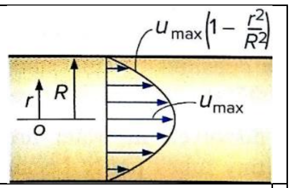
8. LAMINAR FLOW

LAMINAR FLOW: A flow in which layers (Laminas) of fluid slides over one another with relative velocities by the virtue of viscous forces.

CHARACTERISTICS OF LAMINAR FLOW:	**EXAMPLES OF LAMINAR FLOW:**
1. Re & KE of low is relatively lower. 2. No intermixing of fluid particles between the layers. 3. No random fluctuation of velocity of fluid particles with respect time. 4. Newtons law of viscosity is enough to calculate shear stress. 5. Flow is rotational. 6. Surface roughness of pipe does not affect losses in laminar flow.	1. Flow past dust particles settling of impurities. 2. Capillary flow in soil. 3. Flow of blood in veins. 4. Flow through pipe. 5. Flow between parallel plates. 6. Open channel flow: $Re \leq 500$ 7. Flow past sphere (stokes law): $Re \leq 1$

LAMINAR FLOW THROUGH PIPE: $Re \leq 2000$



1. Local Velocity:

Here, $y = R - r \Rightarrow dy = -dr$

$$\tau = \mu\frac{du}{dy} = \mu\left(\frac{-du}{dr}\right) = \left(\frac{-dP}{dx}\right)\frac{r}{2} \Rightarrow du = \frac{1}{2\mu}\left(\frac{dP}{dx}\right)rdr\ (From\ Previous\ Chp.)$$

$u = \frac{1}{4\mu}\left(\frac{dP}{dx}\right)r^2 + C$	$C = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)R^2 (\because BC\ @r = R, u = 0)$	$\boldsymbol{u = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)(R^2 - r^2)}$

This is equation of local velocity in laminar flow through pipes.

OBSERVATIONS:

1. $u = f(r)$.
2. Local Velocity decreases parabolically with respect to “r”.
3. Maximum Local Velocity is attaining at centreline. $r = 0$

$$\boldsymbol{u_{max} = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)R^2}$$

$$\boldsymbol{\therefore \frac{u}{u_{max}} = 1 - \frac{r^2}{R^2}}$$

2. Discharge:

$dQ = udA = u_{max}\left(1 - \frac{r^2}{R^2}\right)2\pi rdr$	$\boldsymbol{Q = \pi R^2\frac{u_{max}}{2}}$
Average velocity: $\boldsymbol{V_{avg.} = Q/A}$	$\boldsymbol{V_{avg.} = u_{max}/2}$

NOTE: At $u = V_{avg.}$ (Local Velocity = Avg. Velocity), $R = \sqrt{2}r$.

3. Pressure Drop Equation (Hagen-Poiseuille Equation):

$V_{avg.} = \frac{u_{max}}{2} = \frac{1}{8\mu}\left(\frac{-dP}{dx}\right)R^2 = \frac{1}{8\mu}\left(\frac{\Delta P}{L}\right)R^2$	$\boldsymbol{\because \frac{-dP}{dx} = \frac{\Delta P}{L}}$	$\Delta P = \frac{32\mu V_{avg.}L}{D^2} = \frac{128\mu QL}{\pi D^4}$

4. Head Loss:

$$\boldsymbol{h = \frac{\Delta P}{\gamma}} = \frac{32\mu V_{avg.}L}{\gamma D^2} = \frac{128\mu QL}{\gamma\pi D^4}$$

5. Equation of Power Loss:

$$P = h\gamma Q = \frac{128\mu Q^2L}{\pi D^4}$$

6. Fanning’s Friction coefficient(f'):

$f' = \frac{\tau_0}{(1/2)\rho V^2} = \frac{16}{Re}$	$\because \tau_0 = \left(\frac{-dP}{dx}\right)\frac{R}{2}$ & $V_{avg.} = \frac{1}{8\mu}\left(\frac{-dP}{dx}\right)R^2$

7. Darcy Friction Factor(f): $f = 4f' = 64/Re$

SUMMARY

1. Local Velocity	2. Avg. Velocity ($V_{avg.} = u_{max}/2$)	3. Discharge($Q = V_{avg.}A$)
4. Drop in Pressure ($-dP/dx = \Delta P/L$)	5. Head Loss ($h = \Delta P/\gamma$)	6. Power Loss ($P = h\gamma Q$)
7. $f' = 16/Re$	8. $f = 4f' = 64/Re$	

NOTE: If pipe is inclined, Replace Pressure with piezometric pressure. $P = P^*$& x is axis of flow.

