1. Suppose X is a random variable with mean 64.5 and variance 144. Use Chebyshevs inequality to approximate the following probabilities.

$$P[44 < X < 85]$$

$$P[36 < X < 93]$$

2. Consider a vector random variable $X = [X1, X2]^T$. These random variables have unit variance and are uncorrelated. Now the transformed vector $Y = [Y1, Y2]^T$ is obtained as $Y = AX$, where A is the transformation matrix. Find the matrix A so that Y has the covariance matrix

$$C_Y = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

3. Suppose that $X1$ and $X2$ have joint pdf $f_{X_1,X_2}(x_1, x_2) = 2, 0 < x_1 < x_2 < 1$ and zero otherwise. Compute the joint pdf of random variables $Y_1 = \frac{X_1}{X_2}$ and $Y_2 = X_2$.

4. Let the pdf of X_1 and X_2 be given by

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & for x_1 \geq 0 and x_2 \geq 0 \\ 0, & elsewhere. \end{cases}$$

Consider two RVs Y_1 and Y_2 be defined as

$$Y_1 = X_1 + X_2$$

$$Y_2 = \frac{X_1}{X_1 + X_2}$$

, Find the joint pdf of Y_1 and Y_2.