Challenging problem

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Matrices

***Abstract*—This document contains the solution of decomposition of matrix .**

Download all python codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment/Codes
```

Download latex-tikz codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment2/
    Assignment.tex
```

1 Problem

$\mathbf{V} = \mathbf{PDP^T}$, with $\mathbf{P^TP} = \mathbf{I}$. So $\mathbf{P}$is an orthogonal matrix. For what matrices $\mathbf{V}$ do you get this kind of decomposition where $\mathbf{P}$ is an orthogonal.

2 Solution

$\mathbf{V}$ must be symmetric matrix.

3 Proof

Given, $\mathbf{V}$ a $n \times n$ matrix. P is orthogonal matrix $\mathbf{P}$ such that, $\mathbf{P^TVP}$ is a diagonal matrix $\mathbf{D}$, then $\mathbf{V}$ is said to be orthogonally diagonalizable and P is said to orthogonally diagonalize V.Hence,

$$\mathbf{P^TVP} = \mathbf{P^{-1}VP} = \mathbf{D} \tag{3.0.1}$$

multiply equation (3.0.1) left by $\mathbf{P}$ we get

$$\mathbf{PP^TVP} = \mathbf{PD} \tag{3.0.2}$$

multiply equation (3.0.2) right by $\mathbf{P^T}$ we get.

$$\mathbf{PP^TVPP^T} = \mathbf{PDP^T} \tag{3.0.3}$$

Now since P is orthogonal Hence,

$$\mathbf{PP^T} = \mathbf{P^TP} = \mathbf{I} \tag{3.0.4}$$

Hence from (3.0.3)we have:

$$\mathbf{V} = \mathbf{PDP^T} \tag{3.0.5}$$

If the matrix $\mathbf{V}$ has non repeated eigen values then an orthogonal matrix P always exists. Such that$\mathbf{V} = \mathbf{PDP^T}$
Now Transposing both sides of equation (3.0.5)we have:

$$\mathbf{V}^T = (\mathbf{PDP^T})^\mathbf{T} \tag{3.0.6}$$

$$\implies (\mathbf{P^T})^\mathbf{T}\mathbf{D^TP^T} \tag{3.0.7}$$

$$\implies \mathbf{PDP^T} = \mathbf{V} \tag{3.0.8}$$

Since $\mathbf{V}^T = \mathbf{V}$
Hence $\mathbf{V}$ should be symmetric matrix for this decomposition.