CT303 - Digital Communications

Lecture 24 : 7 December 2020

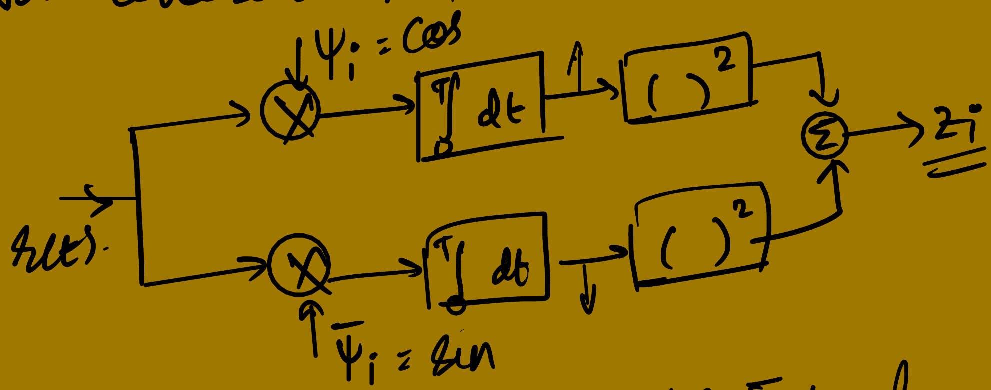
* Lecture 23 recap
  - Difference between Coherent and Non-Coherent detection

  1. Differential DPSK (Differential detection of differentially encoded PSK).

  2. Non-Coherent Orthogonal FSK

$$f_{i+1} - f_i = \frac{1}{T}$$

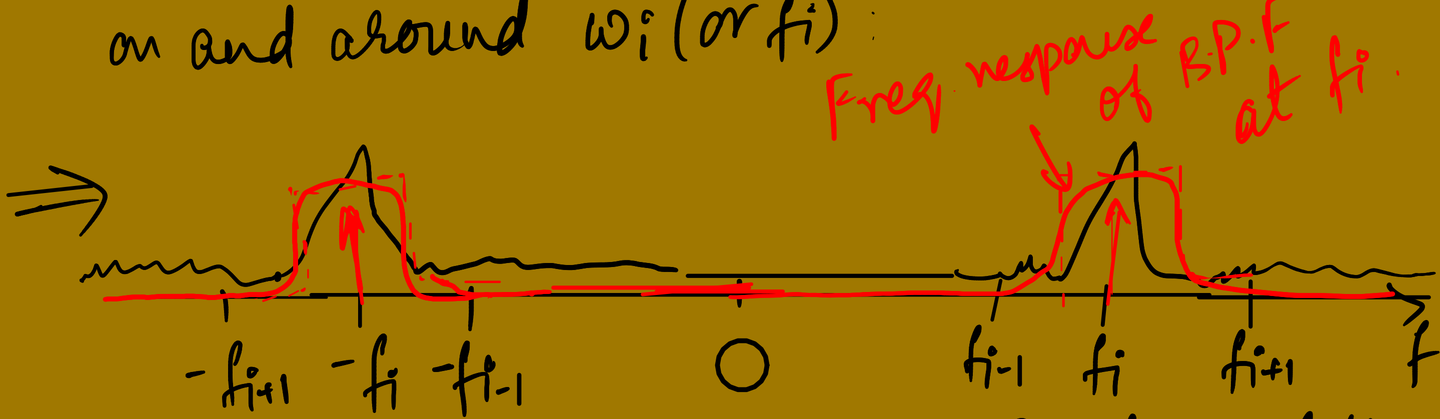
  - Non-Coherent FSK demodulator.



