CT303 - Digital Communications
Autumn 2020

MidSem 1 — 29 October 2020
Duration: 75 mins — Total Marks 35

Instructions

1.
2. Total marks for this section: 20. Total questions in this section: 10.
3. You get 2 marks for every correct answer, and negative 1 mark for an incorrect answer.
4. Fill in the blanks with numbers with only as many decimals as required, for example: if your answer is 0.25 do not enter 0.250, or if your answer is 10, do not enter 10.0. If your answer is ∞, enter only `inf` (small letters) instead of Infinity or Infinite.

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1 Fill in the blanks

1. The variance of Additive white Gaussian noise with Power spectral density 1 W/Hz is _____.

Answer: inf

2. Let

$$x(t) = \cos(t),\ t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$
$$= 0,\ else$$

be an energy signal. The Energy spectral density of $x(t)$ at $f = 0$ is _____.

Answer: 4.

3. Let x represent a periodic square wave with period 0.5 secs, with amplitudes alternating between $+5$ and -5 V. The Power Spectral density of x at $f = 11Hz$ is _____.

Answer: 0

4. Let

$$x(t) = \cos\left(\frac{\pi}{2}t\right),\ t \in [-1, 1]$$
$$= 0,\ else.$$

The range of values of τ for which the autocorrelation function of x, R_x is non-zero is from $\tau =$ _____ to $\tau =$ _____.

Answer: $-2, 2$

5. Assume that a Binary PCM scheme uses the same symbol waveforms to transmit bit 0 and bit 1, i.e., s_1 and s_2. Given that $P(s_1) = P(s_2)$, the probability of bit error of this scheme is _____.

Answer: 0.5.

6. The minimum number of bits per sample needed for a uniform quantizer to achieve an SNR equal to 75 is _____

Answer: 3

7. Let $X(t)$, a Wide-sense stationary random process be given as an input to an LTI system with impulse response $h(t) = 1$, for $0 \leq t \leq 3$, and 0 elsewhere. Given that $R_{XX}(0) = 5$ and variance of $X(t)$, σ_X^2 is 1, the mean of the output stochastic process of the LTI system is _____.

Answer: 6

8. A binary PCM signaling scheme consists of transmitting $p(t) = 1, 0 \leq t \leq 1$ and $q(t) = 1, 0 \leq t \leq 0.5$ and $q(t) = -1, 0.5 < t \leq 1$. Assuming equiprobable symbols, the bit-error probability $P_B = Q\left(\sqrt{\frac{x}{N_0}}\right)$, where x = _____.

Answer: 1

9. Let $s_1(t) = t,\ 0 \leq t \leq 3$ and $s_2(t) = 1, 0 \leq t \leq 3$. be the two symbol waveforms to be used for Binary PCM. With a single matched filter as discussed in class, the maximum SNR at $t = 3$ at the output of this matched filter is $\frac{x}{N_0}$, where x is _____.

Answer: 6

10. Let $s_1(t) = 2,\ 0 \leq t \leq 1$ and $s_2(t) = 0$ be the two symbol waveforms to be used in a binary PCM system. Given that both symbols are equi-probable, the MAP based optimal decision threshold for the sampled matched filter output is _____.

Answer: 2

2 Multiple Choice Questions

Instructions:

(a) Total marks for this section: 15. Total questions in this section: 5.

(b) None, some or all of the given choices may be correct.

(c) You will get 3 marks for a question only if you get all answer correct, and for every incorrect choice you make, you get a negative 1 mark.

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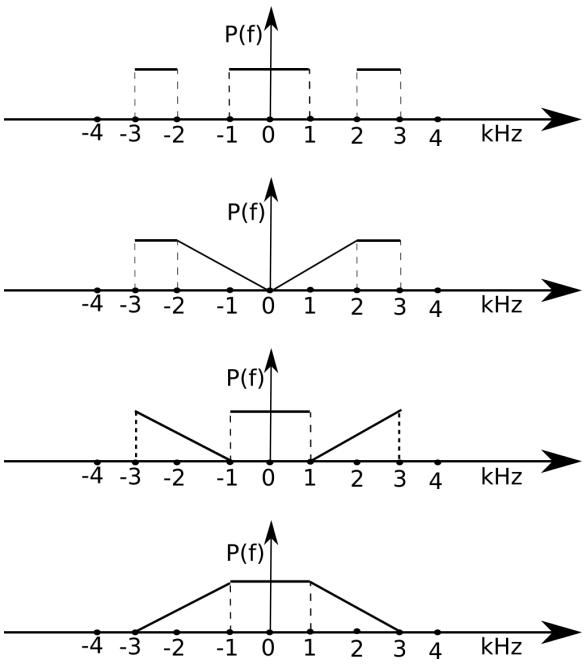
11. Let us denote a pulse waveform as $p(t)$ and its Fourier transform as $P(f)$. Examples of Fourier transforms of a few pulses are shown below. Given that we want to transmit symbols at a rate 4000 symbols/sec, choose the ones which will not cause any Inter-Symbol Intereference. Assume the peak value in the figures below to be $1/4000$.

(a)

(b)

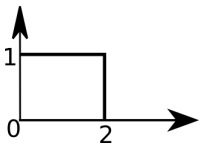


(c)

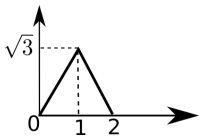
(d)

Answer: (a),(c),(d)

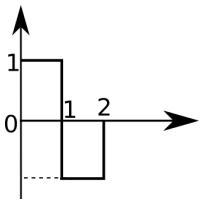
12. Let a Binary PCM signaling scheme transmit the pulses $p(t)$ and $-p(t)$, where $p(t)$ could be some of the waveforms given below. With respect to minimizing the bit-error probability, which of the following pulse waveforms would you choose? Note that for all figures given in this question, the x-axis is the time axis.



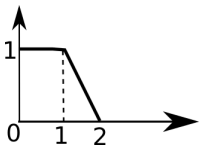
(a)



(b)



(c)



(d)

Answer: (a),(b),(c)

13. A sound signal is sampled at $20KHz$ and quantized using 16 bits per sample. With Binary PCM using Raised Cosine waveforms of roll-off factor $r = 0.25$, what is the minimum channel bandwidth required to transmit the signal in KHz?

(a) 160

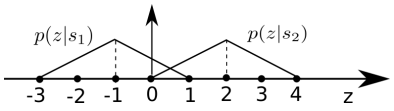
(b) 200

(c) 320

(d) 360

Answer: (b)

14. The likelihood of the sampled output of the matched filter z given that symbols s_1 and s_2 were transmitted are shown in the figure below. What is the probability of bit error



given that both symbols are equally likely to be transmitted.

(a) $\frac{1}{8}$

(b) $\frac{1}{16}$

(c) $\frac{1}{10}$

(d) $\frac{1}{32}$

Answer: (d)

15. Which of the following is a better practical solution for converting analog to discrete signals?

(a) Pass the signal through a low order analog anti-aliasing filter, over sample with a rate that is 10 times the Nyquist sampling rate, pass the discrete sequence through a high order digital anti-aliasing filter, and downsample the resultant discrete sequence.

(b) Over sample the analog signal with a rate that is 10 times the Nyquist sampling rate

(c) Pass the signal through a high order analog anti-aliasing filter and them sample using the Nyquist sampling rate.

(d) Pass the signal through a low order analog anti-aliasing filter, over sample with a rate that is 10 times the Nyquist sampling rate, and downsample the resultant discrete sequence.

Answer: (a)