6. HEAT EXCHANGER

HEAT EXCHANGER: Tt is a device which is used to exchange thermal energy from one fluid to another fluid either direct or indirect contact. E.g. Boiler, Condenser, Evaporator, Etc…

AREA DENSITY (β): $\beta = \dfrac{Surface\ Area\ of\ Heat\ Exchanger}{Volume\ of\ Heat\ Exchanger} = \dfrac{A_s}{V}$	For Car Radiator, $\beta \approx 1000$ For Ceramic Glass Turbine, $\beta \approx 6000$ For Generator of Stirling engine, $\beta \approx 15000$

If $\beta > 700\ m^2/m^3$, HE is called Compact Type HE.

PARALLEL FLOW: Both the Fluids moving in same direction.

COUNTER FLOW: Both the Fluids moving in opposite direction.

OVERALL HEAT TRANSFER COEFFICIENT (OHTC) ($U\ in\ W/m^2K$):

It's Experimentally determined Quantity.

$Q = \dfrac{\Delta T}{R_{Total}} = UA\Delta T \left(\begin{matrix} Extension\ of\ Newton's \\ Law\ of\ Cooling \end{matrix}\right)$	$Q = \dfrac{\Delta T}{R_{Total}} = (UA)_i\Delta T = (UA)_o\Delta T$
Fouling Resistance ($R_F\ in\ K/W$) $= R_{Old\ Surface} - R_{New\ Surface}$	$R_F = 1/(UA)_{Old} - 1/(UA)_{New}$
Fouling Factor ($in\ m^2K/W$) $= 1/U_{Old} - 1/U_{New}$	If $A_i \approx A_o, R_{cond} \approx 0$ & $U_o = U_i$

Here, Q remains the along the direction of heat transfer.

HOLLOW CYLINDER	HOLLOW SPHERE	PLANE WALL
$U_i r_i = U_o r_o$	$U_i r_i^2 = U_o r_o^2$	$U_i = U_o\ Because\ A_i = A_o$

GENERALISED RATE OF THERMAL ENERGY BALANCE:

From SFEE, $Q(in\ J/kg) = h_2 - h_1$ $Q(in\ J/s) = \dot{m}(h_2 - h_1) = \dot{m}C_P(T_2 - T_1)$ For Both the Fluid of HE, $Q = Q_{loss} = Q_{gain}$ $Q = -\dot{m}_h C_h(T_{h2} - T_{h1}) = \dot{m}_c C_c(T_{c2} - T_{c1})$ $= UA(\Delta T)_{Avg}$	$\dot{m}_h, \dot{m}_c =$ Mass Flow rate of hot fluid and cold fluid (kg/s) $C_h, C_c =$ Specific heat of hot fluid and cold fluid ($J/kg\ K$) $T_{h1}, T_{h2} =$ Inlet and outlet temperature of hot fluid, $T_{c1}, T_{c2} =$ Inlet and outlet temperature of cold fluid

HEAT CAPACITY RATE (C): $C\ (in\ W/K) = \dot{m}C_P$

- It denotes at given time how much energy required to change a temperature by $1K$ or $1°C$.
- Fluid undergoes large temperature variation at a given time for a **fluid having minimum heat capacity rate**.

$C_{min} = \min\{\dot{m}_h C_h, \dot{m}_c C_c\}$	$C_{max} = \max\{\dot{m}_h C_h, \dot{m}_c C_c\}$

HEAT CAPACITY RATIO (R): $R = C_{min}/C_{max}$

- If $R = 1$, Temperature change in hot fluid and cold fluid will be same.
- In Phase change (Boiling, Evaporation, Condensation), $C_{max} = \infty \Rightarrow R = 0$

ANALYSIS OF HEAT EXCHANGER:

1. LMTD METHOD:

Assumptions:

