3. FLUID STATICS

PRESSURE: Intensity of applied force per unit area. Or intensity of compressive work per unit volume is pressure.

PRESSURE	STRESS
Always Compressive in nature	It may be tensile or maybe compressive or shear.
It's scalar Quantity.	It's Tensor Quantity.

Units: $1\ bar = 10^5\ Pa, 1\ atm = 1.01325\ bar = 10.3\ mWater = 760\ mmHg$

PRESSURE IN TERMS OF HEAD OF FLUID: Height of a fluid column required to create a particular amount of pressure. E.g. $1\ mmHg = 133.33\ Pa = 1\ tor$

STANDARD PRESSURES:

1. **ABSOLUTE ZERO:** The pressure at which the molecular activity in matter completely stops.
2. **STANDARD ATMOSPHERIC:** The pressure exerted by atmosphere at mean sea level on earth.
3. **LOCAL ATMOSPHERIC PRESSURE:** the pressure exerted by atmosphere at particular location is local atmospheric pressure at that location. It's non uniform reference.

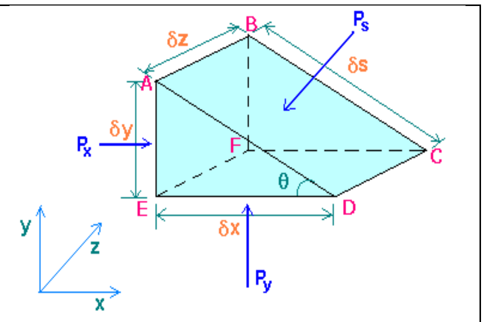
ABSOLUTE SCALE: It's measured w. r. t. absolute zero as reference. It's always non negative.
GAUGE SCALE: it's measured w. r. t. local atmospheric pressure as reference. It may be negative, 0 or positive.

PASCAL'S LAW: When constrained fluid is pressured, every point in the fluid experience a rise in pressure by same magnitude in all direction.
At equilibrium condition, $a_x = a_y = 0 \Rightarrow \sum F_x = \sum F_y = 0$

$$P_x = P_y = P_s \neq f(\theta)$$

REST: $\boldsymbol{\tau = 0}$ ($\boldsymbol{Constrained\ fluid}$)
It's valid for,
1) Always valid for ideal fluid ($\mu = \tau = 0$).
2) Real Fluid:

Rest condition ($du/dy = 0$)	Rigid Body motion ($a = Const.$)



PASCAL'S MACHINE: Any machine which functions on principle of pascal's law is known as pascal's machine. E.g. Hydraulic lift.

$$Mec.Adv. = \frac{Load\ Lifted}{Force\ Applied}$$

Pressure at a point in a fluid at rest,

1. Uniform intensity in all direction	2. Compressive in nature	3. Zero shear stress.

HYDROSTATIC LAW:
Aim: To identify the rate of change in pressure w. r. t. elevation for fluid at rest.

$$\frac{dP}{dz} = \gamma$$

Hydrostatic Law is valid for compressible and incompressible law.

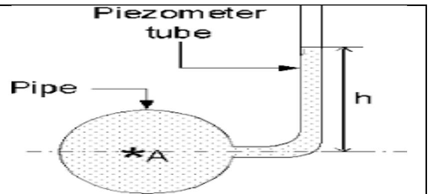
HYDROSTATIC LAW FOR	
INCOMPRESSIBLE FLUID	**COMPRESSIBLE FLUID**
$\Delta P = \gamma \Delta h\ (Liner\ Variation)$	$\Delta P = g[\rho_0 h + (\alpha h^2/2)]\{\because \rho = \rho_0 + \alpha h\}(Parabolic\ Variation)$

VARIATION OF PRESSURE IN ATMOSPHERE FOR				
ISOTHERMAL CONDITION		**VARIATION THERMAL CONDITION**		
$\frac{dP}{dz} = -g\left(\frac{P}{RT}\right)$	$P = P_0 e^{\frac{g}{RT}(h_0 - h)}$	$\frac{dP}{dz} = \frac{-g}{R(T_0 - \alpha h)}$	$P = P_0\left(1 - \frac{\alpha z}{R_0}\right)^{\frac{g}{R\alpha}}$	Where, α = Lapse Rate, Generally, $6 \leq \alpha \leq 9\ K/km$ Std. Value $6.5\ K/km$

