

Assignment 19

Shivangi Parashar
AI20MTECH14012

Download latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment19/Assignment_19.tex

1 QUESTION

Let A be a 3×3 matrix with real entries. Identify the correct statements.

1. A is necessarily diagonalizable over \mathbf{R}
2. If A has distinct real eigen values than it is diagonalizable over \mathbf{R}
3. If A has distinct eigen values than it is diagonalizable over \mathbf{C}
4. If all eigen values are non zero than it is diagonalizable over \mathbf{C}

2 SOLUTION

Statement 1.	A is necessarily diagonalizable over \mathbf{R}
False statement Example:	<p>Matrix A is diagonalizable if and only if there is a basis of \mathbf{R}^3 consisting of eigenvectors of A.</p> <p>Consider a matrix</p> $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix} \quad (2.0.1)$ <p>Eigen values are:</p> $\begin{pmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{pmatrix} = 0. \implies \lambda_1 = 1, \lambda_2 = 4 \quad (2.0.2)$ <p>$\lambda_1 = 1$ has eigen vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\lambda_2 = 4$ has eigen vector $\begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}$ (2.0.3)</p> <p>We have found only two linearly independent eigenvectors for A, not diagonalisable</p>
Statement 2.	If A has distinct real eigen values then it is diagonalizable over \mathbf{R}
True statement Proof 1:	<p>Distinct real eigenvalues implies linearly independent eigenvectors spanning the entire space. and if a matrix has n linearly independent vectors then it is diagonalizable.</p> <p>Distinct eigen values implies linearly independent vectors that spans entire space.</p> <p>Consider 3 eigen vectors \mathbf{v}, \mathbf{w} and \mathbf{u} with eigen values λ, μ, ν respectively. such that $\lambda \neq \mu \neq \nu$</p> $\alpha(\mathbf{v}) + \beta(\mathbf{w}) + \gamma(\mathbf{u}) = 0 \quad (2.0.4)$ $\alpha A(\mathbf{v}) + \beta A(\mathbf{w}) + \gamma A(\mathbf{u}) = 0 \quad (2.0.5)$ $\alpha \lambda \mathbf{v} + \beta \mu \mathbf{w} + \gamma \nu \mathbf{u} = 0 \quad (2.0.6)$ <p>Multiplying (2.0.4) with $-\lambda$ and subtracting from (2.0.6) we have,</p> $\beta(\mu - \lambda)\mathbf{w} + \gamma(\nu - \lambda)\mathbf{u} = 0 \quad (2.0.7)$ <p>From equation (2.0.7) we have, $\beta = \gamma = 0$ substituting $\beta = \gamma = 0$ in equation (2.0.4) we have, $\alpha = \beta = \gamma = 0$ which proves that vectors are linearly independent.</p>
Proof 2:	<p>If a matrix has n linearly independent vectors then it is diagonalizable</p> <p>If $(\mathbf{p}_1 \ \mathbf{p}_2 \ \cdots \ \mathbf{p}_n)$ are n independent eigen vectors then, $A\mathbf{p}_1 = \lambda\mathbf{p}_1, \dots, A\mathbf{p}_n = \lambda\mathbf{p}_n$</p> $D = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix} P = (\mathbf{p}_1 \ \mathbf{p}_2 \ \cdots \ \mathbf{p}_n) \quad (2.0.8)$ <p>Now, $A\mathbf{p}_i = \lambda_i\mathbf{p}_i \implies AP = PD$ so, $P^{-1}AP = D$ is a diagonal matrix.</p>

TABLE 1: Solution

Statement 3.	If A has distinct real eigen values than it is diagonalizable over \mathbb{C}
True statement	If A is an $N \times N$ complex matrix with n distinct eigenvalues, then any set of n corresponding eigenvectors form a basis for \mathbb{C}^n
Proof:	It is sufficient to prove that the set of eigenvectors is linearly independent which is proved in statement 2.
Example:	$A = \begin{pmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{pmatrix} \quad (2.0.9)$ <p>Eigen values of A are:</p> $\lambda_1 = 2, \lambda_2 = 3, \lambda_3 = 6 \quad (2.0.10)$ <p>Eigen vectors are:</p> $x_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, x_3 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \quad (2.0.11)$ <p>Matrix A is diagonalizable because there is a basis of \mathbb{C}^3 consisting of eigenvectors of A.</p>
Statement 4.	If all eigen values are non zero than it is diagonalizable over \mathbb{C}
False Statement:	Matrix would be diagonalizable if and only if it has linearly independent eigenvectors spanning the entire space.
Example:	<p>Consider a matrix</p> $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix} \quad (2.0.12)$ <p>Eigen values are:</p> $\begin{pmatrix} 1 - \lambda & 1 & 0 \\ 0 & 1 - \lambda & 1 \\ 0 & 0 & 4 - \lambda \end{pmatrix} = 0. \implies \lambda_1 = 1, \lambda_2 = 4 \neq 0 \quad (2.0.13)$ <p>$\lambda_1 = 1$ has eigen vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\lambda_2 = 4$ has eigen vector $\begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}$ (2.0.14)</p> <p>We have found only two linearly independent eigenvectors for A, not diagonalisable</p>

TABLE 2: Solution