Loop pivoting

- This is the technique that is used to make some basic variable non-basic and possibly some non-basic variable basic.

How this works?

- Suppose somehow we know a non-basic variable that wants to enter the basis.

Method:

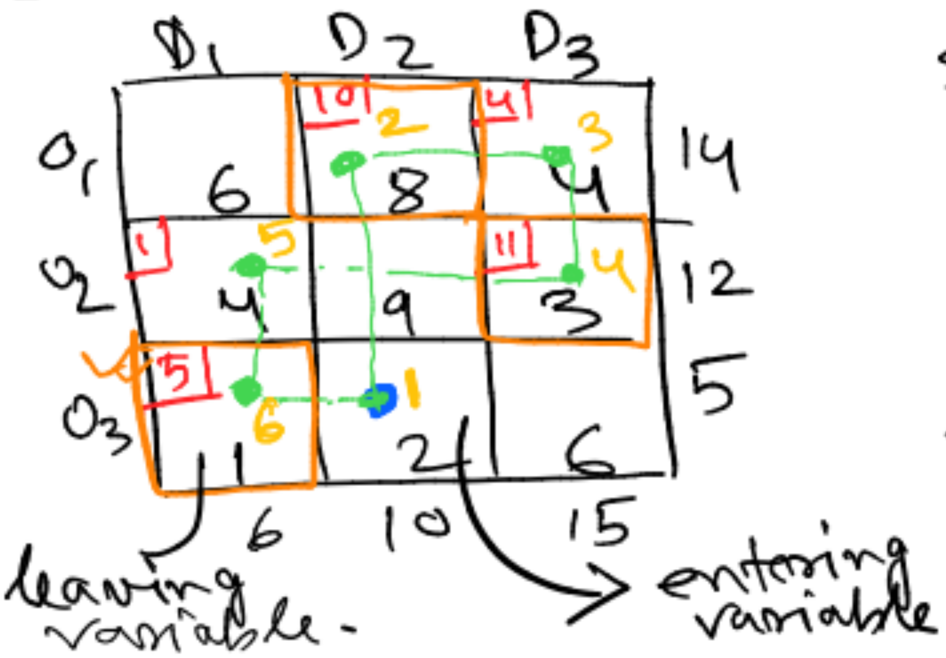
1. Determine the entering variable
2. Find the only loop starting at the entering variable cell and some of the basis variables cells.
3. Number the cells in the loop starting from 1
4. Find an even cell in the loop with smallest assignment (break the ties arbitrarily).

   Let this value be α.

   This cell corresponds to the leaving variable.
5. Decrease the values in each even cell in the loop by α.

   Increasing the values in each odd cell in the loop by α.

Example



Step 1: suppose (3,2) cell is the entering variable cell.

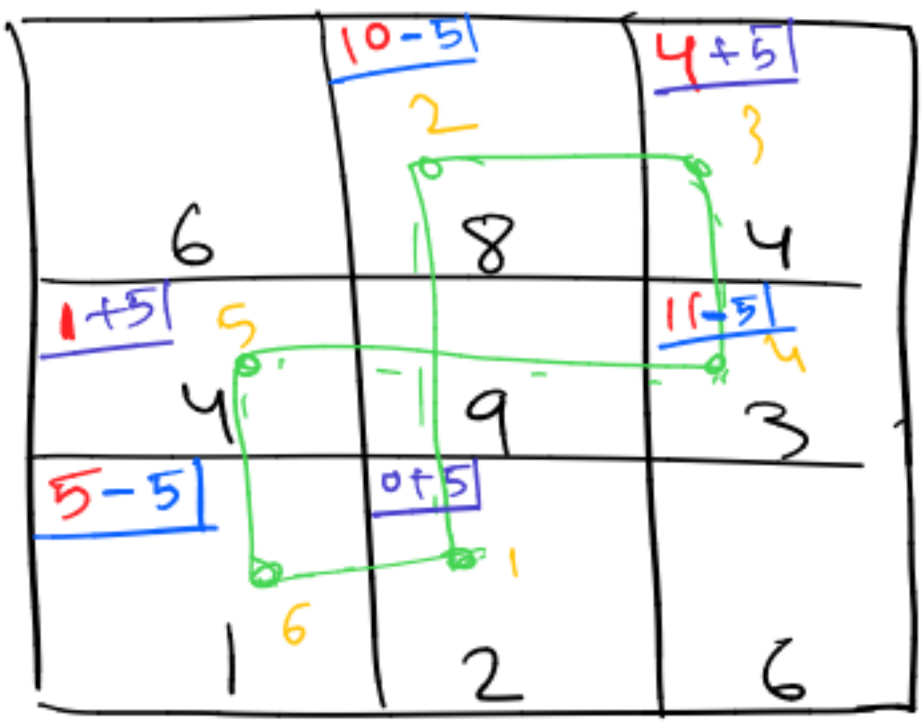
Step 2: Loop identified.

Step 3: Numbering cells in the loop.

Step 4: identifying min assignment cell.

