2. DESIGN FOR STATIC LOADING

LOAD: anything externally acting on the component produces deformation in the component is said to be load.

CLASSIFICATION OF LOAD:

BASED ON LOADING WITH RESPECT TO TIME	
Static loading (Constant w.r.t. time)	Dynamic loading (Varying w.r.t. time)

BASED ON RATE OF LOADING		
Gradually applied loading	Sudden loading	Impact loading

Draw Graphs of above loading.

BASED ON LOADING ON SURFACE	
NORMAL LOADING	**SHEAR LOADING**
Component of force perpendicular to the surface	Component of force parallel to the surface
Eg. liner deformation	Eg. Angular deformation

<table>
<tr><th colspan="6">BASED ON DIRECTION OF LOADING</th></tr>
<tr><th colspan="2">AXIAL LOADING</th><th colspan="2">LONGITUDINAL LOADING</th><th colspan="2">RADIAL LOADING</th></tr>
<tr><td>AXIAL</td><td>ECCENTRIC AXIAL</td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>LOA of force passing through axis of member</td><td>LOA of force doesn’t pass through axis of member</td><td rowspan="2">Only Axial load</td><td rowspan="2">Axial load + Twisting</td><td></td><td></td></tr>
<tr><td>Only Axial load</td><td>Bending + Axial load</td><td></td><td></td></tr>
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Normal Principle Stress: When there is no shear stress.
Plane Stress condition: All Stresses are present in only one plane. Eg. only XY/ YZ/ ZX plane.
Plane Strain condition: All Strains are present in only one plane. Eg. only XY/ YZ/ ZX plane.

(Machine > Component > Particle => All need to be safe)
Normal Stress: It’s normal load acting on the smallest particle of the component.
Normal Strain: It’s the linear deformation experienced by smallest particle in the component.
Due to Normal load, Changes in Size and Shape.
Hook’s Law: Normal Stress $\sigma \propto$ Normal Strain δ

Shear Stress: It’s load acting on the surface of particle which is perpendicular to the surface.
Shear Strain: It’s the angular deformation experienced two perpendicular surfaces of particle. $\gamma = \sum\theta$
Hook’s Law: Shear Stress $\tau \propto$ Normal Strain γ

Bending: Rotation of member about axis parallel to the cross section.
Twisting: Rotation of member about axis perpendicular to the cross section.

Stress And Strain Representation:
X-plane: The plane whose surface normal vector is in X- direction.
Stress: 2nd order tensor.

$\begin{array}{cc} & Direction \\ \sigma_{ij} = Plane & \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \end{array}$	**Plane Stress Condition: Stresses are only in one plane.** $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$	$\begin{array}{cc} & Direction \\ Plane & \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$
$\begin{array}{cc} & Direction \\ \varepsilon_{ij} = Plane & \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy}/2 & \gamma_{xz}/2 \\ \gamma_{yx}/2 & \varepsilon_{yy} & \gamma_{yz}/2 \\ \gamma_{zx}/2 & \gamma_{zy}/2 & \sigma_{zz} \end{bmatrix} \end{array}$	**Plane Stress Condition: Strains are only in one plane.** $\varepsilon_{zz} = \gamma_{xz}/2 = \gamma_{yz}/2 = 0$	$\begin{array}{cc} & Direction \\ Plane & \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy}/2 & 0 \\ \gamma_{yx}/2 & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$

Poisson’s Ratio:
Deformation in perpendicular direction $\propto$ Deformation in direction of load

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$$

Principle Stress and Maximum Shear Stress:

$\sigma = C \pm R$, $\tau = R = \frac{\sigma_1 - \sigma_2}{2}$ Where $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$ And $C = \frac{\sigma_x + \sigma_y}{2}$

Principle Strain and Maximum Shear Strain:

$\varepsilon = C \pm R$, $\gamma/2 = R = \frac{\varepsilon_1 - \varepsilon_2}{2}$ Where $R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + (\gamma_{xy}/2)^2}$ And $C = \frac{\varepsilon_x + \varepsilon_y}{2}$

Combined Bending and Twisting:

$\boldsymbol{M_{eq} = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right], \quad T_{eq} = \sqrt{M^2 + T^2}}$

Hydrostatic loading (Change in Size, No change in Shape):

$\varepsilon_{Vh} = \frac{3\sigma_h}{E}[1 - 2\mu]$ Where, $\sigma_h = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$ ($\sigma_h = Bulk\ modulus * \varepsilon_{Vh}$, Hooks law)

DESIGN OF MACHINE ELEMENT UNDER STATIC LOADING:

ASSUMPTIONS FOR DESIGNING:

1) Material is homogeneous and isotropic.
2) Gradual loading (Unidirectional). And loading is in the elastic region.
3) Material is linear elastic.

