

# Matrix theory Assignment 14

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**Abstract**—This document contains the concept of linear transformations.

Download all python codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/tree/master/Assignment14/Codes](https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment14/Codes)

Download latex-tikz codes from

[https://github.com/shivangi-975/EE5609-Matrix\\_Theory/blob/master/Assignment14/Assignment\\_14.tex](https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment14/Assignment_14.tex)

Hence,  $\beta'$  as matrix

$$\beta' = \begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix} \quad (2.0.4)$$

Let  $\mathbf{T}$  be the transformation defined by

$$\mathbf{T}(\mathbf{x}) = \mathbf{A}\mathbf{x} \quad (2.0.5)$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix} \mathbf{x} \quad (2.0.6)$$

we have to write  $\mathbf{T}(\epsilon_1)$  and  $\mathbf{T}(\epsilon_2)$  as linear combinations of  $\alpha_1, \alpha_2$ . Hence,

$$\begin{pmatrix} 1 & -i & 1 & 0 \\ i & 2 & 0 & 0 \end{pmatrix} \xrightarrow{R_2=R_2-iR_1} \begin{pmatrix} 1 & -i & 1 & 0 \\ 0 & 1 & -i & 0 \end{pmatrix} \quad (2.0.7)$$

$$\xleftarrow{R_1=R_1+iR_2} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -i & 0 \end{pmatrix} \quad (2.0.8)$$

Therefore matrix of relative to the pair  $\beta, \beta'$

$$\mathbf{T}(\beta) = \begin{pmatrix} 2 & 0 \\ -i & 0 \end{pmatrix} \beta \quad (2.0.9)$$

## 1 PROBLEM

Let  $\mathbf{T}$  be the linear operator on  $\mathbb{C}^2$  defined by :

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$

. Let  $\beta$  be the standard ordered basis for  $\mathbb{C}^2$  and let

$$\beta' = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

be the ordered basis defined by

$$\alpha_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix}$$

What is the matrix of  $\mathbf{T}$  relative to the pair  $\beta, \beta'$ ?

## 2 SOLUTION

$$\beta = \{\epsilon_1, \epsilon_2\} \implies \epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.1)$$

Hence,  $\beta$  as matrix

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.2)$$

$$\beta' = \{\alpha_1, \alpha_2\} \implies \alpha_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix} \quad (2.0.3)$$