The graphical method

Two steps

1. Finding feasible region
2. Finding optimal solution from the feasible region

extreme/corner point method

Iso profit/ Iso cost method.

Example: Max $Z = 3x_1 + 4x_2$

s.t. $x_1 - x_2 \geq 0$ —— (1)

$-x_1 + 3x_2 \leq 3$ —— (2)

$x_1, x_2 \geq 0$

If we take it as minimize

Solution:

constraint 1

$x_1 - x_2 = 0$

when $x_1 = 0$, $x_2 = 0$

when $x_1 = 1$, $x_2 = 1$

$(0, 0)$ and $(1, 1)$

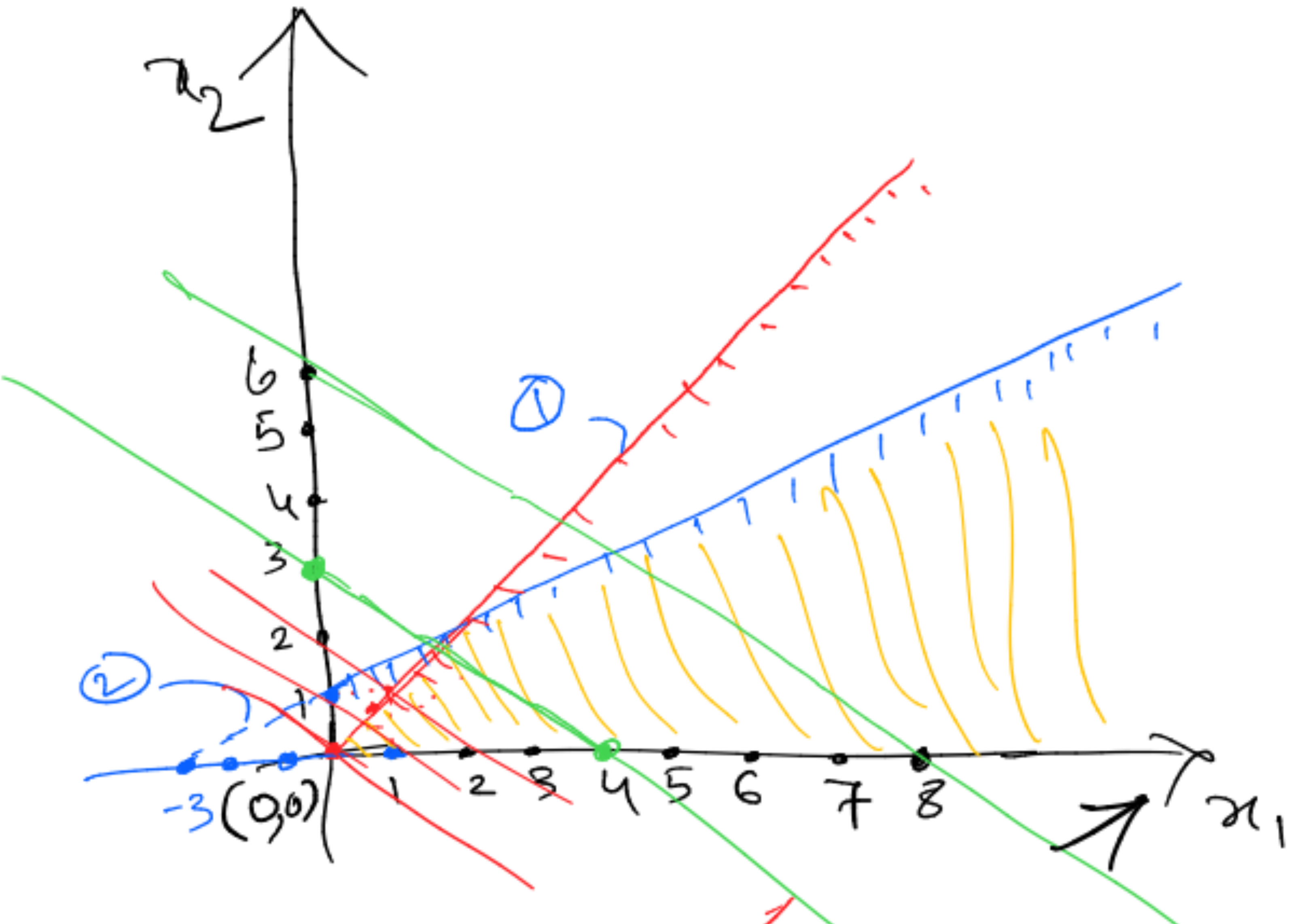
constraint 2

$-x_1 + 3x_2 = 3$

when $x_1 = 0$, $x_2 = 1$

when $x_2 = 0$, $x_1 = -3$

$(0, 1)$ and $(-3, 0)$



$Z=12$

$x_1 - x_2 \geq 0$

$0 \geq 0$

$-x_1 + 3x_2 \leq 3$

$0 \leq 3$

direction will be changed for minimization objective.

The feasible region is not a closed polygon.

Draw the profit line at $Z=12$

$3x_1 + 4x_2 = 12$

when $x_1 = 0$, $x_2 = 3$ $(0,3)$

when $x_2 = 0$, $x_1 = 4$ $(4,0)$

Since it is a maximization problem the profit line can be moved far away indefinitely from the origin and parallel to the profit line $3x_1 + 4x_2 = 12$

Therefore no finite maximum value of Z can be possible.

In this case we get an unbounded solution.

Feasible region is unbounded solution is unbounded

For minimization objective.

