PROBABILITY

SAMPLE SPACE: The set of all possible outcomes of an experiment is known as sample space. It's denoted by S.
EVENT: Any subset of the sample set is known as event.

COUNTABLY INFINITE SET:	**UN-COUNTABLY INFINITE SET:**	**UN-COUNTABLY FINITE SET:**
E.g. $S = \{x \mid x = 0,1,2,3,\ldots\}$	**E.g.** $S = \{x \mid x \geq 0\}$	**E.g.** $S = \{x \mid 0 \leq x \leq 5\}$

PROBABILITY: If the sample space S of an experiment consist of finitely many outcomes that are equally likely, then the probability of an event A is given by $p(A)$	$p(A) = \frac{No.of\ Values\ in\ A}{No.of\ Values\ in\ S} = \frac{n(A)}{n(S)}$

GENERAL DEFINITION OF PROBABILITY: Given a sample space S, with each event A of S, there is a number $p(A)$ called the probability of A, such that the following axioms of probability are satisfied,

$0 \leq p(A) \leq 1$	$p(S) = 1$	Exclusive Events: $A \cap B = \emptyset$ hence, $p(A \cup B) = p(A) + p(B)$

COMPLEMENT RULE: For an event A and it's Compliment A^C in a sample space S, $p(A^C) = 1 - p(A)$ $p(A \cap A^C) = \emptyset$ | $p(A \cup A^C) = S$	**ADDITIONAL RULES FOR MUTUALLY EXCLUSIVE EVENTS:** n mutually exclusive events A_n $p(A_0 \cup A_1 \cup A_2 \cup \ldots) = p(A_0) + p(A_1) + p(A_2) + \cdots$ $A_i \cap A_j = \emptyset, Where\ i \neq j$

ADDITIONAL RULE FOR ARBITRARY EVENTS: For events A and B in the sample space S,

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$	$p(A \cup B \cup C) = p(A) + p(B) - p(A \cap B) - p(A \cap B) - p(A \cap B) + p(A \cap B \cap C)$

ADDITIONAL RULE FOR "n" ARBITRARY EVENTS:

$$p(A_0 \cup A_1 \cup A_2 \cup \ldots) = \sum_{i=1}^{n} p(A_i) - \sum_{i1<i2} p(A_{i1} \cap A_{i2}) + \sum_{i1<i2<i3} p(A_{i1} \cap A_{i2} \cap A_{i3}) \ldots + (-1)^{n+1} p(A_0 \cap A_1 \cap A_2 \cap \ldots)$$

CONDITIONAL PROBABILITY: $p(A \mid B) = \frac{p(A \cap B)}{p(B)}\ And\ p(B \mid A) = \frac{p(A \cap B)}{p(A)}$	**MULTIPLICATION THEOREM:** $p(A \cap B) = p(A \mid B)\, p(B) = p(B \mid A)\, p(A)$
MUTUALLY EXCLUSIVE EVENTS	**INDEPENDENT EVENTS**
Mutually exclusive events only have significance if we consider one particular performance of one particular experiment.	Independent Events can only be considered in multiple performances of the same experiment or different experiment or different experiments together.
$p(A \mid B) = p(B \mid A) = 0\ [\because\ p(A \cap B) = 0\ ]$	$p(A \mid B) = p(A)\ And\ p(B \mid A) = p(B)$
$p(A \cup B) = p(A) + p(B)[Can\ be\ used\ for\ n\ events]$	$p(A \cap B) = p(A)\, p(B)[Can\ be\ used\ for\ n\ events]$

Bay's Rule: $p(A) = \sum_{j=1}^{n} p\left(A \middle| B_j\right) p(B_j)$ **Here,** $B_i \cap B_j = \emptyset, Where\ i \neq j$ [Mutually Exclusive]	$p(B_i \mid A) = \frac{p(A \mid B_i)\, p(B_i)}{\sum_{j=1}^{n} p\left(A \middle| B_j\right) p(B_j)}$

SAMPLING: Randomly drawing objects from a given set of objects.	
With Replacement	**Without Replacement**
The object that was drawn at random is placed back to the given set and the set is mixed thoroughly.	The object that was drawn is put aside.

CARD CLASSIFICATIONS				**FACE CARDS**
52 Cards	26 Cards	13 Heart Suits	A, 2, 3, …., 10, K, Q, J	K, Q, J
		13 Diamond Suits	A, 2, 3, …., 10, K, Q, J	
	26 Cards	13 Spades Suits	A, 2, 3, …., 10, K, Q, J	
		13 Clubs Suits	A, 2, 3, …., 10, K, Q, J	

RANDOM VARIABLE: Random Variable is a function whose domain is a sample space and whose range is same set of real numbers.

