Matrix theory Assignment 13

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***Abstract*—This document contains the concept of linear transformation.**

Download all python codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/tree/master/Assignment13/Codes

Download latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Assignment13/Assignment_13.tex

1 Problem

Let $\mathbf{T}$ and $\mathbf{U}$ be the linear operators on $\mathbb{R}^2$ defined by $\mathbf{T}(x_1, x_2)$ =(x_2, x_1) and $\mathbf{U}(x_1, x_2)$=$(x_1, 0)$.

Give rules like the ones defining $\mathbf{T}$ and $\mathbf{U}$ for each of the transformations $\mathbf{U}+\mathbf{T}$, $\mathbf{UT}$, $\mathbf{TU}$, $\mathbf{T}^2$, $\mathbf{U}^2$. $\mathbb{R}^2$ into $\mathbb{R}^2$ is linear transformation?

2 Solution

Given,

$$\mathbf{T} : \mathbb{R}^2 \to \mathbb{R}^2$$
$$\mathbf{U} : \mathbb{R}^2 \to \mathbb{R}^2$$

Let ,

$$\alpha = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}; \quad \alpha \in \mathbb{R}^2 \tag{2.0.1}$$

For Transformation $(\mathbf{U}+\mathbf{T})$ we have,

$$(\mathbf{U}+\mathbf{T})\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{U}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \mathbf{T}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$
$$(\mathbf{U}+\mathbf{T})\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} + \begin{pmatrix} x_2 \\ x_1 \end{pmatrix}$$
$$(\mathbf{U}+\mathbf{T})\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 \end{pmatrix} \tag{2.0.2}$$

For Transformation $\mathbf{UT}$ we have,

$$\mathbf{UT}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{U}\left[\mathbf{T}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$
$$\mathbf{UT}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{U}\begin{pmatrix} x_2 \\ x_1 \end{pmatrix}$$
$$\mathbf{UT}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ 0 \end{pmatrix} \tag{2.0.3}$$

For Transformation $\mathbf{TU}$ we have,

$$\mathbf{TU}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{T}\left[\mathbf{U}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$
$$\mathbf{TU}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{T}\begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$
$$\mathbf{TU}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ x_1 \end{pmatrix} \tag{2.0.4}$$

For Transformation $\mathbf{T}^2$ we have,

$$\mathbf{T}^2\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{T}\left[\mathbf{T}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$
$$\mathbf{T}^2\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{T}\begin{pmatrix} x_2 \\ x_1 \end{pmatrix}$$
$$\mathbf{T}^2\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.0.5}$$

For Transformation $\mathbf{U}^2$ we have,

$$\mathbf{U}^2\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{U}\left[\mathbf{U}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]$$
$$\mathbf{U}^2\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{U}\begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$
$$\mathbf{U}^2\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} \tag{2.0.6}$$