BALANCING OF ROTATING AND RECIPROCATING MASSES

BALANCING:

- The process of either removing or reducing the unbalanced force or couple from a system is known as balancing.
- Balancing can be done either by adding the counter masses or by removing the extra masses present in the system.

A. STATIC BALANCING:

- A system is said to be statically balanced if there is no unbalanced force in the system.
- The centre of mass will lie on the axis of rotation.
- The force polygon will be completely closed. $\sum F_{Any\ Direction} = 0$

B. DYNAMIC BALANCING:

- A system is said to be dynamically balanced if the force polygon as well as couple polygon both are closed.
- Since the force polygon is closed: $\sum F_{Any\ Direction} = 0$
- Since the couple polygon is closed: $\sum M_{Any\ Point} = 0$
- A system which is dynamically balanced will also be statically balanced whereas reverse is not true.

BALANCING OF ROTATING MASSES (SHAFT CARRYING A ROTOR):

$m =$ Mass of rotor	$r =$ Eccentricity of unbalanced mass	$F_{un} = mr\omega^2$

METHOD-1: ADDING THE BALANCING MASSES IN THE SAME PLANE:

a. Using Single counter mass: There will be no unbalanced force in the system due to Adding/ Removal of mass from opposite direction in Same plane. Hence, force polygon will be completely closed.

$m_1 =$ Counter Mass added/ Removed from rotor	r_1 =Radius of Counter Mass	From force balance, $mr = m_1 r_1$

b. Balancing by adding more than one counter mass (in Same plane): The system is statically balanced therefore,

$\sum F_{xi} = \sum_{i=1}^{n} m_i r_i \cos\theta_i = 0$	$\sum F_{yi} = \sum_{i=1}^{n} m_i r_i \sin\theta_i = 0$

Here, we can use sine rule or cosine rule according to the diagram/ Polygon generated.

METHOD-2: ADDING THE BALANCING MASSES IN THE PARALLEL PLANE:

a. By adding a single balancing mass: Here the 2 rotors will be baled in normal plane of axial direction of shaft. But there will be couple which creates additional reaction force on the support.

NOTE: The reaction which are equal in magnitude but opposite in direction are known as dynamic reactions. Total dynamic reaction in a system will be zero (Support reactions).

BALANCING OF RECIPROCATING MASSES:

ASSUMPTION: Inertia of crank, crank pin and connecting rod are negligible.

Let, ω is constant. And obliquity ratio, $n = l/r =$ Ratio of connecting rod length to the radius of the crank.

$a_{slider} = r\omega^2[\cos\theta + \cos 2\theta/n]$	$F_{un} = (F_i)_{slider} = m\, a_{slider}$

F_{un} is always along the line of stroke.

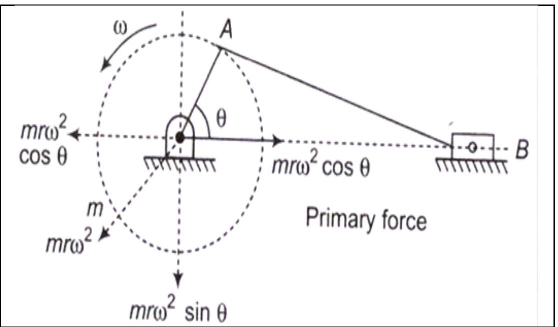
A. Primary inertia force $\left(\boldsymbol{mr\omega^2[\cos\theta]}\right)$: In a cycle primary inertia force will be maximum 2 times.

B. Secondary inertia force $\left(\boldsymbol{mr\omega^2[\cos 2\theta/n]}\right)$: In one cycle, it will be maximum 4 times. Hence, the frequency of the force is twice the frequency of primary inertia force. And the magnitude of the force is less than primary inertia force.

- Neglecting the secondary inertia force for analysis.
- Maximum force before balancing is $mr\omega^2$ acting along the line of stroke. After balancing Maximum force before balancing is $mr\omega^2$ acting along perpendicular to the line of stroke.
- This is why this method is known as **the complete balancing**.

PARTIAL BALANCING: In this method, the balancing mass is considered as a fraction of unbalanced slider mass to reduce unbalanced mass. Hence, Resultant unbalanced force,

$$R = mr\omega^2\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$$



Same way secondary inertia force can be balanced.

	Primary inertia force $\left(mr\omega^2[\cos\theta]\right)$	**Secondary inertia force** $\left(mr'\omega'^2[\cos\theta'/n]\right)$
Mass	m	m
Radius	r	$r/4n$
Angular Position	θ	2θ
Speed	ω	2ω

NOTE:

1. The Rotating unbalance force is always constant in magnitude but its direction continuously changes, whereas reciprocating unbalance force magnitude changes continuously but its always along the line of stoke.

