

GCD

~~1~~
~~0~~
Not defined
~~20~~

$$\frac{0}{20} ?$$

$$= 0 \times 20 = 0$$

$$= 0 = \frac{0}{20} =$$

$$\textcircled{3} \times -1 = \underline{-5}$$

gcd(a, b)

$$\Rightarrow \underline{5}, \underline{15} \rightarrow 5$$

$$\underline{12}, \underline{8} \rightarrow 4$$

$$\underline{\underline{20}}, \underline{0} \rightarrow 20$$

(20)

gcd(A, 0) \rightarrow A

$$\Rightarrow \boxed{-5, -10, \textcircled{5}} \quad 1$$

$$6, \textcircled{1} \rightarrow \cancel{1}$$

gcd(A, 1) \rightarrow 1

$$\begin{aligned} \overbrace{\text{GCD}(A, B, C)}^{\downarrow} &= \text{GCD}(\text{GCD}(A, B), C) \\ &= \text{GCD}(\text{GCD}(A, C), B) \\ &= \text{GCD}(\text{GCD}(B, C), A) \end{aligned}$$

$$\text{GCD}(A, B) = \text{GCD}(B, A)$$

(A, B)
 L.

$\rightarrow \min\text{-gcd} = b$

max-gcd = $\min(A, B)$

\dots
 $\text{gcd} = 1;$
 $\text{for } (i = \min(A, B); i \geq 1; i--) \{$
 { if ($A \% i == 0$
 &
 $\rightarrow B \% i == 0$)
 ret $i;$
 $\}$
 $\}$
 gcd

$\underline{\mathcal{O}(n)}$

$$\begin{aligned}
 & A, B \\
 & B > A \\
 & B - A = C \\
 & \cancel{\text{gcd}(A, B)} = \text{gcd}(A, A+C) \\
 & \quad \quad \quad \boxed{A \% g = 0} \\
 & \quad \quad \quad \boxed{(A+C) \% g = 0} \\
 & \quad \quad \quad \boxed{C \% g = 0} \\
 & \quad \quad \quad \boxed{\text{gcd}(A, B-A) = g} \\
 & \Rightarrow \boxed{\text{gcd}(A, B) = \text{gcd}(A, B-A)}
 \end{aligned}$$

$\text{gcd}(a, b) \{$
 LO, 8
 $\min \quad \max \Rightarrow \min = \text{Min}(a, b),$
 $8 \quad \underline{10} \quad \max = \text{Max}(a, b);$
 $\rightarrow \text{if } \min == \max \text{ return } \max$

$\begin{array}{r} 8 \\ 2 \\ 2 \\ 2 \\ 2 \end{array}$ \Rightarrow if $(\min = 1 \mid \max = 1)$ then $\boxed{1}$
 $\begin{array}{r} 2 \\ 6 \\ 4 \\ 2 \\ 0 \end{array}$ then $\text{gcd}(\min, \max - \min)$

