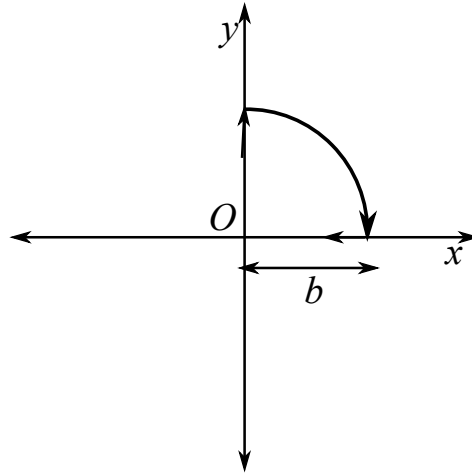


1. Find $\vec{\nabla}(\sin(\vec{n} \cdot \vec{r}))$, $\vec{\nabla} \times (\vec{n} \times \vec{r})$ where \vec{n} is a constant vector. (5)
2. If \vec{A} and \vec{B} are vectors prove that $\vec{A} \cdot \vec{B}$ transforms as a scalar under rotation. (5)
(You may use two dimensional case.)
3. A wheel, lying in the xy plane with its center at the origin, rotates clockwise about its center with an angular velocity ω about the z axis. The velocity of any point on the wheel can be described by a vector field $\vec{v}(x, y)$. Determine $\vec{v}(x, y)$ and $\vec{\nabla} \times \vec{v}$. (5)
4. Verify stokes' theorem for the vector field $\vec{A} = (y\hat{i} - x\hat{j})$ over the region shown in the figure. The loop consists of a quarter arc of a circle of radius b and two straight paths along the y and the x axes traversed clockwise. (5)



5. Consider a cubical surface S formed by the planes $x = a \pm l$; $y = b \pm l$; $z = c \pm l$. Let $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$. Verify that at the point (a, b, c) ,

$$\vec{\nabla} \cdot \vec{A} = \lim_{l \rightarrow 0} \frac{1}{8l^3} \oint_S \vec{A} \cdot d\vec{a}$$

(5)

Gradient, divergence and curl

$$\vec{\nabla} F = \hat{i} \frac{\partial F}{\partial x} + \hat{j} \frac{\partial F}{\partial y} + \hat{k} \frac{\partial F}{\partial z} \quad \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \vec{A} = \hat{i} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{j} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{k} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2}$$