Matrix theory Assignment 15

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***Abstract*—This document contains the concept of linear functionals.**

Download all python codes from

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https://github.com/shivangi-975/EE5609-
    Matrix_Theory/tree/master/Assignment15/
    Codes
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Download latex-tikz codes from

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https://github.com/shivangi-975/EE5609-
    Matrix_Theory/blob/master/Assignment15/
    Assignment_15.tex
```

1 Problem

In R^3, let $\alpha_1 = \begin{pmatrix}1\\0\\1\end{pmatrix}$, $\alpha_2 = \begin{pmatrix}0\\1\\-2\end{pmatrix}$ and $\alpha_3 = \begin{pmatrix}-1\\-1\\0\end{pmatrix}$. Describe a linear functional f on R^3 such that $f(\alpha_1) = f(\alpha_2) = 0$ but $f(\alpha_3) \neq 0$.If α=$\begin{pmatrix}2\\3\\-1\end{pmatrix}$ such that $f(\alpha) \neq 0$

2 Solution

Given, $\alpha = \begin{pmatrix}a\\b\\c\end{pmatrix}$. Let,

$$\mathbf{A} = \begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{AX} = \alpha \tag{2.0.2}$$

$$\begin{pmatrix}1 & 0 & -1\\0 & 1 & -1\\1 & -2 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix} \tag{2.0.3}$$

$\mathbf{X} = A^{-1}\alpha$ will give solution of the equation.

$$\begin{pmatrix}1 & 0 & -1 & 1 & 0 & 0\\0 & 1 & -1 & 0 & 1 & 0\\1 & -2 & 0 & 0 & 0 & 1\end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 - R_1} \tag{2.0.4}$$

$$\begin{pmatrix}1 & 0 & -1 & 1 & 0 & 0\\0 & 1 & -1 & 0 & 1 & 0\\0 & -2 & 1 & -1 & 0 & 1\end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 + 2R_2} \tag{2.0.5}$$

$$\begin{pmatrix}1 & 0 & -1 & 1 & 0 & 0\\0 & 1 & -1 & 0 & 1 & 0\\0 & 0 & -1 & -1 & 2 & 1\end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3/(-1)} \tag{2.0.6}$$

$$\begin{pmatrix}1 & 0 & -1 & 1 & 0 & 0\\0 & 1 & -1 & 0 & 1 & 0\\0 & 0 & 1 & 1 & -2 & -1\end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 + R_3} \tag{2.0.7}$$

$$\begin{pmatrix}1 & 0 & 0 & 2 & -2 & -1\\0 & 1 & -1 & 0 & 1 & 0\\0 & 0 & 1 & 1 & -2 & -1\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + R_3} \tag{2.0.8}$$

$$\begin{pmatrix}1 & 0 & 0 & 2 & -2 & -1\\0 & 1 & 0 & 1 & -1 & -1\\0 & 0 & 1 & 1 & -2 & -1\end{pmatrix} \tag{2.0.9}$$

Thus,

$$A^{-1} = \begin{pmatrix}2 & -2 & -1\\1 & -1 & -1\\1 & -2 & -1\end{pmatrix} \tag{2.0.10}$$

$$\mathbf{X} = A^{-1}\alpha = \begin{pmatrix}2 & -2 & -1\\1 & -1 & -1\\1 & -2 & -1\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix} \tag{2.0.11}$$

Given, f is a linear functional on R^3,

$$\alpha = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 \tag{2.0.12}$$

$$\implies f(\alpha) = \mathbf{X}^T \begin{pmatrix}f(\alpha_1)\\f(\alpha_2)\\f(\alpha_3)\end{pmatrix} \tag{2.0.13}$$

Given, $f(\alpha_1) = 0$, $f(\alpha_2) = 0$ and $f(\alpha_3) \neq 0$.

$$f(\alpha) = \mathbf{X}^T \begin{pmatrix} 0 \\ 0 \\ f(\alpha_3) \end{pmatrix} \tag{2.0.14}$$

$$\implies f(\alpha) = \begin{pmatrix} a \\ b \\ c \end{pmatrix}^T \begin{pmatrix} 2 & 1 & 1 \\ -2 & -1 & -2 \\ -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ f(\alpha_3) \end{pmatrix} \tag{2.0.15}$$

$$f(\alpha) = \begin{pmatrix} a \\ b \\ c \end{pmatrix}^T \begin{pmatrix} f(\alpha_3) \\ -2f(\alpha_3) \\ -f(\alpha_3) \end{pmatrix} \tag{2.0.16}$$

Hence,

$$f(\alpha) = f(\alpha_3)(a - 2b - c) \tag{2.0.17}$$

Given α=$\begin{pmatrix} 2 & 3 & -1 \end{pmatrix}$ Substituting value of α in equation (2.0.17) we have,

$$f(\alpha) = -3f(\alpha_3) \neq 0 \tag{2.0.18}$$

Hence proved,$f(\alpha) \neq 0$