Lecture-18

i) sign up on Moodle with your DA account.

ii) Upload your RECENT photo/mugshot on Moodle moodle.daiict.ac.in

iii) Enrol in SC222

please complete this by 12 noon tomorrow.

Recap:

Jointly distributed random variables: independence

$$P(X \in A, Y \in B) = P(X \in A)\, P(Y \in B)$$

$$p_{X,Y}(X = x, Y = y) = P_X(X = x)\, P_Y(Y = y)$$

$$f_{X,Y}(x,y) = f_X(x)\, f_Y(y)$$

Sums of independent random variables

e.g. X : $(0,1)$ uniform

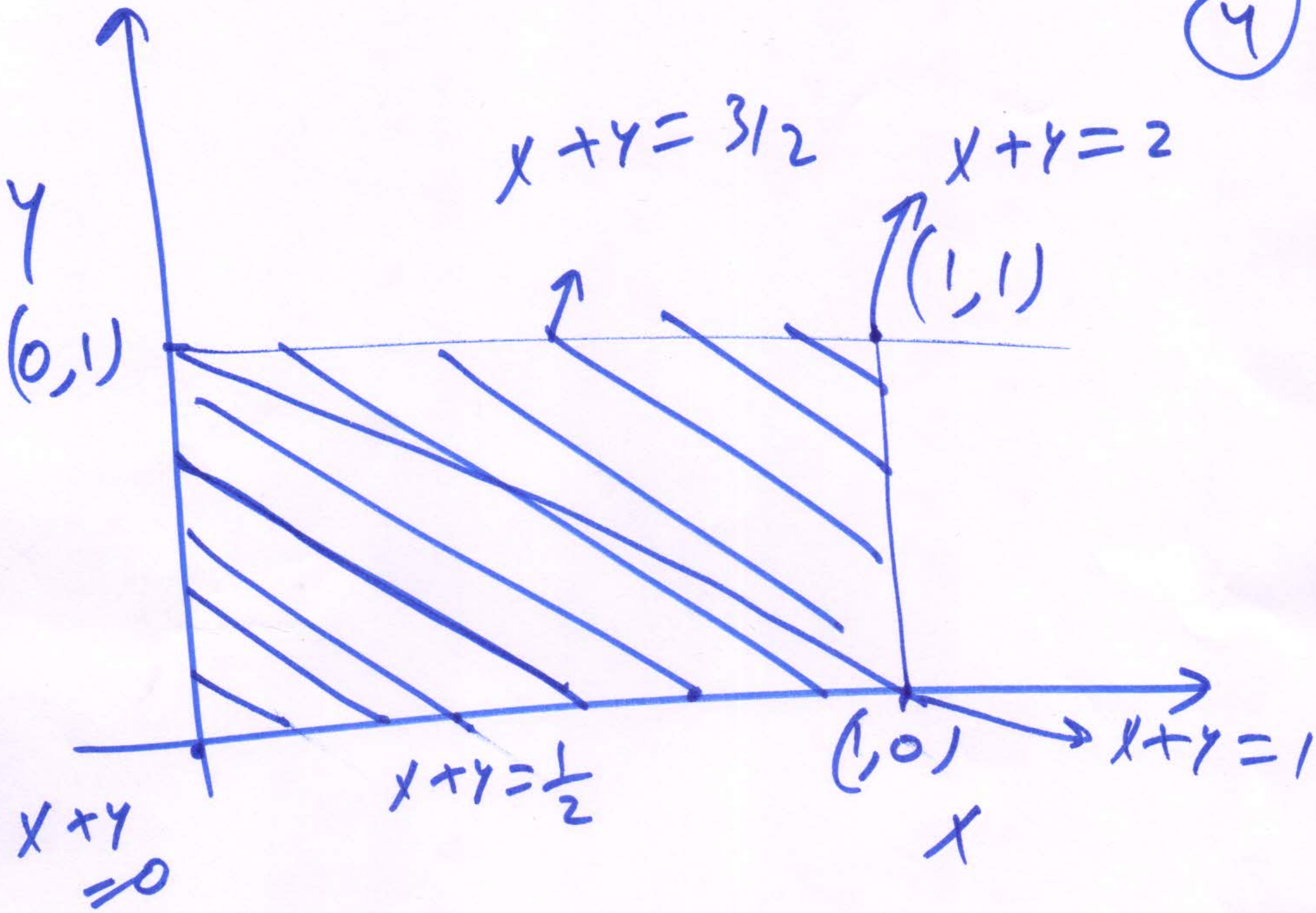
Y : $(0,1)$ uniform

$Z = X + Y$

X, Y are independent

$P(Z \leq a)$

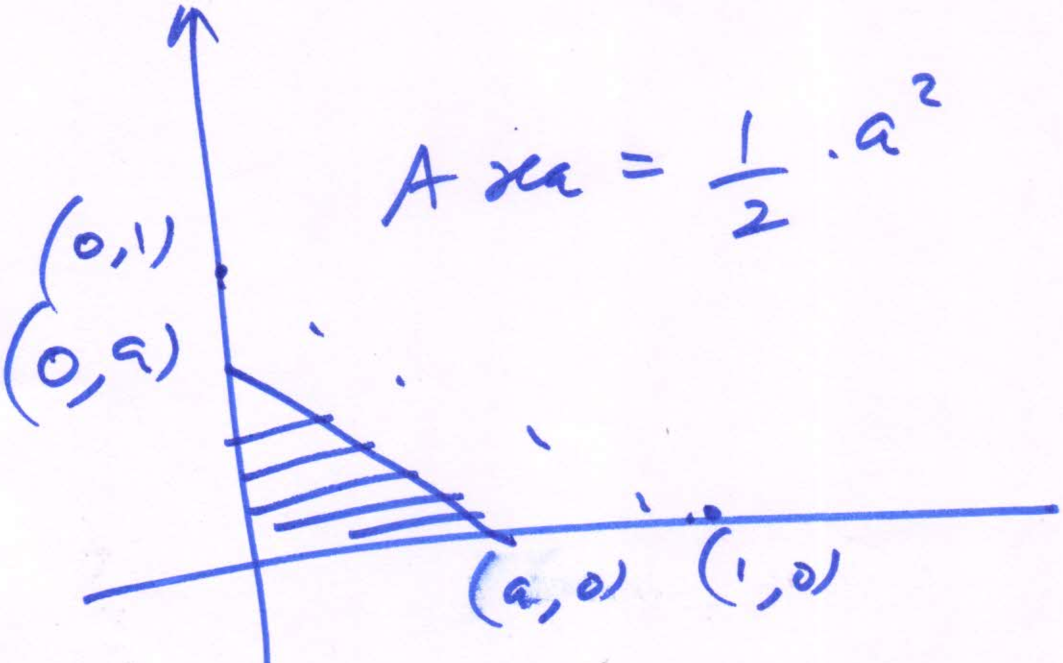
$P(X + Y \leq a)$



