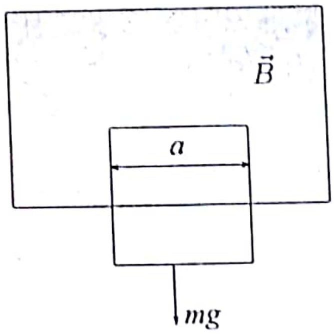
SC217: Electromagnetic Theory, **AUTUMN'16 ENDSEM**

DA-IICT, B.Tech, Sem III 14:00-16:30 PM 30^{th}Nov'16 **50 marks**

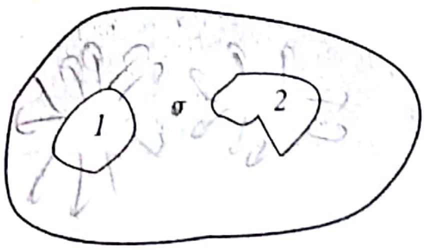
1. A linear dielectric material with permitivity $\epsilon = 1.5\epsilon_0$ has free charge embedded into it given by the charge distribution $\rho_f(r)$. The polarization in the dielectric due to this charge distribution is found to be $\vec{P}(\vec{r}) = \frac{k}{r^2}\hat{r}$. Find the total free charge that is embedded inside the dielectric material. **(5)**

2. A square loop of mass m and side a falls due to gravity in a uniform magnetic field $\vec{B}$ as shown in the figure below. The total resistance of the loop is R. The plane of the loop is perpendicular to the magnetic field. Find the terminal velocity of the loop. (Terminal velocity is the constant velocity that the loop attains as it falls under gravity through the magnetic field.) **(5)**

$\frac{mg}{Ba}$



3. Two metal objects, 1 and 2, are embedded in a weakly conducting material of conductivity σ as shown in the figure. If C is the capacitance of the arrangement of the metal objects, find the resistance between them. **(10)**



4. The electric field in a region is $E_0\hat{k}$. A conducting sphere of radius a with a total charge Q is placed in it. Find the resultant electric field outside the sphere and the surface charge density on the surface of the sphere. **(10)**

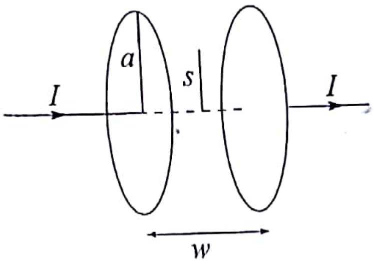
$\epsilon_0\left[\frac{Q}{4\pi\epsilon_0 a^2} - 3E_0\cos\theta\right]$

5. A uniform surface current $\vec{K} = K_0\hat{i}$ flows over the xy plane. Find the magnetic field $\vec{B}$ and the vector potential $\vec{A}$ due to this. **(10)**

6. A parallel plate capacitor is charged by a thin wire carrying a constant current I as shown in the figure. The radius of the plates is a and the distance between the plates is $w << a$. Assume that the surface charge on the plates is always uniform though changing with time.

$E = \frac{\sigma}{2\epsilon_0}$

$R = \frac{V}{I} = \frac{V}{\sigma E} = \frac{V 2\epsilon_0}{\sigma^2}$ $C = \frac{Q}{V}$



$\vec{J} = \sigma \vec{E}$

(a) Find the electric field in between the plates as a function of time t. **(5)**

(b) Find the magnetic field within the capacitor at a distance s from the axis($s << a$).
hint: Use Maxwell's modfied current density $\vec{J}' = \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}$. **(5)**

Gradient, divergence and curl

Spherical polar coordinates :

$\vec{\nabla} F = \frac{\partial F}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial F}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial F}{\partial \phi}\hat{\phi}$, $\quad \vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$

$$\vec{\nabla}\times\vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\phi}$$

Cylindrical coordinates:

$\vec{\nabla} F = \frac{\partial F}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial F}{\partial\phi}\hat{\phi} + \frac{\partial F}{\partial z}\hat{z}$, $\quad \vec{\nabla}\cdot\vec{A} = \frac{1}{s}\frac{\partial}{\partial s}(sA_s) + \frac{1}{s}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$

$$\vec{\nabla}\times\vec{A} = \left[\frac{1}{s}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_s}{\partial\phi}\right]\hat{z}$$

Legendre Polynomials

$P_0(\cos\theta) = 1, \quad P_1(\cos\theta) = \cos\theta, \quad P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}, \quad P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta$

$\int_0^\pi P_l(\cos\theta)P_m(\cos\theta)\sin\theta d\theta = \frac{2}{2l+1}\delta_{lm}$

$I = \frac{dQ}{dt}$

$Q = \int_0^t I\,dt$

$Q = It$