

Challenging problem

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Matrices

Abstract—This document contains the solution of decomposition of matrix .

Download all python codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment/Codes

Download latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment2/Assignment.tex

Now Transposing both sides of equation (3.0.5) we have:

$$\mathbf{V}^T = (\mathbf{PDP}^T)^T \quad (3.0.6)$$

$$\Rightarrow (\mathbf{P}^T)^T \mathbf{D}^T \mathbf{P}^T \quad (3.0.7)$$

$$\Rightarrow \mathbf{PDP}^T = \mathbf{V} \quad (3.0.8)$$

Since $\mathbf{V}^T = \mathbf{V}$

Hence \mathbf{V} should be symmetric matrix for this decomposition.

1 PROBLEM

$\mathbf{V} = \mathbf{PDP}^T$, with $\mathbf{P}^T \mathbf{P} = \mathbf{I}$. So \mathbf{P} is an orthogonal matrix. For what matrices \mathbf{V} do you get this kind of decomposition where \mathbf{P} is an orthogonal.

2 SOLUTION

\mathbf{V} must be symmetric matrix .

3 PROOF

Given, \mathbf{V} a $n \times n$ matrix. \mathbf{P} is orthogonal matrix \mathbf{P} such that, $\mathbf{P}^T \mathbf{V} \mathbf{P}$ is a diagonal matrix \mathbf{D} , then \mathbf{V} is said to be orthogonally diagonalizable and \mathbf{P} is said to orthogonally diagonalize \mathbf{V} . Hence,

$$\mathbf{P}^T \mathbf{V} \mathbf{P} = \mathbf{P}^{-1} \mathbf{V} \mathbf{P} = \mathbf{D} \quad (3.0.1)$$

multiply equation (3.0.1) left by \mathbf{P} we get

$$\mathbf{PP}^T \mathbf{V} \mathbf{P} = \mathbf{PD} \quad (3.0.2)$$

multiply equation (3.0.2) right by \mathbf{P}^T we get.

$$\mathbf{PP}^T \mathbf{V} \mathbf{PP}^T = \mathbf{PDP}^T \quad (3.0.3)$$

Now since \mathbf{P} is orthogonal Hence,

$$\mathbf{PP}^T = \mathbf{P}^T \mathbf{P} = \mathbf{I} \quad (3.0.4)$$

Hence from (3.0.3) we have:

$$\mathbf{V} = \mathbf{PDP}^T \quad (3.0.5)$$