

6. GEARS & GEAR TRAINS

GEARS: Gears are the example of higher pair mechanism. They are used to transfer constant velocity from one shaft to another shaft.

Gear: The wheel which is larger in size is known as gear. Due to its bigger size, Mass moment of inertia is more. Therefore, it prefers as driven element.

Pinion: The wheel which is smaller in size is known as Pinion. Due to its smaller size, Mass moment of inertia is less. Therefore, it prefers as driving element.

CLASSIFICATION OF GEARS: Gears may be Classified according to the relative position of axes of shafts to be connected. It may be,

Parallel Axes	Intersecting Axes	Neither Parallel nor intersecting Axes E.g. skew axes
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1. PARALLEL AXES:

SPUR GEAR:

- Straight spur gear is the simplest form of gear having teeth parallel to the gear axis.
- The contact of two teeth takes place over the entire width along a line parallel to the axes of rotation.
- As gear rotates, the line of contact goes on shifting parallel to the shaft.
- Spur gears generate noise in high speed applications due to sudden contact over the entire face width between two meshing teeth.
- Spur gears are cheapest.

Uses: Automobiles, High speed turbines.

HELICAL GEARS:

- In this gear, teeth are part of helix instead of straight across the gear parallel to the axis.
- The mating gears will have same helix angle but in opposite direction for proper mating.
- As the gear rotates, the contact shifts along the line of contact in involute helicoid across the teeth.
- There is gradual engagement at the beginning of any individual tooth, starting at the point on leading edge and progressing across the face of the gear as it rotates.
- This results in reduced dynamic effect and less noise.
- This inclined tooth develops thrust load and bending couples which are not present with spur gears.

HERRINGBONE GEARS:

- Single helical gear will produce an axial thrust on the shaft bearing due to the inclination of the teeth it can be eliminated by employing the double helical gears with two sets of teeth back to back, each set cut to opposite hand.
- Herringbone gears are also known as double helical gears.
- Herringbone gears made of two helical gears with opposite helix angles, which can be used up to 45°.
- Helical gears used to obtain herringbone have same module, number of teeth and pitch circle diameter, but with teeth having opposite hand of helix.
- In Herringbone gears there is a groove known as tool run out where as in herringbone gear this groove is not present.

Uses: High power applications such as ship drive and large turbines.

RAKE AND PINION:

- In these gears the spur rack can be considered to be spur gear of infinite pitch radius with its axis of rotation placed at infinitely parallel to that of pinion.
- The pinion rotates while the rack translates.

2. INTERSECTING AXES:

STRAIGHT BEVEL GEARS:

- Straight bevel gears are provided with straight teeth, radial teeth, radial to the point of intersection of the shaft axes and vary in cross section through the length inside generator of the cone.
- Straight bevel gears can be seen as modified version of straight spur gear in which teeth are made in conical direction instead of parallel to axis.
- Produce noise in high speed application.

SPIRAL BEVEL GEARS:

- Bevel gears are made with their teeth are inclined at an angle to face of the bevel and forms a circular arc.
- Spiral gears are also known as helical bevel gears.
- It gives the same advantage of helical teeth.
- Spiral bevels are difficult to design and costly to manufacture.
- They have smooth teeth engagement which results in quiet application even at high speeds.
- Spiral bevel gears have better strength so use for high power transmission application.

NOTE: When the axes are at right angles the large gear is called crown wheel and smaller pinion.

a. MITRE GEARS:

- Two identical bevel gears mounted on shafts, which are intersecting at right angles.
- The pinion and gear have same dimensions namely addendum, dedendum, pitch circle diameter, number of teeth and module.
- The pinion and gear rotate at same speed.

b. CROWN GEARS:

- When one of the gears has pitch angle 90° then that gear is called crown gear.
- Such gears are mounted on shafts which are intersecting at an angle that is more than 90° .

c. HYPOID GEARS:

- Similar to spiral bevel gears that are mounted on shafts which are non-parallel, non-intersecting.
- Hypoid gears are based upon pitch surfaces which are hyperboloid of revolution.

