5. CONVECTION HEAT TRANSFER

CONDUCTION	CONVECTION
No bulk motion of particles (Stationary Fluid).	Bulk motion in fluid particles.
	$Convection = Conduction\ (At\ bottom\ layer) + Advection\ (Fluid\ motion\ due\ to\ \Delta P)$

Boiling & Condensation are convection heat transfer with phase change due to vapour bubble movement.

Free Convection	Forced Convection	Internal Flow	External Flow
Flow due to $\Delta\rho$	Flow due to ΔP	Flow inside the body	Flow over the body

NEWTON'S LAW OF COOLING: The convective rate of heat transfer from a solid surface is directly proportional to temperature difference between solid surface and surrounding fluid and surface area. $Q_{Conv} \propto A_s \Delta T$

HEAT TRANSFER COEFFICIENT (h): h is a quantity of rate of transfer convected from a unit surface area for a unit temperature difference. It's not a property of fluid, it's a experimental determined parameter.

It depends on, 1. Thermophysical properties of fluid (K, C_P, ρ, μ). 2. Type of flow (Laminar or Turbulent).	3. Type of surface (Smooth or Rough). 4. Position of Surface. 5. Geometry of Surface.

$$h_{\substack{Boiling\ or\\ Condentation}} > h_{\substack{Forced\ Convection\\ in\ Liquid}} > h_{\substack{Forced\ Convection\\ in\ Gas}} > h_{\substack{Free\ Convection\\ in\ Liquid}} > h_{\substack{Free\ Convection\\ in\ Gas}}$$

$h = f(x)$ $Q_{Conv.} = \bar{h}\ (T_s - T_{Fluid})$ $Q_{Local\ Conv.} = h\ (T_s - T_{Fluid})$	For External Flow, $T_{Fluid} = T_\infty =$ Free Stream Temp. $Q_{Conv.} = \bar{h}\ (T_s - T_\infty)$	For Internal Flow, $T_{Fluid} = T_m =$ Mean Flow Temp. $Q_{Conv.} = \bar{h}\ (T_s - T_m)$

In Some Cases, h values vary.

FORCED CONVECTION EXTERNAL FLOW:

CONCEPT OF BOUNDARY LAYER:

Assumption:

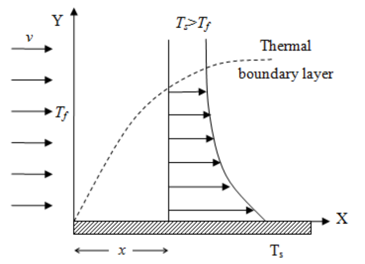
Steady State	Incompressible Fluid Flow	Flow Parallel to Surface

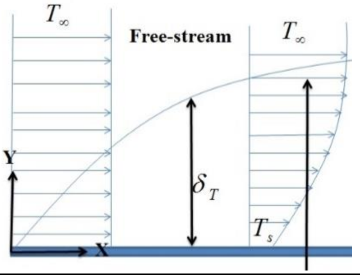
From FM:

$\tau = \mu \dfrac{du}{dx} = \dfrac{C_f}{2} \rho U_\infty^2, C_f = \begin{matrix} Skin \\ Friction \end{matrix}$	$\vartheta = \dfrac{\mu}{\rho}$	$Re_x = \dfrac{U_\infty L_C}{\vartheta}$	
Laminar Flow		**Turbulent Flow**	
$Re_x < 5 * 10^5$		$Re_x > 10^7$	
$\delta = \dfrac{5x}{\sqrt{Re_x}}$	$C_f = \dfrac{0.664}{\sqrt{Re_x}}$	$\delta = \dfrac{0.358x}{Re_x^{1/5}}$	$C_f = \dfrac{0.06}{Re_x^{1/5}}$

CONCEPT OF THERMAL BOUNDARY LAYER:

The necessary condition to developed thermal boundary layer,





1. $T_s \gg T_\infty$ (Cold Fluid Flow over a hot plate)
OR
2. $T_s \ll T_\infty$ (Hot Fluid Flow over a Cold plate)

