5. FLUID DYNAMICS

BERNOULLI'S EQUATION:

$$\sum F_s = m\, a_s$$

Forces acting on the fluid element shown in the figure,

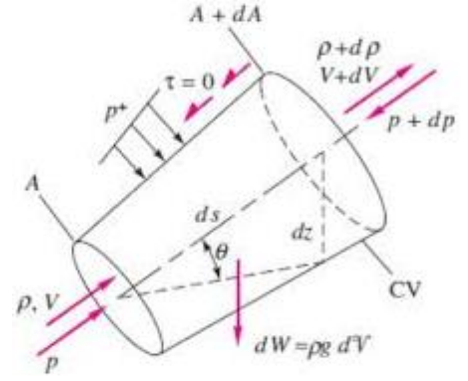
F_g = Gravitational Force	F_P = Pressure Force
F_V = Viscous Force	F_T = Turbulent Force
F_S = Surface Tension Force	F_C = Compressible Force

$$\left(F_g + F_P + F_V + F_T + F_S + F_C\right)_s = m\, a_s$$

$$\therefore \left(F_g + F_P + F_V + F_T\right)_s = m\, a_s \ (Reynold's\ Equation)$$

$$\therefore \left(F_g + F_P + F_V\right)_s = m\, a_s \ (Navier - Stoke's\ Equation)$$

$$\therefore \left(F_g + F_P\right)_s = m\, a_s \ (Euler's\ Equation)$$



$$\therefore -dW \sin\theta + P\, dA - \left(P + \frac{\partial P}{\partial s} ds\right) dA = dm\, a_s \Rightarrow -\gamma\, d\forall \frac{dz}{ds} - \frac{\partial P}{\partial s} ds\, dA = \rho\, d\forall\, V_s \frac{\partial V_s}{\partial s}$$

$$\left(\because \sin\theta = \frac{dz}{ds} \,\&\, a_s = \frac{\partial V_s}{\partial t} + V_s \frac{\partial V_s}{\partial s} = V_s \frac{\partial V_s}{\partial s}\ (Steady\ Flow)\ \&\ d\forall = ds\, dA\right)$$

$\frac{dP}{\rho} + V_s dV_s + g\, dz = 0\ (Euler's\ Equation)$	$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = Const.\ (Bernoulli's\ Eq. For\ Steady\ Flow)$
Bernoulli's Eq. For Steady and incompressible Flow, $Pressure\ Head + Kinetic\ Head + Potential\ or\ Datum\ Head = Total\ Head = Const.$	$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H$

- Net mechanical energy of fluid in an ideal flow remains constant.

VARIOUS FORMS OF BERNOULLI'S EQUATION:

Energy Per unit mass, $\frac{P}{\rho} + \frac{V^2}{2} + gz = Constant$	**Energy Per unit Volume,** $P + \rho\frac{V^2}{2} + \rho gz = Constant$	
P = Static Pressure	$\rho(V^2/2)$ = Dynamic Pressure (Rise in pressure due to drop in K.E.)	ρgz = Hydrostatic Pressure (Rise in pressure due to drop in P.E.)
Energy Per unit weight, $\frac{P}{\gamma} + \frac{V^2}{2g} + z = Constant$	At stagnation point, Stagnation Pressure = Static Pressure + Dynamic Pressure Piezometric Pressure = Static Pressure + Hydrostatic Pressure	

LIMITATIONS/ ASSUMPTIONS IN BERNOULLI'S EQUATION

1. Flow is Steady.
2. Incompressible Flow.
3. Heat Transfer effects are neglected.
4. In Irrotational Flow, B. Eq. is valid across any two stream line but for Rotational flow, B. Eq. is valid only along stream line not across the stream line.
5. Involvement of shaft work (Pump & Turbine): Heads need to balance when energy per unit mass is added or removed. E.g. Pump is adding head & turbine is Using head to gain shaft work.
6. Valid only for ideal flow. But by introducing head loss, we can use the B. Eq. for real flow.

HEAD LOSS: Energy given by the fluid to overcome resistance against the flow per unit weight is called head loss.

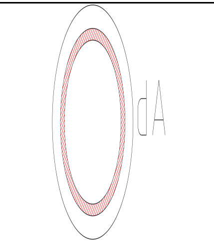
NOTE: In the absence of a pump, Real flow takes place from higher total head to lower total head.

KINETIC ENERGY CORRECTION FACTOR (α):

When the velocity profile is non uniform, Total velocity head and avg. velocity head are not equal.

