4. FLUID KINEMATICS

FLUID KINEMATICS: Study of the fluid flow without considering the forces causing the flow.

APPROACHES	
LAGRANGIAN APPROACH	**EULERIAN APPROACH**
Partial Oriented Approach	Position Oriented Approach
$\vec{S} = f(\vec{S_0}, t)$ $Where, \vec{S}$ = Identity of Partial, $\vec{S_0}$ = Initial Identity of Partial	$\vec{S} = f(\vec{P}, t)$ $Where, \vec{S}$ = Identity of Partial, $\vec{P}$ = Position Vector or Space Coordinates
Conservations Laws are applied to partials	Conservations Laws are applied to Control volume
It's system approach	It's Control volume approach
	Most Commonly used due to simplicity of analysis.

VELOCITY: Time rate of change of displacement. It's vector quantity.

Velocity in Cartesian Co-Ordinate System(x, y, z), $\vec{V} = u\hat{\imath} + v\hat{\jmath} + w\hat{k} = f(x, y, z, t)$ $u = \frac{dx}{dt} = f_1(x, y, z, t)$ $v = \frac{dy}{dt} = f_2(x, y, z, t)$ $w = \frac{dz}{dt} = f_3(x, y, z, t)$	Velocity in Cylindrical Polar Co-Ordinate System (r, θ, z), $\vec{V} = V_r\hat{\imath} + V_\theta\hat{\jmath} + V_z\hat{k} = f(r, \theta, z, t)$ $V_r = \frac{dr}{dt} = f_1(r, \theta, z, t)$ $V_\theta = r\frac{d\theta}{dt} = f_2(r, \theta, z, t)$ $V_z = \frac{dz}{dt} = f_3(r, \theta, z, t)$

CLASSIFICATION OF FLOW	
STEADY FLOW	**UNIFORM FLOW**
It's Flow in which velocity and other hydrodynamic parameters don't fluctuate **w. r. t. time**. $\vec{V} \neq f(t)$	It's Flow in which velocity doesn't fluctuate or change **w. r. t. Space**. $\vec{V} \neq f(x, y, z)$
VISCOUS FLOW	**1D, 2D, 3D**
It's Flow in which Newtons Law is valid. It's non uniform flow. Because $u = f(y)(du/dy\ \ term)$	It's Flow in which Flow is function of No. of Space Parameters.

MASS FLOW RATE ($\dot{m}$): Amount of mass crossing a section per unit time.

$$\dot{m} = \rho AV, Where\ V = Avg. Velocity\ of\ flow\ through\ the\ cross\ section$$

VOLUME FLOW RATE/ DISCHARGE (Q): Amount of volume crossing a section per unit time.

$$Q = AV = \dot{m}/\rho, Where\ V = Avg. Velocity\ of\ flow\ through\ the\ cross\ section$$

CONTINUITY EQUATION: The result of law of conservation of mass in a fluid flow is continuity equation.

Continuity equation in cartesian co-ordinate system: $\boldsymbol{\nabla \cdot (\rho \vec{V}) + \dot{\rho} = 0}$

For Steady flow in cartesian co-ordinate system: $\nabla \cdot (\rho \vec{V}) = 0$	For Incompressible $\nabla \cdot \vec{V} = 0$

CONSERVATION OF MASS: $\dot{m}_{in} - \dot{m}_{out} = \dot{m}_{gen}$

$$\dot{m}_{in} = \dot{m}_{out} \Rightarrow (\rho AV)_{in} = (\rho AV)_{out}\ (\because Neither\ mass\ generation\ nor\ mass\ consumtion)$$

ACCELERATION: Time rate of change of velocity.

Velocity in Cartesian Co-Ordinate System(x, y, z), $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} = g(x, y, z, t)$ $a_x = \frac{du}{dt} = g_1(x, y, z, t)$ $a_y = \frac{dv}{dt} = g_2(x, y, z, t)$ $a_z = \frac{dw}{dt} = g_3(x, y, z, t)$	Velocity in Cylindrical Polar Co-Ordinate System (r, θ, z), $\vec{V} = a_r\hat{\imath} + a_\theta\hat{\jmath} + a_z\hat{k} = g(r, \theta, z, t)$ $a_r = \frac{dV_r}{dt} = g_1(r, \theta, z, t)$ $a_\theta = r\frac{dV_\theta}{dt} = g_2(r, \theta, z, t)$ $a_z = \frac{dV_z}{dt} = g_3(r, \theta, z, t)$

$$du = (\delta u)_x + (\delta u)_y + (\delta u)_z + (\delta u)_t = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz + \frac{\partial u}{\partial t}dt \Rightarrow \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}\frac{dt}{dt}$$

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$	$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$	$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$

