3. TRANSIENT HEAT CONDUCTION

UNSTEADY OR TRANSIENT HEAT CONDUCTION: Rate of heat transfer changes w. r. t. time.

$mC =$ Thermal Capacitance	**Steady State**	**Unsteady or Transient State**
$\frac{\partial u}{\partial t} = Q_{stored} = mC\frac{\partial T}{\partial t}$	$\frac{\partial T}{\partial t} = 0, Q_{stored} = 0, \frac{\partial u}{\partial t} = 0$	$\frac{\partial T}{\partial t} \neq 0, Q_{stored} \neq 0, \frac{\partial u}{\partial t} \neq 0$
1. K, C, ρ	2. V, A_s	3. h

CHARACTERISTIC LENGTH (L_C):

$L_C = \frac{V}{A_s}$	For Plate, $L_C = \frac{HWL}{2[HW + HL + WL]}$	For thin plate, $L_C = \frac{L}{2}$	For Solid Cube, $L_C = \frac{a}{6}$	For Solid Sphere, $L_C = \frac{R}{3}$	For Short Solid Cylinder, $L_C = \frac{RL}{2[R + L]}$	For Long Solid Cylinder, $L_C = \frac{R}{2}$

FOURIER NUMBER (F_o): It represents the ratio of rate of heat conduction in given volume to the rate of thermal energy storage in given volume. Fourier number is dimensionless number which is also called dimensionless time. Higher Fourier number means faster will be heat propagation higher will be thermal response.

$F_o = \frac{Q_{Cond}}{Q_{Stored}} = \frac{\alpha t}{L_C^2} = \frac{Operating\ Time\ (t)}{Diffusion\ Time\ (t_d)}$	$t_d = \frac{L_C^2}{\alpha}$	$\alpha =$ Thermal diffusivity $t =$ Operating time

BIOT NUMBER (Bi): It's ratio of R_{Cond} to R_{Conv}. Biot number represents temperature gradient within the body larger Biot number, larger temperature gradient.

$Bi = \frac{T_{s1} - T_{s2}}{T_{s2} - T_\infty} = \frac{Q_{Conv}}{Q_{Cond}} = \frac{R_{Cond}}{R_{Conv}} = \frac{\overline{h}\, L_C}{K_{Solid}}$			$\overline{h}$ = Avg. heat transfer coefficient, $K_{Solid} =$ Thermal Conductivity of solid.		
$Bi = 0$	$R_{Cond} = 0$	$T_{s1} = T_{s2}$	$B = 1$	$R_{Cond} = R_{Conv}$	$T_{s1} - T_{s2} = T_{s2} - T_\infty$
$Bi \ll 1$	$R_{Cond} \ll R_{Conv}$	$T_{s1} - T_{s2} \ll T_{s2} - T_\infty$	$Bi \gg 1$	$R_{Cond} \gg R_{Conv}$	$T_{s1} - T_{s2} \gg T_{s2} - T_\infty$

Make Temperature Profiles on paper.

REAL BODY	**LUMPED BODY**
$T = f(Location, t)$	$T = f(t)$
Internal temp. Gradient present.	No Internal temp. Gradient.
Temp. Distribution is not uniform.	Temp. Distribution is uniform.
Internal Conductive resistance is not equal to zero.	Internal Conductive resistance is equal to zero.
$Bi \neq 0$ ($\because R_{Cond} \neq 0$)	$Bi = 0$ ($\because R_{Cond} = 0$)

CONDITIONS FOR LUMPED BODY:

K should be high	Size should be small	h should be small

ASSUMPTIONS:

1. No internal heat generation within a body.
2. Heat transfer coefficient value is constant.
3. No radiation heat loss.
4. Surrounding fluid temperature is constant.
5. All the thermophysical properties are constant.

$T_0 =$ Initial Temp. of body ($t = 0$)	$T =$ Temp. of body at particular time ($t = t$)
$T_\infty =$ Surrounding fluid Temperature	$h =$ Average heat transfer coefficient
$m, \rho, C, V, A_s =$ Mass, Density, Specific Heat, Volume, Surface Area of body.	
$T_0 \gg T_\infty$ Colling of body ($Q_{in} = 0$)	$T_0 \ll T_\infty$ Heating of body ($Q_{out} = 0$)

Temperature Varies Exponentially w. r. t. time.

