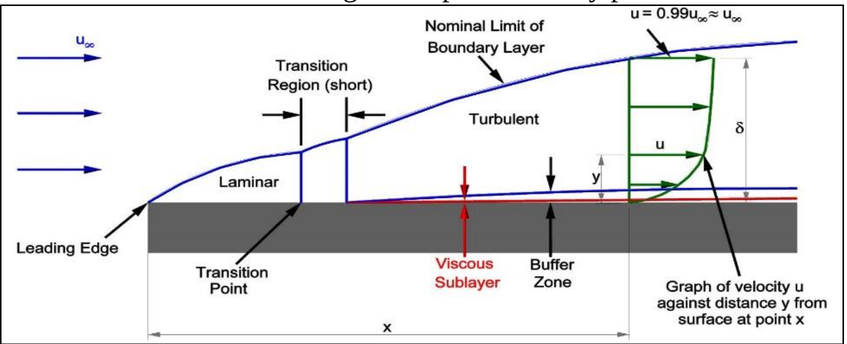
11. BOUNDARY LAYER THEORY

EXTERNAL FLOW	
BOUNDARY LAYER THEORY	**FLOW OVER SUBMERGE BODY**

As we go away from the wall the shear stress decreases because of increasing the slope of velocity profile.

1. When a real fluid flows and Passes a solid boundary, viscous effects get concentrated in a very thin region adjacent to the surface.
2. The flow in this thin region is known as boundary layer flow. (Real, Rotational, No slip condition)
3. The flow beyond the boundary layer. (Ideal, Irrotational, with almost uniform velocity profile.)



BOUNDARY LAYER THEORY ON FLAT PLATES:

U_∞ = Free stream Velocity	$x = 0$ =Leading Edge	$x = L$ =Trailing Edge
u = Local Velocity	B = Width of the plate	
δ = Boundary Layer/ Nom. thickness	x = Dist. Measured Along the plate from LE	

$Re_x = \frac{\rho U_\infty x}{\mu} = \frac{U_\infty x}{\vartheta}$	$Re_L = \frac{\rho U_\infty L}{\mu} = \frac{U_\infty L}{\vartheta}$

CLASSIFICATION OF FLOW INSIDE BOUNDARY LAYER			
LAMINAR	**TRANSITIONAL**		**TURBULENT**
	HLBL	**LTBL**	
$Re_x \le 2 * 10^5$	$Re_x \le 5 * 10^5$	$Re_x > 5 * 10^5$	$Re_x > 6 * 10^5$

LAMINAR BOUNDARY LAYER (LBL):

1. BOUNDARY LAYER THICKNESS/ NOMINAL THICKNESS (δ):

The distance measured perpendicular or normal from the wall of a plate till the point velocity is almost U_∞.

$\boldsymbol{\frac{\delta}{x} = \frac{K}{\sqrt{Re_x}}, Where\ K = \ Constant}$	1. K Depends on the velocity profile inside the BL 2. $K = 5$ (if not given)

2. VELOCITY PROFILE:

EXACT VELOCITY PROFILE	**APPROXIMATE VELOCITY PROFILE**	
Blasius Velocity profile.	E.g. Linear, Parabolic, Cubic, sinusoidal Velocity Profile	
$K = 5$ (Blasius Constant)	$Linear\ VP: \frac{u}{U_\infty} = \frac{y}{\delta}, K = 3.46$	$Sinusoidal\ VP: \frac{u}{U_\infty} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right), K = 4.79$

3. WALL SHEAR STRESS (τ_0):

$Friction\ Coeff. = \frac{Wall\ Shear\ Stress}{Dynamic\ Pressure} = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2} = C_f = Local\ Skin\ Friction$	In LBL, $\boldsymbol{C_f = (Re_x)^{-0.5}}$ $\therefore \tau_0 \propto (x)^{-0.5}$

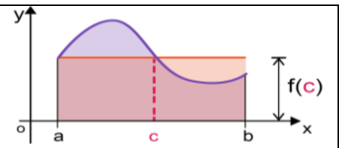
CASE-I: Exact Velocity Profile	CASE-II: Approximate Velocity Profile
$\boldsymbol{C_f = \frac{0.664}{\sqrt{Re_x}}} \Rightarrow \tau_0 = \frac{0.664}{\sqrt{Re_x}} * \frac{1}{2}\rho U_\infty^2$	$\tau_0 = \mu \left.\frac{du}{dy}\right\|_{y=0}$