At the boundary layer,

$$\frac{T - T_s}{T_\infty - T_s} = 0.99$$

Special Case, At $T_s = 0$

$$T = 0.99 T_\infty$$

Conduction heat loss at S/c, $q = -K_f \left.\dfrac{dT}{dy}\right\|_{y=0}$	Convection above Surface, $q = h(T_s - T_\infty)$ $\boldsymbol{\alpha} \uparrow \Rightarrow \boldsymbol{\delta_T} \uparrow$	Along the length direction, Temperature gradient decreasing at particular distance from the surface. $x \uparrow \Rightarrow \delta \uparrow \Rightarrow \delta_T \uparrow \Rightarrow dT/dy \downarrow \Rightarrow q \downarrow \Rightarrow h \downarrow$

PRANDTL NUMBER: It's dimesionless number which represents ratio of momentum diffusivity (Kinematic Viscosity) to Thermal Diffusivity.	$Pr = \dfrac{\vartheta}{\alpha} = \dfrac{\mu\, C_P}{K_f} = \dfrac{\delta}{\delta_T}$

NOTE: It's property of fluid which indicates relation between velocity boundary layer and thermal boundary layer.

RANGE OF PRANDTL NUMBER		For Laminar Flow,	For Turbulent Flow,
Liquid metal: $0.004 - 0.030$ ($Pr \ll 1$)	Gases: $0.7 - 1.0$ ($Pr \approx 1$)	$\dfrac{\delta}{\delta_T} = Pr^{\frac{1}{3}}$	$\delta \approx \delta_T$ Due to rapid mixing.
Water: $1.7 - 13.7$	Liquid organic fluids: $5 - 50$		
Oil: $50 - 1{,}00{,}000$ ($Pr \gg 1$)	Glycerine: $2000 - 1{,}00{,}000$		

NUSSELT NUMBER: It's dimensionless number which represent ratio of convective flux to the conductive heat flux (Conductive heat flux) is calculated by assuming motionless fluid.
Higher the Nusselt number higher will be convective heat transfer.

$\frac{T-T_s}{T_\infty - T_s} = \theta^*$	$\frac{y}{L_C} = y^*$	At Surface of Plate, $q_{Cond.} = q_{Conv.}$	$q = -K_f \frac{dT}{dy}\Big\|_{y=0} = h(T_s - T_\infty)$	$h = K_f \frac{d\theta^*}{dy}\Big\|_{y=0}$
$\frac{dy^*}{dy} = \frac{1}{L_C}$	$h = K_f \frac{d\theta^*}{dy^*}\frac{dy^*}{dy}\Big\|_{y=0} = \frac{K_f}{L_C}\frac{d\theta^*}{dy^*}\Big\|_{y^*=0}$		$Nu = \frac{hL_C}{K_f} = \frac{d\theta^*}{dy^*}\Big\|_{y^*=0}$	$\frac{d\theta^*}{dy^*} = \begin{matrix} Non\ Dimensional \\ Temp. Gradient \end{matrix}$
$\boldsymbol{Nu = \frac{q_{Conv.}}{q_{Cond.}} = \frac{h\Delta T}{K_f\,\Delta T/L_C} = \frac{d\theta^*}{dy^*}\Big\|_{y^*=0}}$	$\boldsymbol{Nu_x = \frac{h_x x}{K_f}}$		$\boldsymbol{\overline{Nu} = \frac{\overline{h}\ L}{K_f}}$	Nu_x = Local Nusselt Number, $\overline{Nu}$ = Local Nusselt Number,

NOTE: Bi is same like Nu but only change is K.

