Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)

First In Semester Examination

CT314 (Statistical Communication Theory)

Date of Examination: Feb 08, 2013

Duration: 2 Hours

Maximum Marks: 20

Instructions:

1. Attempt all questions.
2. Use of scientific non programmable calculator is permitted.
3. Figures in brackets indicate full marks.
4. All the acronyms carry their usual meaning.

Q1 Consider the quadratic equation $Ax^2 + Bx + c = 0$. The coefficients of this equation are found by throwing a 'die' three times. Find the probability that the roots are real.

(5 marks)

Q2: Consider three random variables X_1, X_2, X_3. Let the event A be defined as $A = \{\max(X_1, X_2, X_3) \leq 5\}$. Express the probability of A i.e., $P(A)$, using the cumulative distribution function $F_{X,Y,Z}(x, y, z)$. (2 marks)

Q3: Let $f_{X,Y}(x,y) = \begin{cases} \frac{1}{2}, & 0 \leq x \leq y \ \ and \ \ 0 \leq y \leq 2 \\ 0 & otherwise \end{cases}$

Find $f_{Y/X}(y/1)$ and $f_{Y/X}(y/1.5)$. Without using marginals, comment on the dependency between X and Y. (5 marks)

Q4: Let $f_{X,Y}(x,y) = \dfrac{a}{1 + x^2 + x^2y^2 + y^2}$. Find the marginals. Are X and Y independent?

(4 marks)

Q5. Which of the following (2 by 2) matrices are valid covariance matrices and why?

a. $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ (2 marks)

Q6: Let us say that you are generating a Gaussian distributed random variable using MATLAB. The default is $N(0,1)$. Let this be X. However, my work needs $N(1,2)$ i.e., Y. How do you get it from the generated values (i.e., write the mathematical operation on X to get Y) (2 marks)

ANSWERS TO I INSEM· QUESTIONS
WINTER 2012-13.

Q.1. For the roots to be real we need discriminant $B^2-4AC \geq 0$

or $\frac{B^2}{4} \geq AC$: NOW A, B, C can take any value in the range 1–6.

Consider $B = 1$, $\frac{1}{4} \ngeq AC$ for any values of A, C

$B = 2$, $\frac{B^2}{4} \geq AC$, true for $A = C = 1$

$B = 3$, True for $A = C = 1$; $A = 2, C = 1$; $A = 1, C = 2$ } 3 times

$B = 4$, True for above 3 cases and also 1, 3; 3, 1; 2, 2; 1, 4; 4, 1 } 5 + 3 = 8 times

$B = 5$ — 14 times

$B = 6$ — 17 times

So total ~~number~~ number of times $\frac{B^2}{4} \geq AC$ is 43

So $P[\text{real roots}] = \frac{43}{\text{total possibilities}} = \frac{43}{6^3}$.

Q 2. Maximum of 3 numbers is less than 5 if each of the ~~thr~~ 3 numbers is less than 5

So $P[A] = P[x_1 \leq 5, x_2 \leq 5, x_3 \leq 5]$
$= F_{xyz}(5, 5, 5)$

Q3. $f_{Y/X}(y/x) = \frac{f_{xy}(x,y)}{f_x(x)}$

$f_x(x)$ can be obtained from $f_{xy}(x,y)$ by integrating out y.

~~for~~ we know that $0 \leq y \leq 2$ and also $y \geq x$

$\therefore \; f_X(x) = \int_x^2 \frac{1}{2}\, dy = 1 - \frac{x}{2}, \quad 0 \le x \le 2$

$\therefore$

$\therefore \quad f_{Y/X}(y/1) = \left.\frac{1/2}{f_X(x)}\right|_{x=1} = \frac{1/2}{1/2} = \begin{cases} 1 & 1 \le y \\ 0 & \text{other} \end{cases}$ bec

$f_{Y/X}(Y/1.5) = \frac{\frac{1}{2}}{\frac{1}{4}} = \begin{cases} 2, & 1.5 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$

From the problem statement y should be $\geq x$ so y depends on x

We see that $f_{Y/X}(y/x)$ depends on x where as if X & Y are independent $f_{Y/X}(y/x) =$

So X and Y are dependent.

Q.4 $f_{XY}(x,y) = \frac{a}{1+x^2+x^2y^2+y^2} = \frac{a}{(1+x^2)(1+y^2)}$

$f_{X,Y}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\, dx\, dy = a\pi^2 = 1 \quad \therefore a = \frac{1}{\pi^2}$

$\therefore \; f_X(x) = \frac{1}{\pi(1+x^2)}, \quad f_Y(y) = \frac{1}{\pi(1+y^2)}$, X & Y are independent

Q.5:

a — determinant -ve NOT Valid $C_{\underline{x}}$

b — valid — satisfies the properties $C_{\underline{x}}$

c — valid — "

d — Not valid $C_{\underline{x}}$ — not symmetric

Q.6 $Y = \sqrt{2}X + 1$