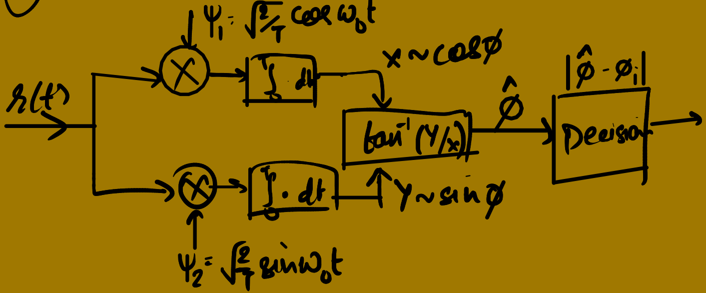
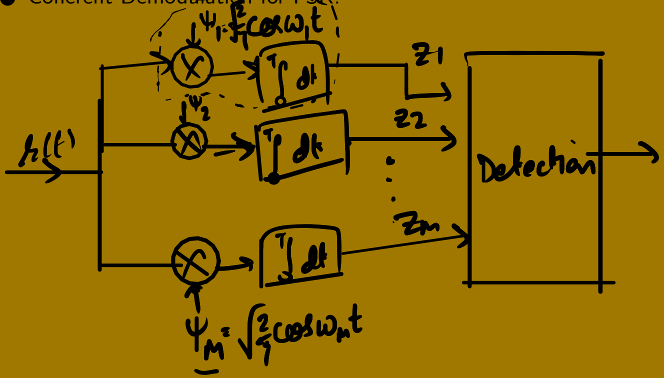
CT303 Lecture 23: December 2, 2020

- Lecture 22 Recap:
  - Bandpass Modulation - ASK, PSK, FSK, APK.
  - Coherent Demodulation for PSK:



- Coherent Demodulation for FSK:



- Orthogonality: $\int \cos(\omega_i t) \cos(\omega_j t)\ dt = 0$?

$$\int_{-\infty}^{\infty} s_i(t)\, s_j(t)\, dt \,, \quad \text{with } s_i(t) = \cos\omega_i t \; \text{rect}\left(\frac{t - T/2}{T}\right)$$

$$= \int_0^T \cos\omega_i t \cos\omega_j t \, dt$$

$$= \frac{1}{2} \int_0^T \cos((\omega_i + \omega_j)t) + \cos((\omega_i - \omega_j)t) \, dt$$

$$= \frac{1}{2} \left[\frac{\sin(\omega_i + \omega_j)t}{(\omega_i + \omega_j)} + \frac{\sin((\omega_i - \omega_j)t)}{(\omega_i - \omega_j)} \right]_0^T$$

$$= \frac{1}{2} \left[\frac{\sin((\omega_i + \omega_j)T)}{(\omega_i + \omega_j)} + \frac{\sin((\omega_i - \omega_j)T)}{(\omega_i - \omega_j)} \right]$$

since $\omega_i + \omega_j >> 1$

$$\Rightarrow \frac{\sin((\omega_i+\omega_j)T)}{(\omega_i+\omega_j)} \approx 0$$

$\langle s_i, s_j \rangle =$

$$\therefore \boxed{\int_{-\infty}^{\infty} s_i(t)\, s_j(t)\, dt} = \frac{1}{2} \frac{\sin((\omega_i-\omega_j)T)}{(\omega_i-\omega_j)}$$

$\therefore$ for $\langle s_i, s_j \rangle = 0$, with $\omega_i \neq \omega_j$

$$\Rightarrow \sin((\omega_i-\omega_j)T) = 0$$

$$\Rightarrow (\omega_i-\omega_j)T = k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow 2\pi T(f_i - f_j) = k\pi$$

$$\Rightarrow \boxed{f_i - f_j = \frac{k}{2T}}$$

$T \rightarrow$ symbol duration

* Minimum difference in frequency between 2 consecutive Orthogonal FSK (OFSK) symbols in case of Coherent detection is $f_i - f_{i+1} = \frac{1}{2T}$

* Given M symbols, the minimum B.W required for Coherent OFSK

$$= (M-1) \cdot \frac{1}{2T}$$

$$\approx \frac{M}{2T}.$$

Coherent vs. Non-Coherent

* Let transmitted symbol waveform be

$$s(t) = \sqrt{\frac{2E}{T}} \cos \omega_0 t \quad 0 \le t \le T$$

with $f_0 = 10^9 \text{ Hz}$ $(= 1 \text{ GHz})$

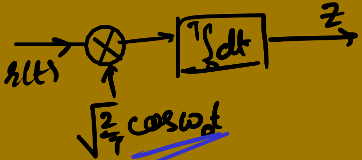
* Ignoring noise and attenuation, the received signal is

$$\rightarrow r(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0(t - t_d))$$

