# Matrix theory Assignment 16

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Abstract—This document contains the concept of algebra of ploynomials.

Download all python codes from

https://github.com/shivangi-975/EE5609-Matrix\_Theory/tree/master/Assignment16/ Codes

Download latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix\_Theory/blob/master/Assignment16/ Assignment 16.tex

#### 1 Problem

If f and g are independent polynomials over a field **F**and h is a non-zero polynomial over **F**, show that fh and gh are independent.

#### 2 Definition

Polynomials  $(f_1, f_2, ..., f_m) \in k(x_1, ..., x_n)$  are called algebraically independent over a field  $\mathbf{F}$ , if there is no nonzero m-variate polynomial  $A \in k[y_1, ..., y_m]$  such that  $A(f_1, ..., f_m) = 0$ 

### 3 Solution

Given f and g are independent polynomials over a field  $\mathbf{F}$ . Consider scalars a and  $b \in \mathbf{F}$  Hence,

$$af + bf = 0$$
 (3.0.1)

Since f and g are independent Hence f and  $g \neq 0$ 

$$\implies a, b = 0. \tag{3.0.2}$$

Given h a non zero polynomial over **F**. Substituting in equation (3.0.1) we have,

$$a(fh) + b(gh) = 0 (3.0.3)$$

$$(af)h + (bg)h = 0$$
 (3.0.4)

$$(af + bg)h = 0$$
 (3.0.5)

$$af + bg = 0 \tag{3.0.6}$$

f and g are independent polynomial. Also from equation (3.0.2) a=0 and b=0.

Hence proved fh and gh are independent.