Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)
End Semester Examination
CT314 (Statistical Communication Theory)
Date of Examination: May 01, 2017
Duration: 2:30 Hours
Maximum Marks: 45

Instructions:

1. Attempt all questions.
2. Use of scientific non programmable calculator is permitted.
3. Figures in brackets indicate full marks.
4. All the acronyms carry their usual meaning.

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Q1: What is the difference between a random variable and a random process? Consider the example of fair die experiment. How many sample functions are there for this experiment and for what time interval they exist? Give the probability of occurrence of each sample function. Are the sample functions random? If your answer is NO, where is the randomness?

(7 marks)

Q2: Give the definitions of SSS and WSS. Show that $R_X(\tau) \le R_X(0)$. Consider $R_X(\tau) = \cos(\tau)$, for $|\tau| \le \frac{\pi}{2}$. Check if this is a valid autocorrelation function. Give proper reason for your answer.

WSS

(12 marks)

Q3: Let $Y(t) = X(t)\cos(2\pi f_c t + \Theta)$ where $X(t)$ is a WSS process with known $R_X(\tau)$ and $S_X(f)$. Θ is uniformly distributed random variable in the range $(0 - 2\pi)$. $X(t)$ (now a random variable at t) and Θ are statistically independent. Find Autocorrelation and power spectral density (PSD) of $Y(t)$.

(6 marks)

Q4: In AWG noise, what does each term i.e., Additive, White, Gaussian represent. Let this noise with PSD of $\frac{N_0}{2}$ is applied as input to an ideal low-pass filter of bandwidth B. Find the autocorrelation function of the output process and plot it. What is the interval between those samples (of the process) that are statistically independent? Let Y be the random variable obtained by sampling the output process at t=2 secs. Find the probability density function of the random variable Y (i.e., $f_Y(y)$).

(10 marks)

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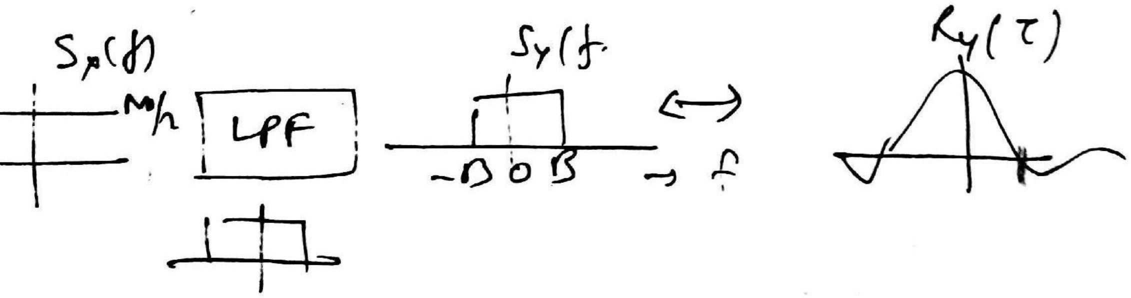


$C_Y(\tau) = R_Y(\tau)$

$R_Y(\tau) = E[Y(t)\,Y(t+\tau)]$

$= E$

$\mathrm{Cov}(XY) = E(X - m_X)(Y - m_Y)$

$C_Y(\tau) = E(XY)$

$Y(t) \quad Y(t+\tau)$

$= E(Y^2) - [E(Y)]^2$

Q5: Using the concepts of the pre-envelope and complex envelope, prove the following:

$$x_R(t) = x(t)\cos(2\pi f_c t) + \hat{x}(t)\sin(2\pi f_c t)$$
$$x_I(t) = \hat{x}(t)\cos(2\pi f_c t) - x(t)\sin(2\pi f_c t)$$

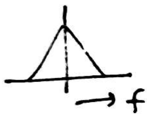
(3 marks)

Q6: Let $x(t) = A_c(1 + am(t))\cos(2\pi f_c t)$, where a is the modulation index, $m(t)$ is the modulating signal ~~and~~ $A_c\cos(2\pi f_c t)$ represents carrier. Plot $X(f)$, $X_{pe}(f)$, $X_{ce}(f)$. Find $x_{pe}(t)$, $x_{ce}(t)$, $A(t)$ and $\Phi(t)$

(7 marks)

Take $m(t) = \cos 2\pi f_m t$

$A_c \cos 2\pi f_c t$ + A_c [illegible]



Answers End Sem May 1 2017.