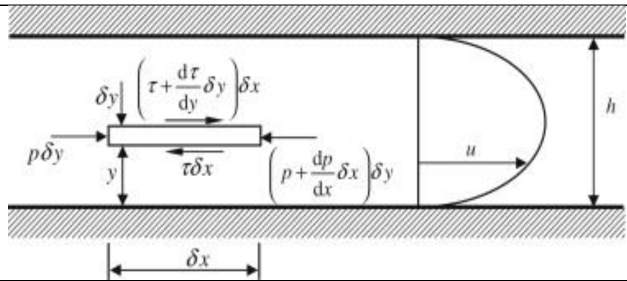
FLOW BETWEEN TWO FIXED PARALLEL PLATES: $Re \leq 1000$

From continuity equation, $V_1 = V_2$
From momentum conservation equation,

$$PdyW - (P + dP)dyW + (\tau + d\tau)dxW - \tau dxW = 0$$

$$\frac{dP}{dx} = \frac{d\tau}{dy} (Equation\ Governing\ The\ Flow)$$

$$\tau = \mu \frac{du}{dy} \Rightarrow \frac{d\tau}{dy} = \frac{dP}{dx} = \frac{d^2u}{dy^2}$$



$u = \frac{1}{2\mu}\left(\frac{dP}{dx}\right)y^2 + C_1 y + C_2$	$C_1 = \frac{1}{2\mu}\left(\frac{-dP}{dx}\right)B \begin{pmatrix} \because BC@\ y = B/2\,, \\ u = u_{max} \Rightarrow du/dy = 0 \end{pmatrix}$	$C_2 = 0$ $(\because BC\ @\ y = 0, u = 0)$

$\boldsymbol{u = \frac{1}{2\mu}\left(-\frac{dP}{dx}\right)(By - y^2)}(Parabolic)$	$\boldsymbol{u_{max} = \frac{1}{8\mu}\left(-\frac{dP}{dx}\right)B^2} \left(\because u = u_{max}\ @y = \frac{B}{2}\right)$

Discharge:

$dQ = udA = \frac{1}{2\mu}\left(-\frac{dP}{dx}\right)(By - y^2)dyW$	$Q = \frac{1}{12\mu}\left(-\frac{dP}{dx}\right)B^3W$
Average velocity: $\boldsymbol{V_{avg.} = Q/A}$	$\boldsymbol{V_{avg.} = (2/3)\ u_{max}}$

Pressure Drop:

$\Delta P = \frac{12\ \mu V_{avg.}L}{B^2} = \frac{12\ \mu QL}{B^3W} \begin{pmatrix} \because From\ V_{avg.}\ Formula \\ From\ Discharge\ Formula \end{pmatrix}$	$\because \boldsymbol{\frac{-dP}{dx} = \frac{\Delta P}{L}}$

Head Loss:	Power Loss:
$\boldsymbol{h = \frac{\Delta P}{\gamma}} = \frac{12\ \mu V_{avg.}L}{\gamma B^2} = \frac{12\ \mu QL}{\gamma B^3 W}$	$P = h\gamma Q = \frac{12\ \mu Q^2 L}{B^3 W}$

Shear Stress:	Wall Shear Stress: $\tau = \tau_0 @\ y = 0$
$\tau = \mu \frac{du}{dy} = \left(-\frac{dP}{dx}\right)\left(\frac{B - 2y}{2}\right)(Linear\ Profile)$	$\tau_0 = \left(-\frac{dP}{dx}\right)\left(\frac{B}{2}\right)$

NOTE: Here, y is distance measured from wall not centre.

