Problem Set 1

① a) There are people who have taken 6.042 & have gotten A's in 6.042

ans) $\exists x \; (S(x) \wedge A(x)$

(b) All people who ~~have taken~~ are 6.042 TA's & have taken 6.042 got A's in 6.042

$$\forall x \; (T(x) \wedge S(x) \wedge A(x)$$

(c) There are no people who are 6.042 TA's who did not get A's in 6.042

$$\forall x \; T(x) \Rightarrow A(x) \quad \text{or}$$

(d) There are atleast 3 people who are TA's in 6.042 & have not taken 6.042

$$\exists x \; \exists y \; \exists z \; (\cancel{\neq} \, x \neq y \wedge y \neq z \wedge z \neq x \wedge T(x) \wedge T(y) \wedge T(z) \wedge \sim S(x) \wedge \overline{S(y)} \wedge \overline{S(z)}).$$

② $\overline{(P \vee (Q \wedge R))} = (\bar{P}) \wedge (\bar{Q} \vee \bar{R})$.

P	Q	R	$\overline{P \vee (Q \wedge R)}$	$\bar{P} \wedge (\bar{Q} \vee \bar{R})$
T	T	T	F	F
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

LHS = RHS.

2(b) $\left(\overline{P \wedge (Q \vee R)}\right) = \bar{P} \vee (\bar{Q} \vee \bar{R})$.

P	Q	R	$\overline{P \wedge (Q \vee R)}$	$\bar{P} \vee (\bar{Q} \vee \bar{R})$
T	T	T	F	F
T	T	F	F	T
T	F	T	F	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

LHS $\neq$ RHS.

(3) nand.

P	Q	P nand Q	$A \vee Q$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

(a) $A \wedge B = \overline{A \text{ nand } B}$

(b) $A \vee B = \overline{\bar{A} \wedge \bar{B}}$ or $\bar{A}$ nand $\bar{B}$

(c). $A \Rightarrow B$

A	B	$A \Rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

$A \wedge \bar{B} \Rightarrow$

A	B	$A \wedge \bar{B}$
T	T	F
T	F	T
F	T	F
F	F	F

A nand $\bar{B}$ $\Rightarrow$

A	B	A nand $\bar{B}$
T	T	T
T	F	F
F	T	T
F	F	T

(b) $\bar{A}$ using only Nand

A nand A

A	A	A nand A	$\bar{A}$
T	T	F	F
F	F	T	T

$\therefore \bar{A}$ = A nand A

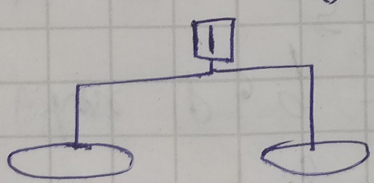
(c) construct always "true" & always "false".

A	A	A nand A	A nand (A nand A)	A and (A nand A)
T	T	F	T	F
F	F	T	T	F

always true   always false

always true = A nand (A nand A)
always false = A and (A nand A).

(4) 12 coins, one fake.   Atmost 3 trials.



1st attempt:
6 vs 6.
& select the lighter of the two 6's.

2nd attempt: divide into 3 vs 3.
select the lighter of the two 3's.

3rd attempt: divide into 1 vs 1, & leave the last one out.

if 1 vs 1 are equal wt. then last one is fake.
if 1 vs 1 : are unequal, then lighter 1 is fake.

(5). ~~if r is~~

proposition: if r is irrational, then $r^{1/5}$ is irrational.

Contrapositive: if $r^{1/5}$ is not irrational then r is not irrational.

~~as~~ this is an example of implication.

$r^{1/5}$ is not irrational.

i.e. $r^{1/5} = p/q$

where $p, q \in$ Natural nos.

$\Rightarrow r = p^5/q^5$

if $p, q \in$ Natural nos. then.

p^5, q^5 also $\in$ Natural nos.

thus $r = \frac{p^5}{q^5}$ can be expressed as ratio of 2 natural nos.

$\Rightarrow$ r is rational.

or r is not irrational.

thus if $r^{1/5}$ is not irrational, then r is not irrational

(6) given,

$$w^2 + x^2 + y^2 = z^2 \quad ; \; w, x, y, z \text{ always +ve integers}$$

proposition : z is even if & only if w, x, y are even.

Case 1 : consider w, x, y = even.

$w = 2i_1$, $x = 2i_2$, $y = 2i_3$.

$\Rightarrow w^2 + x^2 + y^2 = z^2$

$\Rightarrow 2^2 [i_1^2 + i_2^2 + i_3^2] = z^2$

i.e $z = 2\sqrt{i_1^2 + i_2^2 + i_3^2}$

thus z is also even.

Case 2 if any one of w, x, y is odd.

$w = 2i_1 + 1$, $x = 2i_2$, $y = 2i_3$.

Then $w^2 + x^2 + y^2 = z^2$

$\Rightarrow (2i_1+1)^2 + (2i_2)^2 + (2i_3)^2 = z^2$.

$\Rightarrow \underbrace{(4i_1^2 + 4i_1 + 4i_2^2 + 4i_3^2)}_{\text{even}} + \underbrace{1}_{\text{odd}} = z^2$.

thus z^2 = odd , i.e z is odd.

Case 3 if any 2 of w, x, y are odd.

$w = 2i_1 + 1$, $x = 2i_2 + 1$, $y = 2i_3$.

$\Rightarrow w^2 + x^2 + y^2 = z^2$

$\Rightarrow 4i_1^2 + 4i_1 + 4i_2^2 + 4i_2 + 4i_3^2 + 2 = z^2$.

$\Rightarrow \underbrace{2}_{\text{even}} \underbrace{[2i_1^2 + 2i_2^2 + 2i_3^2 + 2i_1 + 2i_2 + 1]}_{\text{odd}} = z^2$.

$\Rightarrow z^2 = 2 \times \text{odd}$ $\therefore$ z is irrational

but given z is +ve $\therefore$ this case is not possible.

Case (iii)

$$w^2 + x^2 + y^2 = z^2$$

where, w, x, y, z are all odd.

$$\Rightarrow (2i_1+1)^2 + (2i_2+1)^2 + (2i_3+1)^2 = z^2$$

$$\Rightarrow \underbrace{4i_1^2 + 4i_1 + 4i_2^2 + 4i_2 + 4i_3^2 + 4i_3}_{\text{even}} + \underbrace{3}_{\text{odd}} = z^2$$

$\Rightarrow z^2 = \text{odd} \Rightarrow z = \text{odd}$.

$\therefore$ z is even only in case I i.e w, x, y are <u>even</u>.