = 0, len(arr) - 1
while left < right:
    if condition:
        # process
```

```
    left += 1
else:
    right -= 1
```

2. Same Direction (Fast and Slow)

- **Use case:** When need to maintain a window or remove duplicates
- **Pattern:** Both pointers start at beginning, fast pointer explores, slow pointer maintains valid position
- **Applications:** Remove duplicates, partition array, remove elements

```
slow = 0
for fast in range(len(arr)):
    if condition:
        arr[slow] = arr[fast]
        slow += 1
```

3. Sliding Window (Variable Size)

- **Use case:** Finding subarrays with specific properties
- **Pattern:** Expand window with right pointer, shrink with left pointer
- **Applications:** Longest substring without repeating characters, minimum window substring

```
left = 0
for right in range(len(arr)):
    # add arr[right] to window
    while window_invalid:
        # remove arr[left] from window
        left += 1
    # update result
```

4. Fixed Window Size

- **Use case:** When window size is predetermined
- **Pattern:** Maintain constant distance between pointers

- **Applications:** Maximum sum of k consecutive elements, average of subarrays of size k

```
window_sum = sum(arr[:k])
max_sum = window_sum
for i in range(k, len(arr)):
    window_sum += arr[i] - arr[i-k]
    max_sum = max(max_sum, window_sum)
```

Common Two Pointer Problems

- **Pair with target sum:** Find two numbers that add up to target (sorted array)
- **Three sum:** Find triplets that sum to target
- **Container with most water:** Find two lines that form container with maximum area
- **Trapping rain water:** Calculate water trapped between bars
- **Remove duplicates:** Remove duplicates from sorted array in-place
- **Longest substring without repeating:** Find length of longest substring with unique characters
- **Minimum window substring:** Find smallest window containing all characters of pattern
- **Subarray with product less than k:** Count subarrays with product less than k
- **Fruits into baskets:** Maximum fruits with at most 2 types
- **Longest subarray with sum $\leq k$:** Find longest subarray with sum at most k

Time and Space Complexity

- **Time complexity:** Typically $O(n)$ as each element is visited at most twice
- **Space complexity:** Usually $O(1)$ extra space, sometimes $O(n)$ if using hash map for window tracking

Classification of Data Structures

Data structures can be classified in multiple ways based on their organization, access patterns, and characteristics. Understanding these classifications helps

in choosing the right data structure for specific problems.

1. Linear vs Non-Linear Classification

Linear Data Structures

Data structures where elements are arranged in a sequential manner, with each element connected to its previous and next element.

- **Arrays:** Fixed-size sequential collection with contiguous memory allocation
- **Linked Lists:** Dynamic sequential collection with non-contiguous memory
- **Stacks:** LIFO (Last In First Out) structure
- **Queues:** FIFO (First In First Out) structure
- **Characteristics:** Elements are traversed sequentially, only one level exists

Non-Linear Data Structures

Data structures where elements are arranged in a hierarchical or interconnected manner, not in sequence.

- **Trees:** Hierarchical structure with root and child nodes (Binary Tree, BST, AVL, etc.)
- **Graphs:** Network structure with nodes and edges connecting them
- **Heaps:** Complete binary tree with heap property (min-heap or max-heap)
- **Tries:** Tree-like structure for storing strings efficiently
- **Characteristics:** Elements are on multiple levels, hierarchical relationships exist

2. Classification by Container Type

Sequential Containers

Store elements in a linear sequence, maintaining the order of insertion.

- **Arrays:** Fixed-size contiguous storage
- **Vectors/Dynamic Arrays:** Resizable arrays with contiguous storage
- **Linked Lists:** Node-based sequential storage
- **Deques:** Double-ended queues allowing insertion/deletion at both ends

Associative Containers

Store elements in a sorted order based on keys, allowing fast lookup.

- **Sets:** Store unique elements in sorted order
- **Maps/Dictionaries:** Store key-value pairs in sorted order by key
- **Multisets:** Allow duplicate elements in sorted order
- **Multimaps:** Allow duplicate keys in sorted order

Unordered Associative Containers

Store elements using hash tables for fast average-case lookup.

- **Hash Sets:** Store unique elements with $O(1)$ average lookup
- **Hash Maps:** Store key-value pairs with $O(1)$ average lookup
- **Unordered Multisets:** Allow duplicates with hash-based storage
- **Unordered Multimaps:** Allow duplicate keys with hash-based storage

Container Adapters

Provide restricted interfaces to underlying sequential containers.

- **Stacks:** Adapter providing LIFO access
- **Queues:** Adapter providing FIFO access
- **Priority Queues:** Adapter providing access to highest priority element

3. Classification by Indexing

Direct Indexing (Random Access)

Elements can be accessed directly using an index in $O(1)$ time.

- **Arrays:** $\text{arr}[i]$ provides immediate access to i th element
- **Vectors:** Dynamic arrays with direct indexing capability
- **Hash Tables:** Direct access via hash function (average $O(1)$)
- **Time complexity:** $O(1)$ for access
- **Advantage:** Fast element retrieval at any position

Sequential Indexing

Elements must be accessed sequentially from the beginning or a known position.

- **Linked Lists:** Must traverse from head to reach specific position