FAILING	Ductile Material	Brittle Material
Tension test	Yield Strength in tension σ_{yt}	Ultimate Strength in tension σ_{ut}
Compressive test	Yield Strength in compression $\sigma_{yc} \cong \sigma_{yt}$	Ultimate Compression Strength σ_{uc}
Shear Test	Shear Yield Strength τ_{yt}	Shear Ultimate Strength τ_{ut}

Factor of Safety	**At Component level: (Force)** **FOS =** Design Capacity / Allowable force	**At Particle level: (Stress)** **FOS =** Design Stress / Allowable Stress

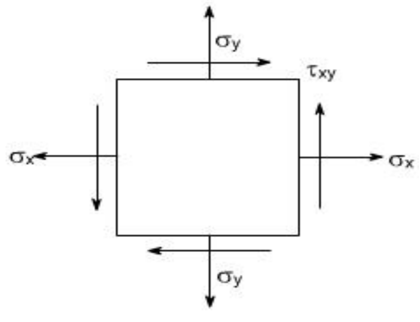
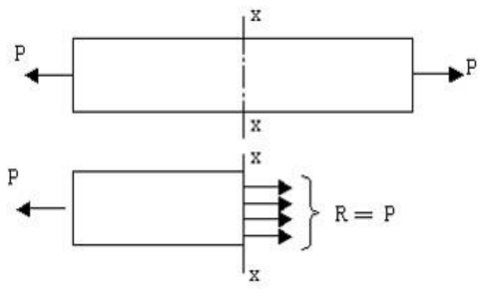
1. DESIGN OF COMPONENTS SUBJECTED TO UNI-AXIAL LOADING:

Normal Stress in Component	F/A	My_{max}/I
Shear Stress in Component		Tr_{max}/J

2. DESIGN OF COMPONENT SUBJECTED TO BI-AXIAL AND TRI-AXIAL LOADING:

There is no direct way to find safe load so scientists have converted combined loading to Uni-axial loading.

THEORIES OF FAILURE CLASSIFICATION AS PER FAILURE CRITERIA		
STRESS	**STRAIN**	**ENERGY**
1) Maximum normal stress theory (Principle Stress theory) **2) Maximum Shear stress theory**	**1) Principle Strain Theory**	**1) Detorsion energy theory (Shear Strain Energy theory)** 2) Total Strain Energy theory

COMPLEX STATE OF STRESS	SIMPLE (UNI-AXIAL) STATE OF STRESS
 Here, $\sigma_z = \tau_{xz} = \tau_{yz} = 0$	 Here, $\sigma_y = \sigma_z = \tau_{xz} = \tau_{xy} = \tau_{yz} = 0$
From the maximum principle stress plane, We can find σ_1 and σ_2. $\sigma = C \pm R$; $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$ $\tau = R = \frac{\sigma_1 - \sigma_2}{2}$; $C = \frac{\sigma_x + \sigma_y}{2}$ Here $\tau_{max} = \max\left\lvert\frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_1}{2}, \frac{\sigma_2}{2}\right\rvert$	From the maximum principle stress plane, We can find σ_1 and σ_2. Here, $\sigma_y = \tau_{xy} = 0$ Hence, $\sigma_1 = \sigma_2 = \sigma_x$; $C = R = \frac{\sigma_x}{2}$ $\tau_{max} = \max\left\lvert 0, \frac{\sigma_1}{2}, \frac{\sigma_2}{2}\right\rvert = \frac{\sigma_x}{2}$

MAXIMUM NORMAL STRESS/ PRINCIPLE STRESS/ RANKINE'S THEORY:

Statement: The failure of a component subjected to complex stress occurs when the **"Maximum Principle Stress"** at any point in the body reaches the **"Maximum Principle Stress"** of a material in simple tension test when failure occurs.

$$\sigma_{MPST} < (\sigma_{yt}\ or\ \sigma_{ut})/FOS\ , where\ \sigma_{MPST} = \max(|\sigma_1|, |\sigma_2|)$$

It's Used for Brittle Materials: Because brittle materials fail in normal stress (Not in shear stress) and this theory considers maximum normal stress in the analysis. Whereas Ductile materials are weak in shear so it's used to design using shear failure criteria.

Note:

1. This theory is neither accurate nor conservative. Because by experiment $\sigma_{MPST} < 0.57\sigma_{yt}$
2. This theory considers normal stress as failure stress hence, change in shape & size is considered.
3. This theory can't use for hydrostatic loading & pure shear loading.

MAXIMUM SHEAR STRESS/ GUEST & TRESCA'S THEORY:

Statement: The failure of a component subjected to complex stress occurs when the **"Maximum Shear Stress"** at any point in the body reaches the **"Maximum Shear Stress"** of a material in simple tension test when failure occurs.

