CT303 Lecture 14: 7 October 2020

- Review of Lecture 13.
  - Bandwidth efficiency: $\eta_B = \frac{R_s}{B}$.
  - Duobinary coding - Use *sinc* waveforms.
  - Closer to being realizable - Use *sinc* waveforms to encode $y_k = x_k + x_{k-1}$.
  - Issues? 1. Still not realizable

2. $y_k \in \{-2V, 0, 2V\}$.
$\underbrace{\longleftrightarrow}_{2V} \underbrace{\longleftrightarrow}_{2V}$

Binary $x_k \in \{-V, V\}$
$\underbrace{\longleftrightarrow}_{2V}$

↑ η_B at the cost of ↑ Power

3. $y_k = 2V \rightarrow x_k = V$
$y_k = -2V \rightarrow x_k = -V$
$y_k = 0, \; x_k = -x_{k-1}$

error may propagate

Pre-coding

$x_k \rightarrow$ bit stream

- Define $\tilde{w}_k = x_k \oplus \tilde{w}_{k-1}$.
- Convert the bit sequence $\tilde{w}_k$ into bipolar amplitude sequence w_k.
- $y_k = w_k + w_{k-1}$.
- Transmit $\sum_k y_k sinc(Tt - k)$.
- Decoding:

$\tilde{w}_k \rightarrow \ldots 1\,1\,0\,1\,0\,0 \ldots$

$w_k \rightarrow \ldots V\; V\; -V\; V\; -V\; -V \ldots$

$y_k = w_k + w_{k-1}$

$1 = x_k \oplus \tilde{w}_{k-1}$

$x_k = ?$

$x_k = ?$

$y_k \in \{-2V, 0, 2V\}$

$y_k = 2V \longrightarrow w_k = V, \; \tilde{w}_k = 1$

$y_k = -2V \longrightarrow w_k = -V, \; \tilde{w}_k = 0$

$\tilde{w}_{-1} = 1/0$

k	0	1	2	3	4	5	6
x_k	1	0	1	0	0	1	1
$\tilde{w}_k$	0/1	0/1	1/0	1/0	1/0	0/1	1/0
w_k	$-V$	$-V$	V	V	V	$-V$	V
y_k		$-2V$	0	$2V$	$2V$	0	0
$\tilde{x}_k$ (estimate of x_k)		0	1	0	0	1	1

$\tilde{x}_k = 1$ if $y_k = 0$

$\tilde{x}_k = 0$ if $y_k \in \{2V, -2V\}$

$\Rightarrow$

- Decoding:

$$\text{If } y_k = \pm 2V, \hat{x}_k = 0$$
$$\text{If } y_k = 0, \hat{x}_k = 1$$

$\Rightarrow$ Ensures that the error does not propagate !!

Duobinary coding with Precoding

$x_k \in \{V, -V\} \xrightarrow{y_k = x_k + x_{k-1}} y_k \in \{2V, -2V, 0\}$.

$y_k = x_k + x_{k-1} + x_{k-2}$ $\quad y_k \in \{3V, 2V, V, 0, -V, -2V, -3V\}$

Polybinary

End of Chapter 2

Chapter 3: Baseband Demodulation & Detection

- Binary transmission: Bit-1: $s_1(t)$, Bit-0: $s_2(t)$.
- Assumptions: (1) The channel is modeled as an LTI system with impulse response $h_c(t)$, and frequency response $H_c(f)$. (2) Noise - AWGN.
  - Received signal: $r(t) = s_i(t) * h_c(t) + \eta(t)$, with $G_\eta(f) = \frac{N_0}{2}$.

    2 sided PSD
- Demodulation: Process of recovering the transmitted waveform $[r \to \tilde{s}_i]$ → Approximation estimate of s_i
- Detection: Determining the digital message (bits) from the recovered waveform.

Demodulation

Analog: Corrupted w/f → Approx of the original w/f.
LTI system. Restorate. (Improve SNR!)

- Receiving filter to improve SNR.
- In some cases, another filter is used to compensate for channel induced ISI. This is called *Equalization filter*.

▶ Let $h_r(t)$ denote the impulse response of the receiving filter. Let $z(t) = (h_r * r)(t)$ denote the output of the receiving filter.

▶ Since s_i corresponds to one bit for every $\boxed{T} = 1/R_b$ secs, r and z also correspond to the received signal and output of the receiving filter for one transmitted bit every T secs, resp.

$z(t), t \in [0, T]$

▶ The output of the receiving filter z is sampled every T secs to give a real number $\boxed{z(T)}$.

▶ The detector takes in $z(T)$ and determines which bit was sent for $t \in [0, T]$.

Detection

- $r(t) = h_c(t) * s_i(t) + \eta(t).$
  - $z(t) = \underbrace{h_r(t) * h_c(t) * s_i(t)} + \underbrace{h_r(t) * \eta(t).}$ → zero mean.

$z(t) = a_i(t) + \eta_o(t)$ ← Is η_o AWGN? NO!

$Z(T) = a_i(T) + \eta_o(T)$

1 RN ← $Z = a_i + \eta_o$ → Gaussian RV

a_i: a_1, a_2 deterministic

[Linear transf of GRV's given GRN.]

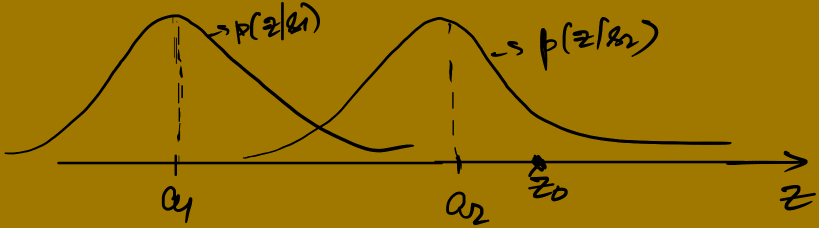
$y = Ax + b$

$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

$$p_{\eta_o}(x) = \frac{1}{2\pi\,\sigma_0}\, e^{-x^2/2\sigma_0} \quad \text{(zero mean GRV)}$$

$$p(z/s_1) = \frac{1}{2\pi\sigma_0} e^{-\frac{(x-a_1)^2}{2\sigma_0}} \rightarrow \text{Likelihood of } z \text{ given } s_1$$

$$p(z/s_2) = \frac{1}{2\pi\sigma_0} e^{-\frac{(x-a_2)^2}{2\sigma_0}} \rightarrow \text{Likelihood of } z \text{ given } s_2$$



* $d(z_0, a_2) < d(z_0, a_1) \Rightarrow s_2$

* $p(z_0/s_2) > p(z_0/s_1) \Rightarrow s_2$

$$-\ln(p(z_0/s_2)) = k + \frac{d(z_0, a_2)^2}{2\sigma_0}$$

MLE and MAP estimation

- $\boxed{\text{MLE: } s_k = \arg\max p(z/s_i).}$ → Maximum likelihood estimate.
- MAP: $s_k = \arg\max p(s_i/z)$.

$p(s_1/z)$ $p(s_2|z)$ → Posterior distribution.

$$\left.\begin{array}{l} p(s_1/z) > p(s_2|z) \Rightarrow s_1 \\ p(s_2|z) > p(s_1/z) \Rightarrow s_2 \end{array}\right\}$$ Maximum Aposteriori probability (MAP).