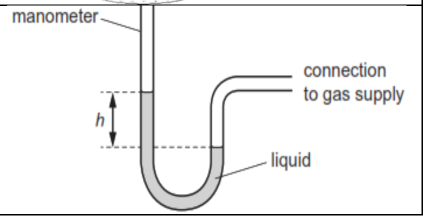
Step1:
First Mark Reference.
Step 2:

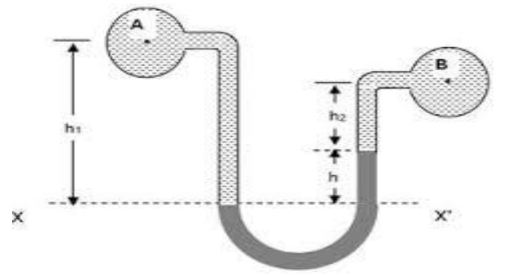
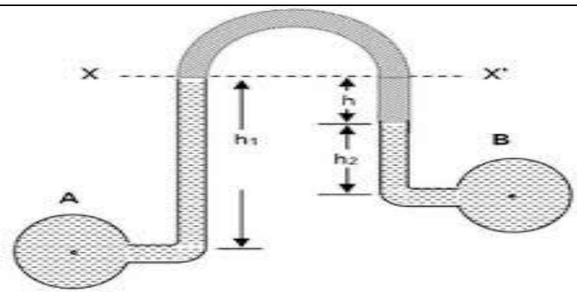
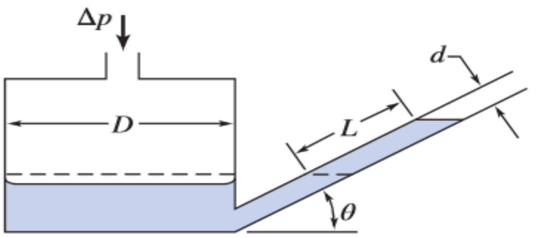
Going down	$+P$
Going up	$-P$

PIEZOMETER: $P_{abs} = P_{atm} + \gamma h$
Limitations:
1. Not suitable for gases fluids.
2. Not suitable for high pressure measurement.
3. Not suitable for vacuum or suction pressure.



SIMPLE MANOMETER	
Advantages	**Disadvantages**
• Suitable for gas and slightly high pressure can be measure • Vacuum Pressure can be measure.	• Very high pressure can't measure.



DIFFERENTIAL MANOMETER	INVERTED DIFFERENTIAL/ “U” TUBE MANOMETER
	
Aim: To measure difference in pressure between two point having relatively high pressure.	**Aim:** To measure relatively low pressure difference between two point.
$\boldsymbol{\Delta P = h_m(\gamma_m - \gamma_w)}$	$\boldsymbol{\Delta P = h_m(\gamma_w - \gamma_m)}$
$h_{Water\ Col} = h_m\left(\frac{\gamma_m}{\gamma_w} - 1\right) = h_m\left(\frac{S_m}{S_w} - 1\right)$	$h_{Water\ Col} = h_m\left(1 - \frac{\gamma_w}{\gamma_m}\right) = h_m\left(1 - \frac{S_w}{S_m}\right)$
	**INCLINED TUBE MANOMETER** Any manometer having at least one limb inclined w. r. t. the vertical plane. $L = h_m \sin\theta$ **Advantage:** It increase sensitivity of pressure measurement. **Sensitivity:** Ability of sense least amout of change. $Sensitivity = \frac{L}{h_m} = \frac{1}{\sin\theta}$

FORCES ON INCLINED SUBMERGED BODY IN THE FLUID:

