UNIT III (BINARY SEARCH TREES, AVL TREES, SPLAY TREES & RED BLACK TREES)

TREE: A tree is recursively defined as a set of one or more nodes and branches where one node is designated as 'root' of the tree and all the remaining nodes can be partitioned into non-empty sets, each of which is a sub-tree with respect to root of the tree. It is a non-linear data structure compared to arrays, linked lists.

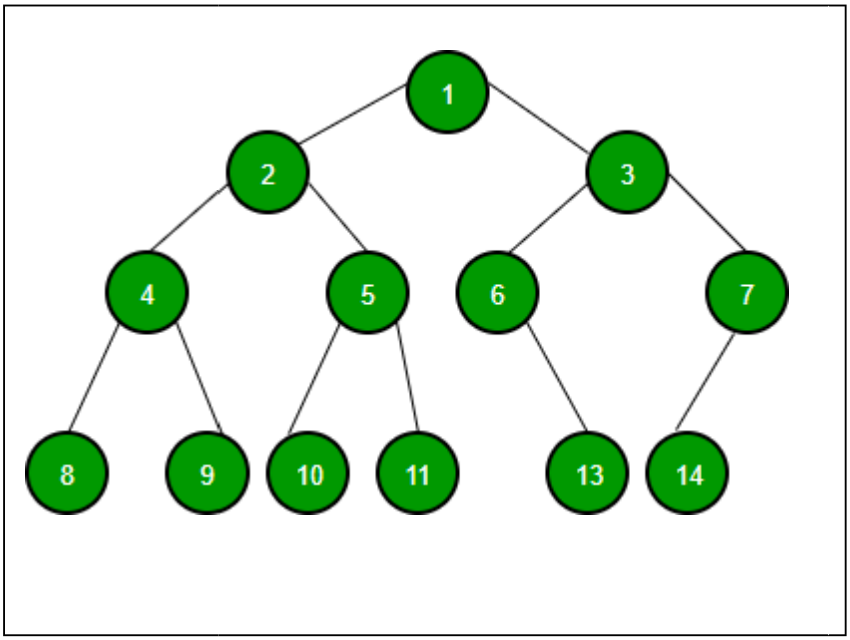


Fig. Example of a Tree

BASIC TERMINOLOGY:

Node: A data unit to store information.
Branch or edge: The Link between two nodes.
Root node: It is the top most node in the tree.
Leaf Node: The node which does not have any children.

Degree of the Node: It is the number of edges connected to that node.
In-Degree: Number of edges arriving at that node.
Out-Degree: Number of edges leaving that node.
Level: It is the length of the path from the root to that node.

- Root node is said to be at level 0.

Depth or Height of the Tree: It is the maximum level of any node in the tree.
Internal node: A non-leaf node.

Parent: Node with out-degree greater than 0.
Child: Node with in-degree greater than 0.
Siblings: Nodes with the same parent.
Path: Sequence of consecutive edges is called a path.
Degree of the Tree: Maximum degree of a node in that tree.

IMPORTANT POINTS TO BE NOTED:

1. **Is Root node considered as internal node?**
   Any vertex(node) for which there exist one or more children are called as internal vertices, the root of a tree is an internal vertex unless it is the only vertex in the tree.

2. **Degree of a node = In-Degree + Out-Degree**

- Any node in a tree has a maximum of in-degree 'one'.
- for any node in a binary tree, the maximum out-degree 'two'.
- root node has in-degree 'zero' .
- leaf node has out-degree 'zero'.

Applications of trees:

1. Used to represent hierarchies.
2. Used to represent simple as well as complex data.
3. Used for implementing other data structures like hash tables, sets, and maps.
4. Used for compiler construction.
5. Used in data base design.
6. Used in file system directories.
7. Used in symbol tables.

BINARY TREE: A tree whose elements have at most 2 children is called a binary tree. Since each element in a binary tree can have only 2 children, we typically name them the left and right child.

- For every node inbinary tree, In-degree is always 'one'.
  Out-degree is always less than or equal to 'two'. Maximum degree of any node is 'three'.
- Every binary tree is a tree, but every tree is not a binary tree.

PROPERTIES OF A BINARY TREE:

1. The maximum number of nodes at level 'L' of a binary tree is 2^L-1.
2. Maximum number of nodes in a binary tree of height 'h' is $2^h - 1$.
3. In Binary tree where every node has 0 or 2 children, number of leaf nodes is always one more than nodes with two children.

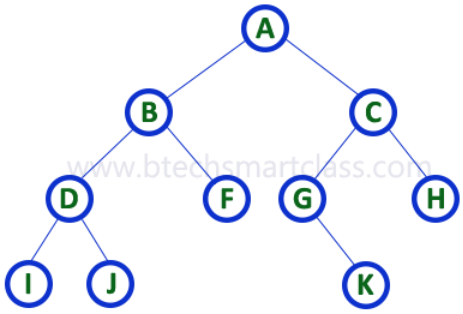
IMPLEMENTATION OF A BINARY TREE USING DOUBLY LINKED LIST:

Binary Tree Representations

A binary tree data structure is represented using two methods. Those methods are as follows...

1. **Array Representation**
2. **Linked List Representation**

Consider the following binary tree...



1. Array Representation of Binary Tree

In array representation of a binary tree, we use one-dimensional array (1-D Array) to represent a binary tree.

Consider the above example of a binary tree and it is represented as follows...

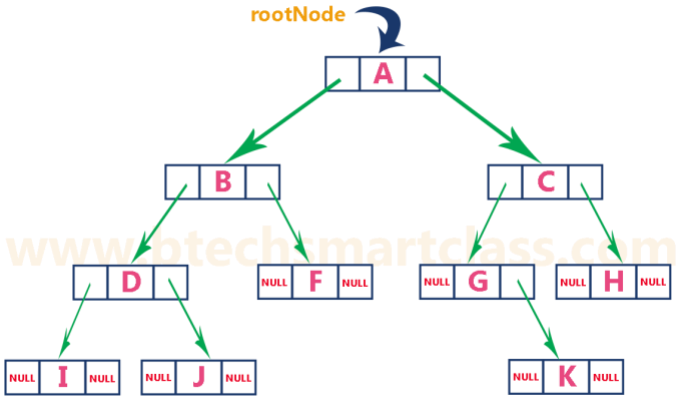
To represent a binary tree of depth '**n**' using array representation, we need one dimensional array with a maximum size of **2n + 1**.

2. Linked List Representation of Binary Tree

We use a double linked list to represent a binary tree. In a double linked list, every node consists of three fields. First field for storing left child address, second for storing actual data and third for storing right child address.

In this linked list representation, a node has the following structure...

The above example of the binary tree represented using Linked list representation is shown as follows...



Typically, creation of a node is done as, A Node contains:

1. Data
2. Pointer to left child
3. Pointer to right child

```
struct node {
   int data;
   struct node *leftChild;
   struct node *rightChild;
};
```

Then respective links are given to the nodes, creating branches between the nodes by using two self-referencing structure pointers.

TYPES OF BINARY TREES:

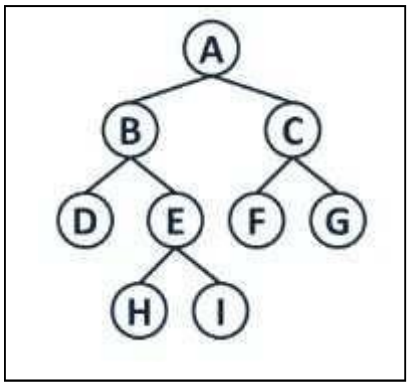
There are different types of binary trees,

1. Full or Strictly Binary Tree
2. Perfect Binary Tree
3. Complete Binary Tree
4. Degenerate or Pathological Tree
5. Skewed Binary Tree
6. Balance Binary Tree

1. FULL OR STRICTLY BINARY TREE

A Binary Tree is said to be a Full or Strictly Binary Tree, If all the nodes other than leaf nodes has 0 or 2 children, then that is Full Binary Tree.

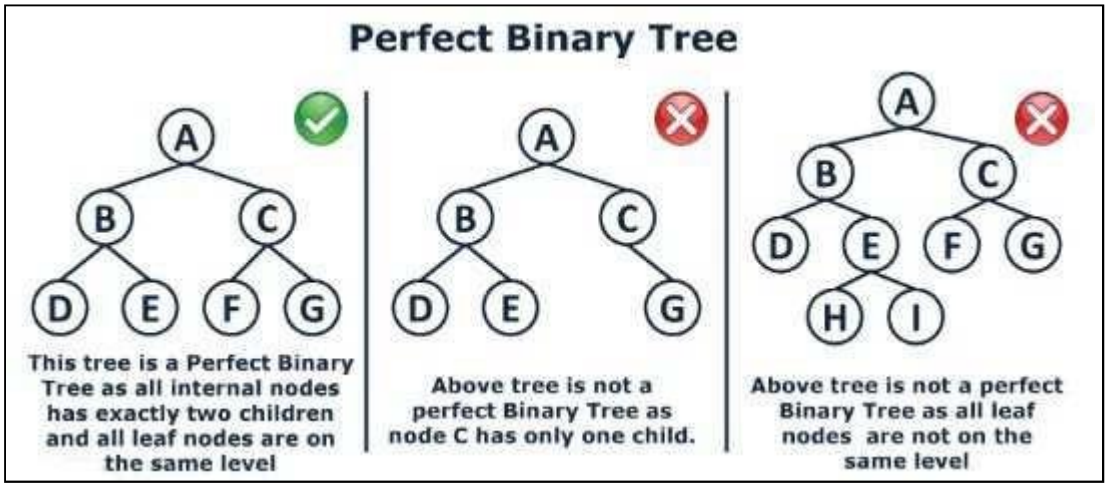
- All the nodes in a Full or Strictly Binary Tree are of out-degree zero or two, never degree one.



2. PERFECT BINARY TREE

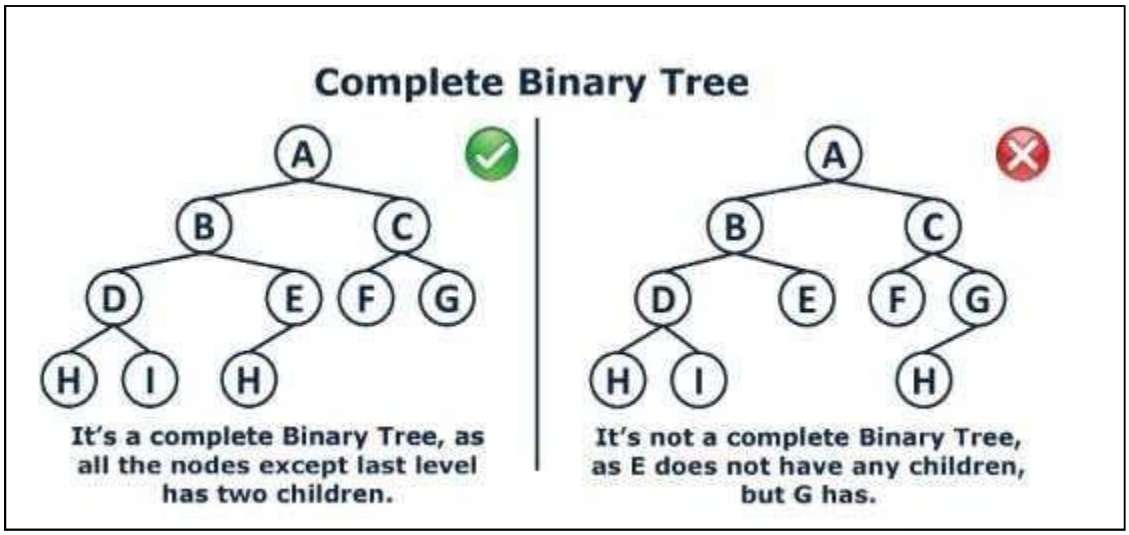
A Binary Tree is said to be a Perfect Binary Tree, if all its internal nodes has exactly 2 children.

- In Perfect Binary Tree, all leaf nodes are on the same level or depth.

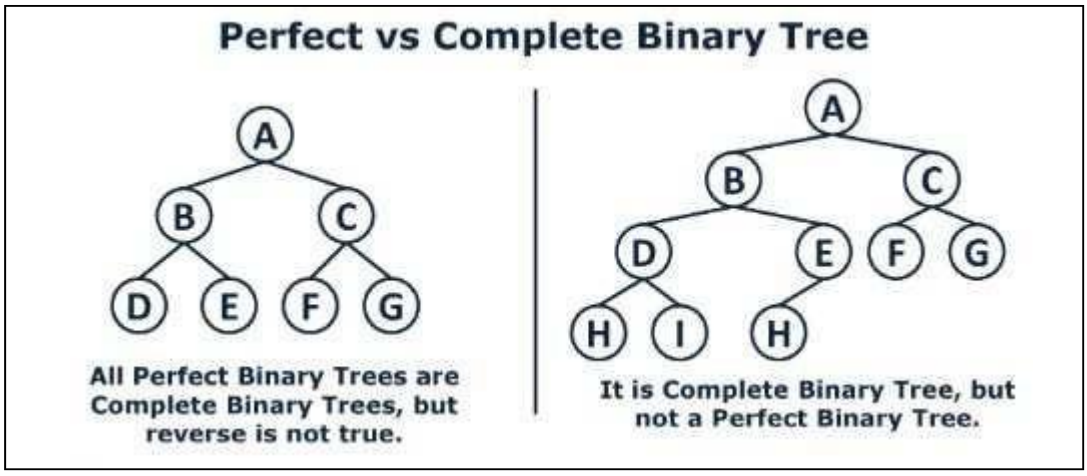


3. COMPLETE BINARY TREE

A Binary Tree is said to be Complete Binary Tree if all levels are completely filled except possibly the last level and the last level has all the nodes as left as possible.



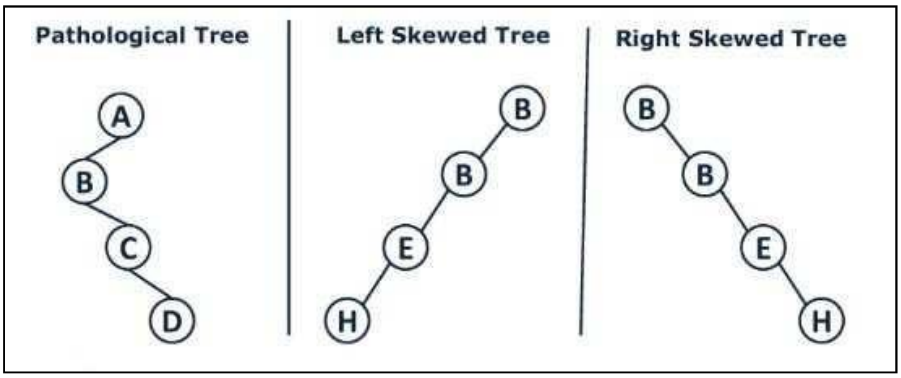
<u>NOTE</u>: Perfect VS Complete Binary Tree



4. DEGENERATE OR PATHOLOGICAL TREE

A Degenerate or Pathological Tree is a Tree where every parent node has only one child either left or right.

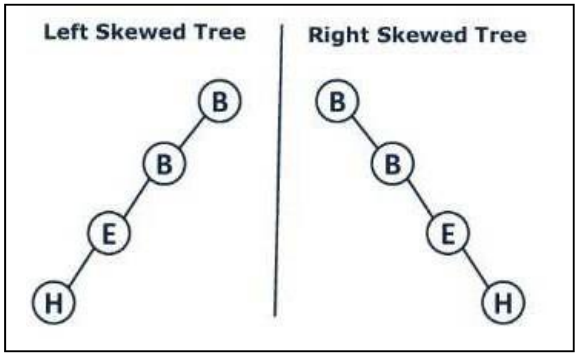
- Such trees are performance-wise same as Linked List. In-fact, even slower while traversing because you need to check whether tree has left child or right child and then move to next node.



5. SKEWED BINARY TREE

A Binary Tree, Which is dominated solely by left child nodes or right child nodes, is called Skewed Binary Tree.

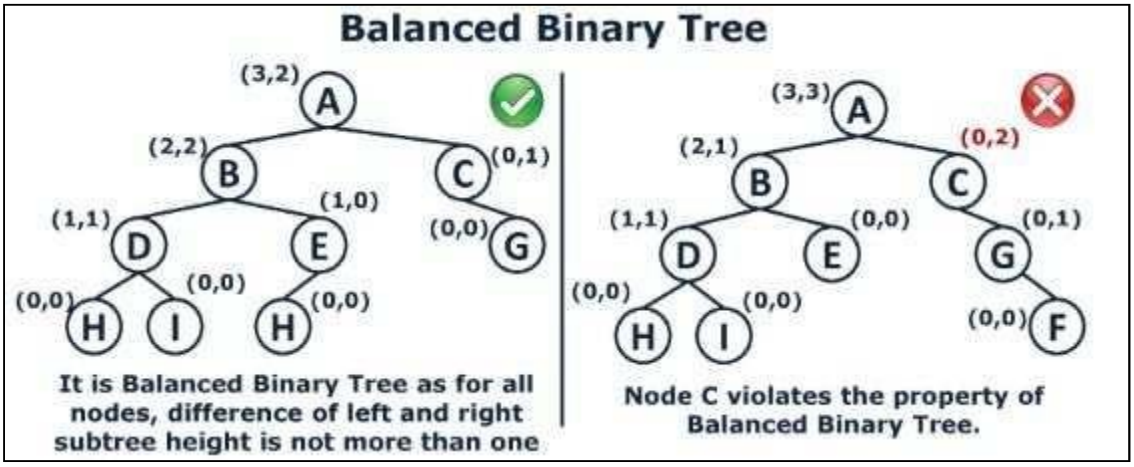
- All Skewed Trees are Pathological Trees, But all Pathological Trees are not Skewed Trees.



6. BALANCED BINARY TREE

A Binary Tree is called Balanced Binary Tree, If difference of left and right subtree depth have the values [-1,0,1].

- If any nodes violate, Then it is not a Balanced Binary Tree, It can be balanced by using suitable methods (rotations, etc).