$t_d \rightarrow$ Propagation delay, $t_d = \text{distance} / 3 \times 10^8 \text{ m/s}$

Correlate $r(t)$ with $\sqrt{\frac{2}{T}} \cos \omega_0 t$

Rx



$$z = \int_0^T \sqrt{\frac{2}{T}}\cos\omega_0 t \, \sqrt{\frac{2E}{T}}\cos(\omega_0 t - \omega_0 t_d)\, dt$$

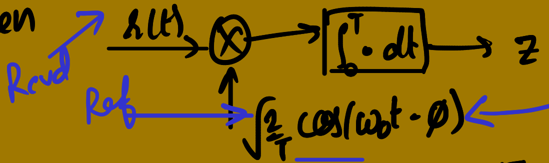
* What is z if $\omega_0 t_d = \pi/2$? $\quad z = \int_0^T \cos\omega_0 t \sin\omega_0 t \, dt = 0$

* For $\omega_0 t_d = \pi/2 \Rightarrow 2\pi\times 10^9 \times \frac{dist}{3\times 10^8} = \pi/2$

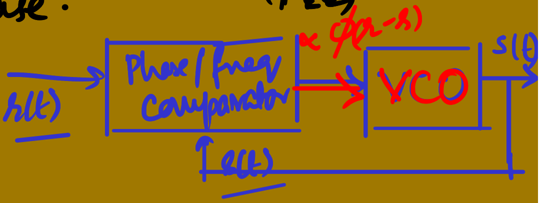
$\Rightarrow dist = 0.3/4 \text{ m} = 7.5 \text{ cm}$!!!

* If the receiver knows the phase of the received signal $r(t)$, i.e., $r(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t - \underbrace{\omega_0 t_d}_{\phi})$

then



* This is called COHERENT DETECTION
* Requires circuits like "PHASE LOCKED LOOPS" (PLL) to estimate the phase.



* Non-Coherent Detection – Does not assume or estimate the phase of the received signal.

1. DPSK (Differential PSK)
   – "Differentially Coherent" detection of "Differentially encoded" PSK

* Differential encoding (M-ary)
  – Let the previous transmitted symbol waveform be $\boxed{s(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \theta_j)}$
  – Then to transmit symbol i / message i in the current pulse duration, we will transmit

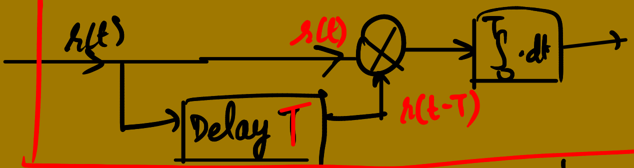
$$\sqrt{\frac{2E}{T}} \cos\left(\omega_0 t + \left[\theta_j + \boxed{\frac{2\pi}{M}(i-1)}\right]\right)$$

i.e., instead of using an absolute phase, we use 'relative' phase.

- "**Differential detection**" (Cannot use phase inf.)

If $s(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \theta_i)$, let the received signal be $r(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \theta_i + \alpha)$

where α is due to propagation delay.



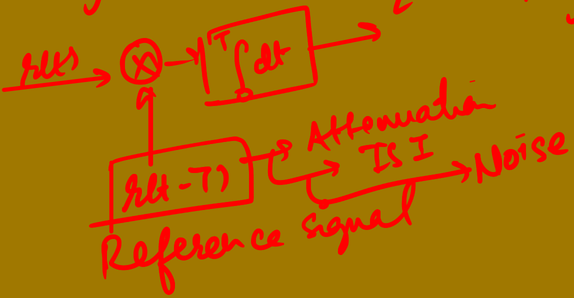
Z — Not be dependent on α!!!

* If α does not vary much over '2T' secs z will be independent of α.

$r(t) \rightarrow \theta_i, \alpha$

$r(t-T) \rightarrow \theta_j, \alpha$

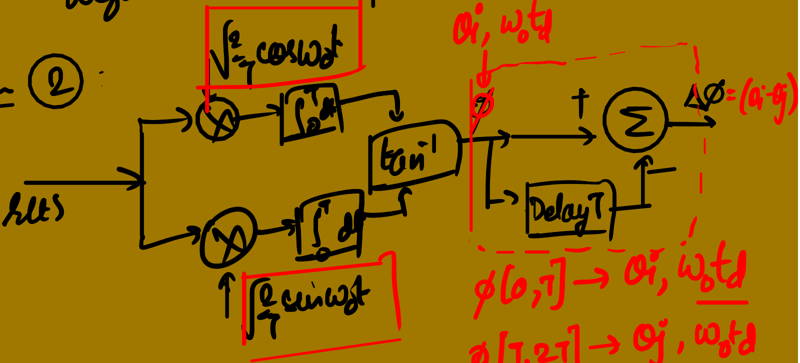
$z \rightarrow \theta_i - \theta_j$



* Issues: (1) Error will propagate because of Relative encoding

(2) The received signal is used as a reference. If there is noise (usual case) higher error expected.