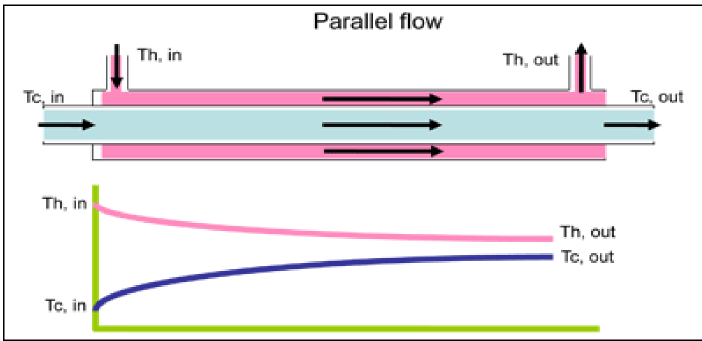
1. 1D Heat Flow (Radial Flow)
2. Steady State
3. No internal heat generation
4. Neglect KE & PE changes
5. All thermophysical properties of fluid are constant.
6. Overall heat transfer coefficient value is constant.
7. Outer surface of heat exchanger is well insulated.
8. Radiation heat loss is neglected.
9. No Partial phase changes.

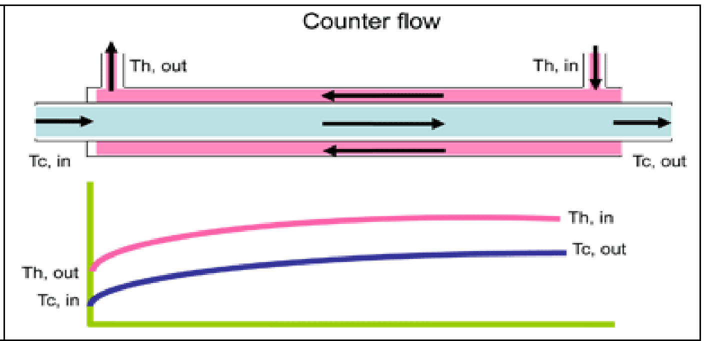
CASE-I: DOUBLE PIPE PARALLEL FLOW HEAT EXCHANGER

$\boldsymbol{dQ = -\dot{m}_h C_h dT_h = \dot{m}_c C_c dT_c = UdA(\Delta T)_{Avg}}$	$Q = -\dot{m}_h C_h(T_{h2} - T_{h1}) = \dot{m}_c C_c(T_{c2} - T_{c1}) = UA(LMTD)$
$d(T_h - T_c) = -dQ\ \mu_P$ $\boldsymbol{d(\Delta T) = -dQ\ \mu_P \cdots (1)}$ $\mu_P = \dfrac{1}{\dot{m}_h C_h} + \dfrac{1}{\dot{m}_c C_c}$ By integration, $\boldsymbol{\mu_P = (\Delta T_1 - \Delta T_2)/Q \cdots (2)}$	$dQ = UdA(\Delta T)_{Avg}$ By integration & Eq. (1), $\boldsymbol{\ln(\Delta T_1/\Delta T_2) = UA\mu_P \cdots (3)}$
From Equation (2) & (3), $\boldsymbol{(\Delta T)_{Avg} = LMTD}$	$\boldsymbol{LMTD = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}}\left(\begin{matrix} \boldsymbol{Always\ Same\ in} \\ \boldsymbol{°C\ or\ K\ \&> 1} \end{matrix}\right)$

CASE-II: DOUBLE PIPE COUNTER FLOW HEAT EXCHANGER

$dQ = -\dot{m}_h C_h dT_h = -\dot{m}_c C_c dT_c = UdA(\Delta T)_{Avg}$	$Q = -\dot{m}_h C_h(T_{h2} - T_{h1}) = -\dot{m}_c C_c(T_{c2} - T_{c1}) = UA(LMTD)$
$d(T_h - T_c) = -dQ\ \mu_C$ $\boldsymbol{d(\Delta T) = -dQ\ \mu_C \cdots (1)}$ $\mu_C = \dfrac{1}{\dot{m}_h C_h} - \dfrac{1}{\dot{m}_c C_c}$ By integration, $\boldsymbol{\mu_C = (\Delta T_1 - \Delta T_2)/Q \cdots (2)}$	$dQ = UdA(\Delta T)_{Avg}$ By integration & Eq. (1), $\boldsymbol{\ln(\Delta T_1/\Delta T_2) = UA\mu_C \cdots (3)}$
From Equation (2) & (3), $\boldsymbol{(\Delta T)_{Avg} = LMTD}$	$\boldsymbol{LMTD = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}}\left(\begin{matrix} \boldsymbol{Always\ Same\ in} \\ \boldsymbol{°C\ or\ K\ \&> 1} \end{matrix}\right)$