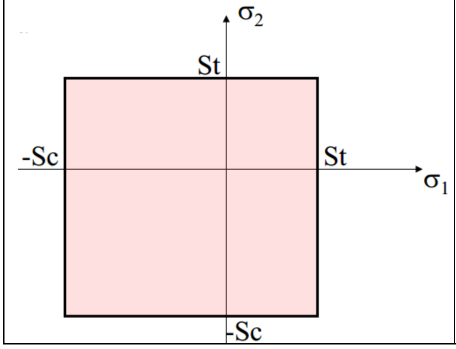
$$\tau_{MSST} < (\sigma_{yt}\ or\ \sigma_{ut})/2\ FOS\ , Where\ \tau_{MSST} = \tau_{max}$$

It's Used for Ductile Materials: Ductile materials are weak in shear so it's used to design using shear failure criteria.

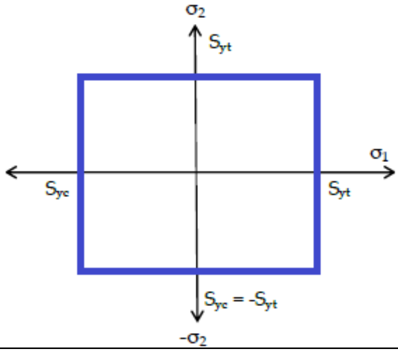
Note:

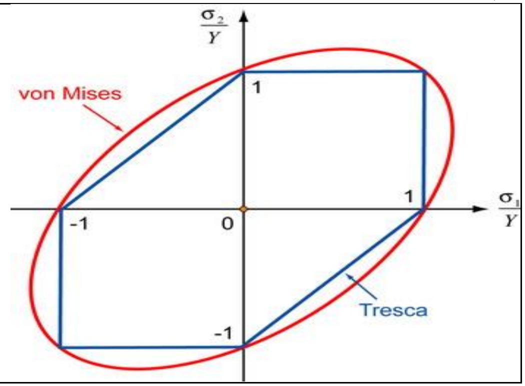
1. This theory considers Shear stress as failure stress hence, change in shape & size is not considered.
2. This theory can't use for hydrostatic loading.
3. This theory can used for pure shear loading.
4. This theory is conservative and accurate than MPST (It can be used for Brittle material but it's not cost effective).

Rankine: For Brittle material



Rankine: For Ductile material.





DISTORTION ENERGY/ VON MISES AND HENCKY'S THEORY:

$$U_T = U_v + U_d, Hence\ U_d = U_T - U_v$$

$U_T = \sum \frac{1}{2}\sigma_i\varepsilon_i$	$\varepsilon_x = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$
$\varepsilon_v = \frac{3\sigma_v}{E}[1-2\mu]$	$\varepsilon_v = \frac{\sum \sigma_{id}}{E}[1-2\mu] = 0$

Here, $\sigma_i = \sigma_{id} + \sigma_v$

U_T= Total Energy,
U_v= Energy Used to change the Size,
U_d= Energy Used to change the Shape,
ε_v =Volumetric Strain,
σ_i =Principle Stress,
σ_{id} =Stress Responsible to change the Shape,
σ_v =Volumetric Stress,

$$U_T = \frac{1}{2E}\left[\left(\sum \sigma_i^2\right) - 2\vartheta\left(\sum \sigma_i\sigma_j\right)\right], Where\ i < j$$

$$U_v = \frac{[1-2\mu]}{6E}\left(\sum \sigma_i\right)^2$$

$$U_d = \frac{1+\vartheta}{6E}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2]$$

$$= \frac{1+\vartheta}{6E}\left[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]$$

For our condition: $\sigma_3 = 0$

$$\boldsymbol{D.E.T: U_d = \frac{1+\vartheta}{3E}[(\sigma_1)^2 + (\sigma_2)^2 - \sigma_1\sigma_2] = \frac{1+\vartheta}{3E}\left[(\sigma_x)^2 + (\sigma_y)^2 - \sigma_x\sigma_y + 3(\tau_{xy}^2)\right]}$$

Statement: The failure of a component subjected to complex stress occurs when the **"DE/Unit Volume"** at any point in the body reaches the **"DE/Unit Volume"** of a material in simple tension test when failure occurs.

$$\sigma_{Van} < \sigma_{yt}\ or\ \sigma_{ut}/FOS\ , Where\ (\sigma_{Van})^2 = (\sigma_1)^2 + (\sigma_2)^2 - \sigma_1\sigma_2 = (\sigma_x)^2 + (\sigma_y)^2 - \sigma_x\sigma_y + 3(\tau_{xy}^2)$$

EQUATION OF ELLIPSE: $x^2 + y^2 - xy = a^2$	**Semi Minor Axis=** $\boldsymbol{\sqrt{2/3}\ a}$ **Semi Major Axis=** $\boldsymbol{\sqrt{2}\ a}$

Note:

1. This theory is more accurate but not conservative than MSST.
2. This theory considers only shape deformation at failure. hence, change in shape & size is not considered.
3. This theory can't use for hydrostatic loading & pure shear loading.

