HYDRAULIC MACHINES

Pelton Turbine	Francis Turbine	Kaplan Turbine
Similarity Law	Cavitation	

1. PELTON TURBINE:

$$H_g = H + H_f$$
$$H = \left(\frac{P}{\rho g} + \frac{V^2}{2g}\right)_{inlet\ of\ nozzle} = \frac{V^2}{2g} + H_{fn}$$
$$V = \sqrt{2g\left(H - H_{fn}\right)} = C_V\sqrt{2gH}$$

$H_g =$ Gross Head
$H =$ Head Available at the inlet of the nozzle
$H_f =$ Head Loss in the Penstock
$H_{fn} =$ Head Loss in the Nozzle
$C_V =$ Coefficient of velocity (0.9-0.98)
$\alpha =$ Vane Angle = Angle between $\vec{u}$ & $\vec{V}$
$\beta =$ Blade Angle = Angle between $\vec{u}$ & $\vec{V_r}$
$\vec{V} =$ Absolute Velocity of Water
$\vec{V_r} =$ Relative Velocity of Water w.r.t. Bucket
$\vec{u} =$ Bucket Velocity
$K =$ Bucket Friction Coefficient (0.9-0.98)

Force exerted by jet on the bucket:
$$F = \begin{pmatrix} Mass\ Striking\ on\ the \\ Bucket\ per\ unit\ time \end{pmatrix}\begin{pmatrix} Decreasing\ in\ the\ vel. \\ in\ direction\ of\ force \end{pmatrix}$$

$\boldsymbol{\vec{V} = \vec{u} + \vec{V_r}}$	$V_{r1} = V - u$	$V_{r2} = K(V - u)$

$$F = \dot{m}\left(\overrightarrow{V_{r1}} - \overrightarrow{V_{r2}}\right) = \dot{m}[(V-u) + V_{r2}\cos\beta_2]$$
$$F = \dot{m}(V-u)[1 + K\cos\beta_2]$$

Torque $T = FR = \dot{m}(V-u)[1 + K\cos\beta_2]R$
Power Generated by Runner:
$$RP = T\omega = Fu$$

Mass Striking,
For Single Bucket, $\dot{m} = \rho a(V-u)$
For Multiple Bucket, $\dot{m} = \rho aV$

Wheel Efficiency:
$$\eta_w = \frac{RP}{KE} = \frac{2(V-u)[1 + K\cos\beta_2]}{V^2} = f(u)$$
Condition For maximum η_w, $u = V/2$

$KE = \frac{1}{2}\dot{m}V^2 =$ Kinetic Energy per unit time,
$$\eta_{w,max} = \frac{1 - K\cos\beta_2}{2}$$

$N_R =$ Run Away Speed = Maximum Possible Speed of Turbine Bucket (It can be Theoretical or Actual)
Turbines are designed at N_R for Safety Aspect. Because of No load condition Turbines have max. Stress at N_R

Nozzle Efficiency	Hydraulic Efficiency	Mechanical Efficiency	
$\eta_n = \frac{KE}{HP} = C_V^2$	$\eta_h = \frac{RP}{HP} = \eta_n\eta_w$	$\eta_m = \frac{RP}{SP}$	$HP = \rho gQH =$ Hydraulic Power $RP =$ Runner Power $SP =$ Shaft Power

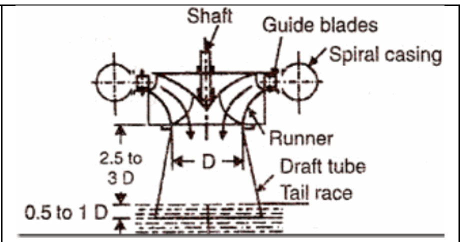
Overall Efficiency: $\eta_o = SP/HP = \eta_h\eta_m = \eta_n\eta_w\eta_m$

Jet Ratio	Speed Ratio	No of Buckets on Wheel
$m = \frac{Wheel\ Diameter}{Jet\ Diameter} = \frac{D}{d}$	$K_u = \frac{u}{V_{th}} = \frac{u}{\sqrt{2gH}} \cong \frac{u}{V}$	$Z = 0.5m + 15$ It's Tygun's Formula

Relation between frequency of AC & Runner Speed:
1 Rev. = 1 Cycle of AC
$$f = \frac{PN}{60}, Where\ P = No. of\ Poles.$$

2. FRANCIS TURBINE:

Spiral Casing: Distribute water equally to all guide vanes.
Guide Vanes: It guides water for smooth entry of water. It acts like nozzle (Convers pressure partially into KE). It Controls discharge through the runner.
Runner: To generate torque/ Power output. Pressure decreases (Nozzle Effect & Centrifugal Force) and KE is also decreases.
Old Turbine: Radial Entry & Radial Exit.
Morden Turbine: Radial Entry & Axial Exit.