$\boldsymbol{\Delta T_1 = T_{h1} - T_{c1}}$	$\boldsymbol{\Delta T_2 = T_{h2} - T_{c2}}$	$\boldsymbol{\Delta T_1 = T_{h1} - T_{c2}}$	$\boldsymbol{\Delta T_2 = T_{h2} - T_{c1}}$

LMTD	**AMTD**	By series expansion of ln term in LMTD,
$LMTD = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$	$AMTD = \dfrac{\Delta T_1 + \Delta T_2}{2}$	$LMTD = \dfrac{AMTD}{1 + \frac{1}{3}\left[\frac{\Delta T_1 - \Delta T_2}{\Delta T_1 + \Delta T_2}\right]^2}$
If $\Delta T_1 = \Delta T_2, LMTD = AMTD$(Linear Profile), Else $LMTD < AMTD$		

SPECIAL CASES:

CASE-I: When Both fluids have equal heat capacity rate in counter flow heat exchanger. $C_{min} = C_{max}$

$R = 1$ $d(\Delta T) = 0$	$\dfrac{dT_h}{dx} = -\dfrac{UP(\Delta T)_{Avg}}{\dot{m}_h C_h} = Const.$	$\dfrac{dT_c}{dx} = -\dfrac{UP(\Delta T)_{Avg}}{\dot{m}_c C_c} = Const.$	$LMTD = \Delta T_1 = \Delta T_2$ $= AMTD\ (Linear\ Profile)$

In counter flow when $R = 1$, temperature profile is linear & parallel.

CASE-II: Phase Change Devices ($\boldsymbol{C_{max} = \infty \Rightarrow R = 0}$)

Boiler/ Evaporator:

$\Delta T_1, \Delta T_2, LMTD$ For Parallel Flow and Counter Flow are same for Boiling & evaporation process.

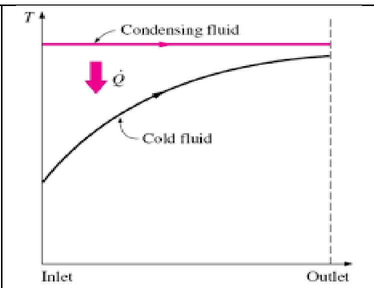
For Evaporator,

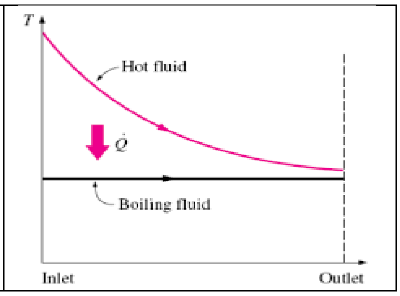
$Q = -\dot{m}_h C_h (T_{h2} - T_{h1}) = \dot{m}_c h_{fg} = UA(LMTD)$

For Condenser,

$Q = \dot{m}_h h_{fg} = \dot{m}_c C_c (T_{c2} - T_{c1}) = UA(LMTD)$

$(\Delta T_1)_P = (\Delta T_2)_C$	$(\Delta T_2)_P = (\Delta T_1)_C$
$(LMTD)_P = (LMTD)_C$	





COMPARISON OF PARALLEL FLOW & COUNTER FLOW:

In parallel Flow, T_{h2} cannot be less than T_{c2}.	In Counter Flow, T_{h2} can be less than T_{c2}.

So, Possibility in Counter Flow,

$C_{min} = \dot{m}_h C_h$	$C_{min} = \dot{m}_c C_c$	$Q_{max} = C_{min}(\Delta T)_{max}$ Where, $(\Delta T)_{max} = T_{h1} - T_{c1}$
It may possible that, $T_{h2} = T_{c1}$	It may possible that, $T_{h1} = T_{c2}$	
$Q_{max} = C_{min}(T_{h1} - T_{c1})$	$Q_{max} = C_{min}(T_{h1} - T_{c1})$	

- Q_{max} is calculated for a fluid having minimum heat capacity rate by assuming counter flow HE of infinite length.
- When Q & U is constant,

$LMTD_{Counter} > LMTD_{Cross} > LMTD_{Parallel}$	$A_{Counter} < A_{Cross} < A_{Parallel}$	$A_s LMTD = Constant$

- In HE, Counter Flow HE is considered as reference.
- $Q_{Cross\ or\ Multipass} = UA(LMTD)_{Cross\ or\ Multipass}$

$$LMTD_{Cross\ or\ Multipass} = LMTD_{Counter} * F, Where\ F = Correction\ Factor \leq 1$$

F Depends on Geometry & Flow Direction.