Feasible region is unbounded solution is unique

Example

$$\max Z = 2x_1 - x_2$$

$$\text{s.t.} \quad x_1 - x_2 \leq 1$$

$$x_1 \leq 3$$

$$x_1, x_2, \geq 0$$

Solution

Constraint 1

$$x_1 - x_2 = 1$$

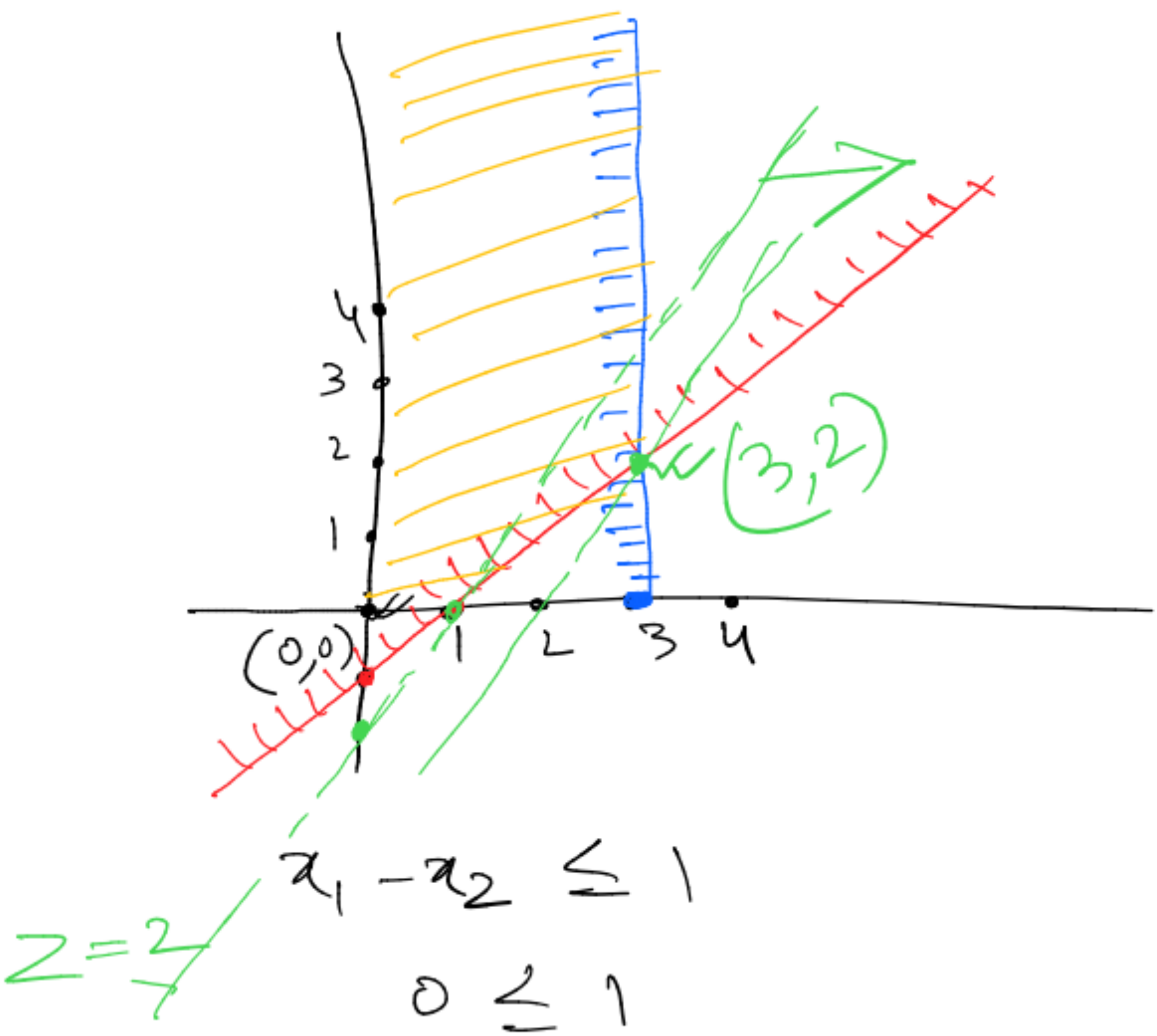
when, $x_1 = 0$, $x_2 = -1$

when $x_2 = 0$, $x_1 = 1$

$(0, -1)$ and $(1, 0)$

Constraint 2

$$x_1 = 3$$



$$x_1 - x_2 \le 1$$

$$0 \le 1$$

$$x_1 \le 3$$

$$0 \le 3$$

The objective function line.

$$2x_1 - x_2 = 2$$

when $x_1 = 0$, $x_2 = -2$

when $x_2 = 0$, $x_1 = 1$

$(0,-2)$ and $(1,0)$

The line $z = 4$ gives the optimum value and the optimum solution can be found by solving the lines for constraint 1 and 2

$$x_1 - x_2 = 1$$

$$x_1 = 3$$

$$-x_2 = 1 - x_1$$

$$= 1 - 3$$

$$-x_2 = -2$$

$$x_2 = 2$$

$$x_1 = 3, \ x_2 = 2$$

Feasible region unbounded
solution is unique

Example: $\max Z = -x_1 + 3x_2$

$$\text{s.t.} \quad x_1 - x_2 \geq -1$$

$$-\frac{1}{2}x_1 + \frac{3}{2}x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