Uses: Automobile Differentials.

d. ZEROL GEAR:

- Spiral bevel gears with zero spiral angel.
- These gears theoretically give more gradual contact and a slightly larger contact ratio.

NOTE: Bevel gears are always designed in Paris.

3. NEITHER PARALLEL NOR INTERSECTING AXES:

HYPOID GEARS:

- The Hypoid gears are made of the frusta of hyperboloids of revolution.
- Two matching hypoid gears are made by revolving the same line of contact, these gears are non-interchangeable.
- Similar to bevel gears except that the shafts are offset and non- intersecting.

WORM GEARS:

- A special from of skew gears which has line contact is the worm and wheel pair.
- Usually, though not necessarily, axis is at right angle.
- Teeth on worm gear are cut continuously like the threads on a screw.
- Worm resembles a screw.
- Direction of rotation of the worm gear (or worm wheel) depends upon the direction of rotation of the worm.

ADVANTAGES OF WORM GEAR DRIVES:

- High speed reduction such as 100:1 can be obtained with a single pair of worm gears.
- Compact with small overall dimensions.
- Smooth and silent operation.
- Self-locking prevision can be made where the motion is transmitted only from worm to worm wheel, this is advantageous in applications like cranes and lifting devices.

DRAW BACKS OF WORM GEAR DRIVES:

- Efficiency is low compared to other gear drives.
- Worm wheel is generally made of Phosphor-bronze, which increases the cost.

RING GEAR OR ANNULUS: The gear which is having teeth in inner side is known as ring gear and annulus.

RAKE GEAR: It's used to convert rotary into translator motion. It is the gear whose pitch circle radius is infinite.

Two gears which are in mesh are known as gear set.

EXTERNAL MESHING	INTERNAL MESHING
Component rotates in opposite direction.	Component rotates in same direction.

VELOCITY RATIO: It indicate the mechanical advantage of gear set. In general, if two gears "A" and "B" are in mesh, the velocity ratio can be written as,

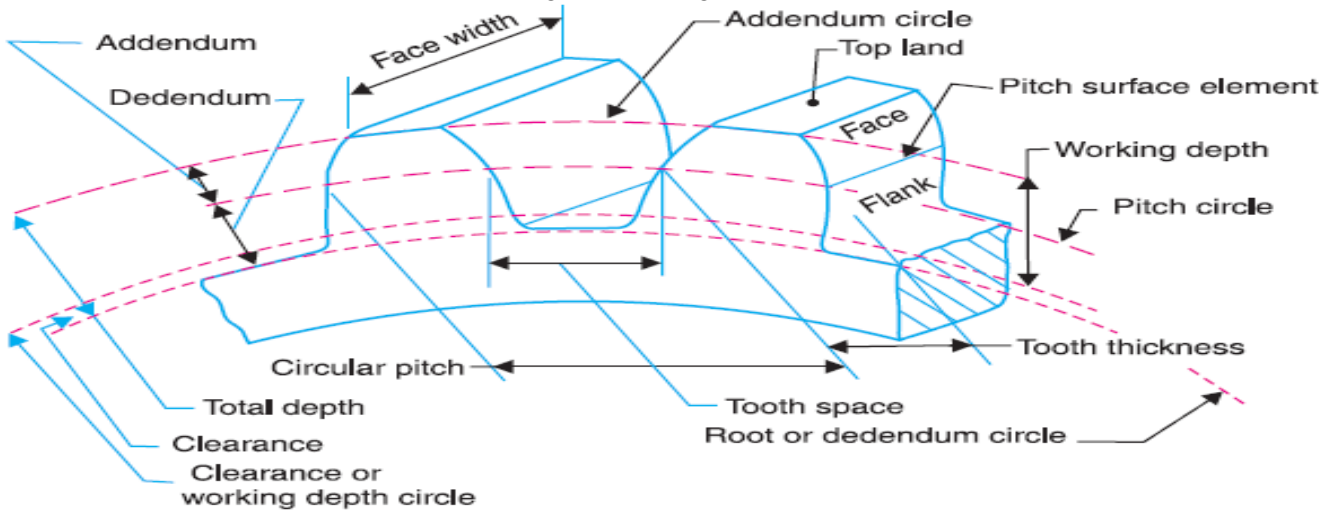
$\lambda = \pm \frac{\omega_A}{\omega_B} = \pm \frac{\omega_{O/P}}{\omega_{I/P}} = \pm \frac{\omega_{Gear}}{\omega_{Pinion}} \text{ (Generally } < 1 \text{)}$	+ve Rotation is in same direction (Internal meshing) -ve Rotation is in Opposite direction (External meshing)
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GEAR RATIO (G. R): $G. R = \frac{NO. OF TEETH ON GEAR}{NO. OF TEETH ON PINION} = \frac{T}{t}$	REDUCTION RATIO (R. R): $R. R = \frac{\omega_{I/P}}{\omega_{O/P}} = \pm \frac{\omega_{Pinion}}{\omega_{Gear}} \text{ (Generally } > 1 \text{)}$
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GEAR TERMINOLOGY:

