Matrix theory Assignment 9

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Abstract—**This document contains the solution to find all solutions of Linear Equation.**

Download all python codes from

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https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment9/Codes
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Download latex-tikz codes from

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https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment9/Assignment_9.tex
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1 Problem

$$\mathbf{A} = \begin{pmatrix} 6 & -4 & 0 \\ 4 & -2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \tag{1.0.1}$$

Find all solutions of $\mathbf{AX} = 2\mathbf{X}$and all solutions of $\mathbf{AX} = 3\mathbf{X}$.The symbol c$\mathbf{X}$ denotes the matrix each entry of which is c times corresponding entry.

2 Solution

$$\mathbf{A} = \begin{pmatrix} 6 & -4 & 0 \\ 4 & -2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \tag{2.0.1}$$

To find all solutions of $\mathbf{AX} = 2\mathbf{X}$ we will look for vectors in the nullspace of $\mathbf{A} - 2\mathbf{I}$:

$$(\mathbf{A} - 2\mathbf{I})\mathbf{X} = 0 \tag{2.0.2}$$

Substituting values in (2.0.2),

$$\begin{pmatrix} 4 & -4 & 0 \\ 4 & -4 & 0 \\ -1 & 0 & 1 \end{pmatrix}\mathbf{X} = 0 \tag{2.0.3}$$

The value of $\mathbf{X}$ is the special solution of $\mathbf{A} - 2\mathbf{I}$ which is nothing but solutions to N($\mathbf{A}$). So to find special solutions,converting $(\mathbf{A} - 2\mathbf{I})$into Row Reduced Echelon Form we have:

$$\begin{pmatrix} 4 & -4 & 0 \\ 4 & -4 & 0 \\ -1 & 0 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 4 & -4 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{4}}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 + R_1} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix} \xleftrightarrow{R_3 \longleftrightarrow R_2}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow -R_2}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.4}$$

Hence from equation (2.0.4) we have:

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}\mathbf{X} = 0 \tag{2.0.5}$$

So, x_3 is a free variable: Let $x_3 = c$. Where c is a constant

$$x_2 - x_3 = 0 \implies x_2 = x_3 = c \tag{2.0.6}$$

$$x_1 - x_3 = 0 \implies x_1 = x_3 = c \tag{2.0.7}$$

So,the solution to $\mathbf{AX} = 2\mathbf{X}$ are all elements of $\mathbf{F}^3$ of form (c, c, c). Hence solutions of $\mathbf{AX} = 2\mathbf{X}$ is,

$$\mathbf{X} = c\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2.0.8}$$

Now To find the solution of $\mathbf{AX} = 3\mathbf{X}$ we will look for vectors in the nullspace of $\mathbf{A} - 3\mathbf{I}$

$$(\mathbf{A} - 3\mathbf{I})\mathbf{X} = 0 \tag{2.0.9}$$

Substituting values in (2.0.9)we have:

$$\begin{pmatrix} 3 & -4 & 0 \\ 1 & -2 & 0 \\ -1 & 0 & 0 \end{pmatrix}\mathbf{X} = 0 \tag{2.0.10}$$

The value of $\mathbf{X}$ is the special solution of $\mathbf{A} - 3\mathbf{I}$ which is nothing but solutions to N($\mathbf{A}$). So to find special solutions,converting ($\mathbf{A} - 3\mathbf{I}$)into Row Reduced Echelon Form we have:

$$\begin{pmatrix} 3 & -4 & 0 \\ 1 & -2 & 0 \\ -1 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & -\frac{4}{3} & 0 \\ 1 & -2 & 0 \\ -1 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R1}$$

$$\begin{pmatrix} 1 & -\frac{4}{3} & 0 \\ 0 & -\frac{2}{3} & 0 \\ -1 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 + R_1} \begin{pmatrix} 1 & -\frac{4}{3} & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & -\frac{4}{3} & 0 \end{pmatrix} \xleftrightarrow{R_1 \longleftrightarrow R_1 - 2R_2}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & -\frac{4}{3} & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 - 2R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - \frac{2}{3}R_2}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.11}$$

Hence from equation (2.0.11),

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\mathbf{X} = 0 \tag{2.0.12}$$

So x_3 is a free variable:

$$x_1 = 0 \tag{2.0.13}$$
$$x_2 = 0 \tag{2.0.14}$$
$$x_3 = c \tag{2.0.15}$$

So,the solution to $\mathbf{AX}$ =3$\mathbf{X}$ are all elements of $\mathbf{F}^3$ of form $(0, 0, c)$. Hence solutions of $\mathbf{AX} = 3\mathbf{X}$ is,

$$\mathbf{X} = c\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.16}$$

Now finding eigen values of A we have:

$$\mathbf{A} = \begin{pmatrix} 6-\lambda & -4 & 0 \\ 4 & -2-\lambda & 0 \\ -1 & 0 & 3-\lambda \end{pmatrix} \tag{2.0.17}$$

calculating we have:

$$-(\lambda - 2)(\lambda^2 - 5\lambda + 6) \tag{2.0.18}$$
$$\implies -(\lambda - 2)(\lambda - 2)(\lambda - 3) \tag{2.0.19}$$

Solving we have eigen values as $\lambda = 2, 3$ so the solutions to $\mathbf{AX}$ =2$\mathbf{X}$ is nothing but eigen vector corresponding to $\lambda = 2$

$$\mathbf{X} = c\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2.0.20}$$

which is same as equation 2.0.8
Now, solutions to $\mathbf{AX}$ =3$\mathbf{X}$ is nothing but eigen vector corresponding to $\lambda = 3$

$$\mathbf{X} = c\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.21}$$

which is same as equation 2.0.16