2. LAPLACE TRANSFORMATION

LAPLACE TRANSFORMATION: It's used to transform time domain function to frequency domain function.

$Given\ x(t), \forall t \geq 0$ $\mathcal{L}$ = Laplace Operator	$\mathcal{L}\{x(t)\} = \int_0^\infty e^{-st}x(t)dt = X(s)$	$x(t)$ = Objective Function, $X(s)$ = Image Function (Frequency Domain Fun.),

$\mathcal{L}\{x(t)\} = X(s)$	$\mathcal{L}^{-1}\{X(s)\} = x(t)$	$\mathcal{L}\{x(t)\} = X(s)$	$\mathcal{L}^{-1}\{X(s)\} = x(t)$
$\mathcal{L}\{1\} = \frac{1}{s}$	$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$	**"n" is Integer**	**"n" is Fraction**
$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$	$\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$	$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$	$\mathcal{L}\{t^n\} = \frac{\sqrt{n+1}}{s^{n+1}}$
$\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$	$\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$	$\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$	$\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{\sqrt{n+1}}$
$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$	$\mathcal{L}^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{\sin at}{a}$	$\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$	$\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$
$\mathcal{L}\{\sinh at\} = \frac{a}{s^2-a^2}$	$\mathcal{L}^{-1}\left\{\frac{1}{s^2-a^2}\right\} = \frac{\sinh at}{a}$	$\mathcal{L}\{\cosh at\} = \frac{s}{s^2-a^2}$	$\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at$

GAMMA FUNCTION: $\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx = (n-1)!\ , for\ n > 0$

$\sqrt{n+1} = n\sqrt{n}$	$\Gamma(1/2) = \sqrt{\pi}$	$\Gamma(-1/2) = -2\sqrt{\pi}$

FIRST SHIFT PROPERTY: If $\mathcal{L}\{x(t)\} = X(s)$, then	$\mathcal{L}\{e^{-at}x(t)\} = X(s+a)$ $\mathcal{L}\{e^{-at}x(t)\} = X(s-a)$	$\mathcal{L}^{-1}\{X(s+a)\} = e^{-at}x(t)$ $\mathcal{L}^{-1}\{X(s-a)\} = e^{at}x(t)$

THEOREM-I: Multiplication by "t"	If $\mathcal{L}\{x(t)\} = X(s)$, then	$\mathcal{L}\{t^n\,x(t)\} = (-1)^n\frac{d^n}{ds^n}X(s)$	$\mathcal{L}^{-1}\left\{\frac{d^n}{ds^n}X(s)\right\} = (-1)^n t^n\,x(t)$

EVALUATION OF DEFINITE INTEGRAL USING L.T. TECHNIQUE:

$At\ s = 0$	$\mathcal{L}\{x(t)\} = \int_0^\infty x(t)dt = X(0) =\ Integration\ of\ x(t)\ from\ 0\ to\ \infty$

THEOREM-II: Division by "t"	If $\mathcal{L}\{x(t)\} = X(s)$, then	$\mathcal{L}\left\{\frac{x(t)}{t}\right\} = \int_s^\infty X(s)\,ds$	$\mathcal{L}^{-1}\left\{\int_s^\infty X(s)\,ds\right\} = \frac{x(t)}{t}$

THEOREM-III: If $\mathcal{L}\{x(t)\} = X(s)$, then	$\mathcal{L}\{x'(t)\} = sX(s) - x(0)$
$\mathcal{L}\{x''(t)\} = s^2X(s) - sx(0) - x'(0)$	$\mathcal{L}\{x'''(t)\} = s^3X(s) - s^2x(0) - sx'(0) - x''(0)$

THEOREM-IV:	If $\mathcal{L}\{x(t)\} = X(s)$, then	$\mathcal{L}\left\{\int_0^t x(t)\,dt\right\} = \frac{X(s)}{s}$	$\mathcal{L}^{-1}\left\{\frac{X(s)}{s}\right\} = \int_0^t x(t)\,dt$

UNIT STEP FUNCTION: $U(t) = \begin{cases} 1; t > 0 \\ 0; t < 0 \\ 1/2\,; t = 0 \end{cases}$		**SHIFTED UNIT STEP FUNCTION:** $U(t-a) = \begin{cases} 0; t > a \\ 1; t < a \\ 1/2\,; t = a \end{cases}$	
THEOREM-A		**THEOREM-B**	
$\mathcal{L}\{U(t)\} = \frac{1}{s}$	$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = U(t)$	$\mathcal{L}\{U(t-a)\} = \frac{e^{-as}}{s}$	$\mathcal{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} = U(t-a)$
$x(t-a)\,U(t-a) = \begin{cases} x(t-a); t > a \\ 0; t < a \\ x(0)/2\,; t = a \end{cases}$		**THEOREM-C** $\mathcal{L}\{x(t-a)\,U(t-a)\} = e^{-as}\,X(s) = e^{-as}\,\mathcal{L}\{x(t)\}$ $\mathcal{L}^{-1}\{e^{-as}X(s)\} = x(t-a)\,U(t-a)$	
$x(t)\,U(t-a) = \begin{cases} x(t); t > a \\ 0; t < a \\ x(a)/2\,; t = a \end{cases}$		**THEOREM-D** $\mathcal{L}\{x(t)\,U(t-a)\} = e^{-as}\,\mathcal{L}\{x(t+a)\}$	

LAPLACE OF SEMI PERIODIC FUNCTION: If $x(t), \forall t \geq 0\ \&\ x(t-T) = x(t)$, then	$\mathcal{L}\{x(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}\,x(t)\,dt$

CONVOLUTION THEOREM: If $\mathcal{L}^{-1}\{X_1(s)\} = x_1(t)$ & $\mathcal{L}^{-1}\{X_2(s)\} = x_2(t)$, then	$\mathcal{L}^{-1}\{X_1(s)X_2(s)\} = \int_0^t x_1(u)\, x_2(t-u)\, du$

INVERSE LAPLACE TRANSFORMATION USING PARTIAL FRACTIONS:

FORM-I: **Non-Repeated Linear Factor**	$\frac{p(s)}{q(s)} = \frac{p(s)}{(s-a)(s-b)} = \frac{A}{(s-a)} + \frac{B}{(s-b)}$
FORM-II: **Repeated Linear Factor**	$\frac{p(s)}{q(s)} = \frac{p(s)}{(s-a)(s-b)^2} = \frac{A}{(s-a)} + \frac{B}{(s-b)} + \frac{C}{(s-b)^2}$
FORM-III: **Quadratic Factor**	$\frac{p(s)}{q(s)} = \frac{p(s)}{(s^2+a^2)(s-b)} = \frac{A}{(s-b)} + \frac{Bs+C}{(s^2+a^2)}$

$$Where, Degree\big(p(s)\big) < Degree(q(s))$$