2. CONDUCTION

GENERALISED ONE DIMENSIONAL HEAT CONDUCTION:

$\boldsymbol{Q_{in} - Q_{out} + Q_{gen} = Q_{stored}}$	$\boldsymbol{\frac{\partial}{\partial x}\left[KA\frac{\partial T}{\partial x}\right]dx + q_g A\,dx = \rho A dx\, C\frac{\partial T}{\partial t}}$
Assumptions: Material is homogenous and isotropic (K is same in all directions). K is constant (Independent of the Temperature).	$\frac{1}{A}\frac{\partial}{\partial x}\left[A\frac{\partial T}{\partial x}\right] + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$ Where, $\alpha =$ Thermal Diffusivity.

CARTESIAN COORDINATE: Prismatic Square bar with heat transfer only along longitudinal axis.

Assumptions: K and A are constant for the component.	$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}\,(For\ 1D)$	Fourier Biot Eq.: $\nabla^2 T + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$
Steady State, K & A constant , Poisson's Eq.: $\nabla^2 T + \frac{q_g}{K} = 0$	K & A constant, No q_g Diffusion Eq.: $\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	Steady State, K & A constant, No q_g Laplace Eq.: $\nabla^2 T = 0$

CYLINDRICAL COORDINATE: Heat transfer in Prismatic Circular bar.

1. Heat Flows Axially: $T = f(x,t)$ & $A = (\pi/4)d^2 \neq f(x)$ $\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	2. Heat Flows Radially: $T = f(r,t)$ & $A = 2\pi r L = f(r)$ $\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

SPHERICAL COORDINATE: Heat transfer in Sphere.

$T = f(r,t)$ & $A = 4\pi r^2 = f(r)$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right] + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

COMPACT HEAT CONDUCTION EQUATION: $\frac{1}{r^n}\frac{\partial}{\partial r}\left[r^n\frac{\partial T}{\partial r}\right] + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	**Cartesian:** $\boldsymbol{r = x, n = 0}$ **Cylindrical:** $\boldsymbol{r = r, n = 1}$ **Spherical:** $\boldsymbol{r = r, n = 2}$

CONCEPT OF THERMAL RESISTANCE: Valid in Steady State Only With $q_g = 0$.

Electrical Conductor	Thermal Conductor	
Voltage (ΔV)	Temperature (ΔT)	
Current (I)	Heat (Q)	
$R_{ele} = \Delta V/I$	$R_{th}(in\ K/W) = \Delta T/Q$	
	$R_{Cond} = L/KA$	$R_{Conv} = 1/\bar{h}A_s$

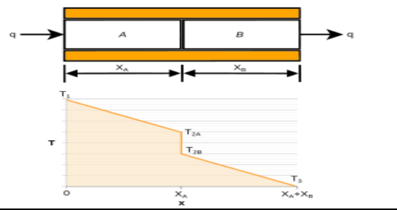
$$We\ can\ judge\ parameter\ Values\ using: R_{th} \propto \Delta T/Q$$

$For\ Series\ Connction: R_{eq} = R_1 + R_2 + \cdots$	$For\ Parallel\ Connction: R_{eq}^{-1} = R_1^{-1} + R_2^{-1} + \cdots$

CONCEPT OF THERMAL CONTACT RESISTANCE: It depends on,
1. Size of roughness
2. Type of fluid inside a cavity
3. Intensity of compressive force applied at the end surface during joining

$R_{fluid} = L_{avg}/K_{fluid}A_{fluid}$ & $R_{Spot} = L_{avg}/K_{least}A_{Spot}$

$$R_{eq} = R_1 + R_2 + R_{contact} = R_1 + R_2 + R_{fluid} + R_{Spot}$$



TEMPERATURE DISTRIBUTION AND HEAT CONDUCTION EQUATION:

ASSUMPTIONS:
1. 1D heat transfer (Radial in case of cylinder)
2. Steady state
3. No internal heat generation
4. Material is homogenous and isotropic
5. Thermal conductivity value is constant
6. Surfaces are isothermal.

PLANE WALL (INFINITE PLANE WALL):

$\frac{\partial^2 T}{\partial x^2} = 0$	$\frac{\partial T}{\partial x} = C_1 = \frac{Constant}{Slop}$	$T = C_1 x + C_2$ $= Linear\ Profile$	$T = \left(\frac{T_2 - T_1}{L}\right)x + T_1\left[\because \begin{matrix} Using\ BC\ x = 0, T = T_1 \\ x = L, T = T_2 \end{matrix}\right]$

Same formula we can find Variable separable method in Fourier's Law of conduction.

