# Matrix theory Assignment 19

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***Abstract*—This document contains the concept of matrix diagonalization.**

Download all python codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment19/
    Codes
```

Download latex-tikz codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment19/
    Assignment_19.tex
```

## 1 Problem

Let A be a $3\times3$ matrix with real entries.Identify the correct statements.

1.A is necessarily diagonalizable over **R**

2.If A has distinct real eigen values than it is diagonalizable over**R**

3.If A has distinct eigen values than it is diagonalizable over **C**

4.If all eigen values are non zero than it is diagonalizable over **C**

## 2 Solution

Refer Table 0.

| | |
|---|---|
| Statement 1. | A is necessarily diagonalizable over **R** |
| False statement<br><br>Example: | matrix A is diagonalizable if and only if there is a basis of $\mathbf{R}^3$consisting of eigenvectors of A.<br>Consider a matrix $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix}$<br>Eigen values are: $\begin{pmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{pmatrix} = 0. \implies \lambda_1 = 1, \lambda_2 = 4$<br>$\lambda_1 = 1$ has eigen vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\lambda_2 = 4$ has eigen vector $\begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}$<br>We have found only two linearly independent eigenvectors for A,Hence not diagonali |
| Statement 2. | If A has distinct real eigen values than it is diagonalizable over**R** |
| True statement<br>Proof : | A has n linearly independent eigenvectors which implies it is diagonalizable.<br>Consider 3 eigen vectors **v**,**w** and **u** with eigen values $\lambda$,$\mu$,$\nu$ respectively.<br>$\alpha(\mathbf{v}) + \beta(\mathbf{w}) + \gamma(\mathbf{u}) = 0$<br>$\alpha \mathrm{A}(\mathbf{v}) + \beta \mathrm{A}(\mathbf{w}) + \gamma \ \mathrm{A} \ (\mathbf{u}) = 0$<br>$\alpha\lambda\mathbf{v} + \beta\mu\mathbf{w} + \gamma\nu\mathbf{u} = 0$<br>$\beta(\mu - \lambda)\mathbf{w} + \gamma(\nu - \lambda)\mathbf{u} = 0$ |
| $\alpha\mathbf{V}+\beta\mathbf{w}+\gamma\mathbf{u}$ Statement 3. | If all eigen values are non zero than it is diagonalizable over **C** |
| True statement<br><br>Example: | A has n linearly independent eigenvectors which implies it is diagonalizable.<br>$\mathrm{A}=\begin{pmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{pmatrix} \implies \lambda_1 = 2, \lambda_2 = 3, \lambda_3 = 6$<br>Eigen vectors are:$x_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, x_3 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ respectively |
| Statement 4. | If all eigen values are non zero than it is diagonalizable over **C** |
| False statement | counter example same as statement 1 ex. eigen values are non zero but not diagonali |

TABLE 0: Explanation