4. DRAG FORCE ($\boldsymbol{F_D}$):

$$F_D = \int dF_D = \int \tau_0\, dA = B \int \tau_0\, dx = Drag\ force\ on\ one\ side\ of\ the\ plate$$

$$F_D = B \int C_f * \frac{1}{2}\rho U_\infty^2 dx = \frac{B}{2}\rho U_\infty^2 \int_0^L C_f * dx = \frac{1}{2}\rho U_\infty^2 C_D (LB)(1\ Side\ of\ Plate)$$

$\overline{C_f}$ = Average Skin Friction = C_D = Coefficient of drag.



$$C_D = \frac{1}{L}\int C_f * dx$$

SUMMARY

$\frac{\delta}{x} = \frac{K}{\sqrt{Re_x}}$	$\frac{\tau_0}{\frac{1}{2}\rho U_\infty^2} = C_f \propto \frac{1}{\sqrt{Re_x}}$	$\tau_0 = C_f \frac{1}{2}\rho U_\infty^2 = \mu \left.\frac{du}{dy}\right\|_{y=0}$	$C_f = \frac{0.664}{\sqrt{Re_x}}$
$F_D = \int \tau_0\, dA = \frac{1}{2}\rho U_\infty^2 C_D (LB)$		$C_D = \frac{1}{L}\int_0^L C_f * dx$	

TURBULENT BOUNDARY LAYERS ON FLAT PLATES: $Re_x > 5 * 10^5$

1. BOUNDARY LAYER THICKNESS/ NOMINAL THICKNESS ($\boldsymbol{\delta}$):

$\boldsymbol{\delta = \frac{0.371x}{(Re_x)^{1/5}} \propto x^{4/5}}$	**NOTE:** Turbulent Boundary layer grows faster along a plate compared to laminar boundary layer.

2. VELOCITY PROFILE:

$\boldsymbol{n^{th} POWER\ LAW}: \frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/n}$	Std. Value, $n = 7$

3. WALL SHEAR STRESS ($\boldsymbol{\tau_0}$):

$\tau_0 = C_f \frac{1}{2} \rho U_\infty^2$	$\boldsymbol{C_f = \frac{0.0579}{(Re_x)^{1/5}}\ (For\ 7^{th}\ Power\ Law)}$

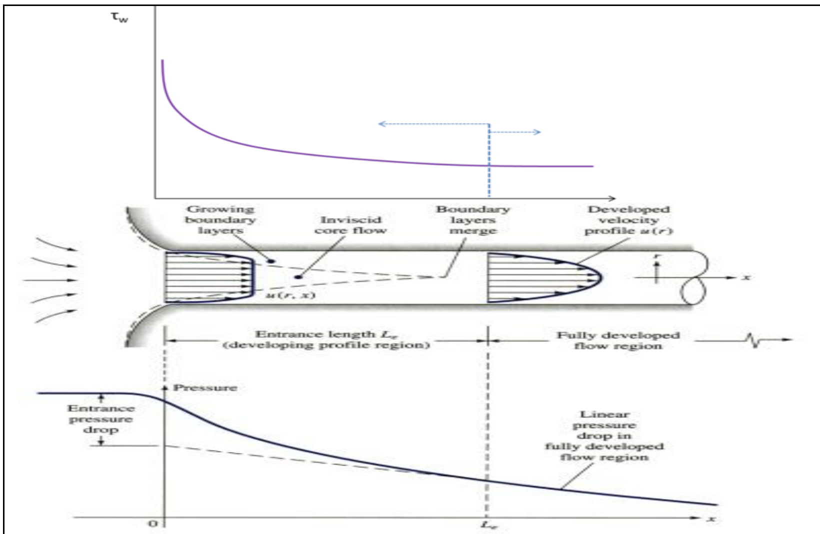
4. DRAG FORCE ($\boldsymbol{F_D}$):

$F_D = \frac{1}{2} \rho U_\infty^2 C_D A, Where\ A = BL$	$C_D = \frac{0.072}{(Re_x)^{1/5}}$ $(For\ 5 * 10^5 \leq Re_x \leq 10^7)$	$C_D = \frac{0.455}{(\log_{10} Re_x)^{2.58}}$ $(For\ 10^7 \leq Re_x \leq 10^9)$

BOUNDARY CONDITIONS:

1. No Slip Condition: At $y = 0, u = 0 \Rightarrow \tau = du/dy = 0$
2. Definition of Boundary Layer: At $y = \delta, u \approx U_\infty \Rightarrow \tau = du/dy = 0$
3. No Variation in velocity outside boundary Layer: At $y \geq \delta, u \approx U_\infty \Rightarrow \tau = du/dy = 0$
4. Linear Velocity Profile or Constant shear stress neat to the surface:
   At $y \approx 0, \tau = du/dy = C \Rightarrow d\tau/dy = d^2u/dy^2 = 0$
5. At $y = \delta, u \approx U_\infty \Rightarrow d\tau/dy = d^2u/dy^2 = 0$

Note: Boundary Conditions must be used in same sequence as given because initial boundary conditions are more fundamental in nature.



