8. TAYLOR'S SERIES

The Taylor's series expansion of $\boldsymbol{f(x)}$ About $\boldsymbol{x = a}$ is given by,

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^n(a)}{n!}(x-a)^n$$

Special Case $(a = 0)$ of Taylor's Series is McLaurin's Series.

MacLaurin's Series of different functions,

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \cdots = \frac{e^x - e^{-x}}{2}$	$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots = \frac{e^x + e^{-x}}{2}$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \cdots$	$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!} + \cdots$

The Taylor's series expansion of $\boldsymbol{f(x, y)}$ About $(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{a}, \boldsymbol{b})$ is given by,

$$f(x,y) = f(a,b) + \frac{\partial f(a,b)}{\partial x}(x-a) + \frac{\partial f(a,b)}{\partial y}(y-b)$$

$$+ \frac{1}{2!}\left[\frac{\partial^2 f(a,b)}{\partial x^2}(x-a)^2 + \frac{\partial^2 f(a,b)}{\partial y^2}(y-b)^2 + 2\frac{\partial^2 f(a,b)}{\partial x \partial y}(x-a)(y-b)\right] + \cdots$$

$$\boldsymbol{f(x,y) = \sum_{n=0}^{\infty}\left(\frac{1}{n!}\sum_{k=0}^{n} {}^{n}C_k \frac{\partial^n f(a,b)}{\partial x^{n-k}\partial y^k}(x-a)^{n-k}(y-b)^k\right)}$$