IMPORTANT CIRCLES:

- A. PITCH CIRCLE:** It denotes the size of the gears, all the important dimensions such as tooth thickness, tooth space, arc of contact etc. are measured on the pitch circle circumference.
- It is an imaginary circle. E.g. It's radius can be changed.
 - Two gears which are in mesh can be represented by their pitch circle. and the point of the contact of the pitch circle is known as pitch point.
- B. BASE CIRCLE:** It's a real circle. E.g. it's radius can't be changed. Circle from where involute profile begins.
- C. ADDENDUM CIRCLE:** The circle which passes through top of the gear teeth is known as addendum circle.
- D. DEDENDUM CIRCLE:** Circle from where gear tooth begins.



Addendum	Pitch	Base	Dedendum	Most common profile, can be used with gears as well as pinions.
Addendum	Pitch	Dedendum	Base	$r_{Dedendum} = r_{Base}$ (Non-involute prof. doesn't exist, non-practical)

In second profile, no chance of interference. It can be used with the wheel that is larger in size E.g. Gears only.

1) Circular Pitch (P_c): It's the distance between two similar points an adjacent tooth measured along the pitch circle circumference.	$P_c = \frac{\text{Pitch Circle Circumference}}{\text{No. of Teeth}}$	
2) Module (m): It's SI unit of gear. In mm .	$m = \frac{PCD}{\text{No. of Teeth}}$	$P_d = \frac{\text{No. of Teeth}}{PCD(\text{in inch})}$
3) Diametral Pitch (P_d): It's F.P.S. unit of gear. It is defined as no. of teeth per inch to PCD.	$P_c P_d = \pi$	
For a gear set the units of meshing elements must be same. For gear and pinion. $m_{Gear} = m_{Pinion}$	$(P_c)_G = \pi m_G = \pi \frac{D}{T}$	$(P_c)_g = \pi m_p = \pi \frac{d}{t}$

SET (A): DIMENSIONS WHICH ARE VISIBLE IN SINGLE GEAR (WITHOUT MESHING)

TOOTH THICKNESS: The thickness of the tooth measured along the pitch circle circumference.

TOOTH SPACE: the distance between adjacent tooth is known as tooth space. During meshing tooth thickness of gear will be in centre in to tooth space of the pinion and vice-versa.

DEDENDUM (b): The radial distance between pitch circle and dedendum circle is known as dedendum.

ADDENDUM (a): The radial distance between pitch circle and addendum circle is known as addendum.

- a & b expressed as a fraction of module.

FULL DEPTH: It's Summation of addendum and dedendum of gear.

FACE WIDTH: The portion of gear tooth above the pitch circle is known as face and below the pitch circle is known as flank.

