Solve the LPP using graphical method.

$$\min Z = 2x - y$$

$$\text{s.t.} \quad x + y \le 5$$

$$x + 2y \le 8$$

$$4x + 3y \ge 12$$

$$x, y \ge 0$$

Solution:

Constraint 1

$$x + y \le 5$$

Consider the equation of constraint 1

$$x + y = 5$$

when $x = 0$, $y = 5$

when $y = 0$ $x = 5$

The two points are $(0, 5)$ and $(5, 0)$

Constraint 2

$$x + 2y = 8$$

when $x = 0$, $y = \frac{8}{2} = 4$

when $y = 0$, $x = 8$

The two points are $(0, 4)$ and $(8, 0)$

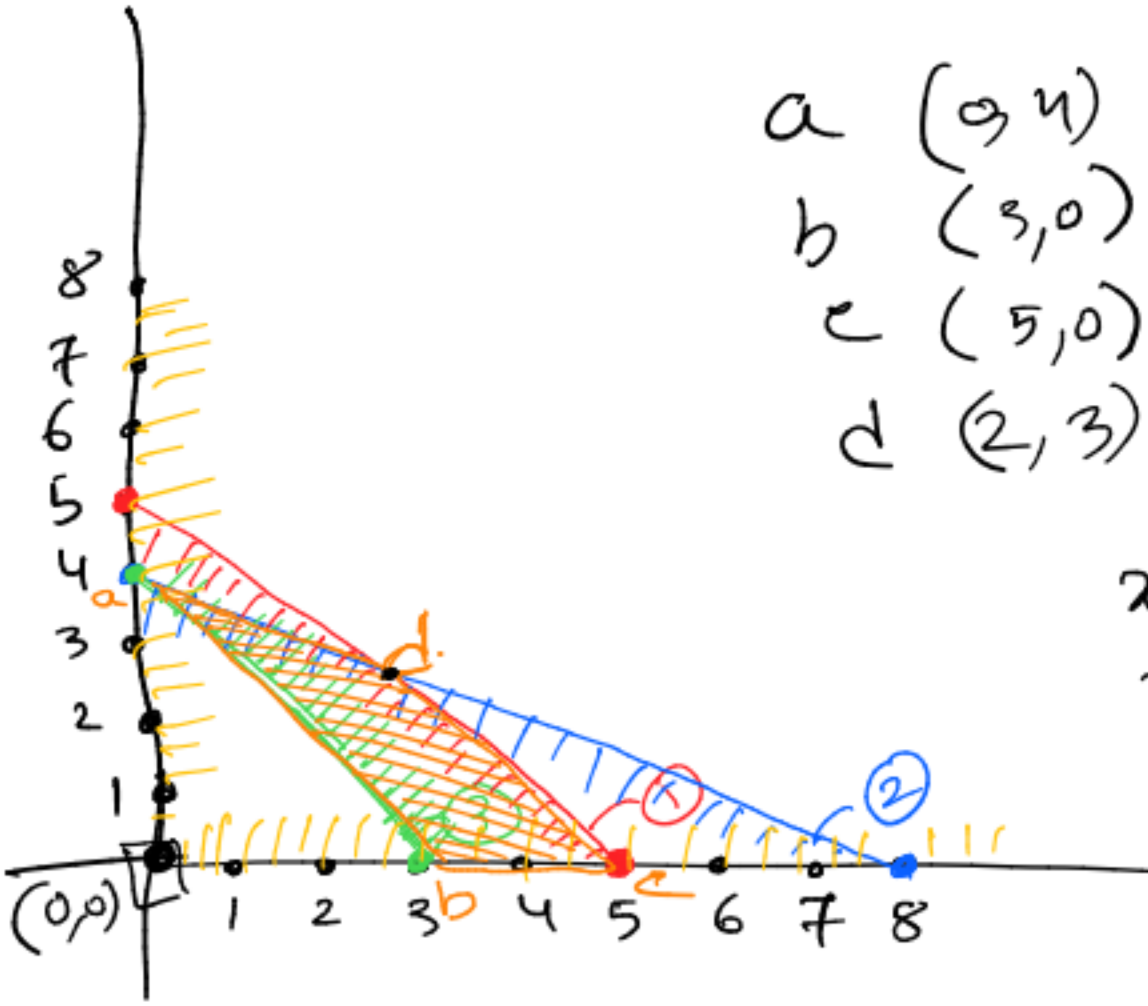
constraint 3

$$4x + 3y = 12$$

when $x = 0$, $y = \frac{12}{3} = 4$

when $y = 0$, $x = \frac{12}{4} = 3$

The two points are, $(0, 4)$ and $(3, 0)$



$x + y = 5$ — ①

$x + 2y = 8$ — ②

solving we get

②

$x = 8 - 2y$

Now from ①

$8 - 2y + y = 5$

$-y = -3$

$y = 3$

from ① we get

$x + y = 5$

$x = 5 - 3$

$= 2$

Now we comute the shaded region of the constraints.

constraint 1

$$x + y \leq 5$$

$$0 + 0 \leq 5 \quad \text{true.}$$

constraint 2

$$x+2y \leq 8$$

$0+0 \leq 8$ true.