PROBABILITY MASS FUNCTION: $PMF = f(x) = p(X = x)$ $\sum p(x_i) = 1$ And $0 \leq f(x) \leq 1, \forall x$	X	= Random Variable
	p(X)	= Probability of R. V.

Cumulative Distribution Function $CDF = F(x) = p(X \leq x) = \sum_{x_i \leq x} p(X = x_i) = \sum_{x_i \leq x} f(x_i)$

CONTINUOUS RANDOM VARIABLE (CRV): A random variable x and it's distribution are of continuous type if it's cumulative distribution (CDF) F(x) is given by,

$F(x) = \sum_{x \le x_i} f(x) = \sum_{x \le x_i} p(X = x) = \int_{-\infty}^{x} f(v)\, dv$	Where, f = Probability density function (PDF) of x Relation Between CDF & PDF: $f(x) = \frac{d}{dx} F(x)$

PROPERTIES OF CRV:

1. **NON-NEGATIVITY:** The PDF $f(x)$ is a Non-Negative function. $f(x) \ge 0$
2. **NORMALIZATION:** The total area under the graph of the PDF is equal to Unity. $\int_{-\infty}^{\infty} f(x)\, dx = 1$

EXPECTED VALUE OF RANDOM VARIABLE		
DISCRETE RANDOM VARIABLE		**CONTINUOUS RANDOM VARIABLE**
$\mathrm{E[X]} = \sum_{\forall x} x\, p(X = x)$		$\mathrm{E[X]} = \int_{-\infty}^{\infty} x\, f(x)\, dx$
PROPERTIES		$\mathrm{E}[g(x)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$
$\mathrm{E}[a\mathrm{X}] = a\mathrm{E[X]}$	$\mathrm{E}[a\mathrm{X} + \mathrm{b}] = a\mathrm{E[X]} + b$	
$\mathrm{E}\big[\mathrm{E[X]}\big] = \mathrm{E[X]}$	$\mathrm{E}[K] = \mathrm{K}$	If $g(x) = x^n$, $\mathrm{E}[g(x)] = \mathrm{n}^{th}$ moment

VARIANCE OR SECOND CENTRAL MOMENT OF A RANDOM VARIABLE:

$$\mathrm{E}[(\mathrm{X} - \mathrm{E[X]})^2] = \mathrm{E}[\mathrm{X}^2] - (\mathrm{E[X]})^2 = Variance = [Std.Deviation\ (\sigma)\ ]^2$$

PROPERTIES	$Var\ (x) \ge 0$
$Var(ax) = a^2\, Var(x)$	$Var(K) = 0$
$Var(ax + b) = a^2\, Var(x)$	$Var(x \pm y) = Var(x) + Var(y) \pm Cov(x, y)$
$Cov(x, y) = \mathrm{E}[xy] - \mathrm{E}[x]\ \mathrm{E}[y]$	

For independent Random variables,

$Cov(x, y) = 0$	$Var(x \pm y) = Var(x) + Var(y)$

FUNCTION	$\boldsymbol{f(x)}$	**E[X]**	$\mathbf{E}\big[\boldsymbol{X^2}\big]$	$\boldsymbol{Var(x)}$
Uniform Random Variable	$= \frac{1}{b-a} (For\ a \le x \le b)$ = 0 (Otherwise)	$\frac{a+b}{2}$	$\frac{a^2 + b^2 + ab}{3}$	$\frac{(b-a)^2}{12}$
Exponential Random Variable	$= \lambda\, e^{-\lambda x} (For\ x \ge 0)$ = 0 (Otherwise)	$\frac{1}{\lambda}$	$\frac{2}{\lambda^2}$	$\frac{1}{\lambda^2}$
Poisson Random Variable (Parameter λ)	$= \frac{\lambda^x\, e^{-\lambda}}{x!} (x = 0,1,2, \dots)$ = 0 (Otherwise)	λ	$\lambda(\lambda + 1)$	λ
Normal or Gaussian Distribution	$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} (For - \infty \le x \le \infty)$	μ	-	σ^2
Std. Normal or Gaussian Distribution ($\sigma^2 = 1, \mu = 0$)	$= \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} (For - \infty \le x \le \infty)$	0	-	± 1

For $\sigma^2 = 1$, Area =0.6827	For $\sigma^2 = 2$, Area = 0.9545	For $\sigma^2 = 3$, Area = 0.996

If x is Normal Random variable with mean μ and variance σ^2 then $Z = (x - \mu)/\sigma$ is also Normal random variable with $\mu = 0$ and $\sigma^2 = 1$.