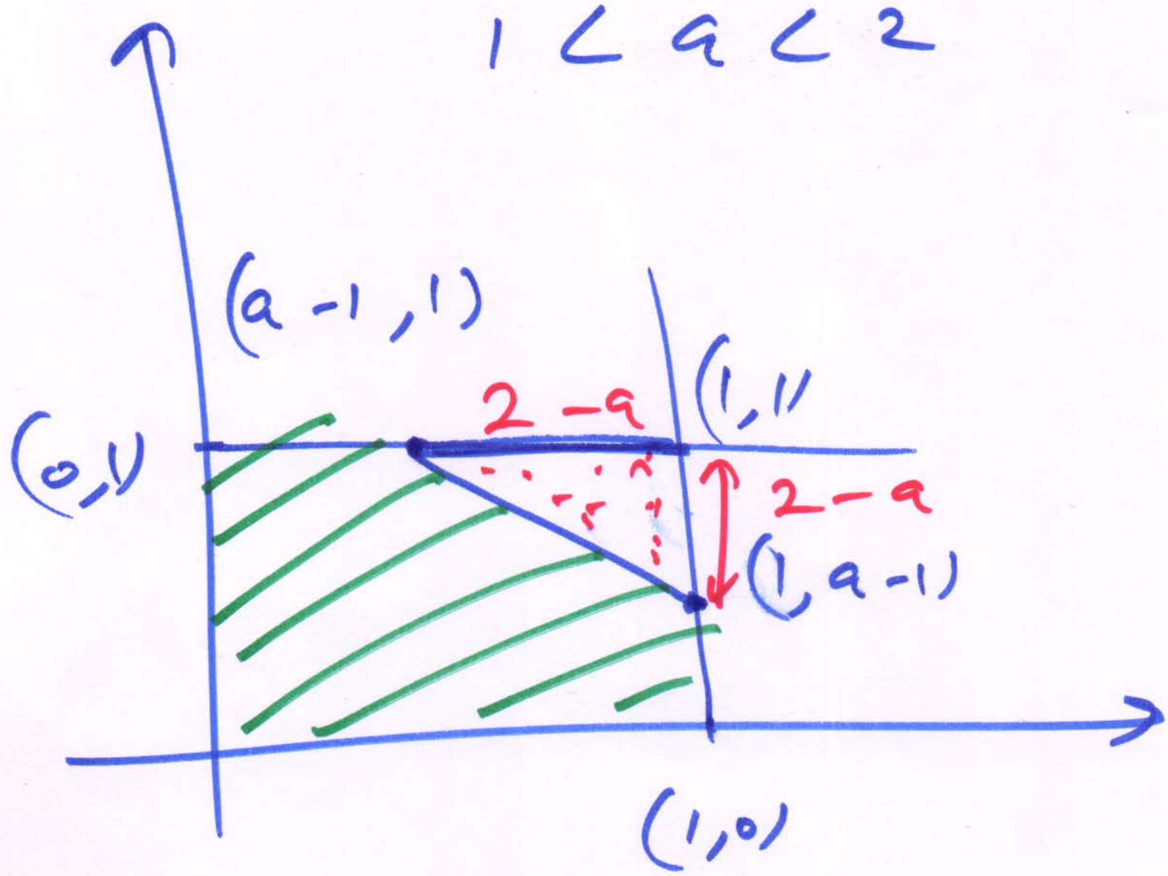
$x+y \le a \quad \in (0,2)$

$0 < a < 1$

$\text{Area} = \frac{1}{2} \cdot a^2$ — 1



$1 < a < 2$ ⑤



$$1 - (a-1) = 2 - a$$

$$A \cancel{x}a = 1 - \frac{1}{2}(2-a)^2$$

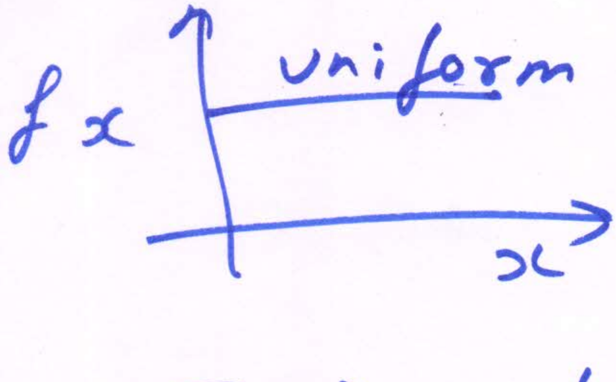
$$= 1 - \frac{1}{2}(4 + a^2 - 4a)$$

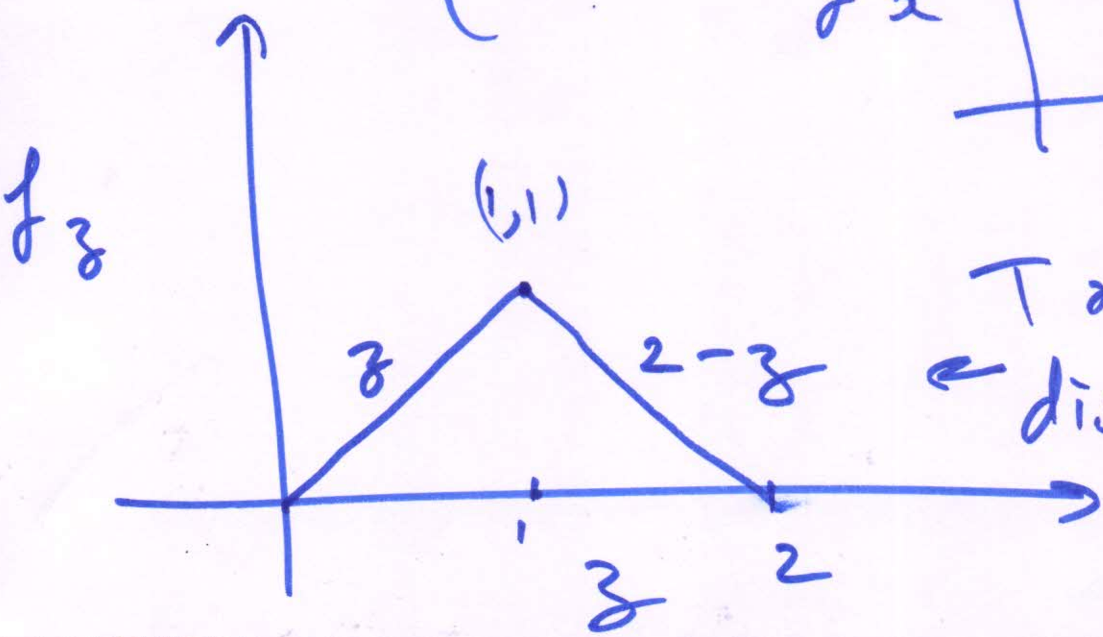
$$= 1 - 2 - \frac{a^2}{2} + 2a$$

$$= 2a - \frac{a^2}{2} - 1 \quad \text{——} \ ②$$

$$P(X+Y \le a) = \begin{cases} \dfrac{a^2}{2} & 0 < a \le 1 \\ 2a - \dfrac{a^2}{2} - 1 & 1 < a < 2 \end{cases}$$

$$f_{X+Y}(x+y) = \begin{cases} a & 0 < a \le 1 \\ 2-a & 1 < a < 2 \end{cases}$$





Triangular distribution