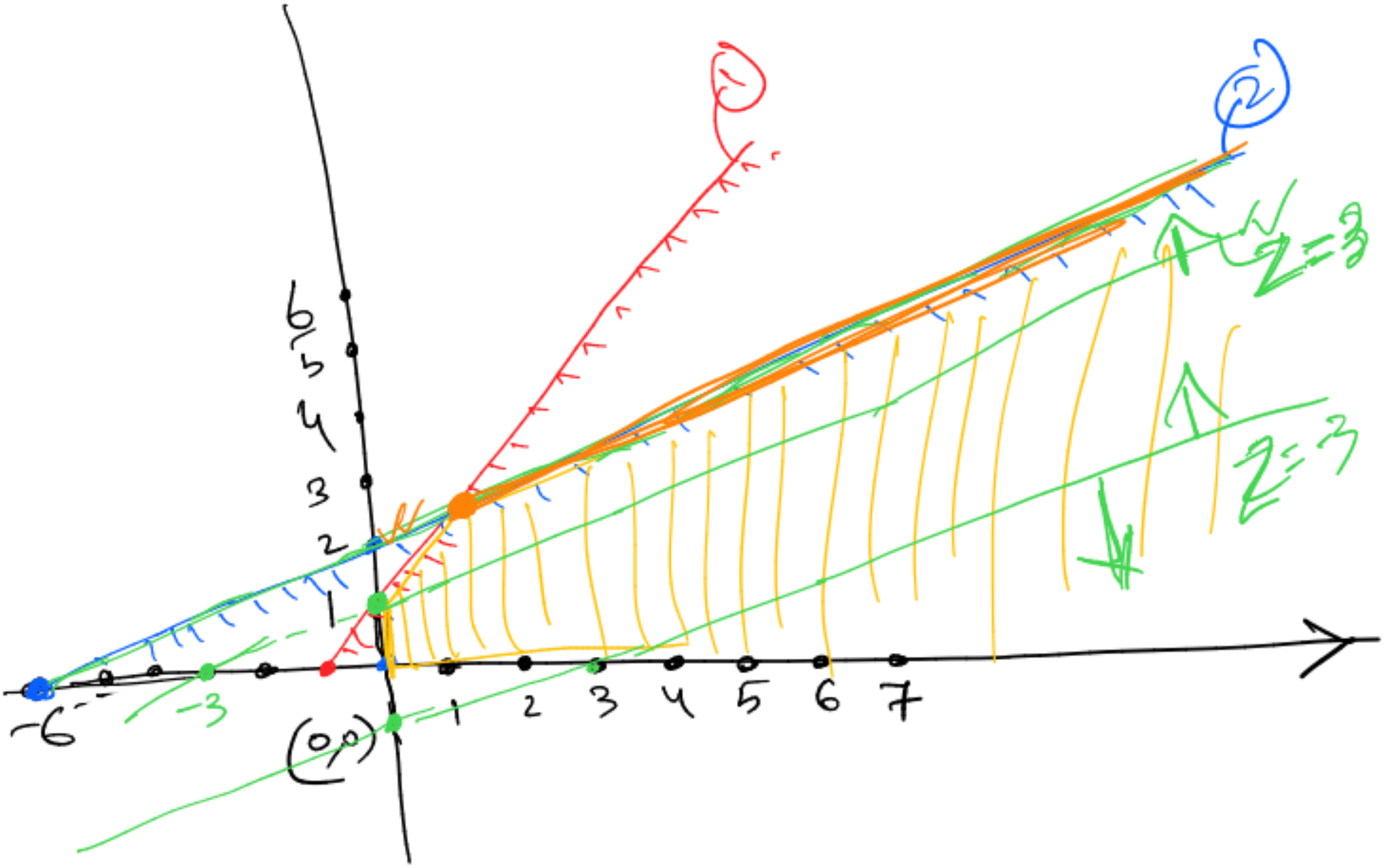
$$\boxed{\begin{array}{l}\text{Feasible region is unbounded} \\ \text{Infinite number of solutions}\end{array}}$$