Q1 Random variable - mapping from random expt space to a real number x. See notes for details.

(a) Random process - mapping from random expt to a ~~real~~ signal $x(t)$. It is a probability space composed of sample space, ensemble of time functions and a probability measure

(b) 6 (c) $-\infty < t < \infty$ (d) $1/6$ (e) No (f) Randomness is the uncertainty as to which waveform will occur in a given trial.

Q2

(a) ~~SSS~~ Process $X(t)$ is SSS if

$$f_{X(t_1), X(t_2), \ldots, X(t_n)}(x_1, x_2, \ldots, x_n) = f_{X(t_1+T), X(t_2+T), \ldots, X(t_n+T)}(x_1, \ldots, x_n)$$

for any shift T & for any finite set ~~time~~ instants $1, \ldots, n$.

WSS

(b) Process $X(t)$ is WSS if $m_x(t) = m_x$ & $R_x(t_k, t_i) = R_x(t_k - t_i) = R_x(\tau)$

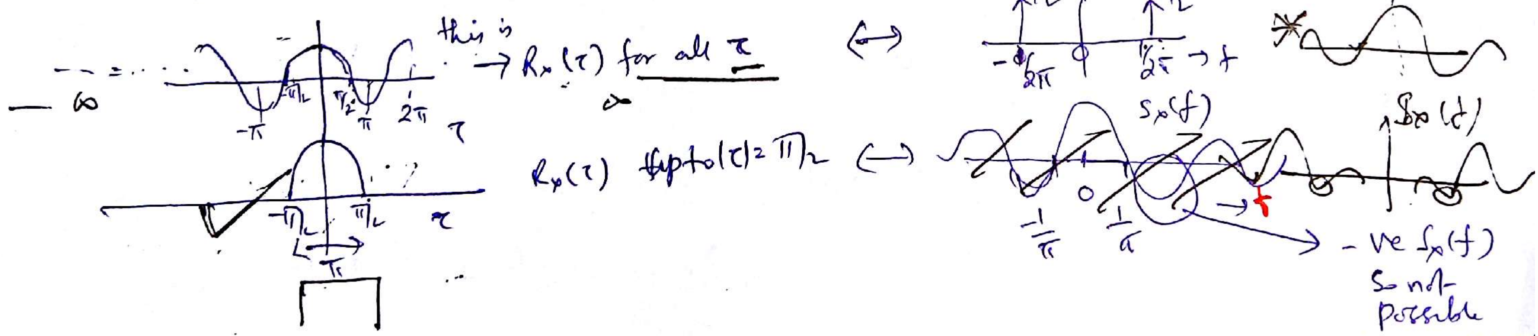
(c) Consider $E\left[\left(X(t) \pm X(t+\tau)\right)^2\right] \geq 0$

$$E\big[\underbrace{X^2(t) + X^2(t+\tau)}_{\text{both are } R_x(0)} \pm 2X(t)X(t+\tau)\big] \geq 0$$

Expectation is linear operator.

So $R_x(0) \geq |R_x(\tau)|$

(d) $R_x(\tau) = \cos(\tau)$ has FT (i.e. PSD $S_x(f)$) that has -ve values so not a valid $R_x(\tau)$. OR because of discontinuity not a valid $R_x(\tau)$



Q3 $Y(t) = X(t)\cos(\omega_c t + \Theta)$, $\omega_c = 2\pi f_c$

$$R_Y(\tau) = E[Y(t)Y(t+\tau)] = E\{X(t+\tau)\cos[\omega_c(t+\tau)+\Theta]\, X(t)\cos[(\omega_c t+\Theta)]\}$$

$$= E[X(t+\tau)X(t)]\, E[\cos(\omega_c(t+\tau)+\Theta)\cos(\omega_c t+\Theta)]$$

$$= \frac{1}{2} R_X(\tau)\cos\omega_c\tau.$$

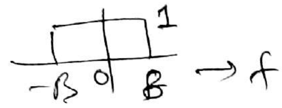
$$\therefore S_Y(f) = F[R_Y(\tau)] = \frac{1}{4}\left[S_X(f-f_c) + S_X(f+f_c)\right]$$

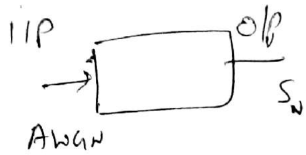
Q4 Additive → Noise gets added up to signal

White → psd is constant

(a) Gaussian → The joint pdf of n random variable is Gaussian distributed.