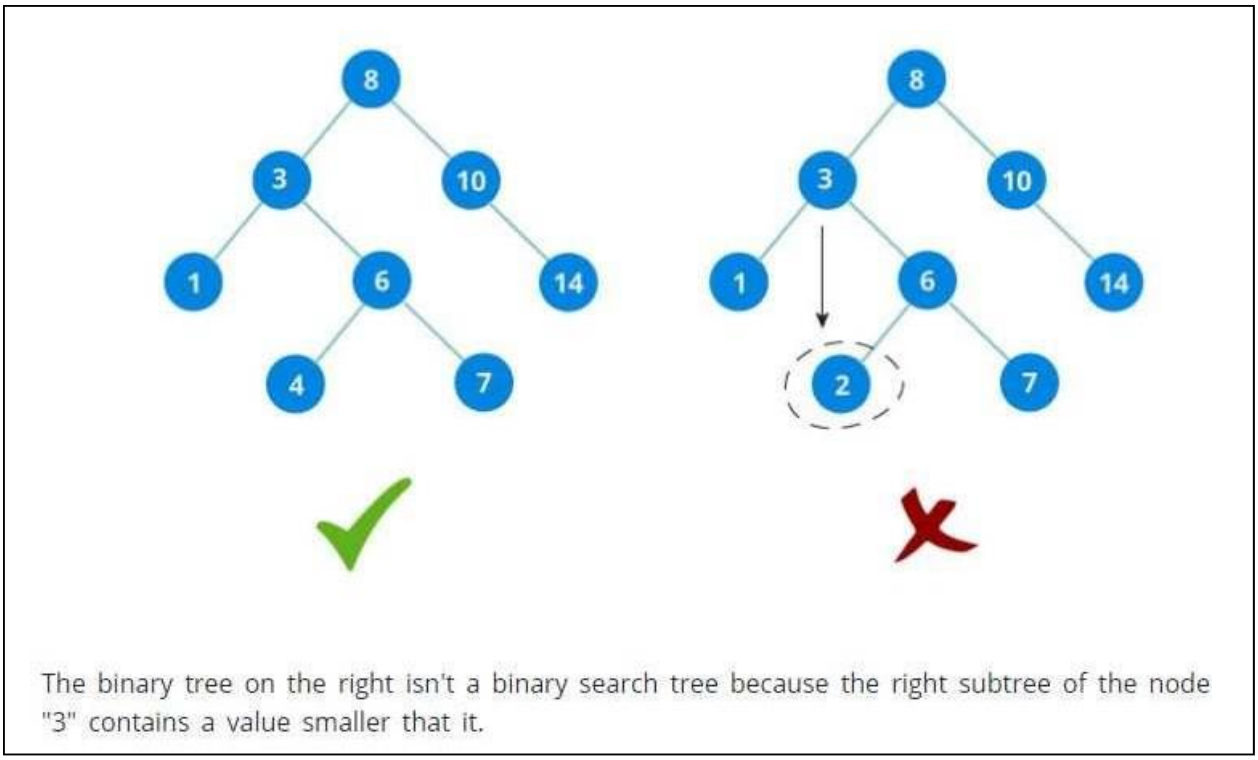
BINARY SEARCH TREES (BST):

Binary Search tree can be defined as a class of binary trees, in which the nodes are arranged in a specific order. This is also called ordered binary tree. **IT HAS ALL THE PROPERTIES OF A BINARY TREE.**

Binary Search Tree is a node-based binary tree data structure which has the following properties:

- The left subtree of a node contains only nodes with keys lesser than the node's key.
- The right subtree of a node contains only nodes with keys greater than the node's key.
- The left and right subtree each must also be a binary search tree.
- This rule will be recursively applied to all the left and right sub-trees of the root.

Rule of Binary Search Tree: Left Child ≤ Root ≤ Right Child



The binary tree on the right isn't a binary search tree because the right subtree of the node "3" contains a value smaller that it.

NOTE: Every Binary Search Tree is a Binary Tree, But not every Binary Tree is a Binary Search Tree.

Tree > Binary Tree > Binary Search Tree

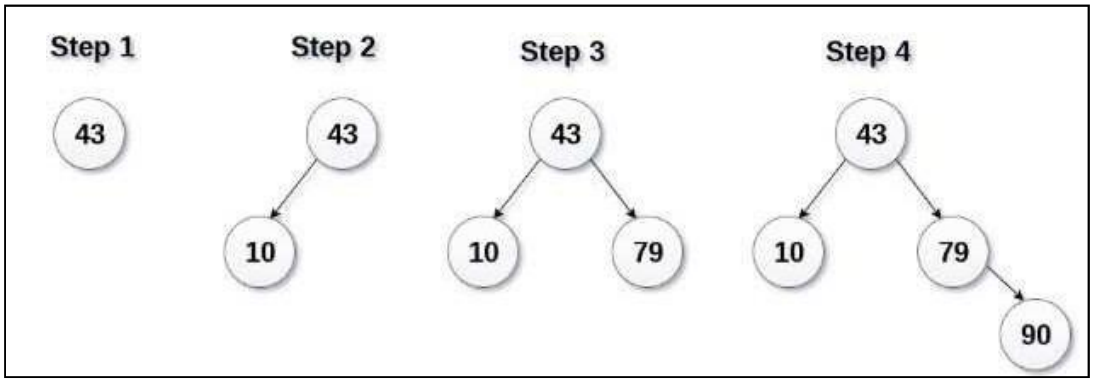
Advantages of using Binary Search Tree:

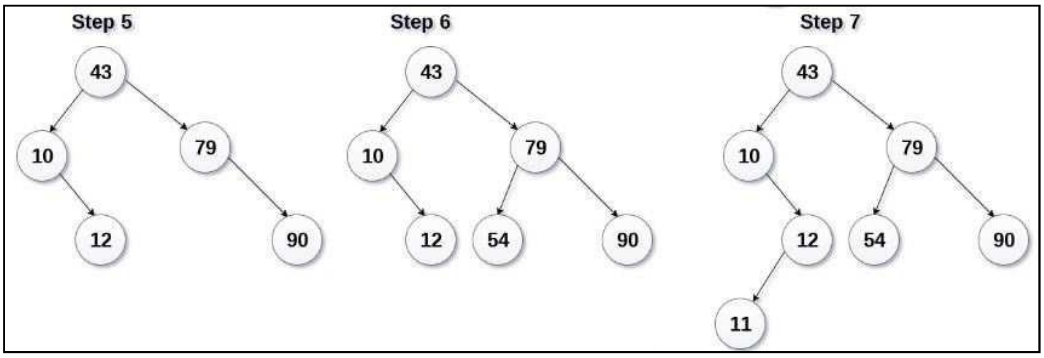
1. Searching become very efficient in a binary search tree since, we skip half of the elements in each step of search process.
2. The binary search tree is considered as efficient data structure in compare to arrays and linked lists. In searching process, it removes half sub-tree at every step. Searching for an element in a binary search tree takes $O(\log_2 n)$ time. In worst case, the time it takes to search an element is O(n).
3. It also speed up the insertion and deletion operations as compare to that in array and linked list.
4. It is called a search tree because it can be used to search for the presence of a number in O(log(n)) time.

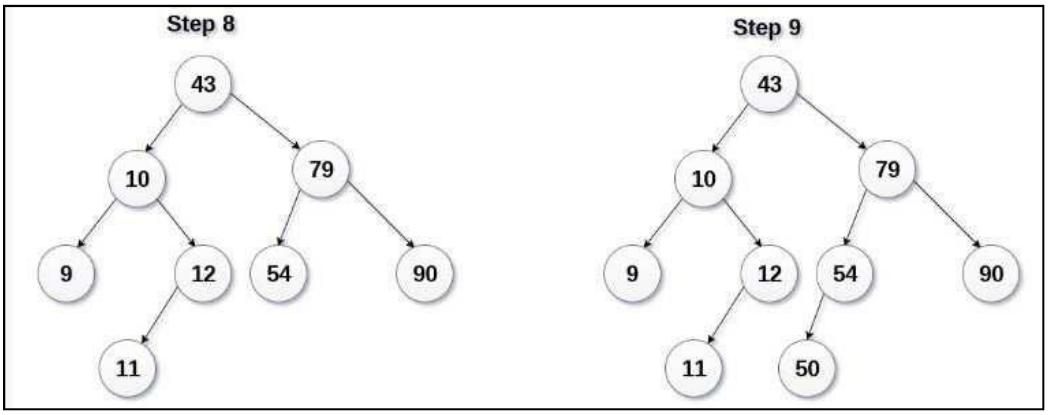
CREATION OF BINARY SEARCH TREE:

Q. Create the binary search tree using the following data elements. 43, 10, 79, 90, 12, 54, 11, 9, 50

1. Insert 43 into the tree as the root of the tree.
2. Read the next element, if it is lesser than the root node element, insert it as the root of the left sub-tree.
3. Otherwise, insert it as the root of the right of the right sub-tree.
4. Continue the process recursively for all the entries.







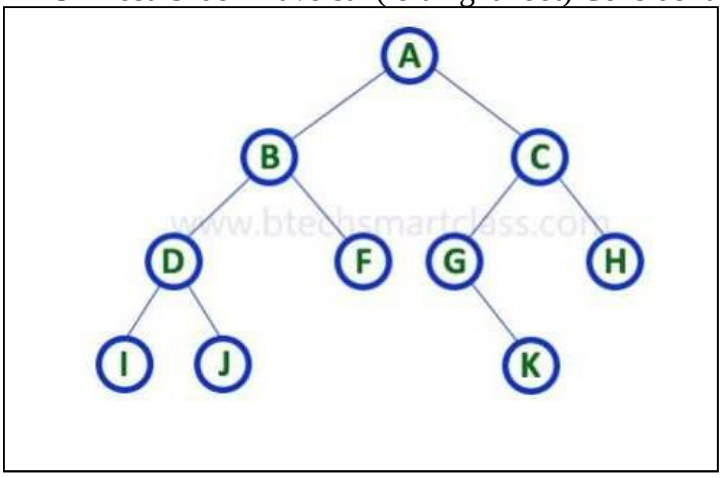
NOTE: If you observe carefully, we can see that the left-most element is the smallest, and right-most element is the largest in a binary search tree.

BINARY SEARCH TREE TRAVERSALS:

Traversal: Displaying (or) visiting order of nodes in a binary tree is called as Binary Tree Traversal.

There are mainly three types of Traversal techniques for BST:

1. Pre-Order Traversal (root-left-right)
2. In-Order Traversal (left-root-right)
3. Post-Order Traversal (left-right-root) Consider the following binary tree...



1. **In - Order Traversal (Left Child - Root - Right Child)**

   In In-Order traversal, the root node is visited between the left child and right child.

   In this traversal, the left child node is visited first, then the root node is visited and later we go for visiting the right child node. This in-order traversal is applicable for every root node of all subtrees in the tree. This is performed recursively for all nodes in the tree.

In the above example of a binary tree, first we try to visit left child of root node 'A', but A's left child 'B' is a root node for left subtree. so we try to visit its (B's) left child 'D' and again D is a root for subtree with nodes D, I and J. So we try to visit its left child 'I' and it is the leftmost child. So first we visit 'I' then go for its root node 'D' and later we visit D's right child 'J'. With this we have completed the left part of node B. Then visit 'B' and next B's right child 'F' is visited. With this we have completed left part of node A. Then visit root node 'A'. With this we have completed left and root parts of node A. Then we go for the right part of the node A. In right of A again there is a subtree with root C. So go for left child of C and again it is a subtree with root G. But G does not have left part so we visit 'G' and then visit G's right child K. With this we have completed the left part of node C. Then visit root node 'C' and next visit C's right child 'H' which is the rightmost child in the tree. So we stop the process.

That means here we have visited in the order of I - D - J - B - F - A - G - K - C - H using In-Order Traversal.

In-Order Traversal for above example of binary tree is

I - D - J - B - F - A - G - K - C – H

2. **Pre - Order Traversal (Root - Left Child - Right Child)**

In Pre-Order traversal, the root node is visited before the left child and right child nodes.

In this traversal, the root node is visited first, then its left child and later its right child. This pre-order traversal is applicable for every root node of all subtrees in the tree.

In the above example of binary tree, first we visit root node 'A' then visit its left child 'B' which is a root for D and F. So we visit B's left child 'D' and again D is a root for I and J. So we visit D's left child 'I' which is the leftmost child. So next we go for visiting D's right child 'J'. With this we have completed root, left and right parts of node D and root, left parts of node B. Next visit B's right child 'F'. With this we have completed root and left parts of node A. So we go for A's right child 'C' which is a root node for G and H.

After visiting C, we go for its left child 'G' which is a root for node K. So next we visit left of G, but it does not have left child so we go for G's right child 'K'. With this, we have completed node C's root and left parts. Next visit C's right child 'H' which is the rightmost child in the tree. So we stop the process.

That means here we have visited in the order of A-B-D-I-J-F-C-G-K-H using Pre-Order Traversal.

Pre-Order Traversal for above example binary tree is

A - B - D - I - J - F - C - G - K - H

3. **Post - Order Traversal (Left Child - Right Child - Root)**

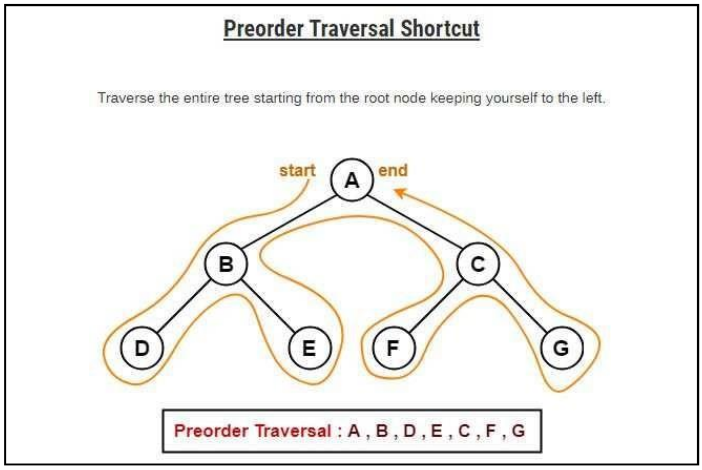
In Post-Order traversal, the root node is visited after left child and right child. In this traversal, left child node is visited first, then its right child and then its root node. This is recursively performed until the right most node is visited.

Here we have visited in the order of I - J - D - F - B - K - G - H - C - A using Post-Order Traversal.

Post-Order Traversal for above example binary tree is

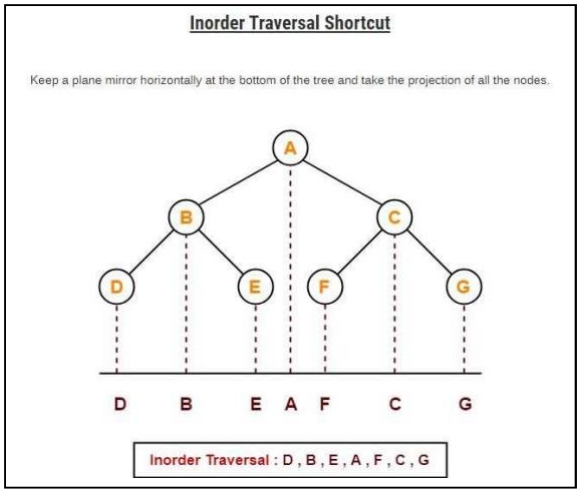
I - J - D - F - B - K - G - H - C - A

IMPORTANT POINTS TO BE NOTED:



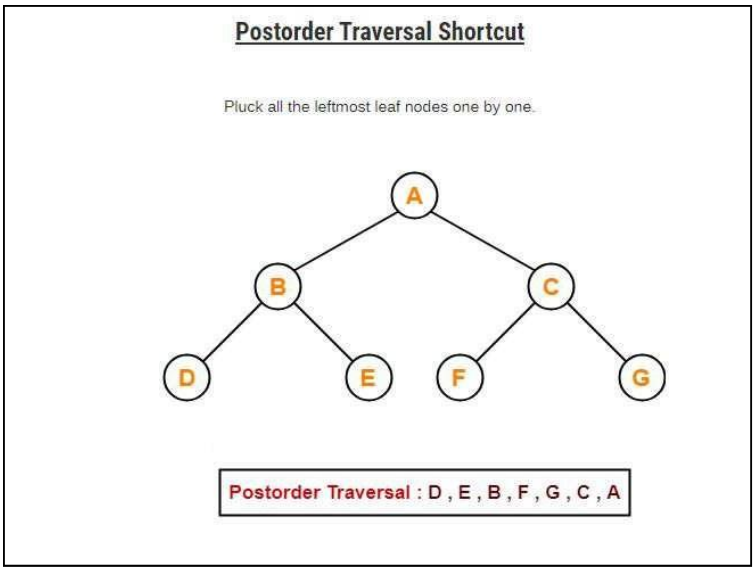
- **Application of Pre-Order Traversal:**

1. used to create a copy of the tree.
2. used to get prefix expression of an expression tree.



- **Application of In-Order Traversal:**

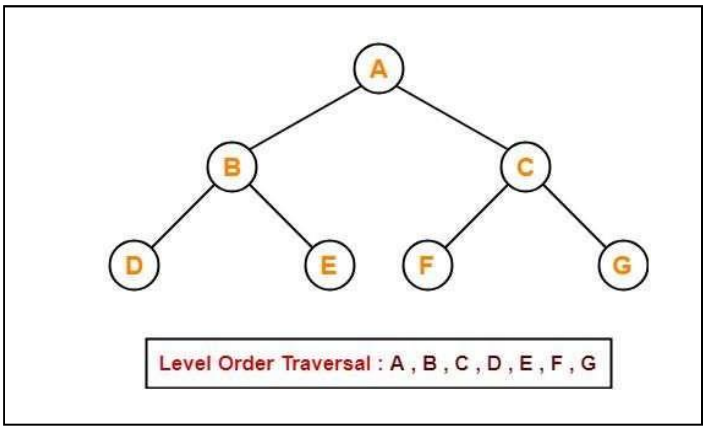
. used to get infix expression of an expression tree.
NOTE: In-Order traversal of BST always produces sorted output.



