Matrix theory Assignment 13

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***Abstract*—This document contains the concept of linear transformation.**

Download all python codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment13/
    Codes
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Download latex-tikz codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment13/
    Assignment_13.tex
```

1 Problem

Let $\mathbf{T}$ and $\mathbf{U}$ be the linear operators on $\mathbb{R}^2$ defined by $\mathbf{T}(x_1, x_2) = (x_2, x_1)$ and $\mathbf{U}(x_1, x_2) = (x_1, 0)$.

Give rules like the ones defining $\mathbf{T}$ and $\mathbf{U}$ for each of the transformations $\mathbf{U}+\mathbf{T}$, $\mathbf{UT}$, $\mathbf{TU}$, $\mathbf{T}^2$, $\mathbf{U}^2$. $\mathbb{R}^2$ into $\mathbb{R}^2$ is linear transformation?

2 Solution

Let $\mathbf{T}$ and $\mathbf{U}$ defined by matrices $\mathbf{A}$ and $\mathbf{B}$ such that ,

$$\mathbf{T}(\mathbf{x}) = \mathbf{A}\mathbf{x}; \quad \mathbf{U}(\mathbf{x}) = \mathbf{B}\mathbf{x} \tag{2.0.1}$$

Where,

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.2}$$

For Transformation $(\mathbf{U}+\mathbf{T})$ we have,

$$(\mathbf{U}+\mathbf{T})\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{U}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \mathbf{T}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 \end{pmatrix} \tag{2.0.3}$$

For Transformation $\mathbf{UT}$ we have,

$$\mathbf{UT}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{U}\left[\mathbf{T}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ 0 \end{pmatrix} \tag{2.0.4}$$

For Transformation $\mathbf{TU}$ we have,

$$\mathbf{TU}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{T}\left[\mathbf{U}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ x_1 \end{pmatrix} \tag{2.0.5}$$

For Transformation $\mathbf{T}^2$ we have,

$$\mathbf{T}^2\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{T}\left[\mathbf{T}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.0.6}$$

For Transformation $\mathbf{U}^2$ we have,

$$\mathbf{U}^2\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{U}\left[\mathbf{U}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} \tag{2.0.7}$$