Assignment 6

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***Abstract*—This document explains the concept of affline transformation of equations when the origin is moved to the point**

Download all latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Assignment6/Assignment6.tex

1 Problem

To what point must origin be shifted so that

$$\mathbf{x}^T \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 10 & -19 \end{pmatrix} \mathbf{x} + 23 = 0 \tag{1.0.1}$$

is transformed to

$$\mathbf{x}^T \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \mathbf{x} = 1 \tag{1.0.2}$$

2 Solution

Given,

$$\mathbf{x}^T \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 10 & -19 \end{pmatrix} \mathbf{x} + 23 = 0 \tag{2.0.1}$$

The general second degree equation can be expressed as follows,

$$\mathbf{x}^{\mathbf{T}} \mathbf{V} \mathbf{x} + 2\mathbf{u}^{\mathbf{T}} \mathbf{x} + f = 0 \tag{2.0.2}$$

From the given second degree equation we get,

$$\mathbf{V} = \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u} = \begin{pmatrix} 5 \\ \frac{-19}{2} \end{pmatrix} \tag{2.0.4}$$

$$f = 23 \tag{2.0.5}$$

Origin which is moved to the point is given by

$$\mathbf{c} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.6}$$

The above equation (2.0.2) can be modified as

$$(\mathbf{x} + \mathbf{c})^T \mathbf{V} (\mathbf{x} + \mathbf{c}) + 2\mathbf{u}^T (\mathbf{x} + \mathbf{c}) + 23 = 0 \tag{2.0.7}$$

From equation (2.0.7) consider,

$$\implies (\mathbf{x} + \mathbf{c})^T \mathbf{V} (\mathbf{x} + \mathbf{c}) \tag{2.0.8}$$

$$\implies \mathbf{x}^T \mathbf{V} \mathbf{x} + \mathbf{c}^T \mathbf{V} \mathbf{x} + \mathbf{x}^T \mathbf{V} \mathbf{c} + \mathbf{c}^T \mathbf{V} \mathbf{c} \tag{2.0.9}$$

we know that

$$\mathbf{x}^T \mathbf{V} \mathbf{c} = \mathbf{c}^T \mathbf{V} \mathbf{x} \tag{2.0.10}$$

Substituting equation (2.0.10) in equation (2.0.9)

$$\implies \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{c}^T \mathbf{V} \mathbf{x} + \mathbf{c}^T \mathbf{V} \mathbf{c} \tag{2.0.11}$$

Equation (2.0.7) is modified as

$$\implies \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{c}^T \mathbf{V} \mathbf{x} + \mathbf{c}^T \mathbf{V} \mathbf{c} + 2\mathbf{u}^T \mathbf{x} 2\mathbf{u}^T \mathbf{c} + 23 = 0 \tag{2.0.12}$$

Equating (2.0.7) and (1.0.2):

$$\implies \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{c}^T \mathbf{V} \mathbf{x} + \mathbf{c}^T \mathbf{V} \mathbf{c} + 2\mathbf{u}^T \mathbf{x} 2\mathbf{u}^T \mathbf{c} + 23 = \mathbf{x}^T \mathbf{V} \mathbf{x} - 1 \tag{2.0.13}$$

From above equation (2.0.13) we have,

$$2\mathbf{c}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} = 0 \tag{2.0.14}$$

and

$$2\mathbf{u}^T \mathbf{x} + \mathbf{v}^T \mathbf{c} = -22 \tag{2.0.15}$$

From (2.0.14) we have:

$$2\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \mathbf{x} = -2\begin{pmatrix} 5 & \frac{-19}{2} \end{pmatrix} \tag{2.0.16}$$

$$2\begin{pmatrix} 2a - \frac{-3}{2}b & \frac{-3}{2}a + 4b \end{pmatrix} \mathbf{x} = 2\begin{pmatrix} -5 & \frac{-19}{2} \end{pmatrix} \mathbf{x} \tag{2.0.17}$$

Equating we have:

$$2a - \frac{3}{2}b = -5 \tag{2.0.18}$$

$$\frac{-3}{2}a + 4b = \frac{19}{2} \tag{2.0.19}$$

Converting above equation in augmented form we have :

$$\begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ \frac{-3}{2} & 4 & \frac{19}{2} \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + \frac{3}{4}R_1} \begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ 0 & \frac{23}{8} & \frac{23}{4} \end{pmatrix} \tag{2.0.20}$$

$$\begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ 0 & \frac{23}{8} & \frac{23}{4} \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 \div 238} \begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.21}$$

$$\tag{2.0.22}$$

$$\begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ 0 & 1 & 2 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 \div 32R_2} \begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.23}$$

$$\tag{2.0.24}$$

$$\begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 2 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 \div 2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.25}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{2.0.26}$$

From equation (2.0.26) The point origin need to be shifted:

$$\mathbf{c} = \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{2.0.27}$$

From (2.0.27) when the origin is moved to the point $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\mathbf{V}$ doesn't change

$$\det(\mathbf{V}) = 5.75 \tag{2.0.28}$$

Since $\det(\mathbf{V}) > 0$ the given equation represents the ellipse The below plot 1 verifies the given ellipse

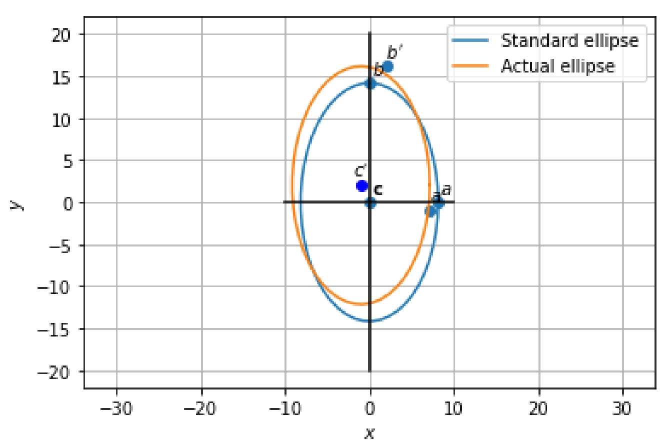

Fig. 1: Ellipse when origin is shifted