9. THERMODYNAMICS RELATIONS

MATHEMATICAL THEOREMS	
$dz = Mdx + Ndy$ *is exact differential equation,* $\left[\frac{\partial M}{\partial y}\right]_x = \left[\frac{\partial N}{\partial x}\right]_y$	*If* $z = f(x, y)$, $\left[\frac{\partial x}{\partial y}\right]_z \cdot \left[\frac{\partial y}{\partial z}\right]_x \cdot \left[\frac{\partial z}{\partial x}\right]_y = -1$

PROPERTIES			
GIBB'S FUNCTION		**HELMHOLTZ FUNCTION**	
Open System Availability Function.		Closed System Availability Function.	
$G = H - TS$	$g = h - Ts$	$F = U - TS$	$f = u - Ts$
COEFFICIENT OF VOLUME EXPANSIVITY		**ISOTHERMAL COMPRESSIBILITY**	
$\beta = \frac{1}{v}\left[\frac{\partial v}{\partial T}\right]_P > 0(Due\ to\ Expansion)$		$K_T = \frac{1}{v}\left[\frac{\partial v}{\partial P}\right]_T < 0(Due\ to\ Compression)$	
$\boldsymbol{C_P = T\left[\frac{\partial s}{\partial T}\right]_P}$		$\boldsymbol{C_V = T\left[\frac{\partial s}{\partial T}\right]_V}$	

MAXWELL'S EQUATIONS	
Derived from 1st TdS Equation, $du = Tds - Pdv\ (It's\ exact\ differential)$ $\therefore \left[\frac{\partial T}{\partial v}\right]_s = \left[\frac{-\partial P}{\partial s}\right]_v$	**Derived from 2nd TdS Equation,** $dh = Tds + vdP\ (It's\ exact\ differential)$ $\therefore \left[\frac{\partial T}{\partial P}\right]_s = \left[\frac{-\partial v}{\partial s}\right]_P$
Derived from $\boldsymbol{g = h - Ts}$ $\Rightarrow dg = dh - d(Ts) = vdP - sdT$ $\therefore \left[\frac{\partial v}{\partial T}\right]_P = \left[\frac{-\partial s}{\partial P}\right]_T$	**Derived from** $\boldsymbol{f = u - Ts}$ $\Rightarrow df = du - d(Ts) = -Pdv - sdT$ $\therefore \left[\frac{\partial P}{\partial T}\right]_v = \left[\frac{\partial s}{\partial v}\right]_T$

$s = f(T,v) \Rightarrow Tds = T\left[\frac{\partial s}{\partial T}\right]_v dT + T\left[\frac{\partial s}{\partial v}\right]_T dv = C_V dT + T\left[\frac{\partial P}{\partial T}\right]_v dv \cdots (1)$ $s = f(T,P) \Rightarrow Tds = T\left[\frac{\partial s}{\partial T}\right]_P dT + T\left[\frac{\partial s}{\partial P}\right]_T dP = C_P dT - T\left[\frac{\partial v}{\partial T}\right]_P dP \cdots (2)$	From, Above mentioned C_P, C_V Equations, 3rd and 4th Maxwell's equation.
$dT = \frac{T}{C_P - C_V}\left\{\left[\frac{\partial v}{\partial T}\right]_P dP + \left[\frac{\partial P}{\partial T}\right]_v dv\right\} \cdots (3)$	Equating above (1) & (2),
$s = f(P,v) \Rightarrow s = \left[\frac{\partial T}{\partial P}\right]_v dP + \left[\frac{\partial T}{\partial v}\right]_P dv \cdots (4)$	From the maths theorem,
$\left[\frac{\partial T}{\partial P}\right]_v = \frac{T}{C_P - C_V}\left[\frac{\partial v}{\partial T}\right]_P \cdots (5)$	Equating above (3) & (4),
$T = f(P,v) \Rightarrow \left[\frac{\partial P}{\partial v}\right]_T \cdot \left[\frac{\partial v}{\partial T}\right]_P \cdot \left[\frac{\partial T}{\partial P}\right]_v = -1 \Rightarrow \left[\frac{\partial T}{\partial P}\right]_v = -1\Big/\left[\frac{\partial P}{\partial v}\right]_T \cdot \left[\frac{\partial v}{\partial T}\right]_P \cdots (6)$	From the maths theorem,
$C_P - C_V = -T\left[\frac{\partial v}{\partial T}\right]_P^2 \left[\frac{\partial P}{\partial v}\right]_T$	From (5) & (6)

MAYER'S EQUATION: $C_P - C_V = -T\left[\frac{\partial v}{\partial T}\right]_P^2 \left[\frac{\partial P}{\partial v}\right]_T = \frac{Tv\beta^2}{K_T}$	From definition of K_T & β.

INTERNAL ENERGY EQUATION: It's valid for Real & ideal Gas. $du = C_V dT + \left[T\left[\frac{\partial P}{\partial T}\right]_v - P\right] dv = \boldsymbol{f(dT, dv)}$	From Mayer's equation, equation (1) and 1st TdS Equation. **For ideal Gas,** $\boldsymbol{T\left[\frac{\partial P}{\partial T}\right]_v = P}$
ENTHALPY EQUATION: It's valid for Real & ideal Gas. $dh = C_P dT + \left[T\left[\frac{\partial v}{\partial T}\right]_P - v\right] dP = \boldsymbol{f(dT, dP)}$	From Mayer's equation, equation (2) and 2nd TdS Equation. **For ideal Gas,** $\boldsymbol{T\left[\frac{\partial v}{\partial T}\right]_P = v}$
JOULE-THOMSON COEFFICIENT EQUATION: It's valid for Real & ideal Gas. $\mu = \left[\frac{dT}{dP}\right]_h = \frac{1}{C_P}\left[T\left[\frac{\partial v}{\partial T}\right]_P - v\right]$	From Enthalpy equation, & for Throttling process $dh = 0$ **For ideal Gas,** $\boldsymbol{\mu = 0}$