$$39 \cdot 10 = 0$$

$$\frac{A, B}{g}$$

$$\begin{array}{c} B > A \\ B - A = C \end{array}$$

$$\begin{array}{l} A \% g = 0 \\ B \% g = 0 \end{array}$$

$$\begin{array}{l} (A+C) \% g = 0 \\ C \% g = 0 \end{array}$$

$$g'?$$

$$\frac{B}{g'} \rightarrow (A+C) \% g' = 0$$

$$A \% g' = 0$$

$$\boxed{B > A}$$

$$B = A + C$$

$$\Rightarrow \boxed{\frac{B}{g} = \frac{C_1 A + C_2}{g}} =$$

$$\text{gcd}(A, B) = \text{gcd}(A, \underline{(C_1 A + C_2)})$$

$$\boxed{g = \text{gcd}(A, B)}$$

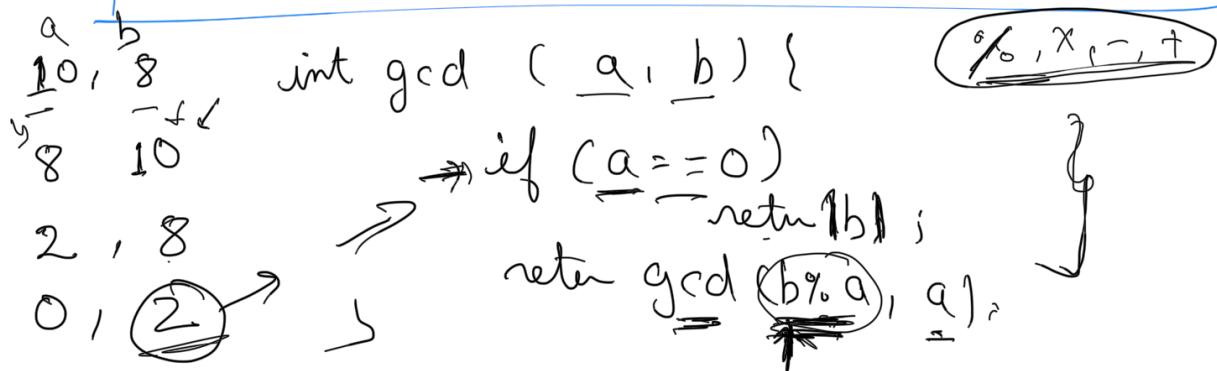
$$\begin{array}{l} A \% g = 0 \\ C_1 A \% g = 0 \end{array}$$

$$\boxed{C_2 \% g = 0}$$

$$\text{if } C_2 = 1$$

$$\boxed{g = \text{gcd}(A, B \% A)}$$

$$87.(D) \quad \boxed{\gcd(A, B) = \gcd(A, B \% A)} \quad B > A$$



$$\gcd(A, B)$$

$$\gcd(A, B \% A)$$

\downarrow

$$\boxed{A < B/2}$$

$$\boxed{B \% A < A}$$

$$A < B$$

$$B/2$$

$$A < B/2$$

$\log n$

$\log(\text{max})$

$$\boxed{A \geq B/2}$$

$$\boxed{B \% A < B/2}$$

?

$$\frac{A}{B} \quad (1)$$

$$\gcd(A; B) \rightarrow \gcd(A, \boxed{B \% A}) < B/2$$

$$A < B$$

$$A < B/2$$

$$\boxed{A \geq B/2}$$

$$\boxed{B \% A < B/2}$$

$$\boxed{B \% A < A}$$

$$\boxed{B \% A < B/2}$$

$$\boxed{6) 10 (1}$$

4

$$\{3, 4, 6\}$$

$$\frac{[3, 4, 8, 6]}{[3!, 4!, 8!, 6!]} \quad \text{---} \\ \underline{\underline{6}}$$

$$13, 24, 8, 5 \quad -$$

$$\Rightarrow 120$$

$$6, 12, 18$$

$$\begin{array}{c} 2 \times 3 \\ 2 \times 2 \times 3 \\ 2 \times 3 \times 3 \end{array}$$

$$2 \times 3$$

$$\overline{3, 4, 8, 6}$$

$$\begin{aligned} 3! &= 1 \times 2 \times 3 \\ 4! &= 1 \times 2 \times 3 \times 4 \\ 8! &= 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \\ 6! &= 1 \times 2 \times 3 \times 4 \end{aligned}$$

$$1 \times 2 \times 3$$

$$\begin{array}{c} 3 \times 4 \\ 3! \times 4 \times \\ 3! \times 4 \times 5 \end{array}$$

$$\rightarrow \frac{2, 4, 8, 10}{4, 6, 3, 10} \Rightarrow F$$

$$\text{gcd(Sub Sq)} = 1 ?$$

$$4, 3$$

$$\rightarrow (4, 3) \rightarrow 1 \quad T$$

$$\rightarrow \boxed{14, 9, 26} \quad -T$$

$$\rightarrow \text{gcd}(14, 9) = 1$$

$$2^N$$

$$\Rightarrow [A_1, A_2, A_3, A_4, A_5, A_6]$$

$$n \log M$$

$$\Rightarrow \boxed{\text{gcd}(A_2, A_3, A_6) = 1}$$

$$(a, b) \rightarrow \log M$$

$$\boxed{\text{gcd}(\text{Array}) = \text{gcd}(\cancel{\text{gcd}(A_2, A_3, A_6)}, \cancel{\text{gcd}(A_2, A_4, A_5)})}$$

$$1, 4, 5 \rightarrow \textcircled{1}$$

$$\boxed{\text{gcd}(\text{array}) = 1} \quad \cancel{2, 10, 8} \rightarrow \underline{-1}$$

