Matrix theory Assignment 14

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***Abstract*—This document contains the concept of linear transformations.**

Download all python codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/tree/master/Assignment14/Codes

Download latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Assignment14/Assignment_14.tex

1 Problem

Let $\mathbf{T}$ be the linear operator on $\mathbf{C}^2$ defined by :

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$

. Let β be the standard ordered basis for $\mathbf{C}^2$ and let

$$\beta' = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

be the ordered basis defined by

$$\alpha_{\mathbf{1}} = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_{\mathbf{2}} = \begin{pmatrix} -i \\ 2 \end{pmatrix}$$

What is the matrix of $\mathbf{T}$ relative to the pair β,β'?

2 Solution

$$\beta = \{\epsilon_1, \epsilon_2\} \implies \epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.1}$$

Hence,β as matrix

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.2}$$

$$\beta' = \{\alpha_1, \alpha_2\} \implies \alpha_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix} \tag{2.0.3}$$

Hence, β' as matrix

$$\beta' = \begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix} \tag{2.0.4}$$

Geometrically, $\mathbf{T}$ is a projection onto the x-axis.

For projection, let Consider Matrix A as

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.5}$$

The matrix $\mathbf{A}$ is representation of the linear transformation $\mathbf{T}$ that is projection on x-axis. After applying linear operator $\mathbf{T}$ on it,

$$\mathbf{T}(\beta) = \mathbf{A}\beta = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.6}$$

$$\begin{pmatrix} 1 & -i & 1 & 0 \\ i & 2 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2=R_2-iR_1} \begin{pmatrix} 1 & -i & 1 & 0 \\ 0 & 1 & -i & 0 \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_1=R_1+iR_2} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -i & 0 \end{pmatrix} \tag{2.0.8}$$

Therefore matrix of relative to the pair β , β'

$$\mathbf{T}(\beta) = \begin{pmatrix} 2 & 0 \\ -i & 0 \end{pmatrix}\beta' \tag{2.0.9}$$