Matrix theory Assignment 14

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***Abstract*—This document contains the concept of linear transformations.**

Download all python codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment14/
    Codes
```

Download latex-tikz codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment14/
    Assignment_14.tex
```

1 Problem

Let $\mathbf{T}$ be the linear operator on $\mathbf{C}^2$ defined by :

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$

. Let β be the standard ordered basis for $\mathbf{C}^2$ and let

$$\beta^{'} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

be the ordered basis defined by

$$\alpha_{\mathbf{1}} = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_{\mathbf{2}} = \begin{pmatrix} -i \\ 2 \end{pmatrix}$$

What is the matrix of $\mathbf{T}$ relative to the pair β,$\beta^{'}$?

2 Solution

Let,

$$\beta = \{\epsilon_1, \epsilon_2\} \implies \epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.1}$$

$$\beta^{'} = \{\alpha_1, \alpha_2\} \implies \alpha_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix} \tag{2.0.2}$$

Let $\mathbf{T}$ be defined by matrices $\mathbf{A}$ such that Let $\mathbf{T}$ be the transformation defined by

$$\mathbf{T}(\mathbf{x}) = \mathbf{A}\mathbf{x} \tag{2.0.3}$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix}\mathbf{x} \tag{2.0.4}$$

we have to write $\mathbf{T}(\epsilon_{\mathbf{1}})$ and $\mathbf{T}(\epsilon_{\mathbf{2}})$ as linear combinations of $\alpha_{\mathbf{1}}$, $\alpha_{\mathbf{2}}$.Hence,

$$\begin{pmatrix} 1 & -i & 1 & 0 \\ i & 2 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 = R_2 - iR_1} \begin{pmatrix} 1 & -i & 1 & 0 \\ 0 & 1 & -i & 0 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_1 = R_1 + iR_2} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -i & 0 \end{pmatrix} \tag{2.0.6}$$

Therefore matrix of relative to the pair β , $\beta^{'}$

$$\mathbf{T}(\beta) = \begin{pmatrix} 2 & 0 \\ -i & 0 \end{pmatrix}\beta \tag{2.0.7}$$