Autumn2016

- 1. Find the volume of the tetrahedron whose vertices are (0,0,0), (a,0,0), (0,b,0), (0,0,c).
- 2. Evaluate $\int_P^Q \vec{\mathbf{A}} \cdot \vec{\mathbf{dl}}$ for $\vec{\mathbf{A}} = y\hat{\mathbf{i}} x\hat{\mathbf{j}}$ along the following arcs of a circle of radius a: $P \equiv (-a, 0)$; $Q \equiv (a, 0)$.
 - (a) $(-a,0) \to (0,a) \to (a,0)$
 - **(b)** $(-a,0) \to (0,-a) \to (a,0)$
 - (c) a loop, forward along (a) and backward along (b)
 - (d) Let *I* be the value of the loop integral evaluated in (c). Verify that at the origin

$$\vec{\nabla} \times \vec{A} = \lim_{a \to 0} I / (\pi a^2)$$

- 3. Consider $\vec{\mathbf{A}} = x^2 \hat{\mathbf{i}} + y^2 \hat{\mathbf{j}} + z^2 \hat{\mathbf{k}}$
 - (a) Evaluate $\oint_S \vec{\mathbf{A}} \cdot \vec{\mathbf{da}}$ where S is a cubical surface given by the planes $x = a \pm l$; $y = b \pm l$; $z = c \pm l$.
 - (b) Verify that at the point (a, b, c),

$$\vec{\nabla} \cdot \vec{A} = \lim_{l \to 0} \frac{1}{8l^3} \oint_S \vec{\mathbf{A}} \cdot \vec{\mathbf{da}}$$

4. Let $\vec{A} = \hat{r}$. Evaluate $\int_S \vec{A} \cdot d\vec{a}$ over the surface of a sphere given by the equation $x^2 + y^2 + z^2 = a^2$.