Lecture - 20

Recap:

Binomial distributions

X Y

(m, p) (n, p)

- $X + Y$ is also binomial

$(m+n, p)$

Normal distributions

X Y

μ_1, σ_1^2 μ_2, σ_2^2

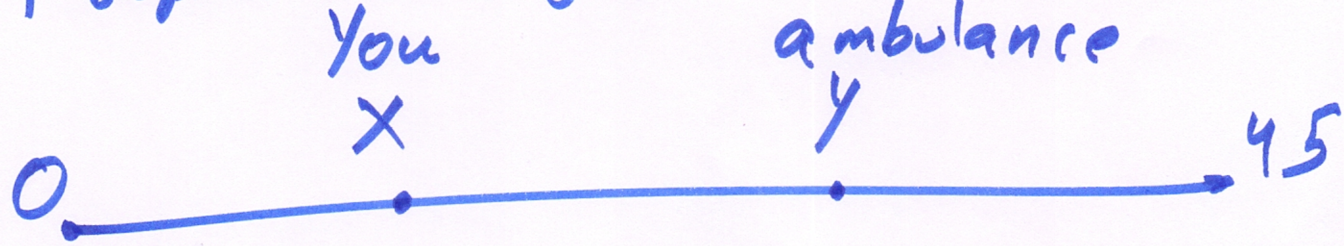
$X + Y$ is normal

$\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2$

Conditional distributions

discrete, continuous

Properties of Expectations.



S.G. Highway

← 45 Kms →

X uniform (0, 45)

Y uniform (0, 45)

X & Y are independent

Z = distance b/w X & Y

$= |X - Y|$

$E[|X-Y|] = ?$

Theorem:

$$E[g(X,Y)]$$

$$= \iint g(X,Y)\, f_{X,Y}(x,y)\, dx\, dy$$

Holds true even if X & Y are <u>not</u> independent

$$g(X,Y) = |X-Y|$$

$$E[|X-Y|] =$$

$$\int_0^{45}\int_0^{45} |X-Y|\, \underbrace{f_{X,Y}(x,y)}_{= \,?}\, dx\, dy$$

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$$f_{X,Y}(x,y) = f_X(x)\, f_Y(y)$$

$$f_{X,Y}(x,y) = \frac{1}{45} \cdot \frac{1}{45}$$

$$= \frac{1}{45^2} \iint |X-Y|\, dx\, dy$$

$\longleftarrow Y \longrightarrow X \longleftarrow y \longrightarrow$

0 to X $\qquad$ x to 45

$$= \frac{1}{45^2}\left[\int_0^{45}\left(\int_0^{x}(x-y)\, dy\right) dx + \int_0^{45}\left[\int_x^{45}(y-x)\, dx\right] dy\right]$$

$$= 15$$

e.g.:

$g(X,Y) = \underline{X+Y}$

X & Y are <u>not</u> independent

(will also work when X & Y are independent)

$$E[g(X,Y)] = \iint g(x,y)\, f_{X,Y}(x,y)\,dx\,dy$$

$$= \iint (x+y)\, \underbrace{f_{X,Y}(x,y)}\, dx\, dy$$

$$= \iint x\, f_{X,Y}(x,y)\, dx\, dy \; +$$

simplify

$$\iint y\, f_{X,Y}(x,y)\, dx\, dy$$

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$$\iint x \, f_{X,Y}(x,y)\, dx\, dy$$

$$\int \left[\int f_{X,Y}(x,y)\, dy \right] x\, dx$$

$$\int f_X(x)\, x\, dx = E[X]$$

$$E[X+Y] = E[X] + E[Y]$$

$$\sum E[x_i] = E\left[\sum x_i\right]$$

[sum of expectations = expectation of the sum

True for both discrete & continuous

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e.g. Binomial random variable

(n, p) $\quad E[X] = np$

$1 \; x_1$

$0 \; x_2$

$$x_i = \begin{cases} 1 \text{ if success} \\ 0 \text{ if failure} \end{cases}$$

$1 \; \vdots$ $\quad X = \sum x_i$

$1 \; x_n$ $\quad E[X] = \sum E[x_i]$

$$X_i$$

$$0(1-p) + 1(p) = p$$

$$E[X_i] = p$$

$$E[X] = \sum_{i=1}^{n} E[X_i]$$

$$E[X] = np$$

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toss 4 dice simultaneously

X = sum

$$X \in \{4, 5, 6, \cdots, 24\}$$

$$P(X=4) = \frac{1}{6^4}$$

$$P(X=5) = \frac{4}{6^{4}}$$

$P(X=18) =$

5, 5, 5, 3

4, 4, 4, 6

4, 2, 6, 6

$$E[X] = 4 \cdot \frac{1}{6^4} + 5 \cdot \frac{4}{6^4}$$

$X_1 =$ outcome of dice 1

$X_2 =$ 2

$X_4 =$ 4

$X = X_1 + X_2 + X_3 + X_4 = 14$

e.g.

Hypergeometric r.v.

$E[X] =$

Total N balls

m white $\qquad N-m$ black

Choose n balls randomly

$X =$ no. of white balls in the sample of n balls

$$E[X] = \frac{mn}{N}$$

X_1

X_2

$\vdots$

X_n

$$X_i = \begin{cases} 1 & \text{if } i^{th} \text{ ball is white} \quad \frac{m}{N} \\ 0 & \text{if } i^{th} \text{ ball is black} \quad 1-\frac{m}{N} \end{cases}$$

$$X = \sum_{i=1}^{n} X_i \qquad \boxed{E[X_i] = \frac{m}{N}}$$

$$E[X] = \sum_{i=1}^{n} E[X_i] = \frac{mn}{N}$$

e.g. 10 people in a room

10 hats in the room

All the people throw their hat in a box.

They pick up a hat at random from the box.

X = no. of people who get their hat

$$E[X] = \frac{10}{10} = 1$$

X_i

- 1 : $\left(\frac{1}{10}\right)$ i^{th} person gets his hat
- 0 : $\left(\frac{9}{10}\right)$ i^{th} person doesn't get it

X_1

$\vdots$

X_{10}

$$E[X_i] = \frac{1}{10}$$

Instruction for insem2 13

1. Bring pen, pencil, Calculator
2. Bring your own water
3. Y odd roll nos: 9:45 am
   Even roll nos: 11 am
4. Seating will be informed
   at the venue
5. You cannot leave early
6. No mobiles, bags
7. Negative marking
   1 for correct
   $-\frac{1}{3}$ for wrong
8. Sequential