## Systems of the form: $\boxed{\dfrac{dx}{dt} = a + bx}$

$a, b > 0$

We know when $\boxed{\dfrac{dx}{dt} = a - bx}$ (with $a,b>0$) the solution is $\boxed{x = \frac{a}{b}\left(1 - e^{-bt}\right)}$. When $\boxed{\dfrac{dx}{dt} = a + bx = a - (-b)x}$, we make the transformation $b \longrightarrow -b$.

Hence, $x = \dfrac{a}{-b}\left(1 - e^{bt}\right)$

$\Rightarrow \boxed{x = \frac{a}{b}\left(e^{bt} - 1\right)}$ is the solution of $\boxed{\dfrac{dx}{dt} = a + bx}$.

Writing $x_0 = a/b$ and $\tau = 1/b$, we set

$\boxed{x = x_0\left(e^{t/\tau} - 1\right)}$ OR $\boxed{X = e^{T} - 1}$ $\left(\begin{array}{c} X = x/x_0 \\ \text{and } T = t/\tau \end{array}\right)$

### Limiting behaviour:

i.) when $t \ll \tau$, $e^{t/\tau} \simeq 1 + t/\tau$

$\therefore\ x \simeq x_0\left(\cancel{1} + \frac{t}{\tau} \cancel{-1}\right) = x_0 \frac{t}{\tau} \simeq at$.

$\Rightarrow \boxed{x = at}$ (early growth is linear).

ii.) when $t \gg \tau$, $e^{t/\tau} - 1 \simeq e^{t/\tau}$

$\therefore\ \boxed{x = x_0 e^{t/\tau}}$ (late growth is exponential)

Consider a hypothetical case when $\boxed{t<0}$.

iii) For $t \to -\infty$, $\boxed{x \to -x_0}$ (limiting terminal value)

iv) For $|t| \ll \tau$, $e^{t/\tau} \simeq 1 + t/\tau$.

$\Rightarrow \quad x \simeq x_0 t/\tau \quad \Rightarrow \quad \boxed{x \simeq at}$ (linear)

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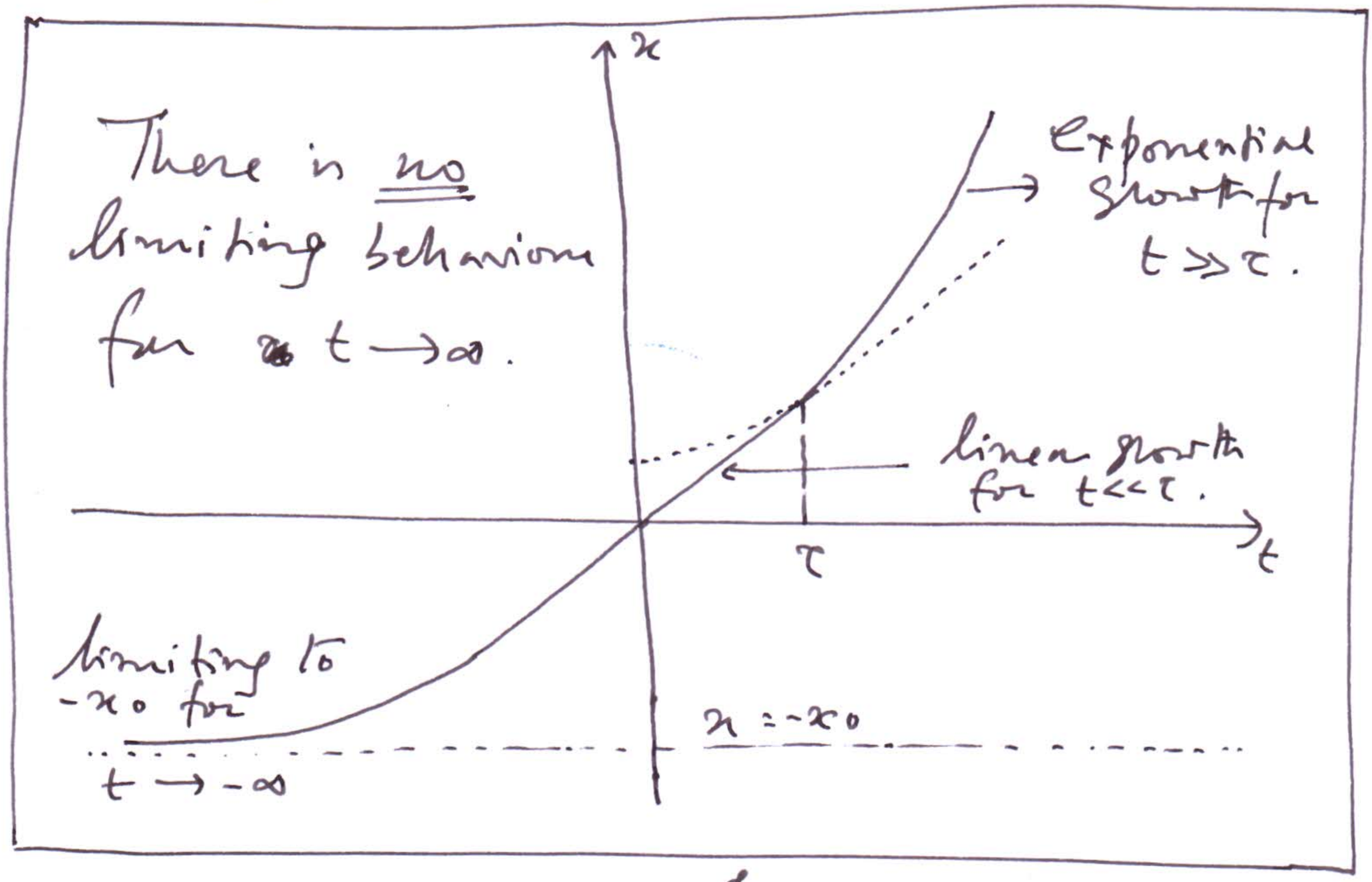
Plotting: $\boxed{x = x_0\left(e^{t/\tau} - 1\right)}$




There is an exchange of the functional behaviour from the first to the third quadrant as $\boxed{\frac{dx}{dt} = a - bx}$ goes to $\boxed{\frac{dx}{dt} = a + bx}$.

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