For constraint 1

$(0,1)$ and $(-1,0)$

For constraint 2

$(0,2)$ $(-6,0)$



$-x_1 + 3x_2 = 3$

$(0,1) \quad (-3,0)$

$-x_1 + 3x_2 = -3$

$(0,-1) \quad (3,0)$

$\boxed{Z = 6}$

obtimum value = 6

Example

$$\max Z = 2x_1 - 3x_2$$

s.t. $x_1 + x_2 \le 2$

$2x_1 + 2x_2 \ge 8$

$x_1, x_2 \ge 0$

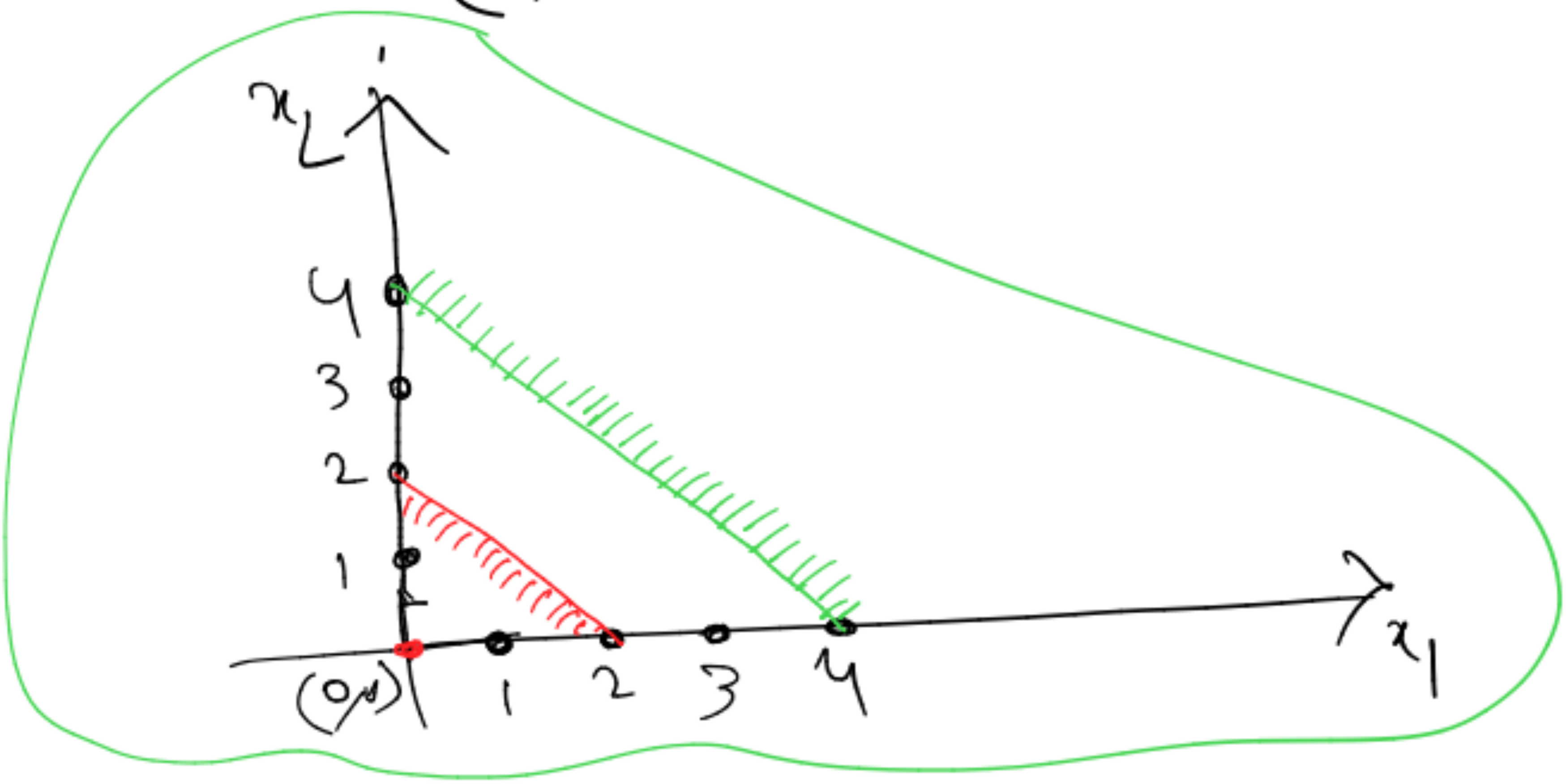
Solution

constraint 1

(0, 2) and (2, 0)

constraint 2

(0, 4) and (4, 0)



$0 + 0 \le 2$ true

$2x_1 + 2x_2 \ge 8$

$0 + 0 \ge 8$ not true.

The feasible region is an empty set
No point satisfies all the constraints simultaniously.
So no feasible solution is possible and hence no optimum solution.