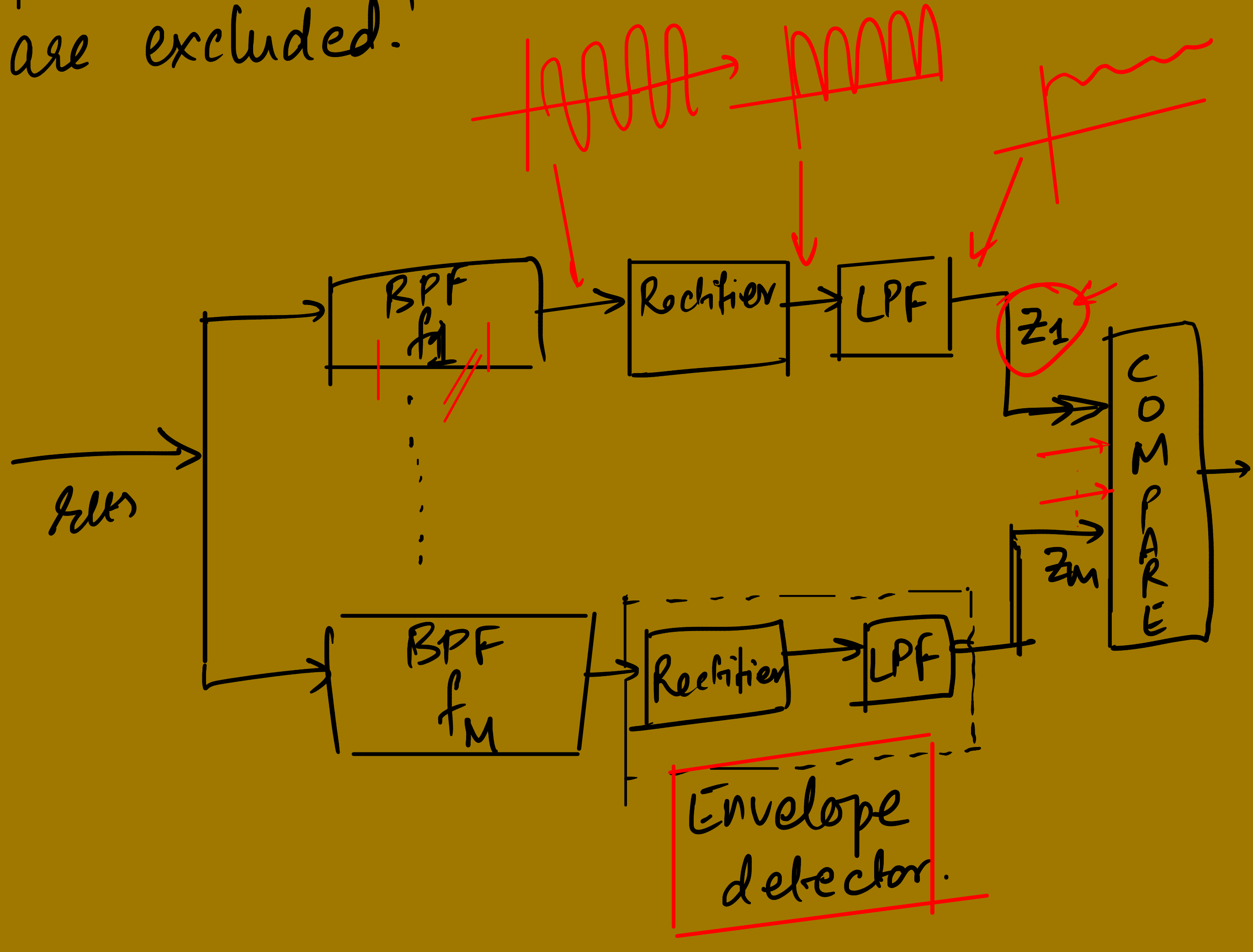
1 - Aim of the NC FSK demod.

* Non-Coherent FSK demodulator 2.

* We expect the amplitude spectrum of the received signal $r(t) = \sqrt{\frac{2E}{T}} \cos(w_i t + \phi) + \eta(t)$ (AWGN) when s_i was transmitted, to have higher values on and around w_i (or f_i):



* This can be detected using a Band pass filter centered at f_i, with a cut-off frequency that ensures frequencies close to f_{i-1} & f_{i+1} are excluded.