- Width of the face is known as face width; it is an important dimension w.r.t gear tooth design.

$$\text{Tooth Thickness} + \text{Tooth Space} = \text{Circular Pitch}$$

$$a = fm \text{ \& } b = fm$$

For 20° full depth tooth,

$$a = m \text{ \& } b = 1.25m$$

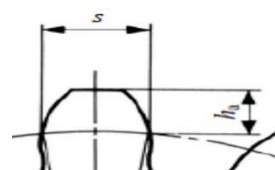
For 20° stub tooth,

$$a = 0.8m \text{ \& } b = m$$

For Pinion, $a_p = r_a - r$ & $b_p = r - r_b$

For Gear, $a_g = R_a - R$ & $b_g = R - R_b$

CHORDAL TOOTH THICKNESS:



SET (B): DIMENSIONS WHICH ARE VISIBLE DURING MESHING.

WORKING DEPTH: It's Summation of addendum of gear and dedendum of pinion.

CLEARANCE: The distance between addendum circle of gear and dedendum circle of pinion is known as clearance.

- Clearance is absent then it will result into interference.

$$\text{Working depth} = a_p + a_g$$

$$\text{Clearance} = a_g - b_p$$

BACKLASH: the amount by which the tooth space of a gear is larger than tooth thickness of mating element is known as backlash.

- If the backlash is absent then rubbing will take place in both the surface which will result in more friction & more wear, due to rubbing there will be more heat generation and a thermal expansion is prevented as no space is available will further result in thermal stress and stress concentration due to which the tooth become weak.
- If backlash increases there will be more noise and more vibration.

PRESSURE ANGLE (ϕ): It is the angle between common normal (line which is tangential to both base circle) and velocity vector at pitch point rotated in a direction similar to driven element. **Or** the angle between common normal and common tangent to the pitch circle of the gear as well as the pitch circle of pinion at pitch point rotated in a direction similar to drive element.

- It is the measure of effectiveness of gear set.
- ϕ remains same for the gear set (Gear & Pinion which are in mesh).

POINT OF BEGINNING OF ENGAGEMENT:

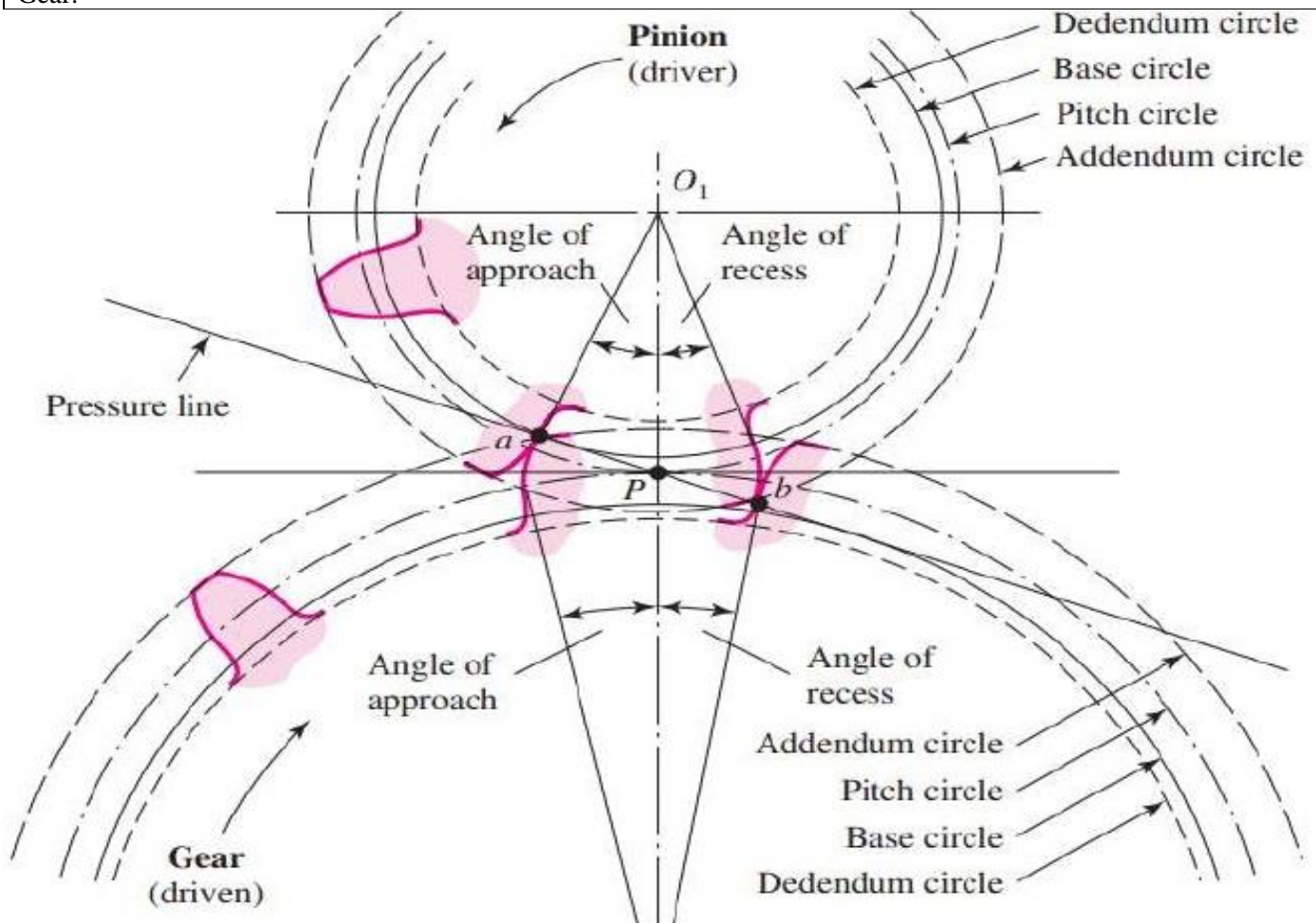
It's point where addendum circle of gear (Driven element) intersect with common normal.