$h = x\sin\theta$ | $dF = PdA = \gamma h dA = \gamma\, x \sin\theta\ dA$

$$dF = \gamma \sin\theta\, x\, dA$$

$\int x\, dA = \bar{x} A$	$\bar{x}\sin\theta = \bar{h}$

$$\boldsymbol{F = \gamma \bar{h} A = Avg.Pressure * Area}$$

$$dM_0 = \gamma \sin\theta\, x^2 dA$$

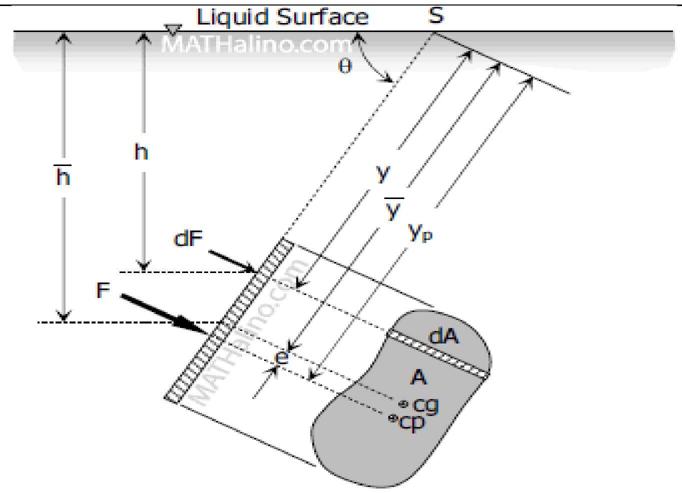
$\int x^2\, dA = I_{YO}$	$I_{YO} = I_{YG} + A\bar{x}^2$

$$M_0 = \gamma\sin\theta\, I_{YO} = \gamma\sin\theta\,(I_{YG} + A\bar{x}^2)$$

Here, $M_0 = F * x^* = \gamma\bar{h}\, A\, x^*$

$x^* = \frac{I_{YG}}{\bar{x}\, A} + \bar{x}$	$\boldsymbol{h^* = \frac{I_{YG}}{\bar{h}\, A}\sin^2\theta + \bar{h}}$

$$\boldsymbol{e = x^* - \bar{x} = \frac{I_{YG}}{\bar{h}\, A}\sin\theta}$$



When $e = 0$ (Centre of Pressure = Centre of Gravity),

1. $\sin\theta = 0 \Rightarrow$ Plate is in horizontal condition.
2. $\bar{h}\, A \gg I_{YG}\sin\theta \Rightarrow$ Plate is considered at very high depth.

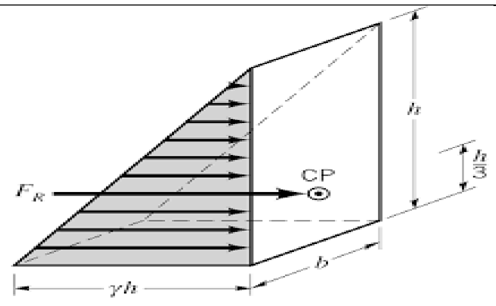
PRESSURE PRISM DIAGRAM:

1. Volume of pressure prism indicates net force.

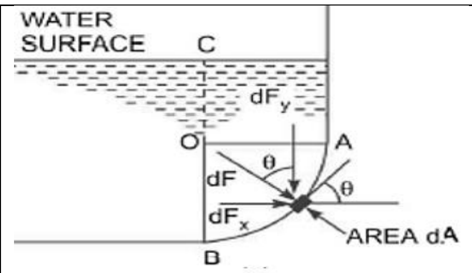
$$\forall_{Pre.\ Prism} = F$$

2. The projection of centroid of pressure prism on the surface of place represents centre of pressure.

Note: When area is not uniform to the page of plate avoid this method.



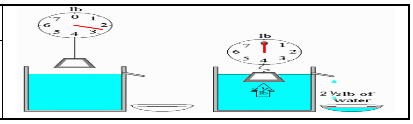
CURVED SURFACE	
HORIZONTAL FORCE	**VERTICAL FORCE**
$F_H = \gamma(\bar{h}\, A)_{Proj.}$	$F_V = \gamma(\forall)_{above}$
F_H is concentrated at the centre of pressure of projected area.	F_V is weight equivalent thrust of the fluid above curved surface.
$R = \sqrt{F_H^2 + F_V^2}$ $\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)$	$(h^*)_P = \frac{I_P}{(\bar{h}\, A)_P} + \bar{h}_P$



- Volume should be measured form the interface to the curved surface, regardless of the presence of fluid.
- F_V passes through the centroid of the volume considered.

