Matrix theory Assignment 19

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***Abstract*—This document contains the concept of matrix diagonalization.**

Download all python codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment19/
    Codes
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Download latex-tikz codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment19/
    Assignment_19.tex
```

1 Problem

Let A be a 3×3 matrix with real entries.Identify the correct statements.

1.A is necessarily diagonalizable over $\mathbf{R}$

2.If A has distinct real eigen values than it is diagonalizable over$\mathbf{R}$

3.If A has distinct eigen values than it is diagonalizable over $\mathbf{C}$

4.If all eigen values are non zero than it is diagonalizable over $\mathbf{C}$

2 Solution

Statement 1.	A is necessarily diagonalizable over **R**
False statement Example:	matrix A is diagonalizable if and only if there is a basis of $\mathbf{R}^3$consisting of eigenvectors of A. Consider a matrix $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ Eigen values are: $\begin{pmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{pmatrix} = 0. \implies \lambda_1 = 1, \lambda_2 = 4$ $\lambda_1 = 1$ has eigen vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\lambda_2 = 4$ has eigen vector $\begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}$ We have found only two linearly independent eigenvectors for A,Hence not diagonalisable
Statement 2.	If A has distinct real eigen values than it is diagonalizable over**R**
True statement Proof	A has n linearly independent eigenvectors which implies it is diagonalizable. **Suppose A is Diagonalizable**.Than,$P^{-1}AP = D$ and hence,AP=PD where P is invertible matrix and D is a diagonal matrix. D=$\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$ and P=$\begin{pmatrix} P_1 & P_2 & \cdots & P_n \end{pmatrix}$ Since AP=PA $\implies \begin{pmatrix} AP_1 & AP_2 & \cdots & AP_n \end{pmatrix} = \begin{pmatrix} \lambda_1 P_1 & \cdots & \lambda_n P_n \end{pmatrix}$ So,AP_i=$\lambda_i P_i$ where $i = 1, \cdots, n$ Now,P is invertible .Hence,P_i is eigen vector of A for λ Also rank(P)=n.So it's columns $\begin{pmatrix} P_1 & P_2 & \cdots & P_n \end{pmatrix}$ are linearly independent **Now,converse**. If$\begin{pmatrix} p_1 & p_2 & \cdots & p_n \end{pmatrix}$are n independent eigen vectors then, $AP_1 = \lambda P_1, \cdots, AP_n = \lambda P_n$ D=$\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$ and P=$\begin{pmatrix} P_1 & P_2 & \cdots & P_n \end{pmatrix}$ Now, $AP_i = \lambda_i P_i \implies AP = PD$ so,$P^{-1}AP = D$ is a diagonal matrix.
Statement 3.	If all eigen values are non zero than it is diagonalizable over **C**
True statement Example:	A has n linearly independent eigenvectors which implies it is diagonalizable. A=$\begin{pmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{pmatrix} \implies \lambda_1 = 2, \lambda_2 = 3, \lambda_3 = 6$ Eigen vectors are:$x_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, x_3 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ respectively
Statement 4.	If all eigen values are non zero than it is diagonalizable over **C**
False statement	counter example same as statement 1 ex. eigen values are non zero but not diagonalizable.