Question Can be twist in the form of integral problem.

BERNOULLI RANDOM VARIABLE	**BINOMIAL RANDOM VARIABLE**	
Suppose that a trail or an experiment whose outcome can be classified as either success or failure is performed. If we let X= 1, when the outcomes is success and X= 0, when outcome is failure, the probability mass function of X is given by, $p(X = 1) = p, p(X = 0) = 1 - p,$ Where $0 \le p \le 1$	Suppose now that "n" independent trails each if which results in a success with probability p and in a failure with probability $1 - p$ are to be performed. If X represents the number of success that occurrence in the "n" trails then X is said to Binomial Random variable with parameters (n, p). It's Probability mass function given by, $p(X = x) = {}^{n}C_x p^x (1-p)^{n-x}, Where\ x = 0,1,2, \dots, n$	
It's Special case of binomial random variable with x = 0,1	$\mathrm{E[X]} = np$	$Var(x) = np(1-p)$
${}_{n}P_r = (n!) / (n-r)!$	${}^{n}C_x = {}_{n}P_r / r!$	

NORMAL APPROXIMATION TO BINOMIAL RANDOM VARIABLE:

When "n" is large, a binomial random variable with parameters "n" and "p" will have approximately the same distribution as a normal random variable with the same mean and variance as the binomial. If X denotes the number of success that occur when "n" independent trails each resulting in a success with probability "p" are performed then for any $a < b$,

$p\left(a \le \frac{X-np}{\sqrt{np(1-p)}} \le b\right) = \int_a^b \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$	$p\left(a \le \frac{X-\mu}{\sigma} \le b\right) = p(a \le X \le b)$ Here, Mean $\mu = np$ And Variance $\sigma^2 = np(1-p)$

POISSON APPROXIMATION TO BINOMIAL RANDOM VARIABLE:

When "n" is large and "p" is small then binomial distribution is very closely approximated by Poisson distribution, Poisson distribution is a limiting case of binomial distribution as $n \longrightarrow \infty$ and $p \longrightarrow 0$.

$p(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} = {}^nC_x p^x (1-p)^{n-x}$	Here, λ is mean of Poisson Random variable. Hence, $\lambda = np$

Exponential Random Variable (Parameter λ)	Poisson Random Variable
In practice, It's often arises as the distribution of the amount of time until some specific event occurs. a. amount of time until an earthquake occurs. b. amount of time until a new war breaks out. c. amount of time until a telephone call you receive turns out to be a wrong number.	a. The number of misprints on a page of book. b. The number of people in a community who survive to age 100. c. The number of Wrong telephone numbers that are dialled in a day. d. The number of customers entering in a day in office.

- If X and Y are independent Poisson R.V. with respective parameters $\lambda_1$and λ_2, then X+Y has a Poisson Distribution with parameter $\lambda_1 + \lambda_2$.
- If X and Y are independent Binomial R.V. with respective parameters (n, p) and (m, p), then X+Y has a Poisson Distribution with parameter $(n + m, p)$.
- If X and Y are independent Normal R.V. with respective parameters (μ_1, σ_1^2) and (μ_2, σ_2^2)then X+Y has a Poisson Distribution with parameter $(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

SIGNIFICANCE VARIANCE OR SECOND CENTRAL MOMENT OF A RANDOM VARIABLE:

It's just description about how closely data is scattered about it's mean value.

STATISTICS

MEAN	MEDIAN	MODE
$\bar{X} = \left(\sum x_i\right)/n$	It's the middle value of the sample, if sample points are arranged in ascending order. If the number of elements in the sample is even, then the arrange of the two middle values can be taken as median.	The sample point that occurs with highest frequency.
Let X be a discrete random variable having the possible values x_n, If $p(X = x_i) = f(x_i)$ E[X] $= \sum x_i\ f(x_i) = \int_{-\infty}^{\infty} x\, f(x)\, dx$	The median of probability distribution is the point at which the distribution function has the value of 0.5. $\boldsymbol{F(x) = p(X \le x) = 0.5 = p(X \ge x)}$ In case of a continuous distribution, the median corresponds to a point "x" which separates the density curve into two parts having equal areas.	For a discrete random variable, mode is the value x at which it's probability mass function takes maximum value. For continuous random variable, mode is the x at which it's probability density function has maximum value. So any peak is a mode.

$Std.Deviation\ \sigma = \sqrt{\left[\sum (x_i - \bar{X})^2\right]/n}$	Mode = 3 Median – 2 Mean