3. DESIGN FOR FLUCTUATING LOADING

FLUCTUATING LOADING: When the variation of load w. r. t. time is sinusoidal then the dynamic loading is said to be fluctuating loading.

P_{max} =Maximum Load	P_{min} =Minimum Load
P_{mean} =Mean Load= $(P_{max} + P_{min})/2$	P_{amp} =Amplitude Load= $(P_{max} - P_{min})/2$

σ_{max} =Maximum Stress $(+ve\ or\ -ve\ or\ 0)$	σ_{mean} =Mean Stress = $(\sigma_{max} + \sigma_{min})/2(+ve\ or\ -ve\ or\ 0)$
σ_{min} =Minimum Stress $(+ve\ or\ -ve\ or\ 0)$	σ_{amp} =Amplitude Stress = $(\sigma_{max} - \sigma_{min})/2(+ve\ or\ 0)$

AMPLITUDE RATIO: $A = \dfrac{\sigma_{amp}}{\sigma_{mean}} = \dfrac{1-R}{1+R}$	**STRESS RATIO**: $R = \dfrac{\sigma_{min}}{\sigma_{max}}$

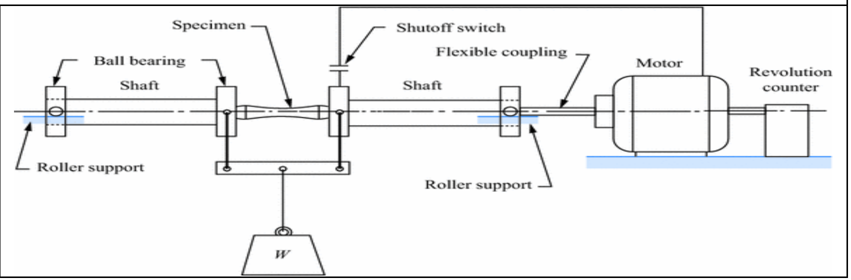
TYPES OF FLUCTUATING LOAD	
REPEATED LOADING	**REVERSED LOADING**
$\sigma_{max}\ or\ \sigma_{min} = 0 \Leftrightarrow \lvert\sigma_{mean}\rvert = \sigma_{amp}$	$\sigma_{mean} = 0$
Design is based on maximum load.	Experimental Results are used for design.

FATIGUE LOADING: It's the weakening of component due to reversal of loading or cyclic loading. And material fails suddenly (Ductile and brittle both).

RR MOORE'S /ROTATING BEAM EXPERIMENTAL ANALYSIS

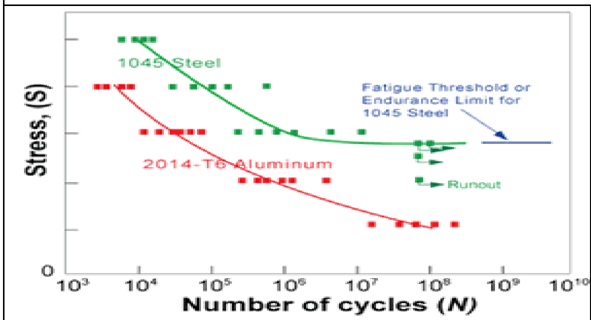
Specimen: Mild Steel
1. Free from defects.
2. No sudden reduction in cross section.
3. Mirror Finished Surface.

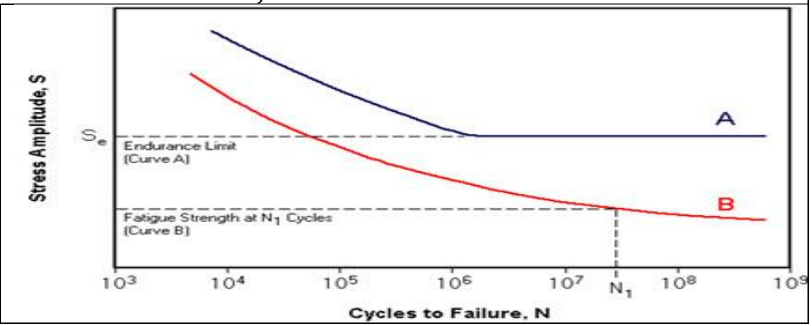
Radius: 3.5in to 10in (88.9 mm to 254 mm)
Min. Dia.: 0.3in (7.62 mm)
Max. Dia.: 0.481in (12.217 mm)
Total Length: 3.4375in (87.312 mm)



- Specimen is simply supported beam. And subjected to reversed pure bending (As shown in fig.).

