Theory of games

- Two person zero sum strictly determined game.
- mixed strategy.

Today

How to solve $2 \times n$ game (or $m \times 2$)

Let the pay-off matrix be given as

		B				
		B_1	B_2	B_3	B_4	B_5
A	A_1	1	4	-1	-5	6
	A_2	3	2	6	4	-3

This is a 2×5 game.

		y_1	y_2	y_3	y_4	y_5
		B_1	B_2	B_3	B_4	B_5
x	A_1	1	4	-1	-5	6
$1-x$	A_2	3	2	6	4	-3

This game does not contain a saddle point.

Let the optimal strategy for player A is $X = (x, 1-x)$, $0 \leq x \leq 1$

and the optimal strategy for player B is $Y = (y_1, y_2, y_3, y_4, y_5)$

where, $\sum_{i=1}^{5} y_i = 1$ and $0 \leq y_i \leq 1$

Expected pay-off to player-A where player B adopts B_1 is $x + 3(1-x)$

$$= x + 3 - 3x$$
$$= 3 - 2x$$

Expected pay-off to player-A when player-B adopts B_2 is $4x + 2(1-x)$

$$= 4x + 2 - 2x$$
$$= 2x + 2$$

Expected payoff ~~to~~ player A when player B adopts B_3 is $-1 \cdot x + 6(1-x)$

$$= -x + 6 - 6x$$
$$= 6 - 7x$$

Expected pay-off to player A when player B adopts B_4 is $-5x+4(1-x)$

$$= -5x+4-4x$$
$$= 4-9x.$$

Expected pay-off to player A when player B adopts B_5 is $6x-3(1-x)$

$$= 6x-3+3x$$
$$= 9x-3$$

Let these expected pay-offs are

$E_1(x) = 3-2x$

$E_2(x) = 2x+2$

$E_3(x) = 6-7x$

$E_4(x) = 4-9x$

$E_5(x) = 9x-3$

we now apply the graphical method

for each expected pay-off $E_i(x)$ find two point on $E_i(x)$ and joining them to get a line segment.

Now our goal is to find x such that $0 \leq x \leq 1$ and maximizing $\min_i E_i(x) = u$

This solved using graphical method.

Draw two vertical lines and Scale the two lines uniformly [Scaled between $(-r, r)$] and then draw the line segments of each $E_i(x)$ considering the distance between the two vertical lines as 1.

~~Two points on $E_1(x) = 3-2x$~~ are

~~$\therefore$ $y = 3-2x$.~~

We take two points when
$x = 0$ and $x = 1$

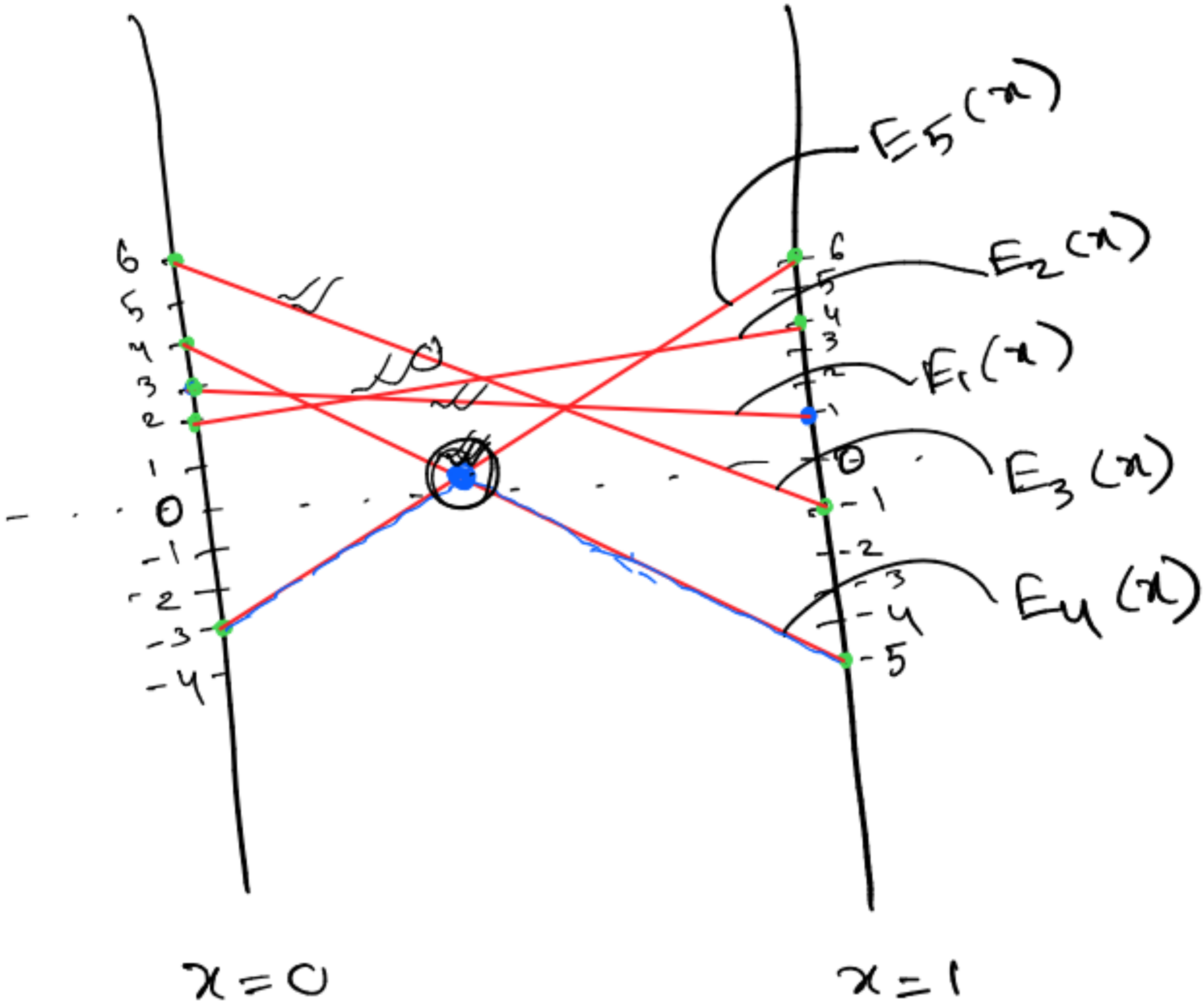
Two points on $E_1(x) = 3-2x$ are
$(0,3)$ and $(1,1)$

