

Matrix theory Assignment 11

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Abstract—This document contains the concept of sub space V over a field F.

Download all python codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment10/Codes

Download latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment10/Assignment_10.tex

Now, $c(\alpha) + \beta$

$$\begin{aligned} &= c(a_1, a_2, a_3, \dots, a_n) + (b_1, b_2, b_3, \dots, b_n) \\ &= (ca_1, ca_2, ca_3, \dots, ca_n) + (b_1, b_2, b_3, \dots, b_n) \end{aligned} \quad (2.0.3)$$

Now, using condition $a_1 + 3(a_2)$ we have,

$$\begin{aligned} ca_1 + b_1 + 3(ca_2 + b_2) &= ca_1 + b_1 + 3ca_2 + 3b_2 \\ \implies c(a_1 + 3a_2) + (b_1 + 3b_2) &= ca_3 + b_3 \end{aligned} \quad (2.0.4)$$

From equation (2.0.4) it is closed under addition and scalar multiplication, hence it is a subspace.

c) All α such that $a_2 = a_1^2 \in \mathbf{R}^n$ ($n \geq 3$),
Let,

$$\alpha = (-1, 1, 0, \dots, 1) \in \mathbf{S}$$

$$\beta = (1, 1, 0, \dots, 0) \in \mathbf{S}$$

$$\implies \alpha + \beta = (0, 2, 0, \dots, 1) \notin \mathbf{S} \quad (2.0.5)$$

From equation (2.0.5) $2 \neq 0^2$. Hence, it is not closed with respect to additions and hence not a subspace.

d) All α such that $a_1 a_2 = 0 \in \mathbf{R}^n$ ($n \geq 3$), Let,

$$\alpha = (0, 1, 0, \dots, 0) \in \mathbf{S}$$

$$\beta = (1, 0, 0, \dots, 0) \in \mathbf{S}$$

$$\implies \alpha + \beta = (1, 1, 0, \dots, 0) \notin \mathbf{S} \quad (2.0.6)$$

From (2.0.6) $1.1 = 1 \neq 0$ it is not closed with respect to addition and hence not a subspace.

e) All α such that a_2 is rational $\in \mathbf{R}^n$ ($n \geq 3$), Let,

$$\alpha = (0, 1, 0, \dots, 0) \in \mathbf{S}$$

$$b = \sqrt{2} \in \mathbf{R}$$

$$b.\alpha = (0, \sqrt{2}, 0, \dots, 0) \notin \mathbf{S} \quad (2.0.7)$$

From (2.0.7) it is not closed with respect to scalar multiplication and hence not a subspace.

1 PROBLEM

Which of the following set of vectors

$$\alpha = (a_1, a_2, \dots, a_n)$$

in \mathbf{R}^n are subspace of \mathbf{R}^n ($n \geq 3$)?

- a) All α such that $a_1 \geq 0$
- b) All α such that $a_1 + 3a_2 = a_3$
- c) All α such that $a_2 = a_1^2$
- d) All α such that $a_1 a_2 = 0$
- e) All α such that a_2 is rational

2 SOLUTION

- a) All α such that $a_1 \geq 0 \in \mathbf{R}^n$ ($n \geq 3$)

Let,

$$\alpha = (1, 0, 0, 0, \dots, 0) \in \mathbf{S}$$

$$(-\alpha) = (-1, 0, 0, 0, \dots, 0) \notin \mathbf{S} \quad (2.0.1)$$

From equation (2.0.1) it is not closed with respect to scalar multiplication and hence not a subspace.

- b) All α such that $a_1 + 3(a_2) \in \mathbf{R}^n$ ($n \geq 3$),

Let $c \in \mathbf{R}$

$$\alpha = (a_1, a_2, a_3, \dots, a_n) \in \mathbf{S}$$

$$\beta = (b_1, b_2, b_3, \dots, b_n) \in \mathbf{S}$$

$$(2.0.2)$$