$\vec{V_w} =$ Velocity of Whirl = Component of $\vec{V}$ along $\vec{u}$
$\vec{V_f} =$ Velocity of Flow = Component of $\vec{V}$ Perpenticular to $\vec{u}$

$\alpha =$ Guide Vane = Angle between $\vec{u}$ & $\vec{V}$
$\beta =$ Runner Blade = Angle between $\vec{u}$ & $\vec{V_r}$

Euler's Turbine Equation:
$$\begin{pmatrix} Torque\ Generated \\ By\ Runner \end{pmatrix} = \begin{pmatrix} Angular\ Momentum\ lost \\ By\ Water\ Per\ unit\ time \end{pmatrix}$$
$$T = \left(\vec{r} \times \dot{m}\vec{V}\right)_1 - \left(\vec{r} \times \dot{m}\vec{V}\right)_2$$

$$T = r_1\,\dot{m}\overrightarrow{V_{w1}} - r_2\,\dot{m}\overrightarrow{V_{w2}}$$
$T = \rho Q\left(r_1\overrightarrow{V_{w1}} - r_2\overrightarrow{V_{w2}}\right)$ Euler eq. for Turbo M/c.
$T = \rho Q\left(r_2\overrightarrow{V_{w2}} - r_1\overrightarrow{V_{w1}}\right)$ Euler eq. for Pumps.

Inertial Frame has Zero acceleration.

Discharge Through Francis Turbine:	$Q_i = \pi D_1B_1V_{f1}$	$Q_e = \pi D_2B_2V_{f2}$
Torque Generated by Runner:	$T = \rho Q\ [r_1V_{w1}]$	
Power Generated by Runner:	$RP = T\omega = \rho Q\ V_{w1}u_1$	

Various losses in Francis Turbine:

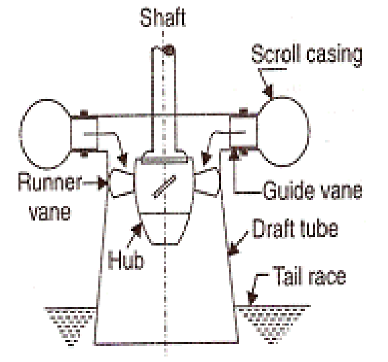
Hydraulic Efficiency	Mechanical Efficiency	Overall Efficiency
$\eta_h = \frac{RP}{HP} = \frac{V_{w1}u_1}{gH}$	$\eta_m = \frac{RP}{SP}$	$\eta_o = \frac{SP}{HP} = \eta_h \eta_m$
Speed Ratio (0.75-0.82)	**Flow Ratio (0.15-0.30)**	**Head Developed by Turbine**
$K_u = \frac{u_i}{\sqrt{2gH}} = f\begin{pmatrix} Geometric \\ Shape \end{pmatrix}$	$K_f = \frac{V_{fi}}{\sqrt{2gH}} = f\begin{pmatrix} Geometric \\ Shape \end{pmatrix}$	$\boldsymbol{H_e = \frac{V_{w1}u_1}{g}}$

3. KAPLAN TURBINE:

1. Spiral Casing: Same as Francis Turbine
2. Guide Vanes: Same as Francis Turbine
3. Swirl/ Whirl Chamber: At exit of Guide Vanes, 2 Component of velocities are present in the flow. 1) Radial 2) Tangential (Responsible for the whirling of the flow). Hence, Angular momentum of fluid partial is conserved in swirl chamber. $V_w r = Const.$ (Free Vortex Flow).
4. Runner: $u_r = r\omega$ hence, velocities are changing w.r.t radius. So, velocity diagram will be different at different radial location. Here, Runner blades are adjustable. So, it's very costly.

Propeller Turbines has fixed blades.

