Matrix theory Assignment 13

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***Abstract*—This document contains the concept of linear transformations.**

Download all python codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment13/
    Codes
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Download latex-tikz codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment13/
    Assignment_13.tex
```

1 Problem

Let $\mathbf{T}$ and $\mathbf{U}$ be the linear operators on $\mathbb{R}^2$ defined by $\mathbf{T}(x_1, x_2)$ =(x_2, x_1) and $\mathbf{U}(x_1, x_2)$=$(x_1, 0)$.

Give rules like the ones defining $\mathbf{T}$ and $\mathbf{U}$ for each of the transformations $\mathbf{U}+\mathbf{T}$, $\mathbf{UT}$, $\mathbf{TU}$, $\mathbf{T}^2$, $\mathbf{U}^2$. $\mathbb{R}^2$ into $\mathbb{R}^2$ is linear transformation?

2 Solution

Let $\mathbf{T}$ and $\mathbf{U}$ defined by matrices $\mathbf{A}$ and $\mathbf{B}$ such that ,

$$\mathbf{T}(\mathbf{x}) = \mathbf{A}\mathbf{x}; \quad \mathbf{U}(\mathbf{x}) = \mathbf{B}\mathbf{x} \tag{2.0.1}$$

Where,

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.2}$$

Table 0 lists the summary of each Transformations.

Transformations	**Summary**
$\mathbf{U}+\mathbf{T}$	$(\mathbf{U}+\mathbf{T})\begin{pmatrix}x_1\\x_2\end{pmatrix}=\mathbf{U}\begin{pmatrix}x_1\\x_2\end{pmatrix}+\mathbf{T}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}+\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}x_1+x_2\\x_1\end{pmatrix}$
$\mathbf{UT}$	$\mathbf{UT}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\mathbf{U}\left(\mathbf{T}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)=\begin{pmatrix}1&0\\0&0\end{pmatrix}\left(\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)=\begin{pmatrix}x_2\\0\end{pmatrix}$
$\mathbf{TU}$	$\mathbf{TU}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\mathbf{T}\left(\mathbf{U}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)=\begin{pmatrix}0&1\\1&0\end{pmatrix}\left(\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)=\begin{pmatrix}0\\x_1\end{pmatrix}$
$\mathbf{T}^2$	$\mathbf{T}^2\begin{pmatrix}x_1\\x_2\end{pmatrix}=\mathbf{T}\left(\mathbf{T}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)=\begin{pmatrix}0&1\\1&0\end{pmatrix}\left(\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)=\begin{pmatrix}x_1\\x_2\end{pmatrix}$
$\mathbf{U}^2$	$\mathbf{U}^2\begin{pmatrix}x_1\\x_2\end{pmatrix}=\mathbf{U}\left(\mathbf{U}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)=\begin{pmatrix}1&0\\0&0\end{pmatrix}\left(\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)=\begin{pmatrix}x_1\\0\end{pmatrix}$

TABLE 0: Summary