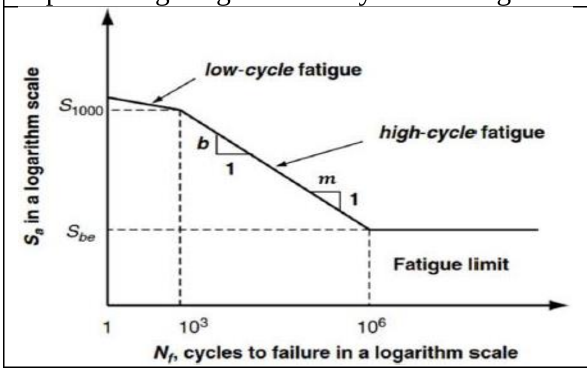
S N DIAGRAM: $log\ N\ Vs\ log\ \sigma_f\ Graph$





For aluminium, there is no endurance limit region where as mild steel has endurance limit region.

ENDURANCE LIMIT: It's the point of loading at which the component starts experiencing fatigue due to cyclic loading.



ENDURANCE STRENGTH: It's maximum amplitude stress that the standard specimen can withstand for a minimum of 10^6 cycle when subjected to completely reversed loading without fatigue.

For Steel,

$For\ N = 10^3, \sigma_e' = 0.5\sigma_{ut}$	$For\ N = 10^6, \sigma_e' = 0.9\sigma_{ut}$

For Iron,	**For Al, Cu,**
$\sigma_e' = 0.4\sigma_{ut}$	$\sigma_e' = 0.4\sigma_{ut}$
	Maximum life $N = 5 * 10^8$

IMP POINTS RELATED TO STANDARD SPECIMEN USED IN EXPERIMENT:

1. Min. diameter $d_{min} \approx 8\,mm$. 2. Surface of the specimen is polished to mirror finishing. 3. No sudden reduction in cross section. 4. Experiment is conducted at room temperature.	5. Loading is pure bending. 6. Specimen may or may not fail at endurance limit. 7. The failure stress corresponding to 10^6 cycles is endurance strength σ_e'.

ACTUAL COMPONENT USED IN THE MACHINE:

$\sigma_e = K_a K_b K_c K_d K_e \frac{1}{K_f} \sigma_e'$	σ_e = Corrected Endurance Strength.

SURFACE FINISHED FACTOR (K_a)		**SIZE FACTOR (K_b)**
Shrigley Equation, $K_a = A\,\sigma_{ut}^B$ A, B Find using experiments.	If $K_a > 1$,use $K_a = 1$ For cast iron $K_a = 1$	$K_b = 1, for\ d \le 8\,mm$ Shrigley and Mitchel equation; $K_b = 1.189 d^{-0.097}, For\ 8\,mm < d \le 250mm$ $K_b = 0.6, For\ d > 250mm$

LOAD FACTOR (K_c)		**RELIABILITY FACTOR (K_d)**
For reversed Bending, $K_c = 1$ For reversed axial loading, $K_c = 0.7$ For reversed torsional loading, $K_c = 0.577$ [$\because$ From distortion energy theory, $\tau_e' = \sigma_e'/\sqrt{3}$]		
TEMPERATURE FACTOR (K_e)		
$K_e = 1$	$T \le 450\,°C$	
Shrigley and Mitchel equation; $K_e = 1 - 0.0058[T - 450]$	$450\,°C < T \le 550\,°C$	

% Success	**Reliability**
50%	$K_d = \mathbf{1}$
90%	$K_d = 0.897$
99%	$K_d = 0.814$
99.9%	$K_d = 0.759$
99.9999%	$K_d = 0.620$

Shrigley and Mitchel equation;
Can't use for Cu, Mg, Al, Etc…

STRESS CONCENTRATION:

$\sigma_0 = \frac{F}{(w-d)t}$	$\sigma_{SC} > \sigma_0$ σ_{SC} depends material, reduction rate, Dimensions of cut.	a = Length of semi major axis, b = Length of semi minor axis [Longitudinal], σ_0 = Normal Stress/ Stress at minimum cross section, σ_{SC} = Maximum stress at minimum cross section due to stress concentration,
Gradual reduction of Cross section induces less stress concentration. And sudden reduction causes high stress concentration.		
STRESS CONCENTRATION FACTOR (K_t): It's maximum concentration factor for a given component and shape of cut.		$\sigma_{SC} = K\sigma_0$ "K" depends on dimension of cut, material.
$(\sigma_{SC})_{max} = K_t\sigma_0, (\sigma_{SC})_{max}\ is\ maximum\ \forall\ material$ K_t depends on shape of cut and it's not dependent on material.		$K_t = 1 + 2\frac{lateral\ Length\ of\ Cut}{Longitudinal\ Length\ of\ Cut}\left(\frac{d}{w} \approx 0\right)$