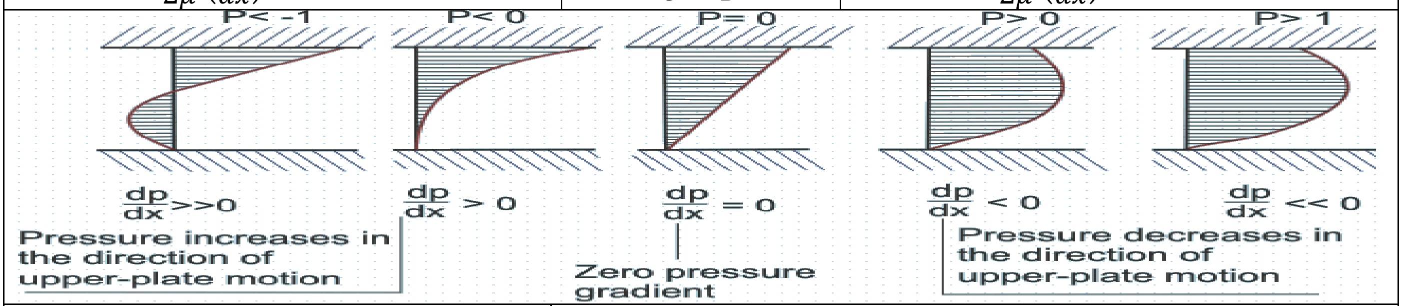
SUMMARY:

1. Local Velocity	2. Avg. Velocity $[V_{avg.} = (2/3)\ u_{max}]$	3. Discharge$(Q = V_{avg.}A)$
4. Drop in Pressure $(-dP/dx = \Delta P/L)$	5. Head Loss $(h = \Delta P/\gamma)$	6. Power Loss $(P = h\gamma Q)$
7. Shear Stress		

NOTE: If Plates are inclined, Replace Pressure with piezometric pressure. $P = P^*$ & x is axis of flow.

COUETTE FLOW: Laminar Flow Between two parallel plates such that one plate is moving relative to the other.

TYPES OF COUETTE FLOW		
$\frac{dP}{dx} > 0(Parabolic)$	$\frac{dP}{dx} = 0(Linear)$	$\frac{dP}{dx} < 0(Parabolic)$
Backward Couette Flow: Laminar Flow is in the direction opposite to the moving Plate.	Simple/ Pure/ Plane Couette Flow	Forward Couette Flow: Laminar Flow is in the direction of the moving Plate.
$u = \frac{1}{2\mu}\left(\frac{dP}{dx}\right)y^2 + C_1 y + C_2$	$\frac{u}{U} = \frac{y}{B}$	$u = \frac{1}{2\mu}\left(\frac{dP}{dx}\right)y^2 + C_1 y + C_2$



$V_{CF} = V_{Simple\ Flow} + V_{Flow\ Btw.2 fixed\ plate}$	$Q_{CF} = Q_{Simple\ Flow} + Q_{Flow\ Btw.2 fixed\ plate}$	
$\boldsymbol{V_{CF} = U\left(\frac{y}{B}\right) + \frac{1}{2\mu}\left(-\frac{dP}{dx}\right)(By - y^2)}$	$\boldsymbol{Q_{CF} = \frac{1}{2}UBW + \frac{1}{12\mu}\left(-\frac{dP}{dx}\right)B^3W}$	$\boldsymbol{V_{avg.} = \frac{Q}{A} = \frac{Q}{WB}}$
$\tau_{CF} = \tau_{Simple\ Flow} + \tau_{Flow\ Btw.2 fixed\ plate}$	$\boldsymbol{\tau_{CF} = \mu \frac{dV_{CF}}{dy} = \mu\left(\frac{U}{B}\right) + \left(-\frac{dP}{dx}\right)\left(\frac{B - 2y}{2}\right)}$	

$$\boldsymbol{u_{max} = u|_{\frac{du}{dy}=0}}$$