DA-IICT

Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)

First In Semester Examination

CT314 (Statistical Communication Theory)

Date of Examination: Feb 08, 2017

Duration: 2 Hours

Maximum Marks: 20

Instructions:

1. Attempt all questions.
2. Use of scientific non programmable calculator is permitted.
3. Figures in brackets indicate full marks.
4. All the acronyms carry their usual meaning.

Q1 A ball is selected from an urn containing two black balls numbered 1 and 2, and two white balls numbered 3 and 4. The number and the color of the ball is noted. (a). Write the sample space. Assuming that the outcomes are equally likely, find P(A/B) and P(A/C). Are the events A and B independent? Are the events A and C independent? Are A and C disjoint? Following are the events A, B and C:
A= black ball is selected. B= Even numbered ball is selected C= number of ball is greater than 2. (4 marks)

Q2:Two numbers x and y selected at random are between 0 and 1. Let the events A, B and C be defined as follows. A={x>0.5}, B={y>0.5}, and C={x>y}. Are the events A and B independent? Are A and C independent? (4 marks)

Q3. Consider a continuous random variable X. If $f_X(x)=0$ for $x<0$, show that $P\{X>\sqrt{m_X}\}\leq\sqrt{m_X}$. Here m_X is the expected (mean) value of X (2 marks)

Q4: As discussed in class, show that the expected value (mean) of random variable Y=g(X) can be obtained using the density function of X itself as $E(Y)=\int_{-\infty}^{+\infty} g(x)f_X(x)dx$.
That is no need to compute the PDF of g(X) to get the mean value of g(X) (2 marks)

Q5: Using the result in Q4, obtain E(Y) and $E(Y^2)$ when the random variable Y is obtained as $Y=a\cos(wt+\Theta)$ where a,w,t are constants and Θ is a uniformly distributed random variable in the interval $(0,2\pi)$. Here, the random variable Y is obtained by taking the values of Θ between 0 and 2π (randomly) and substituting in the expression for Y, keeping a,w,t fixed. (4 marks)

Q6: The life time X of a machine has a continuous CDF $F_X(x)$ and PDF $f_X(x)$. Find the conditional CDF and PDF in terms of CDF and PDF of X, given the event A=(X>t).
(4 marks)

I Insem Feb 08, 2017 SOLUTIONS

Sample space $S = \{(1,b),(2,b),(3,w),(4,w)\}$

$A = \{(1,b),(2,b)\}$, $B = \{(2,b),(4,w)\}$, $C = \{(3,w),(4,w)\}$

$P(A/B) = P(AB)/P(B) = P\{(2,b)\}/P\{(2,b),(4,w)\} = \frac{1}{4}/\frac{2}{4} = \frac{1}{2}$

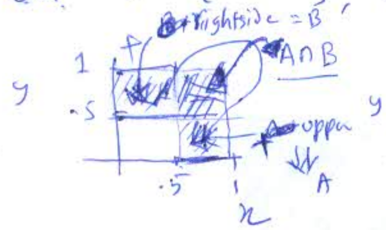
$P(A|C) = P(AC)/P(C) = \frac{0}{2/4} = 0$

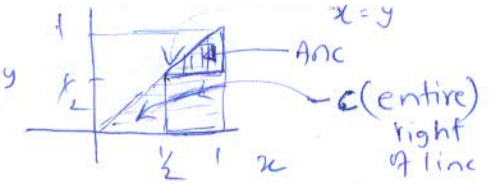
$P(A) = 2/4 = 1/2 = P(A|B)$, So A and B are independent

$P(A|C) = 0 \neq P(A)$, A and C not independent

But A and C are ME (disjoint), since $A \cap C = \phi$.

Q2 $A = \{x > .5\}$, $B = \{y > .5\}$, $C = \{x > y\}$





$P[A|B] = \frac{P(AB)}{P(B)} = \frac{1/4}{1/2} = \frac{1}{2}$

$P[A] = \frac{1}{2}$ So A & B are independent.

A & B can be drawn independently.