(3, 1) cell has min assigment of 5

so $\alpha = 5$



$\Rightarrow$

6	[5] 8	[9] 4	14
[6] 4	9	[6] 3	12
[0] 1	[5] 2	6	5
6	10	15	

Finding an optimal solution of a TP

The MODI method or the u-v method

⇓

modified distribution method.

Observations :-

⇒ The cell where the positive allocation is available is called the basic cell.

There will be $(m+n-1)$ basic cells if the solution (ibfs) is non-degenerate.

⇒ The cells having zero allocations are called non-basic cells.

[There will be basic cells having zero allocation if the solution (ibfs) is degenerate.]

$\boxed{10}$ 5	9	6	~~10~~
2	$\boxed{12}$ 4	$\boxed{0}$ 1	~~12~~
8	$\boxed{3}$ 3	$\boxed{5}$ 4	~~8~~ ~~3~~ 0
~~10~~	~~15~~ ~~3~~ 0	~~5~~	

$m+n-1 = 3+3-1 = 5$

Solution is degenerate.

Cell where no loop can be formed.

MODI method

computational procedure

Step 1: First we find an initial basic feasible solution by any method already discussed. There are $m+n-1$ positive allocations for non-degenerate case.

Step 2: For all occupied cells (i, j), we determine a set of $(m+n)$ numbers u_i and v_j, for $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$ such that, $c_{ij} = u_i + v_j$

Note: In practice we do the following. we choose any one of u_i or v_j equal to 0. we choose that u_i or v_j to be zero for which the corresponding row or column contains maximum number of allocations.

Example:

				a_i
$\boxed{8}$ 6	9	8	7	
4	$\boxed{2}$ 6	$\boxed{5}$ 4	$\boxed{5}$ 7	
9	5	4	$\boxed{6}$ 2	
$\boxed{1}$ 3	$\boxed{9}$ 1	6	8	
b_j				

cost of the solution

$$8\times6 + 2\times6 + 5\times4 + 5\times7 + 6\times2 + 1\times3 + 9\times1$$

$$= 48+12+20+35+12+3+9$$

$$= 139$$

Step 1 : Done by any method.

there are $4+4-1 = 7$ allocations.

so the solution is non-degenerate.

Step 2 : Finding u_i and v_j.

					u_i
	$\boxed{8}$ 6	9	8	7	-2
	4	$\boxed{2}$ 6	$\boxed{5}$ 4	$\boxed{5}$ 7	0
	9	5	4	$\boxed{6}$ 2	-5
	$\boxed{1}$ 3	$\boxed{9}$ 1	6	8	-5
v_j	8	6	4	7	

$$\boxed{c_{ij} = u_i + v_j}$$

Step 3: The cell evaluation for the unoccupied cells are made by the formula

$$\Delta_{ij} = C_{ij} - (u_i + v_j) \quad \text{for cell } (i, j)$$

Example contd..

	D_1	D_2	D_3	D_4	u_i
O_1	$\boxed{8}$; 6	5 ; 9	6 ; 8	2 ; 7	-2
O_2	$\boxed{-4}$; 4	$\boxed{2}$; 6	$\boxed{5}$; 4	$\boxed{5}$; 7	0
O_3	6 ; 9	4 ; 5	5 ; 4	$\boxed{6}$; 2	-5
O_4	$\boxed{11}$; 3	$\boxed{9}$; 1	7 ; 6	6 ; 8	-5
v_j	8	6	4	7	

Step 3: contd...

There are 3 cases based on the values of Δ_{ij}

Case 1: If all $\Delta_{ij} > 0$

Then the solution is unique and optimal.

Case 2: If all $\Delta_{ij} > 0$ with one $\Delta_{ij} = 0$,

then the solution is optimal but not unique.

Case 3: If at least one $\Delta_{ij} < 0$

then the solution is not optimal.

and we are to seek a new or improved basic feasible solution.

If case 3 occurs then we pass on to the next step.

Step 4: To find an improved bfs we try to enter the variable for which δ_{ij} is most negative. (minimum δ_{ij}) and we make loop pivoting by taking this cell as starting cell.

This leads an improved solution.

Step 5: perform step 2 and Step 3 repeatedly on the resulting table until we reach an optimal solution.

Example contd...

	1	2	3	4
1	8 / 6	5 / 9	6 / 8	2 / 7
2	1, 4 / 4	2 −1 / 6	5 / 4	5 / 7
3	6 / 9	4 / 5	5 / 4	6 / 2
4	1 −1 / 3	9 +1 / 1	7 / 6	6 / 8

Step 4: loop pivoting

$\alpha = 1$

add 1 to odd cells

subtract 1 to even cells

8			
1	1	5	5
			6
	10		

cost of the solution.

$8 \times 6 + 1 \times 4 + 1 \times 6 + 5 \times 4 + 5 \times 7 + 6 \times 2 + 10 \times 1$

$= 48 + 4 + 6 + 20 + 35 + 12 + 10$

$= 135$

Now we perform step 2 and step 3 again on the new table.

H. W.

Final value 125