# P.S 4

(1) $G = (V, E)$ be a graph.
$M_1, M_2$ are 2 matchings of $G$.

$$G' = (V, M_1 \cup M_2)$$

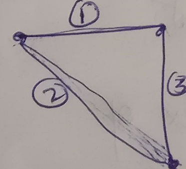
To show: $G'$ is bipartite:

→ Bipartite: A graph $G = (V, E)$ is bipartite if vertices can be split into 2 sets [left & right set] so that all ~~nodes~~ edges connect a node in the left set ~~is connected to~~ to that in the ~~left~~ right set.

Sol): Since, $M_1, M_2$ are 2 matchings of $G$ each of the vertices in $M_1$ & $M_2$ have a degree 1.
⇒ each of the vertices have at most $M_1 \cup M_2$ ⇒ each of the vertices have at most degree 2.

Next,
Lemma 1: $G'$ doesn't have any odd length cycles.

Proof: By contradiction, assume it has a odd length cycle. that means, one ~~part~~ set of even edges & one set of odd edges belong to $M_1$ or $M_2$




But if the even edges can't belong to only one of $M_1$ or $M_2$.
~~Because~~ Assume ① & ② are even edges belonging to $M_1$ & ③ is odd edge belonging to $M_2$.
→ But $M_1$ ~~can~~ is a matching & hence ~~can't~~ contain ① & ② as vertex A has degree 1.
→ ∴ $G'$ has only even cycles.
→ Color a vertex $V_0$ with blue, ~~as $G'$ has even~~ ~~of~~ and successive verteices with red & blue.
→ ~~As~~ As $G'$ has even cycles, it will color ~~to~~ $V_0$ again with blue. meaning $G'$ is bipartite

(2) $G = (V, E)$

(a) $2|E| = \sum_{v \in V} d_v$

$d_v$ = degree of a vertex v.

→ Edge is a line from vertex v to vertex w.

→ ~~The so~~ if we count the edges, the same edge gets counted twice, once from v & once from w.

→ ∴ total edges = $\frac{\sum \text{degree of vertex}}{2}$.

(b). Each student would have shook hands with exactly 110 others.

It's not possible to shake hands with only 17. as total no. of students = 111. & every student shakes hand with every other.

(c). No. of edges = $^{111}C_2 = 111 \times 50 = \underline{\underline{5550}}$

(3) (a). (1) G has an even number of vertices [preserved]

(2) None of vertices of G is an even integer [Not preserved]

(3) G has a vertex of degree 3 [preserved]

(4) G has exactly one vertex of degree 3 [preserved].

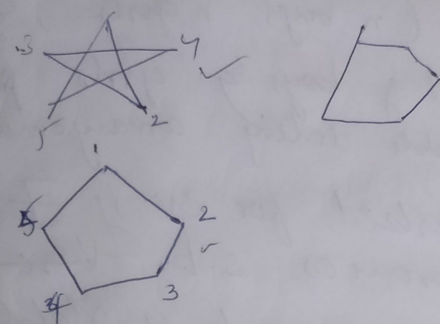
(b)

G

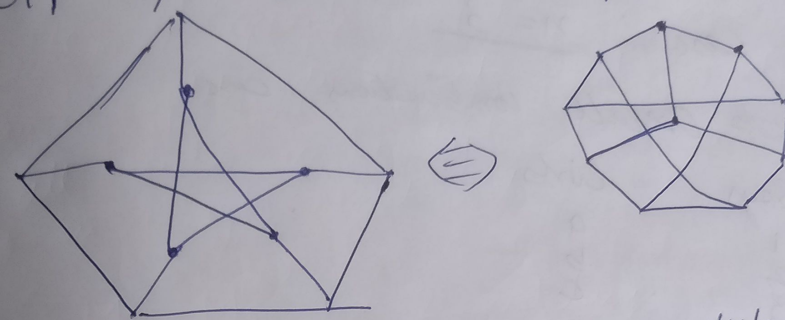
(4) (a

(b

(b)

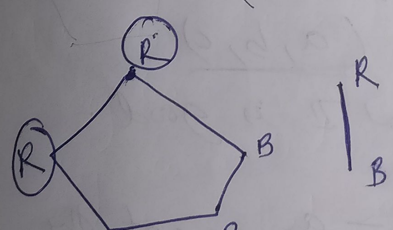



(a) $G_1$, $G_4$ $G_3$ are isomorphic.