POINT OF ENDING OF ENGAGEMENT:

It's point where addendum circle of gear (Driver element) intersect with common normal.

POINT OF TANGENCY OF PINION: It's the point where common normal is in contact with the base circle of the pinion.

POINT OF TANGENCY OF GEAR: It's the point where common normal is in contact with the base circle of the Gear.



LAW OF GEARING:

- The ratio of first kinematic coefficient. E.g. angular velocity must be constant.
- Since the pitch circle is an imaginary circle that its radius can be changed. Therefore, from here, we can't conclude that angular velocity will be constant.
- The pitch point "P" will be a fixed point and it will divide that common normal in a constant ratio.
- All the elements satisfying law of gearing are known as gears. Presence of teeth is not a necessary condition.
- The pitch point "P" is an I-centre. Therefore, there will be pure rolling. E.g. rolling without slipping at a point of contact.

From $\triangle O_G DP$, $R_b = R \cos \phi$ From $\triangle O_P CP$, $r_b = r \cos \phi$	From the angular velocity theorem, $V_{I23} = I_{12} I_{23} \omega_G = I_{13} I_{23} \omega_P$	$\frac{\omega_G}{\omega_P} = \frac{I_{13} I_{23}}{I_{12} I_{23}} = \frac{O_G P}{O_P P} = \frac{r}{R} \dots (1)$
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From the similar triangle $\Delta O_G DP$ & $\Delta O_P CP$,

$\frac{O_G P}{O_P P} = \frac{O_G D}{O_P C} = \frac{PD}{PC} \Rightarrow \frac{R}{r} = \frac{R_b}{r_b} = \frac{PD}{PC} = \text{Constant} \dots (2)$	$\frac{\omega_G}{\omega_P} = \frac{r}{R} = \frac{r_b}{R_b} = \frac{PD}{PC} = \text{Constant}$
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Gear-set has unique module,

$$\frac{\omega_G}{\omega_P} = \frac{r}{R} = \frac{r_b}{R_b} = \frac{t}{T} = \text{Constant (Valid for any gear set)}$$

INTERFERENCE:

- Whenever non-conjugate meshing (meshing of involute profile with non-involute profile) will take place gears will join in each other and they will not be able to transfer the velocity ratio.
- Interference should be avoided.