DEVELOPMENT OF FLOW:

As shown in the graphs,

1. Wall shear stress decreases during entrance length. And in the turbulent region wall shear stress remains constant.
2. Pressure or Head decreases throughout the length of the pipe. And in the turbulent region Pressure dopes linearly decreasing.
3. u_{max} increasing in the entrance length and it remains maximum in the turbulent region.

In the turbulent region,

$$u_{max} = 2u_{avg} = 2U_\infty$$

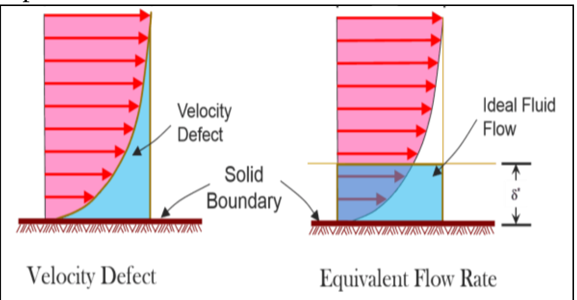
Entrance Length	
Max. for Laminar	Min. for Turbulent

DISPLACEMENT THICKNESS ($\boldsymbol{\delta^*}$): The distance measured perpendicular or normal to the wall, by which a stream line following adjacent to boundary layer gets displaced in order to compensate reduced mass flux.

$\dot{m}_1$ = Issued Mass Flux,
$\dot{m}_2$ = Mass Flux Allowed to Flow through BL,
$\dot{m}_3$ = Mass Flux Reduced/ Not Allowed to Flow through BL,
From continuity equation, $\dot{m}_3 = \dot{m}_1 - \dot{m}_2$

$$(\delta^* B)U_\infty = \int_0^\delta U_\infty B\, dy - \int_0^\delta uB\, dy = \int_0^\delta B(U_\infty - u)\, dy$$

$$\boldsymbol{\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy}$$



Note: For power law profile, $\delta^* = \delta/(n+1)$

CONCLUSION:	1. $\delta^* < \delta$	2. For linear Velocity profile, $\delta^* = 0.5\ \delta$

Alternative Definition of $\boldsymbol{\delta^*}$: The distance measured perpendicular to the surface by which the surface should be displaced to compensate the reduction in mass flux inside the boundary layer.

MOMENTUM THICKNESS (Θ): It's the distance measured perpendicular or normal to the wall of a plate by which it should be displaced in order to compensate the reduction in momentum flux inside the boundary layer.

$\boldsymbol{\Theta = \int_0^{\delta} \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy}$		$\boldsymbol{SHAPE\ FACTOR\ = H = \frac{\delta^*}{\Theta}\ (Always > 1)}$
For LBL, $H \epsilon\ (2.5 - 3)$	For Blasius Velocity Profile $H = 2.59$	For TBL, $H \epsilon\ (1.3 - 1.4)$

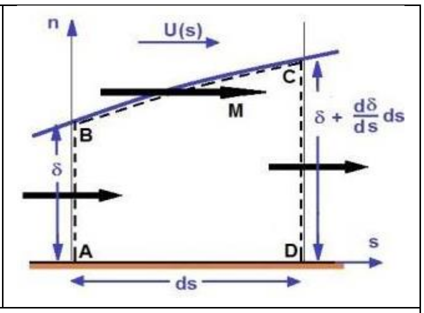
NOTE: Higher the shape factor, higher the chances of adverse pressure gradient. Therefore, higher chances of boundary layer separation.