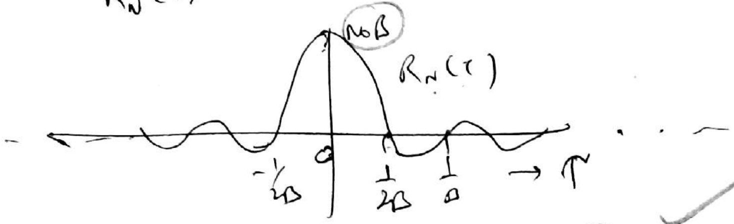
(b) . Filter





$S_N(f) = \begin{cases} \frac{N_0}{2}, & -B \le f \le B \\ 0 & \text{otherwise} \end{cases}$

$\therefore R_N(\tau) = N_0 B \operatorname{Sinc}(2B\tau)$



(c) Interval $\tau = \frac{k}{2B}$, k integer (d)

Since Gaussian - uncorrelatedness implies independent

$C_N(\tau) = R_N(\tau) = E[N(t)N(t+\tau)]$ because $E(N(t)) = 0 = E[N(t+\tau)]$ by def^n of whitening

(d) $E(Y) = 0$ & $E(Y^2) - (E(Y))^2 = \sigma_Y^2$ So $\sigma_Y^2 = E(Y^2)$

$R_Y(0) = E(Y(t))$ $= R_Y(0) = N_0 B$.

So Y is $\underline{N(0, N_0 B)}$.

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Q5. $x_{ce}(t) = x_{pe}(f+f_c)$ ⇒ spectrum of complex envelope = shifted version of spectrum of preenvelope

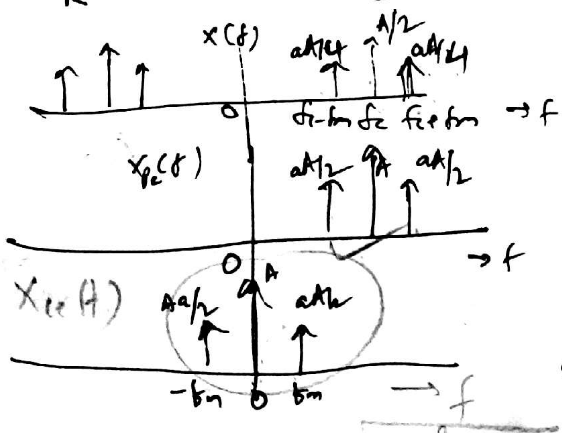
$\therefore x_{ce}(t) = x_{pe}(t)e^{-j2\pi f_c t}$

$x_R(t) + jx_I(t) = [x(t) + j\hat{x}(t)][\cos\omega_c t - j\sin\omega_c t]$

$= x(t)\cos\omega_c t + \hat{x}(t)\sin\omega_c t + j\hat{x}(t)\cos\omega_c t - jx(t)\sin\omega_c t$

$\therefore x_R(t) =$ —— $x_I(t) =$ ——

Q6.



$x_{pe}(t) = Ae^{j\omega_c t} + \frac{aA}{2}e^{j(\omega_c - \omega_m)t} + \frac{aA}{2}e^{j(\omega_c+\omega_m)t}$

$= e^{j\omega_c t}\left(A + \frac{Aa}{2}e^{-j\omega_m t} + \frac{Aa}{2}e^{+j\omega_m t}\right)$

$= x_{ce}(t)\, e^{j\omega_c t}$

So $x_{ce}(t) = A + Aa\cos\omega_m t$

complex envelope

$x_R(t) = A(1 + a\cos\omega_m t)$ envelope of $x(t)$

$x_I(t) = 0$

$A(t) = |x_{ce}(t)|$ & $\phi(t) = 0$

$A(t) = \sqrt{x_R(t)^2 + x_I(t)^2}$