- **Application of Post-Order Traversal:**

1. used to get postfix expression of an expression tree.
2. used to delete the tree. This is because it deletes the children first and then it deletes the parent.

BREADTH FIRST SEARCH TRAVERSAL:
Breadth First Traversal of a tree prints all the nodes of a tree level by level. Breadth First Traversal is also called as Level Order Traversal.
EXAMPLE:



APPLICATION OF BFS TRAVERSAL:

Level order traversal is used to print the data in the same order as stored in the array representation of a complete binary tree.

ALGORITHM FOR BINARY TREE TRAVERSAL:

Pre-Order

- Visit and print the node.
- Traverse the left sub-tree, (recursively call In-Order(root->left))
- Traverse the right sub-tree, (recursively call In-Order(root->right))

```
1. void preorder(struct node *root) 2.  {
3. if(root!=NULL) 4.  {
5. printf("\n%d",root->data);
6. preorder(root->left);
7.          preorder(root->right); 8.  }
9.  }
```

In-Order

- Traverse the left sub-tree, (recursively call In-Order(root->left))
- Visit and print the node.
- Traverse the right sub-tree, (recursively call In-Order(root->right))

```
1. void inorder(struct node *root) 2.  {
3. if(root!=NULL) 4.  {
5. inorder(root->left);
6. printf("\n%d",root->data);
7.          inorder(root->right); 8.  }
9.  }
```

Post-Order

- Traverse the left sub-tree, (recursively call In-Order(root->left))
- Traverse the right sub-tree, (recursively call In-Order(root->right))
- Visit and print the node.

```
1. void postorder(struct node *root) 2.  {
3.          if(root!=NULL) 4.          {
5.      postorder(root->left);
6.      postorder(root->right);
7.          printf("\n%d",root->data); 8.         }
9.  }
```

OPERATIONS ON A BINARY SEARCH TREE:

The following operations are performed on a binary search tree...

1. Search
2. Insertion
3. Deletion

Search Operation in BST

In a binary search tree, the search operation is performed with O(log n) time complexity. The search operation is performed as follows...

- Step 1 - Read the search element from the user.
- Step 2 - Compare the search element with the value of root node in the tree.
- Step 3 - If both are matched, then display "Given node is found!!!" and terminate the function
- Step 4 - If both are not matched, then check whether search element is smaller or larger than that node value.
- Step 5 - If search element is smaller, then continue the search process in left subtree.
- Step 6- If search element is larger, then continue the search process in right subtree.
- Step 7 - Repeat the same until we find the exact element or until the search element is compared with the leaf node
- Step 8 - If we reach to the node having the value equal to the search value then display "Element is found" and terminate the function.

- Step 9 - If we reach to the leaf node and if it is also not matched with the search element, then display "Element is not found" and terminate the function.

Insertion Operation in BST

In a binary search tree, the insertion operation is performed with O(log n) time complexity. In binary search tree, new node is always inserted as a leaf node. The insertion operation is performed as follows...

- Step 1 - Create a newNode with given value and set its left and right to NULL.
- Step 2 - Check whether tree is Empty.
- Step 3 - If the tree is Empty, then set root to newNode.
- Step 4 - If the tree is Not Empty, then check whether the value of newNode is smaller or larger than the node (here it is root node).
- Step 5 - If newNode is smaller than or equal to the node then move to its left child. If newNode is larger than the node then move to its right child.
- Step 6- Repeat the above steps until we reach to the leaf node (i.e., reaches to NULL).
- Step 7 - After reaching the leaf node, insert the newNode as left child if the newNode is smaller or equal to that leaf node or else insert it as right child.

Deletion Operation in BST

In a binary search tree, the deletion operation is performed with O(log n) time complexity. Deleting a node from Binary search tree includes following three cases...

- Case 1: Deleting a Leaf node (A node with no children)
- Case 2: Deleting a node with one child
- Case 3: Deleting a node with two children

Case 1: Deleting a leaf node

We use the following steps to delete a leaf node from BST...

- Step 1 - Find the node to be deleted using search operation
- Step 2 - Delete the node using free function (If it is a leaf) and terminate the function.

Case 2: Deleting a node with one child

We use the following steps to delete a node with one child from BST...

- Step 1 - Find the node to be deleted using search operation
- Step 2 - If it has only one child then create a link between its parent node and child node.
- Step 3 - Delete the node using free function and terminate the function.

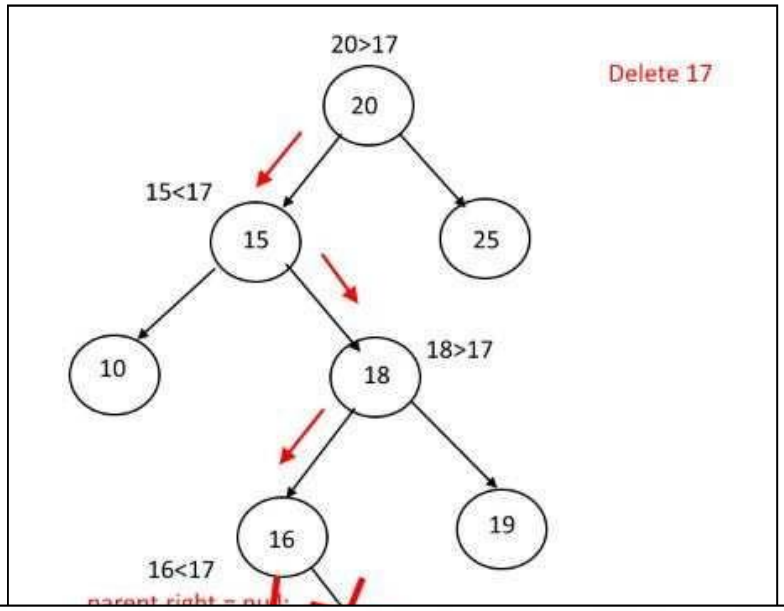
Case 3: Deleting a node with two children

We use the following steps to delete a node with two children from BST...

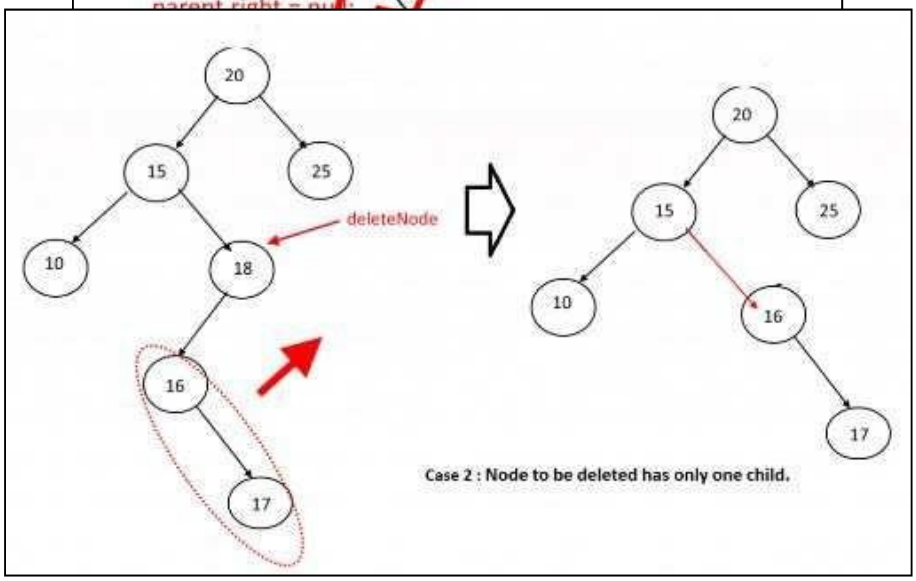
- Step 1 - Find the node to be deleted using search operation
- Step 2 - If it has two children, then find the largest node in its left subtree (OR) the smallest node in its right subtree.
- Step 3 - Swap both deleting node and node which is found in the above step.
- Step 4 - Then check whether deleting node came to case 1 or case 2 or else goto step 2
- Step 5 - If it comes to case 1, then delete using case 1 logic.
- Step 6- If it comes to case 2, then delete using case 2 logic.
- Step 7 - Repeat the same process until the node is deleted from the tree.

ILLUSTRATION OF DELETE OPERATION IN BST:

1.



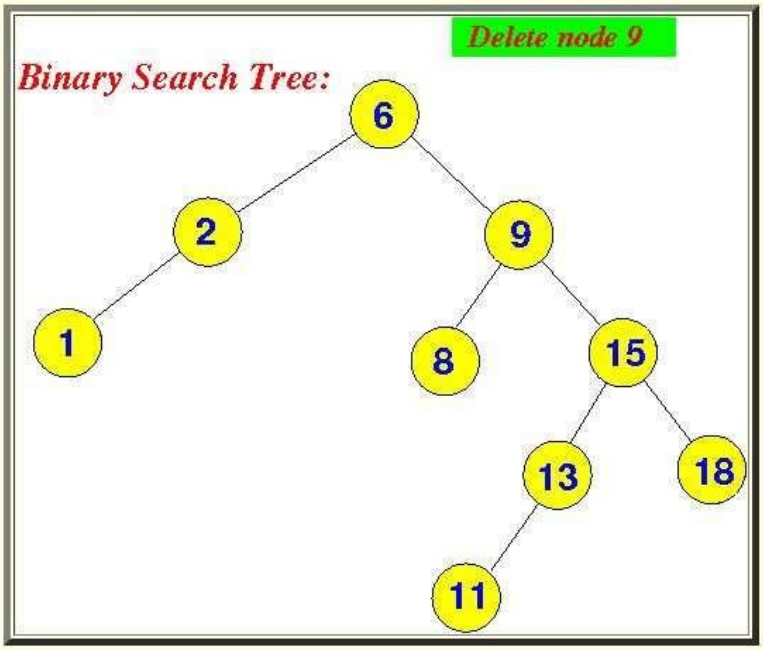
2.



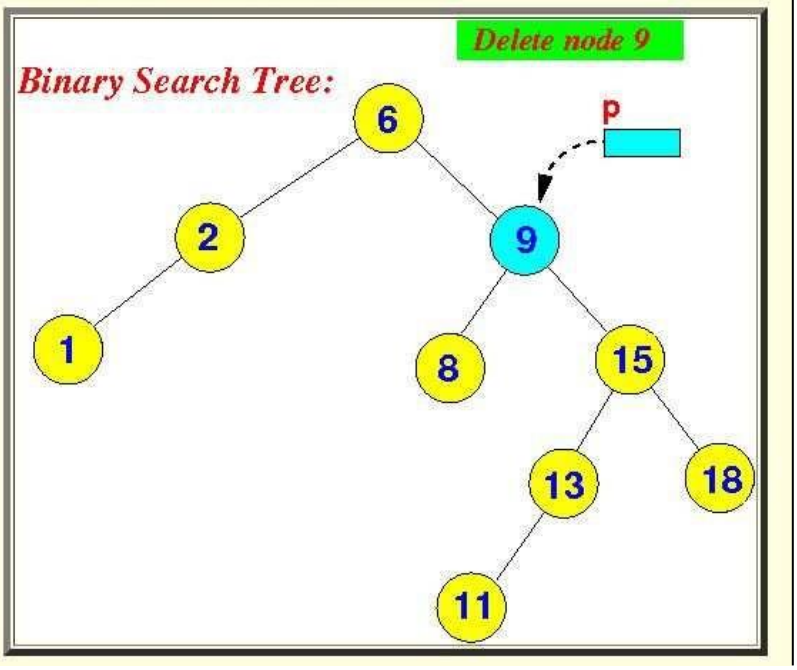
DELETION OF NODE WITH TWO CHILDREN(CASE 3):

EXAMPLE 1:

- Delete **node 9** in the following **BST**:

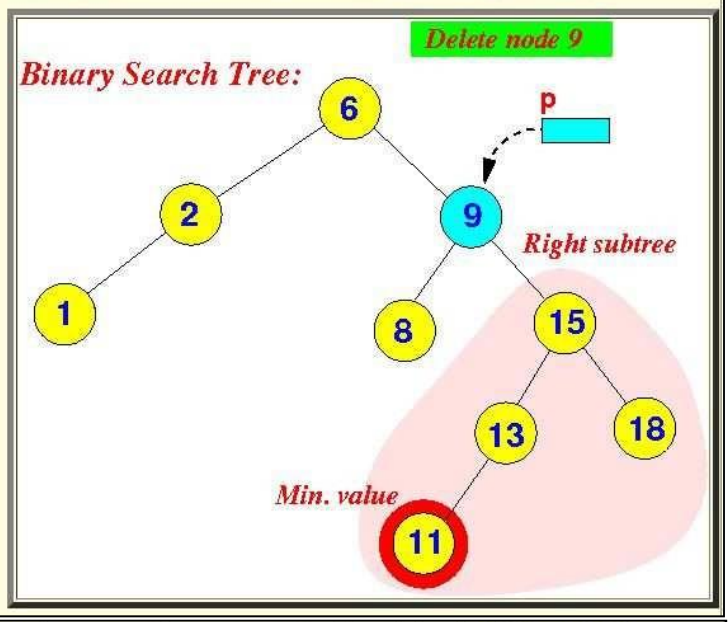


- *First*, we must **find** the **node** with the value **9**:

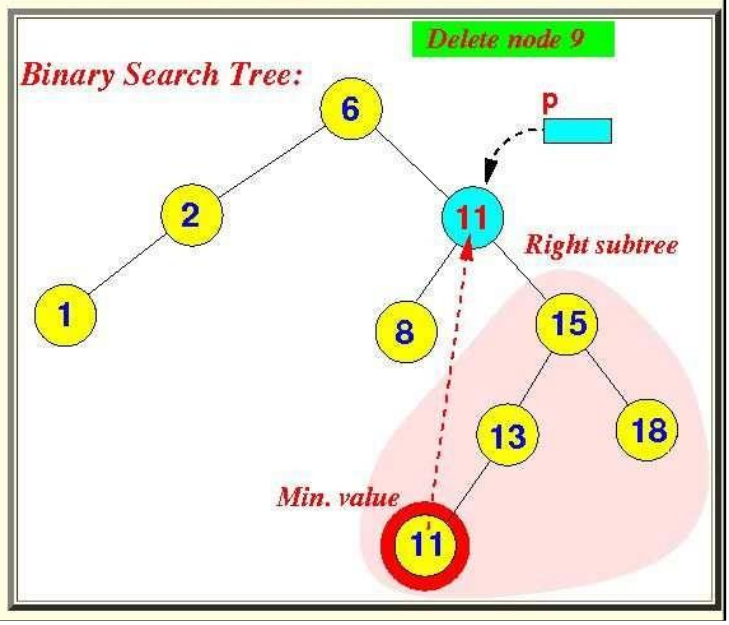


- *Next*, we **find** the ***successor* node** of the node **9**.

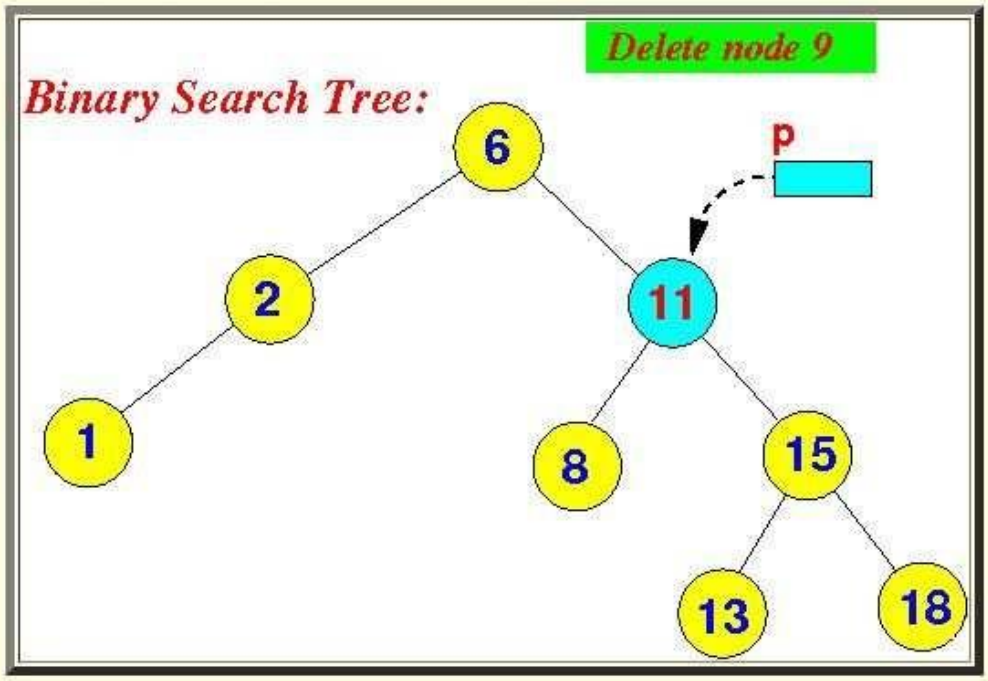
  The ***successor* node** is the **node** with the ***minimum* value** in the ***right* subtree**:



- We **copy** the **content** of the ***successor* node** of the node into the ***deletion* node** (**p**):



- The **last step** is **deleting** the ***successor* node** from the **BST**:

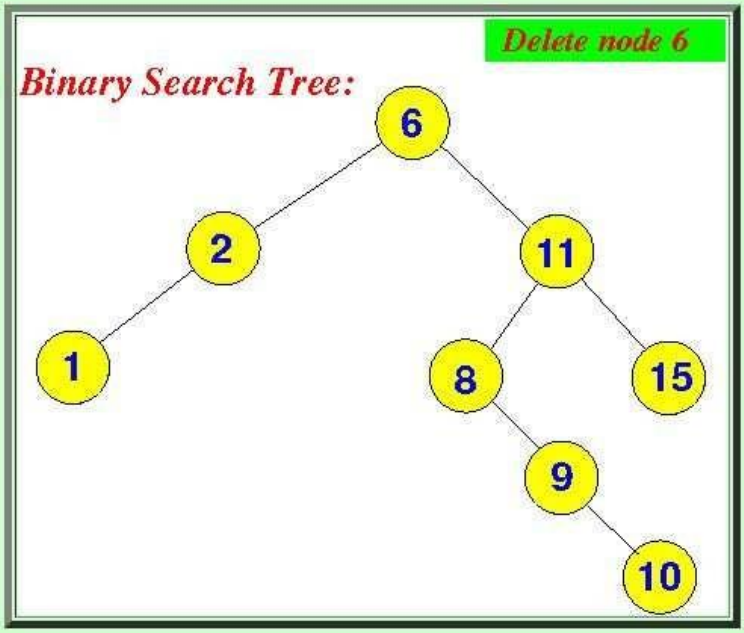


Notice that the tree satisfies the **Binary Search Tree** property:

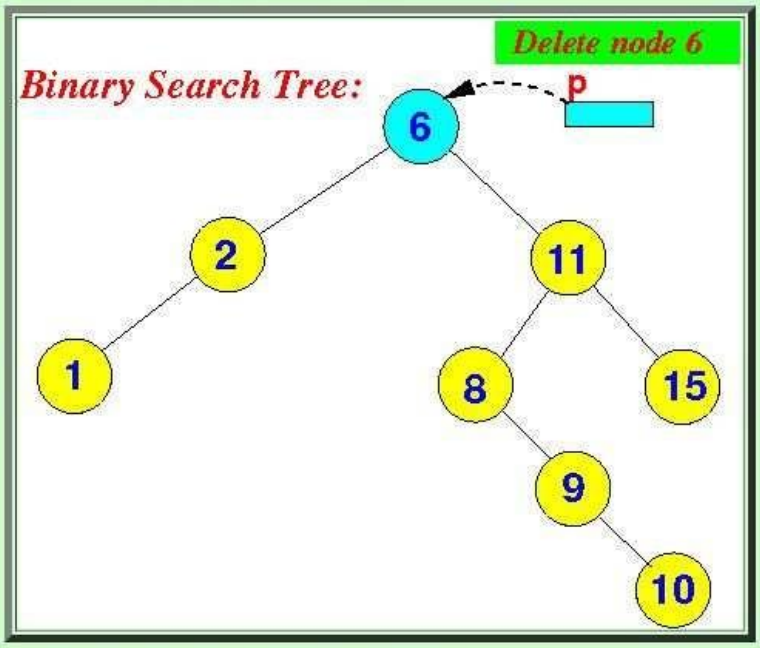
- all nodes in ***left* subtree** has ***smaller* values** and
- all nodes in the ***right* subtree** has ***larger* values**.