REYNOLD'S ANALOGY FOR LAMINAR FLOW OF GASES OVER A FLAT PLATE: $Pr \approx 1, For\ Gases$

$\frac{q}{\tau} = \frac{-K_f\,dT/dy\|_{y=0}}{\mu\,du/dy\|_{y=0}} = -C_P\frac{dT}{du}$ $\because Pr \approx 1 \Rightarrow K_f/\mu = C_P$	$@\ y=0, u=0$ $@\ y=\delta, u=u_\infty$ $@\ y=0, T=T_S$ $@\ y=\delta, T=T_\infty$	By integrating equation and substituting BCs & τ from Fluid Mechanics in q/τ Equation,	$St = \frac{h}{\rho U_\infty C_P} = \frac{C_f}{2}$ It's valid for gases but we can also use for turbulent flow.

STANTON NUMBER (St): It's dimensionless number that measures ratio of heat transfer into a fluid to the thermal capacity of fluid.	**PECLET NUMBER (Pe):** It's ratio of rate of energy carried out due to fluid motion to rate of energy carried due to diffusion.
$St = \frac{q_{Conv.}}{q_{Stored}} = \frac{h}{\rho U_\infty C_P} = \frac{Nu}{Re \cdot Pr} = \frac{Nu}{Pe} = \frac{C_f}{2}$	$Pe = Re \cdot Pr = \frac{\rho U_\infty C_P}{K_f/L} = \frac{Advection\ Transport\ rate}{Diffusion\ Transport\ Rate}$

Note: Pe is used for liquid metals or low Pr.

BOUNDARY LAYER CONDITIONS USED IN CONVECTION: $q = h(T_s - T_\infty), Where\ T_\infty = Const.$

$T_s = Const. \Rightarrow\ q \neq Const.$	$q = Const. \Rightarrow T_s \neq Const.$

1. Dirichlet Boundary Condition: In this condition, surface maintained at constant temperature.

$\overline{h} = \frac{1}{A}\int_0^A h_A dA$	$For\ Flat\ Plate, \overline{h} = \frac{1}{L}\int_0^L h_x dx \begin{pmatrix} For\ Flow\ Along \\ the\ Length \end{pmatrix}$	For $h_x = C\,x^{-0.1}$, $\boldsymbol{\overline{h} = 1.11 h_{x=L}}$

2. Neumann Boundary Condition: In this condition, Surface maintained at constant heat flux.

$Q = qA = hA(T_s - T_\infty) = \overline{h}\,A(\overline{T - T_\infty})$ $\overline{T - T_\infty} = \frac{1}{L}\int_0^L (T - T_\infty)dx = \frac{1}{L}\int_0^L \frac{q_x}{h_x}dx \begin{pmatrix} If\ Required\ use \\ Nusselt\ Number \end{pmatrix}$	Here, the question can be asked to find total heat transfer or temperature of the surface.

TO DEVELOP TEMPERATURE PROFILE IN FLUID FOLLOWING EQUATION USED:

1. Navier Stoke's Equation 2. Conservation of Mass (Continuity)	3. Conservation of Momentum 4. Conservation of Energy

CORRELATION USED IN FORCED CONVECTION EXTERNAL FLOW:

CASE-I: Laminar Flow over a flat plate and plate surface maintained at constant temperature. $T_s = Const.$

$\boldsymbol{\delta = \frac{5x}{\sqrt{Re_x}}}$	$\boldsymbol{C_f = \frac{0.664}{\sqrt{Re_x}}}$	$\boldsymbol{\delta_T = \frac{4.53\,x}{Re_x^{1/2} Pr^{1/3}}}$	$\boldsymbol{\frac{T - T_s}{T_\infty - T_s} = \frac{3}{2}\left(\frac{y}{\delta_T}\right) - \frac{1}{2}\left(\frac{y}{\delta_T}\right)^3}$
$\frac{dT}{dy}\Big\|_{y=0} = \frac{3}{2}\frac{(T_\infty - T_s)}{\delta_T}$		$q = -K_f\frac{dT}{dy}\Big\|_{y=0} = h_x(T_s - T_\infty)$	$\boldsymbol{h_x} = \frac{3}{2}\frac{K_f}{\delta_T} = \frac{0.332\,Re_x^{1/2}Pr^{1/3}K_f}{L} \propto \boldsymbol{\frac{1}{\sqrt{x}}}$
$Nu_x = \frac{h_x x}{K_f} = 0.332\,Re_x^{1/2}Pr^{1/3}$		$\overline{Nu} = \frac{\overline{h}\ L}{K_f} \neq \frac{1}{L}\int_0^L Nu_x dx$	$\boldsymbol{\overline{h} = 2h_{x=L}}$ $\boldsymbol{\overline{Nu} = 2Nu_{x=L}}$

