Assignment 4

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Geometry

***Abstract*—This documnet contains the solution to prove angles of a equilateral triangles are 60 degrees through Linear Algebra .**

Download all python codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment4/
    Codes
```

Download latex-tikz codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment4/
    Assignment4.tex
```

1 Problem

To prove angles of equilateral triangles are 60 °each.

2 Solution

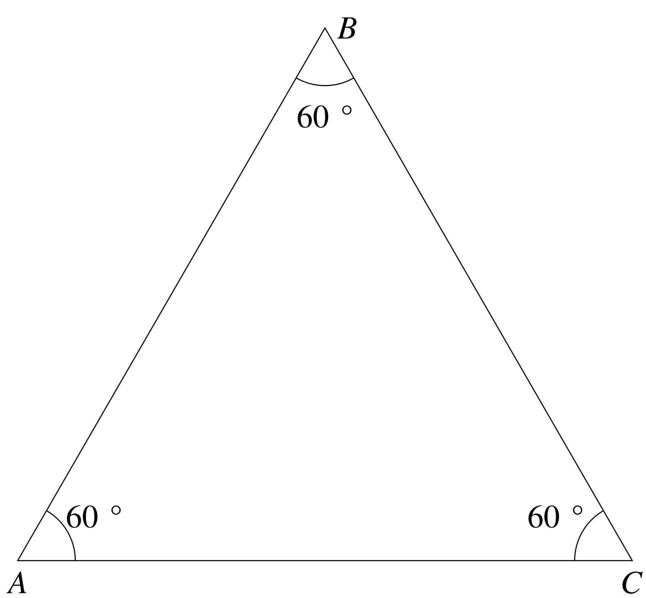

Fig. 1: Equilateral $\triangle ABC$ with A,B and C as vertices

Considering A,B and C as the vertices of triangle:

$$A = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

In equilateral triangle all sides are equal.Hence,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.1}$$

Putting $\mathbf{B}$= 0 in 2.0.1 we have,

$$\|\mathbf{A}\| = \|\mathbf{C}\| \tag{2.0.2}$$

$$\|\mathbf{A}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.3}$$

Squaring equation 2.0.2

$$\|\mathbf{A}\|^2 = \|\mathbf{C}\|^2 \tag{2.0.4}$$

Squaring equation 2.0.3

$$\|\mathbf{A}\|^2 = \|\mathbf{A}\|^2 - 2\mathbf{A}^T C + \|\mathbf{C}\|^2$$
$$\implies \|\mathbf{A}\|^2 = 2\mathbf{A}^T C \tag{2.0.5}$$

Taking the inner product of sides AB,BC we have:

$$(\mathbf{A} - \mathbf{B})^T(\mathbf{B} - \mathbf{C}) = \|\mathbf{A} - \mathbf{B}\| \|\mathbf{B} - \mathbf{C}\| \cos ABC \tag{2.0.6}$$

The angle ABC from the above equation is:

$$\cos ABC = \frac{(\mathbf{A} - \mathbf{B})^T(\mathbf{B} - \mathbf{C})}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{B} - \mathbf{C}\|} \tag{2.0.7}$$

Substituting value in 2.0.7 and putting we have:

$$\cos ABC = \frac{(\mathbf{A})^T(\mathbf{C})}{\|\mathbf{A}\|^2} \tag{2.0.8}$$

From 2.0.5 we have:

$$\cos ABC = \frac{(\mathbf{A})^T(\mathbf{C})}{2(\mathbf{A})^T(C)}$$
$$\implies \cos ABC = 1/2$$
$$\implies \angle ABC = 60° \tag{2.0.9}$$

Taking the inner product of sides AB,AC we have:

$$(\mathbf{B} - \mathbf{A})^T(\mathbf{C} - \mathbf{A}) = \|\mathbf{B} - \mathbf{A}\| \|\mathbf{C} - \mathbf{A}\| \cos BAC \tag{2.0.10}$$

The angle BAC from the above equation is:

$$\cos BAC = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{C}-\mathbf{A})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{C}-\mathbf{A}\|} \tag{2.0.11}$$

Substituting value in 2.0.11 and putting we have:

$$\cos BAC = \frac{(\mathbf{A})^T(\mathbf{C}-\mathbf{A})}{\|\mathbf{A}\|^2} \tag{2.0.12}$$

From 2.0.5 we have:

$$\begin{aligned} \cos BAC &= \frac{(\mathbf{A})^T(\mathbf{C}-\mathbf{A})}{2(\mathbf{A})^T(C)} \\ \implies \cos BAC &= 1/2 \\ \implies \angle BAC &= 60^\circ \end{aligned} \tag{2.0.13}$$

Taking the inner product of sides AC,BC we have:

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{C}\|\cos ACB \tag{2.0.14}$$

The angle ACB from the above equation is:

$$\cos ACB = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.15}$$

Substituting value in 2.0.15 and putting we have:

$$\cos ACB = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{C})}{\|\mathbf{A}\|^2} \tag{2.0.16}$$

From 2.0.5 we have:

$$\begin{aligned} \cos ACB &= \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{C})}{2(\mathbf{A})^T(C)} \\ \implies \cos ACB &= 1/2 \\ \implies \angle ACB &= 60^\circ \end{aligned} \tag{2.0.17}$$

Hence from equation 2.0.9,2.0.13 and2.0.17 we have:

$$\angle ABC = \angle BAC = \angle ACB = 60^\circ \tag{2.0.18}$$