Degeneracy in TP

In the initial stage or any subsequent iteration if the number of allocations is less than $(m+n-1)$, then we have a degeneracy.

Question:

How to resolve degeneracy??

Allocate a very small positive quantity ε to one or more cells (as many allocations are required to have $(m+n-1)$ allocations) and consider those cells as occupied cells.

- The quantity ε is chosen such that

$$0 < \varepsilon < x_{ij} \quad , \quad \varepsilon + 0 = \varepsilon$$

$$x_{ij} \pm \varepsilon = x_{ij}$$

Example

			Supply
8	[ε] 7	[60] 3	60
[50] 3	8	[20] 9	70
11	[80] 3	5	80
50	80	80	

Somehow we get this assignment.

$m+n-1 = 5$

The solution is degenerate.

⇒ We assign ε to a cell such that it does not form a loop through the basic cells.

⇒ This is beacuse of finding u_i and v_j uniquely.

After adding ε, use the u-v method to verify optimality and proceed further.

				u_i
	11 8	ε 7	60 3	0
	50 3	−5 8	20 9	6
	18 11	80 3	6 5	−4
v_j	−3	7	3	

$$\boxed{C_{ij} = u_i + v_j}$$

$$\Delta_{ij} = C_{ij} - (u_i + v_j)$$

$\Delta_{22} < 0$, The solution is not optimal.

				u_i
	11 8	5 7	60+ε 3	−6
	50 3	0+ε 8	20−ε 9	0
	13 11	80 3	1 5	−5
v_j	3	8	9	

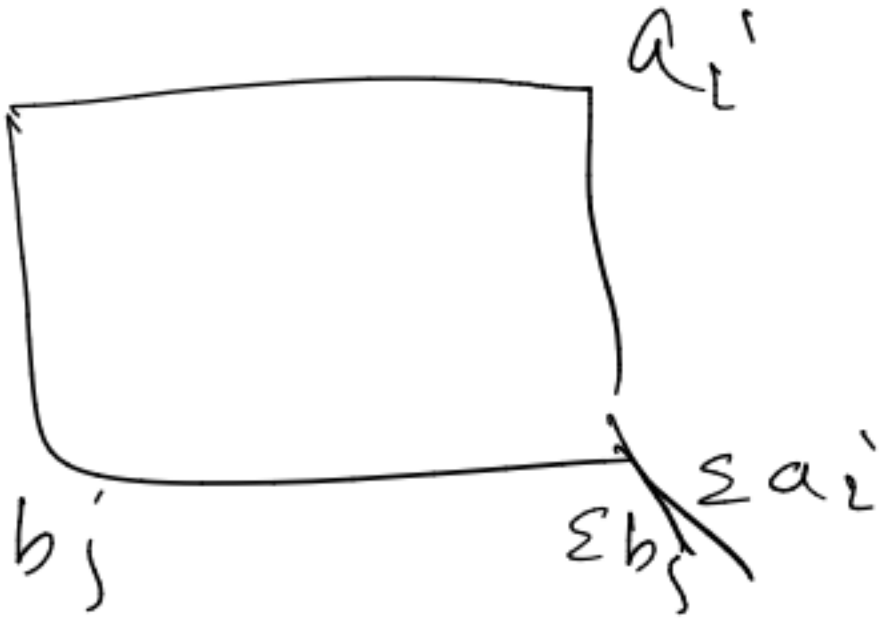
cost of the solution.

$60 \times 3 + 50 \times 3 + 20 \times 9$
$+ 80 \times 3$

$= 180 + 150 + 180 + 240$

$= 750$

Variations of TP



$$\boxed{\Sigma a_i = \Sigma b_j}$$

(a) Unbalanced TP

if $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$

Exm

	D_1	D_2	D_3	a_i
O_1	15	8	9	80
O_2	20	34	26	50
b_j	30	30	40	

100 130

	D_1	D_2	D_3	D_4	
O_1	15	8	9	0	80
O_2	20	34	26	0	50
	30	30	40	30	

130 130

(b) Maximisation TP

Convert it to a minimisation TP.

Subtract all the costs from the highest cost of the matrix.

$$\begin{bmatrix} 5 & 9 \\ 24 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} -19 & -15 \\ 0 & -21 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 2 & 6 \\ 21 & 0 \end{bmatrix}$$

