1) a) $f(x) = \frac{x}{1-x^2}$ $f'(x) = ?$

$$f'(x) = \frac{(1-x^2)\cdot 1 - x(-2x)}{(1-x^2)^2} = \frac{1+x^2}{(1-x^2)^2}$$

b) $f(x) = \ln(\cos x) - \frac{1}{2}(\sin x)^2$, $f'(x) = ?$

$$f'(x) = \frac{1}{\cos x} \times -\sin x - \frac{1}{\cancel{2}} \times \cancel{2}(\sin x)' \times \cos x$$

$$= -\tan x - \sin x \cos x$$

$$= -\tan x\,(1 + \cos^2 x).$$

c) $f^5(x) = ?$, $f(x) = xe^x$

$\Rightarrow f'(x) = xe^x + e^x$

$f''(x) = xe^x + e^x + e^x$

$f'''(x) = xe^x + e^x + e^x + e^x$

$f^4(x) = xe^x + e^x + e^x + e^x + e^x$

$f^5(x) = xe^x + 5e^x$

(2) $x^{2/3} + y^{2/3} = 4$, $\left.\frac{dy}{dx}\right|_{(-\sqrt{27},1)} = ?$

$\Rightarrow \frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \times \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{-1/3}$

@ $(-\sqrt{27}, 1)$

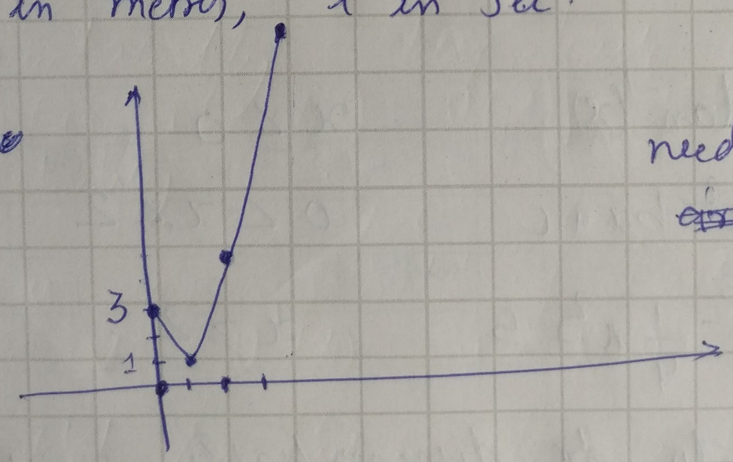
$\frac{dy}{dx} = -\left(\frac{-\sqrt{27}}{1}\right)^{-1/3} = -\times(-1)^{-1/3} \times \left(3^{3/2}\right)^{-1/3}$

$= 1 \times 3^{-1/2}$

$= \frac{1}{\sqrt{3}}$

(3) $y(t) = t^3 - 3t + 3, \quad t \geq 0$

y in metres, t in sec.



need to compute the ~~dist~~ length of the curve.

As it is given the particle is moving ~~alon~~ ALONG vertical axis.

$\therefore$ distance covered $= (3-1) + (y(3) - y(1))$

$= 2 + 21 - 1 = \underline{22}$

(4) $f(x) = x, \quad g(x) = e^x$.

$$\frac{d}{dx}(fg) = \frac{d}{dx}(xe^x)$$

→ using product rule $= xe^x + e^x$. —(1)

→ from definition

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)\, e^{(x+\Delta x)} - xe^x}{\Delta x}$$

$$= \lim \frac{xe^{(x+\Delta x)} - xe^x}{\Delta x} + \lim_{x \to 0} \frac{\Delta x\, e^{(x+\Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} x\left[\frac{e^{(x+\Delta x)} - e^x}{\Delta x}\right] + e^x.$$

$$= xe^x + e^x$$

(5) $$f(x) = \begin{cases} \tan^{-1}(x) & x \leq 0 \\ ax^2 + bx + c & 0 < x < 2 \\ x^3 - \frac{1}{4}x^2 + 5 & x \geq 2. \end{cases}$$

for the fn to be differentiable everywhere.
it has to be continuous everywhere & differentiable everywhere.

Consider @ $x = 0$

$$\lim_{x \to 0^-} f(x) = \tan^{-1}(x) \quad = \quad \lim_{x \to 0^+} ax^2 + bx + c \quad = f(0$$

this is possible when $a \times 0 + b \times 0 \ldots$

$\Rightarrow \quad C = f(0) = \tan^{-1}(0) = 0 \quad \therefore \quad C = 0.$

Consider $x = 2$.

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2).$

$\Rightarrow \quad a \times 4 + b \times 2 + 0 = 8 - \frac{1}{4} \times 2^2 + 5$

$4a + 2b = \cancel{4}\,12.$

~~yes so for all points~~

$2a + b = 6$ & $C = 0$, so under these constraints the fn is continuous.

$\therefore$ to ensure differentiability @ $x = 0$ & 2.

$f'(x)\Big|_{x=2^+} = 3x^2 - \frac{1}{2}x \Big|_{x=2} = 11$

$f'(x)\Big|_{x=2^-} = 4a + b$

thus $4a + b = 11$

$2a + b = 6.$

$a = \frac{5}{2}, \quad b = 1.$

need to confirm if these hold true @ $x = 0$ as well.

yes they indeed hold true

6). $f(x+y) = f(x) + f(y) + x^2y + xy^2$

& $\lim_{x \to 0} \frac{f(x)}{x} = 1$.

a) $f(0) = ?$

$x=0,\ y=0$

$\Rightarrow f(0) = f(0) + f(0) + 0$

$\Rightarrow f(0) = \underline{\underline{0}}$.

b). $y = x$.

$\Rightarrow f(2x) = 2f(x) + 2x^3$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim \frac{f(x) + f(\Delta x) + x^2\Delta x + x\Delta x^2 - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(\Delta x)}{\Delta x} + x^2 + x\Delta x.$$

$$= \underline{1 + x^2}$$

$\therefore f'(0) = \underline{\underline{1}}$

$f'(x) = 1+x^2$