- **Stacks:** Only top element is directly accessible
- **Queues:** Only front and rear elements are accessible
- **Time complexity:** $O(n)$ for access to arbitrary position
- **Advantage:** Efficient insertion/deletion without shifting

Key-Based Indexing

Elements are accessed using keys rather than numerical indices.

- **Hash Maps:** Access via key using hash function
- **Binary Search Trees:** Access via key with $O(\log n)$ complexity
- **Tries:** Access via string keys
- **Time complexity:** $O(1)$ average for hash-based, $O(\log n)$ for tree-based

4. Classification by Priority

Priority-Based Data Structures

Elements are organized and accessed based on priority rather than insertion order.

Heap (Priority Queue)

- **Max Heap:** Highest priority (maximum) element at root
- **Min Heap:** Lowest priority (minimum) element at root
- **Operations:** Insert $O(\log n)$, Extract max/min $O(\log n)$, Peek $O(1)$
- **Applications:** Task scheduling, Dijkstra's algorithm, heap sort

Priority Queue Implementations

- **Binary Heap:** Complete binary tree, most common implementation
- **Fibonacci Heap:** Better amortized time for decrease-key operation
- **Binomial Heap:** Collection of binomial trees
- **Pairing Heap:** Simplified implementation with good practical performance

Comparison: FIFO vs LIFO vs Priority

- **FIFO (Queue):** First element inserted is first to be removed, no priority
- **LIFO (Stack):** Last element inserted is first to be removed, no priority
- **Priority Queue:** Element with highest/lowest priority is removed first, regardless of insertion order

Summary Table

Classification Type	Categories	Examples
Organization	Linear, Non-Linear	Array (Linear), Tree (Non-Linear)
Container	Sequential, Associative, Unordered, Adapters	Vector (Sequential), Map (Associative)
Indexing	Direct, Sequential, Key-Based	Array (Direct), Linked List (Sequential)
Priority	FIFO, LIFO, Priority-Based	Queue (FIFO), Stack (LIFO), Heap (Priority)

Operations of Data Structures

Data structures support various fundamental operations that allow manipulation and access of data. Understanding these operations is crucial for effective use of data structures in problem-solving.

1. Traversal

Accessing each element of a data structure exactly once for processing.

- **Linear structures:** Sequential traversal from start to end
- **Trees:** Inorder, preorder, postorder, level-order traversal
- **Graphs:** DFS (Depth-First Search), BFS (Breadth-First Search)
- **Time complexity:** $O(n)$ where n is number of elements

2. Searching

Finding the location of an element with a given value in the data structure.

- **Linear Search:** $O(n)$ - checking each element sequentially

- **Binary Search:** $O(\log n)$ - dividing search space in sorted structures
- **Hash-based Search:** $O(1)$ average - direct access using hash function
- **Tree Search:** $O(\log n)$ in balanced BST, $O(n)$ in worst case

3. Insertion

Adding a new element to the data structure at a specified position.

- **Arrays:** $O(n)$ - may require shifting elements
- **Linked Lists:** $O(1)$ at head/tail, $O(n)$ at arbitrary position
- **Hash Tables:** $O(1)$ average case
- **Trees:** $O(\log n)$ in balanced trees, $O(n)$ in worst case
- **Heaps:** $O(\log n)$ - maintain heap property

4. Deletion

Removing an element from the data structure.

- **Arrays:** $O(n)$ - requires shifting elements after deletion
- **Linked Lists:** $O(1)$ if node reference known, $O(n)$ to find node
- **Hash Tables:** $O(1)$ average case
- **Trees:** $O(\log n)$ in balanced trees with restructuring
- **Heaps:** $O(\log n)$ - maintain heap property after removal

5. Sorting

Arranging elements in a specific order (ascending or descending).

- **Comparison-based:** $O(n \log n)$ - Merge Sort, Quick Sort, Heap Sort
- **Linear time:** $O(n)$ - Counting Sort, Radix Sort (specific conditions)
- **Simple sorts:** $O(n^2)$ - Bubble Sort, Insertion Sort, Selection Sort

6. Merging

Combining two data structures into a single structure.

- **Sorted arrays:** $O(n + m)$ - merge two sorted arrays
- **Linked lists:** $O(1)$ to $O(n)$ depending on implementation

- **Trees:** $O(n + m)$ - merge two trees

Underflow and Overflow

Underflow and overflow are error conditions that occur when operations violate the capacity constraints of a data structure.

Overflow

Overflow occurs when attempting to insert an element into a data structure that is already full or has reached its maximum capacity.

When Overflow Occurs

- **Stack Overflow:** Pushing element when stack is full (fixed-size stack)
- **Queue Overflow:** Enqueueing element when queue is at maximum capacity
- **Array Overflow:** Trying to insert beyond allocated size in fixed-size array
- **Buffer Overflow:** Writing data beyond buffer boundaries (security risk)
- **Heap Overflow:** Insufficient memory available for dynamic allocation

Consequences of Overflow

- **Program crash:** Unhandled overflow can terminate program execution
- **Data corruption:** Overwriting adjacent memory locations
- **Security vulnerabilities:** Buffer overflow exploits in attacks
- **Unpredictable behavior:** System instability and errors

Handling Overflow

- **Check before insertion:** Verify available space before adding elements
- **Dynamic resizing:** Automatically increase capacity (vectors, dynamic arrays)
- **Return error codes:** Indicate failure to caller for appropriate handling
- **Throw exceptions:** Use exception handling mechanisms
- **Circular implementation:** Overwrite oldest data in circular buffers

