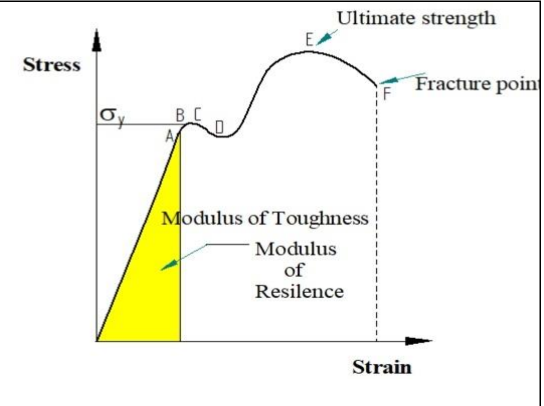
STRAIN ENERGY

Strain Energy = Stored energy due to deformation
= Work done for deformation (Within Elastic limit/Proportionality limit)

"Strain Energy is capable to doing some work"
Resilience: Strain energy stored within elastic limit.
$U_R = [\ A_{\sigma\text{-}\varepsilon}\ ] * \text{Volume}$
$= (1/2)\ \sigma\ \varepsilon * V$
By hook's Law,
$U_R = (1/2)\ (\sigma^2 V / E)$
Proof Resilience: Maximum Strain energy stored up to elastic limit.
$U_{PR} = (1/2)\ (\sigma^2\ V / E)$, here σ = Stress at elastic limit
Toughness: Strain energy stored up to Fracture.
Toughness is maximum amount of shock energy absorbed before fracture.
Toughness is useful for while designing accidental over loading.



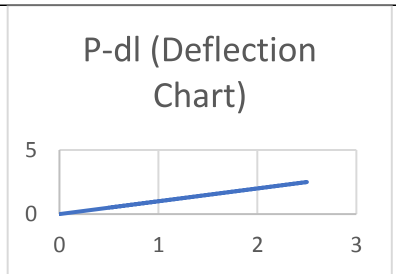
Strain Energy / Volume	
Modulus of Resilience	**Modulus of Toughness**
(Strain Energy/Volume) up to elastic limit/ Proportionality limit	(Strain Energy/Volume) till fracture
$[\ A_{\sigma\text{-}\varepsilon}]$ up to Proportionality limit	$[\ A_{\sigma\text{-}\varepsilon}]$ till fracture

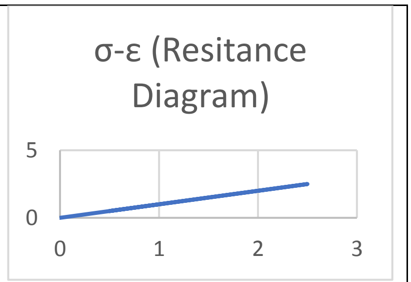
<u>Can we change resilience and toughness</u>? Yes, By Alloying we can change property.
Toughness depends on **Strength** and **ductility.**
Eg. Hard Steel (0.6%C), Soft Steel (0.1%C), Structure Steel (0.2%C) (Moderate Toughness)
Young's Modulus remains constant when alloying material.

Strain Energy Stored due to following loading			
Axial Loading	Bending	Torsion	Shear

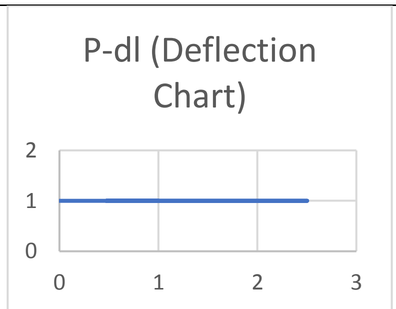
<u>Strain Energy Due to Axial Loading:</u>

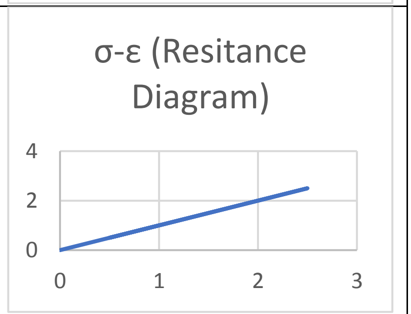
For Gradually Applied Load,
Work Done = Work Stored
$(1/2)\ P\ \delta L = (1/2)\ \sigma\ \varepsilon * \text{Volume}$
From Young's Modulus,
$\sigma = P / A$
$U_G = (1/2)\ (P^2 V / A^2 E)$





For Suddenly Applied Load,
Work Done = Work Stored
$P\ \delta L = (1/2)\ \sigma\ \varepsilon * \text{Volume}$
From Young's Modulus,
$\sigma = 2P / A$
$U_S = 4\ U_G$

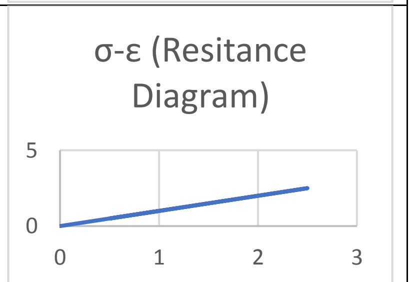




For Impact Load,
Work Done = Work Stored ===> $P\ (h + \delta L) = (1/2)\ \sigma\ \varepsilon * \text{Volume}$
From Young's Modulus,

$$\sigma = \frac{P}{A}\left[1 + \sqrt{1 + \frac{2AEh}{PL}}\right]$$

$\sigma = \sigma_G$ * Impact Factor, here σ_G = Stress at Gradually Applied
δL can be found from Young's modulus equation.



U = (1/2) (σ^2 V / E), here σ put from above derivation.

Strain Energy Stored due to following loading (Gradually Applied)			
Axial Loading	**Bending (For Long Beam)**	**Torsion**	**Shear (For Short Beam)**
U = $\int_0^L \frac{P2}{2EA} dx$	U = $\int_0^L \frac{M2}{2EI} dx,$	U = $\int_0^L \frac{T2}{2GJ} dx,$	U = $\int_0^L \frac{V2}{2GA} dx$
U = (1/2) P δL	U = (1/2) M θ	U = (1/2) T θ	U = (1/2) τ ϕ

Castigliano's Theorem:

If an elastic Structure is in equilibrium under the action of different forces (P_i where i = 1,2,….)

Theorem 1:

The Partial Derivative of Strain Energy with respect to the **point load** is a deflection of a structure **at the point** of application of load **in the direction** of applied load.

$\partial U / \partial Pi = \delta i$

Theorem 2:

The Partial Derivative of Strain Energy with respect to **concentrated bending moment/ point couple** is the slope of a structure **at the point** of application of moment **in the sense** of applied load.

$\partial U / \partial Mi = \theta i$

Tip:

1) If no load present at the desired location to find deflection, then add dummy load 'Q' find,

$\delta_Q = \partial U / \partial Q \,||\, (Q=0)$

2) If no moment present at the desired location to find slope, then add dummy moment 'M_Q' find,

$\theta_Q = \partial U / \partial M_Q || \, (M_Q = 0)$