Modulators

* Irrespective of whether we are using frequency, phase or amplitude modulation, the transmitted waveform is of the form:

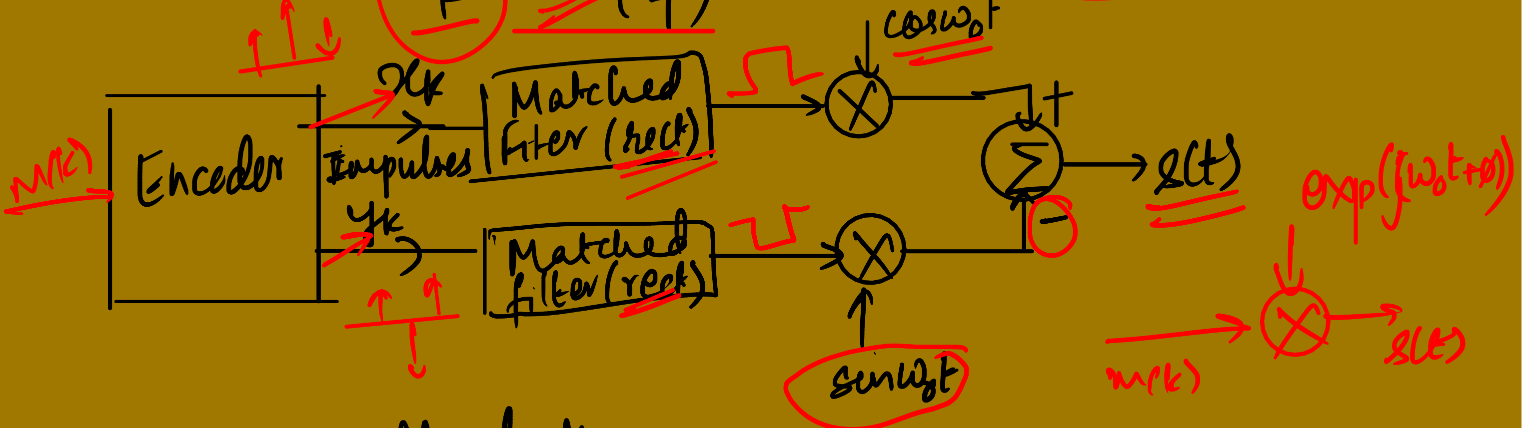
$$\boxed{s(t) = A\cos(\omega t+\phi).} = \mathrm{Re}\{e^{j(\omega t+\phi)}\}$$

* $s(t) = A(\cos\omega_0 t\cos\phi - \sin\omega_0 t\sin\phi)$

$$= \underbrace{A\cos\phi}_{x_k}\,\underbrace{\cos\omega_0 t}_{\substack{\text{carrier}\\ \text{(In phase component)}}} - \underbrace{A\sin\phi}_{y_k}\,\underbrace{\sin\omega_0 t}_{\substack{\text{carrier}\\ \text{(Quadrature component)}}}$$

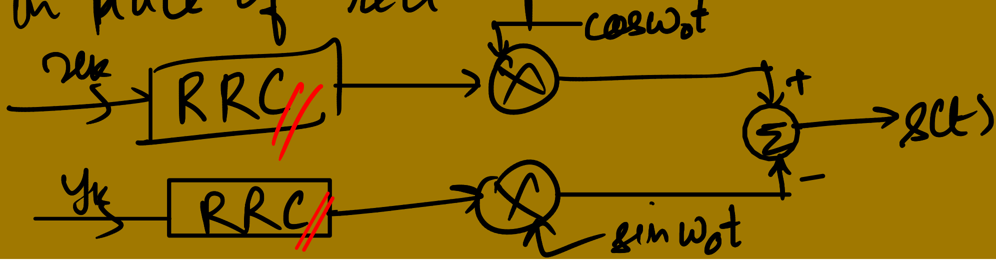
$$= x_k\cos\omega_0 t - y_k\sin\omega_0 t \qquad 0 \le t \le T$$

$$= x_k\,\mathrm{rect}\left(\frac{t}{T}\right)\cos\omega_0 t - y_k\,\mathrm{rect}\left(\frac{t}{T}\right)\sin\omega_0 t$$



* This is called the "Quadrature Modulator."