EXAMPLE 2:

- Delete **node 6** in the following **BST**:



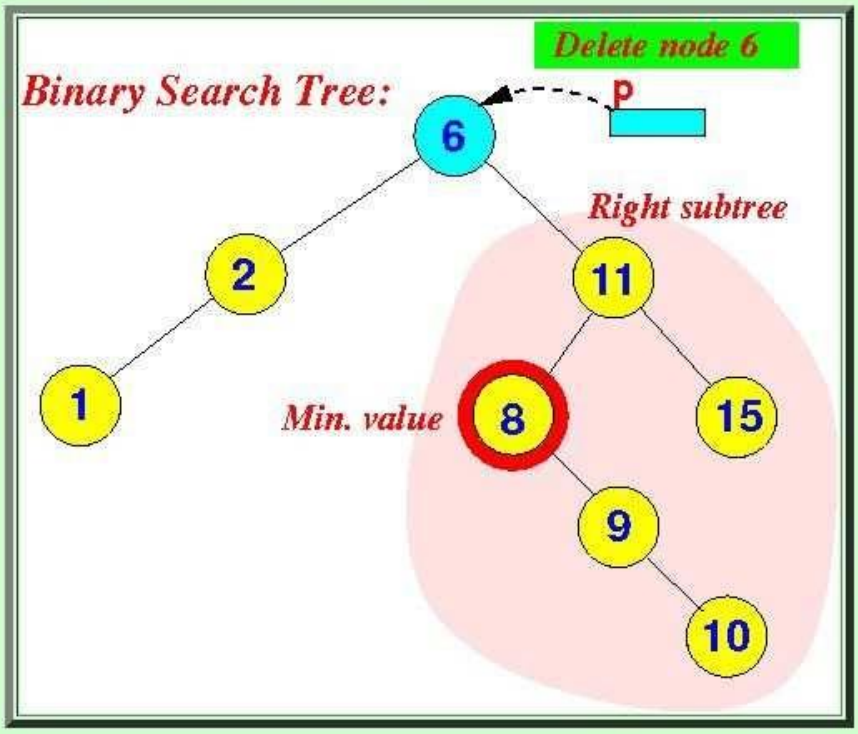
- ***First***, we must **find** the **node** with the value **6**:



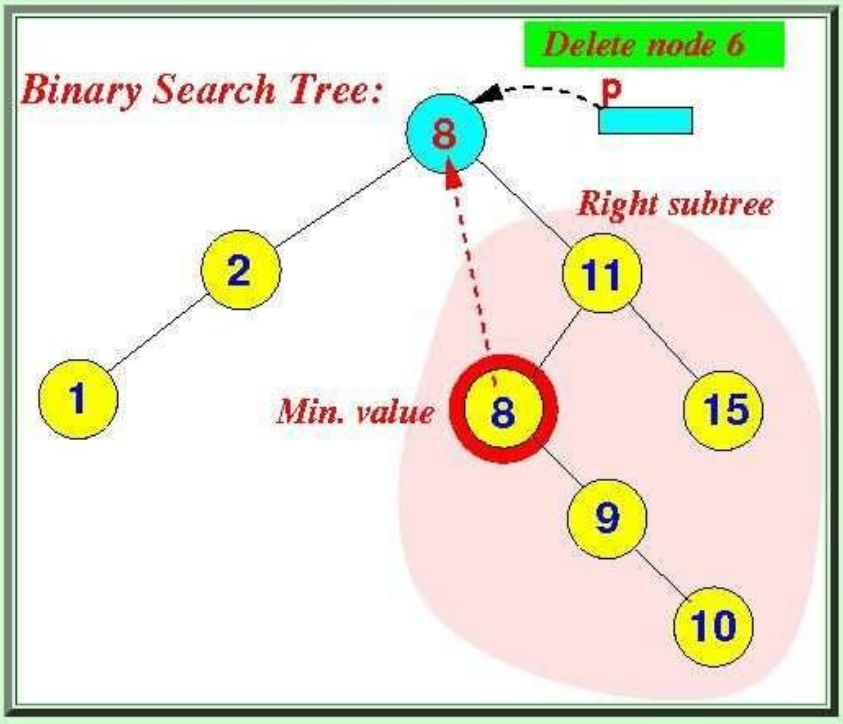
(We use the variable p to point to the **deletion node**)

- *Next*, we **find** the *successor* **node** of the node **6**.

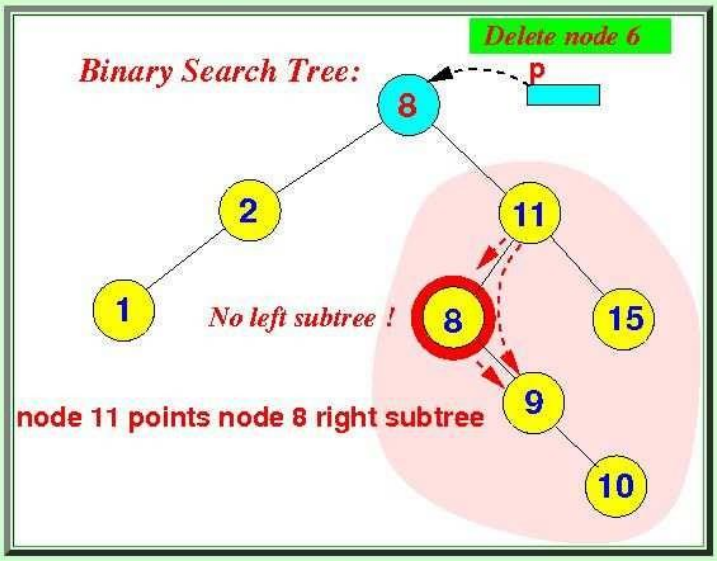
  The *successor* **node** is the **node** with the *minimum* **value** in the *right* **subtree**:



- We **copy** the **content** of the *successor* **node** of the node into the *deletion* **node** (p):

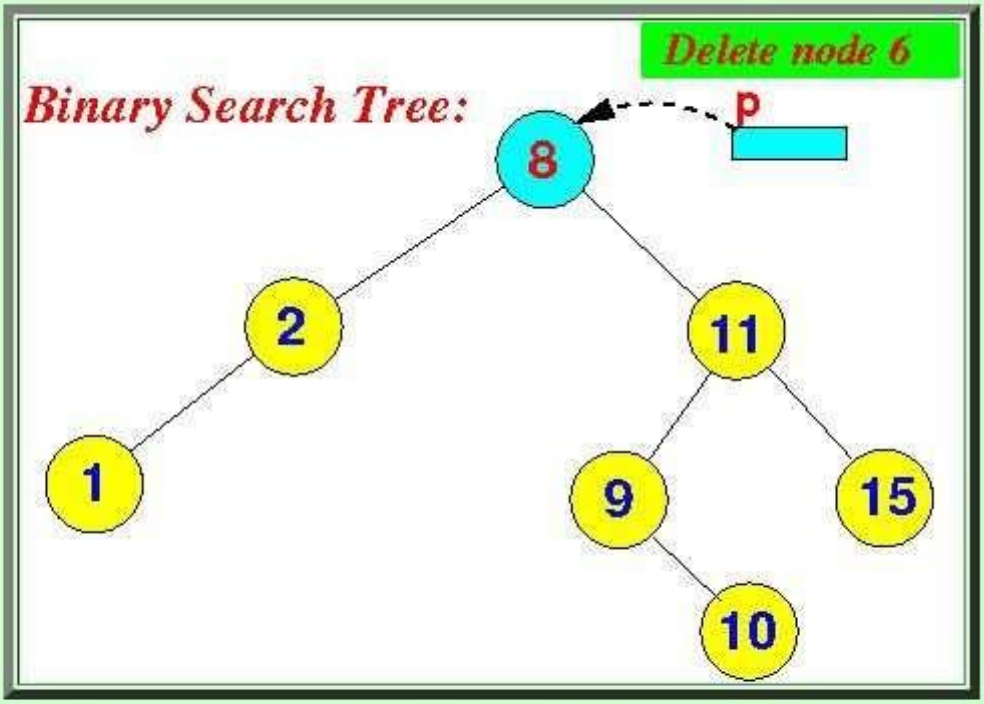


- The **last step** is **deleting** the *successor* **node** from the **BST**:



Because the *successor* **node** does not have a *left* **subtree**, we can **delete** the *successor* **node** using the *short-circuit* method (of **Case 2**)

Result:



Notice that the tree satisfies the Binary Search Tree property:

- all nodes in *left* **subtree** has *smaller* **values** and
- all nodes in the *right* **subtree** has *larger* **values**.

AVL TREE:

AVL tree is a height-balanced binary search tree. That means, an AVL tree is also a binary search tree but it is a balanced tree. A binary tree is said to be balanced if, the difference between the heights of left and right subtrees of every node in the tree is either -1, 0 or +1. In other words, a binary tree is said to be balanced if the height of left and right children of every node differ by either -1, 0 or +1. In an AVL tree, every node maintains an extra information known as balance factor. The AVL tree was introduced in the year 1962 by G.M. Adelson-Velsky and E.M. Landis.

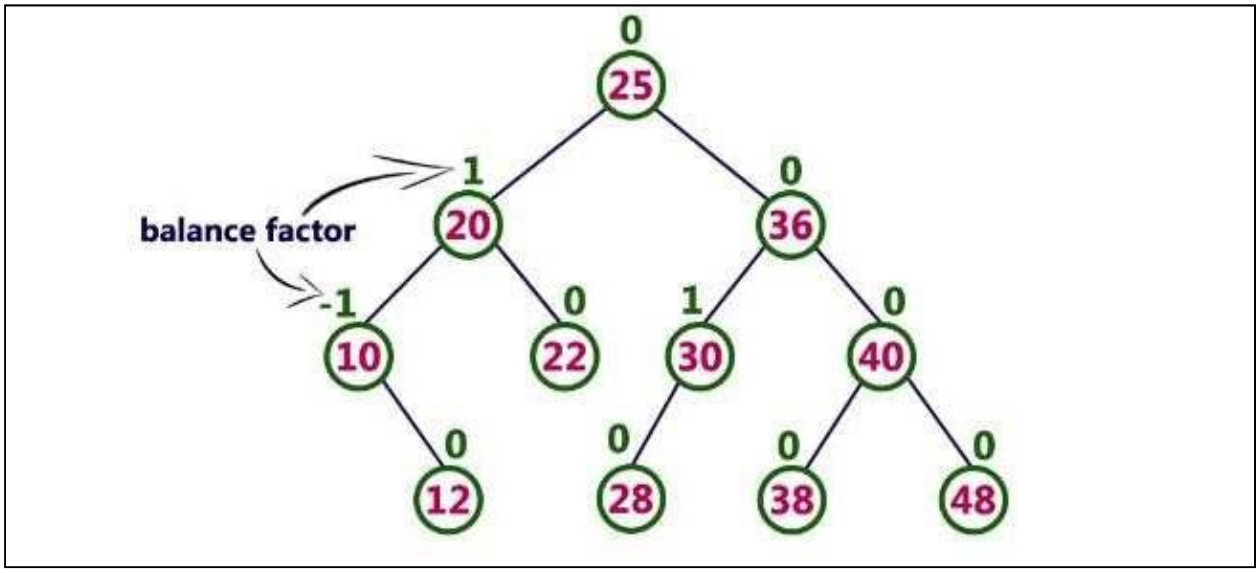
RULE OF AVL TREE:

An AVL tree is a balanced binary search tree. In an AVL tree, balance factor of every node is either -1, 0 or +1.

Where, Balance Factor = (Height of left subtree - Height of right subtree)

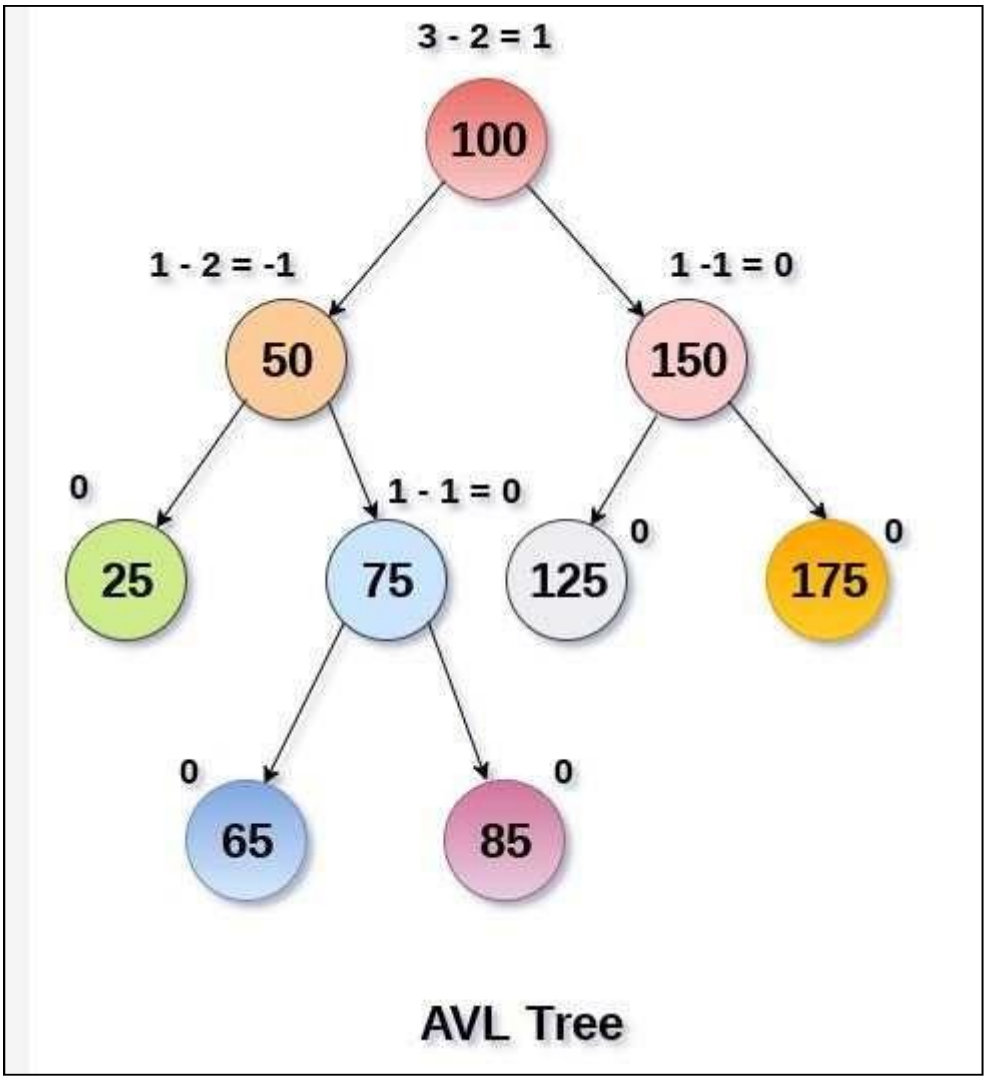
EXAMPLE OF AVL TREE:

The above tree is a binary search tree and every node is satisfying balance



factor condition. So this tree is said to be an AVL tree.

NOTE: Every AVL Tree is a binary search tree but every Binary Search Tree need not be AVL tree.

