Matrix theory Assignment 19

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***Abstract*—This document contains the concept of matrix diagonalization.**

Download all python codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment19/
    Codes
```

Download latex-tikz codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment19/
    Assignment_19.tex
```

1 Problem

Let A be a 3×3 matrix with real entries.Identify the correct statements.

1.A is necessarily diagonalizable over $\mathbf{R}$

2.If A has distinct real eigen values than it is diagonalizable over$\mathbf{R}$

3.If A has distinct eigen values than it is diagonalizable over $\mathbf{C}$

4.If all eigen values are non zero than it is diagonalizable over $\mathbf{C}$

2 Solution

Given	A 3×3 matrix with real entries.
Statement 1.	A is necessarily diagonalizable over **R**
False statement	matrix A is diagonalizable if and only if there is a basis of $\mathbf{R}^3$consisting of eigenvectors of A. Counter example : $\begin{pmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 4\end{pmatrix}$
Statement 2.	If A has distinct real eigen values than it is diagonalizable over**R**
True statement Proof	A has n linearly independent eigenvectors which implies it is diagonalizable. Consider any invertible matrix P with columns $\left(v_1, v_2, \cdots, v_n\right)$ and any diagonal matrix D with diagonal entries $\left(\lambda_1, \lambda_2, \cdots, \lambda_n\right)$ Now,AP=A$\left(v_1, v_2, \cdots, v_n\right)$=$\left(Av_1, Av_2, \cdots, Av_n\right)$ PD=$\left(v_1, v_2, \cdots, v_n\right)\begin{pmatrix}\lambda_1 & 0 & \cdots & 0\\0 & \lambda_2 & \cdots & 0\\\vdots & \vdots & \vdots & \vdots\\0 & 0 & \cdots & \lambda_n\end{pmatrix} \implies \left(\lambda_1 v_1 \quad \lambda_2 v_2 \quad \cdots \lambda_n v_n\right)$ Suppose that A has n linearly independent eigen vectors. Now,$Av_i = \lambda v_i$. Thus,AP=PD Hence we have,$D = P^{-1}AP$ and so A is diagonalisable with diagonalising matrix P. Now suppose A is diagonalizable . Than there is invertible matrix P and a diagonal matrix D with entries $\lambda_1, \lambda_2, \cdots, \lambda_n$. such that $D = P^{-1}AP$ So $PD = AP$, which means$Av_i = \lambda_i v_i$ for each $i = 1, 2, \cdots, n$ that is, each v_i is an eigenvector of A.Since P is invertible, the columns of P form an independent set of vectors, and therefore **A has n linearly independent eigenvectors which implies it is diagonalisable**.
Statement 3.	If all eigen values are non zero than it is diagonalizable over **C**
True statement	A has n linearly independent eigenvectors which implies it is diagonalizable.
Statement 4.	If all eigen values are non zero than it is diagonalizable over **C**
False statement	matrix A is diagonalizable if and only if there is a basis of $\mathbf{R}^3$consisting of eigenvectors of A.

TABLE 1: Summary