5. MAXIMA AND MINIMA

Critical Point: Point at slop is not defined and point at which f'(x) = 0. Hence, f'(x) is 0 or undefined.

Extreme Value Theorem:

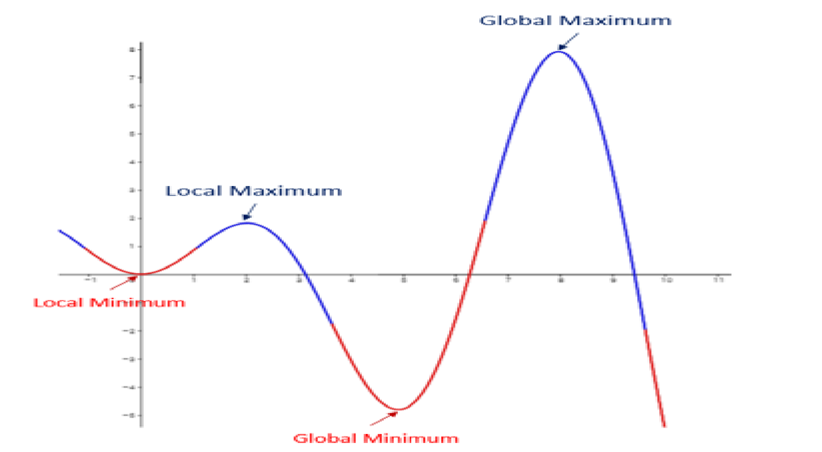
For a continuous function f(x) on [a, b] there exists Maxima and Minima.

- The maxima and minima values occur at either at the end point or at the critical point (f'(x) = 0).

For a continuous function f(x) on (a, b), maxima or minima may or may not exist.

- If maxima or minima exists, then will occur only at critical point.

Concave Up/Down:
Cone Cave
Absolute/Local Maxima/Minima:
Local Maxima/Minima: x at f'(x) = 0
Point of Inflection:
Point on the graph of a function at which concavity changes from up to down or vice versa.
1. f''(x) = 0
2. f''(x) should switch it's sign about the inflection point.
No maxima or No minima at point of inflection.



Maxima And Minima For Function Of Two Variable:

Let $f(x,y)$ be the function of two variable for which maxima and minima is to be determined.

Let, $p = \frac{\partial f}{\partial x}, q = \frac{\partial f}{\partial y}, r = \frac{\partial^2 f}{\partial x^2}, s = \frac{\partial^2 f}{\partial x\, \partial y}, t = r = \frac{\partial^2 f}{\partial y^2}$

1. Find p, q, r, s, t.
2. Obtain stationary points by evaluating p and q to zero.
3. Find r, s, t at stationary points.

Concluding Remarks:

1. If $rt - s^2 > 0$, and $r > 0$, f (x, y) has Minimum at that stationary points.
2. If $rt - s^2 > 0$, and $r < 0$, f (x, y) has Maximum at that stationary points.
3. If $rt - s^2 < 0$, there is no extremum at that stationary points and such that point is called saddle points.
4. If $rt - s^2 = 0$, test is un-conclusive. Need to go with higher order derivative.

All critical points are not stationary points but all stationary points are critical points.

Lagrange Multiplier (Constrained Optimisation):

Type of Constrain		
Unconstrained Optimization	Inequality Constrained Optimization	Equality Constrained Optimization

When we want to Maximise (or Minimise) $f(x,y,z)$,
Subjected to constrain that $g(x,y,z) = c$,
Follow these steps,

1. Introduce a new variable λ and defined a new function $\mathcal{L}$ as follows;
   $\mathcal{L} = f(x,y,z) - \lambda(g(x,y,z) - c) = 0$
   This function is called Lagrangian, and new variable is referred as Lagrange multiplier.
2. Set gradient of $\mathcal{L}$ equal to zero vector. $\nabla\mathcal{L}(x,y,z,\lambda) = 0$
   In other words, find the stationary points of $\mathcal{L}$.
3. Consider each solution, substitute it in f.
   Whichever one gives the greatest (or smallest) value is the maximum (or minimum) point you are seeking.

Solution of function: $ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, x_1 + x_2 = \frac{-b}{a}, x_1 x_2 = \frac{c}{a}$$