COMPARISON OF THEORIES OF FAILURE:

$\boldsymbol{ROS_{MPST} > ROS_{DET} > ROS_{MSST}}$	$\boldsymbol{ROS \propto \frac{1}{Safety} \propto \frac{1}{Size} \propto \frac{1}{Cost}}$
Most Economic Theory: MPPT	**Most Conservative Theory: MSST**
Most Safe: MSST	**Most Accurate Theory: DET**

CHOOSING THEORY OF FAILURES								
Based on Material			Type of loading					
Brittle	Ductile		1D	2D		3D (Hydrostatic)		
MPST	DET Accurate	MSST Easy in calculation	ALL TOF	DET Accurate	MSST Easy in calculation	MPST	TSET	MPStT
						Don't Use: MSST & DET		

Default: MSST

TOTAL STRAIN ENERGY/ HAIGH'S THEORY:

Statement: The failure of a component subjected to complex stress occurs when the **"TSE/ Unit Volume"** at any point in the body reaches the **"TSE/ Unit Volume"** of a material in simple tension test when failure occurs.

$$\sigma_{TStET} < \sigma_{yt} \ or \ \sigma_{ut}/FOS \, , Where \ (\sigma_{TStET})^2 = \left[\left(\sum \sigma_i^2\right) - 2\vartheta\left(\sum \sigma_i\sigma_j\right)\right], Where \ i < j$$

Note: This theory is neither accurate nor conservative. Because by experiment $\tau_{MStET} = 0.6\sigma_{yt}$

MAXIMUM PRINCIPLE STRAIN/ ST. VENANT'S THEORY:

Statement: The failure of a component subjected to complex stress occurs when the **"Maximum Principle Strain"** at any point in the body reaches the **"Maximum Principle Strain"** of a material in simple tension test when failure occurs.

$$\sigma_{MPStT} < \sigma_{yt} \ or \ \sigma_{ut}/FOS \, , Where \ \sigma_{MPStT} = \sigma_1 - \vartheta\sigma_2$$

Safety region is rhombus.

Note:

1. This theory is neither accurate nor conservative. Because by experiment $\tau_{MPStT} > \tau_{yt\ experiment}$
2. This theory considers normal stress as failure stress hence, change in shape & size **is** considered.
3. This theory can't use for hydrostatic loading & pure shear loading.

ALL THEORIES AT ONE PLACE			
THEORY	**COMPONENT**	**TENSILE TEST**	**DESIGN CONDITION**
Maximum Normal Stress/ Principle Stress/ Rankine's Theory	$\sigma_{max} = \max(\sigma_1, \sigma_2, \sigma_3)$ Let, $\sigma_{max} = \sigma_1$	$(\sigma_{max})_{T.T.} = \sigma_{yt}$	$\sigma_1 = \frac{\sigma_{yt}}{FOS}$
Maximum Shear Stress/ Guest & Tresca's Theory	$\tau_{max} = \max(\tau_{12}, \tau_{23}, \tau_{31})$ Let, $\tau_{max} = \tau_{12}$	$(\tau_{max})_{T.T.} = \frac{\sigma_{yt}}{2}$	$\sigma_1 - \sigma_2 = \frac{\sigma_{yt}}{FOS}$
Maximum Principle Strain/ St. Venant's Theory	$\varepsilon_{max} = \max(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ $\varepsilon = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$	$(\varepsilon_{max})_{T.T.} = \frac{\sigma_{yt}}{E}$	$\sigma_1 - \mu\sigma_2 = \frac{\sigma_{yt}}{FOS}$
Total Strain Energy/ Haigh's Theory	$U_T = \frac{1}{2E}\left[\left(\sum \sigma_i^2\right) - 2\vartheta\left(\sum \sigma_i\sigma_j\right)\right], Where \ i < j$	$(U_T)_{T.T.} = \frac{(\sigma_{yt})^2}{E}$	$\sigma_1^2 + \sigma_2^2 - 2\vartheta\sigma_1\sigma_2 = \frac{(\sigma_{yt})^2}{FOS}$
Distortion Energy/ Von Mises And Hencky's Theory	$U_d = \frac{1+\vartheta}{3E}[\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2]$	$(U_d)_{T.T.} = \frac{1+\vartheta}{3E}(\sigma_{yt})^2$	$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \frac{(\sigma_{yt})^2}{FOS}$