TUTE-4 Soln

Soln 1) N distinct type of coupons. Probability of getting any coupon in equaly likely.
Random variable T. = No. of coupons needed to be collected until one obtains complete set of atleast one of each type.

Derive $P\{T=n\}$

First of all let's find $P\{T>n\}$ An define some events $A_1, A_2, \ldots, A_N$

A_i : No (i) coupon is collected among first n coupons are collected.

$$\therefore P(T>n) = P\left(\bigcup_{i=1}^{N} A_i\right)$$

(Now using the inclusion exclusion argument).

$$= \sum_{i} P(A_i) - \sum_{i_1<i_2}\sum P(A_{i_1} A_{i_2}) + \ldots + (-1)^{k} \times \sum_{i_1<i_2<\ldots<i_k}\sum\sum P(A_{i_1} A_{i_2} \ldots A_{i_k})$$

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$+ \dots + (-1)^{N+1} P(A_1 A_2 \cdots A_N)$

Here A_i will occur if there is not i type coupon in first n trials. for single trial it is $\left(\frac{N-1}{N}\right)$ & for n ~~types~~ times...

$$\boxed{P(A_i) = \left(\frac{N-1}{N}\right)^n}$$

Same way $A_{i_1} A_{i_2}$ will occur if none of that kind of coupons will be there.

$$\therefore \boxed{P(A_{i_1} A_{i_2}) = \left(\frac{N-2}{N}\right)^n}$$

And so $\boxed{P(A_{i_1} A_{i_2} \dots A_{i_n}) = \left(\frac{N-k}{N}\right)^n}$

$$\therefore P(T > n) = N\left(\frac{N-1}{N}\right)^n - \binom{N}{2}\left(\frac{N-2}{N}\right)^n + \binom{N}{3}\left(\frac{N-3}{N}\right)^n$$
$$+ \dots + (-1)^N \binom{N}{N-1}\left(\frac{1}{N}\right)^n$$

$$\boxed{\therefore P(T > n) = \sum_{i=1}^{N-1} \binom{n}{i}\left(\frac{N-i}{N}\right)^n (-1)^{i+1}}$$

Now $P(T > n-1) = P(T > n) + P(T = n)$

$$\boxed{\therefore P(T = n) = P(T > n-1) - P(T > n)}$$

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Sol^n-2 A product is sold seasonally
profit = b \$/unit (sold)
loss = l \$/unit (unsold).
Random variable → Number of units of the 'i' product that are ordered during any season.
probability mass function
$p(i)\ , i \geq 0$.

→ Stock the product in advance. How to maximize the profit?
X: number of ~~unti~~ units ordered.
s = units stocked.
$\therefore$ profit $= P(s) = bX - (s-X)l$ if $X \leq s$
$= sb$ if $X > s$

Now finding the expected value of profit.

$$E[P(s)] = \sum_{i=0}^{s} [bi - (s-i)l]\underset{\text{Probability}}{p(i)} + \sum_{i=s+1}^{\infty} sbp(i)$$

$$= \cancel{(b+l)\sum_{i=0}^{s} ip(i) - (b+l)s\sum_{i=0}^{s}}$$

$$= (b+l)\sum_{i=0}^{s} ip(i) - sl\sum_{i=0}^{s} p(i) + sb\left[1 - \sum_{i=0}^{s} p(i)\right]$$

$$= (b+l)\sum_{i=0}^{s} ip(i) - (b+l)s\sum_{i=0}^{s} p(i) + sb$$

$$\boxed{\therefore E[P(s)] = sb + (b+l)\sum_{i=0}^{s} (i-s)p(i)}$$

→ Now to find the optimum value of s let's check what happens if we increase

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it with 1.

$$\therefore E(P(s+1)) = b(s+1) + (b+l)\sum_{i=0}^{s+1}(i-s-1)\,p(i)$$

$$= b(s+1) + (b+l)\sum_{i=0}^{s}(i-s-1)\,p(i)$$

$$\therefore E[P(s+1)] - E[P(s)] = b - (b+l)\sum_{i=0}^{s} p(i)$$

we will increase s by s+1 if above term is > 0

$$\therefore b - (b+l)\sum_{i=0}^{s} p(i) > 0$$

$$\therefore \boxed{\sum_{i=0}^{s} p(i) < \frac{b}{b+l}}$$

In above eq^n L.H.S. is increasing with increase of s & R.H.S. is constant.

$\therefore s^* =$ largest value of s that can satisfy above eq^n.

$$\therefore E[P(0)] < E[P(1)] < \ldots < E[P(s^*)] < E[P(s^*+1)] > E[P(s+2)] > \ldots$$

$\Rightarrow$ We will ~~lead~~ increase it to $\boxed{s^*+1}$ to maximise the profit.

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Solⁿ-3 2 coins

Coin 1 $P_{Head} = 0.6$

Coin 2 $P_{Head} = 0.3$

One of the coin is choosen and flipped you can bet upto 10 \$ on Head. If Head than win that amount & tail then lose that amount.

→ In Amount of C an insiders can tell ~~you~~ you which coin is choosen.

Expected ~~py~~ pay off if we buy this offer?

• If we bet x \$ then expected earnings will be

$$\underset{\substack{\uparrow \\ \text{Head/~~Hea~~ win}}}{xp} - x\underset{\substack{\uparrow \\ \text{Tail/lose}}}{(1-p)} = \boxed{(2p-1)x}$$

→ Here $(2p-1)x$ will be positive iff $\boxed{p > 1/2}$

So to maximize the profit if $p > 1/2$ we can bet 10 \$ (maximum amount) over that.

⇒ we must choose 0.6 Coin.

(If we choose 0.3 coin then we will bet 0 \$ as there is ~~no~~ negative expected value".)

Hence expected pay off.

$$\frac{1}{2}(1.2-1)\times 10 + \frac{1}{2}(0) - C = \boxed{1 - C}$$

$P(\text{winning}) = \frac{0.6+0.3}{2} < \frac{1}{2}$ If information cost $\boxed{(C < 1)}$ then we should buy that

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Soln-4 A & B are playing a game.
P(A win each game) = p (independent)
One who win 3 games first will be the winner

(a) A : A wins the series
B : A wins the first game.
P(A|B) = ?
Here A will have to win 2 games in any of 2^{nd}, 3^{rd} or 4^{th} trial.

$$\boxed{\therefore P(A|B) = \sum_{i=2}^{4} \binom{4}{i} p^i (1-p)^{4-i}.}$$

(b) E : A win the first game.
F : A win the series

$$P(E|F) = \frac{P(EF)}{P(F)}$$

$$= \frac{P(E|E)\ P(E)}{P(F)}$$

$$= \frac{p \sum_{i=2}^{4} \binom{4}{i} p^i (1-p)^{4-i}}{\sum_{i=3}^{5} \binom{5}{i} p^i (1-p)^{5-i}.}$$

$$\boxed{= \frac{\sum_{i=2}^{4} \binom{4}{i} p^{i+1} (1-p)^{4-i}}{\sum_{i=3}^{5} \binom{5}{i} p^i (1-p)^{5-i}}}$$