Solution (2):



$\phi[0,T] \rightarrow \theta_i, \omega_0 t_d$

$\phi[T,2T] \rightarrow \theta_j, \omega_0 t_d$

Soln 1 Pre-coding:

Example: Binary DPSK → Phase shifts = $\{0, \pi\}$

Message bits: $m(k)$, Encoded bits $c(k)$.

Let $c(0) = 0$, $c(k) = c(k-1) \oplus m(k)$

K	0	1	2	3	4	5	6	7	8	9
$m(k)$		1	1	0	0	0	1	0	1	1
$c(k)$	0	1	0	0	0	0	1	1	0	1
Phase Shift	0 $(\omega_0 t)$	π $\omega_0 t+\pi$	0 $\omega_0 t+\pi$	0 $\omega_0 t+\pi$	0 $\omega_0 t+\pi$	0 $\omega_0 t+\pi$	π $\omega_0 t$	π $\omega_0 t+\pi$	0 $\omega_0 t+\pi$	π $\omega_0 t$
Detected Phase Shift		[π]	[0]	[0]	[0]	[0]	e [0]	[π]	[0]	[π]
Detected Message $\hat{c}$	0	1	0	0	0	0	0	1	0	1

- Non-Coherent Orthogonal FSK

$$\langle s_i, s_j \rangle = \int_0^T \underbrace{\cos(\omega_i t)}_{s_i} \underbrace{\cos(\omega_j t + \phi)}_{s_j} dt$$

$$= \int_0^T \left(\cos\phi \cos\omega_j t - \sin\phi \sin\omega_j t\right) \cos(\omega_i t)\, dt$$

$$= \cos\phi \int_0^T \cos\omega_j t \cos\omega_i t\, dt - \sin\phi \int_0^T \sin\omega_j t \sin\omega_i t\, dt$$

$$= \frac{\cos\phi}{2} \int_0^T \left[\cos((\omega_i + \omega_j)t) + \cos((\omega_i - \omega_j)t)\right] dt$$

$$- \frac{\sin\phi}{2} \int_0^T \sin((\omega_i + \omega_j)t) + \sin((\omega_i - \omega_j)t)\, dt$$

$$\langle s_i, s_j \rangle = \frac{\cos\phi}{2}\left[\frac{\operatorname{sen}((\omega_i+\omega_j)t)}{(\omega_i+\omega_j)} + \frac{\operatorname{sen}((\omega_i-\omega_j)t)}{(\omega_i-\omega_j)}\right]_0^T$$

$$+\frac{\operatorname{sen}\phi}{2}\left[\frac{\cos((\omega_i+\omega_j)t)}{(\omega_i+\omega_j)} + \frac{\cos((\omega_i-\omega_j)t)}{(\omega_i-\omega_j)}\right]_0^T$$

$$= \frac{\cos\phi}{2}\left[\frac{\operatorname{sen}((\omega_i+\omega_j)T)}{(\omega_i+\omega_j)} + \frac{\operatorname{sen}((\omega_i-\omega_j)T)}{(\omega_i-\omega_j)}\right]$$

$$+\frac{\operatorname{sen}\phi}{2}\left[\frac{\cos((\omega_i+\omega_j)T)-1}{(\omega_i+\omega_j)} + \frac{\cos((\omega_i-\omega_j)T)-1}{(\omega_i-\omega_j)}\right]$$

Assuming $\omega_i+\omega_j >> 1$

$[-2,0]$

$$\frac{\operatorname{sen}((\omega_i+\omega_j)T)}{\omega_i+\omega_j} \approx \frac{\cos((\omega_i+\omega_j)T)-1}{(\omega_i+\omega_j)} \approx 0$$

Thus,

$$\langle s_i, s_j \rangle = \frac{\cos\phi}{2} \frac{\sin((\omega_i-\omega_j)T)}{(\omega_i-\omega_j)} + \frac{\sin\phi}{2} \frac{(\cos((\omega_i-\omega_j)T)-1)}{(\omega_i-\omega_j)}$$

For orthogonality,

$$\langle s_i, s_j \rangle = \cos\phi \sin((\omega_i-\omega_j)T) + \sin\phi\left[\cos((\omega_i-\omega_j)T)-1\right] = 0$$

This should be true for any ϕ,

$$\therefore \sin((\omega_i-\omega_j)T) = \cos((\omega_i-\omega_j)T) - 1 = 0$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$(\omega_i-\omega_j)T = k\pi \quad \text{AND} \quad (\omega_i-\omega_j)T = 2k\pi, \; k \in \mathbb{Z}$$

$$\Rightarrow 2\pi(f_i-f_j)T = 2k\pi \Rightarrow \boxed{f_i - f_j = \frac{k}{T}}$$

Coherent OFSK	Non Coherent OFSK
- $\min\lvert f_i - f_j \rvert = \frac{1}{2T}$	$\min\lvert f_i - f_j \rvert = \frac{1}{T}$
- min BW $= (M-1) \cdot \frac{1}{2T}$ ✓	min BW $= \frac{M-1}{T}$
- Requires Phase Inf	- Does not require Phase Inf.