KARMAN MOMENTUM INTEGRAL EQUATION:

$\dot{m}_{AB} = \int_0^{\delta} \rho u dy$	$\dot{m}_{CD} = \dot{m}_{AB} + \frac{\partial \dot{m}_{AB}}{\partial x} dx = \int_0^{\delta} \rho u dy + \frac{\partial}{\partial x}\int_0^{\delta} \rho u dy dx$

From Continuity equation, $\dot{m}_{BC} = \dot{m}_{CD} - \dot{m}_{AB} = \frac{\partial}{\partial x}\int_0^{\delta} \rho u dy dx$

$(\dot{m}V)_{AB} = \int_0^{\delta} \rho u^2 dy$	$(\dot{m}V)_{CD} = \int_0^{\delta} \rho u^2 dy + \frac{\partial}{\partial x}\int_0^{\delta} \rho u^2 dy\, dx$
$(\dot{m}V)_{BC} = \frac{\partial}{\partial x}\int_0^{\delta} \rho u dy dx\, U_\infty$	



From Momentum Equation,

$$\sum F_x = -\tau_0 dx = \Delta(\dot{m}V)_x = (\dot{m}V)_{CD} - (\dot{m}V)_{AB} - (\dot{m}V)_{BC} = \frac{\partial}{\partial x}\int_0^{\delta} \rho(u^2 - uU_\infty) dy dx$$

$$\frac{\tau_0}{\rho U_\infty^2} = \rho \frac{\partial}{\partial x}\int_0^{\delta}\left(\frac{u}{U_\infty} - \frac{u^2}{U_\infty^2}\right) dy = \boldsymbol{\frac{\partial \Theta}{\partial x}} = \boldsymbol{\frac{1}{2} C_f}(Karman\ Momentum\ Integral\ Equation)$$

ASSUMPTIONS IN DERIVATION:

The flow is incompressible.	Applicable for 2D flow.	$dP/dx = 0$

ENERGY THICKNESS ($\boldsymbol{\delta_E\ or\ \delta^{**}}$): The distance measured perpendicular or normal to the wall of a plate by which it should be displaced in order to compensate the reduction in kinetic energy flux.

$\boldsymbol{\delta_E = \int_0^{\delta} \frac{u}{U_\infty}\left[1 - \left(\frac{u}{U_\infty}\right)^2\right] dy}$	For Linear Velocity Profile,			
	$\delta^* = \delta/2$	$\Theta = \delta/6$	$H = 3$	$\delta_E = \delta/4$

BOUNDARY LAYER SEPARATION:

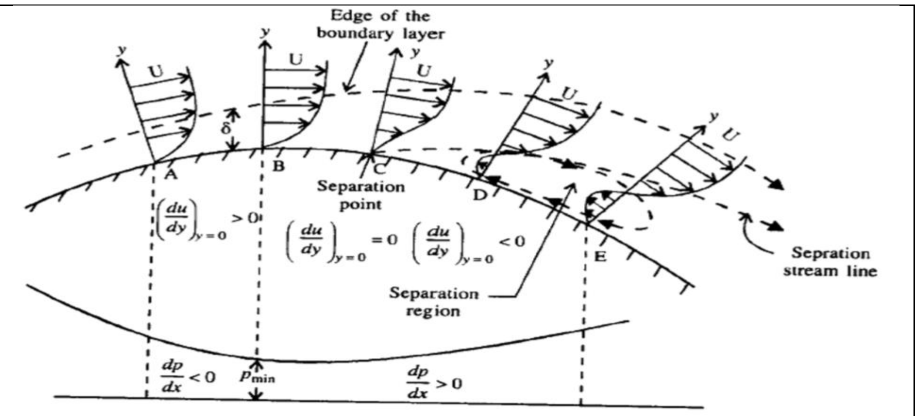
Forces Consideration in the Boundary layer Separation,

F_I = Inertia Force	F_V = Viscous Force	F_P = Pressure Force

Conditions in the flows,

1	$\frac{dP}{dx} < 0 (Favorable\ Pressure)$	Accelerated Flow	F_I & F_P Opposing F_V
		In this Flow Condition, no eddy formations taking place. (Low loss, high η)	
2	$\frac{dP}{dx} > 0 (Adverse\ Pressure)$	Decelerated Flow	F_V & F_P Opposing F_I
		In this Flow Condition, eddy formations/ Flow reversal taking place. (Higher loss, lower η)	

1. When a BL encounters adverse Pressure Gradient, flow experiences deceleration.
2. The BL thickness drastically increases.
3. A portion of BL near the surface separates from the wall. This region is characterised by formation of eddies and known as wake region.
4. This leads to increase in loss and decrease in efficiency.



Factor influencing BL separation,

1. Curvature of the surface.	3. Roughness (TBL) or Smoothness (LBL) of the surface.
2. Re	4. Turbulent flow has lesser boundary layer separation.