

Assignment 6

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Abstract—This document explains the concept of affine transformation of equations when the origin is moved to the point

Download all latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment6/Assignment6.tex

1 PROBLEM

To what point must origin be shifted so that

$$\mathbf{x}^T \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \mathbf{x} + (10 \quad -19) \mathbf{x} + 23 = 0 \quad (1.0.1)$$

is transformed to

$$\mathbf{x}^T \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \mathbf{x} = 1 \quad (1.0.2)$$

2 SOLUTION

Given,

$$\mathbf{x}^T \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \mathbf{x} + (10 \quad -19) \mathbf{x} + 23 = 0 \quad (2.0.1)$$

The general second degree equation can be expressed as follows,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.2)$$

From the given second degree equation we get,

$$\mathbf{V} = \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{u} = \begin{pmatrix} 5 \\ \frac{-19}{2} \end{pmatrix} \quad (2.0.4)$$

$$f = 23 \quad (2.0.5)$$

Origin which is moved to the point is given by

$$\mathbf{c} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.6)$$

The above equation (2.0.2) can be modified as

$$(\mathbf{x} + \mathbf{c})^T \mathbf{V} (\mathbf{x} + \mathbf{c}) + 2\mathbf{u}^T (\mathbf{x} + \mathbf{c}) + 23 = 0 \quad (2.0.7)$$

From equation (2.0.7) consider,

$$\Rightarrow (\mathbf{x} + \mathbf{c})^T \mathbf{V} (\mathbf{x} + \mathbf{c}) \quad (2.0.8)$$

$$\Rightarrow \mathbf{x}^T \mathbf{V} \mathbf{x} + \mathbf{c}^T \mathbf{V} \mathbf{x} + \mathbf{x}^T \mathbf{V} \mathbf{c} + \mathbf{c}^T \mathbf{V} \mathbf{c} \quad (2.0.9)$$

we know that

$$\mathbf{x}^T \mathbf{V} \mathbf{c} = \mathbf{c}^T \mathbf{V} \mathbf{x} \quad (2.0.10)$$

Substituting equation (2.0.10) in equation (2.0.9)

$$\Rightarrow \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{c}^T \mathbf{V} \mathbf{x} + \mathbf{c}^T \mathbf{V} \mathbf{c} \quad (2.0.11)$$

Equation (2.0.7) is modified as

$$\Rightarrow \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{c}^T \mathbf{V} \mathbf{x} + \mathbf{c}^T \mathbf{V} \mathbf{c} + 2\mathbf{u}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{c} + 23 = 0 \quad (2.0.12)$$

Equating (2.0.7) and (1.0.2):

$$\Rightarrow \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{c}^T \mathbf{V} \mathbf{x} + \mathbf{c}^T \mathbf{V} \mathbf{c} + 2\mathbf{u}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{c} + 23 = \mathbf{x}^T \mathbf{V} \mathbf{x} - 1 \quad (2.0.13)$$

From above equation (2.0.13) we have,

$$2\mathbf{c}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} = 0 \quad (2.0.14)$$

and

$$2\mathbf{u}^T \mathbf{x} + \mathbf{v}^T \mathbf{c} = -22 \quad (2.0.15)$$

From (2.0.14) we have:

$$2 \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 4 \end{pmatrix} \mathbf{x} = -2 \begin{pmatrix} 5 & \frac{-19}{2} \end{pmatrix} \mathbf{x} \quad (2.0.16)$$

$$2 \left(2a - \frac{3}{2}b \quad \frac{-3}{2}a + 4b \right) \mathbf{x} = 2 \left(-5 \quad \frac{-19}{2} \right) \mathbf{x} \quad (2.0.17)$$

Equating we have:

$$2a - \frac{3}{2}b = -5 \quad (2.0.18)$$

$$\frac{-3}{2}a + 4b = \frac{19}{2} \quad (2.0.19)$$

Converting above equation in augmented form we have :

$$\begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ \frac{-3}{2} & 4 & \frac{19}{2} \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + \frac{3}{4}R_1} \begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ 0 & \frac{23}{8} & \frac{23}{4} \end{pmatrix} \quad (2.0.20)$$

$$\begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ 0 & \frac{23}{8} & \frac{23}{4} \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 \div \frac{23}{8}} \begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ 0 & 1 & 2 \end{pmatrix} \quad (2.0.21)$$

$$(2.0.22)$$

$$\begin{pmatrix} 2 & \frac{-3}{2} & -5 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 \div \frac{3}{2} R_2} \begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 2 \end{pmatrix} \quad (2.0.23)$$

$$(2.0.24)$$

$$\begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 \div 2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix} \quad (2.0.25)$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (2.0.26)$$

From equation (2.0.26) The point origin need to be shifted:

$$\mathbf{c} = \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (2.0.27)$$

From (2.0.27) when the origin is moved to the point $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ \mathbf{V} doesn't change

$$\det(\mathbf{V}) = 5.75 \quad (2.0.28)$$

Since $\det(\mathbf{V}) > 0$ the given equation represents the ellipse The below plot 1 verifies the given ellipse

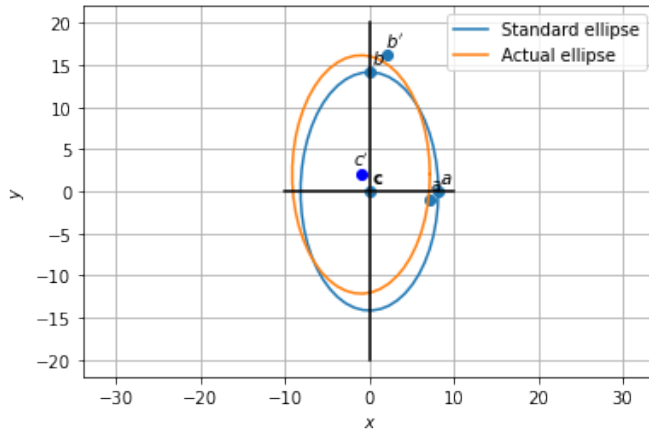


Fig. 1: Ellipse when origin is shifted