$$\boxed{\text{No feasible region} \quad \text{infeasible solution}}$$

Example:

$$\max Z = x_1 + x_2$$

$$\text{s.t. } x_1 - x_2 \geq 0$$

$$2x_1 - x_2 \leq -2$$

$$x_1, x_2 \geq 0$$

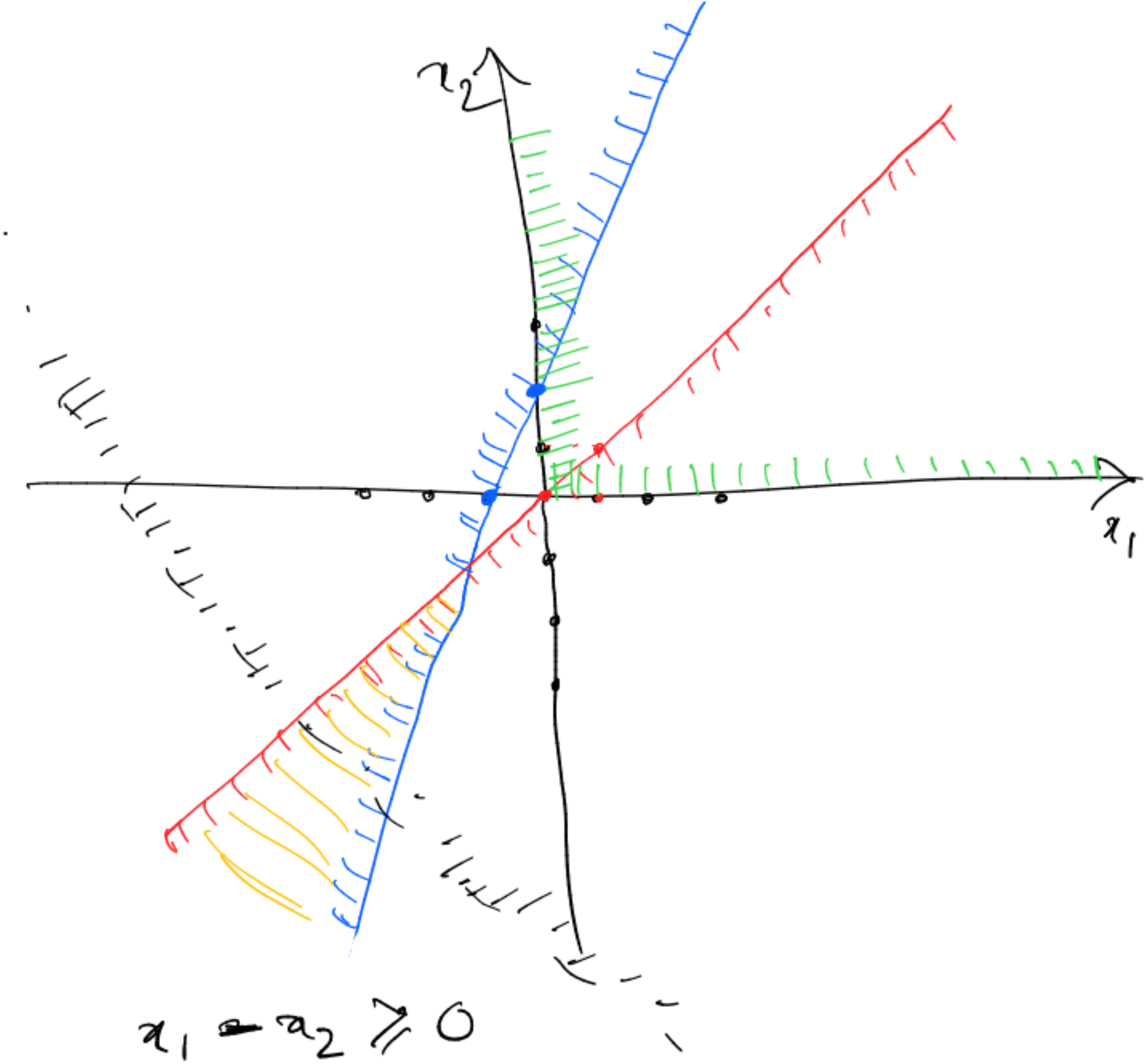
Solution

constraint 1

(0, 0) and (1, 1)

constraint 2

(0, 2) and (-1, 0)



$x_1 - x_2 \geq 0$

$0 - 0 \geq 0$ true or $1 - 1 \geq 0$ true.

$2x_1 - x_2 \leq -2$

$2 \cdot 1 - 1 \leq -2$

$1 \leq -2$ not true.

