Matrix theory Assignment 18

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***Abstract*—This document contains the concept of ideal ploynomials.**

Download all python codes from

```
https://github.com/shivangi−975/EE5609−
    Matrix_Theory/tree/master/Assignment18/
    Codes
```

Download latex-tikz codes from

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https://github.com/shivangi−975/EE5609−
    Matrix_Theory/blob/master/Assignment18/
    Assignment_18.tex
```

1 Problem

Let A be an $n \times n$ matrix over a field $\mathbf{F}$. Show that the set of all polynomials f in $\mathbf{F}[x]$ such that f(A) = 0 is an ideal.

2 Definition

Let $\mathbf{F}$ be a field. An ideal in $\mathbf{F}[x]$ is a subspace $\mathbf{M}$ of $\mathbf{F}[x]$ such that fg belongs to $\mathbf{M}$ whenever f is in $\mathbf{F}[x]$ and g is in $\mathbf{M}$.

3 Solution

Given a square matrix of order n and $f(A) = 0$. Let $\mathbf{I}$ be an ideal of $C[z1, \cdots, zn]$.

$$\mathbf{I} = \{\mathbf{F} \in \mathbf{F}(x) | f(A) = 0\} \tag{3.0.1}$$

$$f, g \in \mathbf{I} \;\; \& \;\; k \in \mathbf{F}.$$

Now,Consider the polynomials $f = f_0 + f_1 z_n + \cdots f_d {z_n}^d$ and $g = g_0 + g_1 z_n + \cdots g_e {z_n}^e$ of degree d and e respectively .

$$(kf + g)(A) = kf(A) + g(A) \tag{3.0.2}$$
$$\implies c.0 + 0 = 0 \tag{3.0.3}$$

From (3.0.3) $\mathbf{I}$ is a subspace of $\mathbf{F}(x)$

Now,f$\in$ $\mathbf{I}$ and g $\in F(x)$ then,

$$(gf)A = g(A)f(A) \tag{3.0.4}$$
$$\implies g(A).0 = 0 \tag{3.0.5}$$

Hence,proved $\mathbf{I}$ is ideal