<u>constraint 3</u>

$$4x+3y \geq 12$$

$0+0 \geq 12$ not true

<u>Finding the optimal solution</u>

Extreme point	value of the objective function $2x-y$
$a(0,4)$	$2 \cdot 0 - 4 = -4$
$b(3,0)$	$2 \cdot 3 - 0 = 6$
$c(5,0)$	$2 \cdot 5 - 0 = 10$
$d(2,3)$	$2 \cdot 2 - 3 = 1$

Therefore the minimum value of the LPP is -4

and the optimum solution is

$$x=0, \; y=4$$

solve the following LPP using graphical method.

$$\begin{cases} \text{Max } Z = 5x_1 + 7x_2 \\ \text{s.t.} \quad x_1 + x_2 \le 4 \\ 3x_1 + 8x_2 \le 24 \\ 10x_1 + 7x_2 \le 35 \\ x_1, x_2 \geq 0 \end{cases}$$

<u>solution</u>

constraint 1

$$x_1 + x_2 = 4$$

when $x_1 = 0$, $x_2 = 4$

when $x_2 = 0$, $x_1 = 4$

The two points are $(0,4)$ and $(4,0)$

<u>constraint 2</u>

$$3x_1 + 8x_2 = 24$$

when, $x_1 = 0$, $x_2 = \frac{24}{8} = 3$

when $x_2 = 0$, $x_1 = \frac{24}{3} = 8$

The two points are $(0,3)$ and $(8,0)$

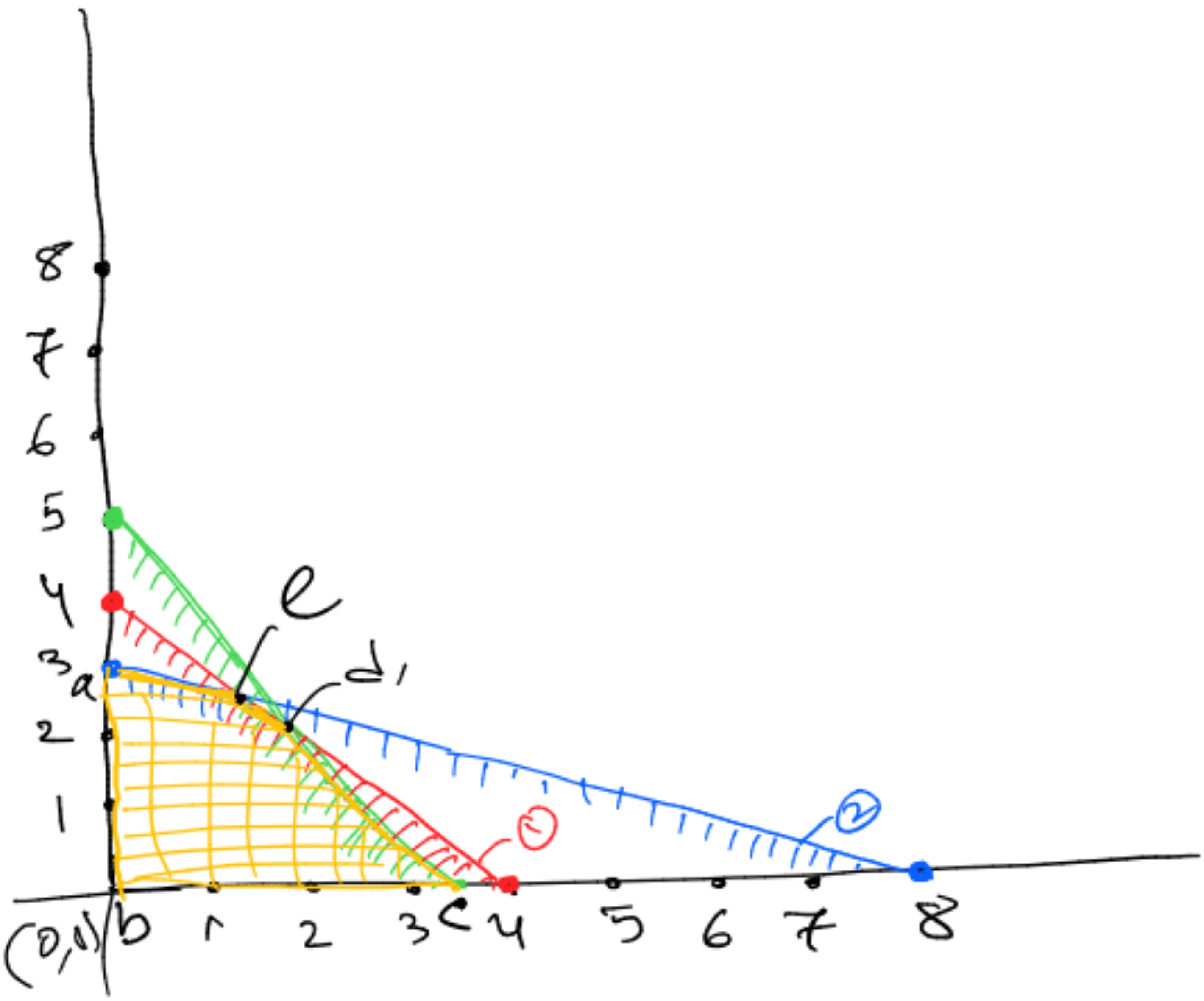
constraint 3

$$10x_1 + 7x_2 = 35$$

when, $x_1 = 0$, $x_2 = \frac{35}{7} = 5$

when, $x_2 = 0$, $x_1 = \frac{35}{10} = 3.5$

The two points are, (0,5) and (3.5,0)



constraint 1

$$x_1 + x_2 \le 4$$

$0 + 0 \le 4$ true.

Finding the optimal solution

extreme points	value of the objective function $5x_1+7x_2$
$a\ (0,3)$	$5\cdot 0+7\cdot 3 = 21$
$b\ (0,0)$	$5\cdot 0+7\cdot 0 = 0$
$c\ (3.5,0)$	$5\times 3.5+7\cdot 0 = 17.5$
$d\ (7/3,\ 5/3)$	$5\times 7/3+7\cdot\frac{5}{3} = \frac{35+35}{3} = \frac{70}{3}$ $\boxed{= 23.33}$