Local Velocity = u & Avg. Velocity = V

$d(\dot{KE}) = \frac{1}{2} d\dot{m}\, u^2 \Rightarrow \dot{KE} = \frac{1}{2}\iint u^3 dA$	$\dot{KE} = \alpha \frac{1}{2} \rho A V^3$	$\boldsymbol{\alpha = \frac{1}{AV^3}\iint u^3 dA}$
Always $\alpha \geq 1$ For uniform Flow $\alpha = 1$	For Laminar Flow through pipe $\alpha = 2$	For turbulent flow through pipes, $1.1 \leq \alpha \leq 1.3$

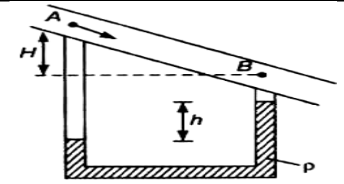


DIFFERENCE IN PIEZOMETRIC HEAD:

$$\boldsymbol{P_A = P_B} \Rightarrow P_1 + \gamma_w z_1 = P_2 + \gamma_w (z_2 - h_m) + \gamma_m h_m$$

$$\therefore \Delta P^* = h_m(\gamma_m - \gamma_w)$$

$\therefore \Delta h^* = h_m\left(\frac{\gamma_m}{\gamma_w} - 1\right) = h_m\left(\frac{S_m}{S_w} - 1\right)$	$S_m \gg S_w \Rightarrow \Delta h^* = h_m\left(\frac{S_m}{S_w}\right)$



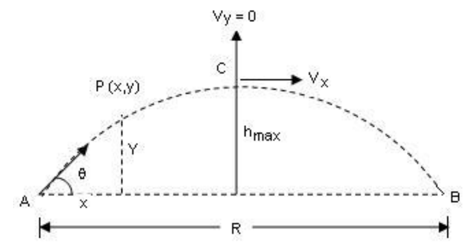
EQUATION OF POWER: $P(in\ J) = h\rho g Q$ & $P(in\ W) = h\rho g \dot{Q}$

FREE LIQUID JET:

$u = V\cos\theta$	$v = V\sin\theta$	H = Max. Height	T = Time of Flight
$s = ut + \frac{1}{2}at^2$	$x = V\cos\theta\ t$	$y = V\sin\theta\ t + \frac{1}{2}(-g)t^2$	

From above both equations,

$$y = x(\tan\theta) - \left(\frac{g\sec^2\theta}{2V^2}\right)x^2 (Parabolic\ Eq.)$$



$H = \frac{V^2\sin^2\theta}{2g} = \frac{v^2}{2g}(\because V^2 - U^2 = 2aS)$	$T = \frac{2v}{g}(\because V = U + at\ \&\ T = t_a + t_d)$	$R = \frac{2uv}{g}$

VELOCITY MEASUREMENT	$\frac{V^2}{2g} = H - h^*$

1. PITOT TUBE + PIEZOMETER:

$h_{stag} = \frac{P}{\gamma} + \frac{V^2}{2g}$	$h_{stat} = \frac{P}{\gamma}$	$h^* = h_{stat} + z$ $H = h_{stag} + z$

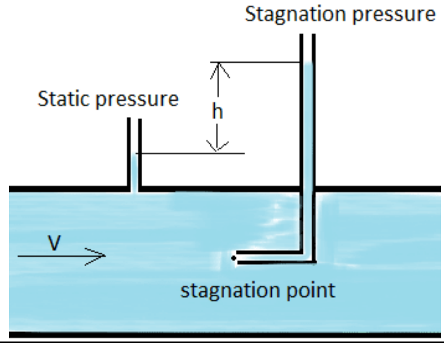
From above equations, we can find velocity head or velocity in the pipe.

$$V_{th} = \sqrt{2g(H - h^*)} = \sqrt{2gh}\ (From\ the\ Fig.)$$

$$\boldsymbol{Coefficient\ Of\ Velocity\ or\ Prob\ Factor\ C_V} = \frac{Actual\ Velocity}{Theoretical\ Velocity}$$

$$V_{act} = C_V\sqrt{2g(H - h^*)} = C_V\sqrt{2gh}$$

It can't use for gaseous fluid.



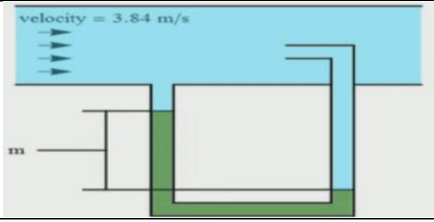
NOTE: If C_V is not give, $C_V = 1$.

