

# Matrix theory Assignment 11

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**Abstract**—This document contains the concept of subspace.

Download all python codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/tree/master/Assignment11/Codes](https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment11/Codes)

Download latex-tikz codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/blob/master/Assignment11/Assignment\\_11.tex](https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment11/Assignment_11.tex)

Now,  $c(\alpha) + \beta$

$$\begin{aligned} &= c(a_1, a_2, a_3, \dots, a_n) + (b_1, b_2, b_3, \dots, b_n) \\ &= (ca_1, ca_2, ca_3, \dots, ca_n) + (b_1, b_2, b_3, \dots, b_n) \end{aligned} \quad (2.0.3)$$

Now, using condition  $a_1 + 3(a_2)$  we have,

$$\begin{aligned} ca_1 + b_1 + 3(ca_2 + b_2) &= ca_1 + b_1 + 3ca_2 + 3b_2 \\ \implies c(a_1 + 3a_2) + (b_1 + 3b_2) &= ca_3 + b_3 \end{aligned} \quad (2.0.4)$$

From equation (2.0.4) it is closed under addition and scalar multiplication, hence it is a subspace.

c) All  $\alpha$  such that  $a_2 = a_1^2 \in \mathbf{R}^n$  ( $n \geq 3$ ),

Let,

$$\alpha = (a_1, a_1^2, a_3, a_4, \dots, a_n) \in \mathbf{S}$$

$$\beta = (b_1, b_1^2, b_3, b_4, \dots, b_n) \in \mathbf{S}$$

$$\implies \alpha + \beta = (a_1 + b_1, a_1^2 + b_1^2, a_3 + b_3, \dots, a_n + b_n) \notin \mathbf{S} \quad (2.0.5)$$

From equation (2.0.5)  $(a_1 + b_1)^2 \neq a_1^2 + b_1^2$  it is not closed with respect to additions and hence not a subspace.

d) All  $\alpha$  such that  $a_1 a_2 = 0 \in \mathbf{R}^n$  ( $n \geq 3$ ), Let,

$$\alpha = (0, a_1, 0, \dots, 0) \in \mathbf{S}; \quad a_1 \neq 0$$

$$\beta = (b_1, 0, 0, \dots, 0) \in \mathbf{S}; \quad b_1 \neq 0$$

$$\implies \alpha + \beta = (b_1, a_1, 0, \dots, 0) \notin \mathbf{S} \quad (2.0.6)$$

From equation (2.0.6)  $a_1 b_1 \neq 0$  it is not closed with respect to addition and hence not a subspace.

e) All  $\alpha$  such that  $a_2$  is rational  $\in \mathbf{R}^n$  ( $n \geq 3$ ), Let,

$$\alpha = (0, a_1, 0, \dots, 0) \in \mathbf{S}$$

$$b = \sqrt{2} \in \mathbf{R}$$

$$b \cdot \alpha = (0, \sqrt{2}a_1, 0, \dots, 0) \notin \mathbf{S} \quad (2.0.7)$$

From equation (2.0.7) it is not closed with respect to scalar multiplication and hence not a subspace.

## 1 PROBLEM

Which of the following set of vectors

$$\alpha = (a_1, a_2, \dots, a_n)$$

in  $\mathbf{R}^n$  are subspace of  $\mathbf{R}^n$  ( $n \geq 3$ )?

- a) All  $\alpha$  such that  $a_1 \geq 0$
- b) All  $\alpha$  such that  $a_1 + 3a_2 = a_3$
- c) All  $\alpha$  such that  $a_2 = a_1^2$
- d) All  $\alpha$  such that  $a_1 a_2 = 0$
- e) All  $\alpha$  such that  $a_2$  is rational

## 2 SOLUTION

a) All  $\alpha$  such that  $a_1 \geq 0 \in \mathbf{R}^n$  ( $n \geq 3$ )

Let,

$$\alpha = (a_1, a_2, a_3, a_4, \dots, a_n) \in \mathbf{S}$$

$$(-\alpha) = (-a_1, -a_2, -a_3, \dots, -a_n) \notin \mathbf{S} \quad (2.0.1)$$

From equation (2.0.1) it is not closed with respect to scalar multiplication and hence not a subspace.

b) All  $\alpha$  such that  $a_1 + 3(a_2) \in \mathbf{R}^n$  ( $n \geq 3$ ),

Let  $c \in \mathbf{R}$

$$\alpha = (a_1, a_2, a_3, \dots, a_n) \in \mathbf{S}$$

$$\beta = (b_1, b_2, b_3, \dots, b_n) \in \mathbf{S}$$

$$(2.0.2)$$