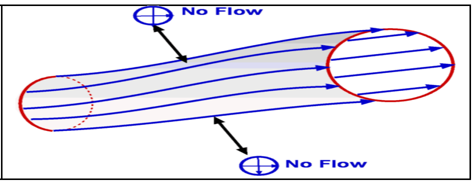
MATERIAL DERIVATIVE: $\boldsymbol{DM/Dt = (\vec{V} \cdot \nabla)\, M + \dot{M}}$

Local/ Temporal Change Terms: $\dot{M}$	Convective Change Terms: $(\vec{V} \cdot \nabla)\, M$

$a_r = V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_z \frac{\partial V_r}{\partial z} + \frac{\partial V_r}{\partial t} - \frac{V_\theta^2}{r}(Centri.)$	$a_\theta = V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_z \frac{\partial V_\theta}{\partial z} + \frac{\partial V_\theta}{\partial t} - \frac{V_\theta V_r}{r}(Coriolli.)$	$a_z = V_r \frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z} + \frac{\partial V_z}{\partial t}$

STREAM LINE: A set of imaginary curves drawn in a flow field at given instant of time such that tangent at any point represent the direction of velocity vector for the same position.

STREAM TUBE: A bundle of stream line forming a passage through which flow can be visualized.



PROPERTIES OF STREAMLINES AND STREAM TUBES:

1. The streamlines cannot intersect each other nor can a streamline interest itself.
2. Flow is possible only along the stream line. It's impossible across stream line.
3. In Steady flow streamlines don't fluctuate w. r. t. time.

EQUATION OF STREAM LINES: $\vec{V} \times \vec{dS} = 0$ (*Because Both are in same direction*)	$\therefore \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$
Slope of stream line = Slope of velocity vector	

ACCELERATIONS W.R.T. STREAM LINES: There are two mutually perpendicular accelerations developed,

1. Tangential Acceleration (a_s) 2. Normal Acceleration (a_n)	$a = \sqrt{a_s^2 + a_n^2}$	$a_s = \frac{\partial V_s}{\partial t} + V_s \frac{\partial V_s}{\partial s}$	$a_n = \frac{\partial V_n}{\partial t} + V_s \frac{V_s^2}{r}$

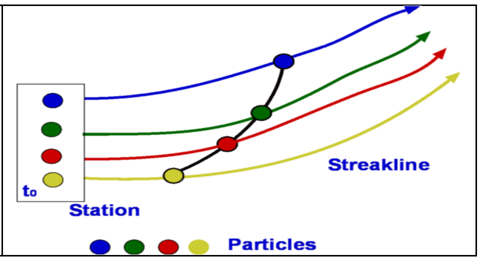
PATH LINE: The path travelled by a particular particle (Lagrangian Approach) in a flow field over a period of time.

PROPERTIES OF PATH LINES:

1. The Path lines can be intersected each other and it can interest itself but only in unsteady flow.
2. In Steady flow Path lines don't fluctuate w. r. t. time and *Path lines = Stream lines*.

STREAK LINES: It's instantaneous line obtained by joining position of different fluid particles which have sequentially passed thorough same point in the flow. E.g. Die injected in the flow.

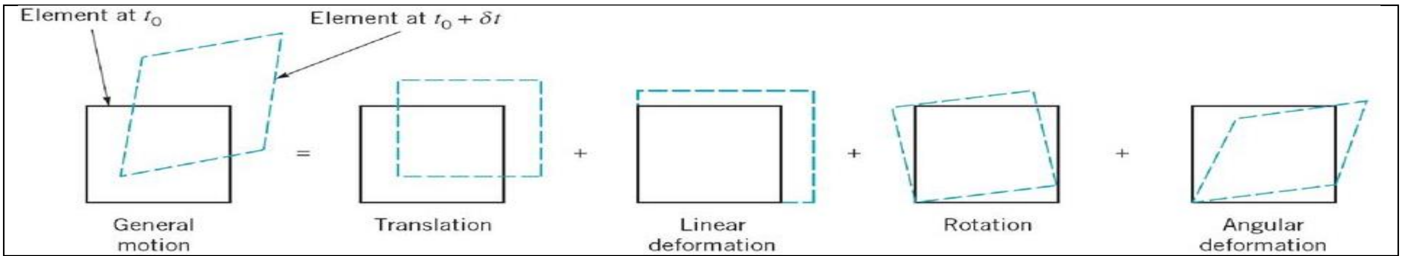
1. A Family of streak lines drawn in the flow field is known as **Rake of Streak lines.**
2. In Steady Flow Streak lines don't fluctuate w. r. t. time and *Streak lines = Stream lines = Path lines (For Stready Flow)*



TIMELINE: A set of curves drawn in a flow field which represent the positions of a set of neighbouring particles at which various instance of times. E.g. hydrogen bubbles forming velocity profiles.