Theorem

X, Y are independent, continuous random variables.

~~$f(x$~~

$$f_{X+Y}(a) = \int_{-\infty}^{\infty} f_X(a-y)\, f_Y(y)\, dy$$

(Proof in the tutorial)

X, Y uniform (0, 1)

How is $\underline{X+Y}$ distributed?

$f_X(x) = 1 \quad 0 < x < 1$

$f_Y(y) = 1 \quad 0 < y < 1$

(8)

$$f_{X+Y}(a)$$

$$= \int_{-\infty}^{\infty} f_X(a-y)\, f_Y(y)\, dy$$

$$= \int_0^1 f_X(a-y)\, dy$$

$$0 < y < 1$$

$$0 < a - y < 1$$

$0 < a < 1$	$1 < a < 2$
$\int_0^a f_X(a-y)\, dy$	$\int_{a-1}^1 f_X(a-y)\, dy$
$= a$	$= 2-a$

eg Discrete random variables, Poisson

$$Z = X + Y$$

X : Poisson, λ_1
Y : Poisson, λ_2
independant

$$P(X+Y=n) = \sum_{k=0}^{n} P(X=k, Y=n-k) = \sum_{k=0}^{n} P(X=k)\,P(Y=n-k)$$

X	Y
0	n
1	$n-1$
2	$n-2$
$\vdots$	
n	0

$$= \sum_{k=0}^{n} \frac{e^{-\lambda_1} \lambda_1^{k}}{k!} \frac{e^{-\lambda_2} \lambda_2^{n-k}}{(n-k)!}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{n!} \sum_{k=0}^{n} \frac{\lambda_1^{k} \lambda_2^{n-k} \, n!}{k! \, (n-k)!}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{n!} \boxed{\sum_{k=0}^{n} \binom{n}{k} \lambda_1^{k} \lambda_2^{n-k}}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{n!} (\lambda_1+\lambda_2)^{n}$$

= Poisson, with parameter $\lambda_1 + \lambda_2$

e.g.: Binomial

X : (n, p)

Y : (m, p)

$X + Y$