Two points on $E_2(x) = 2x+2$ are
$(0,2)$ and $(1,4)$

Two points on $E_3(x) = 6-7x$ are
$(0,6)$ and $(1,-1)$

Two points on $E_4(x) = 4-9x$ are
$(0,4)$ and $(1,-5)$

Two points on $E_5(x) = 9x-3$ are
$(0,-3)$ and $(1,6)$



Thus to find

$$\max_{Y}\left[\min_{Y} E(x, Y)\right]$$

we have a x_0 such that

$$E_4(x_0) = E_5(x_0)$$

$$\Rightarrow \quad 4 - 9x_0 = 9x_0 - 3$$

$$\Rightarrow \quad -18x_0 = -7$$

$$\Rightarrow \quad x_0 = \frac{7}{18}$$

Hence, the optimal strategy for player A is

$$\boxed{X = \left(\frac{7}{18}, \frac{11}{18}\right)}$$

and the value of the game is

$$v = 4 - 9x_0$$
$$= 4 - 9 \cdot \frac{7}{18}$$
$$= 4 - \frac{7}{2}$$
$$= \frac{8-7}{2}$$
$$= \frac{1}{2}$$

Now we need to find the optimal strategy for player-B.

Note that here only y_4 and y_5 are positive and y_1, y_2, y_3 are 0, as they are not controlling (no effect) on the game now.

Hence $Y = (0, 0, 0, y_4, y_5)$ is the optimal strategy for player B.

we need to find y_4 and y_5 now.

since y_1, y_2, y_3 is zero we can consider the following 2x2 game with A_1, A_2 and B_4 B_5

	B_4	B_5
A_1	-5	6
A_2	4	-3

Now we have arrived at a situation where we need to solve a 2x2 game.

$$\begin{array}{cc|cc} & & y & 1-y \\ & & B_4 & B_5 \\ \hline x & A_1 & -5 & 6 \\ 1-x & A_2 & 4 & -3 \end{array}$$

$$\boxed{\begin{aligned} y_4 &= y \\ y_5 &= 1-y. \end{aligned}}$$

$$E(X,Y) = E(x,y) = -5xy + 6x(1-y) + 4(1-x)y - 3(1-x)(1-y)$$

$$= -5xy + 6x - 6xy + 4y - 4xy - 3 + 3x + 3y - 3xy$$

$$= (-5-6-4-3)xy + (6+3)x + (4+3)y - 3$$

$$= -18xy + 9x + 7y - 3$$

$$\boxed{C + D(x-K)(y-l)}$$

$$= C + D(xy - Ky - lx + Kl)$$

(1) $D = -18$

(2) $-DK = 7$

$\Rightarrow K = \frac{7}{18}$

(3) $-Dl = 9$

$\Rightarrow l = \frac{9}{-(-18)} = \frac{1}{2}$

④ $C + D\kappa l = -3$

$$\Rightarrow C = -3 - D\kappa l$$
$$= -3 - (-18)\left(\frac{7}{18}\right)\cdot\left(\frac{1}{2}\right)$$
$$= -3 + 7\cdot\frac{1}{2}$$
$$= \frac{7}{2} - 3$$
$$= \frac{7-6}{2} = \frac{1}{2}$$

value of the game is $C = \frac{1}{2}$

$y = \frac{1}{2}$ $\quad \boxed{x = 7/18}$

Therefore, the optimal strategy for player-B is

$$\left(0, 0, 0, \frac{1}{2}, \frac{1}{2}\right)$$

optimal strategy for player-A is

$$\left(\frac{7}{18}, \frac{11}{18}\right)$$

Solving m×2 game

	B_1	B_2
A_1	0	-2
A_1	7	-1
A_3	-1	4
A_4	-2	6
A_5	5	-3

H.W.

	B_1	B_2	B_3	B_4
A_1	6	5	2	3
A_2	1	2	6	3

H.W