DELHI TECHNOLOGICAL UNIVERSITY
MA – 101 (MATHEMATICS - I)
ASSIGNMENT – 5
ODD SEM 2020-21

Q.1. If $\boldsymbol{A} = 5t^2\boldsymbol{I} + t\boldsymbol{J} - t^3\boldsymbol{K}$, $\boldsymbol{B} = Sint\boldsymbol{I} - Cost\boldsymbol{J}$ **,** find ${}^{d}/_{dt}\,(A.B)$; and ${}^{d}/_{dt}\,(A \times B)$.

Answer: $5t^2Cost + 11tSint - Cost$, and $(t^3Sint - 3t^2Cost)\boldsymbol{I} - (t^3Cost - 3t^2Sint)\boldsymbol{J} + [(5t^2 - 1)Sint - 11tCost]\boldsymbol{K}$

Q.2. Prove that $\nabla r^n = nr^{n-2}\boldsymbol{R}$, where $\boldsymbol{R} = x\boldsymbol{I} + y\boldsymbol{J} + z\boldsymbol{K}.$

Q.3. If $\nabla u = 2r^4\boldsymbol{R}$, find u, if $\boldsymbol{R} = x\boldsymbol{I} + y\boldsymbol{J} + z\boldsymbol{K}.$

Answer: $u = \frac{1}{3}\left[r^{3/2}\right] + c$

Q.4. If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = yz + zx + xy$, prove that *grad u*, *grad v*, and *grad w* are coplanar.

Q.5. Find the directional derivative of $f(x, y, z) = xy^3 + yz^3$ at the point *(2, -1, 1)* in the direction of vector $\boldsymbol{I} + 2\boldsymbol{J} + 2\boldsymbol{K}.$

Answer: $(-\frac{11}{3})$

Q.6. Find the directional derivative of $f(x, y, z) = x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line PQ where Q is the point (5,0,4). Also, calculate the magnitude of the maximum directional derivative.

Answer: ${}^{28}/_{\sqrt{21}}$ *and* $\sqrt{164}$

Q.7. Find div **F** and curl **F** where $\boldsymbol{F} = grad(x^3 + y^3 + z^3 - 3xyz).$

Answer: div = $6(x + y + z)$, curl = 0

Q.8. Prove the following:

(i) $curl\ curl\ \boldsymbol{F} = grad\ div\ \boldsymbol{F} - \nabla^2\boldsymbol{F},$ i.e., $\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla.\mathbf{F}) - \nabla^2\boldsymbol{F}$

(ii) $curl\ grad\ \boldsymbol{f} = \nabla \times \nabla\mathbf{f} = 0.$

(iii) $curl\ (f\boldsymbol{G}) = (grad\ f) \times \boldsymbol{G} + f(curl\ \boldsymbol{G})$, i.e., $\nabla \times (f\boldsymbol{G}) = \nabla f \times \mathbf{G} + f\,\nabla \times \mathbf{G}$

(iv) div $(\mathbf{F} \times \boldsymbol{G}) = \boldsymbol{G}.(curl\ \boldsymbol{F}) - \boldsymbol{F}.(curl\ \boldsymbol{G})$, i.e.$\nabla.(\mathbf{F} \times \mathbf{G}) = \mathbf{G}.(\boldsymbol{\nabla} \times \mathbf{F}) - \mathbf{F}.(\boldsymbol{\nabla} \times \mathbf{G})$

Q.9. If r is the distance of a point (x,y,z) from the origin, prove that $curl\left(\boldsymbol{K} \times \text{grad}\ \frac{1}{r}\right) + grad\left(\boldsymbol{K}.\text{grad}\ \frac{1}{r}\right) = 0,$ where $\boldsymbol{K}$ is the unit vector in the direction OZ.

Q.10. Given $\boldsymbol{R}(t) = 3t^2\boldsymbol{I} + t\boldsymbol{J} - t^3\boldsymbol{K},$ evaluate $\int_0^1 (R \times \frac{d^2\boldsymbol{R}}{dt^2})\, dt.$

Answer: $-2\boldsymbol{I} + 3\boldsymbol{J} - 3\boldsymbol{K}$

Q.11. If $\boldsymbol{F} = 3xy\boldsymbol{I} - y^2\boldsymbol{J},$ evaluate $\int \boldsymbol{F}.d\boldsymbol{R}$, where C is the curve in the xy-plane $y = 2x^2$ from (0, 0) to (1, 2).

Answer: $-\frac{7}{6}$

Q.12. Find the work done in moving a particle in the force field $\boldsymbol{F} = 3x^2\boldsymbol{I} + (2xz - y)\boldsymbol{J} + z\boldsymbol{K}$, along

(i). straight line from (0, 0, 0) to (2, 1, 3) Answer: 16

(ii). curve defined by $x^2 = 4y,\ 3x^3 = 8z$ from x=0 to x=2 Answer: 16

Q.13. Evaluate $\int \boldsymbol{F}.\boldsymbol{N}\, ds$, over surface S where $\boldsymbol{F} = 2x^2y\boldsymbol{I} - y^2\boldsymbol{J} + 4xz^2\boldsymbol{K}$ and S is the closed surface of the region in the first octant bounded by cylinder $y^2 + z^2 = 9$ and planes $x = 0, x = 2, y = 0, z = 0.$

Answer: 180

Q.14. Verify Green's Theorem for $\int[(xy + y^2)dx + x^2dy]$ over the curve C, where C is bounded by $y = x\ and\ y = x^2$.

Q.15. Using Green's Theorem, evaluate $\int[(y - Sinx)dx + Cosx\ dy]$ over the curve C, where C is plane triangle enclosed by lines $y = 0, x = \frac{\pi}{2}, y = \frac{2x}{\pi}$.

Answer: $-(\frac{\pi}{4} + \frac{2}{\pi})$

Q.16. Verify Stoke's Theorem for $\boldsymbol{F} = (x^2 + y^2)\boldsymbol{I} - 2xy\boldsymbol{J}$, taken around the rectangle bounded by lines $x = a, x = -a, y = 0, y = b$.

Q.17. Using Stoke's Theorem, evaluate $\int[(x + y)dx + (2x - z)dy + (y + z)dz]$ over C, where C is boundary of triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6).

Answer: 21

Q.18. Verify Gauss Divergence Theorem for $\boldsymbol{F} = (x^2 - yz)\boldsymbol{I} + (y^2 - zx)\boldsymbol{J} + (z^2 - xy)\boldsymbol{K}$ taken over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$.

Q.19. Evaluate $\int \boldsymbol{F}.d\boldsymbol{s}$ over surface S where $\boldsymbol{F} = 4x\boldsymbol{I} - 2y^2\boldsymbol{J} + z^2\boldsymbol{K}$, and S is the surface bounding the region $x^2 + y^2 = 4, z = 0\ and\ z = 3$.

Answer: 84π