Challenge Problem

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Curve Fitting

***Abstract*—This document contains the solution to interpolate the curve**

Download all python codes from Download latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Challenge/Challenge.tex

1 Problem

Given n distinct pairs of points, is there always a polynomial of degree at most n−1 which passes through all these points.

2 Construction

Given n points, we construct n equations in n unknowns. Let a_k be the coefficient for x^kin the unknown polynomial, and let (x_k, y_k)be the data point (given in the problem). The equation is given as:

$$(a^n)(x_K^n) + ...(a_1)(x_K) + a_0 = y_K \tag{2.0.1}$$

In matrix-vector form the equation looks like:

$$\begin{pmatrix} x_1^n & x_1^{n-1} & \cdots & 1 \\ x_2^n & x_2^{n-1} & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_n^n & x_n^{n-1} & \cdots & 1 \end{pmatrix} \begin{pmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_0 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \tag{2.0.2}$$

If the points x_i are distinct, then the matrix would have inverse,and that will always gives us the coefficients of our polynomial and hence the equation of curve. We will always get Vandermonde matrix . The non-vanishing of the Vandermonde determinant for distinct points shows that, for distinct points, the map from coefficients to values at those points is a one-to-one correspondence, and thus that the polynomial interpolation problem is solvable with a unique solution.

3 Solution

Let us consider $P(x) = a_0 + a_1x + a_2x^2$. Taking $P(2) = 17$,$P(3) = 11$ and $P(7) = 2$
From above we will get equations like

$$a_0 + 2(a_1) + 4(a_2) = 17 \tag{3.0.1}$$

$$a_0 + 3(a_1) + 9(a_2) = 11 \tag{3.0.2}$$

$$a_0 + 7(a_1) + 49(a_2) = 2 \tag{3.0.3}$$

Now converting above equations in matrix form we have:

$$\begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 4 \\ 1 & 7 & 49 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 17 \\ 11 \\ 12 \end{pmatrix} \tag{3.0.4}$$

Now writing equation in form $X = A^-1B$ we have

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = A^{-1} \begin{pmatrix} 17 \\ 11 \\ 12 \end{pmatrix} \tag{3.0.5}$$

On solving 3.0.5 we have :

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \frac{67}{2} \\ \frac{-39}{4} \\ \frac{3}{4} \end{pmatrix} \tag{3.0.6}$$

Hence equation of the curve is
$y = 3/4x^2 - 39/4x + 67/2$