Lecture - 19

P (1)

Recap:

Sums of independent random variables

X, Y: uniform over $(0,1)$

$X+Y$: triangular over $(0,2)$

$$f_{X+Y}(a) = \int_{-\infty}^{\infty} f_X(a-y)\, f_Y(y)\, dy$$

X, Y: Poisson with parameters $\lambda_1\ \lambda_2$

$X+Y$: Poisson with parameter $\lambda_1 + \lambda_2$

e.g. Binomial, independent (2)

$X : (n, p)$

$Y : (m, p)$

$Z = X + Y$

$P(Z = k)$

X	Y
0	k
1	k-1
⋮	⋮
k	0

$$= \sum_{i=0}^{k} P(X = i, Y = k - i)$$

$$= \sum_{i=0}^{k} \underline{P(X = i)}\, P(Y = k - i)$$

$$= \sum_{i=0}^{k} P(X=i)\, P(Y=k-i)$$

$$= \sum_{i=0}^{k} \binom{n}{i} p^{i} (1-p)^{n-i} \binom{m}{k-i} p^{k-i} (1-p)^{m-k+i}$$

$$= \sum_{i=0}^{k} \frac{n!}{i!(n-i)!}\, p^{k} (1-p)^{n+m-k} \frac{m!}{(k-i)!(m-k+i)!}$$

$$= p^{k} (1-p)^{n+m-k} \sum_{i=0}^{k} \binom{n}{i} \binom{m}{k-i}$$

$$= p^{k} (1-p)^{m+n-k} \binom{m+n}{k}$$

= Binomial with parameters $(m+n, p)$

Theorem:

X : normal, independent

Y : normal, independent

$X: \mu_1, \sigma_1^2$

$Y: \mu_2, \sigma_2^2$

$X+Y: \mu_1+\mu_2, \sigma_1^2+\sigma_2^2$

This completes your

syllabus for insem 2

tutorials: 1 to 7

lectures : 1 to 19

Conditional distribution: 5

discrete case

e.g.:

X: λ_1 Poisson, independent

Y: λ_2 Poisson, independent

what is the conditional distribution of X given $X+Y=n$?

$$P(X=k \mid X+Y=n)$$

$$= \frac{P(X=k,\ X+Y=n)}{P(X+Y=n)}$$

$$= \frac{P(X = k,\ X + Y = n)}{P(X + Y = n)}$$

$$= \frac{P(X = k,\ Y = n - k)}{P(X + Y = n)}$$

$$= \frac{P(X = k)\ P(Y = n - k)}{P(X + Y = n)}$$

$$= \frac{\dfrac{e^{-\lambda_1} \lambda_1^{k}}{k!} \cdot \dfrac{e^{-\lambda_2} \lambda_2^{n-k}}{(n-k)!}}{\dfrac{e^{-(\lambda_1+\lambda_2)} (\lambda_1+\lambda_2)^{n}}{n!}}$$

$$= \frac{\lambda_1^{k} \lambda_2^{n-k}}{(\lambda_1+\lambda_2)^n} \cdot \frac{n!}{k!\,(n-k)!}$$

$$= \binom{n}{k} \frac{\lambda_1^{k}}{(\lambda_1+\lambda_2)^{k}} \cdot \frac{\lambda_2^{n-k}}{(\lambda_1+\lambda_2)^{n-k}}$$

$$= \binom{n}{k} p^{k} (1-p)^{n-k}$$

= Binomial with parameters

$$\left(n, \frac{\lambda_1}{\lambda_1+\lambda_2}\right)$$

$$p = \frac{\lambda_1}{\lambda_1+\lambda_2}$$

e.g.

$$f(x,y) = \begin{cases} \frac{12}{5} x(2-x-y) & 0 < x < 1 \\ & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

What is the conditional density of X given $Y = y$?

$$f_{X|Y}(x|y) = ?$$

$$= \frac{f(x,y) = \text{know}}{f_Y(y) = \cancel{\text{tod}}\ \text{marginal density of } Y.}$$

$$f_Y(y) = \int_0^1 f(x,y)\, dx$$

9

Home work