Tutorial 11

1. Let $X_1, ..., X_n$ be independent and identically distributed random variables having expected value μ and variance σ^2, let $\overline{X} = \sum_{i=1}^{n} X_i/n$ be the sample mean. The quantities $X_i - \overline{X}, i = 1, ..., n$, are called deviations, as they equal the differences between the individual data and the sample mean. The random variable

$$S^2 = \sum_{i=1}^{n} \frac{(X_i - \overline{X})^2}{n-1}$$

is called the sample variance. Find (a) $\mathrm{Var}(\overline{X})$ and (b) $\mathrm{E}[S^2]$.

2. If X and Y are independent binomial random variables with identical parameters n and p, calculate the conditional expected value of X given that $X + Y = m$.

3. Suppose that the number of people entering a department store on a given day is a random variable with mean 50. Suppose further that the amounts of money spent by these customers are independent random variables having a common mean of 8 dollar. Finally, suppose also that the amount of money spent by a customer is also independent of the total number of customers who enter the store. What is the expected amount of money spent in the store on a given day?

4. Consider a gambling situation in which there are r players, with player i initially having n_i units, $n_i > 0$, $i = 1, ..., r$. At each stage, two of the players are chosen to play a game, with the winner of the game receiving 1 unit from the loser. Any player whose fortune drops to 0 is eliminated, and this continues until a single player has all $n = \sum_{i=1}^{r} n_i$ units, with that player designated as the victor. Assuming that the results of successive games are independent and that each game is equally likely to be won by either of its two players, find the average number of stages until one of the players has all n units.