

# Matrix theory Assignment 10

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**Abstract**—This document contains the concept of vector space  $\mathbf{V}$  over a field  $\mathbf{F}$ .

Download all python codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/tree/master/Assignment10/Codes](https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment10/Codes)

Download latex-tikz codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/blob/master/Assignment10/Assignment\\_10.tex](https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment10/Assignment_10.tex)

## 1 PROBLEM

If  $\mathbf{V}$  is a vector space over field  $\mathbf{F}$ , verify that:

$$(\alpha_1 + \alpha_2) + (\alpha_3 + \alpha_4) = [\alpha_2 + (\alpha_3 + \alpha_1)] + \alpha_4$$

## 2 THEORY

vector space  $\mathbf{V}$  is an Abelian group over field  $\mathbf{F}$  on operation  $+$  (vector addition) as it satisfies following properties.

Closure law: If  $\mathbf{u}, \mathbf{v} \in \mathbf{V}$ ,

$$\mathbf{u} + \mathbf{v} \in \mathbf{V}$$

Commutative law:  $\forall \mathbf{u}, \mathbf{v} \in \mathbf{V}$ ,

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

Associative law:  $\forall \mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbf{V}$ ,

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$$

Additive identity:  $\exists \mathbf{v} \in \mathbf{V}$ ,

$$\mathbf{0} + \mathbf{v} = \mathbf{v}$$

$$\mathbf{v} + \mathbf{0} = \mathbf{v}$$

Additive inverses:  $\forall \mathbf{v} \in \mathbf{V}$ ,

$$\mathbf{v} + \mathbf{x} = \mathbf{0}$$

$$\mathbf{x} + \mathbf{v} = \mathbf{0}$$

have a solution  $\mathbf{x}$  in  $\mathbf{V}$ , called an additive inverse of  $\mathbf{v}$ , and denoted by  $-\mathbf{v}$ .

## 3 SOLUTION

Using property of commutativity of  $(+)$  in  $\mathbf{V}$

$$(\alpha_1 + \alpha_2) + (\alpha_3 + \alpha_4) = (\alpha_2 + \alpha_1) + (\alpha_3 + \alpha_4) \quad (3.0.1)$$

Using property of associativity of  $(+)$  in  $\mathbf{V}$

$$(\alpha_2 + \alpha_1) + (\alpha_3 + \alpha_4) = \alpha_2 + [\alpha_1 + (\alpha_3 + \alpha_4)] \quad (3.0.2)$$

Using property of commutativity of  $(+)$  in  $\mathbf{V}$

$$\alpha_2 + [\alpha_1 + (\alpha_3 + \alpha_4)] = \alpha_2 + (\alpha_3 + \alpha_1) + \alpha_4 \quad (3.0.3)$$

Using property of associativity of  $(+)$  in  $\mathbf{V}$

$$\alpha_2 + (\alpha_3 + \alpha_1) + \alpha_4 = [\alpha_2 + (\alpha_3 + \alpha_1)] + \alpha_4 \quad (3.0.4)$$