HOLLOW CYLINDER: $r_2 > r_1$

$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] = 0$	$\frac{\partial T}{\partial r} = \frac{C_1}{r}$	$T = C_1 \ln r + C_2$
$Using\ BC\ r = r_1, T = T_1$ $r = r_2, T = T_2$	$\frac{T - T_1}{T_2 - T_1} = \frac{\ln(r/r_1)}{\ln(r_2/r_1)} = Log\ Profile$	$R_{th} = \frac{\ln(r_2/r_1)}{2\pi KL}$

CONCEPT OF MEAN AREA: It's area of plane wall when Hollow cylinder is analysed as plane wall.

$Q_{Hollow\ Cyl.} = Q_{Plane\ Wall}$	$-2\pi KL\dfrac{(T_2 - T_1)}{\ln(r_2/r_1)} = -KA_m\dfrac{(T_2 - T_1)}{r_2 - r_1}$	$r_m = \dfrac{r_2 - r_1}{\ln(r_2/r_1)}$	$A_m = \dfrac{A_2 - A_1}{\ln(A_2/A_1)}$
$A_m = 2\pi r_m L =$ Logarithmic Mean Area.		$r_m =$ Logarithmic Mean Radius	

SOLID CYLINDER: Consider Only Axial Flow in the solid cylinder.

Note: Either Pipes/Cylinder or Plane Wall connected in the series connected use Electric Analogy for solution.

EFFECT OF INTERCHANGING LAYER ON HEAT TRANSFER: In order to reduce heat loss from pipe, 1st layer of insulation should have low thermal conductivity than 2nd layer of insulation.

HOLLOW SPHERE: $r_2 > r_1$

$\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left[r^2\dfrac{\partial T}{\partial r}\right] = 0$	$\dfrac{\partial T}{\partial r} = \dfrac{C_1}{r^2}$	$T = -\dfrac{C_1}{r} + C_2$
$Using\ BC\ r = r_1, T = T_1$ $r = r_2, T = T_2$	$\dfrac{T - T_1}{T_2 - T_1} = \dfrac{1/r - 1/r_1}{1/r_2 - 1/r_1} = Hyperbolic\ Profile$	$R_{th} = \dfrac{1}{4\pi K}\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right]$

CONCEPT OF MEAN AREA: It's area of plane wall when Hollow cylinder is analysed as plane wall.

$Q_{Hollow\ Sph.} = Q_{Plane\ Wall}$	$-4\pi K r_1 r_2\dfrac{(T_2 - T_1)}{r_2 - r_1} = -KA_m\dfrac{(T_2 - T_1)}{r_2 - r_1}$	$r_m = \sqrt{r_1 r_2}$	$A_m = \sqrt{A_1 A_2}$
$A_m = 2\pi r_m L =$ Geometric Mean Area		$r_m =$ Geometric Mean Radius	

CONE WITH CIRCULAR CROSS SECTION:

$D = ax$	$A = (\pi/4)a^2x^2$	$\dfrac{T - T_1}{T_2 - T_1} = \dfrac{1/x - 1/x_1}{1/x_2 - 1/x_1} = \begin{matrix} Hyperbolic \\ Profile \end{matrix}$	$R_{th} = \dfrac{4}{\pi K a^2}\left[\dfrac{1}{x_1} - \dfrac{1}{x_2}\right]$

CRITICAL OR OPTIMUM THICKNESS OF INSULATION:

ASSUMPTIONS:

1. One dimensional heat flow (Radial Flow for cylinder).
2. Steady state.
3. No internal heat generation in insulating material.
4. Insulating material is homogenous and isotropic.
5. K value for insulating material is constant.
6. Assume perfect contact between base and insulation.
7. Heat transfer coefficient value is constant.
8. Radiation loss is neglected.
9. Thermal resistance of base material is neglected.
10. ΔT is fixed.

HOLLOW CYLINDER OR HOLLOW SPHERE WITH INSIDE INSULATION: No need of to check for critical insulation thickness because Total thermal resistance always increases.

