Matrix theory Assignment 18

Shivangi Parashar

***Abstract*—This document contains the concept of ideal ploynomials.**

Download all python codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/tree/master/Assignment18/Codes

Download latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Assignment18/Assignment_18.tex

1 Problem

Let A be an $n \times n$ matrix over a field $\mathbf{F}$. Show that the set of all polynomials f in $\mathbf{F}[x]$ such that f(A) = 0 is an ideal.

2 Definition

Let $\mathbf{F}$ be a field. An ideal in $\mathbf{F}[x]$ is a subspace $\mathbf{M}$ of $\mathbf{F}[x]$ such that fg belongs to $\mathbf{M}$ whenever f is in $\mathbf{F}[x]$ and g is in $\mathbf{M}$.

3 Solution

Given a square matrix of order n and $f(A) = 0$. Let $\mathbf{I}$ be an ideal of $C[z1, \cdots, zn]$.

$$\mathbf{I} = \{\mathbf{F} \in \mathbf{F}(x) | f(A) = 0\} \tag{3.0.1}$$

Now,Consider the polynomials,

$$f = f_0 + f_1 z_n + \cdots f_d {z_n}^d \tag{3.0.2}$$

$$g = g_0 + g_1 z_n + \cdots g_e {z_n}^e \tag{3.0.3}$$

$$f, g \in \mathbf{I} \ \ \& \ \ k \in \mathbf{F}.$$

$$(kf + g)(A) = kf(A) + g(A) \tag{3.0.4}$$
$$\implies c.0 + 0 = 0 \tag{3.0.5}$$

From equation(3.0.5) $\mathbf{I}$ is a subspace of $\mathbf{F}(x)$

Now,$f \in$ $\mathbf{I}$ and $g \in F(x)$ then,

$$(gf)A = g(A)f(A) \tag{3.0.6}$$
$$\implies g(A).0 = 0 \tag{3.0.7}$$

Hence,proved $\mathbf{I}$ is ideal