

Assignment 7

Shivangi Parashar

Abstract—This document explains the method of performing QR decomposition on a 2×2 matrix.

Download all python codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment7/codes

and latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/edit/master/Assignment7/Assignment7.tex

1 PROBLEM

Find the QR decomposition on a given 2×2 matrix.

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \quad (1.0.1)$$

2 SOLUTION

The QR decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. QR decomposition of a square matrix is given by,

$$\mathbf{A} = \mathbf{QR} \quad (2.0.1)$$

Here \mathbf{Q} is an orthogonal matrix and \mathbf{R} is an upper triangular matrix.

Given matrix,

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \quad (2.0.2)$$

The column vectors of the matrix is given by,

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad (2.0.3)$$

Equation (2.0.2) can be written in QR form as:

$$\mathbf{QR} = (\mathbf{q}_1 \quad \mathbf{q}_2) \begin{pmatrix} u_1 & u_3 \\ 0 & u_2 \end{pmatrix} \quad (2.0.4)$$

Now,

$$u_1 = \|\mathbf{a}\| = \sqrt{1^2 + 2^2} = \sqrt{5} \quad (2.0.5)$$

$$\mathbf{q}_1 = \frac{\mathbf{a}}{u_1} = \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} \quad (2.0.6)$$

$$u_3 = \frac{\mathbf{q}_1^T \mathbf{b}}{\|\mathbf{q}_1\|^2} = \left(\frac{2}{\sqrt{5}} \quad \frac{1}{\sqrt{5}} \right) \begin{pmatrix} 1 \\ -2 \end{pmatrix} = 0 \quad (2.0.7)$$

$$\mathbf{q}_2 = \frac{\mathbf{b} - u_3 \mathbf{q}_1}{\|\mathbf{b} - u_3 \mathbf{q}_1\|} = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} \end{pmatrix} \quad (2.0.8)$$

$$u_2 = \mathbf{q}_2^T \mathbf{b} = \left(\frac{1}{\sqrt{5}} \quad -\frac{2}{\sqrt{5}} \right) \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \sqrt{5} \quad (2.0.9)$$

Substituting equation (2.0.5) to (2.0.9) in (2.0.4), to the QR Decomposition of the given matrix is:

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & 0 \\ 0 & \sqrt{5} \end{pmatrix} \quad (2.0.10)$$