They are isomorphic because any cycle in both $G_1$ $G_4$ $G_3$ have 5 edges...

(4)(a) Consider the below graph.




clearly, it satisfies the constraints, max degree = 2 and a vertex with degree = 1

But it's not k-colorable as 2 vertices will always have same color.

(b) Base case is wrong:

$n = 1$: G has only 1 vertex so 1 colorable.

the predicate says, G with n vertices & max degree k is k-colorable.

→ In base case, degree is 0 ∴ it should be 0 colorable but we know its 1 colorable.

→ Thus, it fails the base case.

(5) for some $n \geq 3$ ($n$ boys, $n$ girls, & total $2n$ people)

There exists a set of boys & girls preferences
s.t. ~~there is a~~ every ~~stable~~ dating arrangement is stable

Sol). Here we are asked, ~~for ev~~ if $\exists$ a set of ~~there is a~~ every
boys & girls preferences s.t.
~~stable~~ dating arrangement is stable.

Base case: $\underline{n = 3}$

~~Any to~~ Overall combinations are

| Boys | Girls |
|---|---|
| 1 | a |
| 2 | b |
| 3 | c |

| | | | | | |
|---|---|---|---|---|---|
| 1a | 1a | 1b | 1b | 1c | 1c |
| 2b | 2c | 2a | 2c | 2a | 2b |
| 3c | 3b | 3c | 3a | 3b | 3a |

for all these arrangements to be stable,
no arrangement exists.
imagine if a likes 1 & 1 likes a the most
or a = $\underline{(1,2,3)}$, 1 = $\underline{(a,b,c)}$

Arrangement I is good, Arrangment II is good.
But in arrangment 3,
1 – b & 2 – a, not the
favorite choices of 1, a $\therefore$ they form a rogue
couple.

Only way the every arrangment is stable.
is if everyone are equal
i.e. there is NO HIERARCHY of
preferences.

in that case, there is no motivation to
chose one ahead of the other.
So whatever the arrangement be,
Everyone is OK. There's no incentive to
$\underline{\text{switch}}$

6). $(s_1, s_2 \ldots s_n)$. arbitrary sequence of $(1, 2, \ldots n-1, n)$.
if $n = 5$, one such seq = $\underline{5, 3, 4, 2, 1}$

(1) $V = \{v_1, v_2 \ldots v_n\}$.

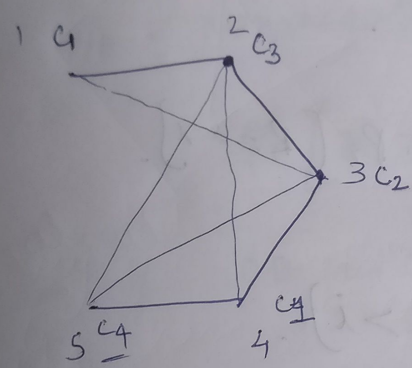
$e = (v_i, v_j) \in E$. s.t. [Proof by example]

(a) $j = i + 1$

(b) $i = s_k$, $j = s_{k+1}$ for $1 \le k \le n-1$

T.P: graph is 4-colorable for any $(s_1, s_2 \ldots s_n)$.

(a).




$(1, 3, 5, 2, 4)$ max. degree $= 4$.

$= 4$ colorable

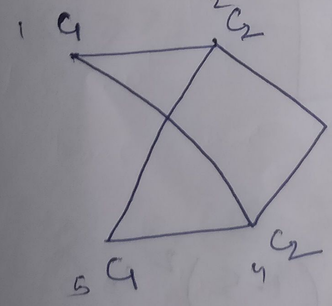
$\rightarrow$ ~~V~~ =. $(1, a_1, 3, a_2, 5, a_3 \ldots)$.

(b) $(s_1, s_2 \ldots s_n) =$

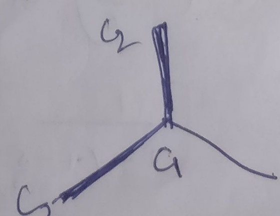
$a_1, a_2 \ldots$ sequence of even nos.

$(1, \underline{4, 3, 2, 5})$.

$= 2$ colorable max degree $= 3$




but the vertex with degree 3 always has a ⅄ star configuration.




and these adjacent vertices always have a degree $= 2$ connected to another unique set of vertices