HOLLOW CYLINDER OR HOLLOW SPHERE WITH OUTSIDE INSULATION:

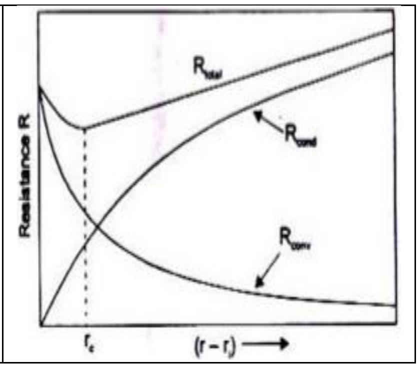
R_{ins} Increases with increasing the thickness of insulation.
R_{conv} Decreases with increasing the thickness of insulation.
For Cylinder: $r_{cr} = K_{ins}/h_o$

$R_{th} = R_{ins} + R_{conv} = \dfrac{\ln(r/r_1)}{2\pi K_{ins} L} + \dfrac{1}{2\pi r L h_o}$	$\dfrac{dR_{th}}{dr} = 0$	$\dfrac{d^2R_{th}}{dr^2} > 0$

For Sphere: $r_{cr} = 2K_{ins}/h_o$

$R_{th} = R_{ins} + R_{conv} = \dfrac{1}{4\pi K}\left[\dfrac{1}{r_1} - \dfrac{1}{r}\right] + \dfrac{1}{2\pi r L h_o}$	$\dfrac{dR_{th}}{dr} = 0$	$\dfrac{d^2R_{th}}{dr^2} > 0$

At critical radius, maximum heat loss occurs. It's used in the insulting the electric wires for easy removal of heat as well as electric isolation.



- If $r < r_{cr}$, then due to insulation, heat loss increases & reach to maximum value then start decreasing.
- If $r > r_{cr}$, then due to insulation, heat loss always decreases.
- Condition for maximum heat flow, $r = r_{cr}$

CONDUCTION WITH INTERNAL HEAT GENERATION:

ASSUMPTIONS

1. 1D Heat Flow (Radial flow for cylinder).
2. Steady state.
3. Material is homogeneous and isotropic.
4. Thermal conductivity value is constant.
5. Uniform internal heat generation.
6. Radiation heat loss neglected.

HEAT GENERATION IN PLANE WALL:

$\boldsymbol{\dfrac{d^2T}{dx^2} = -\dfrac{q_g}{K}}$	$\boldsymbol{\dfrac{dT}{dx} = -\dfrac{q_g x}{K} + C_1}$	$\boldsymbol{T = -\dfrac{q_g x^2}{2K} + C_1 x + C_2 = Parabolic}$

CASE-I: BOTH THE SURFACES OF PLANE WALL MAINTAINED AT DIFFERENT TEMPERATURE

$Using\ BC\ x = 0, T = T_1$ $x = L, T = T_2\ \&\ T_2 > T_1$	$T = -\dfrac{q_g x^2}{2K} + \left[\dfrac{T_2 - T_1}{L} + \dfrac{q_g L}{2K}\right]x + T_1$	At Max. temp $\frac{dT}{dx} = 0, x_{max} = (K/q_g)C_1$ At $x_{max}, T = T_{max}$

CASE-II: BOTH THE SURFACES OF PLANE WALL MAINTAINED AT SAME TEMPERATURE

$Using\ BC\ x = 0, T = T_1$ $x = L, T = T_2\ \&\ T_2 = T_1$	$T = -\frac{q_g x^2}{2K} + \left[\frac{q_g L}{2K}\right] x + T_1 = Parabolic$	At Max. temp $\frac{dT}{dx} = 0, x_{max} = L/2$ At $x_{max}, T = T_{max} = q_g L^2/8K + T_1$
$\frac{T - T_1}{T_{max} - T_1} = -4\left(\frac{x^2}{L^2}\right) + \frac{4x}{L}$	Energy Balance, $Q_{gen} = Q_{out}$ $T_1 - T_\infty = q_g L/2h$	$T_{max} = \frac{q_g L^2}{8K} + T_\infty + \frac{q_g L}{2h}$

