7. INTRODUCTION TO INTERNAL FLOW

INTERNAL FLOW: A flow through a conduit (without free surface), which is driven by pressure gradient.

INTERNAL FLOW	OPEN CHANNEL FLOW
No free surface.	Free surface present.
Pressure need not be atmospheric.	Pressure throughout the free surface is atmospheric.
Pressure varies throughout the flow.	Pressure throughout the flow remains constant.
Flow is driven by pressure forces/ Pressure Gradient.	Flow is driven by Gravitational Force.
E.g. Every Flow through pipe, etc…	E.g. flow in canal river, etc…

REYNOLD'S NUMBER ($\boldsymbol{Re}$): It's dimensionless number represent the ratio of inertia force to viscous force.

$Re = \dfrac{F_{Ine.}}{F_{Vis.}} = \dfrac{\rho VL}{\mu}$ Where, L = Characteristic dimension, $L = D\ (in\ pipe\ flow)$	$F_{Ine.} = ma = \rho\forall\left(\dfrac{dV}{dt}\right) \propto \rho V^2 L^2$	$here, \forall \propto L^3\ \&\ \dfrac{dV}{dt} = \dfrac{dV}{ds} dV \propto \dfrac{V^2}{L}$
	$F_{Vis.} = \tau_{Vis.} A_s = \mu\dfrac{du}{dy} A_s \propto \mu VL$	$here, A_s \propto L^2\ \&\ \dfrac{du}{dy} \propto \dfrac{V}{L}$

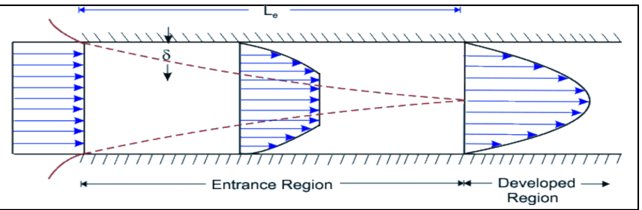
For flow through Pipe/ Duct,		
$Re \le 2000$, Laminar Flow	$2000 < Re \le 4000$, Transition Flow	$Re > 4000$, Turbulent Flow

DEVELOPMENT/ ESTABLISHMENT OF FLOW:

$$\frac{L_e}{D} = 0.07\ Re(For\ Laminar\ Flow)$$

$$\frac{L_e}{D} = 50(For\ Turbulent\ Flow)$$

L_e = Establishment entrance length,



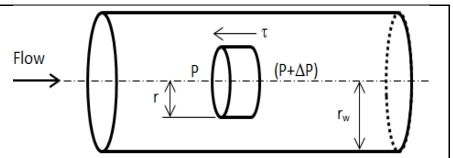
FUNDAMENTALS OF INTERNAL FLOW THROUGH PIPES:

CONDITIONS FOR VALIDITY OF THEORY:

1. Flow must be steady. ($Q = Const.$)
2. Flow must be fully developed.

EQUATION GOVERNING INTERNAL FLOW THROUGH PIPES:

$F = P2\pi rdr$	$F + dF = (P + dP)2\pi rdr$	r =Radial Axis,
$F_{sf} = \tau_r 2\pi rdx$	$F_{sb} = \tau_{r+dr} 2\pi(r + dr)dx$	x = Axis along the flow,



$$\sum F_x = 0 \Rightarrow -r\left(\frac{dP}{dx}\right) = \frac{(\tau_r)_{r+dr} - (\tau_r)_r}{dr} = \frac{d(\tau_r)_r}{dr}$$

$r\left(\dfrac{-dP}{dx}\right) = \dfrac{d(r\tau_r)}{dr}(For\ avg.pressure)$	$r\left(\dfrac{-\partial P}{\partial x}\right) = \dfrac{d(r\tau_r)}{dr}(For\ Local\ pressure)$

PRESSURE DROP:

Pressure decreases linearly along the length of flow,

$P \propto (-x)^1 \Rightarrow P = C_1(-x)^1 + C_2 \Rightarrow \left(\dfrac{-dP}{dx}\right) = C_1 = Constant\ (+ve)$	$\left(\dfrac{-dP}{dx}\right) = \dfrac{\Delta P}{L} = Constant$

