Simplex algorithm – Tabular form

We have the LPP.

$$\max Z = 3x_1 + 4x_2$$

$$\text{s.t.} \quad x_1 + 2x_2 \le 4 \quad \text{——} \ (1)$$

$$3x_1 + 2x_2 \le 6 \quad \text{——} \ (2)$$

$$x_1, x_2 \geq 0$$

Introducing slack variables x_3 and x_4 to (1) and (2) respectively

Then we have,

$$\max Z = 3x_1 + 4x_2 + 0x_3 + 0x_4$$

$$\text{s.t} \quad x_1 + 2x_2 + x_3 \qquad = 4$$

$$3x_1 + 2x_2 \qquad + x_4 = 6$$

$$x_1, x_2 \geq 0.$$

$$A = \begin{array}{c} \begin{matrix} a_1 & a_2 & a_3 & a_4 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix} \end{array} \qquad b = \begin{bmatrix} 4 \\ 6 \end{bmatrix} \qquad B = \text{basic}$$

X_B = basis variables

		C_j	3	4	0	0	b/a_j	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	b/a_{2i} min ratio	operation
0	a_3	x_3	4	1	$\boxed{2}$	1	0	$4/2=2$	
0	a_4	x_4	6	3	2	0	1	$6/2=3$	
$\boxed{z_j - c_j}$			$\boxed{0}$	$\boxed{-3}$	$\boxed{-4}$	0	0		

$$z_1 - c_1 = 0 \cdot 1 + 0 \cdot 3 - 3 = -3$$
$$z_2 - c_2 = 0 \cdot 2 + 0 \cdot 2 - 4 = -4$$
$$z_3 - c_3 = 0 \cdot 1 + 0 \cdot 0 - 0 = 0$$
$$z_4 - c_4 = 0 \cdot 0 + 0 \cdot 1 - 0 = 0$$

optimality reach when all $z_j - c_j \geq 0$

Entering variable is $\min_j \{(z_j - c_j)\}$

		c_j	3	4	0	0	b/a_1	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	min ratio	operation
4	a_2	x_2	$4/2=2$	$1/2$	1	$1/2$	0	$2/\tfrac{1}{2}=4$	$R_1' = R_1/2$
0	a_4	x_4	2	$\boxed{2}$	0	-1	1	$2/2=1$	$R_2' = R_2 - 2R_1'$
$z_j - c_j$			$\boxed{8}$	$\boxed{-1}$	0	2	0		

$4 \times \frac{1}{2} + 0 \cdot 2 - 3 = 2 - 3 = -1$

$6 - 2 \cdot 2 = 6 - 4 = 2$

$3 - 2 \cdot \frac{1}{2} = 3 - 1 = 2$

$0 - 2 \cdot \frac{1}{2} = -1$

		C_j		3	4	0	0		
C_B	B	X_B	b	a_1	a_2	a_3	a_4	min ratio	operation
4	a_2	x_2	$3/2$	0	1	$3/4$	$-1/4$		$R_1' = R_1 - \frac{1}{2}R_2'$
3	a_1	x_1	1	1	0	$-1/2$	$1/2$		$R_2' = R_2/2$
$Z_j - C_j$			$\boxed{9}$	$\boxed{0}$	0	$3/2$	$1/2$		

$$4 \cdot 3/4 - 3 \cdot \frac{1}{2} - 0$$
$$= 3 - \frac{3}{2} - 0$$
$$= 3/2$$
$$4 \cdot \left(-\frac{1}{4}\right) + 3 \cdot \frac{1}{2} - 0$$
$$= -1 + \frac{3}{2}$$
$$= 1/2$$

$$2 - \frac{1}{2} \cdot 1$$
$$= 2 - \frac{1}{2} = \frac{3}{2}$$
$$1 - \frac{1}{2} 0 = 1$$
$$\frac{1}{2} - \frac{1}{2}\left(-\frac{1}{2}\right)$$
$$= \frac{1}{2} + \frac{1}{4}$$
$$= 3/4$$
$$0 - \frac{1}{2} \cdot 1/2$$
$$= -1/4$$

All $Z_j - C_j \geq 0$.
The optimality is reached.
The optimal solution is, $x_1 = 1$, $x_2 = 3/2$
and value of the optimal solution $Z = 9$

Another example

$$\max Z = 3x_1 + 2x_2$$

$$\text{s.t.} \quad 2x_1 + x_2 \leq 5 \quad (1)$$

$$3x_1 - 2x_2 \leq 6 \quad (2)$$

$$x_1 + 4x_2 \leq 8 \quad (3)$$

$$x_1, x_2 \geq 0$$

Introducing slack variables x_3, x_4, and x_5 in (1), (2), and (3), respectively.

Then we have,

$$\max Z = 3x_1 + 2x_2 + 0x_3 + 0x_4 + 0x_5$$

$$\begin{aligned}
\text{s.t.} \quad 2x_1 + x_2 + x_3 &= 5 \\
3x_1 - 2x_2 + x_4 &= 6 \\
x_1 + 4x_2 + x_5 &= 8
\end{aligned}$$

$$x_1, x_2, \ldots, x_5 \geq 0$$

C_B	B	X_B	b	a_1	a_2	a_3	a_4	a_5	min ratio	operation
			c_j	3	2	0	0	0	b/a_1	
0	a_3	x_3	5	2	1	1	0	0	$5/2 = 2.5$	
0	a_4	x_4	6	$\boxed{3}$	-2	0	1	0	$6/3 = 2$ →	
0	a_5	x_5	8	1	4	0	0	1	$8/1 = 8$	
$z_j - c_j$				$\boxed{-3}$ ↑	-2	0	0	0	b/a_2	
0	a_3	x_3	1	0	$\boxed{7/3}$	1	-2/3	0	$1/7/3 = 3/7$ →	$R_1' = R_1 - 2 \cdot R_2'$
3	a_1	x_1	2	1	-2/3	0	1/3	0	$2/-2/3$	$R_2' = R_2/3$
0	a_5	x_5	6	0	14/3	0	-1/3	1	$6/14/3 = 9/7$	$R_3' = R_3 - R_2'$
$z_j - c_j$				0	-4 ↑	0	1	0	b/a_4	
2	a_2	x_2	3/7	0	1	3/7	-2/7	0	- - -	$R_1' = R_1/7/3$
3	a_1	x_1	16/7	1	0	2/7	1/7	0	$16/7/1/7 = 16$	$R_2' = R_2 + \frac{2}{3} R_1'$
0	a_5	x_5	4	0	0	-2	$\boxed{1}$	1	$4/1 = 4$ →	$R_3' = R_3 - \frac{14}{3} R_1'$
$z_j - c_j$				0	0	14/7	-1/7 ↑	0		
2	a_2	x_2	11/7	0	1	-1/7	0	2/7		$R_1' = R_1 + \frac{2}{7} R_3'$
3	a_1	x_1	12/7	1	0	4/7	0	-1/7		$R_2' = R_2 - \frac{1}{7} R_3'$
0	a_4	x_4	4	0	0	-2	1	1		$R_3' = R_3$
$z_j - c_j$			$\boxed{58/7}$	0	0	10/7	0	1/7		

All $z_j - c_j \geq 0$ ∴ optimality is reached

optimal solution is $x_1 = \frac{12}{7}$, $x_2 = 11/7$

value $Z = 58/7$