Assignment 19

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Download latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Assignment19/Assignment_19.tex

1 QUESTION

Let A be a 3×3 matrix with real entries.Identify the correct statements.

1.A is necessarily diagonalizable over **R**

2.If A has distinct real eigen values than it is diagonalizable over**R**

3.If A has distinct eigen values than it is diagonalizable over **C**

4.If all eigen values are non zero than it is diagonalizable over **C**

2 Solution

Statement 1.	A is necessarily diagonalizable over **R**
False statement Example:	Matrix A is diagonalizable if and only if there is a basis of $\mathbf{R}^3$consisting of eigenvectors of A. Consider a matrix $$\begin{pmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 4\end{pmatrix} \tag{2.0.1}$$ Eigen values are: $$\begin{pmatrix}1-\lambda & 1 & 0\\0 & 1-\lambda & 1\\0 & 0 & 4-\lambda\end{pmatrix} = 0. \implies \lambda_1 = 1, \lambda_2 = 4 \tag{2.0.2}$$ $$\lambda_1 = 1 \text{ has eigen vector} \begin{pmatrix}1\\0\\0\end{pmatrix} \text{and} \lambda_2 = 4 \text{ has eigen vector} \begin{pmatrix}1\\3\\9\end{pmatrix} \tag{2.0.3}$$ We have found only two linearly independent eigenvectors for A,not diagonalisable
Statement 2.	If A has distinct real eigen values than it is diagonalizable over**R**
True statement Proof 1: Proof 2:	Distinct real eigenvalues means linearly independent eigenvectors spanning the entire space. and if a matrix has linearly independent vectors than it is diagonalizable. **Distinct eigen values implies linearly independent vectors that spans entire space.** Consider 3 eigen vectors **v**,**w** and **u** with eigen values λ,μ,ν respectively. such that $\lambda \neq \mu \neq \nu$ $$\alpha(\mathbf{v}) + \beta(\mathbf{w}) + \gamma(\mathbf{u}) = 0 \tag{2.0.4}$$ $$\alpha A(\mathbf{v}) + \beta A(\mathbf{w}) + \gamma A(\mathbf{u}) = 0 \tag{2.0.5}$$ $$\alpha\lambda\mathbf{v} + \beta\mu\mathbf{w} + \gamma\nu\mathbf{u} = 0 \tag{2.0.6}$$ Multiplying (2.0.4)with -λ and subtracting from (2.0.6) we have, $$\beta(\mu-\lambda)\mathbf{w} + \gamma(\nu-\lambda)\mathbf{u} = 0 \tag{2.0.7}$$ From equation(2.0.7) we have, $\beta = \gamma = 0$ substituting $\beta = \gamma = 0$ in equation (2.0.4)we have,$\alpha = \beta = \gamma = 0$ **which proves that vectors are linearly independent**. **If vectors are linearly independent than matrix is diagonalizable** If$\begin{pmatrix}\mathbf{p_1} & \mathbf{p_2} & \cdots & \mathbf{p_n}\end{pmatrix}$are n independent eigen vectors then, $A\mathbf{p_1} = \lambda\mathbf{p_1}, \cdots, A\mathbf{p_n} = \lambda\mathbf{p_n}$ $$D = \begin{pmatrix}\lambda_1 & 0 & \cdots & 0\\0 & \lambda_2 & \cdots & 0\\\vdots & \vdots & \vdots & \vdots\\0 & 0 & \cdots & \lambda_n\end{pmatrix} P = \begin{pmatrix}\mathbf{P_1} & \mathbf{P_2} & \cdots & \mathbf{P_n}\end{pmatrix} \tag{2.0.8}$$ Now, $A\mathbf{P_i} = \lambda_i\mathbf{P_i} \implies AP = PD$ so,$P^{-1}AP = D$ is a diagonal matrix.

TABLE 1:Solution

Statement 3.	If A has distinct real eigen values than it is diagonalizable over $\mathbf{C}$
True statement . Proof: Example:	If A is an $N \times N$ complex matrix with n distinct eigenvalues, then any set of n corresponding eigenvectors form a basis for $\mathbf{C}^n$ It is sufficient to prove that the set of eigenvectors is linearly independent which is proved in statement 2. $$A = \begin{pmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{pmatrix} \quad (2.0.9)$$ Eigen values of A are: $$\lambda_1 = 2, \lambda_2 = 3, \lambda_3 = 6 \quad (2.0.10)$$ Eigen vectors are: $$x_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, x_3 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \quad (2.0.11)$$ Matrix A is diagonalizable because there is a basis of $\mathbf{C}^3$consisting of eigenvectors of A.
Statement 4.	If all eigen values are non zero than it is diagonalizable over $\mathbf{C}$
False Statement: Example:	Matrix would be diagonalizable if and only if it has linearly independent eigenvectors spanning the entire space. Consider a matrix $$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix} \quad (2.0.12)$$ Eigen values are: $$\begin{pmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{pmatrix} = 0. \implies \lambda_1 = 1, \lambda_2 = 4 \neq 0 \quad (2.0.13)$$ $$\lambda_1 = 1 \text{ has eigen vector} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{and} \lambda_2 = 4 \text{ has eigen vector} \begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix} \quad (2.0.14)$$ We have found only two linearly independent eigenvectors for A,not diagonalisable

TABLE 2:Solution