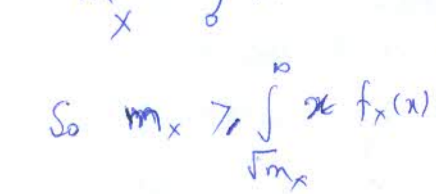
$P(A|C) = P[AC]/P(C) = \frac{3/8}{\frac{1}{2}} = \frac{3}{4} \neq P(A)$, A depends on C (A & C not independent).

Q3 $m_X = \int_0^\infty x f_X(x)\,dx = \underbrace{\int_0^{\sqrt{m_X}} x f_X(x)\,dx}_{I} + \underbrace{\int_{\sqrt{m_X}}^\infty x f_X(x)\,dx}_{II}$

So $m_X \geq \int_{\sqrt{m_X}}^\infty x f_X(x)\,dx$ or $m_X \geq \int_{\sqrt{m_X}}^\infty \sqrt{m_X} f_X(x)\,dx$

or $\frac{m_X}{\sqrt{m_X}} \geq \int_{\sqrt{m_X}}^\infty f_X(x)\,dx$ or $\sqrt{m_X} \geq P(X > \sqrt{m_X})$ or $P(X > \sqrt{m_X}) \leq \sqrt{m_X}$

Q 4



where $y = g(x_1) = g(x_2) = g(x_3)$

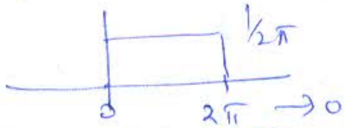
$f_Y(y)\,dy = P(y \leq Y \leq y+dy) = f_X(x_1)|dx_1| + f_X(x_2)|dx_2| + f_X(x_3)|dx_3|$

So $y f_Y(y)\,dy = g(x_1) f_X(x_1)|dx_1| + g(x_2) f_X(x_2)|dx_2| + g(x_3) f_X(x_3)|dx_3|$

If we takes strips g dy covering the entire y, we can integrate LHS from $-\infty$ to ∞. This leads to non overlapping (disjoint) dx strips covering the entire x axis leading to

$\int y f_Y(y)\,dy = \int_{-\infty}^{\infty} g(x) f_X(x)\,dx = E(Y)$

Q5 $Y = g(\Theta) = a\cos(\omega t+\theta)$ So $E(Y)=\int_{-\infty}^{\infty} y\, f_\Theta(\theta)\,d\theta$, But θ is uniform

So 

$\therefore\ E(Y)=\frac{1}{2\pi}\int_0^{2\pi} a\cos(\omega t+\theta)\,d\theta$

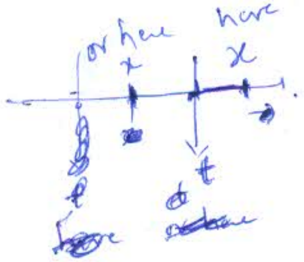
or $E(Y) = +\frac{1}{2\pi}\left(a\sin(\omega t+\theta)\right)_0^{2\pi} = +\frac{a}{2\pi}\left[\sin(\omega t+2\pi) - \sin(\omega t+0)\right] = 0$

and $E(Y^2)=\int_{-\infty}^{\infty} y^2 f_\Theta(\theta)\,d\theta = \frac{1}{2\pi}\int_0^{2\pi}\left(a\cos(\omega t+\theta)\right)^2 d\theta$

$=\frac{1}{2\pi}\left[a^2\int_0^{2\pi}\left[\frac{1+\cos(2\omega t+2\theta)}{2}\right]d\theta\right]$

$=\frac{a^2}{2\cdot 2\pi}\left[2\pi+0\right] = \frac{a^2}{2}$.

Q6. $F_X(x/X>t) = \frac{P(X\le x,\ X>t)}{P(X>t)} = \frac{P(X\le x,\ X>t)}{1-F_X(t)}$



when $x<t$, then $\{X\le x,\ X>t\}=\phi$

when $x\geq t$ then event is $\{t<X\le x\}$

So $F_X(x/X>t) = 0 \qquad x<t$

$= \frac{F_X(x)-F_X(t)}{1-F_X(t)}, \quad x\geq t.$

$\therefore\ f_X(x/X>t)=\frac{f_X(x)}{1-F_X(t)}, \quad x\geq t.$