5. Draft Tube:



$u_1 = u_2 = r\omega$	$V_f = V_{f1} = V_{f2} = \frac{Q}{A_{f1}} = \frac{Q}{A_{f2}}$	$A_{f1} = A_{f2} = \frac{\pi}{4}\left(D_t^2 - D_h^2\right)$	$D_t =$ Blade Tip diameter $D_h =$ Hub diameter

Discharge Through Kaplan Turbine:	$Q = A_{f1}V_{f1} = A_{f2}V_{f2}$
Torque Generated by Runner:	$T = \rho Q\,[r_1 V_{w1}]$
Power Generated by Runner:	$RP = T\omega = \rho Q\, V_{w1}u_1$
Hydraulic Efficiency:	$\eta_h = \frac{RP}{HP} = \frac{V_{w1}u_1}{gH}$

Torque, RP, Hydraulic Efficiency for any radial location will remains exactly same.

Various losses in Kaplan Turbine:

Hydraulic Efficiency	Mechanical Efficiency	Overall Efficiency
$\eta_h = \frac{RP}{HP} = \frac{V_{w1}u_1}{gH}$	$\eta_m = \frac{RP}{SP}$	$\eta_o = \frac{SP}{HP} = \eta_h \eta_m$
Speed Ratio (1.3-2.3)	**Flow Ratio (0.35-0.75)**	**Head Developed by Turbine**
$K_u = \frac{u_{tip}}{\sqrt{2gH}} = f\begin{pmatrix} Geometric \\ Shape \end{pmatrix}$	$K_f = \frac{V_f}{\sqrt{2gH}} = f\begin{pmatrix} Geometric \\ Shape \end{pmatrix}$	$\boldsymbol{H_e = \frac{V_{w1}u_1}{g}}$

4. SIMILARITY LAW:

Valid under homologous condition: 1. Model & Prototype are geometrically similar. 2. Corresponding velocity triangles are also similar. η of the Hydraulic Machine depends on Velocity triangle.	$d \propto D$ $L_r = \frac{D_m}{D_P} = \frac{d_m}{d_P}$	$Velocity \propto \sqrt{H}$ $\frac{(V_{r1})_m}{(V_{r1})_P} = \frac{(u_1)_m}{(u_1)_P} = \frac{V_m}{V_P} = \sqrt{\frac{H_m}{H_P}}$
$u = \pi DN$	$ND \propto \sqrt{H}$	$C_H =$ Head Coefficient
$Q = a\,V = (\pi/4)D^2\sqrt{2gH}$	$Q \propto D^2\sqrt{H}$	$C_Q =$ Discharge Coefficient
$P = \eta_o \rho g Q H$	$P \propto D^2 H^{3/2}$	$C_P =$ Power Coefficient

Important $\boldsymbol{\pi}$ Terms Valid for all Hydraulic Machines:

$\frac{Q}{ND^3}$	$\frac{gH}{N^2D^2}$	$Re = \frac{\rho ND^2}{\mu}$	$\frac{P}{\rho N^3 D^5}$	$\frac{E/\rho}{N^2D^2}$

SPECIFIC SPEED ($\boldsymbol{N_S}$):

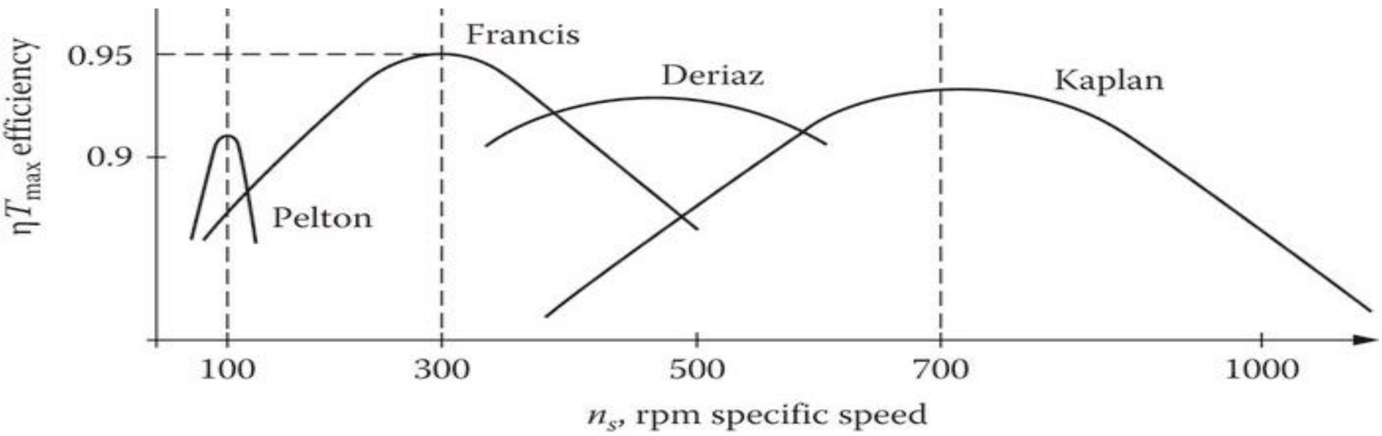
$C_H, C_Q, C_P, C_S = f\begin{pmatrix} Geometric & Shape\ of\ Velocity \\ Shape & Triangle \end{pmatrix}$	$Speed\ Coefficent = C_S = C_H\sqrt{C_P} = \frac{N\sqrt{P}}{H^{5/4}}$

