Matrix theory Assignment 19

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***Abstract*—This document contains the concept of matrix diagonalization.**

Download all python codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment19/
    Codes
```

Download latex-tikz codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment19/
    Assignment_19.tex
```

1 Problem

Let A be a 3×3 matrix with real entries.Identify the correct statements.

1.A is necessarily diagonalizable over $\mathbf{R}$

2.If A has distinct real eigen values than it is diagonalizable over$\mathbf{R}$

3.If A has distinct eigen values than it is diagonalizable over $\mathbf{C}$

4.If all eigen values are non zero than it is diagonalizable over $\mathbf{C}$

2 Solution

Given	A 3×3 matrix with real entries.
To prove	A is necessarily diagonalizable over **R**
Proof	matrix A is diagonalizable if and only if there is a basis of $\mathbf{R}^3$consisting of eigenvectors of A. Consider a matrix $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ Eigen values are given by $\begin{pmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{pmatrix}$=0 Simplifying eigen values are $\lambda_1 = 1$,$\lambda_2 = 4$ Eigen vectors are: $\lambda_1 = 1$ has eigen vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\lambda_2 = 4$ has eigen vector $\begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}$ We have found only two linearly independent eigenvectors for A,Hence not diagonalizable

Given	A 3×3 matrix with real entries.
To prove	If A has distinct real eigen values than it is diagonalizable over**R**
Proof	Consider any invertible matrix P with columns $\left(v_1, v_2, \cdots, v_n\right)$ and any diagonal matrix D with diagonal entries $\left(\lambda_1, \lambda_2, \cdots, \lambda_n\right)$ Now,AP=A$\left(v_1, v_2, \cdots, v_n\right)$=$\left(Av_1, Av_2, \cdots, Av_n\right)$ PD=$\left(v_1, v_2, \cdots, v_n\right) \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$ $\implies \left(\lambda_1 v_1 \quad \lambda_2 v_2 \quad \cdots \lambda_n v_n\right)$ Suppose that A has n linearly independent eigen vectors Now,$Av_i = \lambda v_i$ Thus,AP=PD Hence we have,$D = P^{-1}AP$ and so A is diagonalizable with diagonalizing matrix P. Now suppose A is diagonalizable . Than there is invertible matrix P and a diagonal matrix D with entries $\lambda_1, \lambda_2, \cdots, \lambda_n$. such that $D = P^{-1}AP$ So $PD = AP$, which means$Av_i = \lambda_i v_i$ for each $i = 1, 2, \cdots, n$ that is, each v_i is an eigenvector of A.Since P is invertible, the columns of P form an independent set of vectors, and therefore **A has n linearly independent eigenvectors which implies it is diagonalizable**.

Given	A 3×3 matrix with real entries.
To prove	If A has distinct real eigen values than it is diagonalizable over**C**
Proof	eigenvectors for distinct eigenvalues are linearly independent. If vectors are linearly independent than matrix can be diagonalized

Given	A 3×3 matrix with real entries.
To prove	If all eigen values are non zero than it is diagonalizable over **C**
Proof	matrix A is diagonalizable if and only if there is a basis of $\mathbf{R}^3$consisting of eigenvectors of A.

A is necessarily diagonalizable over **R**.	False statement
If A has distinct real eigen values than it is diagonalizable over**R**	True statement
If A has distinct eigen values than it is diagonalizable over **C**	True statement
If all eigen values are non zero than it is diagonalizable over **C**	False statement

TABLE 5: Summary