Groups and linear algebra (SC220) Autumn 2019
In Sem -I Time: 1hr 30 min

Name: ______________________________

Student I.D.: ______________________________

Section 1. True/False (2 pts. each)

Print "T" if the statement is true, otherwise print "F". In either case give a very brief (One or Two line) justification or a counter example.

_______ $A_4 =< (12)(34), (13)(24) >$

_______ The group $(\mathbb{Q}, +)$ is isomorphic to $(\mathbb{Q}^+, \times)$

_______ S_3 is isomorphic to $Z_2 \times Z_3$

_______ There are exactly 6 automorphisms from $\mathbb{Z}_9$ to $\mathbb{Z}_9$

_______ In S_4 let $\sigma = (123)(134)$ then σ^{2019} is e

_______ If G is a group of order p^k where p is a prime then it has a subgroup of order p^m for some positive integer $m \leq k$.

_______ The number of 4 cycles in S_5 is 24

_______ The elements r^2s and r^3s generate D_5

_______ The remainder when 13^{36} is divided by 18 is 1

_______ D_6 (Group of Symmetries of a hexagon) is isomorphic to A_4 (Group of even permutations of 4 letters)

Section 2. Short Answer (10 pts each)

Answer all problems in as thorough detail as possible.

1. Prove that if H and K are proper subgroups of a finite group G such that $gcd(|H|, |K|) = 1$, then $H \cap K = \{e\}$. Is the converse true ? That is, if H and K are proper subgroups of G such that $H \cap K = \{e\}$ then is it necessary that $gcd(|H|, |K|) = 1$.

2. Show that if G and H are groups and $A \leq G$ and $B \leq H$ then $A \times B$ is a subgroup of $G \times H$. Does every subgroup of $G \times H$ has to be of the form $A \times B$ where $A \leq G$ and $B \leq H$?

3. Let G be a finite group.
i)Show that if $x \in G$ then the map $\lambda_x : G \to G$ given by $\lambda_x(g) = xg$ is a 1-1 and onto map from G to G (that is the map λ_x is a permutation of the elements of G).
ii)Let S_G be the set of all permutations of G. Show that the map $\Phi : G \to S_G$ given by $\Phi(g) = \lambda_g$ is a 1-1 mapping and satisfies $\Phi(x * y) = \Phi(x) * \Phi(y)$. Hence conclude that every finite group is isomorphic to a subgroup of the permutation group S_n.