Feasible region formed

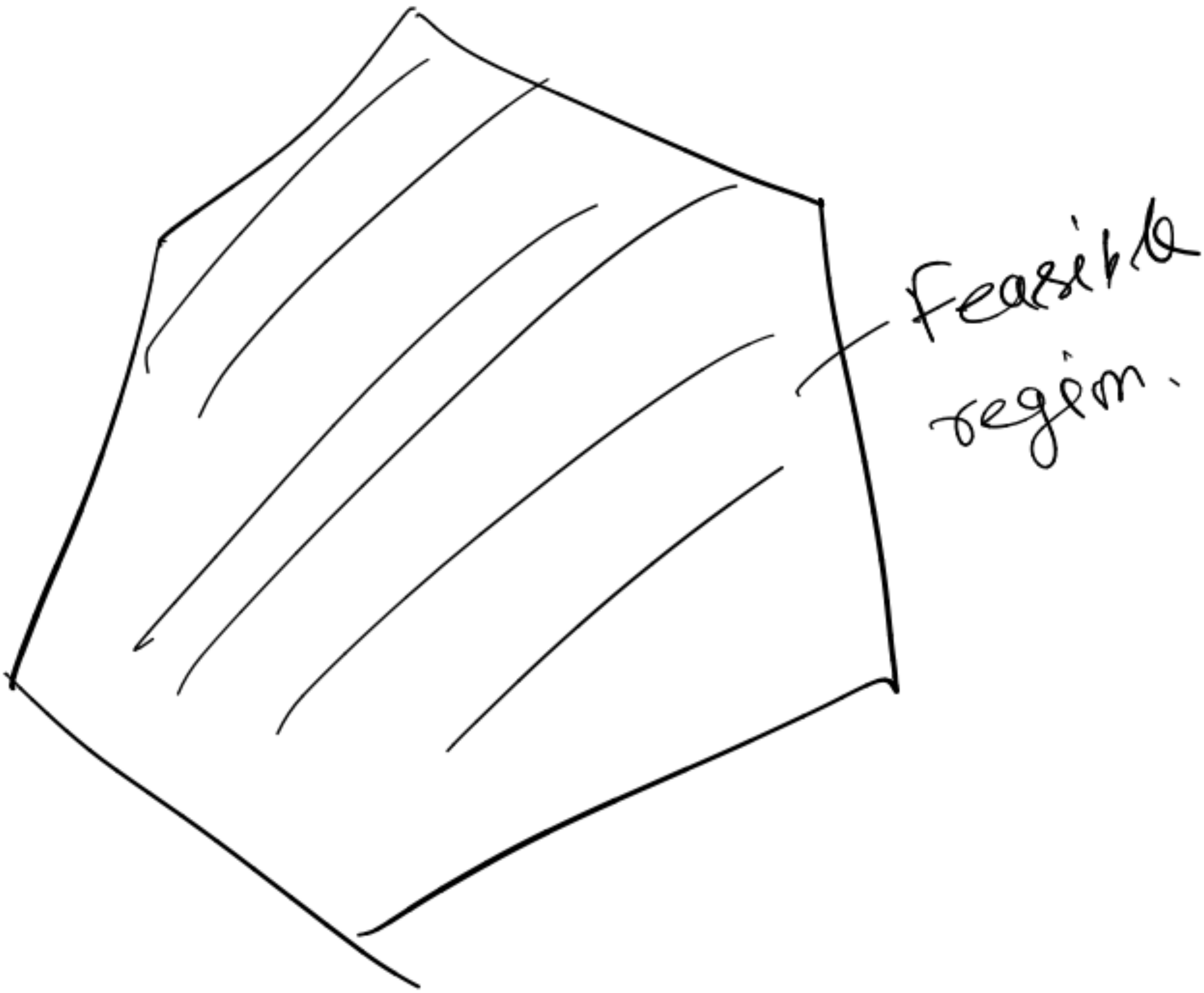
But the feasible region is in the third quadrant

All the constraints are satisfied

But the non-negativity restriction is not satisfied.

Feasible region is unbounded

infeasible solution


feasible
region.