- For counter flow or Phase change devices $F = 1$

NOTE: Parallel Flow HE is used when Oil because with increasing temperature Viscosity decreases and pump power decreases. Hence, Selection is purely depending on the Requirement.

LMTD Method is used to find surface area of HE when outlet Temp. of fluid is known ($T_{h2}\ or\ T_{c2}\ must\ be\ known$).

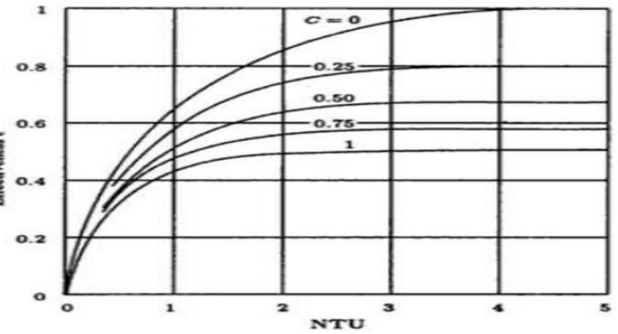
2. EFFECTIVENESS (ϵ) NTU METHOD:

$$\epsilon = \frac{Actual\ Heat\ Loss\ From\ Any\ One\ Fluid}{Maximum\ Heat\ Loss\ From\ Fluid} = \frac{Q_{Act}}{Q_{Max}} \le 1$$

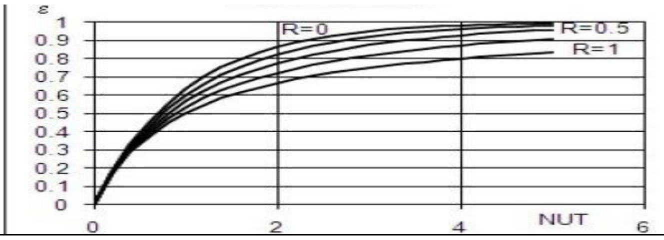
HOT FLUID WITH C_{min}	**COLD FLUID WITH C_{min}**
$\epsilon = \frac{Q_{Act}}{Q_{Max}} = \frac{\dot{m}_h C_h (T_{h1} - T_{h2})}{C_{min}(T_{h1} - T_{c1})} = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}} \cdots (1)$	$\epsilon = \frac{Q_{Act}}{Q_{Max}} = \frac{\dot{m}_c C_c (T_{c2} - T_{c1})}{C_{min}(T_{h1} - T_{c1})} = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \cdots (1)$
$\dot{m}_h C_h (T_{h1} - T_{h2}) = \dot{m}_c C_c (T_{c2} - T_{c1}) = UA(LMTD)$ $R = \frac{C_{min}}{C_{max}} = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{h2}} = \frac{Smaller\ Temp.Drop}{Larger\ Temp.Drop} \cdots (2)$ $NTU = \frac{UA}{C_{min}} = \frac{T_{h1} - T_{h2}}{LMTD} = \frac{Large\ Temp.Diff.}{Avg.Temp.Diff.} \cdots (4)$	$\dot{m}_h C_h (T_{h1} - T_{h2}) = \dot{m}_c C_c (T_{c2} - T_{c1}) = UA(LMTD)$ $R = \frac{C_{min}}{C_{max}} = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}} = \frac{Smaller\ Temp.Drop}{Larger\ Temp.Drop} \cdots (2)$ $NTU = \frac{UA}{C_{min}} = \frac{T_{c2} - T_{c1}}{LMTD} = \frac{Large\ Temp.Diff.}{Avg.Temp.Diff.} \cdots (4)$
From (1) & (2) $\epsilon R = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \cdots (3)$	From (1) & (2) $\epsilon R = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}} \cdots (3)$

