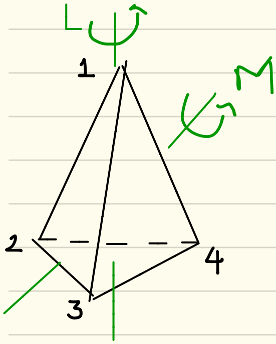
Leture 1 - Symmetries

Rotational symmetries of a regular tetrahedron



There are (4×2) symmetries of type L and 3 types of symmetries of type M, these and the identity give a total of 12 rotational symmetries of a regular tetra-tetrahedron

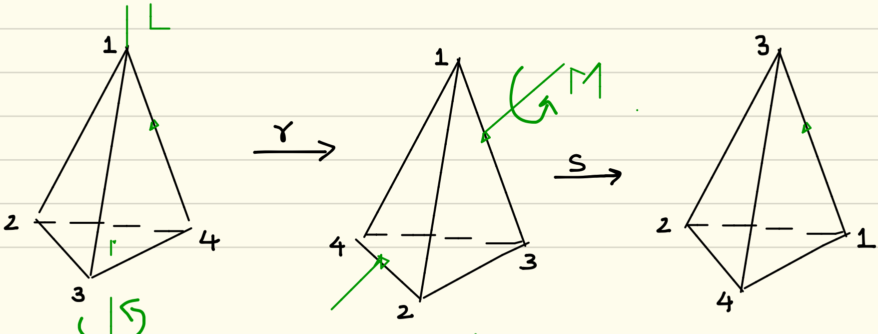
$r \sim$ rotation by an angle of $2\pi/3$ about axis L

$s \sim$ rotation by angle π about axis M, $s^2 = e$

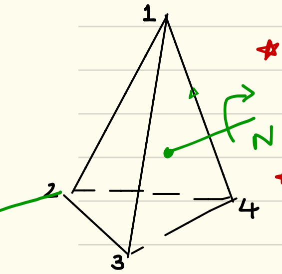
r : $1 \to 1$, $2 \to 3$, $3 \to 4$, $4 \to 1$, In cycle notn (234)

s : $1 \to 4$, $2 \to 3$, $3 \to 2$, $4 \to 1$, In cycle notation $(14)(23)$

Let us see what happens when we apply r followed by s. This



which is the same as rotation about axis N as shown below

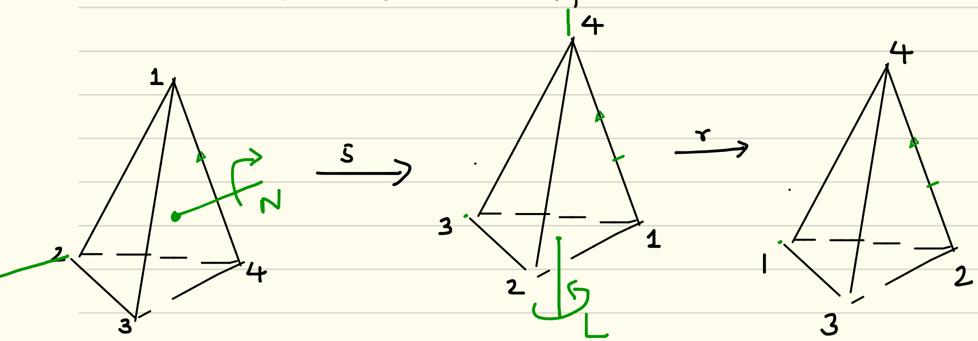


* So combining two rotations u and v gives another rotation w

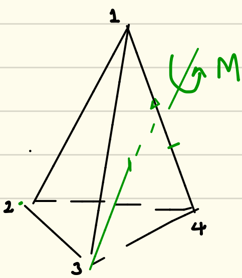
* You will also observe that for rotations u, v, w
$(u * v) * w = u * (v * w)$. This property is known as associativity

* For each rotation u $\exists$ rotation u^{-1} s.t.
$u * u^{-1} = u^{-1} * u = e$

Now let us see what happens when one applies rotation s followed by r i.e. rs



which is a rotation about axis M as shown



i.e. the axis passing through the vertex 3 and the midpoint of the side defined by vertices 1,2,4

So $rs = (124)$ where as $sr = (143)$

So $rs \neq sr$ in general.

The algebraic structure formed by the rotations of the regular tetrahedron is called a group. All the properties marked by $\star$ in the previous discussion will form the axioms of an algebraic structure called group that we shall study in detail in the next lecture.