Matrix theory Assignment 9

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***Abstract*—This document contains the solution to find all solutions of Linear Equation.**

Download all python codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment9/
    Codes
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Download latex-tikz codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment9/
    Assignment_9.tex
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1 Problem

$$\mathbf{A} = \begin{pmatrix} 6 & -4 & 0 \\ 4 & -2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \tag{1.0.1}$$

Find all solutions of $\mathbf{AX} = 2\mathbf{X}$and all solutions of $\mathbf{AX} = 3\mathbf{X}$.The symbol c$\mathbf{X}$ denotes the matrix each entry of which is c times corresponding entry.

2 Solution

$$\mathbf{A} = \begin{pmatrix} 6 & -4 & 0 \\ 4 & -2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \tag{2.0.1}$$

To calculate solution of $\mathbf{AX} = 2\mathbf{X}$and all solutions of $\mathbf{AX} = 3\mathbf{X}$we calculate eigen values of $\mathbf{A}$:

$$(\mathbf{A} - \lambda\mathbf{I})\mathbf{X} = 0 \tag{2.0.2}$$

Substituting values in (2.0.2),

$$\begin{pmatrix} 6-\lambda & -4 & 0 \\ 4 & -2-\lambda & 0 \\ -1 & 0 & 3-\lambda \end{pmatrix}\mathbf{X} = 0 \tag{2.0.3}$$

Simplifying we have:

$$\begin{pmatrix} 6-\lambda & -4 & 0 \\ 4 & -2-\lambda & 0 \\ -1 & 0 & 3-\lambda \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + \frac{4}{\lambda-6}R_1} \begin{pmatrix} 6-\lambda & -4 & 0 \\ 0 & \frac{-\lambda^2+4\lambda-4}{\lambda-6} & 0 \\ -1 & 0 & 3-\lambda \end{pmatrix}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - \frac{1}{\lambda-6}R_1} \begin{pmatrix} -\lambda+6 & -4 & 0 \\ 0 & \frac{-\lambda^2+4\lambda-4}{\lambda-6} & 0 \\ 0 & \frac{4}{\lambda-6} & -\lambda+3 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 + \frac{4}{\lambda^2-4\lambda+4}R_2}$$

$$\begin{pmatrix} -\lambda+6 & -4 & 0 \\ 0 & \frac{-\lambda^2+4\lambda-4}{\lambda-6} & 0 \\ 0 & 0 & -\lambda+3 \end{pmatrix} \tag{2.0.4}$$

From equation (2.0.4) :

$$-\lambda^3 + 7\lambda^2 - 16\lambda + 12 \tag{2.0.5}$$

From equation (2.0.5) eigen values are:

$$\lambda_1 = 2 \tag{2.0.6}$$

$$\lambda_2 = 3 \tag{2.0.7}$$

solution to $\mathbf{AX} = 2\mathbf{X}$ is eigen vector corresponding to $\lambda = 2$

$$\begin{pmatrix} 4 & -4 & 0 & 0 \\ 4 & -4 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{4}} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 4 & -4 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 4R_1}$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_3 \longleftrightarrow R_2}$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow -R_2} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 + R_2}$$

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.8}$$

So, x_3 is a free variable: Let $x_3 = c$.

$$x_2 - x_3 = 0 \implies x_2 = x_3 = c \tag{2.0.9}$$
$$x_1 - x_3 = 0 \implies x_1 = x_3 = c \tag{2.0.10}$$

So,the solution to $\mathbf{AX} = 2\mathbf{X}$is

$$\mathbf{X} = c\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{2.0.11}$$

solution of $\mathbf{AX} = 3\mathbf{X}$ is eigen vector corresponding to $\lambda = 3$

$$(\mathbf{A} - 3\mathbf{I})\mathbf{X} = 0 \tag{2.0.12}$$

$$\begin{pmatrix}3 & -4 & 0\\1 & -2 & 0\\-1 & 0 & 0\end{pmatrix}\mathbf{X} = 0 \tag{2.0.13}$$

$$\begin{pmatrix}3 & -4 & 0 & 0\\1 & -2 & 0 & 0\\-1 & 0 & 0 & 0\end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix}1 & -\frac{4}{3} & 0 & 0\\4 & -5 & 0 & 0\\-1 & 0 & 0 & 0\end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 4R_1} \begin{pmatrix}1 & -\frac{4}{3} & 0 & 0\\0 & \frac{1}{3} & 0 & 0\\-1 & 0 & 0 & 0\end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 + R_1} \begin{pmatrix}1 & -\frac{4}{3} & 0 & 0\\0 & \frac{1}{3} & 0 & 0\\0 & -\frac{4}{3} & 0 & 0\end{pmatrix}$$

$$\xleftrightarrow{R_2 \longleftrightarrow \frac{R_2}{3}} \begin{pmatrix}1 & \frac{-4}{3} & 0 & 0\\0 & 1 & 0 & 0\\0 & -\frac{4}{3} & 0 & 0\end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 - \frac{4}{3}R_2} \begin{pmatrix}1 & \frac{4}{3} & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\end{pmatrix}$$

$$\xleftrightarrow{R_3 \leftarrow R_1 + \frac{4}{3}R_2} \begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\end{pmatrix} \tag{2.0.14}$$

So x_3 is a free variable:

$$x_1 = 0 \tag{2.0.15}$$
$$x_2 = 0 \tag{2.0.16}$$
$$x_3 = c \tag{2.0.17}$$

So,the solution to $\mathbf{AX}$ =3$\mathbf{X}$ is,

$$\mathbf{X} = c\begin{pmatrix}0\\0\\1\end{pmatrix} \tag{2.0.18}$$