Conjugate action: Involute is not meshing with non-involute profile of the meshing gear.

NECESSARY CONDITION FOR INTERFERENCE:

- The presence of non-involute profile is a necessary condition for interference but, it is not the sufficient one.

SUFFICIENT & NECESSARY CONDITION FOR INTERFERENCE:

- If the addendum of gear penetrates into the base circle of pinion or crosses the point of tangency of the base circle of pinion, it will result in interference.
- If clearance is absent the interference will occur certainly (due to non-conjugate action) and if clearance is present, then interference may or may not occur.
- If *full depth* = *working depth*, then interference will occur certainly. And if *full depth* > *working depth*, then interference may or may not occur.

Actual path of approach (EP) (Check the figure),

$EP = DE - DP = \sqrt{R_a^2 - R_b^2} - R \sin \phi = \sqrt{R_a^2 - (R \cos \phi)^2} - R \sin \phi = f(\text{Gear})$	$EP_{max} = CP = r \sin \phi = f(\text{Pinion})$
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Actual path of recess (PF) (Check the figure),

$PF = CF - CP = \sqrt{r_a^2 - r_b^2} - r \sin \phi = \sqrt{r_a^2 - (r \cos \phi)^2} - r \sin \phi = f(\text{Pinion})$	$PF_{max} = PD = R \sin \phi = f(\text{Gear})$
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Actual path of contact (EF) = Actual path of approach + Actual path of recess

$EF = EP + PF = \sqrt{R_a^2 - (R \cos \phi)^2} + \sqrt{r_a^2 - (r \cos \phi)^2} - (R + r) \sin \phi$	$EF_{max} = CD = (R + r) \sin \phi$
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$$\text{Arc of contact} = \text{Arc of Approach} + \text{Arc of Recess} = \frac{\text{Path of Approach}}{\cos \phi} + \frac{\text{Path of Recess}}{\cos \phi} = \frac{\text{Path of contact}}{\cos \phi}$$

CONTACT RATIO (CR): It denotes the no. of pairs of teeth which are in mesh.

E.g. $CR = 1$ means 1 teeth of gear and 1 teeth of pinion remains always in contact at pitch point. &

$CR = 1.2$ means 1 gear pair of teeth remains in complete contact at pitch point. And for 20% of total time of contact another pair is in mesh.

ANGLE OF ACTION (δ): It's angle subtended by arc of contact at centre of either Gear or pinion.

$\delta_G = \text{Arc of Approach} / R$	$\delta_P = \text{Arc of Approach} / r$
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SLIDING VELOCITY:

$\text{At beginning of engagement} = (\text{Path of Approach})(\omega_G + \omega_P)$	$\text{At Ending of engagement} = (\text{Path of Recess})(\omega_G + \omega_P)$
$\text{At pitch point} = 0 \text{ (Because of pure rolling)}$	

ROLLING VELOCITY (RV): $RV = \omega_G R = r \omega_P$

METHODS TO AVOID INTERFERENCE:

1. **By increasing the centre distance:** here, $R_b = R \cos \phi$. Hence, by decreasing pressure angle.
2. **By properly section of No. of teeth on gear as well as pinion:** In a gearset i.e. when gear and pinion are in mesh and have same size of addendum then gear will interfere first.
 - If two wheels are in mesh having same size of addendum then driven wheel will interfere first.
 - If two wheels are in mesh having different size of addendum then the wheel having larger addendum will interfere first.