CASE-II: Laminar Flow over a flat plate and plate surface maintained at constant Heat Flux. $q = Const.$

$\boldsymbol{Nu_x = \frac{h_x x}{K_f} = 0.453\,Re_x^{1/2}Pr^{1/3}}$	$\boldsymbol{\overline{h} = 1.5h_{x=L}}$ $\boldsymbol{\overline{Nu} = 1.5Nu_{x=L}}$	$\boldsymbol{h_x \propto \frac{1}{\sqrt{x}}}$

Note: For particular Re & Pr, Laminar Flow over a flat plate and plate surface maintained at constant Heat Flux boundary conditions Nu is 36 % more than constant surface temperature boundary condition.

For Laminar Flow,

$\boldsymbol{\frac{\delta}{\delta_T} = Pr^{1/3}}$	$\boldsymbol{St_x\,Pr^{2/3} = \frac{C_f}{2}}$ **Colburn Analogy Valid for Any Fluid.**	$\boldsymbol{St_x\,Pr^{2/3} = \frac{C_f}{2}}$ **Reynolds Analogy Valid for Gas**

TURBULENT FLOW OVER FLAT PLATE & PLATE SURFACE MAINTAINED AT CONSTANT TEMPERATURE: $Re_x > 10^7$

$\boldsymbol{\delta = \dfrac{0.358x}{Re_x^{1/5}}}$	$\boldsymbol{C_f = \dfrac{0.059}{Re_x^{1/5}}}$	**From Colburn Analogy,** $Nu_x = 0.0295\, Re_x^{4/5} Pr^{1/3}$	$\boldsymbol{h_x \propto x^{-\frac{1}{5}}}$	$\boldsymbol{\bar{h} = 5/4\, h_{x=L}}$ $\boldsymbol{\overline{Nu} = 5/4\, Nu_{x=L}}$

TURBULENT FLOW OVER FLAT PLATE & PLATE SURFACE MAINTAINED AT CONSTANT HEAT FLUX: $Nu_x = 0.03068 Re_x^{4/5} Pr^{1/3}$

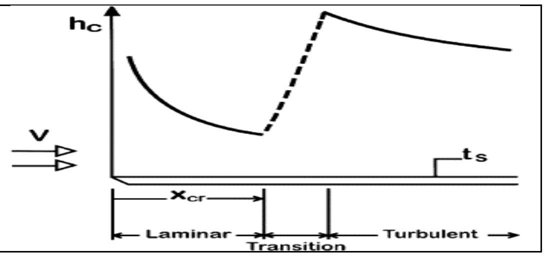
Note: For particular Re & Pr, Turbulent Flow over a flat plate and plate surface maintained at constant Heat Flux boundary conditions Nu is 4 % more than constant surface temperature boundary condition.

VARIATION OF HEAT TRANSFER COEFFICIENT OVER FLAT PLATE:

At surface, $q_{Cond.} = q_{Conv.} \Rightarrow h \approx K/\delta_T$,
In External flow, all the thermophysical properties of fluid are assumed to be constant between they are calculated on mean film temperature,

$$T_{mf} = \frac{T_s + T_\infty}{2}$$

Nu is more in turbulent flow compare to laminar flow.
h is more in turbulent flow compared to laminar flow.



ANALYSIS OF CROSS FLOW OVER SOLID CYLINDER: $Re_D \leq 2 * 10^5$

1. For laminar flow, Separation takes place at $\theta = 80°$(Measured from Stagnation Point)
2. For Turbulent Flow, Separation takes place at $\theta = 140°$(Measured from Stagnation Point)

δ is maximum at separation point hence at separation point h is minimum.

