Tute-5 Sol^n.

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d → dominant gene.
r → recessive gene.
dd → purely dominent.
rr → purely recessive.
rd → Hybrid.

NOTE: The purely dominant and hybrid individuals Are alike in appearance.

- Children receive 1 gene from each parent with respect to perticular trait; 2 Hybrid Parents Have a total 4 children.

E: 3 out of 4 children have the outward appearance of the dominant gene.

P(E) = ?

Here it is given that both the parents are hybrid

Parent 1 → rd
Parent 2 → rd.

So, their individual child will have the gene pair with probability:

$$P(rr) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(rd) = \left(\frac{1}{2} \times \frac{1}{2}\right) \times 2 = \frac{1}{2}$$

$$P(dd) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

But in given event we need to find the probability of outward appearance of the dominant gene. In this case there can be 2 pairs possible.

→ rd or dd.

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$P(rr \wedge rd) p = P(dd \wedge rd) = P(rd) + P(dd)$

$= \frac{1}{4} + \frac{1}{2}$

$\therefore \boxed{p = \frac{3}{4}}$

So here, $P(E) = \binom{4}{3} p^3 (1-p)^1$

$= \cancel{\frac{4}{3}} \binom{4}{3} \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)$

$\boxed{= \frac{27}{64}}$

Solⁿ 2 p = Probability that an individual component is working properly. / component function.

A communication system has n components

~~The system will be able to operate properly~~

The total system will be able to operate effectively if at least half of its components function.

(a) $p = 3.$

P(5-component system is working effectively) $>$ P(3-components system works properly)

$\therefore P(5,p) > P(3,p)$

$\therefore \binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p)^1 + p^5 > \binom{3}{2} p^2 (1-p) + p^3$

$\therefore 10p^3(1-2p+p^2) + 5p^4(1-p) + p^5 > 3p^2(1-p) + p^3$

$\therefore 10p^5 - 20p^4 + 10p^3 + 5p^4 - 5p^5 + p^5 > 3p^2 - 3p^3 + p^3$

$\therefore 6p^5 - 15p^4 + 10p^3 > 3p^2 - 2p^3$

$\therefore 6p^5 - 15p^4 + 12p^3 - 3p^2 > 0$

$\therefore 6p^3 - 15p^2 + 12p - 3 > 0$

(Here $6-15+12-3=0 \Rightarrow$ ~~x~~ $p-1$ is a factor)

$$
\begin{array}{r|l}
 & 6p^2 - 9p + 3 \\
\hline
p-1 & 6p^3 - 15p^2 + 12p - 3 \\
 & -\,6p^3 \mp 6p^2 \\
\hline
 & -9p^2 + 12p \\
 & +\,-9p^2 \pm 9p \\
\hline
 & 3p - 3 \\
 & 3p - 3 \\
\hline
 & 0 \quad 0
\end{array}
$$

$\therefore (p-1)(6p^2 - 9p + 3) > 0$

$\therefore (p-1)(6p^2 - 6p - 3p + 3) > 0$

$\therefore (p-1)(6p(p-1) - 3(p-1)) > 0$

$\therefore (p-1)^2 (6p-3) > 0$

$\therefore \boxed{3(p-1)^2(2p-1) > 0}$

Here $(p-1)^2 \geq 0$ (Here we are assuming $p \neq 0 \land p \neq 1$)

$\Rightarrow (2p-1) > 0$

$\Rightarrow \boxed{p > \frac{1}{2}}$

(b) $p = 2$.

P($(2k+1)$ component system is working effectively)

$> P$($(2k-1)$ component system is working effectively)

$\therefore P_{2k+1}(\text{effective}) > P_{2k-1}(\text{effective})$.

In $2k+1$ component system.

Now, let X denote the number of the first $2k-1$ components that function.

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Now $(2k+1)$-component system will be effective in this 3 cases
(i) $X \geq k+1$;
(ii) $X = k$ and one of the remaining 2 components function.
(iii) $X = k-1$ and both next 2 component function.

$$\therefore \boxed{P_{2k+1}(\text{effective}) = P\{X \geq k+1\} + P\{X=k\}(1-(1-p)^2) + P\{X=k-1\}p^2}$$

For $(2k-1)$ component system to be effective
- $X \geq k$.

$$\therefore \boxed{P_{2k-1}(\text{effective}) = P\{X \geq k\}}$$

or same Random variable

Here Note that we can use the same probability distribution as the probability for any component to function is the same and independent and X is define for $2k-1$ components only.

