CT303 - Digital Communications
Autumn 2020

Lab 2: Random Processes & Stationarity — Submit by: 13/9/20 12 noon.

1. Let $X(n)$ be a random process. Let us verify the wide-sense stationarity of this process computationally. Generate N realizations/sample functions of this random process (use `numpy.random` library), and compute (a) the mean $m_X(n)$, and (b) autocorrelation function $R_X(n_1, n_2)$. Verify if the following stochastic processes are wide-sense stationary or not, and compare them against theoretically obtained mean and autocorrelation. Plot the estimated density functions for a few random variables $X(k)$ using the function `numpy.histogram`.
   (a) $X(n) = \cos(0.2\pi n + \theta), \theta \sim U[-\pi, \pi], n \in [0, 9]$.
   (b) $X(t) = A\cos(0.25\pi n), A \sim U[-5, 5], n \in [0, 7]$.
   (c) $X(n) = A(n)$, where $A(n) \sim \mathcal{N}(0, 1)$[1] are independent random variables.

2. In the file `Gandhinagar_RainfallData.xls`, average rainfall for every month of the year between 1901 and 2001 is available. Treating the average rainfall every month in Gandhinagar as a Stochastic process, estimate the mean and covariance and conclude whether the stochastic process is stationary (either wide-sense or strict-sense). You might have to learn how to import data from excel file into Python for this.

3. A matrix $A \in \mathbb{R}^{n\times n}$ is said to be Symmetric Positive Definite(SPD) if $A = A^T$ and $y^T Ay > 0, \forall y \in \mathbb{R}^n, y \neq 0$, and is said to be Symmetric Positive Semi-definite if $A = A^T$ and $y^T Ay \geq 0, \forall y \in \mathbb{R}^n$. Given n random variables $X_{t_i}, i = 1, \ldots, n$ (belonging to a random process), the autocorrelation and autocovariance function of the random vector $x = (X_{t_i})_{i=1,\ldots,n}$ can be written as a matrix $R = \mathbb{E}[xx^T]$ and $C = \mathbb{E}[(x - m)(x - m)^T]$, where $m \in \mathbb{R}^n$ is the vector of means, respectively.
   (a) Analytically show that the two matrices are symmetric positive semi-definite. Verify the same computationally for the two examples from the previous question[2].
   (b) Try to observe an additional pattern in the autocorrelation/autocovariance matrix of a wide-sense stationary process. What is such a matrix called?

[1] $\mathcal{N}(0, 1)$ denotes a normal distribution with zero mean and unit variance.
[2] An equivalent definition for a symmetric matrix to be positive definite (semi-definite) is that all its eigenvalues are positive (non-negative).