2. PITOT-STATIC TUBE:

$$V_{th} = \sqrt{2g\Delta h^*}$$

From the previous derivations,

$$V_{act} = C_V\sqrt{2g\ h_m\left(\frac{S_m}{S_w} - 1\right)}$$



DISCHARGE/ FLOW MEASUREMENT:

OBSTRUCTION FLOWMETER: E.g. Orifice Meter, Venturi Meter, Nozzle Meter.

Head-loss in Obstruction Flowmeter:

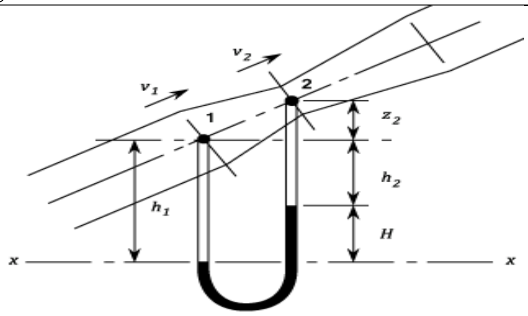
$$\left(\frac{P}{\gamma} + \frac{V^2}{2g} + z\right)_1 = \left(\frac{P}{\gamma} + \frac{V^2}{2g} + z\right)_2 + h_l \Rightarrow h_l = \Delta h^* - \left(\frac{V_2^2 - V_1^2}{2g}\right)$$

From the mass conservation in incompressible flow,

$$h_l = \Delta h^* - \frac{V_2^2 A_2^2}{2g\ \Delta h^*}\left(1 - \frac{A_2^2}{A_1^2}\right)\frac{\Delta h^*}{A_2^2} = \Delta h^* - \frac{Q_{act}^2}{Q_{th}^2}\Delta h^*$$

$$\therefore \boldsymbol{h_l = \Delta h^*(1 - C_d^2)}$$

$Q_{act}^2 = V_2^2A_2^2$	$Q_{th}^2 = 2g\ \Delta h^*[A_1^2A_2^2/(A_1^2 - A_2^2)]$



VENTURI METER:

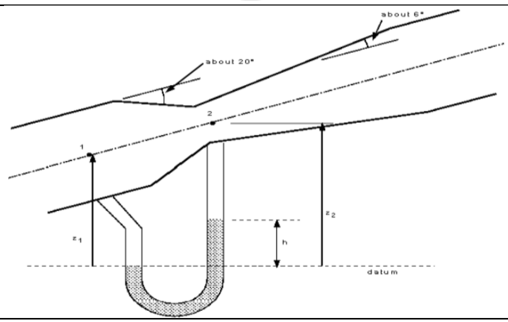
$$\boldsymbol{h_l = \Delta h^*(1 - C_d^2)}$$

$Q_{act}^2 = V_2^2A_2^2$	$Q_{th}^2 = 2g\ \Delta h^*[A_1^2A_2^2/(A_1^2 - A_2^2)]$

By Considering $h_l = 0$,

$$\Delta h^* = \frac{V_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right) \Rightarrow V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\ \Delta h^*}$$

$$Q_{th} = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\ \Delta h^*} \Rightarrow Q_{th} = Q_{act}$$



Q_{th} equation is same for all measuring device.

$Coefficient\ of\ Discharge, C_d = \frac{Q_{act}}{Q_{th}}$	$Coefficient\ of\ Contraction, C_c = \frac{A_c}{A_2} = \frac{V_2}{V_c}$

In Venturi Meter, To Reduce Minor Losses in Diffuser, Converging Angle (20°) > Divergence Angle ($7°\ to\ 15°$)

$PART - I: \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} = \frac{\pi}{4}\frac{D_1^2D_2^2}{\sqrt{D_1^4 - D_2^4}}$	$PART - II: \sqrt{2g\ \Delta h^*} \Rightarrow \Delta h^* = h_m\left(\frac{S_m}{S_w} - 1\right) = \frac{\Delta P^*}{\gamma}$

	Venturi Meter	Nozzle Meter	Orifice Meter
C_d	0.95-0.98 (High)	0.85 (Medium)	0.65 (Low)
h_{loss}	Low	Medium	High
Accuracy	High	Medium	Low
Cost	High	Medium	Low