FORCED CONVECTION INTERNAL FLOW:
Circular pipe geometry withstands high pressure without distortion compared to non-circular geometry.

HYDRAULIC DIAMETER ($\boldsymbol{D_h}$): It's used to analysing the thickness of boundary layer in internal flow.

$D_h = \dfrac{4A_C}{P}$	$A_C =$ Cross section Area, $P =$ Wetted perimeter	Pipe: Circular Cross Section $D_h = D$	Duct: Non-Circular Cross Section For Rectangular Duct, $D_h = 2ab/(a+b)$

For Circular Concentric Annular Pipes, $D_h = D_i - D_o (D_i > D_o)$

Nusselt Number, $Nu = hD_h/K_f$	**HYDRODYNAMIC BOUNDARY LAYER:**
For Laminar Flow, $Re < 2300$	$\dfrac{\partial u}{\partial x} = 0\ (for\ x > L_h(Hydrodynamic\ Entrance\ Length))$
For Turbulent Flow, $Re > 10{,}000$	
For Laminar Flow, $L_h = 0.05\, Re\, D \approx 115D$	$\boldsymbol{u_m = \dfrac{2}{R^2}\int_0^R u(r,x) r dr}$
For Turbulent Flow, $L_h \approx 10D$	

THERMAL BOUNDARY LAYER:

For Fully Developed Profile (Actual Profile),

$$Energy_{Ideal} = Energy_{Actual}$$
$$\dot{m}h = Constant$$
$$\rho A u_m C_P T_m = \int_0^R \rho dA u(r,x) C_P T(r,x)$$
$$\boldsymbol{T_m u_m = \frac{2}{R^2}\int_0^R u(r,x) T(r,x) r dr}$$

Temperature Profiles,

$$Here, \frac{\partial T}{\partial x} \neq 0, \frac{\partial T_m}{\partial x} \neq 0$$
$$\frac{d\theta^*}{dx} = 0 = \frac{d}{dx}\left[\frac{T_s - T}{T_s - T_m}\right]$$

From Energy Balance,

$$q_{in} = q_{Conv}$$
$$K_f \left.\frac{dT}{dr}\right|_{r=R} = h\Delta T = K_f \frac{\Delta T}{R}$$

At $x = 0, \delta_T = 0$,
Hence, Within L_T,

$$\boldsymbol{h_x/K = 1/\delta_T = f(x)}$$

Outside L_T, $\boldsymbol{\delta_T \neq f(x)}$

For constant cross section duct when, fluid properties are constant, then heat transfer coefficient along flow Direction remains constant.

$$Nu = \frac{h_x D_h}{K} = Const.$$

Graph of $h_x\ Vs.\, x$

In case of thermal fully developed flow, Temperature profile may vary with x in the flow direction but dimensionless temperature profile remains unchanged. In the thermal fully developed region convection heat transfer coefficient is constant (Doesn't vary with x).

Thermal Entrance Length ($\boldsymbol{L_T}$),

For laminar Flow, $L_T = 0.05\, Re\, D\, Pr$	For turbulent Flow, $L_T \approx 10D$

$Pr = \frac{L_T}{L_h}\left(\begin{matrix}Valid\ For\ Laminar\\ Flow\ only\end{matrix}\right)$	In turbulent flow, the intense mixture due to random fluctuation of usually overshadows the effect of molecular diffusion. Therefore $L_T \approx L_h$ and L_T is independent of Prandtl Number. L_T is much shorter w.r.t. laminar Flow.