$$\cancel{2, 3, 4} \rightarrow \textcircled{0}$$

$$\cancel{4, 5, 20} \rightarrow \textcircled{0}$$

$$\cancel{5, 10, 3, 9} \rightarrow \textcircled{0}$$

$$\cancel{(4, 10, 3, 9)} \rightarrow \textcircled{-1}$$

$\hookrightarrow 2 \rightarrow 10$

$$\text{gcd}(\text{Array}) = \text{gcd}(g_a, g_b, g_c, g_d, g_e)$$

$$\cancel{2, 8, 16} \rightarrow 2$$

$\hookrightarrow 8 \downarrow$

$$\Rightarrow \cancel{15}, \cancel{12}, \cancel{18}$$

$$12, 18 \rightarrow \textcircled{6}$$

$$15, 18 \rightarrow 3$$

$$12, 18 \rightarrow 2$$

$$\left\{ \begin{array}{l} \cancel{13}, \cancel{2}, 4, 8 \rightarrow 1 \\ \hookrightarrow 2 \end{array} \right.$$

↔ ↔ ↔ ↔ ↔

$\boxed{2, 4, 6, 8 \dots}$ $\boxed{13}$

$A_1, A_2, A_3, A_4, A_5, A_6$

\downarrow \downarrow \downarrow
 $\underline{15}, \underline{12}, \underline{18}$
~~P-gcd~~ $\underline{\circled{15}}, \underline{\circled{3}}, \underline{\circled{3}}$ $\underline{\circled{15}}, \underline{\circled{6}}, \underline{\circled{18}}$
~~S-gcd~~ $\underline{3}, \underline{\circled{6}}, \underline{\circled{18}}$

$\underline{\gcd(15, 18)}$

for each i

$\underline{g} = \underline{\gcd(P_{\text{gcd}}(i-1), S_{\text{gcd}}(i+1))}$

\downarrow \downarrow \downarrow
 $\underline{3}, \underline{6}, \underline{12}, \underline{1}, \underline{18}, \underline{21}$
~~P-gcd~~ $\underline{\circled{3}}, \underline{\circled{3}}, \underline{\circled{3}}, \underline{1}, \underline{1}, \underline{1}$
~~S-gcd~~ $\underline{1}, \underline{\circled{1}}, \underline{\circled{1}}, \underline{\circled{1}}, \underline{\circled{3}}, \underline{21}$
(3)

$\cancel{60 - 40} \quad \cancel{20} \quad 0$

$A \longrightarrow B$
 $a \quad b$
 $a \quad (b-a)$
 $\cancel{14} \longrightarrow \cancel{1}$
 $\cancel{2} \longrightarrow \cancel{12}$

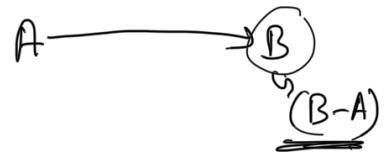
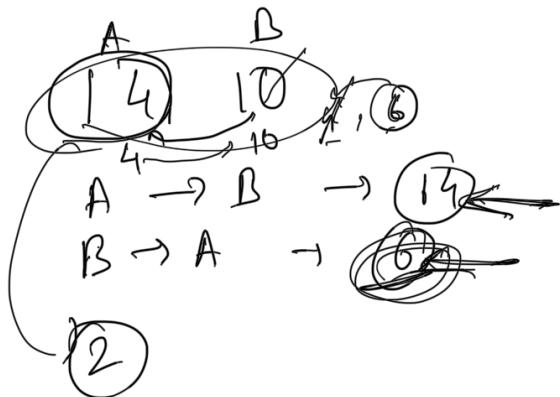
$\underline{1}, \underline{\circled{2}}, \underline{1}, \underline{2}, \underline{8}, \underline{56}$