DYNAMICALLY EQUIVALENT LINK FOR CONNECTING ROD:

- In slider crank mechanism the crank performs rotation and slider perform translation which are easy to analyse.
- The connecting rod which is in general plane motion accept the big end at crank pin in pure rotation and small end at wrist pin in pure translation, difficult to analyse.
- In order to do analysis of connecting rod, we considered its mass to be concentrated only at small end and big end respectively and these ends are connected with the massless rod.
- The two members are equivalent for dynamic analysis purpose if the following 3 conditions are satisfied:
    1. The location of centre of mass must be same.
    2. The total mass of the system must be same.
    3. The mass moment of inertia of actual and equivalent systems must be same with respect to the centre of mass.

Here, $l = l_s + l_b$ **Condition-1:** $m = m_s + m_b$ **Condition-2:** $m_s l_s = m_b l_b$ **Condition-3:** $I_{act} = I_{equivalnet}$ $\Rightarrow mk^2 = m_s l_s^2 + m_b l_b^2$		m = Actual Mass of connecting rod m_s = Mass of connecting rod at small end m_b = Mass of connecting rod at big end k = Radius of gyration of actual system about centre of mass. l_s = Length from CG to small end of rod
$m_s = m\dfrac{l_b}{l_b + l_s}$	$m_b = m\dfrac{l_s}{l_b + l_s}$	l_b = Length from CG to small big of rod Hence, Condition for Dynamically equivalent system, $k = \sqrt{l_s l_b}$

NOTE: Generally, 2/3rd mass of connecting rod is assumed on crack pin and 1/3rd on Gudgeon pin.

$$balancing\ mass \cdot Radius\ of\ crank = m_{Rot.}\, r + c\, m_{Reci.}\, r$$

EFFECT OF UNBALANCED INERTIA FORCE ON LOCOMOTIVES:

- Most of the locomotives have 2 cylinders of same dimension which are placed at right angle to each other in order to have uniform turning moment $(T - \theta)$ diagram. There are 4 types of locomotives:

1. Inside Cylinder Locomotives: The cylinders are placed inside the two wheels.
2. Outside Cylinder Locomotives: The cylinders are placed outside the two wheels.
3. Single or Uncoupled Locomotives: In this the power is transmitted to one pair of wheels.
4. Coupled Locomotives: In this the driving wheel are connected to the leading or trailing wheels with the help of an outside coupling rod.

NOTES:

1. In the locomotive the value of "n" is very large therefore we can neglect the Secondary Inertia Force.
2. In locomotive to balance the primary inertia force the balancing mass is kept on the wheels and partial balancing was done.

The partial balancing results in unbalanced force along the line of stroke (F_H). It results in variation in tractive force along the line of stroke and swaying couple about centre line.

Unbalanced force perpendicular to line of stroke (F_H) due to which there is variation in pressure on the rails which results in hammering actin which is known as "Hammer Blow".

TRACTIVE FORCE: It is the net force along the line of stroke.

$$(F_H)_{Net} = F_{H1} + F_{H2} = (1-c)\, mr\omega^2 \cos\theta + (1-c)\, mr\omega^2 \cos(90^\circ + \theta) = (1-c)\, mr\omega^2[\cos\theta - \sin\theta] = f(\theta)$$

SWAYING COUPLE: It is the net couple due to unbalanced force along the line of stroke.

$$M = F_{H1}\frac{a}{2}(CW) + F_{H2}\frac{-a}{2}(CCW) = (1-c)\, mr\omega^2[\cos\theta + \sin\theta]\frac{a}{2} = f(\theta)$$

TRACTIVE FORCE	**SWAYING COUPLE**
$(F_H)_{Net,Max@\theta=135^\circ} = -\sqrt{2}(1-c)\, mr\omega^2$	$M_{Max@\theta=45^\circ} = 2^{-0.5}(1-c)\, mr\omega^2 a$
$(F_H)_{Net,Max@\theta=315^\circ} = \sqrt{2}(1-c)\, mr\omega^2$	$M_{Max@\theta=135^\circ} = -2^{-0.5}(1-c)\, mr\omega^2 a$
$Variation\ in\ Tractive\ Force = \pm\sqrt{2}(1-c)\, mr\omega^2$	$Variation\ in\ Sw.Couple = \pm 2^{-0.5}(1-c)\, mr\omega^2 a$

HAMMER BLOW: Normal Reaction or Pressure between the wheel and track, $N = W \pm mr\omega^2 \sin\theta$

- Hammer Blow Puts the limit on maximum speed of locomotives. For proper working, $N > 0$

$$\omega = [W/mr\sin\theta]^{0.5}$$