FATIGUE/ ACTUAL STRESS CONCENTRATION FACTOR (K_f)		
For given material, maximum stress due to change in cross section at minimum cross section due to stress concentration given by,		$\sigma_{SC} = K_f\sigma_0$
Failure condition in actual,	$\sigma_0 = \sigma_{SC}/K_f$	$\sigma_e = \sigma_e'/K_f$
$\boldsymbol{K_f = \frac{Endurance\ Strength\ of\ Std.(or\ Notch\ free)\ Specimen}{Corrected\ (or\ Notch\ Specimen)\ Endurance\ Strength}}$		

NOTCH SENSITIVITY(q): It's sensitivity of material toward cuts or notches.

$q = \frac{\Delta\sigma_{act}}{\Delta\sigma_{th}} = \frac{\sigma_{SC} - \sigma_0}{(\sigma_{SC})_{max} - \sigma_0} = \frac{\sigma_0[K_f - 1]}{\sigma_0[K_t - 1]}$	$\therefore K_f = 1 + q[K_t - 1]$ $0 < q < 1, hence\ 1 < K_f < K_t$	$K_t = 1 + 2\frac{a}{b}$

For circular Cut, $K_t = 3$.

METHODS TO REDUCE STRESS CONCENTRATION: Stress concentration is localization of stress due to reduction in cross section. We can reduce stress concentration by Gradually reducing cross section,

Providing fillet radius	Providing Small Holes/ Notches near to vicinity region of the sudden reducing cross section

IMPORTANT POINT:

If K_t and q are given, find K_f	If K_t and q are not given, $K_f = 1$	If K_t given but q is not given, $K_f = K_t$

INFINITE LIFE: For reversed loading, life of component $N > 10^6\ cycles$.

$$To\ avoid\ fatigue(\text{life of component } N > 10^6\ cycles): \frac{F_a}{A_{min}} = \frac{M_a}{Z_{min}} = \frac{\sigma_e}{FOS}$$

FINITE LIFE: For reversed loading, life of component $10^3 < N < 10^6\ cycles$.

At $10^3\ cycles, \sigma_f = \sigma_m = 0.9\ \sigma_{ut}$	At $10^6\ cycles, \sigma_f = \sigma_e$
$\boldsymbol{\sigma = A\,N^B}$($\boldsymbol{Valid\ for\ 10^3 < N < 10^6\ cycles\ and\ reversed\ loading}$)	

$\sum \frac{N_i}{L_i} = 1$	Fatigue Stress	σ_1	σ_2	σ_3	...
	Life Span (in time) before fatigue	L_1	L_2	L_3	...
	No of Revolution Actually Spend (in cycles)	N_1	N_2	N_3	...

SODERBERG THEORY	**GOODMAN THEORY**	
$\frac{\sigma_m}{\sigma_{yt}} + \frac{\sigma_a}{\sigma_e} = \frac{1}{FOS}$	$\frac{\sigma_m}{\sigma_{ut}} + \frac{\sigma_a}{\sigma_e} = \frac{1}{FOS}$	
GERBER THEORY (Parabolic Curve)	**ASME THEORY (Elliptical Curve)**	
$FOS^2\left(\frac{\sigma_m}{\sigma_{ut}}\right)^2 + FOS\frac{\sigma_a}{\sigma_e} = 1$	$\left(\frac{\sigma_m}{\sigma_{ut}}\right)^2 + \left(\frac{\sigma_a}{\sigma_e}\right)^2 = \left(\frac{1}{FOS}\right)^2$	
LANGER THEORY	**MODIFIED GOODMAN THEORY**	
$FOS_1\frac{\sigma_m}{\sigma_{yt}} + FOS_2\frac{\sigma_a}{\sigma_{yt}} = 1$	$Slope\ = \frac{\sigma_a}{\sigma_m} \geq \frac{y}{x}$	$Slope\ = \frac{\sigma_a}{\sigma_m} \leq \frac{y}{x}$
	Use Goodman Theory	Use Langer Theory
	y, x are intersection point of Langer's line and Goodman line	

COMBINED FLUCTUATING LOADING:

1. Use Theory of failures to find σ_a & σ_m. Here all mean and amplitude loads will be given for combined loading condition. 2. Use the theory of fluctuating loading. Find required parameters.	$\sigma_x = \sigma_{x\,min}\ to\ \sigma_{x\,max}$ $\sigma_y = \sigma_{y\,min}\ to\ \sigma_{y\,max}$ $\tau_{xy} = \tau_{xy\,min}\ to\ \tau_{xy\,max}$