GENERALISED THERMAL ANALYSIS OF FULLY DEVELOPED FLOW:

From the energy Balance, $dQ = hdA\,(T_s - T_m) = \dot{m}C_P(T_m + dT_m - T_m)$ $\therefore dQ = hPdx\,(T_s - T_m) = \dot{m}C_P dT_m$ Possibility, $T_s = Const.$ | $q_s = Const.$	T_s = Temperature Surface T_i = Mean Temperature of flow at Inlet ($x = 0$) T_o = Mean Temperature of flow at Outlet ($x = L$) T_m = Mean Temperature of flow at any Location ($x = x$) P = Parimeter

SURFACE MAINTAINED AT CONSTANT TEMPERATURE:

$\theta = T_s - T_m$	$BC\ @x = 0, \theta = T_s - T_i = \Delta T_1$ $@x = L, \theta = T_s - T_o = \Delta T_2$	$\Delta T_1 = T_s - T_i$ $\Delta T_2 = T_s - T_o$	$\frac{d\theta}{dx} = -\frac{dT_m}{dx} = -\frac{hP\theta}{\dot{m}C_P}\left(\begin{matrix}From\ Energy\\ Balance\end{matrix}\right)$

By integration we can get,

$\frac{\Delta T_2}{\Delta T_1} = \frac{T_s - T_m}{T_s - T_i} = e^{-\frac{hPx}{\dot{m}C_P}} = e^{-NTU}$	$No. of\ Transfer\ Units: NTU = \frac{hPx}{\dot{m}C_P}$	Here, T_m Varies exponentially with respect to x.

NOTE: For $NTU \geq 5, T_m \approx T_s$

$dQ = \dot{m}C_P dT_m \Rightarrow Q = \dot{m}C_P(T_o - T_i) = hPL\left[\frac{T_s - T_i - (T_s - T_o)}{\ln{(T_s - T_i)/(T_s - T_o)}}\right] = hA_s LMTD$	$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln \Delta T_1/\Delta T_2}$
$AMTD = \frac{\Delta T_1 + \Delta T_2}{2} = T_s - \left[\frac{T_i + T_o}{2}\right] = T_s - T_{bulk}$	$LMTD$ = Logarithmic Mean Temperature Difference $AMTD$ = Arithmetic Mean Temperature Difference

IMPORTANT POINTS:

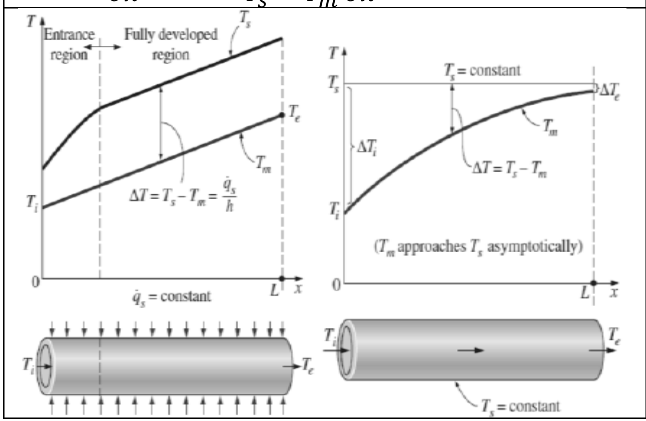
In Case of thermal fully developed flow for constant surface temperature boundary conditions ($T_s = Const.$)

1. Temperature difference between surface and mean temperature of flow continuously varies w. r. t. location.
2. LMTD used as corrected temperature difference in Newton's Law of cooling. AMTD Should not be used.
3. Mean Temperature of fluid varies exponentially with respect to location.
4. Shape of temperature profile varies with respect to location.

SURFACE MAINTAINED AT CONSTANT HEAT FLUX BOUNDARY CONDITION:

$\boldsymbol{q = h(T_s - T_m)}$ $T_s - T_m = q(Const.)/h(Const.)$	$T_s - T_m$ = Constant. $dQ = qPdx = \dot{m}C_P dT_m$	$\frac{\partial T_s}{\partial x} = \frac{\partial T_m}{\partial x} = \frac{qP}{\dot{m}C_P} = Constant$
$T_m = \frac{qPx}{\dot{m}C_P} + C_1$	$BC\ @x = 0, T_m = T_i$ $C_1 = T_m$	$@x = L, T_m = T_o$ $Q = \dot{m}C_P(T_o - T_i) = qA_s$