BUOYANCY: Upward force exerted by fluid on a completely or partially submerged object in the fluid is buoyancy.
ARCHIMEDES PRINCIPLE: **“**Buoyance force is equal to the weight of the fluid displaced.**”**

$F_B = \rho \forall g = \gamma \forall$	F_B passes through centroid of the volume of fluid displaced.

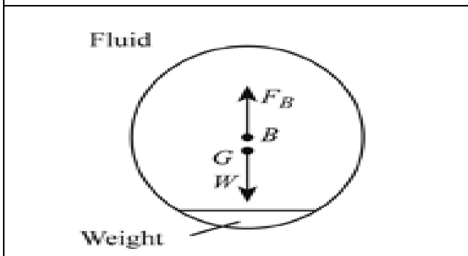
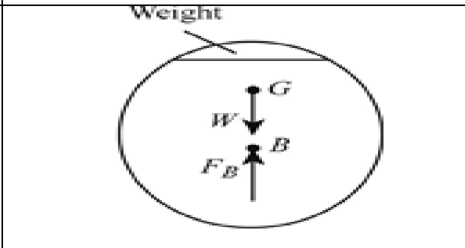
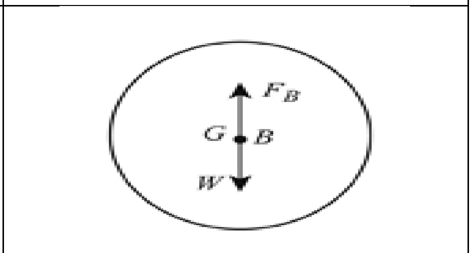
APPARENT WEIGHT		
$T = W - F_B$	$\frac{T_1}{T_2} = \frac{W - F_{B1}}{W - F_{B1}} = \frac{\gamma - \gamma_{f1}}{\gamma - \gamma_{f2}} = \frac{S - S_{f1}}{S - S_{f2}}$	

EQUILIBRIUM AND STABILITY:
EQUILIBRIUM: Condition in which the net unbalanced force & net unbalanced moments equal to zero.
STABILITY: Ability of a body to restore initial equilibrium after disturbance.

TYPES OF EQUILIBRIUM BASED ON STABILITY		
STABLE EQUILIBRIUM	**UNSTABLE EQUILIBRIUM**	**NEUTRAL EQUILIBRIUM**
It Gains original state	Further disturb itself	Finds new equilibrium state
E.g. $\gamma_b < \gamma_f (Floating\ body)$, $\gamma_b = \gamma_f (Neutrally\ buoyant\ body)$	E.g. $\gamma_b > \gamma_f (Accelerating\ body)$	

CONDITIONS REQUIRES FOR EQUILIBRIUM:

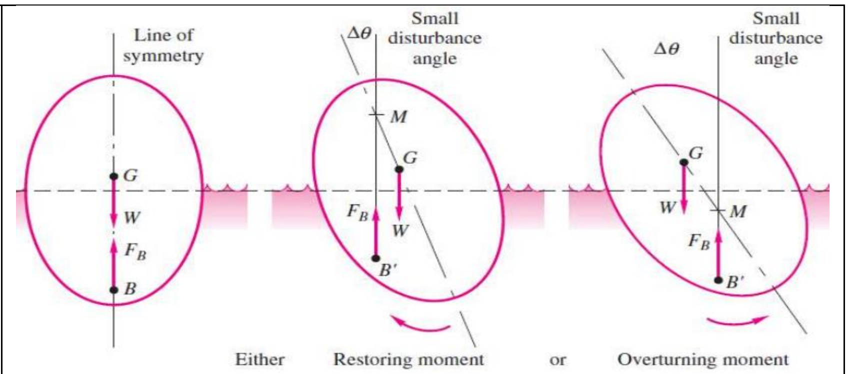
$F_B = W_{body}$	F_B & W_{body} should have same line of action

