Instructions:

- Each question is for 5 marks.
- Understanding a question is part of the evaluation. The invigilators are NOT going to clarify any doubts. If you find a mistake in a question, then please state it in your answer book.

1. What is the probability that the quadratic equation
$$x^2 + 2bx + c = 0$$
   has real roots? [5]
   Assume that both b and c are uniformly distributed over the interval $[-4, 4]$.

2. Duels in a particular town are rarely violent. Each contestant comes at a time uniformly distributed between 5AM and 6AM on the appointed day. And leaves exactly 5 minutes later, honor served, unless his opponent arrives within the time interval, in which case they fight. What fraction of duels lead to violence? [5]

3. In a common carnival game, a player tosses his one-rupee coin from a distance of about 5 feet onto the surface of a table ruled in 1-inch squares. If the coin falls entirely within a square (without touching any of the lines), the player wins 5 rupees but does not get his one-rupee coin back. If the player loses, he loses the one-rupee coin. In reality, some locations where the coin will fall will be more likely than some other locations. However, for the sake of simplicity, assume that the position where the center of the coin falls is uniformly distributed all over the table. The diameter of the coin is ¾ inches. If the coin lands on the table, what is the probability of the player winning? [5]

4. A model for the movement of a stock supposes that if the present price of the stock is s, then, after one day, it will be either us with probability p or ds with probability $1-p$. Assuming that successive movements are independent, approximate the probability that the stock's price will be up at least 30% after 1000 days, if $u = 1.012$, $d = 0.990$, and $p = 0.52$. [5]

5. Each game you play is a win with probability $p = 0.8$. You plan to play 5 games, but if you win the 5th game, then you will keep on playing until you lose.
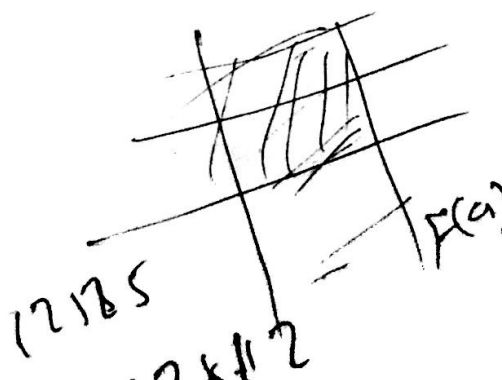
    a. What is the expected number of games that you will play? [3]
    b. What is the expected number of games that you will lose? [2]