* Non-Coherent FSK Detection

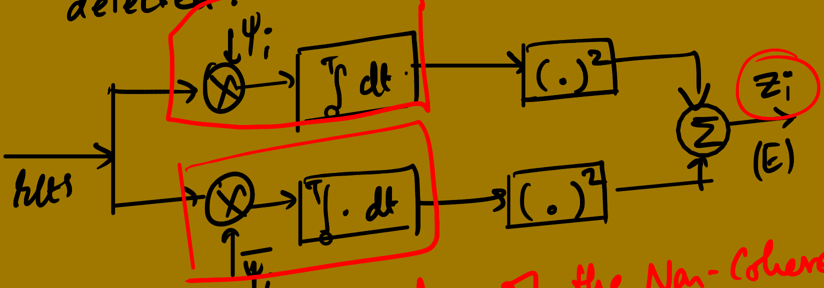
$$r(t) = \sqrt{\frac{2E}{T}} \cos(\omega_i t + \phi)$$

$$= \sqrt{\frac{2E}{T}} \left(\cos\omega_i t \cos\phi - \sin\omega_i t \sin\phi \right)$$

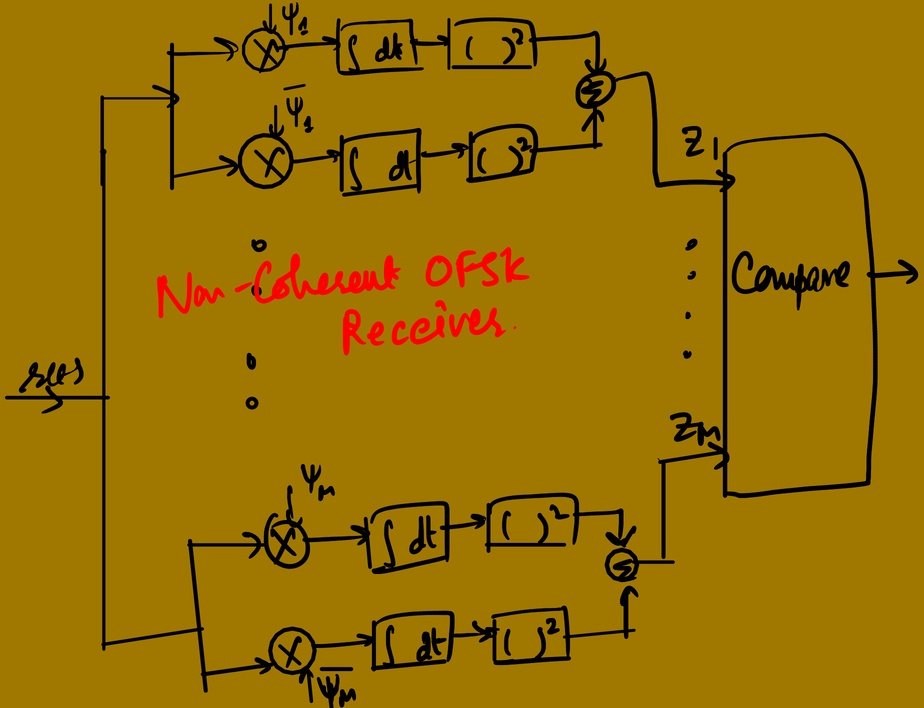
Let $\varphi_i(t) = \sqrt{\frac{2}{T}} \cos\omega_i t$ & $\overline{\varphi}_i(t) = \sqrt{\frac{2}{T}} \sin\omega_i t$.

$\Rightarrow$ $r(t) = \sqrt{E}\left(\cos\phi\, \varphi_i(t) - \sin\phi\, \overline{\varphi}_i(t)\right)$

$\Rightarrow$ Components for both φ_i and $\overline{\varphi}_i$ must be detected.



1 Arm of the Non-Coherent Detector



s(t)
ψ1
ψ̄1
∫ dt
( )²
Σ
Z1
ZM
Compare
ψM
ψ̄M
Non-Coherent OFSK Receiver

Non-Coherent FSK detector 2