Challenge Problem

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Curve Fitting

***Abstract*—This document contains the solution to interpolate the curve**

Download latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Challenge/Challenge.tex

1 Problem

Suppose that we are given n distinct pairs of points $x_1, y_1), (x_2, y_2)........(x_n, y_n)$.How do you check whether all these points lie on a polynomial of degree at most m.Given (m<n)

2 Construction

Given n points, we construct n equations in n unknowns. Let a_k be the coefficient for x^kin the unknown polynomial, and let (x_k, y_k)be the data point (given in the problem). The equation is given as:

$$(a^n)(x_K^n) + ...(a_1)(x_K) + a_0 = y_K \tag{2.0.1}$$

In matrix-vector form the equation looks like:

$$\begin{pmatrix} x_1^n & x_1^{n-1} & \cdots & 1 \\ x_2^n & x_2^{n-1} & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_n^n & x_n^{n-1} & \cdots & 1 \end{pmatrix} \begin{pmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_0 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \tag{2.0.2}$$

The above case is when m=n but if m**<**n than our matrix could have rank of at most m when points are distinct.
Hence characteristic equation of matrix would be a polynomial of degree at most m.
Now if the points $\boldsymbol{x_i}$ are distinct, then the matrix would have inverse,and that will always gives us the coefficients of our polynomial and hence the equation of curve and all the points lying on the curve should satisfy this.

3 Description

Matrix would always be invertible
Inverse would always be possible because we will always get Vandermonde matrix.The non-vanishing of the Vandermonde matrix for distinct points shows that, for distinct points, the map from coefficients to values at those points is a one-to-one correspondence, and thus that the polynomial interpolation problem is solvable with a unique solution.

4 Solution

Let us consider $\boldsymbol{P(x) = a_0 + a_1 x + a_2 x^2}$. Taking $\boldsymbol{P(2) = 17}$,$\boldsymbol{P(3) = 11}$ and $\boldsymbol{P(7) = 2}$
From above we will get equations like

$$\boldsymbol{a_0 + 2(a_1) + 4(a_2) = 17} \tag{4.0.1}$$

$$\boldsymbol{a_0 + 3(a_1) + 9(a_2) = 11} \tag{4.0.2}$$

$$\boldsymbol{a_0 + 7(a_1) + 49(a_2) = 2} \tag{4.0.3}$$

Now converting above equations in matrix form we have:

$$\begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} \\ \mathbf{1} & \mathbf{3} & \mathbf{4} \\ \mathbf{1} & \mathbf{7} & \mathbf{49} \end{pmatrix} \begin{pmatrix} \boldsymbol{a_0} \\ \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{pmatrix} = \begin{pmatrix} \mathbf{17} \\ \mathbf{11} \\ \mathbf{12} \end{pmatrix} \tag{4.0.4}$$

Now writing equation in form $\boldsymbol{X = A^{-1}B}$ we have

$$\begin{pmatrix} \boldsymbol{a_0} \\ \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{pmatrix} = \boldsymbol{A^{-1}} \begin{pmatrix} \mathbf{17} \\ \mathbf{11} \\ \mathbf{12} \end{pmatrix} \tag{4.0.5}$$

On solving 4.0.5 we have :

$$\begin{pmatrix} \boldsymbol{a_0} \\ \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{pmatrix} = \begin{pmatrix} \frac{\mathbf{67}}{\mathbf{2}} \\ \frac{\mathbf{-39}}{\mathbf{4}} \\ \frac{\mathbf{3}}{\mathbf{4}} \end{pmatrix} \tag{4.0.6}$$

Hence equation of the curve is
$\boldsymbol{y = 3/4x^2 - 39/4x + 67/2}$

5 Problem

Given n distinct pairs of points is there always a polynomial of degree at most n**−**1 which passes through all these points.

6 Solution

We can draw obvious generalization that given n points $(x_1, y_1), (x_2, y_2).....(x_n, y_n)$ we can find a polynomial of degree n−1 which passes through those points by doing the following:

- Writing down the general polynomial of degree n−1
- Evaluating the polynomial at the points $x_1.....x_n$
- Solving the resulting system of linear equations as shown in section 4

Hence from above there will always a polynomial of degree at most n−1 which passes through all these n points.