Statistical Communication Theory (CT 314)

Tutorial 1.

21/01/2019

1. Write the sample space for the following experiment.
   a) Select a ball from an urn containing balls numbered 1 to N.
   b) Select a ball from an urn numbered 1 to 4. Suppose the balls 1 and 2 are black and 3 and 4 are white.
   c) Toss the coin 3 times and note the number of heads and tails.
   d) Pick a number at random between 0 and 1.
2. A box contains n numbers 1, 2, 3,. . . . , n. The numbers are taken out at random one by one. Find the probability that they come out in sequence 1, 2, 3,. . . . , n.
3. The same box in Q.2. The numbers are taken out one by one noted and put back. Find the new probability of taking out 1, 2, 3,. . . . , n.
4. The percentage of students who passed courses A, B and C are : A : 50%, B : 40% , C : 30% and A and B : 35%, B and C : 20% , C and A : 25% and all courses 15%. What is the percentage of students who passed at least one of the courses?
5. Items coming out of a production line are marked defective (D) or non-defective (N). Items are observed and their condition is listed. This is continued until two consecutive defective are produced or four items have been checked whichever occur first. Write the sample space for this experiment.
6. In a certain group of computer personal 80% know hardware, 85% know software and 15% know neither hardware nor software. Compute the percentage of people who know both hardware and software.
7. We have 4 boxes. B1 contains 2000 items with 5% defective. B2 contains 500 items with 40% defective. B3 contains 1000 items with 10% defective. B4 contains 1000 items with 10% defective. We select one box at random and remove one component at random from the box. (a) What is the probability that the selected item is defective? (b) We know that the item selected is defective. What is the probability that it comes from B2?
8. Is it possible for A and B to be independent events yet satisfy A=B?
9. A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and then an eight?
10. For independent events A and B, prove that
    a) A and B^c are independent.
    b) A^c and B are independent.
    c) A^c and B^c are independent.

11. In a class of 35 children, 22 like bananas, 18 like cherries and 13 like strawberries. 7 of them like bananas and cherries. 8 of them like bananas and strawberries. 5 of them like cherries and strawberries. They all like at least one of the fruits. What is the probability that a child chosen at random from the class likes cherries only?

12. For each of the following functions Fi(c), state whether or not Fi(c) is the CDF of some random variable. If not state which of the properties of a CDF it violates.

a) $$F_1(c) = \begin{cases} 0, & c \le 0 \\ 0.5c, & 0 < c \le 1 \\ 0.25 + 0.25c, & 1 < c \le 3 \\ 1, & 3 < c \end{cases}$$

b) $$F_2(c) = \begin{cases} 0, & c \le 0 \\ 0.5, & 0 < c \le 1 \\ 0.75, & 1 < c \le 3 \\ 1, & 3 < c \end{cases}$$

c) $$F_3(c) = \begin{cases} 0.5, & c < 1 \\ 0.75, & 0 \le c < 3 \\ 1, & 3 \le c \end{cases}$$

d) $$F_4(c) = \begin{cases} 0, & c < 0 \\ 0.25, & 0 \le c < 1 \\ 0.75, & 1 \le c < 3 \\ 1, & 3 \le c \end{cases}$$

e) $$F_5(c) = \begin{cases} 0, & c < 0 \\ 0.5, & 0 \le c < 1 \\ 0.25, & 1 \le c < 3 \\ 1, & 3 < c \end{cases}$$

f)

$$F_6(c) = \begin{cases} 0, & c \le 0 \\ 0.5c, & 0 < c \le 1 \\ 0.25 + 0.25c, & 1 < c \end{cases}$$