

# Matrix theory Assignment 9

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**Abstract**—This document contains the solution to find all solutions of Linear Equation.

Download all python codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/tree/master/Assignment9/Codes](https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment9/Codes)

Download latex-tikz codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/blob/master/Assignment9/Assignment\\_9.tex](https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment9/Assignment_9.tex)

special solutions, converting  $(\mathbf{A} - 2\mathbf{I})$  into Row Reduced Echelon Form we have:

$$\begin{pmatrix} 4 & -4 & 0 \\ 4 & -4 & 0 \\ -1 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 4 & -4 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{4}}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 + R_1} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_2}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow -R_2}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.4)$$

Hence from equation (2.0.4) we have:

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{X} = 0 \quad (2.0.5)$$

So,  $x_3$  is a free variable: Let  $x_3 = c$ . Where  $c$  is a constant

$$x_2 - x_3 = 0 \implies x_2 = x_3 = c \quad (2.0.6)$$

$$x_1 - x_3 = 0 \implies x_1 = x_3 = c \quad (2.0.7)$$

So, the solution to  $\mathbf{AX} = 2\mathbf{X}$  are all elements of  $\mathbf{F}^3$  of form  $(c, c, c)$ . Hence solutions of  $\mathbf{AX} = 2\mathbf{X}$  is,

$$\mathbf{X} = c \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (2.0.8)$$

Now To find the solution of  $\mathbf{AX} = 3\mathbf{X}$  we will look for vectors in the nullspace of  $\mathbf{A} - 3\mathbf{I}$

$$(\mathbf{A} - 3\mathbf{I})\mathbf{X} = 0 \quad (2.0.9)$$

## 1 PROBLEM

$$\mathbf{A} = \begin{pmatrix} 6 & -4 & 0 \\ 4 & -2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \quad (1.0.1)$$

Find all solutions of  $\mathbf{AX} = 2\mathbf{X}$  and all solutions of  $\mathbf{AX} = 3\mathbf{X}$ . The symbol  $c\mathbf{X}$  denotes the matrix each entry of which is  $c$  times corresponding entry.

## 2 SOLUTION

$$\mathbf{A} = \begin{pmatrix} 6 & -4 & 0 \\ 4 & -2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \quad (2.0.1)$$

To find all solutions of  $\mathbf{AX} = 2\mathbf{X}$  we will look for vectors in the nullspace of  $\mathbf{A} - 2\mathbf{I}$ :

$$(\mathbf{A} - 2\mathbf{I})\mathbf{X} = 0 \quad (2.0.2)$$

Substituting values in (2.0.2),

$$\begin{pmatrix} 4 & -4 & 0 \\ 4 & -4 & 0 \\ -1 & 0 & 1 \end{pmatrix} \mathbf{X} = 0 \quad (2.0.3)$$

The value of  $\mathbf{X}$  is the special solution of  $\mathbf{A} - 2\mathbf{I}$  which is nothing but solutions to  $N(\mathbf{A})$ . So to find

Substituting values in (2.0.9) we have:

$$\begin{pmatrix} 3 & -4 & 0 \\ 1 & -2 & 0 \\ -1 & 0 & 0 \end{pmatrix} \mathbf{X} = 0 \quad (2.0.10)$$

The value of  $\mathbf{X}$  is the special solution of  $\mathbf{A} - 3\mathbf{I}$  which is nothing but solutions to  $N(\mathbf{A})$ . So to find special solutions, converting  $(\mathbf{A} - 3\mathbf{I})$  into Row Reduced Echelon Form we have:

$$\begin{aligned} \begin{pmatrix} 3 & -4 & 0 \\ 1 & -2 & 0 \\ -1 & 0 & 0 \end{pmatrix} &\xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & -\frac{4}{3} & 0 \\ 1 & -2 & 0 \\ -1 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \\ \begin{pmatrix} 1 & -\frac{4}{3} & 0 \\ 0 & -\frac{2}{3} & 0 \\ -1 & 0 & 0 \end{pmatrix} &\xleftrightarrow{R_3 \leftarrow R_3 + R_1} \begin{pmatrix} 1 & -\frac{4}{3} & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & -\frac{4}{3} & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftrightarrow R_1 - 2R_2} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & -\frac{4}{3} & 0 \end{pmatrix} &\xleftrightarrow{R_3 \leftarrow R_3 - 2R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - \frac{2}{3}R_2} \\ &\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.11) \end{aligned}$$

Hence from equation (2.0.11),

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{X} = 0 \quad (2.0.12)$$

So  $x_3$  is a free variable:

$$x_1 = 0 \quad (2.0.13)$$

$$x_2 = 0 \quad (2.0.14)$$

$$x_3 = c \quad (2.0.15)$$

So, the solution to  $\mathbf{AX} = 3\mathbf{X}$  are all elements of  $\mathbf{F}^3$  of form  $(0, 0, c)$ . Hence solutions of  $\mathbf{AX} = 3\mathbf{X}$  is,

$$\mathbf{X} = \mathbf{c} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2.0.16)$$