SC217: Electromagnetic Theory, **AUTUMN'16 MIDSEM2**

DA-IICT, B.Tech, Sem III 08:30-09:30 AM 17^{th}Oct.2016 **25 marks**

Answers to all parts of a question must be written at the same place

1. In the spherical polar system:

   (a) Evaluate $\frac{\partial \hat{r}}{\partial \theta}$, $\frac{\partial \hat{\theta}}{\partial \theta}$, $\frac{\partial \hat{\phi}}{\partial \theta}$, $\frac{\partial \hat{r}}{\partial \phi}$, $\frac{\partial \hat{\theta}}{\partial \phi}$, $\frac{\partial \hat{\phi}}{\partial \phi}$ **(3)**

   (b) Evaluate $\vec{\nabla} \cdot \hat{r}$, $\vec{\nabla} \cdot \hat{\theta}$ and $\vec{\nabla} \cdot \hat{\phi}$. **(3)**

2. Find the charge distribution for the following electric field :
   (i) $\vec{E} = x\hat{i}$ (ii) $\vec{E} = \frac{\hat{\theta}}{r}$ **(6)**

3. Four point charges q are placed at the points whose coordinates are
   $(1, 1, 1), (1, 1, -1), (1, -1, 1), (-1, 1, 1)$.
   Find the average potential due to these charges over the surface of a sphere given by
   $x^2 + y^2 + z^2 = 1$ **(6)**

4. A point charge q is placed a distance a from the center of a grounded conducting sphere of radius R, $a > R$ (q is outside the sphere).

   (a) Find the electric field over the surface of the sphere using the method of images. **(3)**

   (b) Find the total charge induced on the sphere. **(2)**

   (c) The sphere is now isolated from the ground (its connection to the ground is cut) and the point charge q is removed. Then what will be the potential of the sphere ? **(2)**

Spherical polar system

$$\vec{\nabla} F = \hat{r}\frac{\partial F}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial F}{\partial \theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial F}{\partial \phi}$$

$$\vec{\nabla} \cdot \vec{\mathrm{A}} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\vec{\nabla} \times \vec{\mathrm{A}} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathrm{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\hat{\phi}$$

Cylindrical System

$$\vec{\nabla} F = \hat{s}\frac{\partial F}{\partial s} + \frac{\hat{\phi}}{s}\frac{\partial F}{\partial \phi} + \hat{z}\frac{\partial F}{\partial z}$$

$$\vec{\nabla} \cdot \vec{\mathrm{A}} = \frac{1}{s}\frac{\partial}{\partial s}(sA_s) + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \vec{\mathrm{A}} = \left[\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{\mathrm{s}} + \left[\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_s}{\partial \phi}\right]\hat{\mathrm{z}}$$