Analysis of Multi-Disciplinary Problems

Modelling Traffic Flows: A Study on the Traffic of *Jackson, Alabama, U.S.A.*

This is a set of guidelines for you to follow while you are working on the third numerical assignment. You may recall that in the first assignment you had to work with data pertaining to the growth of *IBM*, and in the second assignment with data taken from the repository of the free-software operating system, *Debian*. The trend that you found in the former case was due to an exponential growth, saturated by nonlinearity, something modelled readily with the help of the logistic equation, and in the latter case, a logistic-type non-autonomous equation. Now in this third assignment, you will work with data pertaining to the traffic flow of a town named *Jackson* in the state of *Alabama* in the U.S.A. As usual the purpose behind this exercise is to give you a hands-on experience in modelling real data, but once again the mathematical principle of the modelling is different. While the modelling in the case of *IBM* was based on an autonomous equation, in this instance, just as in the case of *Debian*, the governing equation is non-autonomus. Furthermore, it will also demonstrate some properties of symmetry breaking. The following items give a description of the contents of the other files in the folder, along with some suggestions for the mathematical modelling.

- **The data file:** There is one data file, named `traffic_time.dat`. The data contained in this file have been taken from the repository of the *Alabama Department of Transportation (ALDOT)*, specifically pertaining to the city of *Jackson* in *Alabama State, U.S.A.* The data show the volume of vehicular traffic as a function of time (measured in hours). There are three columns of data. The first column gives the average volume of traffic flowing towards the east. The second column gives the hour of the day, with the 0 hour being set at midday, 12 pm. So all forenoon hours are negative. The third column gives the average volume of the traffic flowing towards the west. You are required to plot the data in the second column along the x axis. Then in separate graphs plot the data in the first and third columns along the y axis. This will give you a distribution of the traffic volume as a function of time in two separate graphs, one for the eastward traffic and the other for the westward traffic. Both the distributions will be bimodal.

- **A research article:** There is a research article named `am_akr_jphys.pdf`. You will require the formula given in Equation (1) for fitting the data. The outcome of your modelling should be shown in two separate figures, as given in Figures 1 & 2. The former figure shows a bimodal distribution of the traffic flow due west as a function of an hour of the day, and the latter does the same thing for the traffic flow due east. The captions of both the figures give the numerical values of the various fixed parameters.

- **The mathematical modelling:**

  1. Equation (1) is a function of the form, $N(t) = A(\mu + t^2)\exp[-(\lambda t - \beta)^2]$. Of crucial importance in this equation are the two parameters, μ and β. You have to use this function to fit the data provided in the data file. Plot time, t, along the x axis, and the volume of traffic, $N(t)$, along the y axis. Then fit Equation (1) with the data, using the values of the parameters given in Figures 1 & 2. You are welcome to try other values, if they come up with a closer match between the model equation and the data.
  2. There are some points for you to think over. What did the value of μ indicate? You will realise that it indicates bimodality, i.e., a distribution with double peaks. Thereafter, consider the value of β. Test β with some random values and you will be able to see that the role of β is to make the bimodal distribution asymmetric.