* One can also use appropriate Pulse shaping filters in place of "rect" pulses.



* Quadrature Demodulator

Ideally, $r(t) = x_k \cos\omega_0 t - y_k \sin\omega_0 t$

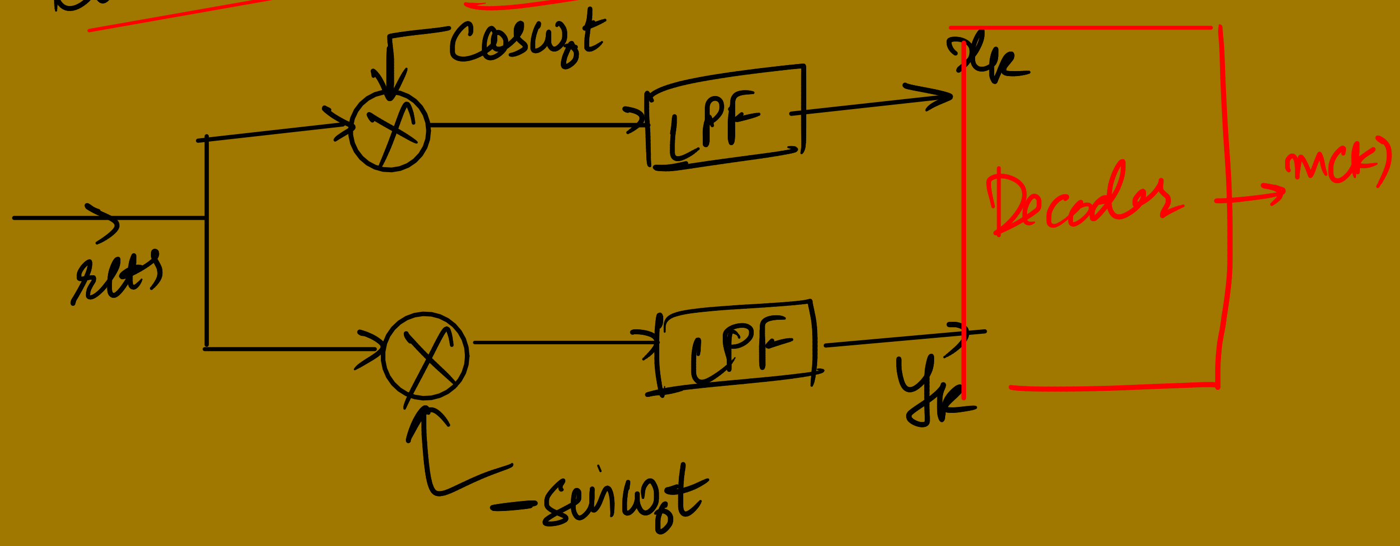
$$r(t)\cos\omega_0 t = x_k \cos^2\omega_0 t - y_k \sin\omega_0 t \cos\omega_0 t$$

$$= x_k \left(\frac{1+\cos 2\omega_0 t}{2}\right) - y_k \frac{\sin(2\omega_0 t)}{2}$$

* Passing $r(t)\cos\omega_0 t$ through a LPF (cut-off $< 2\omega_0 t$) we get $\frac{x_k}{2}$.

* Similarly passing $r(t)\sin\omega_0 t$ through a LPF, gives $-y_k/2$.