- Timelines help in understanding the uniformity or non-uniformity of a flow.

DEFORMATIONS & ROTATIONS:



Linear Deformation	Angular Deformation	
$\dot{\varepsilon}_k = \frac{\partial V_k}{\partial k}$	$Avg.Rate: \frac{\dot{\gamma}_{mn}}{2} = \frac{1}{2}\left(\frac{\partial V_n}{\partial m} + \frac{\partial V_m}{\partial n}\right)$	$Tot.Rate: \dot{\varepsilon}_{mn} = \dot{\gamma}_{mn}$

Rotational Deformation		
$\dot{\theta} = \frac{\partial v}{\partial x}$	$\dot{\phi} = -\frac{\partial u}{\partial y}$	$\omega_z = \frac{1}{2}(\dot{\theta} + \dot{\phi})$
Shear Stress		
$\tau_{mn} = \mu \dot{\varepsilon}_{mn}$		

Rotation

It's the arethematic mean of angular velocities of two mutually perpendicular line segments in a fluid element. It's vector quatity and given by half of curl of velocity vector.

$$\vec{\omega} = (1/2)(\nabla \times \vec{V})$$

VORTICITY ($\vec{\xi}$) : It's curl of velocity Vector. $\vec{\xi}\,(in\ rad/s) = 2\,\vec{\omega} = \nabla \times \vec{V}$

CIRCULATION (Γ): The line integral of tangential component of velocity taken across a closed contour is known as circulation.	$\mathbf{\Gamma} = \oint \vec{V} \cdot d\vec{S} = \iint \vec{\xi} \cdot d\vec{A}$

NOTE: Intensity of circulation per unit area is known as vorticity.

CLASSIFICATION OF FLOW BASED ON ROTATION	
ROTATIONAL FLOW	**IRROTATIONAL FLOW**
A Flow in which Fluid Element rotates about it's mass centre.	A Flow in which Fluid Element doesn't rotates about it's mass centre.
$\vec{\xi} \neq 0$	$\vec{\xi} = \Gamma = \vec{\omega} = 0$

Flux P of Q is defined as $P = \iint \vec{Q} \cdot d\vec{A}$

STREAM FUNCTION (ψ):

A function defined in a 2D Flow field such that it takes a constant value along a particular stream line.

$For\ Given\ Stream\ Line, d\psi = 0\ \&\ u = -\frac{\partial\psi}{\partial y}, v = \frac{\partial\psi}{\partial x}$	$\lvert\Delta\psi\rvert = q = Q/width$	**Note:** Sign is not an issue.

1. The difference in stream function gives the discharge per unit width between corresponding stream lines.
2. Discharge per unit width across any section between two given stream lines is a constant.

POTENTIAL FUNCTION (∅): $\vec{\xi} = \nabla \times \vec{V} = \vec{0}\,(Irrotational\ Flow) \Rightarrow \vec{V} = \nabla \cdot \emptyset\ (From\ the\ Maths\ Basics)$

Irrotational flow is also called as potential flow (From the above equation).

$\emptyset = f(x, y, z)$	$u = -\frac{\partial\emptyset}{\partial x}, v = -\frac{\partial\emptyset}{\partial y}, z = -\frac{\partial\emptyset}{\partial z}$	**Note:** Negative Sign represents that the flow takes place in the direction of decrease in potential. Sign is not an issue.

EQUIPOTENTIAL LINES: A set of curves along which ∅ takes constant value. $d\emptyset = 0$

$d\emptyset = \frac{\partial\emptyset}{\partial x}dx + \frac{\partial\emptyset}{\partial y} + dy = u\,dx + v\,dy$	$\left(\frac{dy}{dx}\right)_{EPL}\left(\frac{dy}{dx}\right)_{StreamLine} = -1, \left(\frac{dy}{dx}\right)_{EPL} = -\frac{u}{v}$

- The stream lines and equipotential lines are orthogonal to each other except stagnation point.

FLOWNET: A grid formed by drawing stream lines and equipotential lines is known as flow net.

STAGNATION POINT: The Point in a flow field at which all the components of velocity drop down to zero.

- At the stagnation point velocity is zero. So, we can't define Slope of equipotential Lines and Slope of stream lines at stagnation point.

LAPLACE EQUATION FOR STREAM FUNCTIONS AND POTENTIAL FUNCTIONS:

$$\mathbf{\nabla^2\emptyset = 0}$$

Laplace equation (Above Equation) represents possible case of incompressible and irrotational flow.

$$\mathbf{\nabla^2\psi = 0}$$

Laplace equation (Above Equation) represents possible case of irrotational flow.