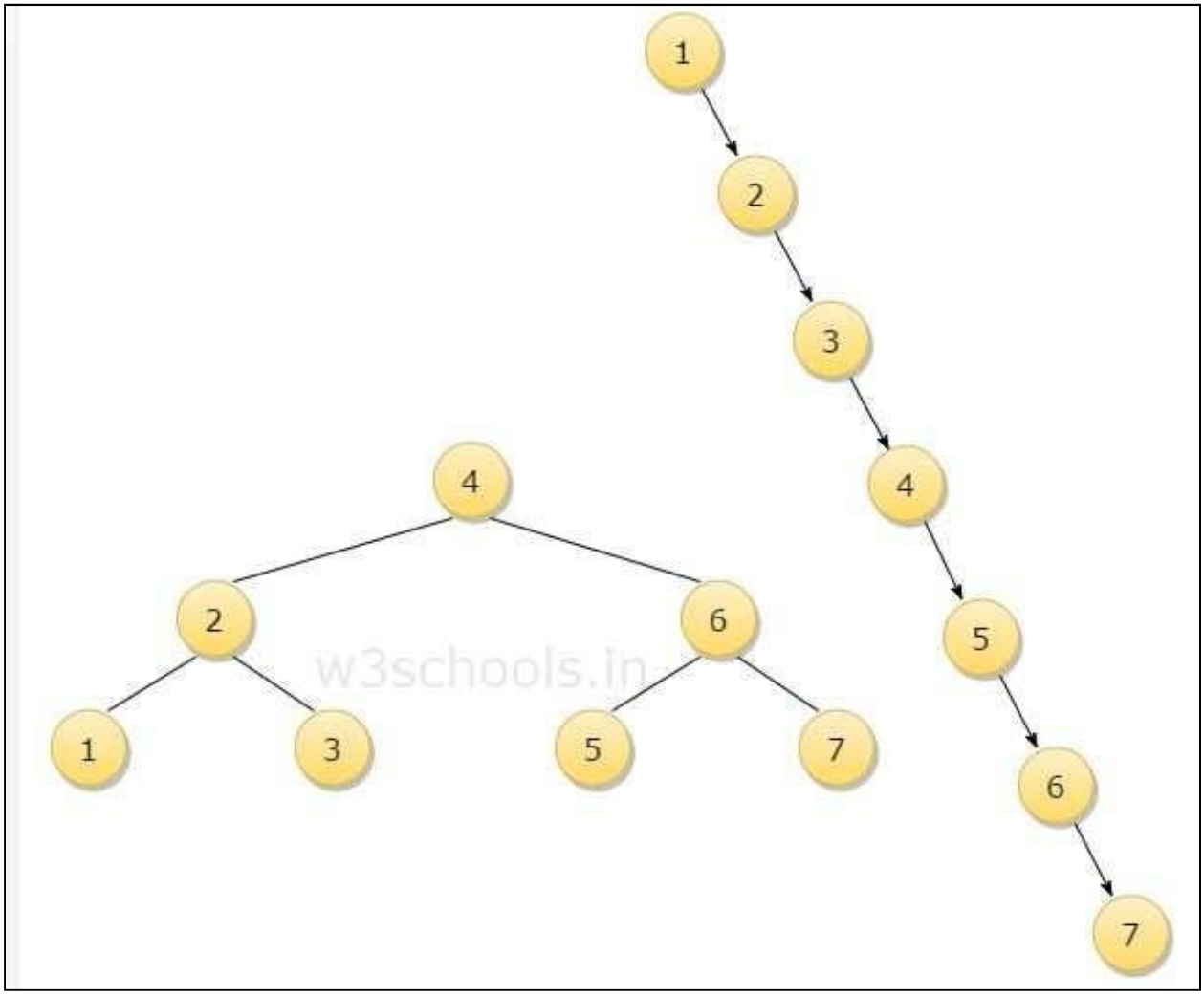


WHY AVL TREES?

Most of the BST operations (e.g., search, max, min, insert, delete.. etc) take O(h) time where h is the height of the BST. The cost of these operations may become O(n) for a skewed Binary tree. If we make sure that height of the tree remains O(Logn) after every insertion and deletion, then we can guarantee an upper bound of O(Logn) for all these operations. The height of an AVL tree is always O(Logn) where n is the number of nodes in the tree

Since AVL trees are height balance trees, operations like insertion and deletion have low time complexity. Let us consider an example:

If you have the following tree having keys 1, 2, 3, 4, 5, 6, 7 and then the binary tree will be like :



To insert a node with a key Q in the binary tree, the algorithm requires seven comparisons, but if you insert the same key in AVL tree, from the above 1st figure, you can see that the algorithm will require three comparisons.

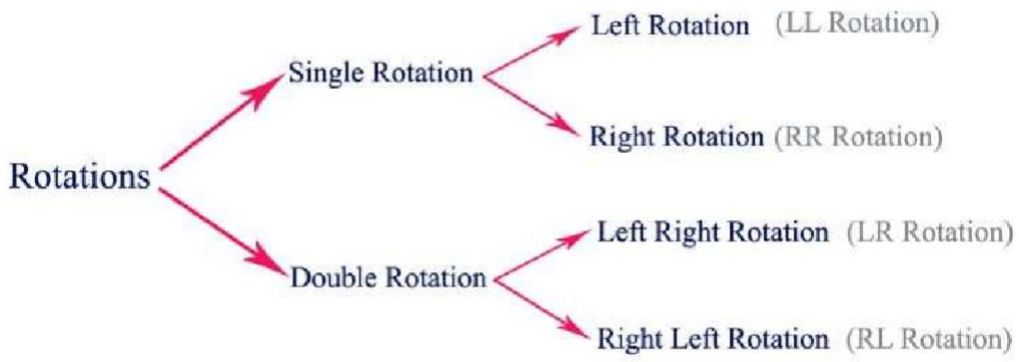
COMPLEXITY

Algorithm	Average case	Worst case
Space	o(n)	o(n)
Search	o(log n)	o(log n)
Insert	o(log n)	o(log n)
Delete	o(log n)	o(log n)

ROTATIONS IN AN AVL TREES:

Rotation is the process of moving nodes either to left or to right to make the tree balanced.

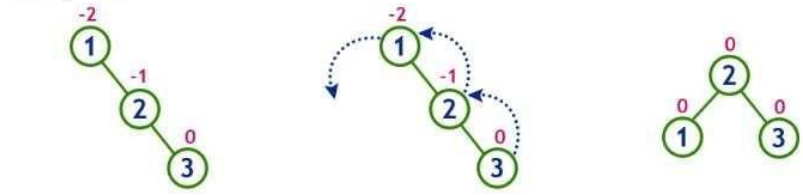
There are **four** rotations and they are classified into **two** types.



Single Left Rotation (LL Rotation)

In LL Rotation, every node moves one position to left from the current position. To understand LL Rotation, let us consider the following insertion operation in AVL Tree...

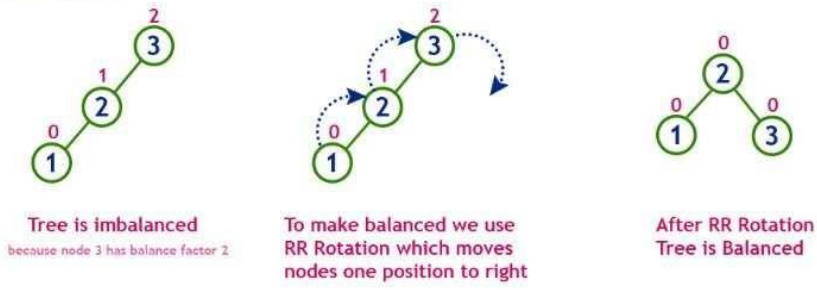
insert 1, 2 and 3



Single Right Rotation (RR Rotation)

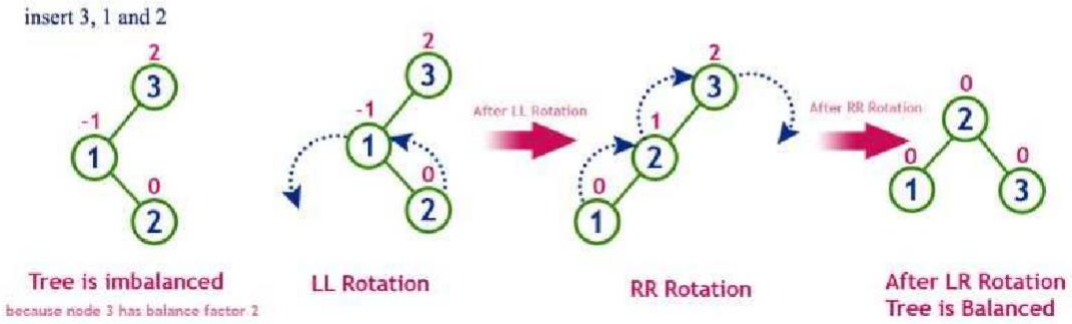
In RR Rotation, every node moves one position to right from the current position. To understand RR Rotation, let us consider the following insertion operation in AVL Tree...

insert 3, 2 and 1



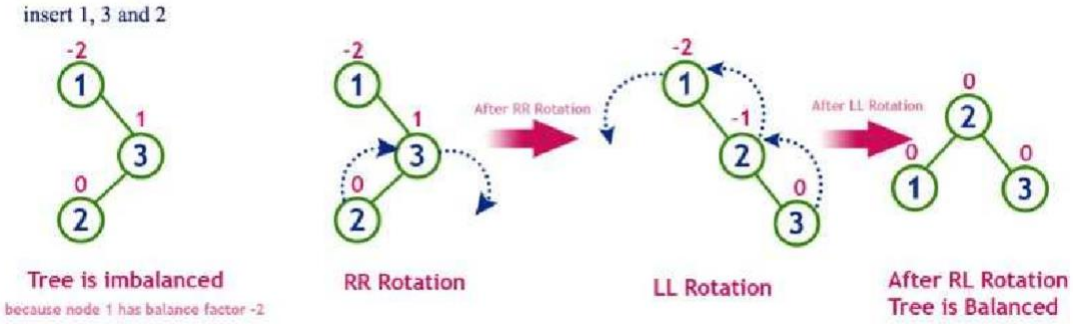
Left Right Rotation (LR Rotation)

The LR Rotation is a sequence of single left rotation followed by a single right rotation. In LR Rotation, at first, every node moves one position to the left and one position to right from the current position. To understand LR Rotation, let us consider the following insertion operation in AVL Tree...



Right Left Rotation (RL Rotation)

The RL Rotation is sequence of single right rotation followed by single left rotation. In RL Rotation, at first every node moves one position to right and one position to left from the current position. To understand RL Rotation, let us consider the following insertion operation in AVL Tree...



OPERATIONS ON AN AVL TREE:

Due to the fact that, AVL tree is also a binary search tree therefore, all the operations are performed in the same way as they are performed in a binary search tree. Searching and traversing do not lead to the violation in property of AVL tree. However, insertion and deletion are the operations which can violate this property and therefore, they need to be revisited.

The following operations are performed on AVL tree...

1. Search
2. Insertion
3. Deletion

Search Operation in AVL Tree

In an AVL tree, the search operation is performed with O(log n) time complexity. The search operation in the AVL tree is similar to the search operation in a Binary search tree. We use the following steps to search an element in AVL tree...

- Step 1 - Read the search element from the user.
- Step 2 - Compare the search element with the value of root node in the tree.
- Step 3 - If both are matched, then display "Given node is found!!!" and terminate the function
- Step 4 - If both are not matched, then check whether search element is smaller or larger than that node value.
- Step 5 - If search element is smaller, then continue the search process in left subtree.
- Step 6 - If search element is larger, then continue the search process in right subtree.
- Step 7 - Repeat the same until we find the exact element or until the search element is compared with the leaf node.
- Step 8 - If we reach to the node having the value equal to the search value, then display "Element is found" and terminate the function.
- Step 9 - If we reach to the leaf node and if it is also not matched with the search element, then display "Element is not found" and terminate the function.

Insertion Operation in AVL Tree

In an AVL tree, the insertion operation is performed with O(log n) time complexity. In AVL Tree, a new node is always inserted as a leaf node. The insertion operation is performed as follows...

- Step 1 - Insert the new element into the tree using Binary Search Tree insertion logic.
- Step 2 - After insertion, check the Balance Factor of every node.
- Step 3 - If the Balance Factor of every node is 0 or 1 or -1 then go for next operation.
- Step 4 - If the Balance Factor of any node is other than 0 or 1 or -1 then that tree is said to be imbalanced. In this case, perform suitable Rotation to make it balanced and go for next operation.

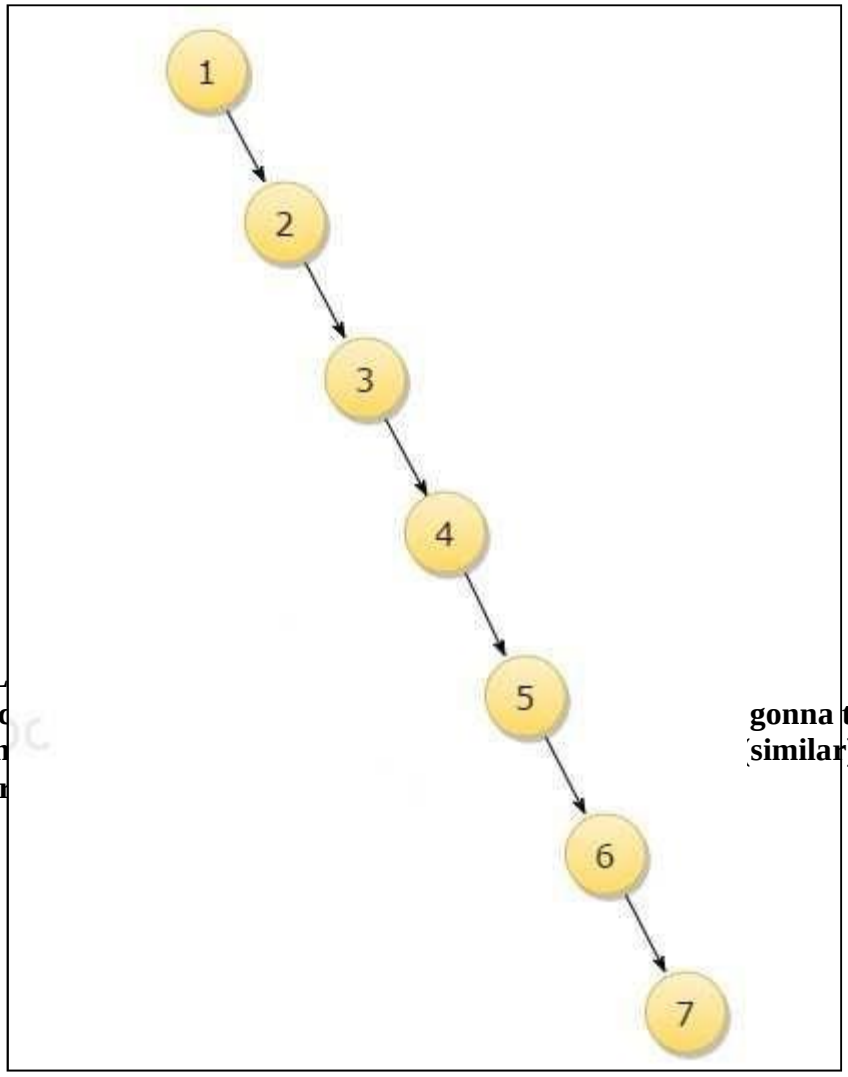
Deletion Operation in AVL Tree

The deletion operation in AVL Tree is similar to deletion operation in BST. But after every deletion operation, we need to check with the Balance

Factor condition. If the tree is balanced after deletion go for next operation otherwise perform suitable rotation to make the tree Balanced.

EXAMPLE PROBLEMS:

Q1.CONSTRUCT BST - 1,2,3,4,5,6,7.