EFFECTIVENESS OF PARALLEL FLOW	**EFFECTIVENESS OF COUNTER FLOW**
HOT FLUID WITH C_{min}	**HOT FLUID WITH C_{min}**
$\mu_P = \frac{1}{\dot{m}_h C_h} + \frac{1}{\dot{m}_c C_c} = \frac{1}{C_{min}}[1 + R]$	$\mu_C = \frac{1}{\dot{m}_h C_h} - \frac{1}{\dot{m}_c C_c} = \frac{1}{C_{min}}[1 - R]$
$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = UA\mu_P \Rightarrow 1 - \frac{\Delta T_2}{\Delta T_1} = K = 1 - e^{-UA\mu_P}$ $\therefore 1 - \frac{\Delta T_2}{\Delta T_1} = \epsilon[1 + R] = K \Rightarrow \epsilon = \frac{K}{1 + R}$ $\therefore\ \boldsymbol{\epsilon = \frac{1 - e^{-UA\mu_P}}{1 + R} = \frac{1 - e^{-NTU[1+R]}}{1 + R} = f(NTU, R)}$	$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = UA\mu_C \Rightarrow \frac{\Delta T_2}{\Delta T_1} = K = e^{-UA\mu_C}$ $\therefore \frac{\Delta T_2}{\Delta T_1} = \frac{1 - \epsilon}{1 - R\epsilon} = K \Rightarrow \epsilon = \frac{1 - K}{1 - RK} = \frac{1 - e^{-UA\mu_C}}{1 - Re^{-UA\mu_C}}$ $\therefore\ \boldsymbol{\epsilon = \frac{1 - e^{-NTU[1-R]}}{1 - R\,e^{-NTU[1-R]}} = f(NTU, R)}$
CASE-I: For Boiling or Condensation (Phase Change), $\epsilon = 1 - e^{-NTU} (\because R = 0)$ For $NTU \to \infty$, $\epsilon_{max} = 1$ **CASE-II:** Both the fluids have equal heat capacity rate, $\epsilon = (1 - e^{-2NTU})/2\ (\because R = 1)$ For $NTU \to \infty$, $\epsilon_{max} = 0.5$ **CASE-III**: For $NTU \to \infty$, $\epsilon_{max} = 1/(1 + R)$ **NOTE:** For more effectiveness $NTU \to \infty$ & $R \to 0$	**CASE-I:** For Boiling or Condensation (Phase Change), $\epsilon = 1 - e^{-NTU} (\because R = 0)$ For $NTU \to \infty$, $\epsilon_{max} = 1$ **CASE-II:** Both the fluids have equal heat capacity rate, $\epsilon = NTU/(1 + NTU)\ (\because R = 1, L'hospital)$ For $NTU \to \infty$, $\epsilon_{max} = 1$

Parallel Flow:



Counter Flow



NTU Method Important Points:

Here, $Q_{Act} = \epsilon\, C_{min}(T_{h1} - T_{c1})$

$$\epsilon = f(NTU, R)$$

$$But\ for\ NTU < 0.3, \epsilon \approx Const. \neq f(R)$$

$NTU = \frac{UA}{C_{min}}$	$R = \frac{C_{min}}{C_{max}}$

$$Q_{Act} = \dot{m}_c C_c (T_{c2} - T_{c1}) = \dot{m}_h C_h (T_{h1} - T_{h2})$$

$$\boxed{NTU \propto Area \propto\ \epsilon \propto Q_{Act}}$$

NTU Measures Size of HE & It's also called as dimensionless area. For Good Thermal & Economical Design NTU is limited up to 1.5. NTU increase rapidly from 0 to 1.5. Beyond 3 NTU is almost Constant.

$$\epsilon_{Counter} > \epsilon_{Cross} > \epsilon_{Parallel}$$

For phase Change, $\epsilon_{Counter} = \epsilon_{Cross} = \epsilon_{Parallel}$

As NTU increases the Size of HE increases (Bulky).

NTU= No. of Heat Transfer Unit

NTU is representation of St Stanton Number in terms of U (Overall heat Transfer Coefficient.)

Same results can be obtained for cold fluid with C_{min}

LMTD Method	**NTU Method**
It's used when Temperature is given and Area required to find for the Heat Exchanger.	It's used when Area is given and Temperature required to find for the Heat Exchanger.