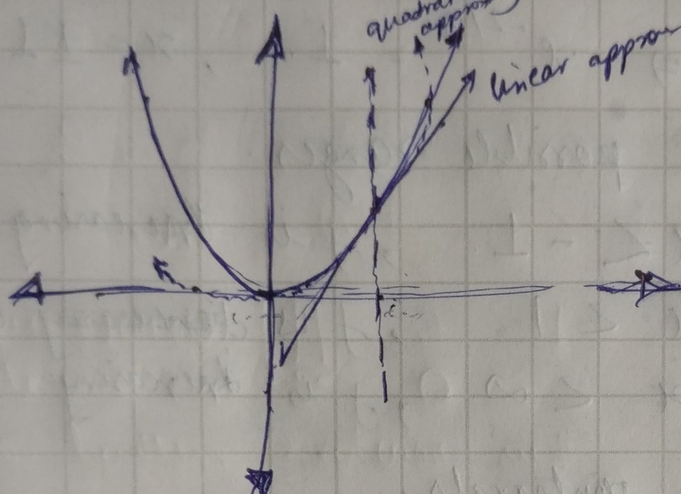
1) a) $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$

~~$x \neq a$~~ as $f(0)$ & $f'(0)$ are not given.
Shifting the origin by a units.

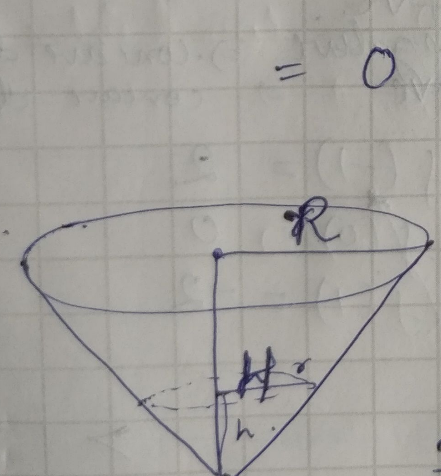
$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$



(b) $\ln(1.2) = \ln(1) + \left.\frac{1}{x}\right|_1 \times 0.2 + \left.\frac{-1}{x^2}\right|_1 (0.2)^2$

$= 0 + 0.2 - 1 \times 0.04 = \underline{\underline{0.16}}$

(2)



$R = 2H$,

~~$\frac{1}{3}\pi r^2 h \Rightarrow$~~

$\frac{r}{h} = \frac{R}{H} = \frac{1}{2} \Rightarrow r = 2h$.

$\frac{1}{3}\pi r^2 h = 30 \times t$, $\left.\frac{dh}{dt}\right|_{t=10} = ?$

$\Rightarrow \frac{4\pi}{3} \times h^3 = 30t$

$\Rightarrow \frac{4\pi}{3} \times \frac{3h^2}{4}\frac{dh}{dt} = 30 \Rightarrow \frac{dh}{dt} = \frac{120}{\pi h^2}$ @ $h = 10$

$\frac{dh}{dt} = \frac{1.2}{\pi}$

③ $f(x) = x - 3x^{1/3}$.

→ fn is defined @ all values of x.

→ $f'(x) = 1 - \cancel{3} \times \frac{1}{\cancel{3}} x^{-2/3}$

$f'(x) = 0 \Rightarrow 1 - x^{-2/3} = 0$

$\Rightarrow x^{-2/3} = 1$. , $x = \pm 1$.

→ so we have 3 possible ranges.

$-\infty < x \leq -1$, f is increasing

$-1 \leq x < 1$, f is ~~increasing~~ decreasing

$1 \leq x < \infty$. , f is increasing

→ $f''(x)$ @ these ~~p~~intervals.

$f''(x) = +\frac{2}{3} x^{-5/3}$

for $0 < x < \infty$ it's always ~~negative~~ +ve ⇒ concave ~~down~~ up.

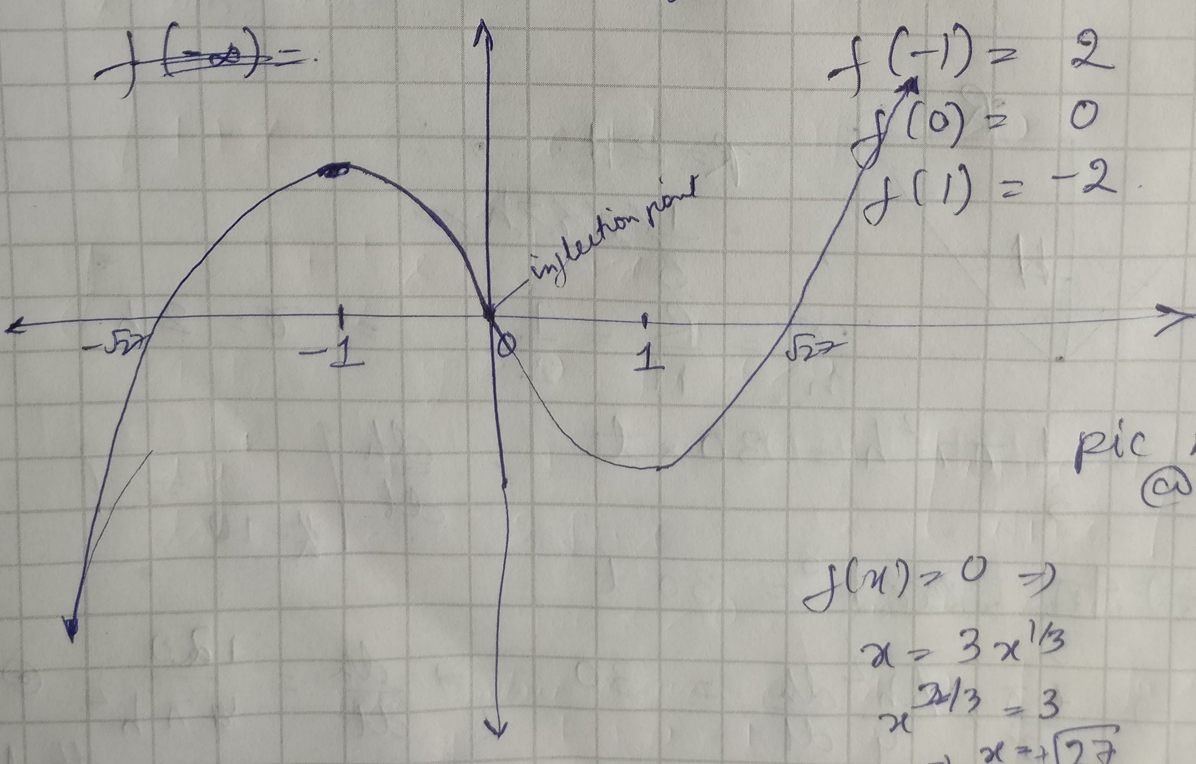
$-\infty < x < 0$ it's always -ve ⇒ concave down.