RESUL

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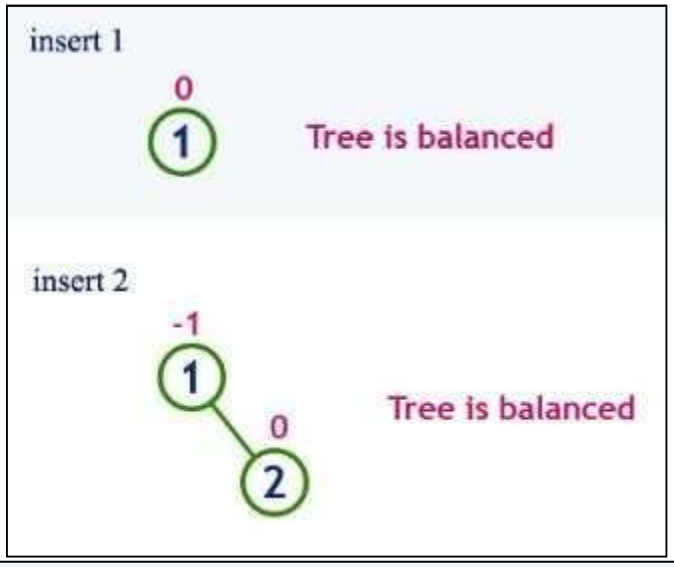
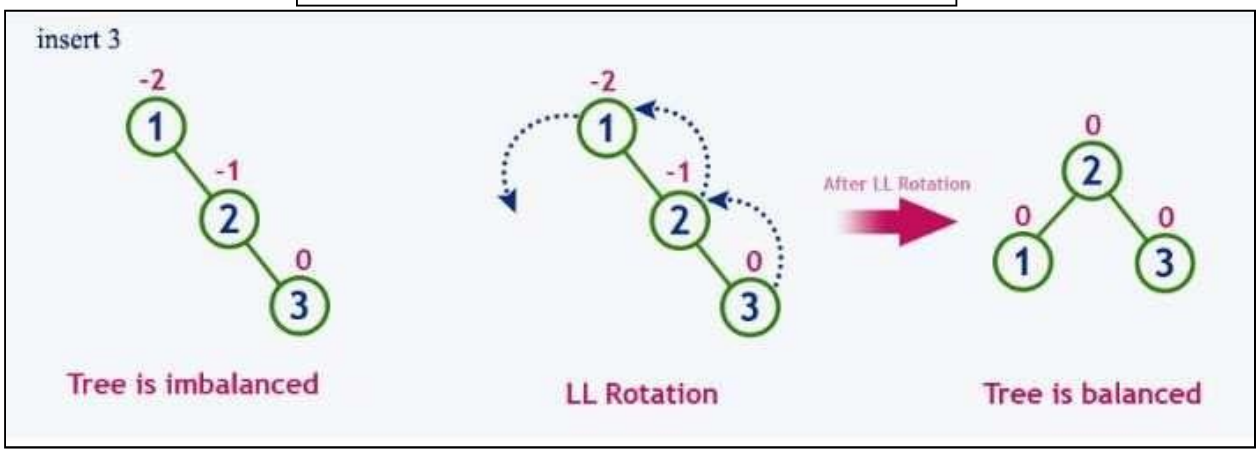
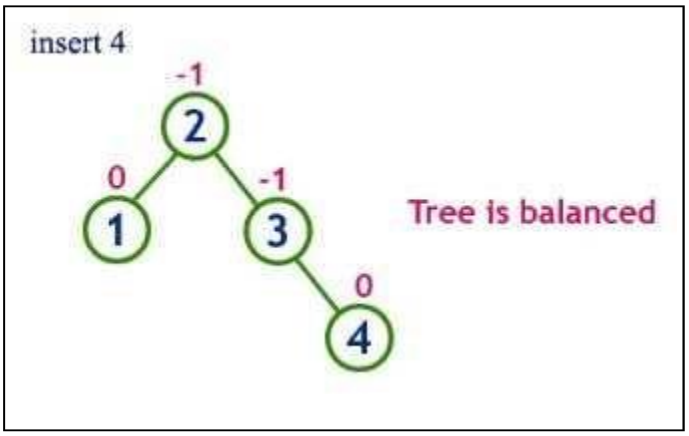
O(n) tin **similar) using**

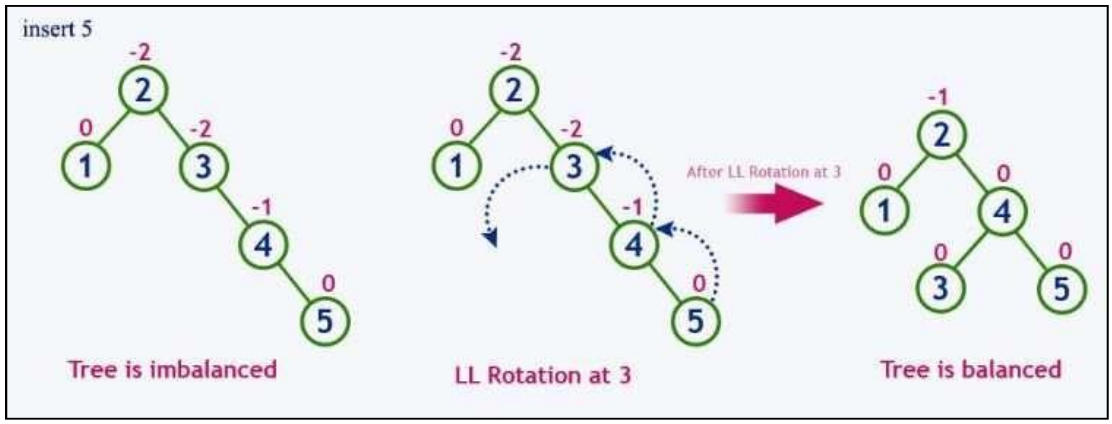
AVL Tr

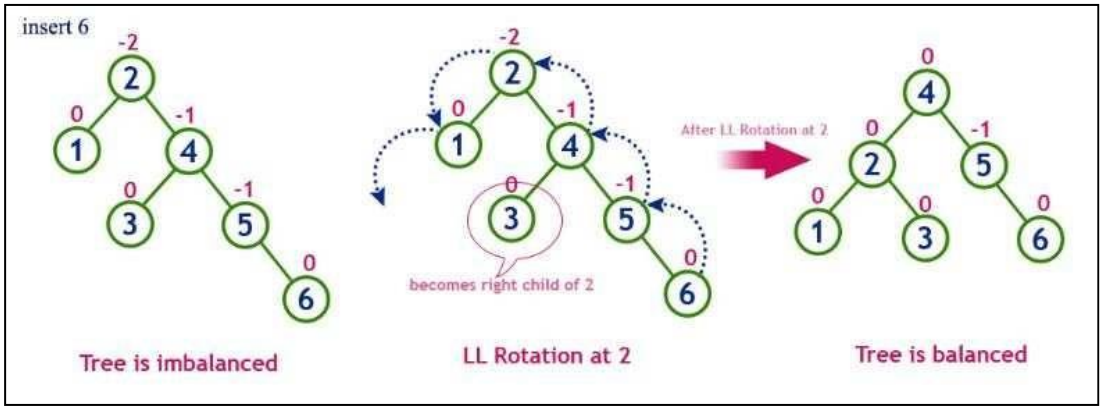
Q2. Construct an AVL Tree by inserting numbers from 1 to 8.

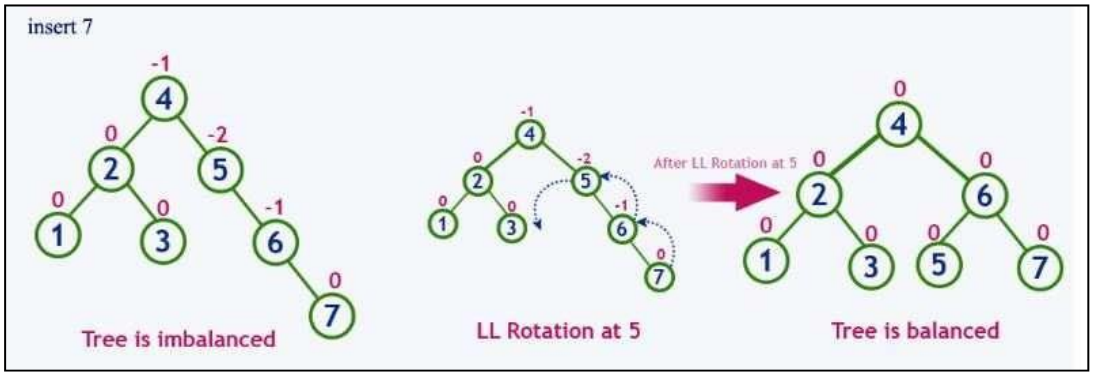


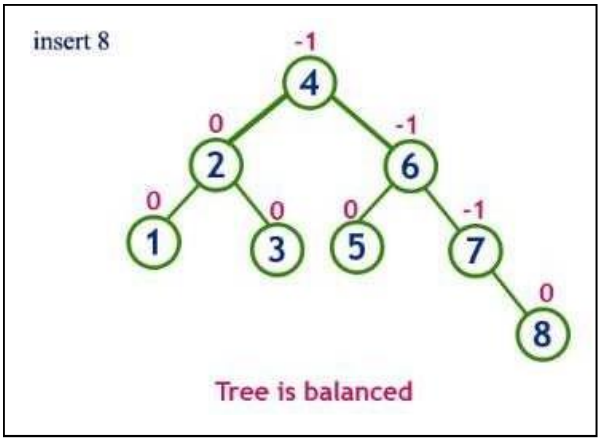





insert 5
-2
2
0
1
-2
3
-1
4
0
5
Tree is imbalanced
-2
2
0
1
-2
3
-1
4
0
5
LL Rotation at 3
After LL Rotation at 3
-1
2
0
1
0
4
0
3
0
5
Tree is balanced


insert 6
becomes right child of 2
After LL Rotation at 2
Tree is imbalanced
LL Rotation at 2
Tree is balanced


insert 7
After LL Rotation at 5
Tree is imbalanced
LL Rotation at 5
Tree is balanced

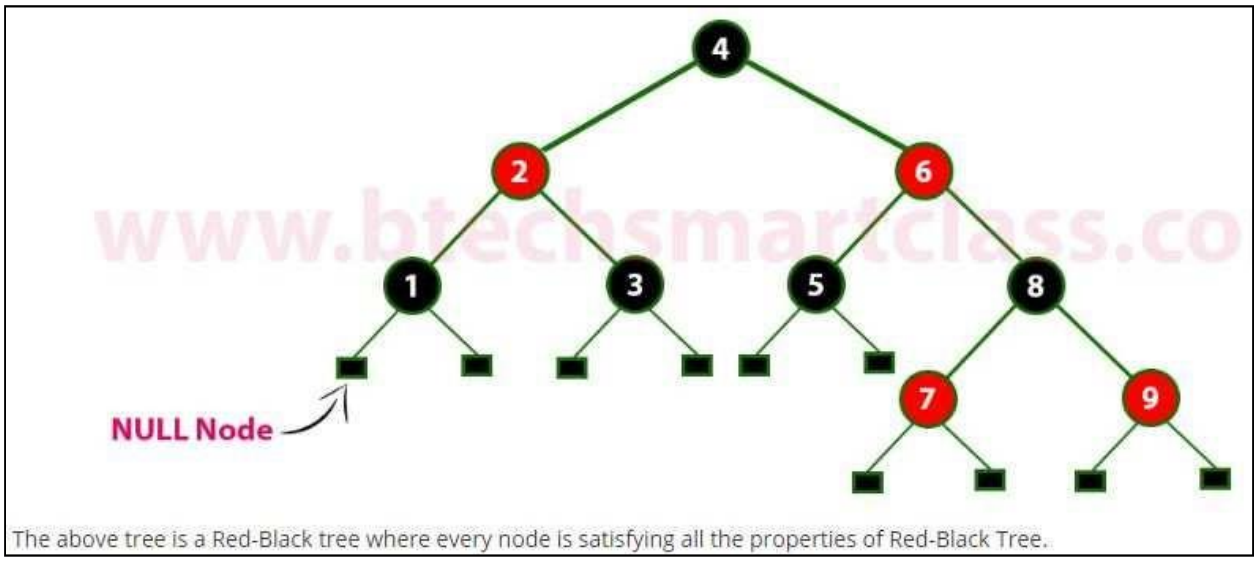

insert 8
Tree is balanced

RED-BLACK TREES

Red-Black Tree is a self-balancing Binary Search Tree (BST) in which every node is colored either RED or BLACK.

In Red Black Tree, the color of a node is decided based on the properties of Red-Black Tree. Every Red Black Tree has the following properties:

- Property #1: Red - Black Tree must be a Binary Search Tree.
- Property #2: The ROOT node must be colored BLACK.
- Property #3: The children of Red colored node must be colored BLACK. (There should not be two consecutive RED nodes).
- Property #4: In all the paths of the tree, there should be same number of BLACK colored nodes.
- Property #5: Every new node must be inserted with RED color.
- Property #6: Every leaf (i.e, NULL node) must be colored BLACK.



The above tree is a Red-Black tree where every node is satisfying all the properties of Red-Black Tree.

NOTE: Every Red Black Tree is a binary search tree but every Binary Search Tree need not be Red Black tree.

How does a Red-Black Tree ensure balance?

A simple example to understand balancing is, a chain of 3 nodes is not possible in the Red-Black tree. We can try any combination of colours and see all of them violate Red-Black tree property.

A chain of 3 nodes is nodes is not possible in Red-Black Trees.

Following are **NOT** Red-Black Trees

Following are different possible Red-Black Trees with above 3 keys

Why Red-Black Trees?

Most of the BST operations (e.g., search, max, min, insert, delete.. etc) take O(h) time where h is the height of the BST. The cost of these operations may become O(n) for a skewed Binary tree. If we make sure that height of the tree remains O(Logn) after every insertion and deletion, then we can guarantee an upper bound of O(Logn) for all these operations. The height of a Red-Black tree is always O(Logn) where n is the number of nodes in the tree.

Comparison with AVL Tree

The AVL trees are more balanced compared to Red-Black Trees, but they may cause more rotations during insertion and deletion. So if your application involves many frequent insertions and deletions, then Red

Black trees should be preferred. And if the insertions and deletions are less frequent and search is a more frequent operation, then AVL tree should be preferred over Red-Black Tree.

Advantages of Red Black Tree

1. Red black tree are useful when we need insertion and deletion relatively frequent.
2. Red-black trees are self-balancing so these operations are guaranteed to be O(logn).
3. They have relatively low constants in a wide variety of scenarios. **NOTE**: Every Red Black Tree with n nodes has height <= 2Log2(n+1) **Applications of Red Black Tree**

1. Most of the self-balancing BST library functions like map and set in C++ (OR TreeSet and TreeMap in Java) use Red Black Tree
2. It is used to implement CPU Scheduling Linux. Completely Fair Scheduler uses it.

Insertion into RED BLACK Tree

In a Red-Black Tree, every new node must be inserted with the color RED. The insertion operation in Red Black Tree is similar to insertion operation in Binary Search Tree. But it is inserted with a color property. After every insertion operation, we need to check all the properties of Red-Black Tree. If all the properties are satisfied then we go to next operation otherwise we perform the following operation to make it Red Black Tree.

1. **Recolor**
2. **Rotation**
3. **Rotation followed by Recolor**

The insertion operation in Red Black tree is performed using the following steps...

- Step 1 - Check whether tree is Empty.
- Step 2 - If tree is Empty then insert the newNode as Root node with color Black and exit from the operation.

- Step 3 - If tree is not Empty then insert the newNode as leaf node with color Red.
- Step 4 - If the parent of newNode is Black then exit from the operation.
- Step 5 - If the parent of newNode is Red then check the color of parentnode's sibling of newNode.
- Step 6 - If it is colored Black or NULL then make suitable Rotation and Recolor it.
- Step 7 - If it is colored Red then perform Recolor. Repeat the same until tree becomes Red Black Tree.
  EXAMPLE:

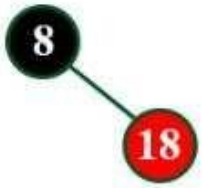
Create a RED BLACK Tree by inserting following sequence of number 8, 18, 5, 15, 17, 25, 40 & 80.

insert (8)

Tree is Empty. So insert newNode as Root node with black color.

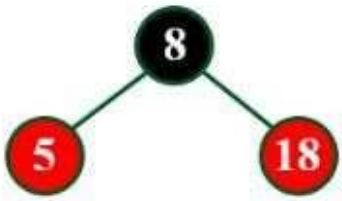
insert (18)

Tree is not Empty. So insert newNode with red color.



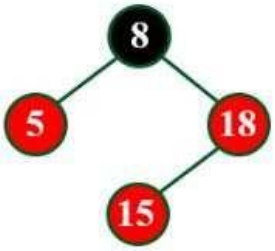
insert (5)

Tree is not Empty. So insert newNode with red color.



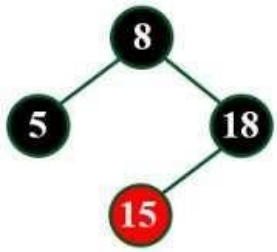
insert (15)

Tree is not Empty. So insert newNode with red color.



Here there are two consecutive Red nodes (18 & 15).
The newnode's parent sibling color is Red
and parent's parent is root node.
So we use RECOLOR to make it Red Black Tree.

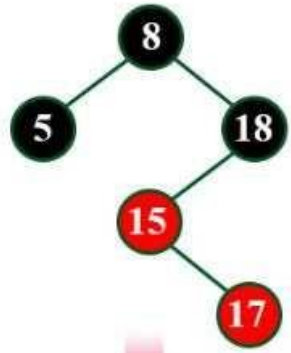
After RECOLOR



After Recolor operation, the tree is satisfying all Red Black Tree properties.

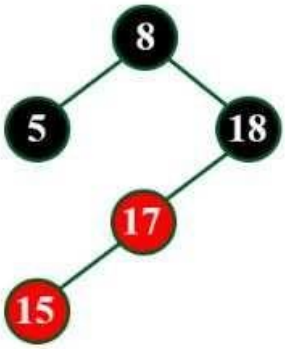
insert (17)

Tree is not Empty. So insert newNode with red color.

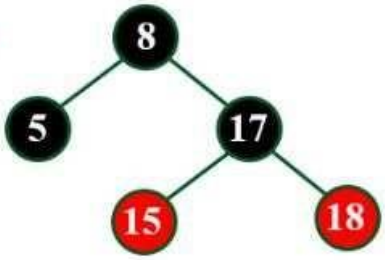


Here there are two consecutive Red nodes (15 & 17).
The newnode's parent sibling is NULL. So we need rotation.
Here, we need LR Rotation & Recolor.

After Left Rotation

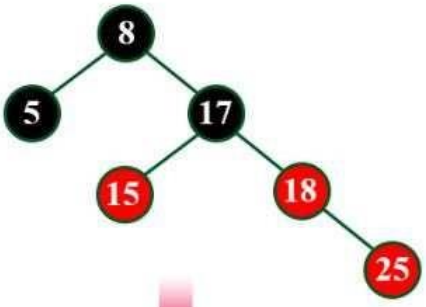


After Right Rotation & Recolor



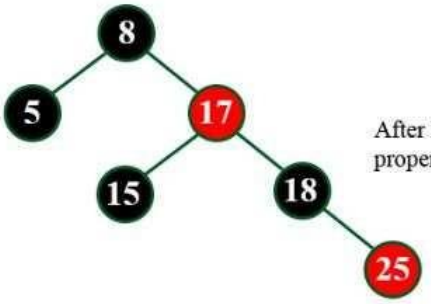
insert (25)

Tree is not Empty. So insert newNode with red color.



Here there are two consecutive Red nodes (18 & 25).
The newnode's parent sibling color is Red
and parent's parent is not root node.
So we use RECOLOR and Recheck.

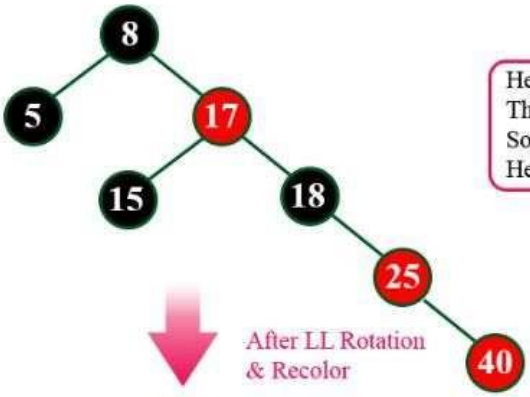
After Recolor



After Recolor operation, the tree is satisfying all Red Black Tree properties.