For fully developed flow,

$\frac{\partial \theta^*}{\partial x} = 0 \Rightarrow \frac{1}{T_s - T_m}\frac{\partial}{\partial x}(T_m - T) = 0$	$\frac{\partial T_s}{\partial x} = \frac{\partial T_m}{\partial x} = \frac{qP}{\dot{m}C_P} = \frac{\partial T}{\partial x} = Constant$
	Here, $q = h(T_s - T_m) = Const.$ Initially $h \to \infty \Rightarrow T_s - T_m \to 0$ After Some Time, $h \downarrow \Rightarrow T_s - T_m \uparrow$ For Fully Developed Flow, $h = Const.$ $T_s - T_m = Const.$ Imp Points for fully developed flow for constant heat flux, 1. Temperature difference between surface and mean temperature of fluid remain constant. 2. Mean temperature of fluid varies linearly with respect to location. 3. Shape of the temperature profile remains unchanged. Just the temperature increases but shape remains same.

CORRELATIONS USED IN FORCED CONVECTION (INTERNAL FLOW):

CASE-I: Laminar Flow through a pipe and pipe surface maintained at Constant Temperature.

$Re < 2300$	$Nu = 3.66$	$h = 3.66\,K/D$

CASE-II: Laminar Flow through a pipe & pipe surface maintained at Constant heat Flux.

$Re < 2300$	$Nu = 4.36 = 48/11$	$h = 4.36\,K/D$

For fully developed laminar flow constant Heat Flux gives 19% more Nusselt number than constant surface temperature.

For Laminar/ Fully Developed Flow, $Nu = Constant$

For fully developed laminar flow Heat loss is independent of diameter due to constant Nusselt number.

CASE-III: Turbulent Flow Through A Pipe.

DITTUS BOILTER EQUATION: $Nu = 0.023\, Re^{0.8} Pr^{n}$ From Colburn Analogy, $St_x\, Pr^{2/3} = f/8\, (\because C_f = f/4)$	For Cooling of Fluid, $h = 0.3$ For Heating of Fluid, $h = 0.4$

For particular $\dot{m}$, in turbulent flow as $D \uparrow, Q \downarrow$ (From above Equation & $Q = hA_s\Delta T$)

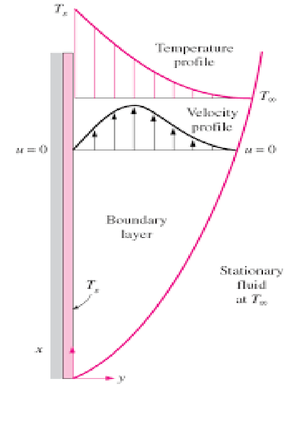
1. DITTUS BOILTER EQUATION valid for both the boundary condition.
2. In turbulent flow Nusselt number is much higher than laminar flow.
3. In turbulent flow entry length is much shorter than laminar flow.
4. In general, constant heat flux boundary condition gives more Nusselt number than constant surface temperature conation.
5. In Internal flow, all the thermophysical properties of fluid are assumed to be constant between they are calculated on mean film temperature, $T_{mf} = (T_i + T_0)/2$

FREE OR NATURAL CONVECTION: Convection process takes place due to density difference and Gravity Force.

Conduction	**Free Convection**	
$T_1 \ll T_2$ & T_2 situated above T_1	$T_1 \gg T_2$ & T_2 situated above T_1	Free Convection takes place where gravity is present. And it has less heat transfer rate and No maintenance cost.
$\rho_1 \ll \rho_2$	$\rho_1 \gg \rho_2$	
Stable Heat Transfer	Unstable Heat Transfer	

CHARACTERISTIC LENGTH (L_C): $L_C = Surface\ Area/Parimeter$

Vertical Plate: $L_C = L$	Vertical Cylinder: $L_C = L$
Sphere: $L_C = D$	Horizontal Cylinder: $L_C = D$
Hot Face Upward: $L_C = A_s/P$ Takes Less time for Colling.	Hot Face Downward: $L_C = A_s/P$ Takes Long time for Colling.
Horizontal Plate: $L_C = A_s/P$	Horizontal Circular Plate: $L_C = A_s/P = D/4$
For Free convection flow within boundary layer, there are majorly Buoyancy force and Viscous forces which plays major role.	



