Winter Semester

Roll No. ________________

Analysis of Multidisciplinary Problems (SC465)

Academic Year 2017-18

End-Semester Examination

Dhirubhai Ambani Institute of Information and Communication Technology, Gandhinagar

Time: 3 Hours

Total Marks: 40

All questions are compulsory. Answer all the sub-parts of a question together. Marks for each question are indicated next to it. All terms and symbols carry their standard textbook meaning. Overdots on variables imply a time derivative.

1. (a) A closed circuit has a d.c. power source of voltage V_0, a resistor R and a capacitor C. Obtain an equation for the charging of the capacitor, and plot it with clear labels.

   (b) The Duckworth–Lewis equation, used to reset targets in interrupted cricket matches, is given as $Z(u,w) = Z_0(w)\left[1 - e^{-b(w)u}\right]$, in which w is the number of wickets lost, u is the number of overs left and Z is the number of runs obtainable. Plot this equation for different values of w. Recast this equation as a first-order autonomous differential equation. [3+2=5]

2. If $x(t)$ is the war potential of a nation, and $y(t)$ is the war potential of its enemy nation, then Richardson's theory of conflict gives $\dot{x} = ky + g - \alpha x$ and $\dot{y} = lx + h - \beta y$. Giving a proper meaning of all the parameters used here, discuss the mathematical conditions and outcomes for:

   A. Mutual disarmament without grievance. B. Mutual disarmament with grievance. C. Unilateral disarmament. D. Arms race (with a plot of x versus y). [1+1+1+2=5]

3. If $x(t)$ is the number of cells in a tumour, then the Gompertz law of tumour growth in an autonomous form is $\dot{x} = -ax\ln(bx)$, in which $a, b > 0$.

   A. Reasoning clearly, draw the phase plot of the equation. B. With scaling and substitution, obtain the integral solution of $x \equiv x(t)$, under the initial condition $t = 0$, $x = x_0$. Also indicate the limit of x when $t \longrightarrow \infty$. [2+3=5] $x = b$

4. Use proper mathematical and practical arguments to establish the following:

   (a) The general coupled equations for the co-existence of a prey species and a predator species with populations x and y, respectively, as per Volterra's predator-prey model. State the optimal equilibrium solutions for both.

   (b) The coupled equations representing the struggle for existence between two similar species with populations x and y, according to the principle of competitive exclusion. Express the mathematical conditions for intense competition and no competition. State the optimal equilibrium solutions for both. [2+3=5] 4

5. (a) Discuss: A. Clustering coefficient in small-world networks. B. Power-law degree distributions.

   (b) The radioactive decay of carbon-14 follows the equation, $\dot{N} = -\lambda N$, with $\lambda > 0$. Using the concept of half life, mathematically argue how this equation is used to measure the age of ancient civilisations. [2(1+1)+3=5]

6. On a field of battle, an x-force and a y-force are engaged in isolated combat. Use Lanchester's combat models.

   A. Obtain the equation for a conventional-conventional combat. Solve this equation to get Lanchester's square law, and state the implication of this law. Provide a plot of x versus y with clear labels. B. Obtain the equation for a conventional-guerilla combat. Solve this equation and provide a plot of x versus y. [3.5+1.5=5]

7. (a) Starting with the differential equation of a damped oscillator, obtain the eigenvalues for the overdamped, underdamped and critically damped conditions.

   (b) Juliet and Romeo are equally cautious about their feelings for each other. Write a set of coupled equation for their feelings, using a "cautiousness" parameter and a "responsiveness" parameter. Show mathematically how their love may either flourish or die out. [2+3=5]

8. A few infected persons introduce an infectious disease in a large population. The disease has a short incubation period, and recovered individuals gain permanent immunity. There are three classes of population — the infected class x, the susceptible class y, and the recovered class z.

   A. Write the time-rate equations of x, y and z following clear practical rules. B. Solve for $x \equiv x(y)$ under the initial condition $t = 0$, $x = x_0$, $y = y_0$ and $z = 0$. Identify the threshold of y for an epidemic to break out. C. Obtain an approximate formula for the number of susceptibles who get infected, if the initial number of susceptibles is slightly higher than the threshold. [1.5+1.5+2=5] 3

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Roll No. 201501157

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Analysis of Multidisciplinary Problems (SC465)

Second In-Semester Examination

Dhirubhai Ambani Institute of Information and Communication Technology

Time: 1 Hour 30 Minutes

Total Marks: 20

All questions are compulsory. Answer all the sub-parts of a question together. Marks for each question are indicated next to it. All terms and symbols carry their standard textbook meaning.

