6. MOMENTUM EQUATION

FORCE: Rate of change in linear momentum is known as force.

$\vec{F}dt = d(m\vec{V})$	$\vec{F}dt$ = Impulse	$d(m\vec{V})$ = Change in Linear Momentum
$\vec{F} = d(\dot{m}\vec{V})$	$\vec{F}$ = Force	$d(\dot{m}\vec{V})$ = Change in Momentum Flux

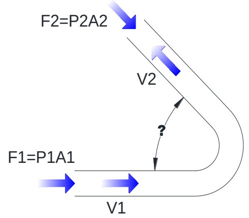
$$\sum F_k = \Delta(\dot{m}V)_k = (\dot{m}V_k)_f - (\dot{m}V_k)_i$$

CONTROL VOLUME: The volume chosen in a fluid flow for analysis (on which the force interaction takes place). The boundary of control volume is known as control surface.

1. Control Volume can be static or moving, but it should not have acceleration.
2. Control volume can have any shape or size, but it should be drawn in such a way that the velocities (initial and final) are supposed to be perpendicular to the control surface.
3. The supports in a control volume should be replaced by equivalent reactions.

$$P_1A_1 - R_x + P_2A_2\cos\theta = \dot{m}(-V_2\cos\theta - V_1)$$
$$\therefore R_x = P_1A_1 + P_2A_2\cos\theta + \dot{m}(V_2\cos\theta + V_1)$$
$$R_y - P_2A_2\sin\theta = \dot{m}V_2\sin\theta$$
$$\therefore R_y = P_2A_2\sin\theta + \dot{m}V_2\sin\theta$$

$Net\ Reaction = \sqrt{R_x^2 + R_y^2}$	$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

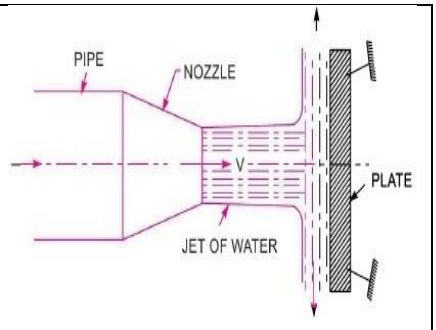


Here, momentum flux creates error if velocity profiles are non-uniform.

$Actual\ momentum\ flux = \beta\dot{m}V = \iint \rho u^2 dA$	$Momentum\ flux\ Correction\ Factor\ \beta = \frac{1}{AV^2}\iint u^2 dA$	
Always For non uni. flow $\beta \geq 1$ & $\alpha > \beta$ For uniform Flow $\beta = 1$ & $\alpha = 1$	For Laminar Flow through pipe $\beta = 4/3$, $\alpha = 2$	α = K.E. Correction factor. $\sum \boldsymbol{F_k} = \Delta(\boldsymbol{\beta\dot{m}V})_{\boldsymbol{k}}$

IMPACT OF LIQUID JET:

Things Already known,	Things to known,
1. ρ 2. A 3. V_{jet} 4. u_{plate}	1. F_{impact} 2. $Power = W/t = F_{impact} \cdot u_{plate}$ 3. $Input = KE/time = (1/2)\dot{m}\,V_{jet}^2$ 4. $\eta_{jet} = Power/Input$



ASSUMPTIONS:

1. Neglect Gravity in horizontal jets.
2. Exit of the jet after striking is tangential to the surface.
3. Neglect friction along the plate surface. $V_{ExitAfterStriking} = V_{jet@Striking}$

For Moving Plate,	$V_{striking} = V_{jet} - u$	$\dot{m}_{exit} = \rho AV_{jet}$	$\dot{m}_{striking} = \rho A(V_{jet} - u)$
For Wheel,			$\dot{m}_{striking} = \rho AV_{jet}$

$\sum \boldsymbol{F_k} = \Delta(\boldsymbol{\dot{m}V})_{\boldsymbol{k}}$	$Power$	$Input = \dot{KE}$	η_{jet}
CASE-I: Horizontal Jet striking normally on a flat vertical plate.			
$F_x = \rho A(V_{jet} - u)^2$	$\rho A(V_{jet} - u)^2 u$	$(1/2)\,\rho AV_{jet}^3$	$2\,u\left[(V_{jet} - u)^2/V_{jet}^3\right]$
CASE-II: Horizontal Jet striking on a series of flat plates mounted on a wheel.			
$F_x = \rho AV_{jet}(V_{jet} - u)$	$\rho AV_{jet}(V_{jet} - u)u$	$(1/2)\,\rho AV_{jet}^3$	$2\,u\left[(V_{jet} - u)/V_{jet}^2\right]$
Condition for $[\eta_{jet}]_{max}$, $Speed\ Ratio\ \emptyset = u/V_{jet} = 1/2$ & $[\eta_{jet}]_{max} = 0.50$			
CASE-III: Horizontal Jet striking normally on symmetric curved vane.			
$F_x = \rho A(\cos\theta + 1)(V_{jet} - u)^2$	$\rho A(\cos\theta + 1)(V_{jet} - u)^2 u$	$(1/2)\,\rho AV_{jet}^3$	$2\left[(\cos\theta + 1)(V_{jet} - u)^2 u\right]/V_{jet}^3$
For semi-circular vane ($\theta = 0°$),$F_x = [F_x]_{max}$= $2\rho A(V_{jet} - u)^2$			
CASE-IV: A Jet is striking tangentially on a fixed curved vane.			
$F_x = \rho A(\cos\theta + \cos\emptyset)V_{jet}^2$		$F_y = \rho A(\sin\emptyset - \sin\theta)V_{jet}^2$	
For symmetric curved vane ($\emptyset = \theta$), $F_x = 2\rho A\cos\theta\, V_{jet}^2$		& $F_y = 0$	
CASE-V: A jet striking on a fixed inclined plate,			
$\sum F_n = \Delta(\dot{m}V)_n$	$F_x = F_n\cos\theta$	$F_x = F_n\sin\theta$	$F_n = \rho AV_{jet}^3\cos\theta$

Note:

1) Net force is normal to the plate. $\sum F_t = \Delta(\dot{m}V)_t = 0 \Rightarrow (\dot{m}V)_f = (\dot{m}V)_i \Rightarrow Q_1 - Q_2 = Q \sin\theta$
2) From the continuity equation, $Q_1 + Q_2 = Q$

$Q_1 = \frac{Q}{2}(1 + \sin\theta)$	$Q_2 = \frac{Q}{2}(1 - \sin\theta)$

CASE-V: A jet striking on a fixed inclined plate,			
$\sum F_n = \Delta(\dot{m}V)_n$	$F_x = F_n \cos\theta$	$F_x = F_n \sin\theta$	$F_n = \rho A\left(V_{jet} - u\right)^2 \cos\theta$

For inclined Plates,

$P = F_x u = \rho A\left(V_{jet} - u\right)^2 \cos^2\theta \ u$	$\dot{KE} = \left(\frac{1}{2}\right) \rho A V_{jet}^3$	$\eta_{jet} = \dfrac{2\left(V_{jet} - u\right)^2 \cos^2\theta \ u}{V_{jet}^3}$