COEFFICIENT OF VOLUME EXPANSION (β):

Same line coefficient of linear expansion, $\beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big\vert_{P=C} = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\Big\vert_{P=C}$	For linear Variation of density with temp. $\beta\rho(T_\infty - T) = \rho - \rho_\infty$ Large Value β gives higher free convection.
For ideal Gas, $\beta = 1/T\ \ (\because P = \rho RT), Where\ T = T_{avg} = (T_s + T_\infty)/2$	

GRASHOF NUMBER (Gr): It represents ratio of buoyancy force to the viscous force acting on fluid. Grashof Number provide the main criteria to determining whether the fluid flow is laminar or turbulent in natural convection.

$Gr = \frac{g\beta(T_s - T_\infty)L_C^3}{\nu^2}$	$Gr^* = Gr \cdot Nu = \frac{g\beta q L_C^4}{K\,\nu^2}$	$Ra = Gr \cdot Pr = \frac{g\beta(T_s - T_\infty)L_C^3}{\nu\,\alpha}$	$Ri = \frac{Gr}{(Re)^2}$

Gr is not suitable for constant heat flux because the surface temperature continuously increases and Gr also changes.

RAYLEIGH NUMBER (Ra):

It viewed a ratio of buoyancy force and product of thermal and momentum diffusivities.

For Vertical Plate & Vertical Cylinder,

For laminar Flow, $Ra \leq 10^9$	For Turbulent Flow, $Ra > 10^9$

MODIFIED GRASHOF NUMBER (Gr^*): It's suitable for constant heat flux boundary condition.

RICHARDSON NUMBER (Ri):

For Free Convection, $Ri \gg 1$	For Forced Convection, $Ri \ll 1$	For Mixed Convection, $Ri \approx 1$

CORRELATION USED IN FREE CONVECTION:

VERTICAL PLATE:

CASE-I: Surface Maintained at constant temperature. $Nu = C[Gr \cdot Pr]^n$

$C =$ Const. (Generally, < 1)	For Laminar Flow, $n = 1/4$	For Turbulent Flow, $n = 1/3$

CASE-II: Surface Maintained at constant Heat Flux. $Nu = C[Gr^* \cdot Pr]^n$

$C =$ Const. (Generally, < 1)	For Laminar Flow, $n = 1/5$	For Turbulent Flow, $n = 1/4$

GENERAL THERMAL ANALYSIS OF LAMINAR FREE CONVECTION OVER A VERTICAL PLATE OR CYLINDER:

CASE-I: Surface Maintained at constant temperature.

For Laminar Flow, $Nu = C[Gr \cdot Pr]^n = hL_C/K\ , Where, n = 1/4$

$Nu \propto (\Delta T)^{1/4}$	$h \propto L^{-1/4}$	$Q = hA_s\Delta T \propto L^{3/4}$	$h_x \propto x^{-1/4}$	$\bar{h}\ = (4/3)h_{x=L}$

For Turbulent Flow, $Nu = C[Gr^* \cdot Pr]^n = hL_C/K\ , Where, n = 1/3$

h is independent of L_C

CASE-II: Surface Maintained at constant Heat Flux. $Nu = C[Gr^* \cdot Pr]^n$

For Laminar Flow, $Nu = C[Gr^* \cdot Pr]^n = hL_C/K\ , Where, n = 1/5$

$Nu \propto (\Delta T)^{1/5}$	$h \propto L^{-1/5}$	$Q = hA_s\Delta T \propto L^{4/5}$	$h_x \propto x^{-1/5}$	$\bar{h}\ = (5/4)h_{x=L}$

For Turbulent Flow, $Nu = C[Gr^* \cdot Pr]^n = hL_C/K\ , Where, n = 1/4$

h is independent of L_C

NOTE: In Turbulent Flow, Free Convection h is independent of characteristic length.