Assignment 19

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Download latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Assignment19/Assignment_19.tex

1 Question

Let A be a 3×3 matrix with real entries.Identify the correct statements.
1.A is necessarily diagonalizable over $\mathbf{R}$
2.If A has distinct real eigen values than it is diagonalizable over$\mathbf{R}$
3.If A has distinct eigen values than it is diagonalizable over $\mathbf{C}$
4.If all eigen values are non zero than it is diagonalizable over $\mathbf{C}$

2 Explanation

Statement 1.	A is necessarily diagonalizable over $\mathbf{R}$
False statement Example:	Matrix A is diagonalizable if and only if there is a basis of $\mathbf{R}^3$consisting of eigenvectors of A. Consider a matrix $$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix} \tag{2.0.1}$$ Eigen values are: $$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{vmatrix} = 0. \implies \lambda_1 = 1, \lambda_2 = 4 \tag{2.0.2}$$ $$\lambda_1 = 1 \text{ has eigen vector} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{and} \lambda_2 = 4 \text{ has eigen vector} \begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix} \tag{2.0.3}$$ We have found only two linearly independent eigenvectors for A,not diagonalisable

Statement 2.	If A has distinct real eigen values than it is diagonalizable over**R**
True statement	Distinct real eigenvalues implies linearly independent eigenvectors . and if a matrix has n linearly independent vectors than it is diagonalizable.

Proof 1:	**Distinct eigen values implies linearly independent vectors that spans entire space.** Consider 2 eigen vectors $\mathbf{v}$,$\mathbf{w}$ with eigen values λ,μ respectively. such that $\lambda \neq \mu$ $\alpha(\mathbf{v}) + \beta(\mathbf{w}) = 0 \quad (2.0.4)$ $\alpha A(\mathbf{v}) + \beta A(\mathbf{w}) = 0 \quad (2.0.5)$ $\alpha\lambda\mathbf{v} + \beta\mu\mathbf{w} = 0 \quad (2.0.6)$ Multiplying (2.0.4)with -λ and subtracting from (2.0.6) we have, $\beta(\mu - \lambda)\mathbf{w} = 0 \quad (2.0.7)$ eigen values are distinct $(\mu - \lambda) \neq 0$. From equation(2.0.7) we have, $\beta = 0$ substituting $\beta = 0$ in equation (2.0.4)we have,$\alpha = 0$.As, $\mathbf{v} \neq 0$ **which proves that vectors are linearly independent**.
Proof 2:	**If a matrix has n linearly independent vectors than it is diagonalizable** If$\begin{pmatrix}\mathbf{p_1} & \mathbf{p_2} & \cdots & \mathbf{p_n}\end{pmatrix}$are n independent eigen vectors then, $A\mathbf{p_1} = \lambda\mathbf{p_1}, \cdots, A\mathbf{p_n} = \lambda\mathbf{p_n}$ $D = \begin{pmatrix}\lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_n\end{pmatrix} P = \begin{pmatrix}\mathbf{P_1} & \mathbf{P_2} & \cdots & \mathbf{P_n}\end{pmatrix} \quad (2.0.8)$ Now, $A\mathbf{P_i} = \lambda_i\mathbf{P_i} \implies AP = PD$ so,$P^{-1}AP = D$ is a diagonal matrix.
Statement 3.	If A has distinct real eigen values than it is diagonalizable over$\mathbf{C}$
True statement .	If A is an $N \times N$ complex matrix with n distinct eigenvalues, then any set of n corresponding eigenvectors form a basis for $\mathbf{C}^n$
Proof:	It is sufficient to prove that the set of eigenvectors is linearly independent which is proved in statement 2.
Example:	$A = \begin{pmatrix}4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5\end{pmatrix} \quad (2.0.9)$ Eigen values of A are: $\lambda_1 = 2, \lambda_2 = 3, \lambda_3 = 6 \quad (2.0.10)$ Eigen vectors are: $x_1 = \begin{pmatrix}-1 \\ 1 \\ 0\end{pmatrix}, x_2 = \begin{pmatrix}1 \\ 1 \\ 1\end{pmatrix}, x_3 = \begin{pmatrix}-1 \\ -1 \\ 2\end{pmatrix} \quad (2.0.11)$ Matrix A is diagonalizable because there is a basis of $\mathbf{C}^3$consisting of eigenvectors of A.

Statement 4.	If all eigen values are non zero than it is diagonalizable over **C**
False Statement:	Matrix would be diagonalizable if and only if it has linearly independent eigenvectors .
Example:	Consider a matrix $$\begin{pmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 4\end{pmatrix} \tag{2.0.12}$$ Eigen values are: $$\begin{pmatrix}1-\lambda & 1 & 0\\0 & 1-\lambda & 1\\0 & 0 & 4-\lambda\end{pmatrix} = 0. \implies \lambda_1 = 1, \lambda_2 = 4 \neq 0 \tag{2.0.13}$$ $$\lambda_1 = 1 \text{ has eigen vector} \begin{pmatrix}1\\0\\0\end{pmatrix} \text{and} \lambda_2 = 4 \text{ has eigen vector} \begin{pmatrix}1\\3\\9\end{pmatrix} \tag{2.0.14}$$ We have found only two linearly independent eigenvectors for A,not diagonalisable.

TABLE 1: Solution summary