2. MEAN VALUE THEOREM

ROLLE'S MEAN VALUE THEOREM:

Let $f(x)$ be defined in [a, b] such that,

1. $f(x)$ is continuous on [a, b].
2. $f(x)$ is differentiable on (a, b).
3. $f(a) = f(b)$

There exists at least one $c \in (a, b)$ such that $f'(c) = 0$.

Geometrically Rolle's theorem gives tangent parallel to X-axis.

Note: differentiable on open interval but why? Hint: Slope can be obtain from one side only at end.

LAGRANGE'S MEAN VALUE THEOREM:

Let $f(x)$ be defined in [a, b] such that,

1. $f(x)$ is continuous on [a, b].
2. $f(x)$ is differentiable on (a, b).

There exists at least one $c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

Geometrically LaGrange's mean value theorem gives tangent parallel to line joining points $(a, f(a))$ and $(b, f(b))$.

CAUCHY'S MEAN VALUE THEOREM:

Let $f(x)$ and $g(x)$ be defined in [a, b] such that,

1. $f(x)$ and $g(x)$ are continuous on [a, b].
2. $f(x)$ and $g(x)$ are differentiable on (a, b).
3. $g'(x) \neq 0, \forall x \in (\mathrm{a}, \mathrm{b})$.

Then there exists at least one point $c \in (a, b)$ such that $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$