MAXIMUM POSSIBLE ADDENDUM RADIUS OF GEAR WITHOUT CAUSING INTERFERENCE:

In ΔO_pCP , Cosine law, $R_{a,max}^2 = R^2 + (r \sin \phi)^2 - 2R(r \sin \phi) \cos(90 - \phi)$	
$R_{a,max} = R\sqrt{1 + \lambda(\lambda + 2) \sin^2 \phi}$ (From Velocity Ratio)	$(a_{Gear})_{max} = R_{a,max} - R = f m_{G,max} = f m_{P,max}$
$m_{P,max} = \frac{2r}{t_{min}} \Rightarrow t_{min} = \frac{2f\lambda}{\sqrt{1 + \lambda(\lambda + 2) \sin^2 \phi} - 1}$	In order to avoid interference, $t_{act.} > t_{min}$
The minimum number of teeth on pinion in order to avoid interference in rake and pinion, $t_{min} = \lim_{\lambda \rightarrow 0} \frac{2f\lambda}{\sqrt{1 + \lambda(\lambda + 2) \sin^2 \phi} - 1} = \frac{2f}{\sin^2 \phi}$	

3. Stubbing (Modifying the addendum): Reducing the addendum is called stubbing.

By stubbing, Length of moment are decreases. Hence the bending stress in the tooth decreases. So, Strength increases.

4. By using cycloidal profile tooth: Cycloid is combination of epicycloid and hypocycloid. But it is difficult to manufacture.

- Here, pressure angle changes as the engagement increases.
- Small change in centre to centre distance may big change in velocity ratio.
- 5. Under cutting: The process of removal of non-involute portion from the flank of pinion is known as under cutting.
- It may take place at the time of manufacturing.
- It may take place as a result of interference.
- It may made purposely to avoid the interference.
- Undercutting always result and stress concentration due to which the tooth become weaker and chances of will/ Failure increases.

FORCE TRANSMISSION: See the figure on page,

$P = F(\cos \phi)$	$T = F \cos \phi r$ & $P = T\omega$
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Hence, as pressure angle increase chances of interference decreases and power transmission decreases.

ϕ	Power Transmission	Chances of Interference
14.5°	Max.	Max.
20° (Optimum)	Moderate	Moderate
22.5°	Min.	Min.

GEAR TRAINS

DOF = 1	DOF = 2 E.g. Epicyclic Gear Tran (Sun & Planet)
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DOF = 1	When 2 Gears are mounted on same shaft.	Reverted Gear Train (RGT)
	All gears are mounted on same plane & different shaft.	Compound Gear Train (CGT)
		Simple Gear Train (SGT)

$$\text{Train Value (TV)} = \frac{\omega_{out}}{\omega_{in}} = \frac{\omega_{driven}}{\omega_{driver}}$$

SIMPLE GEAR TRAIN (SGT):

<p>Centre distance between i/p & o/p element = O_1O_4</p> $m_1 = m_2 = m_3 = m_4 = m$ $O_1O_4 = R_1 + 2R_2 + 2R_3 + R_4 = \frac{m}{2} (T_1 + 2T_2 + 2T_3 + T_4)$ $TV = \frac{\omega_4}{\omega_1} = -\frac{T_1}{T_4} \text{ (Due to external mashing - ve)}$ <p>$TV = -ve$ indicate that input and output shafts are rotating in opposite direction.</p>	
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IDLER WHEELS: These are wheel whose number of teeth does not appear in the final expression of TV. It does not affect magnitude of train value but they can affect the direction of rotation of output w. r. t. input.

- In SGT even, odd number of idler wheels are used, i/p & o/p rotate in opposite, same direction respectively.

COMPOUND GEAR TRAIN (CGT): In this, alternate gears are called driver- driven-driver-driven.

<p>Centre distance between i/p & o/p element = O_1O_4</p> $m_1 = m_2 = m_x \text{ \& } m_3 = m_4 = m_y$ $O_1O_4 = R_1 + R_2 + R_3 + R_4 = \frac{m_x}{2} (T_1 + T_2) + \frac{m_y}{2} (T_3 + T_4)$ $TV = \frac{\omega_4}{\omega_1} = \frac{\omega_4 \omega_3 \omega_2}{\omega_3 \omega_2 \omega_1} = \left(-\frac{T_3}{T_4}\right) \left(-\frac{T_1}{T_2}\right) \left(\because \omega_3 = \omega_2 \text{ external mashing - ve}\right)$ <p>$TV = +ve$ indicate that input and output shafts are rotating in same direction.</p>	
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REVERTED GEAR TRAIN (RGT):

If the input and output shaft are co axial then we use reverted gear train. E.g. Gear box and watches etc.