~~$e\ (d_1, d_2)$~~ $e\ (\frac{8}{5}\ \frac{12}{5})$	$5\cdot\frac{8}{5}+7\cdot\frac{12}{5} = \frac{40+84}{5} = \frac{124}{5} = 24.8$

For d, we need to solve equations corresponding to constraint 1 and 3

ie,

$$x_1+x_2=4 \Rightarrow \underline{x_1=4-x_2}$$

$$10x_1+7x_2=35$$

$$10\cdot(4-x_2)+7x_2=35$$

$$-10x_2+7x_2=35-40$$

$$3x_2 = 5$$

$$x_2 = 5/3$$

Hence the optimum objective value is 24.5 and optimum solution is $x_1=\frac{8}{5}, x_2=\frac{12}{5}$

$$x_1 = 4 - x_2$$
$$= 4 - 5/3$$
$$= \frac{12-5}{3} = 7/3$$

$$3x_1 + 8x_2 = 24 \Rightarrow x_1 = \frac{24 - 8x_2}{3}$$
$$10x_1 + 7x_2 = 35$$
$$10\left(8 - \frac{8}{3}x_2\right) + 7x_2 = 35$$
$$80 - \frac{80}{3}x_2 + 7x_2 = 35$$
$$\frac{-80x_2 + 21x_2}{3} = 35 - 8$$
$$\Rightarrow \frac{59x_2}{3} = 45$$
$$\Rightarrow x_2 = \frac{45 \times 3}{59} = d_1 \checkmark$$
$$x_1 = \frac{24 - 8 \cdot \frac{45\times3}{59}}{3} = d_2 \checkmark$$

This calculation is wrong as e is the intersection of constraint 1 & 2 not 1 and 3 the coordinate of e can be computed by solving the equation of constraints 1 and 2

so we have,

$$x_1 + x_2 = 4 \Rightarrow x_1 = 4 - x_2$$

$$3x_1 + 8x_2 = 24$$

$$3x_1 + 8x_2 = 24$$

$$\Rightarrow 3.(4 - x_2) + 8x_2 = 24$$

$$\Rightarrow 12 - 3x_2 + 8x_2 = 24$$

$$\Rightarrow 5x_2 = 12$$

$$\Rightarrow x_2 = 12/5$$

Then, $x_1 = 4 - x_2$

$$= 4 - \frac{12}{5}$$

$$= \frac{20 - 12}{5}$$

$$= 8/5$$

$$e = \left(\frac{8}{5}, \frac{12}{5}\right)$$