Matrix theory Assignment 11

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***Abstract*—This document contains the concept of sub space.**

Download all python codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/tree/master/Assignment11/Codes

Download latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Assignment11/Assignment_11.tex

1 Problem

Which of the following set of vectors

$$\alpha = (a_1, a_2 \ldots\ldots\ldots a_n)$$

in $\mathbf{R^n}$ are subspace of $\mathbf{R^n}$ $(n \geq 3)$?
a)All α such that $a_1 \geq 0$
b)All α such that $a_1 + 3a_2 = a_3$
c)All α such that$a_2 = a_1^2$
d)All α such that $a_1 a_2 = 0$
e)All α such that a_2 is rational

2 Solution

a) All α such that $a_1 \geq 0 \in \mathbf{R^n}$ $(n \geq 3)$
Let,

$$\alpha = (a_1, a_2, a_3, a_4 \ldots\ldots\ldots, a_n) \in \mathbf{S}$$
$$(-\alpha) = (-a_1, -a_2, -a_3, \ldots\ldots\ldots, -a_n) \notin \mathbf{S} \tag{2.0.1}$$

From equation (2.0.1) it is not closed with respect to scalar multiplication and hence not a subspace.
b) All α such that $a_1 + 3(a_2) \in \mathbf{R^n}$ $(n \geq 3)$,
Let $c \in \mathbf{R}$

$$\alpha = (a_1, a_2, a_3, \ldots, a_n) \in \mathbf{S}$$
$$\beta = (b_1, b_2, b_3, \ldots, b_n) \in \mathbf{S} \tag{2.0.2}$$

Now, c.(α)+β

$$= c(a_1, a_2, a_3, \ldots, a_n) + (b_1, b_2, b_3, \ldots, b_n)$$
$$= (ca_1, ca_2, ca_3, \ldots, ca_n) + (b_1, b_2, b_3, \ldots, b_n) \tag{2.0.3}$$

Now, using condition $a_1 + 3(a_2)$ we have,

$$ca_1 + b_1 + 3(ca_2 + b_2) = ca_1 + b_1 + 3ca_2 + 3b_2$$
$$\implies c(a_1 + 3a_2) + (b_1 + 3b_2) = ca_3 + b_3 \tag{2.0.4}$$

From equation (2.0.4) it is closed under addition and scalar multiplication, hence it is a subspace.
c) All α such that $a_2 = {a_1}^2 \in \mathbf{R^n}$ $(n \geq 3)$,
Let,

$$\alpha = (a_1, a_1^2, a_3, a_4, \ldots a_n) \in \mathbf{S}$$
$$\beta = (b_1, b_1^2, b_3, b_4 \ldots b_n) \in \mathbf{S}$$

$$\implies \alpha + \beta = (a_1 + b_1, a_1^2 + {b_1}^2, \ldots a_n + b_n) \notin \mathbf{S} \tag{2.0.5}$$

From equation (2.0.5) $(a_1 + b_1)^2 \neq a_1^2 + b_1^2$ it is not closed with respect to additions and hence not a subspace.
d) All α such that $a_1 a_2 = 0 \in \mathbf{R^n}$ $(n \geq 3)$, Let,

$$\alpha = (0, a_1, 0, \ldots 0) \in \mathbf{S}; \quad a_1 \neq 0$$
$$\beta = (b_1, 0, 0, \ldots 0) \in \mathbf{S}; \quad b_1 \neq 0$$
$$\implies \alpha + \beta = (b_1, a_1, 0, \ldots 0) \notin \mathbf{S} \tag{2.0.6}$$

From equation(2.0.6) $a_1 b_1 \neq 0$ it is not closed with respect to addition and hence not a subspace.
e) All α such that $a_2$is rational $\in \mathbf{R^n}$ $(n \geq 3)$, Let,

$$\alpha = (0, a_1, 0, \ldots 0) \in \mathbf{S}$$
$$b = \sqrt{2} \in \mathbf{R}$$
$$b.\alpha = (0, \sqrt{2}a_1, 0, \ldots\ldots 0) \notin \mathbf{S} \tag{2.0.7}$$

From equation(2.0.7) it is not closed with respect to scalar multiplication and hence not a subspace.

$\alpha = (a_1, a_2..........a_n)$	
Vector space	**Subspace summary**
$\alpha = (a_1, a_2, a_3, a_4.........., a_n); \quad a_1 \geq 0$	Not a subspace. Scalar multiplication is not satisfied. $-1(\alpha) \neq \alpha$
$\alpha = (a_1, a_2, a_3, a_4.........., a_n); \quad a_1 + 3a_2 = a_3$	It is a subspace
$\alpha = (a_1, a_2, a_3, a_4.........., a_n); \quad a_2 = a_1^2$	Not a subspace. Addition is not satisfied. $(a_1 + b_1)^2 \neq a_1^2 + b_1^2$
$\alpha = (a_1, a_2, a_3, a_4.........., a_n); \quad a_1 a_2 = 0$	Not a subspace. Addition is not satisfied. $a_1 b_1 \neq 0$
$\alpha = (a_1, a_2, a_3, a_4.........., a_n); \quad a_2$ is rational	Not a subspace. Scalar multiplication is not satisfied. $a_2 \neq \sqrt{2} a_1$

TABLE 0: Summary