CS374: Lab Sheet 2

Ex 1) Use Bisection method and write code for finding a root of $e^x - 3x = 0$ correct to four decimal digits.

Ex 2) Perform three iterations for computing $\sqrt{2}$, starting with $x_0 = 1$ using Newton's metthod, and of the Bisection method for $\sqrt{2}$, starting with interval $[1, 2]$. How many iterations are needed for each method in order to obtain 10^{-6} accuracy? Write code for this problem. (Newton's method for $\sqrt{N}$ will be $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$.)

Ex 3) Write code for finding root of $e^x = \sin x$ closest to 0 using Bisection method.

Ex 4) Write a program to solve for a root of the equation $e^{-x^2} = \cos x + 1$ on $[0, 4]$. What happens in Newton's method if we start with $x_0 = 0$ or with $x_0 = 1$?

Ex 5) (**Circuit Problem**) A simple circuit with resistance R, capacitance C in series with a battery of voltage V is given by

$$Q = CV\left(1 - e^{-T/(RC)}\right),$$

where Q is the charge of the capacitor and T is the time needed to obtain the charge. We wish to solve for the unknown C. For example, using Bisection method, solve this exercise

$$f(x) = 10x\left[1 - e^{-0.004/(2000x)}\right] - 0.00001.$$

Plot the curve.

Ex 6) In celestial mechanics, **Kepler's Equation** is important. It reads

$$x = y - \epsilon \sin y,$$

in which x is a planet's mean anomaly, y its eccentric anomaly, and ϵ the eccentricity of its orbit. Taking $\epsilon = 0.9$, construct a table of y for 30 equally spaced values of x in the interval $0 \leq x \leq \pi$. Use Newton's Method to obtain each value of y. The y corresponding to an x can be used as the starting point for the iteration when x is changed slightly.

Ex 7) Using Newton's Method, produce a table of x versus y, where y is defined implicitly as a function of x. Use $G(x, y) = 3x^7 + 2y^5 - x^3 + y^3 - 3$ and start at $x = 0$, proceeding in steps of 0.1 to $x = 10$.

Ex 8) Starting with $(0, 1)$, perform two iterations of Newton's method on the following system

$$\begin{cases} 4x_1^2 - x_2^2 = 0 \\ 4x_1 x_2^2 - x_1 = 1. \end{cases}$$