Note: $\dot{x} \equiv dx/dt$.

1. A. State the three laws of social dynamics. B. Discuss how the Malthusian law and the logistic equation follow from the laws of social dynamics. [1.5+1.5=3]

2. A object falling through a fluid (like air or water) experiences a drag force, $D = kv^{\gamma}$, in which v is the velocity and γ is the drag coefficient.

   A. Write the formula of the Reynold's number, and discuss the kinematic viscosity for both air and water. B. Discuss how γ is determined by the value of the Reynold's number. [1.5+1.5=3]

3. Perturb the fixed point of a first-order autonomous dynamical system $\dot{x} = f(x)$.

   A. Carry out a linear stability analysis and discuss the conditions for stability and instability. Show that convergence can only happen exponentially. B. Show that under the critical condition, the convergence is by a slow power-law. [2.5+1.5=4]

4. A parachutist undergoes free fall through air.

   A. Write a differential equation for the free-fall motion. B. Rescale it to obtain the parameter-free form $(dX/dT) = 1 - X^2$. C. Integrate this equation with the initial condition $T = X = 0$. D. Plot the integral solution, showing clearly its short and long-time features. [1+1+2+1=5]

5. The item response function is given as

$$P(\theta) = c + \frac{1-c}{1+e^{-(\theta-b)/w}},$$

in which θ is the ability, P is the performance index, c is the probability that a candidate with low ability will respond correctly to an item, b is the item difficulty parameter, and w is the item discrimination parameter.

   A. With a suitable transformation, obtain the logistic equation from the $P \equiv P(\theta)$ equation. B. Reasoning clearly, plot the logistic curve, showing the early and late growths. C. Mathematically explain the meaning of the parameters b and w. [2+1.5+1.5=5]

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First In-Semester Examination

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Time: 1 Hour 30 Minutes

Total Marks: 20

All questions are compulsory. Answer all the sub-parts of a question together. Marks for each question are indicated next to it. All terms and symbols carry their standard textbook meaning.

Note: $\dot{x} \equiv \mathrm{d}x/\mathrm{d}t$.

1. Consider the equation $\dot{x} = a - bx$, in which $a, b > 0$.

   A. Rescale and integrate it with the initial condition $x = 0$ at $t = 0$. B. Show when $t \ll b^{-1}$, x varies linearly with t. C. Plot the integral solution for all t with clear labels. [2+1+1=4]

2. Consider an object falling through a very long liquid column, with a velocity v at a depth z.

   A. Mentioning all the forces acting on the object, set down the v–t differential equation. Obtain the terminal velocity from this equation and a natural scale of t. B. Write the integral solution of the v–t equation. For both the small and large limits of t, indicate the approximate dependence of v on t. Integrate to get the approximate solutions of $z(t)$ in both limits. [2.5+2.5=5]

3. Rocks exhibit both elastic and viscous properties under the weight of earth matter.

   A. Write a relation between elastic stress and strain. B. Obtain a similar relation for the viscous effect. C. Taking both effects together, express the differential equation of the viscoelastic deformation of rocks. From the equation get the limiting value of the strain. [0.5+1.5+1=3]

4. Start with $F = ma$.

   A. Derive the formula for the conservation of total energy. B. Argue that conservation also allows time reversal.

   [2+1=3]

5. Lead ore contains radioactive lead-210 (Pb-210) which, with a half-life of 22 years, decays to non-radioactive lead-206 (Pb-206). On the other hand, Pb-210 is replenished by the radioactive decay of radium-226 (Ra-226), which has a half-life of 1600 years. For time scales that are much less than 1600 years, show how the decay rate of Pb-210 is approximated by the linear differential equation $\dot{x} = r - \lambda x$, in which r and λ are constants. [2.5]

6. Consider the equation $\dot{x} = ax - bx^{\alpha+1}$, in which $a, b > 0$ and $\alpha \geq 2$. Use a transformation to derive the standard logistic equation. From it obtain the carrying capacity of x. [2.5]