SC217: Electromagnetic Theory, **AUTUMN'16 MIDSEM1**

DA-IICT, B.Tech, Sem III 14:00-15:00 PM 31^{st}Aug.2016 **25 marks**

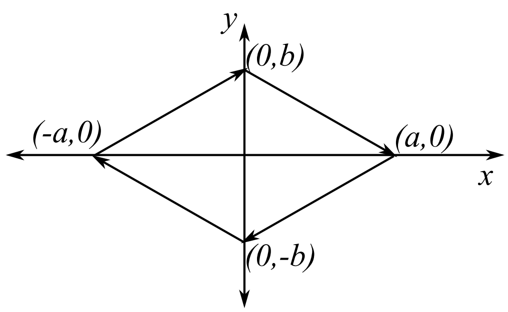
1. If $\vec{A}$ and $\vec{B}$ are two vectors in three dimensions and $\vec{C} = \vec{A} \times \vec{B}$.
   Show that the components of $\vec{C}$ behaves as a vector under a rotation given by the matrix $R = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$. **(5)**

2. Let $f(x,y,z) = \sqrt{x^2+y^2+z^2}$.
   Find $\vec{\nabla} f$. Find the rate of change of f at the point $(1,1,0)$ along a direction specified by the unit vector $\frac{1}{\sqrt{2}}(\hat{\mathrm{i}} - \hat{\mathrm{j}})$. **(5)**

3. Find $\nabla^2(\sin(\vec{\omega}\cdot\vec{r}))$ where $\vec{\omega}$ is a fixed vector. **(5)**

4. For any vector field $\vec{\mathrm{A}}$ and any scalar field F show that
   (i) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{\mathrm{A}}) = 0$; (ii) $\vec{\nabla}\times(\vec{\nabla}F) = 0$. **(5)**

5. Verify Stokes' Theorem for the vector field $\vec{A} = y\hat{i} - x\hat{j}$ over a region enclosed by the path shown in the figure. **(5)**



Gradient, divergence and curl

$$\vec{\nabla}F = \hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2}$$