```

# Stack overflow check example
class Stack:
    def __init__(self, capacity):
        self.capacity = capacity
        self.stack = []

    def push(self, item):
        if len(self.stack) >= self.capacity:
            raise Exception("Stack Overflow: Cannot push, stack is full")
        self.stack.append(item)

    def is_full(self):
        return len(self.stack) == self.capacity

```

Underflow

Underflow occurs when attempting to remove an element from an empty data structure or access data that doesn't exist.

When Underflow Occurs

- **Stack Underflow:** Popping element from empty stack
- **Queue Underflow:** Dequeueing element from empty queue
- **List Underflow:** Removing element from empty linked list
- **Array Underflow:** Accessing negative index or index beyond bounds
- **Heap Underflow:** Extracting element from empty heap

Consequences of Underflow

- **Runtime errors:** Attempting to access non-existent data
- **Null pointer exceptions:** Dereferencing null/undefined references
- **Index out of bounds:** Accessing invalid array positions
- **Logical errors:** Incorrect program behavior and results

Handling Underflow

- **Check before removal:** Verify structure is not empty before deletion
- **Return special values:** Return null, -1, or sentinel values to indicate empty state
- **Throw exceptions:** Raise appropriate exceptions for empty structure access
- **Use size/isEmpty methods:** Check state before performing operations
- **Defensive programming:** Validate all inputs and state conditions

```
# Stack underflow check example
class Stack:
    def __init__(self):
        self.stack = []

    def pop(self):
        if self.is_empty():
            raise Exception("Stack Underflow: Cannot pop, stack is empty")
        return self.stack.pop()

    def peek(self):
        if self.is_empty():
            raise Exception("Stack Underflow: Cannot peek, stack is empty")
        return self.stack[-1]

    def is_empty(self):
        return len(self.stack) == 0
```

Comparison: Overflow vs Underflow

Aspect	Overflow	Underflow
Definition	Inserting into full structure	Removing from empty structure
Common Operations	Push, Enqueue, Insert	Pop, Dequeue, Delete
Cause	Exceeding maximum capacity	No elements available

Prevention	Check capacity before insertion	Check if empty before removal
Solution	Increase capacity or reject operation	Handle empty case gracefully

Best Practices

- **Always validate:** Check preconditions before operations
- **Use built-in methods:** Utilize isEmpty(), isFull(), size() methods
- **Handle gracefully:** Implement proper error handling and recovery
- **Document constraints:** Clearly specify capacity limits and requirements
- **Test edge cases:** Test with empty, full, and boundary conditions
- **Use dynamic structures:** Prefer dynamic over fixed-size when appropriate
- **Provide clear feedback:** Return meaningful error messages to users

Day 2 Complete Summary

Day 2 covered fundamental concepts of Data Structures and Algorithms, focusing on arrays, advanced techniques, and comprehensive understanding of data structure operations and constraints.

Core Topics Covered

- **Arrays Fundamentals:** Learned that arrays are collections of elements stored continuously in memory with zero-based indexing, providing efficient $O(1)$ access to elements through their index position
- **Prefix Sum Technique:** Mastered the technique for efficiently calculating subarray sums using cumulative sum arrays, enabling $O(1)$ range queries with $O(n)$ preprocessing
- **Data Structure Types:** Explored linear structures (arrays, linked lists, stacks, queues), non-linear structures (trees, graphs), and their classifications based on memory allocation and element organization
- **Fundamental Operations:** Studied six key operations - Traversal $O(n)$, Searching $O(1)$ to $O(n)$, Insertion, Deletion, Sorting $O(n \log n)$, and Merging - with their time complexities across different data structures

- **Overflow and Underflow:** Understood error conditions when violating capacity constraints, including causes, consequences, handling strategies, and best practices for prevention

Key Takeaways

- **Memory efficiency:** Continuous memory allocation in arrays enables fast retrieval but may cause overflow in fixed-size implementations
- **Time complexity awareness:** Different data structures offer different performance characteristics for the same operations
- **Error handling:** Always validate preconditions (`isEmpty`, `isFull`) before operations to prevent overflow and underflow
- **Appropriate structure selection:** Choose data structures based on required operations, memory constraints, and performance requirements
- **Dynamic vs Static**