Specific Speed is the speed coefficient at maximum efficiency. Specific Speed is function of geometric shape only.

Selection of turbine based on specific Speed:

$P =$ Based upon quantity of water available or power requirement. $N =$ Based upon gearbox & type of generator used.	$H =$ Based upon heat available.

Based on the graph of Efficiency Vs. Specific Speed, the suitable turbine is selected.



TURBINE	TYPE	FLOW DIRECTION	$N_S\left(rpm\sqrt{KW}/m^{5/4}\right)$	HEAD (M)	DISCHARGE
Pelton	Impulse	Tangential	8-30	High (50-1500)	Low
Francis	Reaction	Radially inward	40-400	Medium (25-350)	Medium
Kaplan	Reaction	Axial	300-900	Low (2-40)	High

Note:

1. For Pelton Turbine Power per jet is considered in case of MultiJet turbine.
2. Power is always considered in “KW”

DRAFT TUBE:

1. Allows installation of turbine above tail-race without effective loss in net head.
2. Recovers KE loss at exit partially.

Head Developed by Turbine $$H_T = B_{1'} - B_{2'} \cong \frac{P_{1'} - P_{2'}}{\rho g}$$	From the Bernoulli Eq. (2’-3), $$\frac{P_{2'}}{\rho g} + \frac{V_{2'}^2}{2g} + (H_S + y) = \left(\frac{P_{atm}}{\rho g} + y\right) + \frac{V_3^2}{2g} + 0 + h_{fd}$$ $$\frac{P_{2'}}{\rho g} = \frac{P_{atm}}{\rho g} - \left(H_S + \frac{V_{2'}^2 - V_3^2}{2g} - h_{fd}\right) \le \frac{P_{atm}}{\rho g}$$
H_S = Recovery of lost Head, $\frac{V_{2'}^2 - V_3^2}{2g}$ = Partial Recovery of KE lost,	

EFFICIENCY OF DRAFT TUBE (η_d):

$$\eta_d = \frac{Actual\ Recovery\ of\ KE\ head}{KE\ head\ at\ inlet\ of\ draft\ Tube} = \frac{\frac{V_{2'}^2 - V_3^2}{2g} - h_{fd}}{\frac{V_{2'}^2}{2g}}$$	$$\eta_d = \frac{Actual\ Recovery\ of\ KE\ head}{Theretical\ Recovery} = \frac{\frac{V_{2'}^2 - V_3^2}{2g} - h_{fd}}{\frac{V_{2'}^2 - V_3^2}{2g}}$$

5. CAVITATION: $P_{min} < P_{Vapour}$ Cavitation starts.

For turbines, Cavitation starts after turbine exit/ before draft tube entry if $P_{2'} < \left(P_{Vapour}\right)_{Water}$.

TOMA’S CAVITATION FACTOR (σ):

$$\sigma = \frac{H_a - H_S - H_V}{H}$$ Critical Cavitation Factor(σ_C) is given by manufacturer. For Cavitation not to happed, $\sigma < \sigma_C$.	H_a = Head of atmospheric pressure, H_S = Height of Draft tube, H_V = Head of Vapour Pressure, H = Net head on the turbine,