CASE-III: ONE OF THE SURFACES OF PLANE WALL IS INSULATED

At insulation end $T_1 = T_{max}$	$Using\ BC\ x = 0, dT/dx = 0$ $x = L, T = T_2$	$T = \frac{q_g}{2K}[L^2 - x^2] + T_2$	$T_{max} = \frac{q_g L^2}{2K} + T_2$
$\frac{T - T_2}{T_{max} - T_2} = 1 - \left(\frac{x}{L}\right)^2$	Energy Balance, $Q_{gen} = Q_{out}$ $T_2 - T_\infty = q_g L/h$	$T_{max} = \frac{q_g L^2}{2K} + \frac{q_g L}{h} + T_\infty$	

HEAT GENERATION IN CYLINDERS:

$\boldsymbol{\frac{1}{r}\frac{d}{dr}\left[r\frac{dT}{\partial r}\right] = -\frac{q_g}{K}}$	$\boldsymbol{r\frac{dT}{dr} = -\frac{q_g r^2}{2K} + C_1}$	$\boldsymbol{T = -\frac{q_g r^2}{4K} + C_1 \ln r + C_2}$

CASE-IV: HEAT GENERATION IN SOLID CYLINDER

$Using\ BC\ r = 0, dT/dr = 0$ $r = R, T = T_2$	$T = \frac{q_g}{4K}[R^2 - r^2] + T_2$	$T_{max} = \frac{q_g R^2}{4K} + T_2$
$\frac{T - T_2}{T_{max} - T_2} = 1 - \left(\frac{r}{R}\right)^2$	Energy Balance, $Q_{gen} = Q_{out}$ $T_2 - T_\infty = q_g R/2h$	$T_{max} = \frac{q_g R^2}{4K} + \frac{q_g R}{2h} + T_\infty$

HEAT GENERATION IN SPHERE:

$\boldsymbol{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right] = -\frac{q_g}{K}}$	$\boldsymbol{r^2\frac{dT}{dr} = -\frac{q_g}{3K}r^3 + C_1}$	$\boldsymbol{T = -\frac{q_g}{6K}r^2 - C_1\frac{1}{r} + C_2}$

CASE-V: HEAT GENERATION IN SOLID SPHERE:

$Using\ BC\ r = 0, dT/dr = 0$ $r = R, T = T_2$	$T = \frac{q_g}{6K}(R^2 - r^2) + T_2$	$T_{max} = \frac{q_g}{6K}R^2 + T_2$
$\frac{T - T_2}{T_{max} - T_2} = 1 - \left(\frac{r}{R}\right)^2$	Energy Balance, $Q_{gen} = Q_{out}$ $T_2 - T_\infty = q_g R/3h$	$T_{max} = \frac{q_g}{6K}R^2 + \frac{q_g}{3h}R + T_\infty$

CASE-VI: HEAT GENERATION IN HOLLOW CYLINDER:

$Using\ BC\ r = r_1, T = T_1$ $r = r_2, T = T_2$	$T_1 = -\frac{q_g r_1^2}{4K} + C_1 \ln r_1 + C_2$	$T_2 = -\frac{q_g r_2^2}{4K} + C_1 \ln r_2 + C_2$
$T_1 - T_2 = -\frac{q_g}{4K}[r_1^2 - r_2^2] + C_1 \ln\left(\frac{r_1}{r_2}\right)$	$C_2 = T_1 + \frac{q_g r_1^2}{4K} - C_1 \ln r_1$	$T - T_1 = -\frac{q_g}{4K}[r_1^2 - r^2] + C_1 \ln\left(\frac{r}{r_1}\right)$
At Max. temp $\frac{dT}{dr} = 0, r_{max}^2 = 2KC_1/q_g$		

CASE-VII: HEAT GENERATION IN HOLLOW CYLINDER WITH INSIDE SURFACE INSULATED:

$Using\ BC\ r = r_1, dT/dr = 0$ $r = r_2, T = T_2$	$C_1 = \frac{q_g r_1^2}{2K}$	$T_2 = -\frac{q_g r_2^2}{4K} + C_1 \ln r_2 + C_2$
$T = T_2 + \frac{q_g}{4K}[r_2^2 - r^2] + \frac{q_g r_1^2}{2K}\ln\left(\frac{r}{r_2}\right)$	$at, r_1, T_1 = T_{max} = T_2 + \frac{q_g}{4K}[r_2^2 - r_1^2] + \frac{q_g r_1^2}{2K}\ln\left(\frac{r_1}{r_2}\right)$	
Energy Balance, $Q_{gen} = Q_{out}$ $T_2 - T_\infty = \left[q_g(r_2^2 - r_1^2)\right]/[2r_2 h]$		