STABILITY OF NEUTRALLY BUOYANT BODY		
BOTTOM HEAVY BODY	**TOP HEAVY BODY**	**HOMOGENOUS BODY**
 (a) Stable	 (c) Unstable	 (b) Neutrally stable

STABILITY OF FLOATING BODY:
Here, M =Metacentre,
BM = Metacentric Radius,
GM = Metacentric Height.
If metacentric Height is positive equilibrium is stable equilibrium. B-G-M

$$GM = BM - BG$$

Stable & Neutral	Unstable
$BM \geq BG$	$BM < BG$

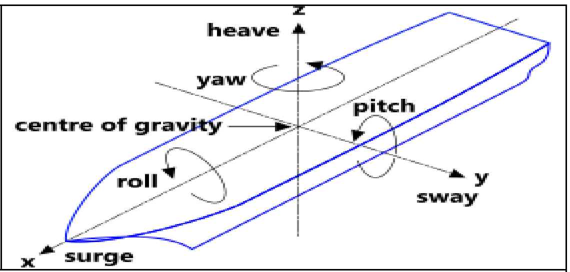


METACENTRE:
Point about which a floating body experiences simple harmonic oscillation when given small angle disturbs. $\Delta\theta \rightarrow 0$
METACENTRIC RADIUS:

$$BM = \frac{I}{\forall}$$

I = AMOI of Top view in the waterline about rolling axis,
$\forall$= Volume of fluid displaced,

$$GM = BM - BG = \frac{I}{\forall} - BG$$



a. Valid for only Simple harmonic Motion only & $\Delta\theta \rightarrow 0$
b. Metacentric Height Depends on geometry of ship, density of fluid and ship.
c. Metacentric Height of particular body in a particular fluid is a constant.

TIME PERIODS OF ROLLING/ OSCILLATIONS:

$Disturbing\ moment = (Mass\ MOI, I)(Acceleration, \alpha)$	$Restoring\ moment = W * GM \sin\theta$

$$I\alpha = W * GM \sin\theta \Rightarrow m\,k^2\alpha = mg\ GM\ \theta \Rightarrow \alpha = -(g\ GM/k^2)\ \theta \Rightarrow \alpha = \omega^2\theta \Rightarrow \alpha \propto -Disp.(\theta) \Rightarrow SHM$$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{k^2}{g\ GM}}$	$T \propto GM^{-1/2} \Rightarrow GM \propto Stability$ $GM_{War\ Ship} > GM_{Passanger\ Ship}$

EXPERIMENT	$GM = \frac{P\ x}{W\tan\theta}(\because P \ll W)$	$GM = \frac{P\ x}{(W+P)\tan\theta}$	$Disturbing\ moment = Px\cos\theta$ $Restoring\ moment = W * GM \sin\theta$

RIGID BODY MOTION	
TRANSLATION	**ROTATION**
E.g. Below Mentioned Cases, etc…	E.g. Vortex Motion.

TRANSLATION RIGID BODY MOTION:

CASE-I: Vertical Translation

$\sum F_z = ma_z$	$F - W = ma_z \Rightarrow F = m(g + a_z) \Rightarrow \boldsymbol{F = mg_{eff}}$ $Here, \boldsymbol{P} = \rho\forall g_{eff}/A = \rho A h g_{eff}/A = \boldsymbol{\rho g_{eff} h}$	Vertical Translation in upward direction.

CASE-II: Vertical Translation

$\sum F_x = ma_x$	$F_1 - F_2 = \gamma A_1\overline{h_1} - \gamma A_2\overline{h_2} = [\gamma(H+h)^2 1 - \gamma(H+h)^2 1]/2$ $\&\ ma_x = \rho BHa_x$ $\therefore \boldsymbol{\theta = \tan^{-1}(a_x/g)} = \tan^{-1}(2h/B)$	$h = Height\ displaced$ $B = Width\ of\ container$ $H = Initial\ height\ of\ liq.$

CASE-III: Translation Along Slope

$\boldsymbol{\theta = \tan^{-1}\left(\frac{a_x}{a_z + g}\right)}$	$a_x = a\cos\alpha\ \ \&\ a_z = a\sin\alpha$ We can use same method like case II for derivation with γ_{eff}	Vertical upward Translation on slope of α angle