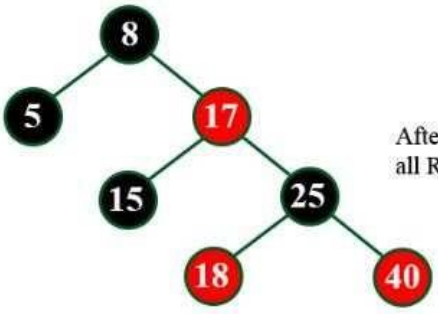
insert (40)

Tree is not Empty. So insert newNode with red color.



Here there are two consecutive Red nodes (25 & 40).
The newnode's parent sibling is NULL
So we need a Rotation & Recolor.
Here, we use LL Rotation and Recheck.

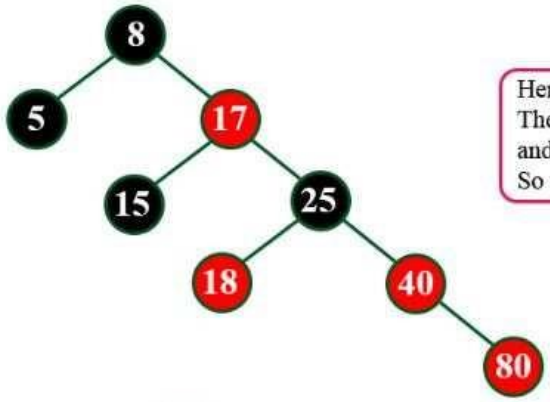
After LL Rotation
& Recolor



After LL Rotation & Recolor operation, the tree is satisfying all Red Black Tree properties.

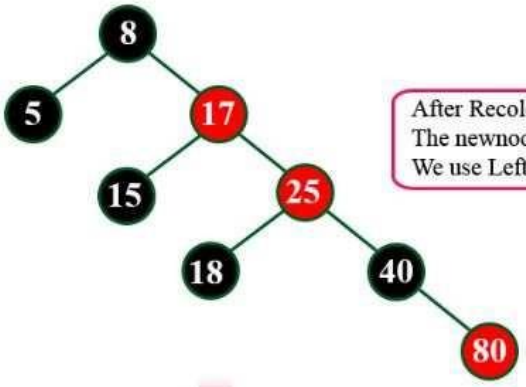
insert (80)

Tree is not Empty. So insert newNode with red color.



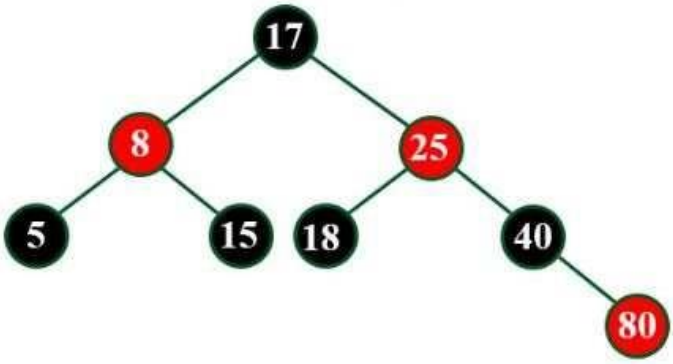
Here there are two consecutive Red nodes (40 & 80).
The newnode's parent sibling color is Red
and parent's parent is not root node.
So we use RECOLOR and Recheck.

After Recolor



After Recolor again there are two consecutive Red nodes (17 & 25).
The newnode's parent sibling color is Black. So we need Rotation.
We use Left Rotation & Recolor.

After Left Rotation
& Recolor



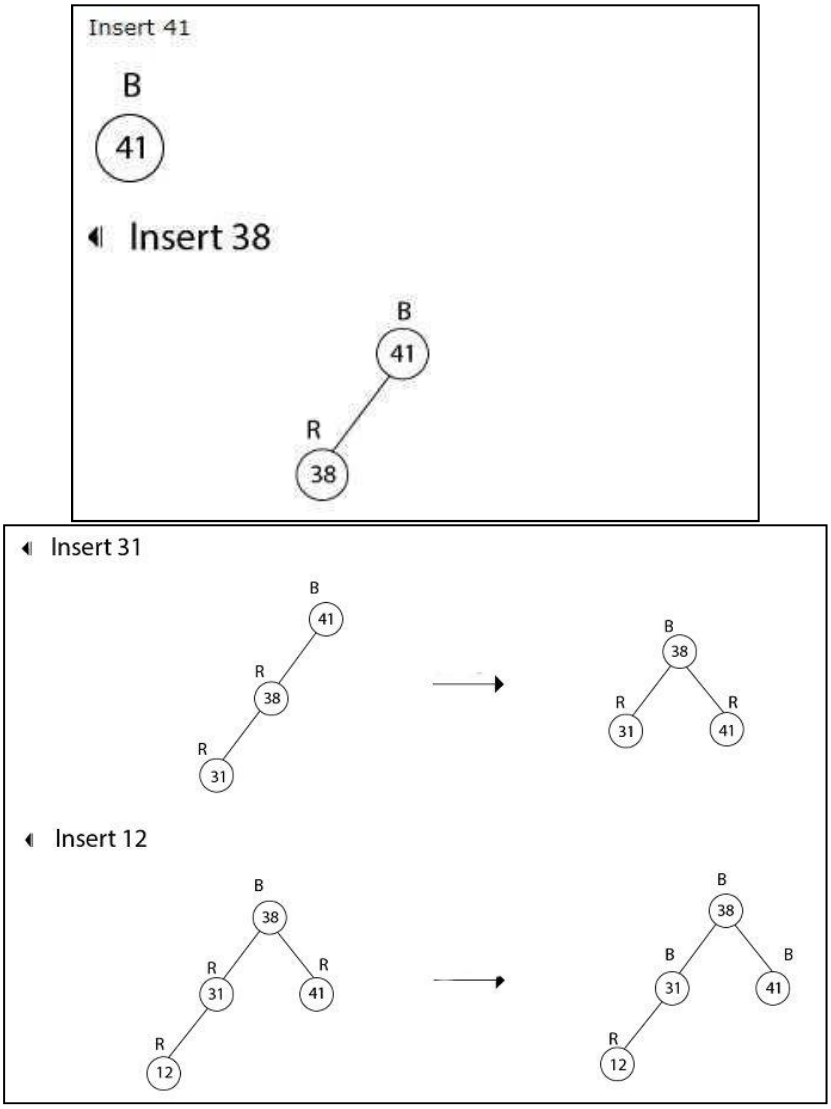
Finally above tree is satisfying all the properties of Red Black Tree and it is a perfect Red Black tree.

Deletion Operation in Red Black Tree

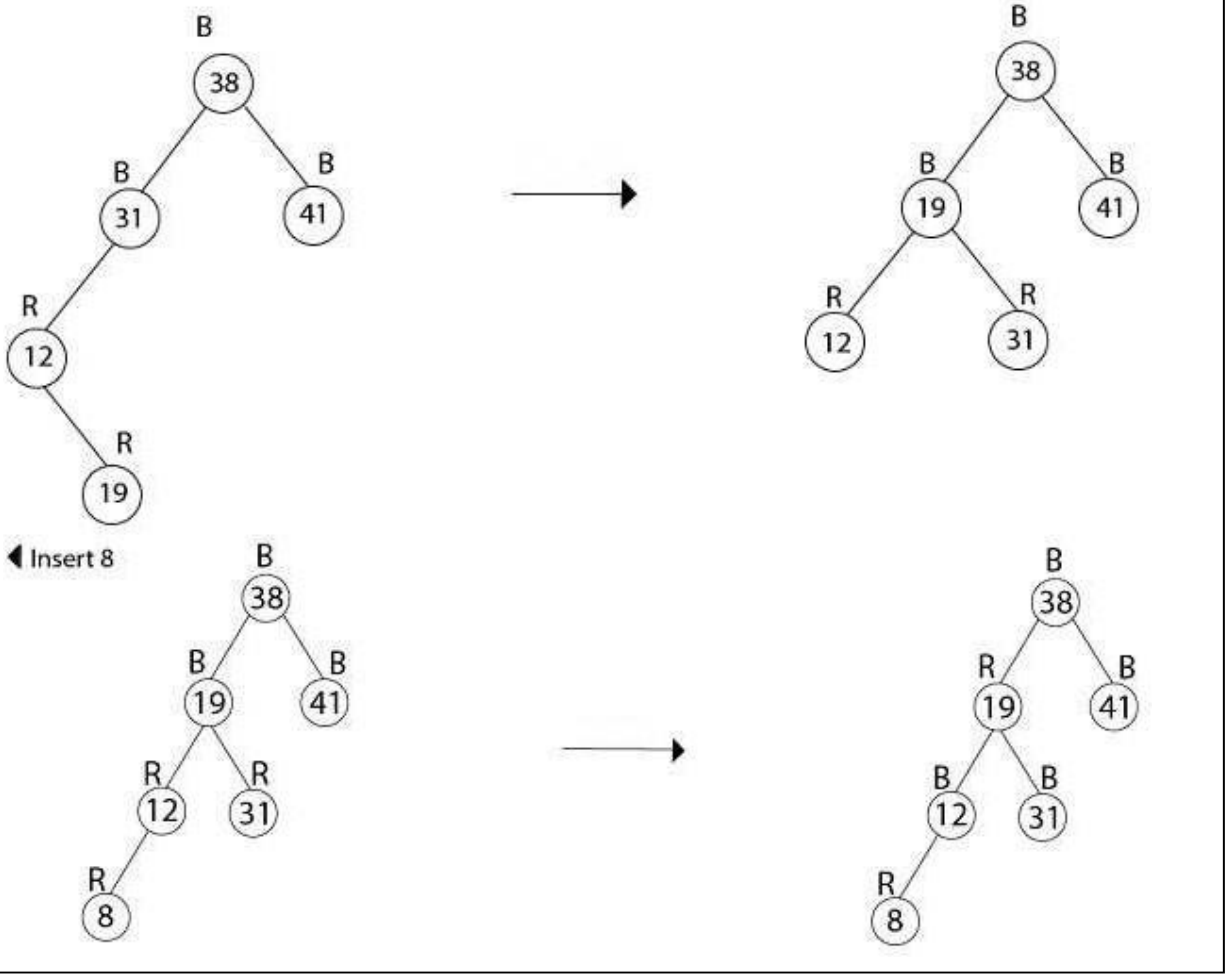
The deletion operation in Red-Black Tree is similar to deletion operation in BST. But after every deletion operation, we need to check with the Red- Black Tree properties. If any of the properties are violated then make suitable operations like Recolor, Rotation and Rotation followed by Recolor to make it Red-Black Tree.

EXAMPLE:

Q. Show the red-black trees that result after successivelyinserting the keys 41,38,31,12,19,8 into an initially empty red-black tree.

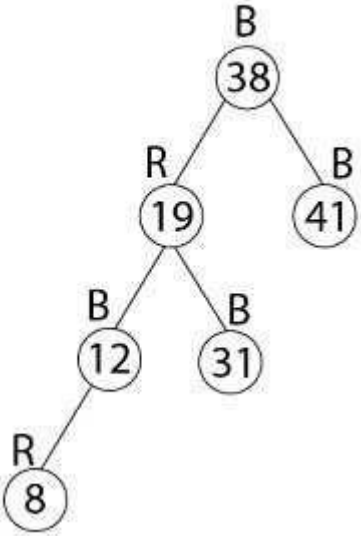


Insert 19

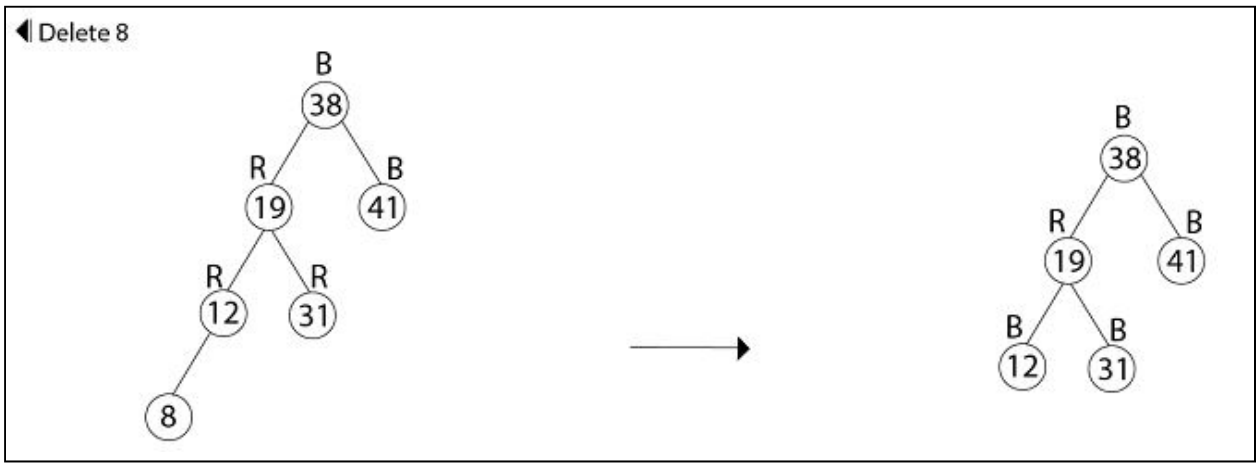


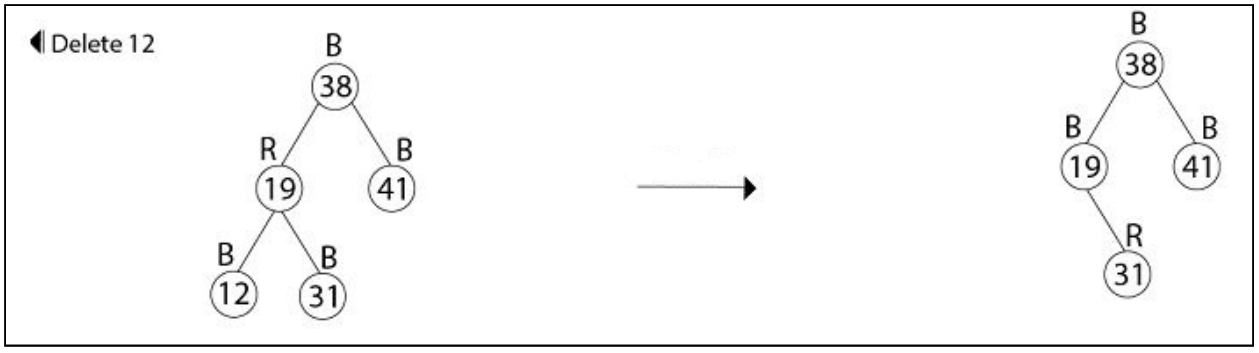
◀ Insert 8

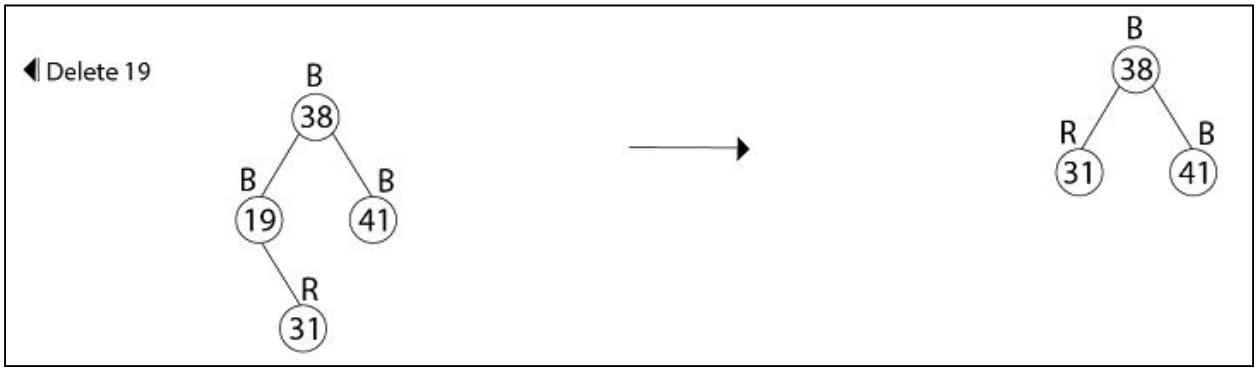
Thus the final tree is

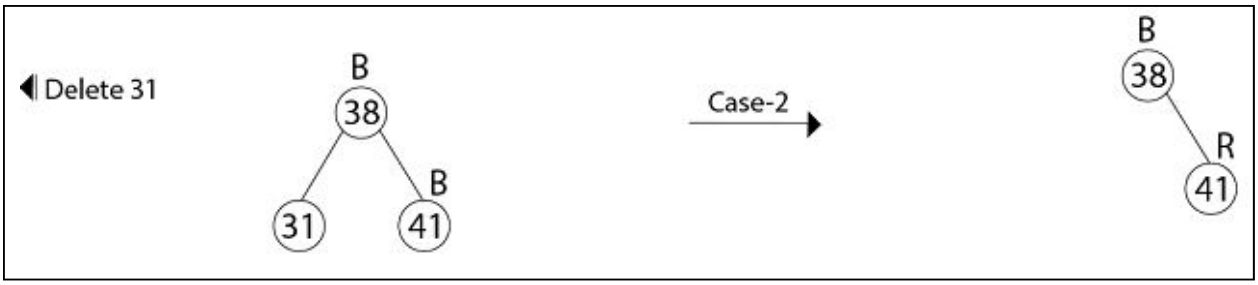


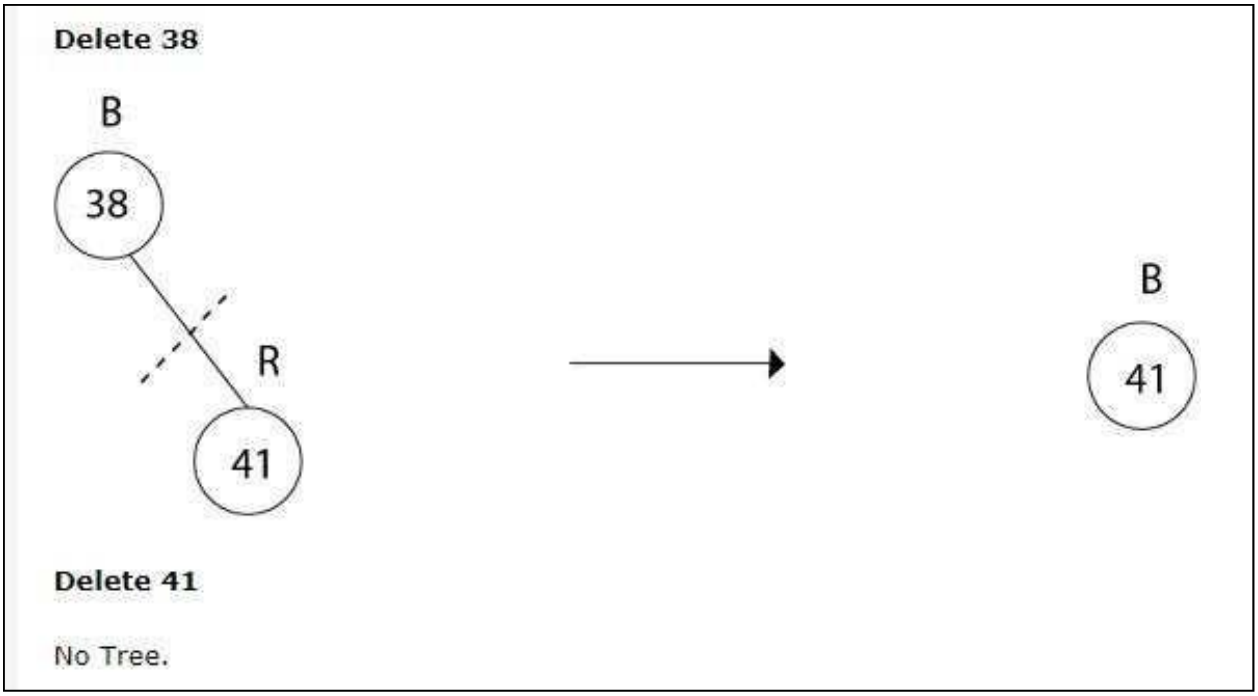
Q. In a previous example, we found that the red-black tree that results from successively inserting the keys 41,38,31,12,19,8 into an initially empty tree. Now show the red-black trees that result from the successful deletion of the keys in the order 8, 12, 19,31,38,41.











