An Oscillator as a Second-Order System

I/. An undamped oscillator: $\boxed{m \frac{d^2x}{dt^2} = -kx}$

II/. A damped oscillator: $\boxed{m \frac{d^2x}{dt^2} = -kx - B\frac{dx}{dt}}$

The damping is proportional to velocity, $\frac{dx}{dt}$.

Now write $\boxed{\frac{dx}{dt} = \dot{x} = v = 0.x + 1.v}$,

and $\frac{d^2x}{dt^2} = \frac{dv}{dt} = \dot{v} = -\frac{k}{m}x - \frac{B}{m}\frac{dx}{dt}$ $\boxed{v = \frac{dx}{dt}}$

$\Rightarrow$ $\boxed{\dot{v} = -\omega^2 x - 2bv}$ $\left(\begin{matrix}\omega^2 = k/m \\ 2b = B/m\end{matrix}\right)$

Hence we have in matrix form $\boxed{\begin{pmatrix}\dot{x}\\ \dot{v}\end{pmatrix} = \begin{pmatrix}0 & 1\\ -\omega^2 & -2b\end{pmatrix}\begin{pmatrix}x\\ v\end{pmatrix}}$

Now we know $\boxed{\frac{d^2x}{dt^2} - \tau\frac{dx}{dt} + \Delta x = 0}$

We use solutions of the type $\boxed{x = x_0 e^{\lambda t}}$

$\Rightarrow$ $\boxed{\dot{x} = dx/dt = \lambda x}$ and $\boxed{d^2x/dt^2 = \ddot{x} = \lambda^2 x}$.

Hence we have $(\lambda^2 - \tau\lambda + \Delta)x = 0$.

$\Rightarrow$ $\boxed{\lambda^2 - \tau\lambda + \Delta = 0}$ Now $\boxed{\tau = -2b}$ and $\boxed{\Delta = \omega^2}$.

$\Rightarrow$ The eigenvalues, (Two eigenvalues) $\boxed{\lambda_{1,2} = \frac{\tau \pm \sqrt{\tau^2 - 4\Delta}}{2}}$

$$\therefore \boxed{\lambda_{1,2} = \frac{-2b \pm \sqrt{4b^2 - 4\omega^2}}{2} = -b \pm \sqrt{b^2 - \omega^2}}.$$

i.) If $\boxed{b^2 > \omega^2}$, then both eigenvalues λ_1, λ_2 are real, and $\boxed{x = x_0 \exp\left[\left(-b \pm \sqrt{b^2-\omega^2}\right)t\right]}$. The oscillator is overdamped.

ii) If $\boxed{b^2 < \omega^2}$, then both eigenvalues λ_1, λ_2 are complex and $\boxed{x = x_0 e^{-bt} \exp\left[\pm i\sqrt{b^2-\omega^2}\, t\right]}$. The oscillator is underdamped, with a decaying amplitude of oscillation.

iii.) If $\boxed{b^2 = \omega^2}$, then both eigenvalues λ_1, λ_2 are real and the same. $\boxed{x = x_0 e^{-bt}}$ The oscillator is critically damped.

Correction to Richardson's Theory.

$\boxed{\frac{dy}{dt} = lx + h - \beta y}$ l indicates the war readiness of x.

Correction to the Predator-Prey Model

The growth rate of the predator population is $\boxed{\frac{dy}{dt} = -Cy + Dxy}$. The term $\boxed{-Cy}$ indicates that even in the linear order the predator population inhibits its own growth.

Additional Points on the Threshold Theorem of Epidemiology

$$\boxed{x = (x_0 + y_0) - y + \frac{B}{A}\ln(y/y_0)}$$

i.) x has a turn (a maximum) when $\boxed{y = B/A}$.

ii.) When $y \to 0$, (i.e. $y \ll B/A$), $x \to -\infty$.

i.e. $\boxed{x \sim \frac{B}{A}\ln\left(\frac{y}{y_0}\right)}$. The logarithmic part dominates.

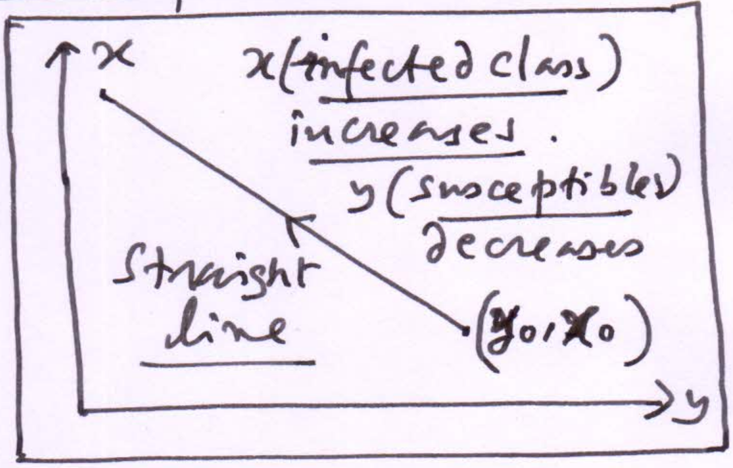
iii.) When $y \to \infty$ (i.e. $y \gg B/A$), then

$\boxed{x \sim -y}$. The linear part dominates.

iv.) For $\boxed{B = 0}$,

$\boxed{\frac{dZ}{dt} = 0}$ (No recovered individual)

and $\boxed{x = (x_0 + y_0) - y}$.



In this case, starting at $t=0$, all susceptibles become infected. No one recovers and no one is removed.

A Correction: $y_0 - y_\infty \simeq 2y_0\left(\frac{y_0}{\rho} - 1\right)$

Now $\boxed{y_0 = \rho + \epsilon}$ $\Rightarrow$ $\boxed{\frac{y_0}{\rho} - 1 = \frac{\epsilon}{\rho}}$. where $\boxed{\epsilon \ll \rho}$.

Hence, $y_0 - y_\infty \simeq 2y_0\epsilon/\rho \simeq 2(\rho + \epsilon)\epsilon/\rho$.

$\Rightarrow$ $y_0 - y_\infty \simeq \frac{2\cancel{\rho}\epsilon}{\cancel{\rho}} + \frac{2\epsilon^2}{\rho} \simeq 2\epsilon$ (neglecting ϵ^2)

$\Rightarrow$ $y_0 - y_\infty \simeq 2\epsilon \simeq 2(y_0 - \epsilon)$ | when y_0 is slightly greater than ρ