Matrix theory Assignment 12

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Abstract—This document contains the concept of linear transformation.

1 PROBLEM

Is the following function $\mathbf{T}$ from $\mathbb{R}^2$ into $\mathbb{R}^2$ is linear transformation?

$$\mathbf{T}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \sin(x_1) \\ x_2 \end{pmatrix}$$

2 SOLUTION

Let,

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}; \quad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad (2.0.1)$$

where,

$$\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$$

Now $\mathbf{T}$ be the transformation,

$$\mathbf{T} = \begin{pmatrix} \sin(x) & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} \pi \\ 0 \end{pmatrix}$$

$$\mathbf{y} = \begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}$$

$$\mathbf{T}(\mathbf{x} + \mathbf{y}) = \begin{pmatrix} \sin(x) & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{3\pi}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{T}(\mathbf{x}) + \mathbf{T}(\mathbf{y}) = \begin{pmatrix} \sin(x) & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \pi \\ 0 \end{pmatrix} + \begin{pmatrix} \sin(x) & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}$$

$$\implies \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.3)$$

From (2.0.2) and (2.0.3) additive transformation property is not satisfied. Hence not a linear transformation. Now,

$$\mathbf{T}(cx_2) = \begin{pmatrix} \sin(x) & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{c\pi}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \sin(\frac{c\pi}{2}) \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$c\mathbf{T}(x_2) = c\begin{pmatrix} \sin(x) & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} c \\ 0 \end{pmatrix} \quad (2.0.5)$$

From (2.0.4) and (2.0.5) scalar multiplication property is not satisfied. Hence not a linear transformation.