1. LIMITS

LIMIT EXISTS: $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

CONTINUITY: A real valued function $f(x)$ is said to be continuous at $x = a$, if $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$

DIFFERENTIABILITY: A function $f(x)$ is said to be differentiable at $x = a$, if $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists.

$$\lim_{x\to a^+} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a^-} \frac{f(x)-f(a)}{x-a} \Leftrightarrow Left\ Slop = Right\ Slop \Leftrightarrow \lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h}$$

If $f(x)$ and $g(x)$ are two continuous/differentiable functions, then

1. $f(x) \pm g(x)$
2. $f(x) * g(x)$
3. $\frac{f(x)}{g(x)}, where\ g(x) \neq 0$

Are also continuous/differentiable functions.

INDETERMINANT FORMS:

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 * \infty, \infty - \infty, 1^\infty, 0^0, \infty^\infty, \infty^0, \mathbf{0}^\infty$$

L'HOSPITAL RULE:

If $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$ and $\lim_{x\to a} \frac{f'(x)}{g'(x)} = L$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = L$.

If $\lim_{x\to a} f(x) = \infty$, $\lim_{x\to a} g(x) = \infty$ and $\lim_{x\to a} \frac{f'(x)}{g'(x)} = L$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = L$.

- L'Hospital rule is a general method for evaluating the indeterminant forms $\frac{0}{0}\ and\ \frac{\infty}{\infty}$.
- L'Hospital rule can also be applied to other indeterminant forms by converting in to $\frac{0}{0}\ or\ \frac{\infty}{\infty}$ using appropriate algebraic transformations.

If $\lim_{x\to a} f(x)^{g(x)}$ is of 1^∞ then, $\lim_{x\to a} f(x)^{g(x)} = e^{\lim_{x\to a} g(x)[f(x)-1]}$

STANDARD LIMITS:

$\lim_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}$	$\lim_{x\to a} \frac{e^{mx} - 1}{x} = m$
$\lim_{x\to a} \frac{a^x - 1}{x} = \log a$	$\lim_{x\to a} \left(1 - \frac{a}{x}\right)^x = e^a$

If $f(x)$ and $g(x)$ are polynomial of degrees "m" and "n" respectively, $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} x^{m-n}$.

LIMITS OF FUNCTION OF TWO VARIABLE:

If $f(x,y) \to L_1$ as $(x,y) \to (a,b)$ along C_1 and If $f(x,y) \to L_2$ as $(x,y) \to (a,b)$ along C_2, where $L_1 \neq L_2$ then $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exists.

CONTINUITY OF TWO VARIABLE:

A function $f(x,y)$ of two variables is called continuous at (a,b), if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.