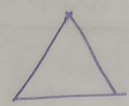
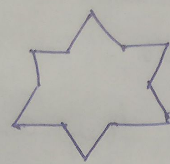
Problem - Set 2

→ $n = 0$



$n = 1$



Complexity of algo

$T(0) = 3$

$T(n) = 4\,T(n-1)$

solving this recurrence, we have.

$x - 4 = 0 \Rightarrow x = 4$

$\therefore$ Solution is of the form.

$\cancel{T(n) = c \times 4^n}$

$T(n) = c \times 4^n$

at $n = 0$, $T(0) = 3$

$\therefore T(0) = c \times 4^0$

$\Rightarrow c = 3$.

$$\boxed{\therefore T(n) = 3 \times 4^n}$$

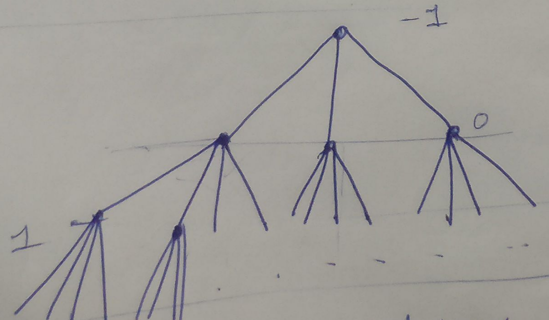
Problem 2-1)

(a) height of recursion tree

$= \cancel{\log_2 T(n)} = \cancel{\frac{n \log_2 4 + \log_2 3}{2}}$

$= \cancel{2n}$

The recursion tree is of the following shape.



$\therefore$ height of the tree $= n$

(b) At level i, no of nodes

$$= \underline{3 \times 4^{i}} \quad \text{as } T(n) = \underline{3 \cdot 4^{n}}$$

$= \cancel{\theta}($

(c) For a "single node" @ each level i, there is only 1 triangle ∴ asymptotic rendering time = $\theta(1)$.

(d) As there are $3 \times 4^{i-1}$ nodes at each level i, & each node has $\theta(1)$ for rendering a triangle.

∴ total rendering time @ each level $= \theta\left(\frac{3}{4} \cdot 4^{i}\right) = \theta(4^{i})$