~~$f(-\infty) =$~~

$f(-1) = 2$

$f(0) = 0$

$f(1) = -2$



pic not @ scale.

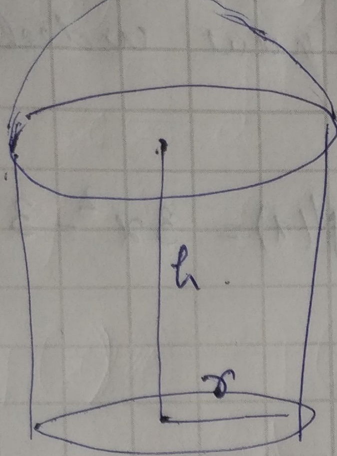
$f(x) = 0 \Rightarrow$

$x = 3x^{1/3}$

$x^{2/3} = 3$

$\Rightarrow x = \pm\sqrt{27}$

(4)



$$V = \pi r^2 h + \frac{2}{3}\pi r^3$$

given V = constant

$$\Rightarrow \cancel{0 = \pi 2r \frac{dh}{dr}} +$$

$$\Rightarrow 0 = \cancel{\pi r^2 \frac{dh}{dr} + \pi 2rh + \frac{2}{3}\pi 3r^2}$$

$$\Rightarrow \pi r^2 \frac{dh}{dr} + \cancel{2\pi r h + 2\pi r^2} = 0$$

$$r^2 \frac{dh}{dr} + 2rh + 2r^2 = 0$$

$$\Rightarrow \cancel{\frac{dh}{dr} = -\frac{2h}{r} - 2}$$

$$\frac{dh}{dr} = -\frac{2h}{r} - 2$$

metal used = T.S.A = $2\pi r h + 2\pi r^2 + \pi r^2$ = *

$$d(T.S.A) = 2\pi h + 2\pi r \frac{dh}{dr} + 4\pi r + 2\pi r$$

$$d(T.S.A) = 0$$

$$\Rightarrow h + r\frac{dh}{dr} + 2r + r = 0$$

$$\frac{dh}{dr} = -\frac{h}{r} - 3$$

but from above, $\frac{dh}{dr} = -\frac{2h}{r} - 2$

$$\Rightarrow -\frac{2h}{r} - 2 = -\frac{h}{r} - 3$$

$$\Rightarrow \frac{h}{r} = 1 \Rightarrow r = h$$

$\therefore$ T.S.A = $5\pi r^2$ & $V = \frac{5}{3}\pi r^3 \rightarrow r = \left(\frac{3V}{5\pi}\right)^{1/3}$

$\therefore$ ~~T.S.A~~ =

min. SA also exists @ boundary conditions.
after checking @ boundary, critical point indeed represents min. T.S.A

(5)

$f(x) = x^3 - 3x + 7$ | $f'(x) = 3x^2 - 3$.

at $x = 2$. $f(2) = \underline{\underline{9}}$.

$x' = x - \frac{f(x)}{f'(x)}$

$\Rightarrow x^{(1)} = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{9}{f'(2)} = 2 - \frac{9}{9} = 1$.

$x^{(2)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{5}{0}$!!! UNDEFINED value...

$x^{(2)}$ is undefined, thus newton's method cann't find the roots.

(1) One failure condition of newton's method occur, when $\underline{\underline{f'(x) = 0}}$, i.e. tangent @ 2 doesn't intersect x-axis, thus, can't use Newton's method.

(6)

$\sqrt{1+x} < \frac{1}{2}x + 1$ if $x > 0$.

$y = 1 + \frac{1}{2}x - \sqrt{1+x}$.

T. Prove $y > 0$, for $x > 0$.

$\frac{dy}{dx} = \frac{1}{2} - \frac{1}{2\sqrt{1+x}} = \frac{1}{2}\left[\frac{\sqrt{1+x} - 1}{\sqrt{1+x}}\right]$

for $x > 0$, this is an increasing fn.

$\therefore$ min value occurs @ $x = 0$

$\Rightarrow y = 1 + 0 - 1 = \underline{0}$.

| $\therefore$ for $x > 0$, $\underline{y > 0}$