For more examples of deletion: https://www.geeksforgeeks.org/red-black-tree-set-3- delete-2/

SPLAY TREES:

Definition: Splay Tree is a self - adjusted Binary Search Tree in which every operation on element rearranges the tree so that the element is placed at the root position of the tree.

The worst case time complexity of Binary Search Tree (BST) operations like search, delete, insert is O(n). The worst case occurs when the tree is skewed. We can get the worst case time complexity as O(Logn) with AVL and Red-Black Trees.

Can we do better than AVL or Red-Black trees in practical situations?

Like AVL and Red-Black Trees, Splay tree is also self-balancing BST. The main idea of splay tree is to bring the recently accessed item to root of the tree, this makes the recently searched item to be accessible in O(1) time if accessed again. The idea is to use locality of reference (In a typical application, 80% of the access are to 20% of the items). Imagine a situation where we have millions or billions of keys and only few of them are accessed frequently, which is very likely in many practical applications.

All splay tree operations run in O(log n) time on average, where n is the number of entries in the tree. Any single operation can take Theta(n) time in the worst case.

Splaying

Splaying an element, is the process of bringing it to the root position by performing suitable rotation operations.

By splaying elements we bring more frequently used elements closer to the root of the tree so that any operation on those elements is performed quickly. That means the splaying operation automatically brings more frequently used elements closer to the root of the tree.

Every operation on splay tree performs the splaying operation. For example, the insertion operation first inserts the new element using the binary search tree insertion process, then the newly inserted element is splayed so that it is placed at the root of the tree. The search operation in a splay tree is nothing but searching the element using binary search process

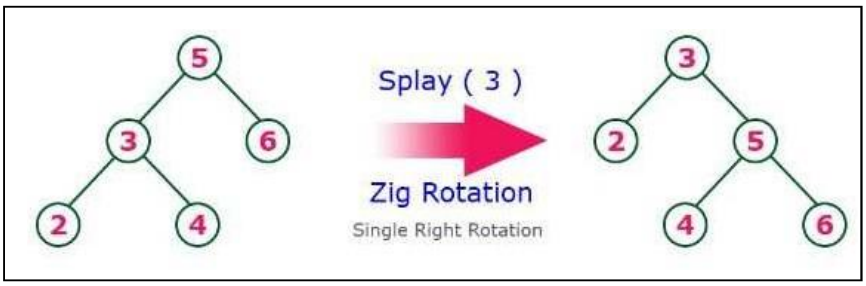
and then splaying that searched element so that it is placed at the root of the tree. In splay tree, to splay any element we use the following rotation operations...

Rotations in Splay Tree

- 1. Zig Rotation
- 2. Zag Rotation
- 3. Zig - Zig Rotation
- 4. Zag - Zag Rotation
- 5. Zig - Zag Rotation
- 6. Zag - Zig Rotation

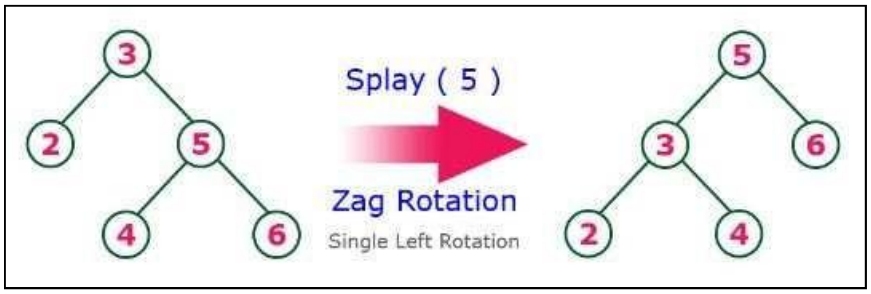
Zig Rotation

The Zig Rotation in splay tree is similar to the single right rotation in AVL Tree rotations. In zig rotation, every node moves one position to the right from its current position. Consider the following example...



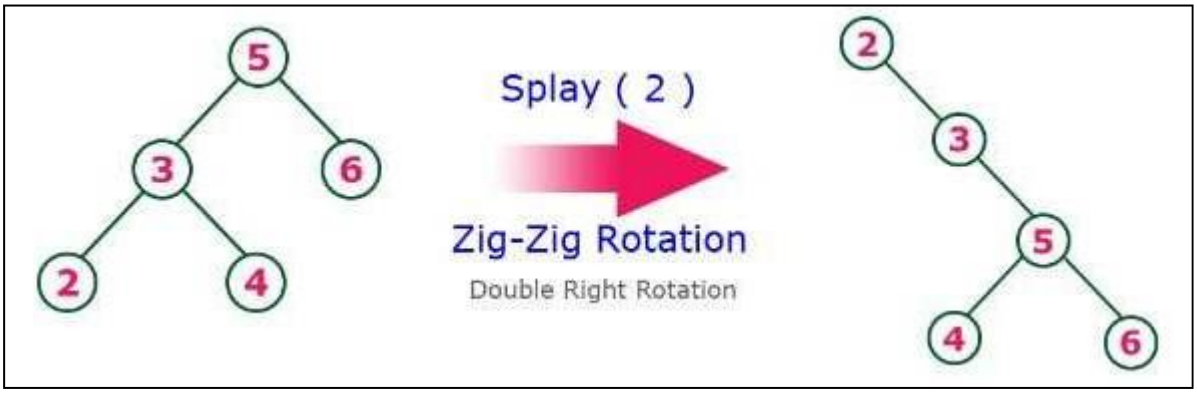
Zag Rotation

The Zag Rotation in splay tree is similar to the single left rotation in AVL Tree rotations. In zag rotation, every node moves one position to the left from its current position. Consider the following example...



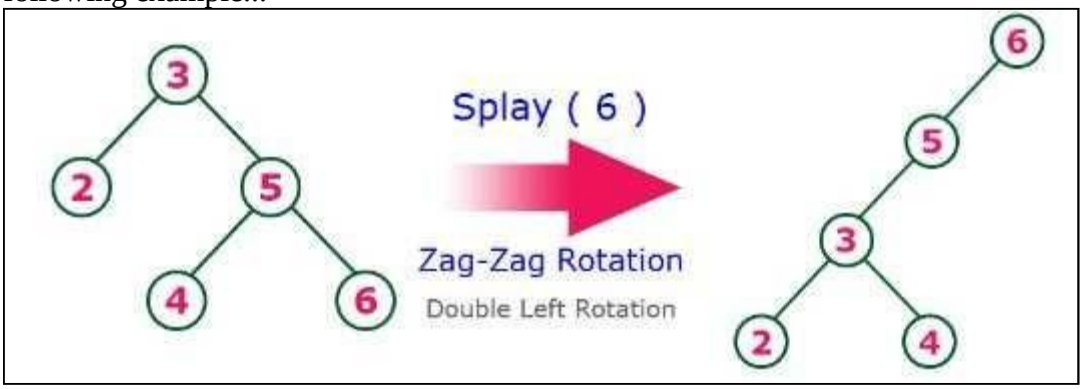
Zig-Zig Rotation

The Zig-Zig Rotation in splay tree is a double zig rotation. In zig-zig rotation, every node moves two positions to the right from its current position. Consider the following example...



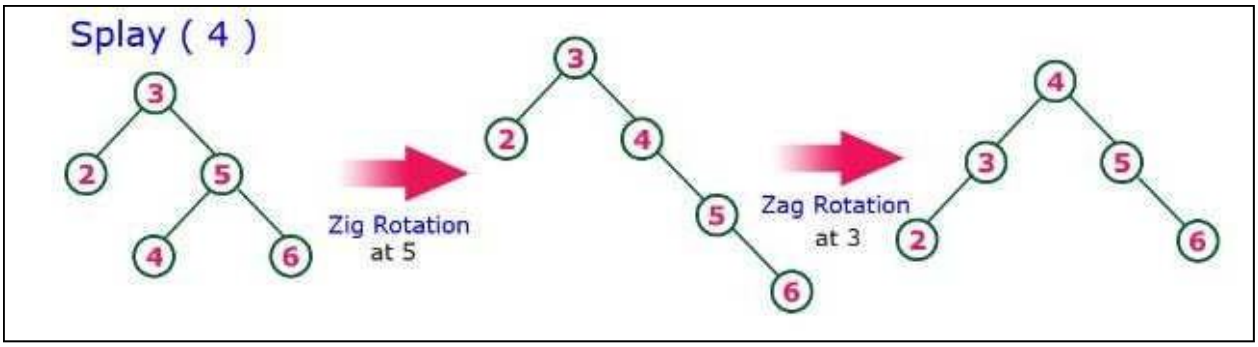
Zag-Zag Rotation

The Zag-Zag Rotation in splay tree is a double zag rotation. In zag-zag rotation, every node moves two positions to the left from its current position. Consider the following example...



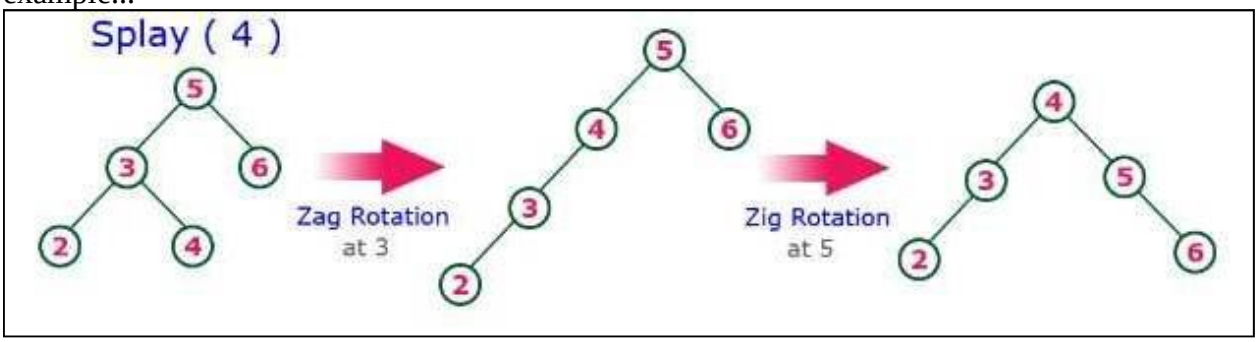
Zig-Zag Rotation

The Zig-Zag Rotation in splay tree is a sequence of zig rotation followed by zag rotation. In zig-zag rotation, every node moves one position to the right followed by one position to the left from its current position. Consider the following example...



Zag-Zig Rotation

The Zag-Zig Rotation in splay tree is a sequence of zag rotation followed by zig rotation. In zag-zig rotation, every node moves one position to the left followed by one position to the right from its current position. Consider the following example...



NOTE: Every Splay tree must be a binary search tree but it is need not to be balanced tree.

Insertion Operation in Splay Tree

The insertion operation in Splay tree is performed using following steps...

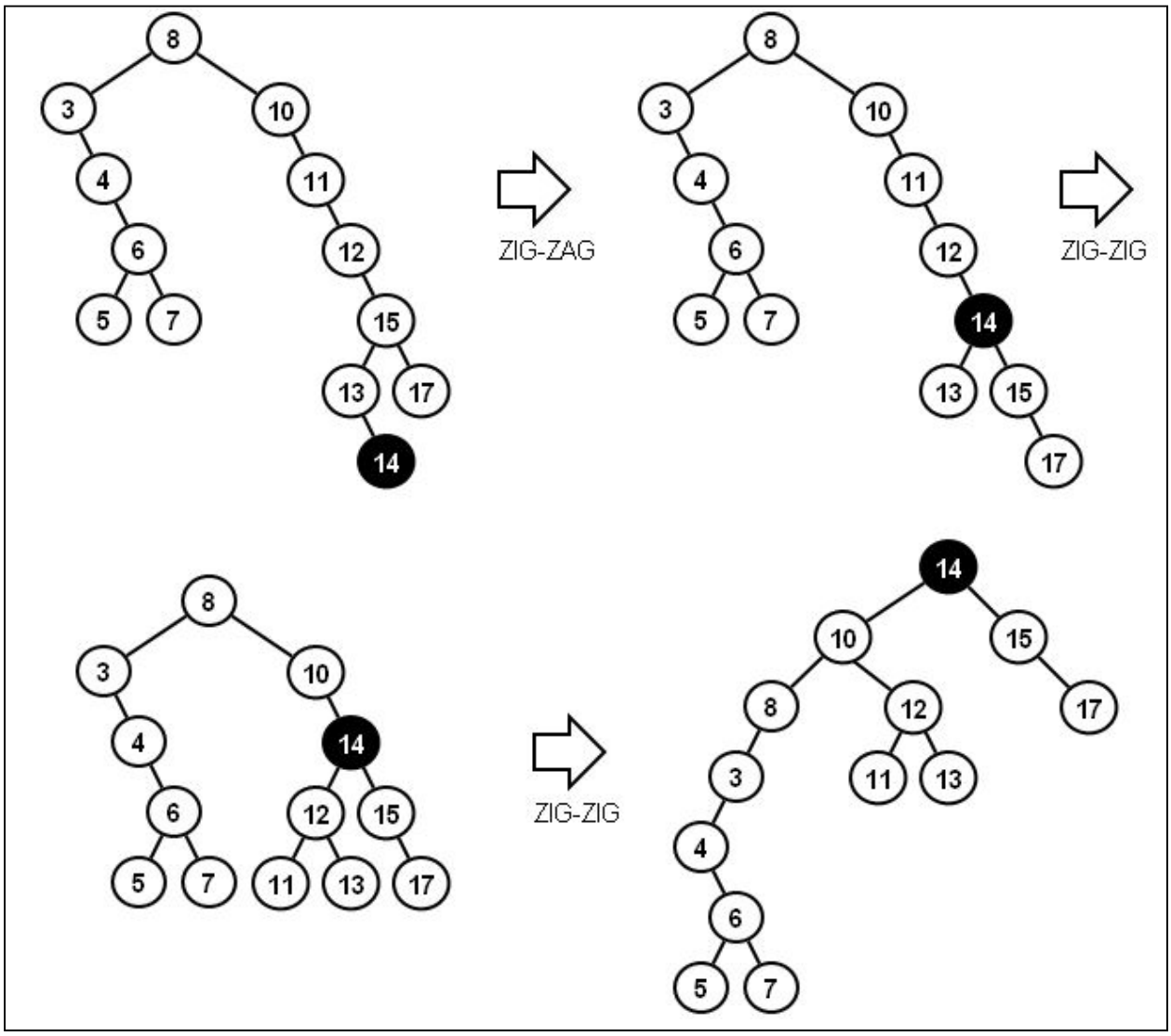
- Step 1 - Check whether tree is Empty.
- Step 2 - If tree is Empty then insert the newNode as Root node and exit from the operation.
- Step 3 - If tree is not Empty then insert the newNode as leaf node using Binary Search tree insertion logic.
- Step 4 - After insertion, Splay the newNode

Deletion Operation in Splay Tree

The deletion operation in splay tree is similar to deletion operation in Binary Search Tree. But before deleting the element, we first need to splay that element and then delete it from the root position. Finally join the remaining tree using binary search tree logic.

EXAMPLE:

Here's an example We're splaying the 14:



VISUALIZATION LINKS:

BINARY SEARCH TREE:
https://www.cs.usfca.edu/~galles/visualization/BST.html

AVL TREE: https://www.cs.usfca.edu/~galles/visualization/AVLtree.html

RED BLACK TREE:
https://www.cs.usfca.edu/~galles/visualization/RedBlack.html

SPLAY TREE: https://www.cs.usfca.edu/~galles/visualization/SplayTree.html