Computing Assignment: Due by Tuesday October 29, 2024, 11:59pm in Crowdmark

Computing Assignment

Required submission: PDF report (**ONE PAGE ONLY**) with your answers to the question here, and any screenshots of your Matlab code **and** its outputs, including calculations and plots, all uploaded to Crowdmark.

There are three bins on Crowdmark to upload to:

- **"Report"** - upload your one-page PDF report here. Your report should make reference to your Matlab code/output/plots below. Your report should not have any MATLAB code or calculations; those belong in the CODE bin.
- **"Plots"** - upload PDF/PNG/JPG screenshots of your MATLAB plots here. You have the option to include your plots in the report instead, if you wish.
- **"CODE"** - upload your Matlab code **and** its output. May upload as a PDF of text, or as PNG/JPG screenshots.

Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

Matlab Essentials:

Matrix Entry Extraction: Suppose we have a matrix stored in memory in Matlab, for example

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$$

To select the entry in row 2, column 2 we use `A(2,2)`.

```
%% Sample Matlab code
A = [1 2 3; 4 5 6];
A(2,2) % extract entry in row 2 column 2
```

output:

```
ans = 5
```

We can use the colon notation to select a sub-matrix consisting of rows i to j, and columns r to s: to do this use `A(i:j,r:s)`.

```
%% Sample Matlab code
A = [1 2 3 4; 5 6 7 8; 9 10 11 12]
A(1:2,2:4)  % extract rows 1 and 2, columns 2 to 4 (inclusive)
```

output:

```
A = 3x4
      1    2    3    4
      5    6    7    8
      9   10   11   12

ans = 2x2
      3   4
      7   8
```

To select all rows or columns use : in the appropriate spot. For example, to extract column 2 from the matrix (that is all row entries in column 2) use `A(:,2)`.

```
%% Sample Matlab code
A(:,2)   % extract all row entries in column 2
```

output:

```
ans = 4x1
         2
         6
        10
```

This technique of extracting sub-matrices from a larger matrix is called *matrix indexing*. You can learn more here:
https://www.mathworks.com/company/technical-articles/matrix-indexing-in-matlab.html

Matrix Inverse: Use `inv` to compute the inverse of a matrix.

```
%% Sample Matlab code
B = [1 1 2; 1 2 2; 2 4 3]
inv(B)
```

output:

```
ans = 3x3
       2   -5    2
      -1    1    0
       0    2   -1
```

We can check it is the inverse by seeing if the identity matrix is the product:

```
%% Sample Matlab code
B*inv(B)
```

output:

```
ans = 3x3
       1    0    0
       0    0    0
       0    1    0
```

Computing Assignment Questions

(see next page for computing assignment questions)

Computing Assignment Questions

Remember that the Report page is only **one page max**. If the report is longer than one page, the marker will only look at the first page.

Make sure your final answers are explicitly stated on the Report page! (Note that plots are allowed to be placed on the "Plots" bin below the Report bin.)

1. (Section 3.4)

   Consider the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 5 \\ 2 & 3 & 1 & 2 & 11 \\ 1 & 1 & 1 & 3 & 7 \\ 1 & 2 & 0 & -1 & 4 \end{bmatrix}$.

   (a) Find a basis for each of the four fundamental spaces of A: $\text{Col}(A)$, $\text{Null}(A)$, $\text{Row}(A)$, and $\text{Null}(A^T)$. To do this use Matlab to row-reduce an appropriate matrix and then extract the necessary basis. In your report indicate what calculation you had Matlab do and explain how you obtained the basis from the information Matlab returned.

   (b) From your answer to part (a) verify the Rank-Nullity theorem holds for both A and A^T.

   (c) Using your answer from part (a) show that each basis vector for $\text{Col}(A)$ is orthogonal to each basis vector of $\text{Null}(A^T)$. Similarly show that each basis vector for $\text{Row}(A)$ is orthogonal to each basis vector of $\text{Null}(A)$.

2. (Section 4.4 and Section 4.6)

   Consider the basis $\mathfrak{B} = \{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3, \vec{\mathbf{v}}_4\}$ where

$$\vec{\mathbf{v}}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{\mathbf{v}}_2 = \begin{bmatrix} 1 \\ 5 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{\mathbf{v}}_3 = \begin{bmatrix} 2 \\ 3 \\ 6 \\ 4 \end{bmatrix}, \quad \vec{\mathbf{v}}_4 = \begin{bmatrix} 1 \\ 3 \\ -1 \\ -1 \end{bmatrix}$$

   (a) Using Matlab, calculate the matrix $P_{\mathfrak{B}\leftarrow\mathcal{S}}$. Then for $\vec{\mathbf{x}} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, find $[\vec{\mathbf{x}}]_{\mathfrak{B}}$. In your Report, state your answer for $P_{\mathfrak{B}\leftarrow\mathcal{S}}$ and $[\vec{\mathbf{x}}]_{\mathfrak{B}}$.

   (b) Let $L : \mathbb{R}^4 \to \mathbb{R}^4$ be the mapping:

$$L(x_1, x_2, x_3, x_4) = (x_2 + 5x_4, \quad x_1 - x_3, \quad x_1 + x_2 + x_3 + x_4, \quad 0)$$

   Using Matlab, calculate the matrix $[L]_{\mathfrak{B}}$. Then for the vector $[\vec{\mathbf{x}}]_{\mathfrak{B}}$ that you found in part (a), use this matrix to calculate the vector $[L(\vec{\mathbf{x}})]_{\mathfrak{B}}$. In your Report, state your answer for $[L]_{\mathfrak{B}}$ and $[L(\vec{\mathbf{x}})]_{\mathfrak{B}}$.