Name:________________________ **id:**______________________

SC217: Electromagnetic Theory, **AUTUMN'15 MIDSEM1**

DA-IICT, B.Tech, Sem III 14:00-15:00 PM 10^{th}Sep'15 **25 marks**

There are 4 questions. Q(1) must be answered in the question paper. Others are to be answered in the answer book. Write your name and ID above on the question paper and return with the answer book.

1. Encircle the correct choice. Answer this in the question paper **($2\times$ 5 = 10)**

(a) Consider two dimensional vectors $\vec{A}$ and $\vec{B}$. Under rotation in this two dimensional space the quantity $A_x B_y - A_y B_x$ transforms as

(a) a scalar, **(b)** a vector,
(c) neither scalar nor vector, **(d)** both, scalar and vector

(b) The velocity vector on the surface of a whirlpool is given as $\vec{v} = \frac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}$. A light tiny boat is found to spin in this whirlpool at a distance of 2 units from the center of the whirlpool. The angular speed of this spin (not the revolution) in radians per second is

(a) 0 **(b)** 2 **(c)** 4 **(d)** $\frac{1}{2}$

(c) $F(\vec{r}) = r^2$ where r is the distance from the origin.
Then over the surface $r = 2$, $\nabla^2 F = \vec{\nabla} \cdot (\vec{\nabla} F)$ is

(a) 6 **(b)** 3 **(c)** 4 **(d)** 0

(d) $F(x, y, z) = f(\frac{x}{\sqrt{x^2+y^2}})$ is a scalar function in three dimension. $\vec{\nabla} F$ is proportional to

(a) $\hat{i} + \hat{j}$ **(b)** $\hat{\phi}$ **(c)** $\hat{\theta}$ **(d)** $\hat{k}$

(e) $\vec{A} = \hat{i}(y - z) + \hat{j}(z - x) + \hat{k}(x - y)$. The line integral $\oint \vec{A} \cdot \vec{dl}$ over a circular loop of radius a on the x-y plane with center at origin, traversed clockwise, is

(a) $2\pi a^2$ **(b)** 0 **(c)** $-2\pi a^2$ **(d)** $4\pi a^2$

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Turn Over

2. An infinitely long wire along the z axis carries a uniform charge with density λ per unit length. Calculate the electric field due to this wire. **(5)**

3. Consider a vector field $\vec{A} = \vec{\omega} \times \vec{r}$. Evaluate

$$\int_S (\vec{\nabla} \times \vec{A}) \cdot \hat{n} da$$

over the upper half surface of an ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$.
(The part that lies above the x-y plane).
You may use the fact that area of the ellipse $x^2/a^2 + y^2/b^2 = 1$ is πab. **(5)**

4. The electric field in a region is given as

$$\begin{aligned} \vec{E}(\vec{r}) &= \frac{ca^2}{\epsilon_0} \frac{\hat{r}}{r^2} \quad ; \quad r \geq a \\ &= 0 \quad ; \quad r < a \end{aligned}$$

Describe the charge densities in the region, i.e, outside, inside and on the sphere $r = a$ **(5)**

Gradient, divergence and curl in spherical and cylindrical co-ordinate systems

Spherical polar system

$$\begin{aligned} \vec{\nabla} F &= \frac{\partial F}{\partial r} \hat{\mathrm{r}} + \frac{1}{r} \frac{\partial F}{\partial \theta} \hat{\theta} + \frac{1}{r \sin\theta} \frac{\partial F}{\partial \phi} \hat{\phi} \\ \vec{\nabla} \cdot \vec{\mathrm{A}} &= \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi} \\ \vec{\nabla} \times \vec{\mathrm{A}} &= \frac{1}{r \sin\theta} \left[\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right] \hat{\mathrm{r}} + \frac{1}{r} \left[\frac{1}{\sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(r A_\phi) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \hat{\phi} \end{aligned}$$

Cylindrical System

$$\begin{aligned} \vec{\nabla} F &= \frac{\partial F}{\partial s} \hat{\mathrm{s}} + \frac{1}{s} \frac{\partial F}{\partial \phi} \hat{\phi} + \frac{\partial F}{\partial z} \hat{\mathrm{z}} \\ \vec{\nabla} \cdot \vec{\mathrm{A}} &= \frac{1}{s} \frac{\partial}{\partial s}(s A_s) + \frac{1}{s} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \\ \vec{\nabla} \times \vec{\mathrm{A}} &= \left[\frac{1}{s} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \hat{\mathrm{s}} + \left[\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[\frac{\partial}{\partial s}(s A_\phi) - \frac{\partial A_s}{\partial \phi} \right] \hat{\mathrm{z}} \end{aligned}$$