Matrix theory Assignment 16

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***Abstract*—This document contains the concept of algebra of ploynomials.**

Download all python codes from

```
https://github.com/shivangi-975/EE5609-
    Matrix_Theory/tree/master/Assignment16/
    Codes
```

Download latex-tikz codes from

```
https://github.com/shivangi-975/EE5609-
    Matrix_Theory/blob/master/Assignment16/
    Assignment_16.tex
```

1 Problem

If f and g are independent polynomials over a field **F**and h is a non-zero polynomial over **F**, show that fh and gh are independent.

2 Definition

Polynomials $(f_1, f_2, \ldots, f_m) \in k(x_1, \ldots, x_n)$are called algebraically independent over a field **F**, if there is no nonzero m-variate polynomial A $\in k[y_1, \ldots, y_m]$ such that $A(f_1, \ldots, f_m) = 0$

3 Example

The smallest degree independent polynomials are: 1+x and 1-x.

$$a(1 + x) + b(1 - x) = 0 \tag{3.0.1}$$

Simplifying,

$$a + b = 0 \tag{3.0.2}$$

$$a - b = 0 \tag{3.0.3}$$

solving,we get a=0 and b=0.So, polynomials are linearly independent.

4 Solution

Given f and g are independent polynomials over a field **F**. Consider scalars a and b $\in$ **F** Hence,

$$af + bg = 0 \tag{4.0.1}$$

Since f and g are independent Hence f and g $\neq 0$

$$\implies a, b = 0. \tag{4.0.2}$$

Given h a non zero polynomial over **F**. Substituting in equation (4.0.1) we have,

$$a(fh) + b(gh) = 0 \tag{4.0.3}$$

$$(af)h + (bg)h = 0 \tag{4.0.4}$$

$$(af + bg)h = 0 \tag{4.0.5}$$

$$af + bg = 0 \tag{4.0.6}$$

f and g are independent polynomial.Also from equation (4.0.2) a=0 and b=0.
Hence proved fh and gh are independent.