BUCKLING OF COLUMN

STRUCT	COLUMN
Member of a structure in **any position** and carrying "an axial Compressive load"	Member of a structure in **vertical position** and carrying "an axial Compressive load"

Types of Columns (Based on Failure Mechanism)		
Short Column	**Long Column**	**Intermediate Colum**
Failure Due to **Crushing**	**Stability/Buckling** failure	**Both** Crushing and Stability Failure
P ---->σ >σ_{yc}	Column Bends in a direction perpendicular of load acting----> **Collapse**	Difficult to analyse----> All formulas are imperial/experimental in nature
Sudden Failure	Buckling Occurs at stress levels much below yield strength of material $P>P_{Cr}$ $((P_{Cr}/A) = \sigma_{Cr}) < \sigma_{yc}$	
High Load Carrying Capacity	Less Load Carrying Capacity	

Types of Equilibrium

Design of Long Column ------> Depends on Stability of Column ------> Depends on Equilibrium Condition

1. Stable Equilibrium ------> Columns go back to original position when load is removed.
2. Neutral Equilibrium ------> about to buckle ------> $P = P_{Cr}$.
3. Unstable Equilibrium ------> Columns permanently deform when load is removed.

For Safe Design, ($P < P_{Cr}$ /FOS) Where P_{Cr} = Critical Load / Crippling Load/ Buckling Load.

Euler's Buckling Formula:

$P_{Cr} = \pi^2 EI / Le^2$ Where, E = Youngs Modulus of Column Material,

I = I_{min} =Minimum area moment of inertia (Column always buckles in minimum area direction)

Le = Equivalent length of column

= It's length of equivalent pinned-end column having same load carrying capacity as the given column with given end condition (Where Pinned end base resists tension and shear)

= Distance between two successive zero moments

= Length of half of the Sin curve

= Le , For Both end hinge

= Le / 2 , For Both fixed

= Le / √2 , For Both one end fixed-one end hinge

= 2 Le , For Both one end free-one end hinge

≠ Distance between two point of contraflexure / Point of Inflection

Observations:

1) $P_{Cr} \propto 1/Le$, Hence Fixed-Fixed Columns are frequently used.

2) I = I_{min} =Minimum area moment of inertia (Column always buckles in minimum area direction)

3) "Le" can be reduced by lateral supports. "Le" depends on position of lateral supports.
Hence, Load carrying capacity can be increased.
For different equivalent length choose P_{Cr} = min (P_{Cr1} ,P_{Cr2} ,P_{Cr3}.................) = P_{Cr} for higher effective length.

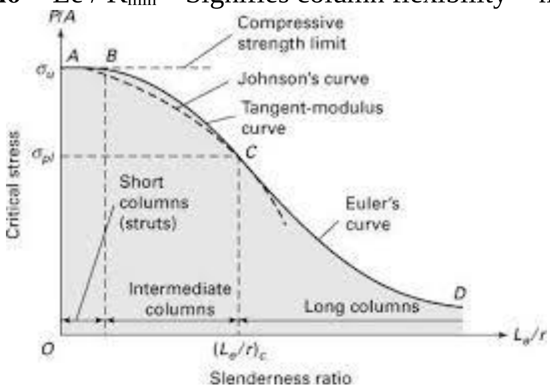
Critical Stress Vs Slenderness Ratio:

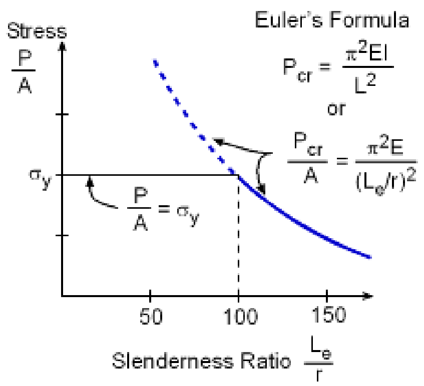
- P_{Cr} Independent of column and depends on E and Length (because it's stability failure).
- **Critical Stress:** Normal Stress in column due to buckling load (at buckling occurs). $((P_{Cr}/A) = \sigma_{Cr}) < \sigma_{yc}$ or $\sigma_{allwable}$.
  $\sigma_{Cr} = P_{Cr}/A = \pi^2 EI_{min} / Le^2 A = \pi^2 EK^2_{min} / Le^2 = \pi^2 E / \lambda^2$
  Where, K = radius of gyration --> Minimum imaginary distance at which entire area is assumed to be concentrated
  λ = Slenderness Ratio = Le / K_{min} = Signifies column flexibility = more value indicates less load carrying

For Steels,

E = 200 GPa, $\sigma_{allwable}$ = 140 MPa

- **λ < 118 => Short Column;**
- **λ > 118 => Long Column.**





Rankine Formula (Valid for all type of columns) (Never Asked in Mechanical):

$(1/P) = (1/P_C) + (1/P_E)$

Where, P = Rankine Crippling load

P_C = Crushing Load = $\sigma_c * A$ where, σ_c = Ultimate Tensile Strength

P_E = Euler's Critical Load

$P = \sigma_c A / [1 + \alpha \lambda^2]$ Where, α = Rankin's Constant = $\sigma_c / \pi^2 E$

Column in Bi-directional Bending (Will Not Asked in Mechanical):

Interior Columns designed as per axial loaded column

Edge Columns designed as per axial loaded + Uni-axial bending column

Corner Columns designed as per axial loaded + Bi-axial bending column

Column or Kernel of a Section:

Is it always possible to have pure axial load on column? ==> No ==> eccentricity bound to be exist ==> Bending moments bound to be exist ==> Bending stress exists ==> Tension exists in the column

Core of Section: Part of column cress section in which load is acting such that there will be no tension in the column. Columns are safe in compression and weak in tension. Due to tension, there is possibility that column is removed from foundation due to tension.

For no tension condition ==> $\sigma_b \leq \sigma_c$ ==> $e \leq Z/A$ (for Axial load and Uni-axial bending)

For Solid circular section, $e \leq d/8$ ==> Middle quadrant (4^{th}) role (because Core/Kernel radius R=d/4)

Area of core = (Area of cross section)/16

For Hollow circular section, $e \leq (D^2 + d^2) / 8D$

For Rectangular section, $e_x \leq b/6$ and $e_y \leq h/6$ ==> this eccentricity creates rhombus ==> Middle third (3^{rd}) role

Area of core = (Area of cross section)/18

Example: For Dam, Resultant (weight & water force) acting on Middle third role there will be no tension develop.

Rigid Struct Supported by Spring:

Rigid Struct ==> No buckling ==> No Euler's Formula applicable

Rigid struct is fails when stability equilibrium lost ==> this gives collapsing force.

Collapsing force, $P = K L$ where K= Stiffness, L = Length of Column.