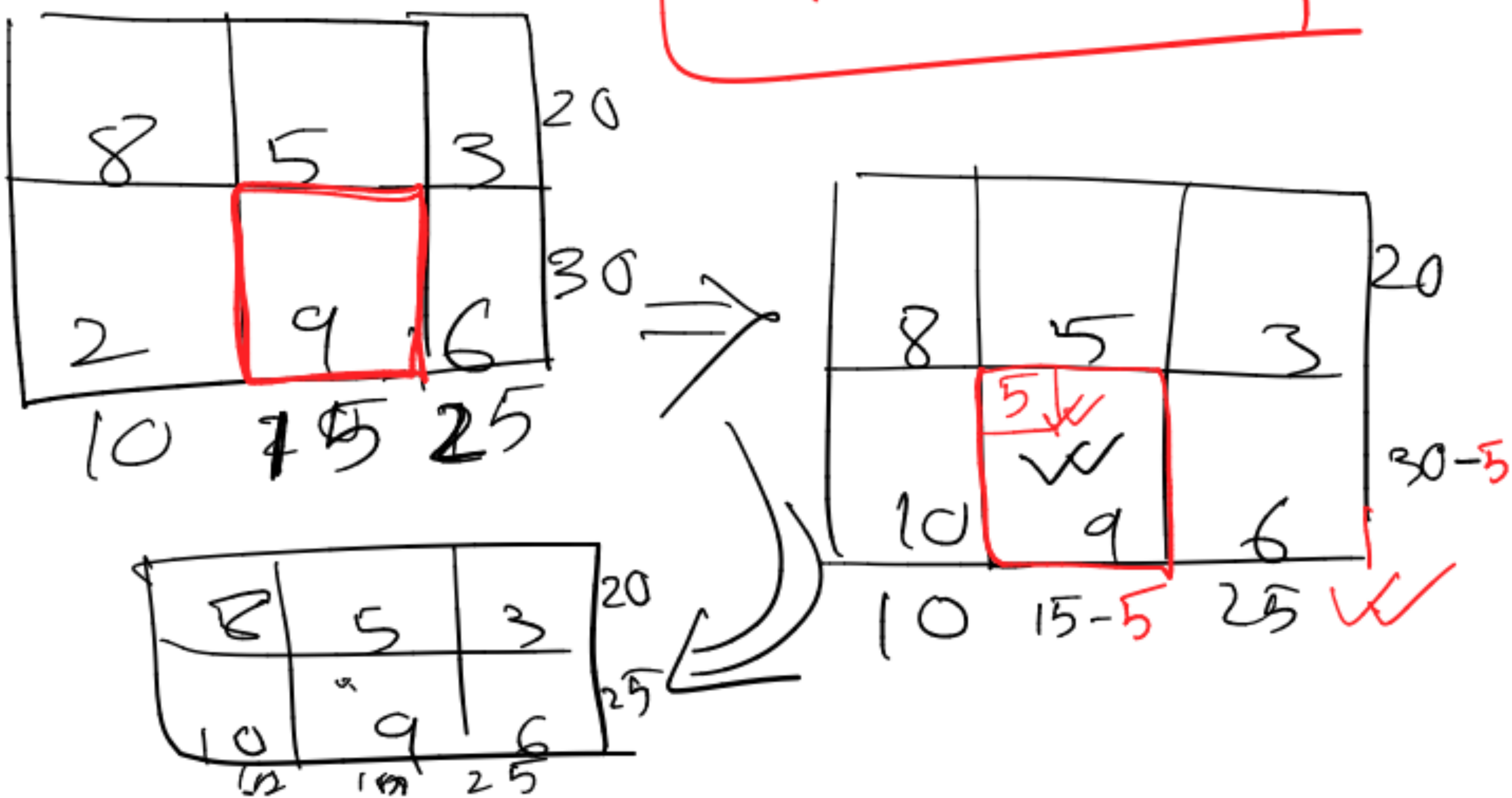
(c) No allocation for a particular cell.

•

M is a very large number.

(D) some positive allocation in a particular cell.

at least 5



Assignment Problem (A P)

An assignment problem is defined by the following table.

facilities

		f_1	f_2	$\cdots$	f_j	$\cdots$	f_m	a_i
	J_1	c_{11}	c_{12}	$\cdots$	c_{1j}	$\cdots$	c_{1m}	1
	J_2	c_{21}	c_{22}	$\cdots$	c_{2j}	$\cdots$	c_{2m}	1
	$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
Jobs	J_i	c_{i1}	$\boxed{c_{i2}}$	$\cdots$	c_{ij}	$\cdots$	c_{im}	1
	$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	J_m	c_{m1}	c_{m2}	$\cdots$	c_{mj}	$\cdots$	c_{mm}	1
	b_j	1	1		1		1	m m

- Problem which arising in following places.
  - assigning men to offices.
  - Jobs to machines
  - classes to schools.

- $J_i \leftarrow$ i-th job, m jobs.
  - $f_j \leftarrow$ j-th facility, m facilities
  - $c_{ij} \leftarrow$ cost of assigning the i-th job to the j-th facility

The table represents that only one unit of job is available for one facility.

The assignment is to be made in such a way that each job can be associated with one and only one facility.

The objective is to find an assignment of jobs to facilities so as to minimise the total cost of assignment.

Mathematical formulation

$c_{ij} \leftarrow$ cost of assigning i-th job to the j-th facility.

$x_{ij} \leftarrow$ assignment of i-th job to the j-th facility.

we want to determine $x_{ij} \geq 0$ $\forall$ $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, m$ such that.

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{m} x_{ij} = 1 \quad \forall\ i = 1, 2, \ldots, m \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad \forall\ j = 1, 2, \ldots, m. \qquad (2)$$

$$x_{ij} = \begin{cases} 1, & \text{if the } i\text{-th job is assigned to the } j\text{-th facility.} \\ 0 & \text{otherwise.} \end{cases}$$

Note: AP is not a linear program as the variable x_{ij} can assume only 0 and 1 values.

$\Rightarrow$ constraints of type 1 ensures that only one job is assigned to one facility

$\Rightarrow$ constraints of type 2 ensures that only one facility is assigned with one job.

Towards a solution of an AP

Observation:

AP is a special case of the TP. in which

$$m = n$$
$$a_i = 1 \quad \forall\ i = 1, 2, \dots m$$
$$b_j = 1 \quad \forall\ j = 1, 2, \dots, m$$

- At the first look we may say that there are $(m+n-1)$ ie $(m+m-1)$ ie, $2m-1$ basic variables.
- But the constraints restricts that each basic solution will consists of m basic variables equal to 1 and $m-1$ basic variables equal to 0.
- A basic ~~fe~~ feasible solution will have high level of degeneracy.