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# Matrix theory Assignment 11

## Shivangi Parashar

Abstract—This document contains the concept of sub space.

Download all python codes from

https://github.com/shivangi-975/EE5609-Matrix\_Theory/tree/master/Assignment11/ Codes

Download latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix\_Theory/blob/master/Assignment11/ Assignment 11.tex

### 1 Problem

Which of the following set of vectors

$$\alpha = (a_1, a_2, \dots, a_n)$$

in  $\mathbb{R}^n$  are subspace of  $\mathbb{R}^n$   $(n \ge 3)$ ?

- a)All  $\alpha$  such that  $a_1 \ge 0$
- b)All  $\alpha$  such that  $a_1 + 3a_2 = a_3$
- c)All  $\alpha$  such that  $a_2 = a_1^2$

Let  $c \in \mathbf{R}$ 

- d)All  $\alpha$  such that  $a_1a_2 = 0$
- e)All  $\alpha$  such that  $a_2$  is rational

## 2 Solution

a) All  $\alpha$  such that  $a_1 >= 0 \in \mathbf{R}^{\mathbf{n}} \ (n \ge 3)$ Let,

$$\alpha = (a_1, a_2, a_3, a_4, \dots, a_n) \in \mathbf{S}$$
  
 $(-\alpha) = (-a_1, -a_2, -a_3, \dots, -a_n) \notin \mathbf{S}$  (2.0.1)

From equation (2.0.1) it is not closed with respect to scalar multiplication and hence not a subspace. b) All  $\alpha$  such that  $a_1 + 3(a_2) \in \mathbb{R}^n$   $(n \ge 3)$ ,

$$\alpha = (a_1, a_2, a_3, ..., a_n) \in \mathbf{S}$$
  
 $\beta = (b_1, b_2, b_3, ..., b_n) \in \mathbf{S}$ 

Now,  $c.(\alpha)+\beta$ 

$$= c(a_1, a_2, a_3, ..., a_n) + (b_1, b_2, b_3, ..., b_n)$$
  
=  $(ca_1, ca_2, ca_3, ..., ca_n) + (b_1, b_2, b_3, ..., b_n)$   
(2.0.3)

Now, using condition  $a_1 + 3(a_2)$  we have,

$$ca_1 + b_1 + 3(ca_2 + b_2) = ca_1 + b_1 + 3ca_2 + 3b_2$$

$$\implies c(a_1 + 3a_2) + (b_1 + 3b_2) = ca_3 + b_3$$
(2.0.4)

From equation (2.0.4) it is closed under addition and scalar multiplication, hence it is a subspace. c) All  $\alpha$  such that  $a_2 = a_1^2 \in \mathbf{R}^n$   $(n \ge 3)$ ,

$$\alpha = (a_1, a_1^2, a_3, a_4, ...a_n) \in \mathbf{S}$$
  
 $\beta = (b_1, b_1^2, b_3, b_4 ...b_n) \in \mathbf{S}$ 

$$\implies \alpha + \beta = (a_1 + b_1, a_1^2 + b_1^2, a_3 + b_3, ...a_n + b_n) \notin \mathbf{S}$$
(2.0.5)

From equation (2.0.5)  $(a_1 + b_1)^2 \neq a_1^2 + b_1^2$  it is not closed with respect to additions and hence not a subspace.

d) All  $\alpha$  such that  $a_1a_2 = 0 \in \mathbf{R}^{\mathbf{n}}$   $(n \ge 3)$ , Let,

$$\alpha = (0, a_1, 0, ...0) \in \mathbf{S}; \quad a_1 \neq 0$$
  
 $\beta = (b_1, 0, 0, ...0) \in \mathbf{S}; \quad b_1 \neq 0$   
 $\implies \alpha + \beta = (b_1, a_1, 0, ...0) \notin \mathbf{S}$  (2.0.6)

From equation(2.0.6)  $a_1b_1 \neq 0$  it is not closed with respect to addition and hence not a subspace.

e) All  $\alpha$  such that  $a_2$  is rational  $\in \mathbb{R}^n$   $(n \ge 3)$ , Let,

$$\alpha = (0, a_1, 0, ...0) \in \mathbf{S}$$

$$b = \sqrt{2} \in \mathbf{R}$$

$$b \cdot \alpha = (0, \sqrt{2}a_1, 0, .....0) \notin \mathbf{S}$$
(2.0.7)

From equation(2.0.7) it is not closed with respect to scalar multiplication and hence not a subspace.

(2.0.2)