* Quadrature Demodulator:



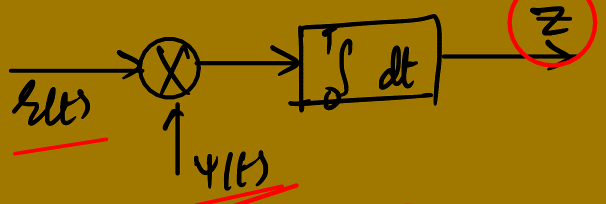
* ERROR PERFORMANCE

1. Binary Bandpass Modulation Systems

(i) Coherently detected BPSK

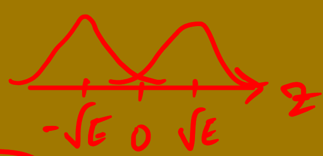
Let $\psi(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t$.

$s_1(t) = \sqrt{E}\,\psi(t)$, $s_2(t) = -\sqrt{E}\,\psi(t)$



$z \geq 0 \rightarrow s_1$

$z < 0 \rightarrow s_2$



$a_1 = E[z/s_1] = \sqrt{E}$, $a_2 = E[z/s_2] = -\sqrt{E}$

* $P_B = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right)$, σ_0^2 = Noise Variance.

$P(s_1) = P(s_2) = 1/2$.

$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$ → energy of the difference signal $(s_1 - s_2)$

$E_d = (2\sqrt{E})^2 = 4E = 4E_b$.

$\therefore P_B = Q\left(\sqrt{\frac{4E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ (Case of Antipodal Signals) → ≈ NRZ-L

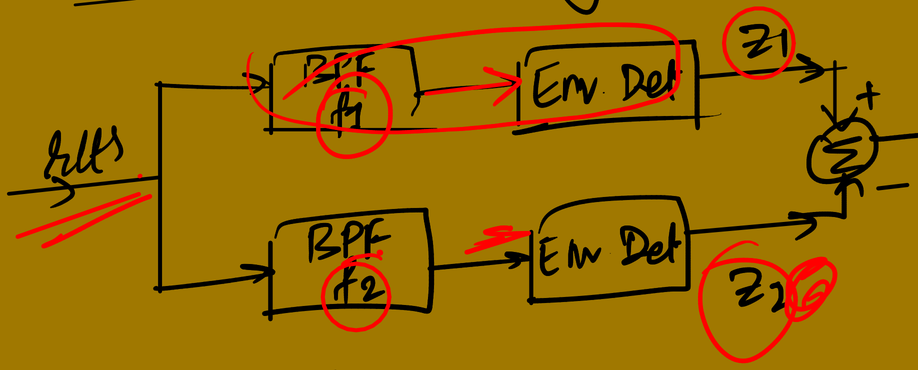
(ii) Coherently detected BFSK (orthogonal)

- $P_B = Q\left(\sqrt{\frac{E(1-\rho)}{N_0}}\right)$ → $\langle s_1, s_2 \rangle$ $(P(s_1) = P(s_2) = 1/2)$

