

# Matrix theory Assignment 18

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Now,  $f \in \mathbf{I}$  and  $g \in F(x)$  then,

**Abstract**—This document contains the concept of ideal polynomials.

$$(gf)A = g(A)f(A) \quad (3.0.4)$$

$$\implies g(A).0 = 0 \quad (3.0.5)$$

Download all python codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/tree/master/Assignment18/Codes](https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment18/Codes)

Hence, proved  $\mathbf{I}$  is ideal

Download latex-tikz codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/blob/master/Assignment18/Assignment\\_18.tex](https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment18/Assignment_18.tex)

## 1 PROBLEM

Let  $A$  be an  $n \times n$  matrix over a field  $\mathbf{F}$ . Show that the set of all polynomials  $f$  in  $\mathbf{F}[x]$  such that  $f(A) = 0$  is an ideal.

## 2 DEFINITION

Let  $\mathbf{F}$  be a field. An ideal in  $\mathbf{F}[x]$  is a subspace  $\mathbf{M}$  of  $\mathbf{F}[x]$  such that  $fg$  belongs to  $\mathbf{M}$  whenever  $f$  is in  $\mathbf{F}[x]$  and  $g$  is in  $\mathbf{M}$ .

## 3 SOLUTION

Given a square matrix of order  $n$  and  $f(A) = 0$ . Let  $\mathbf{I}$  be an ideal of  $C[z_1, \dots, z_n]$ .

$$\mathbf{I} = \{f \in \mathbf{F}(x) | f(A) = 0\} \quad (3.0.1)$$

$$f, g \in \mathbf{I} \ \& \ k \in \mathbf{F}.$$

Now, Consider the polynomials  $f = f_0 + f_1 z^n + \dots + f_d z_n^d$  and  $g = g_0 + g_1 z^n + \dots + g_e z_n^e$  of degree  $d$  and  $e$  respectively.

$$(kf + g)(A) = kf(A) + g(A) \quad (3.0.2)$$

$$\implies c.0 + 0 = 0 \quad (3.0.3)$$

From (3.0.3)  $\mathbf{I}$  is a subspace of  $\mathbf{F}(x)$