(e) total no. of triangles are 4^{i+1} triangles

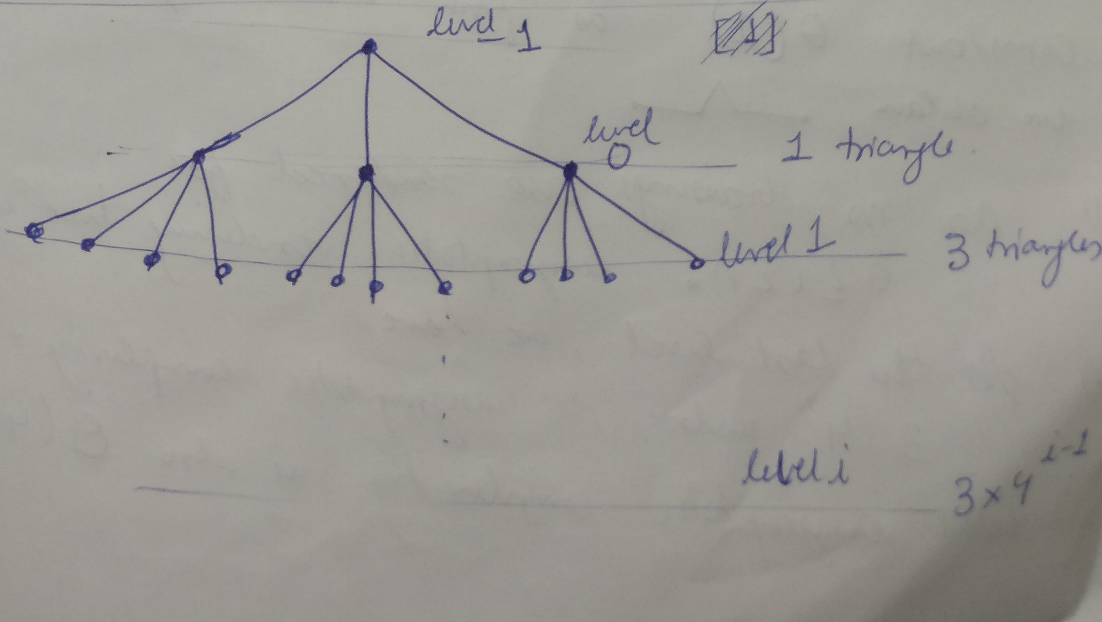
∴ total rendering time

$$= \theta(4^{i+1}) = \theta(4^{i})$$

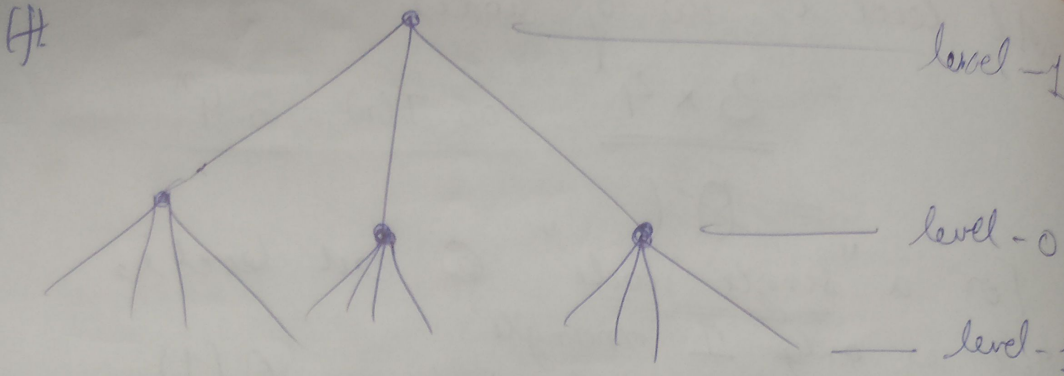
more precisely,

no of triangles $= 1 + 3\times4^{0} + 3\times4^{1} + 3\times4^{2} + \dots + 3\times4^{i}$

$= \boxed{4^{i+1}}$



(f)



Again height of the tree = N, as at each level we reduce the height of tree by "1"

(g). no of nodes at level i

$= 3 \times 4^{i}$

(h). rendering time for a node @ level i is.

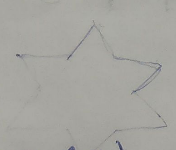
$0 \le i < n$

clearly, only ~~the last part~~ is the tree is drawn only @ the last layer ~~else~~.

$\therefore$ for $0 \le i < n$,

asymptotic rendering time $= 0$.

(i)



for a single node the asymptotic rendering time is constant $\Theta(1)$ as from a line segment we obtain

(j) as no drawings are rendered for $0 \le i < n$, asymptotic rendering time is ~~$\Theta($~~ 0

k) At the last level, we have $3 \cdot 4^{n}$ nodes, $\therefore$ asymptotic complexity $= \Theta(4^{n})$.

l) total asymptotic complexity is also $\Theta(4^{n})$

level-1

level-0

— level-1

each level

i is.

ce is

the
time is

ment

time is $\cancel{\Theta}$ 0.

rity = $\Theta(4^n)$.
$\Theta(4^n)$

m) same height as earlier problem

n) same no. of nodes = $3 \cdot 4^i$

o) ~~length of line segment becomes $\left(\frac{1}{3}\right)^{rd}$ of its length in previous layer.~~
~~$\therefore$ time to render this line segment~~
~~$= \Theta\left(\frac{1}{3}^n\right)$ for layer.~~

o) no lines are rendered $0 \leq i < n$.

$\therefore$ time = 0

p) at the last level, all lines are rendered, the length of each line segment = $\left(\frac{1}{3}\right)^n$.

$\therefore$ time complexity = $\Theta\left(\frac{1}{3}^n\right)$.

q) at each level i, $0 \leq i < n$, no lines are rendered, $\therefore$ time complexity = 0.

r) at the last level n, asymptotic complexity

$$= \underbrace{\left(3 \times 4^n\right)}_{\text{nodes}} \times \underbrace{\Theta\left(\frac{1}{3}^n\right)}_{\text{complexity for each node}}$$

$$= \Theta\left(\left(\frac{4}{3}\right)^n\right).$$

s) total cost of rendering is same as above as Θ cost of rendering from $0 \leq i < n$ is 0 @ layer n is $\Theta\left(\frac{4^n}{3}\right)$

(t, u) total cost of rendering Δ's

$$= \Theta(1) + 3 \times \frac{4^0}{3^2} + 3 \times \frac{4^1}{3^4} + \dots + \frac{3 \times 4^i}{3^{2i+2}}$$

$$= \Theta(1) + \sum_{i=0}^{n-1} 3 \times \underbrace{\left(\frac{4}{9}\right)^i}_{\text{constant} < 1} \in \underline{\Theta(1)}$$