CASE-VIII: HEAT GENERATION IN HOLLOW CYLINDER WITH OUTER SURFACE INSULATED:

$Using\ BC\ r = r_2, dT/dr = 0$ $r = r_1, T = T_1$	$C_1 = \frac{q_g r_2^2}{2K}$	$T_1 = -\frac{q_g r_1^2}{4K} + C_1 \ln r_1 + C_2$
$T = T_1 + \frac{q_g}{4K}[r_1^2 - r^2] + \frac{q_g r_2^2}{2K}\ln\left(\frac{r}{r_1}\right)$	$at, r_2, T_2 = T_{max} = T_1 + \frac{q_g}{4K}[r_1^2 - r_2^2] + \frac{q_g r_2^2}{2K}\ln\left(\frac{r_2}{r_1}\right)$	
Energy Balance, $Q_{gen} = Q_{out}$ $T_2 - T_\infty = \left[q_g(r_2^2 - r_1^2)\right]/[2r_1 h]$		

CASE-IX: HEAT GENERATION IN HOLLOW SPHERE WITH INNER SURFACE INSULATED:

$Using\ BC\ r = r_1, dT/dr = 0$ $r = r_2, T = T_2$	$C_1 = \frac{q_g r_1^3}{3K}$	$T_2 = -\frac{q_g}{6K}r_2^2 - C_1\frac{1}{r_2} + C_2$
$T = T_2 + \frac{q_g}{6K}[r_2^2 - r^2] + \frac{q_g r_1^3}{3K}\left[\frac{1}{r_2} - \frac{1}{r}\right]$	$at, r_1, T_1 = T_{max} = T_2 + \frac{q_g}{6K}[r_2^2 - r_1^2] + \frac{q_g r_1^3}{3K}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$	
Energy Balance, $Q_{gen} = Q_{out}$ $T_2 - T_\infty = \left[q_g(r_2^3 - r_1^3)\right]/[3r_2^2 h]$		

VARIABLE THERMAL CONDUCTIVITY:

For homogenous and isotropic material K varies linearly with the temperature.

$K_{avg} = \frac{\int_{T_1}^{T_2} KdT}{T_2 - T_1}$	$K = K_0[1 + \beta T]$ K_0 = The thermal conductivity at 0 $°C$ $(+ve\ Const.)$ β = Temp. Coefficient of Thermal Conductivity	For linear Profile, $K_{avg} = K_0[1 + \beta T_{avg}]$

$\beta = 0$	$\beta > 0$	$\beta < 0$
$K = Const.$	For, Non-Metal, Gases.	For, Metal, Liquids.

TEMPERATURE DISTRIBUTION AND HEAT CONDUCTION EQUATION:

ASSUMPTION:

1. 1D Heat Flow (Radial Flow for cylinder)
2. Steady State
3. No internal heat generation
4. Material is Homogenous and isotropic.
5. Thermal conductivity value varies linearly with T.
6. Surfaces are Isothermal.

PLANE WALL (INFINITE PLANE WALL):

$\int_0^x \frac{Q}{A} dx = -\int_0^T K_0[1 + \beta T_{avg}]dT$	$Q = -\frac{AK_0(T_2 - T_1)}{L}[1 + \beta T_{avg}]$	$\frac{T - T_1}{T_2 - T_1} = \frac{x\,[1 + \beta T_{avg}]}{L[1 + \beta\,(T + T_1)/2]}$

FOR HOLLOW CYLINDER	**FOR HOLLOW SPHERE**
$\frac{T - T_1}{T_2 - T_1} = \frac{\ln(r/r_1)\ [1 + \beta T_{avg}]}{\ln(r_2/r_1)\,[1 + \beta\,(T + T_1)/2]}$	$\frac{T - T_1}{T_2 - T_1} = \left[\frac{1/r - 1/r_1}{1/r_2 - 1/r_1}\right]\frac{1 + \beta T_{avg}}{1 + \beta\,(T + T_1)/2}$

$\lvert Q/A\rvert = \lvert q\rvert = \lvert K\,dT/dx\rvert$	β	K	dT/dx
	0	$Const.$	$Const.$
	$+ve$	$Increases$	$Decreases$
	$-ve$	$Decreases$	$Increases$

NOTE:

1. Always Check with the equation.
2. It's Valid for the Constant Area Geometry. Not for Sphere.