In first, $m_1 = m_2 = m_x$

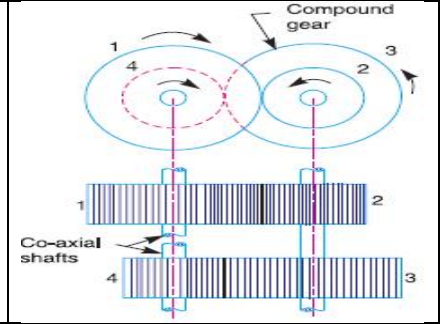
In second stage, $m_3 = m_4 = m_y$

Here, centre distance between compound gear and main gear,

$$O_1O_2 = O_3O_4 \Rightarrow R_1 + R_2 = R_3 + R_4 \Rightarrow m_x(T_1 + T_2) = m_y(T_3 + T_4)$$

$$TV = \frac{\omega_4}{\omega_1} = \frac{\omega_4}{\omega_3} \frac{\omega_3}{\omega_2} \frac{\omega_2}{\omega_1} = \left(-\frac{T_3}{T_4}\right) \left(-\frac{T_1}{T_2}\right) \left(\because \omega_3 = \omega_2\right)$$

$TV = +ve$ indicate that input and output shafts are rotating in same direction.



EPICYCLIC GEAR TRAIN/ SUN AND PLANET GEAR TRAIN: $DOF = 2$ From Kutzbach's equation. So, it's unconstrained gear train ($DOF > 2$).

ADVANTAGES:

- They are compact in size.
- Both static and dynamic forces are balanced if multiple planets are used.
- High torque ratio and velocity ratio can be achieved.
- Bidirectional output can be obtained from a single unidirectional input.
- It's used in automobile transmission system.

ANALYSIS OF THE EPICYCLIC GEAR TRAIN:

1. Kinematic Analysis:

Method-1:

$\omega_{sun} = \omega_{arm} + \omega_{sun/arm}$	$\omega_{planet} = \omega_{arm} + \omega_{planet/arm}$
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Let arm is fixed ($\omega_{arm} = 0$), $DOF = 1$.

$\omega_{sun} = \omega_{sun/arm}$ $\omega_{planet} = \omega_{planet/arm}$	$\frac{\omega_{sun}}{\omega_{planet}} = -\frac{T_P}{T_S} = \frac{\omega_{sun/arm}}{\omega_{planet/arm}} \left(\because \text{arm is fixed } (\omega_{arm} = 0) \text{ \& } \right)$ <i>Constrained Mechanism</i>
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When Arm is moving,

$$\frac{\omega_{sun} - \omega_{arm}}{\omega_{planet} - \omega_{arm}} = -\frac{T_P}{T_S}$$

Method-2:

Sr. No.	Condition	Arm	Sun (T_S)	Planet (T_P)
1	Arm is fixed. $DOF = 1$. Let, Sun is rotating with $+x$ rpm in CW	0	$\omega_S = +x$	$\omega_P = -\frac{T_S}{T_P} \omega_S = -\frac{T_S}{T_P} x$
2	Arm is moving with $+y$ rpm in CW.	$+y$	$\omega_S = +x + y$	$\omega_P = -\frac{T_S}{T_P} x + y$

2. Dynamic/ Torque (Kinetic) Analysis:

Since the train is completely balanced, $\sum \tau_{net} = 0$ $\therefore \tau_{sun} + \tau_{planet} + \tau_{arm} = 0$	$\eta = \frac{\text{Power@O/P}}{\text{Power@I/P}} = \frac{\tau_{out} \omega_{out}}{\tau_{in} \omega_{in}}$
For $\eta = 1$, $\tau_S \omega_S + \tau_P \omega_P + \tau_A \omega_A = 0$	Fixing/ Holding Torque: Torque required to hold the element.