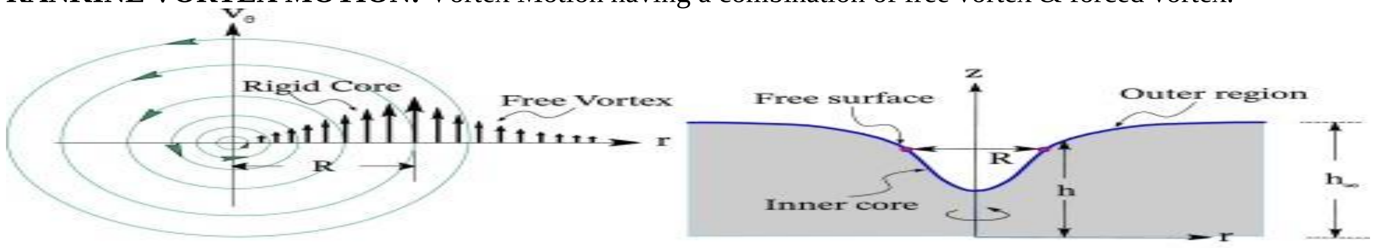
VORTEX MOTION: It's motion of fluid along a curved path is known as vortex motion.
E.g. Whirlpool, Tornado, Water Sink, etc…

1. $V_r = 0, V_\theta$ exists.

2. Stream Lines are curved.

TYPE OF VORTEX FLOW	
FREE VORTEX FLOW	**FORCED VORTEX FLOW**
It's naturally (By the virtue of motion itself) exists.	Appling external torque for generating Vortex flow.
External Torque is equal to zero.	Constant External Torque is applied.
$T = 0 \Rightarrow \frac{d(mv\,r)}{dt} = 0 \Rightarrow mv\,r = Constant$	Angular Velocity of every partial is same. $T = Const. \Rightarrow \omega = Const. \Rightarrow v \propto r$
Singularity Point: $r = 0 \Rightarrow v = not\ defined$	$r = 0 \Rightarrow v = 0$
Flow is irrotational. E.g. Bernoulli's equation can be applied between any point.	Flow is rotational. E.g. Bernoulli's equation can be applied between any point lying on same stream lines.
Valid for any 2 random points, $\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B$	Valid for any 2 random points, $\frac{P_A}{\gamma} - \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} - \frac{V_B^2}{2g} + z_B$

RANKINE VORTEX MOTION: Vortex Motion having a combination of free vortex & forced vortex.



(A) Velocity distribution (B) Free Surface

EQUATION OF PRESSURE:

$$dP = \frac{\partial P}{\partial r}dr + \frac{\partial P}{\partial z}dz\ (\because P = f(r,z))$$

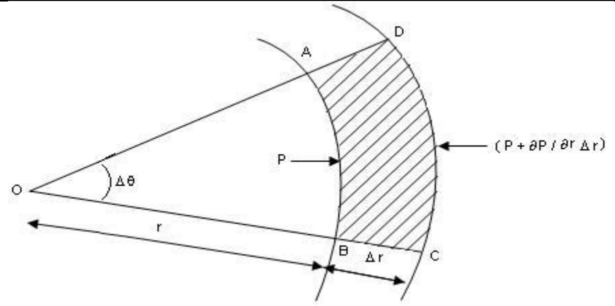
$dm = \rho dA dr$	$a_r = r\omega^2$

From the figure, $\sum F_r = m\,a_r$

$$\frac{\partial P}{\partial r}drdA = dm\,a_r$$

$\frac{\partial P}{\partial r} = \rho\omega^2 r$	$\frac{\partial P}{\partial z} = -\rho g$

$$\therefore dP = \rho\omega^2 r\,dr - \rho g\,dz = \rho\,(a)\,dl$$



Note: Forced Vortex Flow is Part of Rigid Body Motion.

FREE VORTEX FLOW	**FORCED VORTEX FLOW**
$v\,r = r^2\omega = C$	$v = C\,r \Leftrightarrow \omega = Const.$
$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B (From\ dP\ Eq.)$	$\frac{P_A}{\gamma} - \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} - \frac{V_B^2}{2g} + z_B (From\ dP\ Eq.)$
Hence, the flow is irrotational.	Hence, the flow is rotational.

EQUATIONS OF ISOBAR: Constant pressure imaginary curve in vortex (Forced) flow.

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B \Rightarrow h = \frac{V_B^2 - V_A^2}{2g} = \frac{r^2\omega^2}{2g} \propto r^2 \begin{bmatrix} \because z_A - z_B, V_A = 0, V_B = r\omega \\ It's\ Parabolic\ Curve \end{bmatrix}$$

VOLUME OF PARABOLOID: Volume of revolution of parabola. E.g. Circumscribing cylinder with parabola.

Circumscribed refers to a shape surrounding another shape.	$\forall_{par.} = \frac{\pi\omega^2}{g}\frac{R^4}{4} = \frac{1}{2}\forall_{cyl.} (\because r_{max} = R)$