$$14, 2, \underline{26}, \underline{56}$$

→ 2

$$14, 2, 26, 30$$

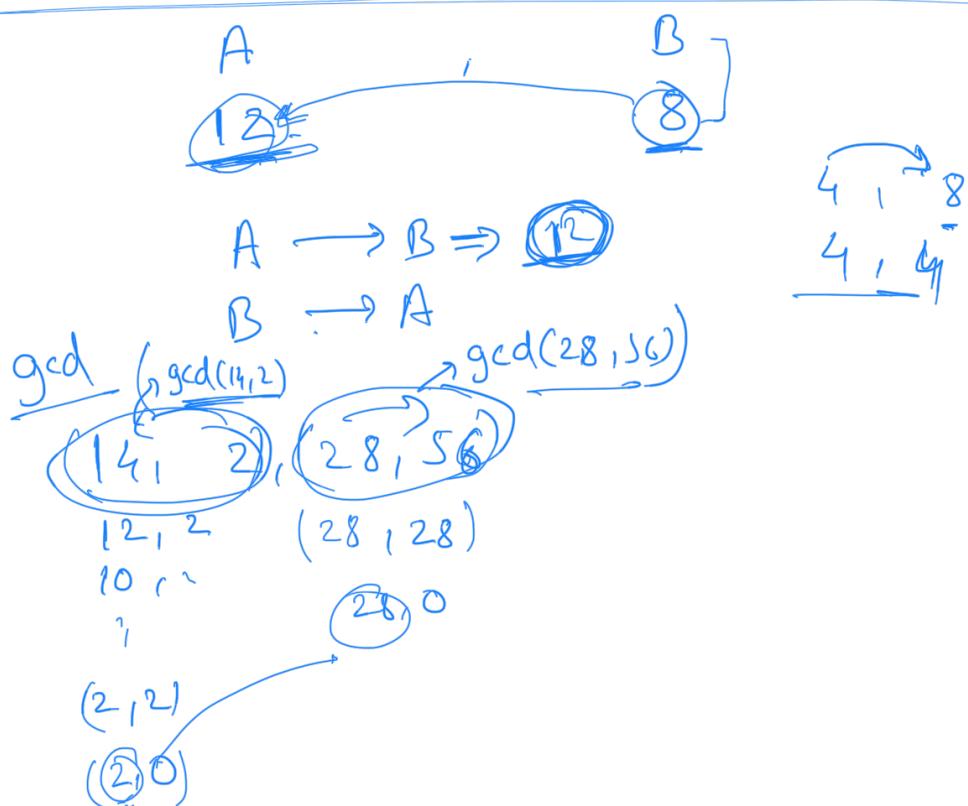
$$14, 2, 28, 56$$



$$\underline{2} \rightarrow 12$$

$$2 \rightarrow \underline{10}$$

$$2 \rightarrow 10$$



① $B \% A$

② ~~Sub S4~~

40%

$\text{GCD}(A, B) \rightarrow 8$

$B > A$

$$B = qA + r$$

A S $\rightarrow A \% g = 0$ -
 x -
 $\cancel{x \geq g}$
 $\cancel{A \% x = 0}$
 $\cancel{r \% x = 0}$
 $\cancel{(qA + r) \% x = 0}$
 \downarrow
 B

$$\boxed{\text{GCD}(A, B) = x}$$

$$\boxed{\text{GCD}(A, B \% A) = g}$$

$$\boxed{\text{GCD}(A, B \% A) = g}$$

~~6, 3, 10, 4, 2~~ $\leftarrow L \neq F$
 A_1, A_2, A_3, A_4, A_5
 ~~2^N~~

$$\Rightarrow \boxed{\text{gcd}(A_2, A_4, A_5) = 1}$$

$$\boxed{\text{gcd}(\text{array}) = \text{gcd}(\boxed{\text{gcd}(A_2, A_4, A_5)}, \boxed{\text{gcd}(A_1, A_3)})}$$

$$\text{gcd}(1, x) = 1$$

$\boxed{\text{gcd}(A, B)}$
 $\boxed{\text{gcd}(A, B \% A)}$
 $\boxed{\text{gcd}(B \% A, A)}$
 $\boxed{\text{gcd}(B \% A, A \% B \% A)}$
 $\boxed{A < B/2}$
 $\boxed{B \% A \leq A}$
 $\boxed{B \% A \leq B/2}$
 $\boxed{B \% A \leq B/2}$
 $\boxed{A \geq B/2}$
 $\boxed{B \% A}$
 $\boxed{A \geq B/2}$

$$\left| \begin{array}{l} \text{Diagram showing a horizontal line segment with a point } B \text{ above it and a point } A \text{ below it.} \\ \frac{B-A}{B-B/2} < B/2 \\ B-B/2 = B/2 \end{array} \right|$$

$$\boxed{B \% A < \underline{\underline{B/2}}}$$