- For orthogonal signals, $\rho = 0$

$\therefore P_B = Q\left(\sqrt{\frac{E}{N_0}}\right)$

(iii) Non-Coherent Binary FSK



$z = z_1 - z_2$

$z \geq 0 \Rightarrow s_1$

$z < 0 \Rightarrow s_2$

$$s_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_1 t)$$

$$s_2(t) = \sqrt{\frac{2E}{T}} \cos(\omega_2 t)$$

$$P_B = P(s_1) \cdot P(e/s_1) + P(s_2) P(e/s_2)$$

Assuming $P(s_1) = P(s_2) = 1/2$

$$P_B = \frac{1}{2} P(e/s_1) + \frac{1}{2} P(e/s_2).$$

$$P(e/s_1) = P(z_2 > z_1 | s_1)$$

$$P(e/s_2) = P(z_1 > z_2 | s_2)$$

Also, $E[z_1|s_1] = \sqrt{\frac{2E}{T}}$, $E[z_2|s_2] = \sqrt{\frac{2E}{T}}$

$E[z_1|s_2] = 0$, $E[z_2|s_1] = 0$

$$P(z = z_1 - z_2 | s_1) = P(-z = z_2 - z_1 | s_2) \rightarrow (1)$$

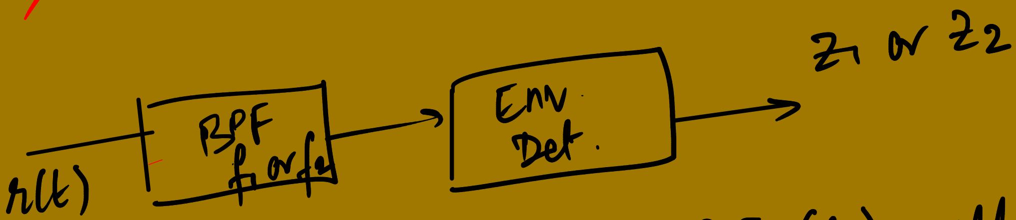
$$\therefore P_B = \frac{1}{2} P(e|s_1) + \frac{1}{2} P(e|s_2)$$

From (1),

$$P(e|s_1) = P(z < 0 | s_1) = P(z > 0 | s_2) = P(e|s_2)$$

$$\therefore P_B = P(e|s_2) = P(z>0|s_2) = P(z_1>z_2|s_2)$$

$$P_B = \int_{-\infty}^{\infty} P(z_2|s_2)\left(\int_{z_2}^{\infty} P(z_1|s_2)\, dz_1\right) dz_2$$



* When s_2 is transmitted, B.P.F (f_2) will allow the signal component of r to pass through $\rightarrow \tilde{z}_2 = s_2 + n$ the signal component of r to pass through the B.W of the filter. along with the noise in the B.W of the filter.

* But the output of B.P.F (f_1) will eliminate s_2, and allow only the noise in the B.W of the filter $f_1 \rightarrow \tilde{z}_1 = n$.

* Since the Envelope detector is a nonlinear device, if the i/p to it is a gaussian RP, the o/p is not gaussian.

* When s_2 is transmitted, the o/p of the Envelope detector connected to B.P.F f_2 is known to have a Rician distribution, i.e.,

$$p(z_2|s_2) = \frac{z_2}{\sigma_0^2}\, e^{\frac{-(z_2^2+A^2)}{2\sigma_0^2}}\, I_0\left(\frac{z_2 A}{\sigma_0^2}\right), \quad z_2 \geq 0$$

$$= 0, \quad z_2 < 0,$$

Where $A = \sqrt{\frac{2E}{T}}$, σ_0^2 is Noise power at B.P.F o/p, ...

... and $I_0(x)$ is the modified zero-order Bessel function of the first kind, defined as

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\theta)\, d\theta .$$

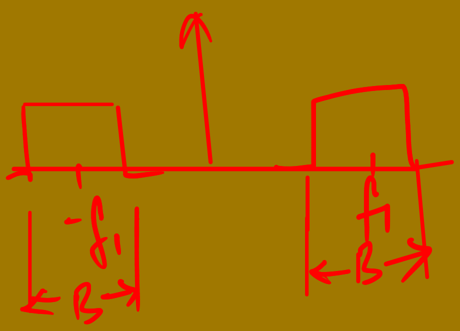
* Similarly, when the o/p of B.P.F f_1 is connected to the Envelope detector, the o/p has a Rayleigh distribution,

$$p(z_1|s_2) = \frac{z_1}{\sigma_0^2}\, e^{-\left(z_1^2/2\sigma_0^2\right)}, \quad z_1 \geq 0$$

$$= 0, \quad z_1 < 0.$$

$$\therefore\ P_B = \int_{-\infty}^{\infty} P(z_2|s_2)\left(\int_{z_2}^{\infty} P(z_1|s_2)\, dz_1\right) dz_2$$

$$= \frac{1}{2}\, e^{\left(-A^2/4\sigma_0^2\right)}$$ with $A = \sqrt{\frac{2E}{T}}$



and $\sigma_0^2 = 2\left(\frac{N_0}{2}\right)\cdot B$, where

B is the B.P.F B.W

$$\Rightarrow P_B = \frac{1}{2}\, e^{-\frac{2E}{T\cdot 4N_0B}} = \frac{1}{2}\, e^{-\frac{E}{2N_0BT}}$$