$-Q_{Stored} = Q_{out} \Rightarrow \int_{T_0}^{T} \frac{dT}{T - T_\infty} = -\frac{hA_s}{mC}\int_0^t dt$	$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\frac{hA_s t}{\rho VC}} (Valid\ for\ Cooling\ \&\ heating)$

INSTANTANEOUS RATE OF COOLING:

$\frac{dT}{dt} = -\frac{hA_s}{\rho VC}(T_0 - T_\infty)e^{-\frac{hA_s t}{\rho VC}}$	$at\ t = 0, -mC\frac{dT}{dt} = hA_s(T_0 - T_\infty)$

INSTANTANEOUS RATE OF HEAT TRANSFER	**TOTAL ENERGY TRANSFER:**
$Q_t(in\ W) = hA_s(T - T_\infty) = hA_s(T_0 - T_\infty)e^{-\frac{hA_s t}{\rho VC}}$	$Q_{Tot.}(in\ J) = \int_0^t Q_t dt = -mC(T_0 - T_\infty)\left[e^{-\frac{hA_s t}{\rho VC}} - 1\right]$

$\frac{\boldsymbol{T-T_\infty}}{\boldsymbol{T_0-T_\infty}}=\boldsymbol{e}^{-\frac{\boldsymbol{hA_st}}{\boldsymbol{\rho VC}}}=\boldsymbol{e}^{-\frac{\boldsymbol{ht}}{\boldsymbol{\rho CL_C}}}=\boldsymbol{e}^{-\boldsymbol{BiF_o}}=\boldsymbol{e}^{-\frac{\boldsymbol{t}}{\boldsymbol{\tau^*}}}$	$Thermal\ Time\ Const.\tau^*=\frac{\rho CL_C}{h}=\frac{\rho VC}{hA_s}$
$\frac{ht}{\rho CL_C}\to 0\Rightarrow T=T_0(No\ Heat\ Transfer)$	$\frac{ht}{\rho CL_C}\to\infty\Rightarrow T=T_\infty(Fully\ Heat\ Transfer)$

- $\frac{ht}{\rho CL_C}$ represent thermal response of body. For higher response, $\frac{ht}{\rho CL_C}$ should be as large as possible this can be achieved by,
  - Select environment having high value of "h"
  - Select a Body having low density, low specific heat.
  - Select the geometry of body having low volume, high surface area (means thin body) (Less value of L_C)

THERMOCOUPLE READING:

Device (Thermometer or Couple) Reading $=\lvert T_0-T\rvert$ Initial Temperature Difference $=T_0-T_\infty$	$\frac{T_0-T}{T_0-T_\infty}=1-e^{-\frac{t}{\tau^*}}=\frac{Q_{Act}}{Q_{Max}}$

THERMAL TIME CONSTANT ($\boldsymbol{\tau^*}$): It's time required for temperature difference between the body and the ambient is 0.368 or 36.8% of initial temperature difference. In other words, temperature difference would be reduced by 63.2%.

$$\tau^*=R_{Conv}\ Thermal\ Capacitance$$

It's time required for Thermocouple or Thermometer to reach 63.2% of initial temperature difference.

$T_0-T=63.2\%\ (T_0-T_\infty)\ at\ t=\tau^*$	$T-T_\infty=36.8\%\ (T_0-T_\infty)\ at\ t=\tau^*$

1. Body reaches almost thermal equilibrium at $t\geq 4\tau^*$.
2. For good thermal response τ^* must be less.

For Practical purpose, Thermocouple or Thermometer reading recorded when $t\geq 4\tau^*$.

VALIDITY/LIMITATION OF LUMPED HEAT CAPACITY METHOD:

1. For Lumped Body: $Bi=0\ \Rightarrow R_{Cond}=0\Rightarrow K=\infty$
2. For Real Body: $Bi\approx 0\ \Rightarrow R_{Cond}\lll R_{Conv.}\Rightarrow K\ high$
3. $Bi\leq 0.1$

HEISLER CHART: It's graphical solution for the problems where lumped approach is not valid.

HEISLER CHART USED TO FIND:	**HEISLER CHART ARE DRAWN WITH THE HELP OF:**
• Centre line Temperature of body (T_C) • Temperature of body at any location (T) • Temperature of body at any particular time.	• Fourier Number (F_o) • Reciprocal of Biot Number (Bi^{-1}) • Dimensionless Position ($x/L\,,r/R$)

CHART-I:	**CHART-II:**
$\frac{T_C-T_\infty}{T_0-T_\infty}=f\left[\frac{1}{Bi},F_o\right]$	$\frac{T-T_\infty}{T_C-T_\infty}=f\left[\frac{1}{Bi},\frac{x}{L}or\frac{r}{R}\right]$

For plane Wall, Total Length $=2L$ Characteristic length $L_C=L$	For Solid Cylinder, Characteristic length $L_C=R$	For Sphere, Characteristic length $L_C=R$

RULES FOR HEISLER CHART:

1. Used for symmetric Figure.
2. There should not be internal heat generation within the body.
3. Half Dimension of body considered as characteristic length (L_C).
4. Centreline or Centre Point considered as reference there for all dimensions are measured w. r. t. centreline.
5. Heat Transfer Coefficient value is constant.
6. $F_o\geq 0.2$