Since $P_{2k+1}(\text{effective}) > P_{2k-1}(\text{effective})$

$$\Rightarrow P_{2k+1}(\text{effective}) - P_{2k-1}(\text{effective}) > 0$$

$$\Rightarrow P\{X \geq k+1\} + P\{X=k\}(1-(1-p)^2) + P\{X=k-1\}p^2 - P\{X \geq k\} > 0$$

$$\Rightarrow \cancel{P\{X \geq k+1\}} + P\{X=k\}(1-(1-p)^2) + P\{X=k-1\}p^2 - (P\{\cancel{X \geq k+1}\} + P(X=k)) > 0.$$

$$\Rightarrow P\{X=k-1\}p^2 - P\{X=k\}(1-p^2) > 0$$

$$\Rightarrow \binom{2k-1}{k-1} p^{k-1}(1-p)^k p^2 - \binom{2k-1}{k} p^k (1-p)^{k-1}(1-p)^2 > 0$$

Since $\binom{2k-1}{k} = \binom{2k-1}{k-1}$

$$\Rightarrow \binom{2k-1}{k} p^{k+1}(1-p)^k - \binom{2k-1}{k} p^k (1-p)^{k+1} > 0$$

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$$\Rightarrow \binom{2k-1}{k} p^{k}(1-p)^{k}(p-(1-p)) > 0$$

$$\Rightarrow \underbrace{\binom{2k-1}{k}}_{>0\checkmark} \underbrace{p^{k}(1-p)^{k}}_{>0\checkmark} \underbrace{(2p-1)}_{?} > 0$$

$$\Rightarrow 2p-1>0$$

$$\Rightarrow \boxed{p > \frac{1}{2}}$$

Sol^n .3 An experiment that ~~consti~~ consist of counting the number of α particles given off in 1 second interval by 1 gram radioactive material.

→ On the average 3.2 α particles are given off.

X = number of α particles given off in 1 seconds.

$P(X \leq 2) = ?$

→ Let's assume that, there are ~~n α particles in~~ n Atoms in 1 gram radio active material. Here, the probability for each particle to ~~g din~~ disintegrate and send off an α particle during that interval is $p = \frac{3.2}{n}$ $\Rightarrow$ ~~$\alpha = np = 3.2$~~ $\boxed{\lambda = np = 3.2}$

NOTE↓
'n' is very large
⇓
Poisson R.V.

→ Here we will be using the X as a poisson random variable ~~as~~ to have very close approximation with $\lambda = 3.2$

$$\therefore P\{X \leq 2\} = e^{-\lambda}\left(\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!}\right)$$

$$= e^{-3.2}\left(1 + 3.2 + \frac{(3.2)^2}{2}\right)$$

$$\boxed{\simeq 0.3799}$$

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Sol^n-4 Independent trials with the probability of success $= p$
What is the probability of r successes before m failures?

- Here r^{th} success should have occured before or at the $(r+m-1)^{th}$ trials.

Using the Negative binomial random variable,
r = number of success
X = number of trials.

$$P\{X=n\} = \binom{n-1}{r-1} p^r (1-p)^{n-r}.$$

So, Here desired probability is

$$P(r \text{ success before } m \text{ failure}) = \sum_{n=r}^{r+m-1} P(X=n)$$

$$\boxed{= \sum_{n=r}^{r+m-1} \binom{n-1}{r-1} p^r (1-p)^{n-r}}$$

Sol^n-5) Expected value and variance of negative binomial random variable with parameters r & p.

- $E[g(x)] = \sum g(x)\, p(x)$

$$\therefore E[X^k] = \sum_{n=r}^{\infty} n^k \binom{n-1}{r-1} p^r (1-p)^{n-r}.$$

$$= \frac{r}{p} \sum_{n=r}^{\infty} n^{k-1} \underbrace{\frac{n}{r}\binom{n-1}{r-1}}_{\binom{n}{r}} p^{r+1} (1-p)^{n-r}$$

$$= \frac{r}{p} \sum_{n=r}^{\infty} n^{k-1} \binom{n}{r} p^{r+1} (1-p)^{n-r}$$

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Now taking $m = n+1$

$$\therefore E[X^k] = \frac{r}{p} \sum_{m=r+1}^{\infty} \frac{(m-1)^{k-1}}{i} \binom{m-1}{r} p^{r+1} (1-p)^{m-(r+1)}$$

$$\boxed{= \frac{r}{p} E[(Y-1)^{k-1}]}$$

- Here Y is negative binomial random variable with parameters $r+1$ & p. Now putting $k=1$

$$\therefore \boxed{E[X] = \frac{r}{p}}$$

- Now putting ~~$X=$~~ $k=2$.

$$\therefore E[X^2] = \frac{r}{p} E(Y-1).$$

$$= \frac{r}{p}[E(Y) - 1]$$

$$\boxed{= \frac{r}{p}\left(\frac{r+1}{p} - 1\right)}$$

(since Y is random variable with parameters $r+1$ & p.)

$$\therefore Var[X] = E[X^2] - (E[X])^2$$

$$= \frac{r}{p}\left(\frac{r+1}{p} - 1\right) - \frac{r^2}{p^2}$$

$$= \frac{r^2+r}{p^2} - \frac{r}{p} - \frac{r^2}{p^2}$$

$$= \frac{\cancel{r^2} + r - rp - \cancel{r^2}}{p^2}$$

$$\therefore \boxed{Var[X] = \frac{r(1-p)}{p^2}}$$