SHEAR STRESS ($\boldsymbol{\tau}$):

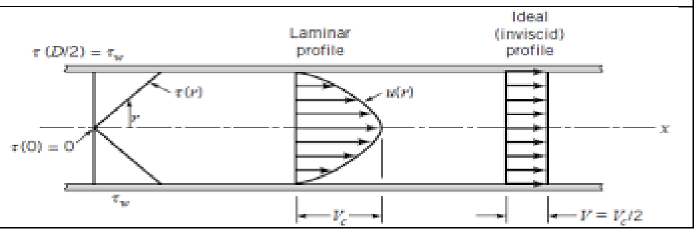
$$\int_{r\tau=0}^{r\tau} d(r\tau) = \left(\frac{-dP}{dx}\right)\int_{r=0}^{r} r\,dr \Rightarrow \tau = \left(\frac{-dP}{dx}\right)\frac{r}{2}$$

This equation is valid for steady and fully developed for any type of flow. E.g. Laminar, Turbulent, etc…

Conclusion:

1. τ increases with respect to radius.
2. At the wall of the pipe shear stress is maximum known as boundary shear or wall shear stress (τ_0).

$$\tau_0 = \left(\frac{-dP}{dx}\right)\frac{R}{2}\ \&\ \frac{\tau}{\tau_0} = \frac{r}{R}$$



FRICTION COEFFICIENT & FRICTION FACTOR: "The ratio of wall shear stress dynamic pressure is known as friction coefficient." **FOR INTERNAL FLOW:** "Friction coefficient if known as fanning's friction coefficient (f')" **DARCY FRICTION FACTOR ($\boldsymbol{f}$):** $f = 4f'$	$Friction\ Coeff. = \frac{\tau_0}{\frac{1}{2}\rho V^2}$ $f' = \frac{\tau_0}{\frac{1}{2}\rho V^2}$

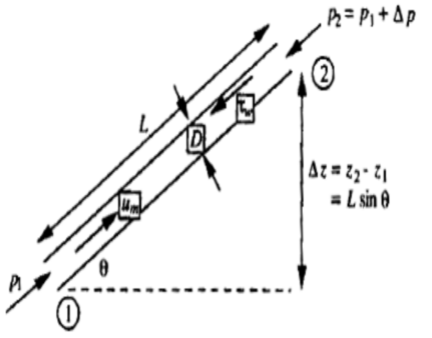
MAJOR HEAD LOSS ($\boldsymbol{h}$): For inclined pipe, by using continuity equation and Bernoulli's Equation, $h = \Delta h^*$(Piezometric head difference)

$$\sum F_x = \Delta(\dot{\boldsymbol{m}}\boldsymbol{V_x}) = 0 \Rightarrow P_1\left(\frac{\pi D^2}{4}\right) - P_2\left(\frac{\pi D^2}{4}\right) - W\sin\theta - \tau_0 A_s = 0$$

$A_s = \pi DL$	$W = \gamma \forall = \gamma \frac{\pi D^2}{4} L$	$\tau_0 = \frac{f}{8}\rho V^2$

$$h = \frac{fLV^2}{2gD} = \frac{4f'LV^2}{2gD} (\because h = \Delta h^*)$$

- Above Equation is known as **Darcy-Weisbach Equation**.



SUMMARY:

1	Flow Governing Equation,	$r\left(-\frac{dP}{dx}\right) = \frac{d(r\tau)}{dr}$
2	Pressure Drop ΔP is positive in the direction of flow.	$\left(\frac{-dP}{dx}\right) = \frac{\Delta P}{L} = Constant\ (+ve)$
3	Shear Stress or Wall shear Stress,	$\tau = \left(\frac{-dP}{dx}\right)\frac{r}{2}$ & $\tau_0 = \left(\frac{-dP}{dx}\right)\frac{R}{2}$ & $\frac{\tau}{\tau_0} = \frac{r}{R}$
4	Friction factor or friction coefficient,	$f = 4f'$
5	Darcy-Weisbach Equation,	$h = \frac{fLV^2}{2gD} = \frac{4f'LV^2}{2gD}$

NOTE: If the flow is non horizontal then, Replace every static pressure term by corresponding piezometric pressure.