Matrix theory Assignment 17

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***Abstract*—This document contains the concept of algebra of ploynomials.**

Download all python codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment17/
    Codes
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Download latex-tikz codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment17/
    Assignment_17.tex
```

1 Problem

Let $\mathbf{F}$ be a field. We have considered certain special linear functionals on $\mathbf{F}[x]$ obtained via 'evaluation at t': $L(f) = f(t)$. Such functionals are not only linear but also have the property that L(fg) =L(f)L(g). Prove that if L is any linear functional on F[x] such that L(fg) =L(f)L(g)for all f and g, then either L = 0 or there is a t in F such that L(f) = f(t) for all f.

2 Solution

Let L be a non zero linear transformation.

$$f(x) = a_0 + a_1(x) + a_2(x^2) + \cdots a_n(x^n) \tag{2.0.1}$$

$$L(f) = L(a_0 + a_1(x) + a_2(x^2) + \cdots a_n(x^n)). \tag{2.0.2}$$

Given,L is any linear functional on f(x). Hence,

$$L(f) = L(f.1) = L(f)L(1) \tag{2.0.3}$$

$\implies L(1) \neq 0.$

Similarly,

$$L(1) = L(1)L(1) \tag{2.0.4}$$

$$\implies L(1) = 1 \tag{2.0.5}$$

Similarly, $\forall a \in \mathbf{F}$.

$$L(a) = L(a.1) \implies aL(1) = a \tag{2.0.6}$$

Simplifying (2.0.2) we have,

$$L(f) = L(a_0) + L(a_1)L(x) + L(a_2)L(x^2) + \cdots L(a_n)L(x^n) \tag{2.0.7}$$

From (2.0.6) we have,$L(x) = t$.

$$L(f) = a_0 + a_1L(x) + a_2L(x)^2 + \cdots a_nL(x)^n \tag{2.0.8}$$

$$\implies L(f) = a_0 + a_1(t) + a_2(t)^2 + \cdots a_n(t)^n \tag{2.0.9}$$

Hence proved,$L(f) = f(t)\ \forall\mathbf{F}$