T → symbol duration

* Let the B.P.F B.W B match the B.W of the transmitted symbols; i.e.,

$$B = R_s(1+r).$$

$$B_{min} = R_s = 1/T \Rightarrow B\cdot T = 1$$

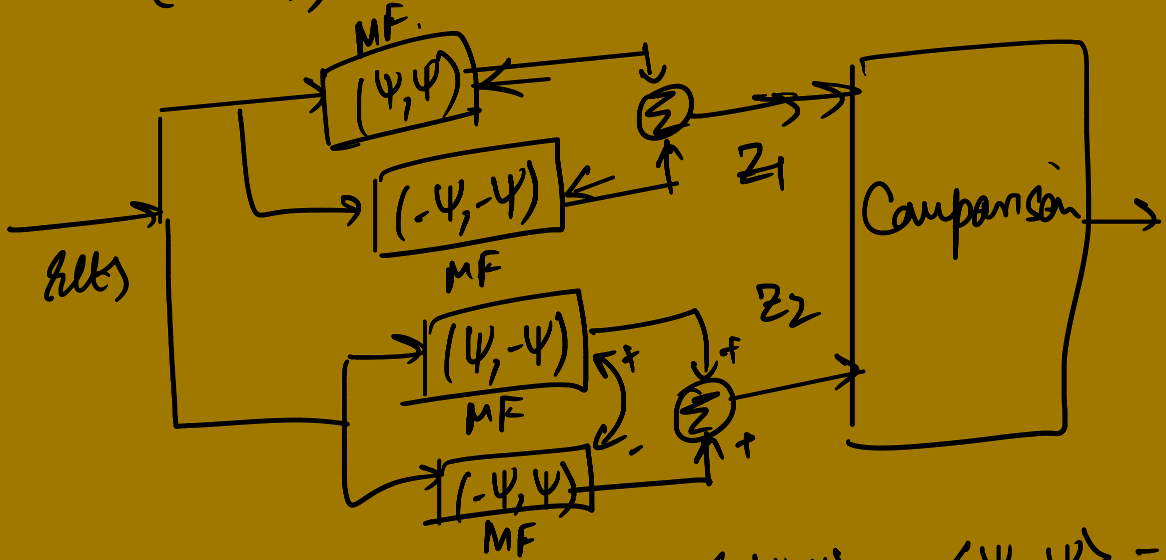
$\Rightarrow \boxed{P_B = \frac{1}{2} e^{-(E/2N_0)}.} = \frac{1}{2} e^{-E_b/2N_0}$

(iv) Binary DPSK (Differential detection of Diff. encoded PSK).

$$\psi(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t \;, \quad \bar{\psi}(t) = \sqrt{\frac{2}{T}} \sin \omega_0 t .$$

Bit 0: $s_1(t) = (\psi, \psi)$ or $(-\psi, -\psi)$ $\quad t \in [0, 2T]$.

Bit 1: $s_2(t) = (\psi, -\psi)$ or $(-\psi, \psi)$ $\quad t \in [0, 2T]$

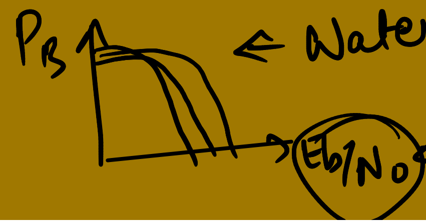


$$\langle s_1, s_2 \rangle_{2T} = \langle (\psi,\psi), (\psi,-\psi) \rangle_{2T} = \langle \psi, \psi \rangle_T + \langle \psi, -\psi \rangle_T = 0$$

$$= 0 .$$

$\Rightarrow s_1 \perp s_2$ $\quad P_B = \frac{1}{2} e^{-\frac{E}{2N_0}}$

$$\boxed{P_B = \frac{1}{2} e^{-2E_b/2N_0} = \frac{1}{2} e^{-E_b/N_0}.}$$



← Waterfall curves/graphs

← dB