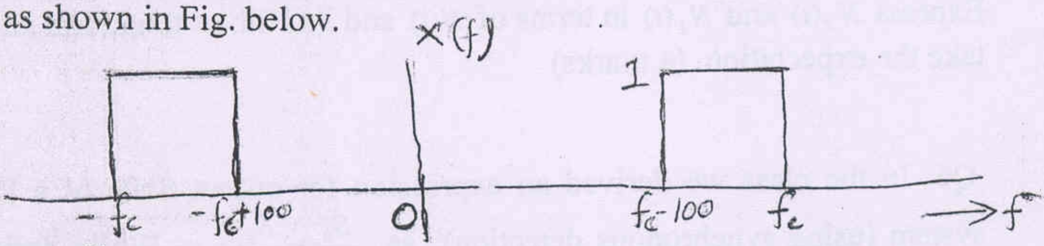
Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)
End Semester Examination
CT314 (Statistical Communication Theory)
Date of Examination: 27-04-2011
Duration: 2:30 hrs
Maximum Marks: 40

Instructions:

1. Attempt all questions.
2. Use of scientific non programmable calculator is permitted.
3. Figures in brackets indicate full marks.
4. All the acronyms carry their usual meaning.
5. Unless specified assume real random process

Q1: Consider a NBBP real signal $x(t)$ with Fourier transform $X(f)$ as shown in Fig. below.



(a) Plot $\tilde{X}(f)$ i.e., Fourier transform of $\tilde{x}(t)$ which is the complex envelope of $x(t)$

(b) Find the expression for $\tilde{x}(t)$

(c) Find the expressions for inphase and quadrature components of $x(t)$

(d) Find expression for the envelope of $x(t)$

(10 marks)

Q2: For each of the following functions of w, state whether it can be a valid psd of a real random process. Give proper reasoning for your answer.

$(a)\quad \delta(w)+\dfrac{1}{w^2+16}$, $\quad (b)\quad \dfrac{w}{w^2+16}$, $\quad (c)\quad j[\delta(w+w_0)+\delta(w-w_0)]$

Here $-\infty < w < \infty$ and w_0 is a constant.

(6 marks)

Q3: Consider a random process $X(t) = A\cos(w_c t + \Theta)$, where w_c is a constant. A and Θ are random variables that are independent and are uniformly distributed in the range (-1, 1) and $(0, 2\pi)$, respectively. Find

(a) $E[X(t)]$

(b) $R_x(t_1, t_2)$

(c) Is the process WSS (Give reason for the answer)

(d) Is the process ergodic in mean. Give reason.

(e) If the process is WSS what is the average power of the random process.

(10 marks)

Q4: Consider NBBP noise process $N(t)$ with zero mean. We know that it can be expressed in terms of inphase and quadrature components $N_R(t)$ and $N_I(t)$, respectively. Prove that $N_R(t)$ and $N_I(t)$ also have zero mean. Hint: Express $N_R(t)$ and $N_I(t)$ in terms of $N(t)$ and its Hilbert transform and then take the expectation. (4 marks)

Q5: In the class we derived an expression for output SNR of a DSB/SC system (using synchronous detection) as $\frac{S_0}{N_0} = \frac{S_i}{NB} = \gamma$. While deriving it we considered the carrier as $\sqrt{2}\cos w_c t$. Derive the expression for the output SNR for the same system considering the carrier as $\cos w_c t$ (at the transmitter and receiver) (6 marks).

Q6: Suppose a message signal $(A + m(t))$ is multiplied with $\sqrt{2}\cos w_c t$ (where A is greater than $m(t)$) in order to obtain the amplitude modulated signal. This signal in the presence of channel noise is available at the input of the receiver input. A synchronous demodulation is used for detection instead of envelope detector. No assumption is made on the amount of noise at the input of the receiver. Obtain the expression for output SNR and show that it is the same as that for envelope detector case (as derived in the class when we considered the small noise case).

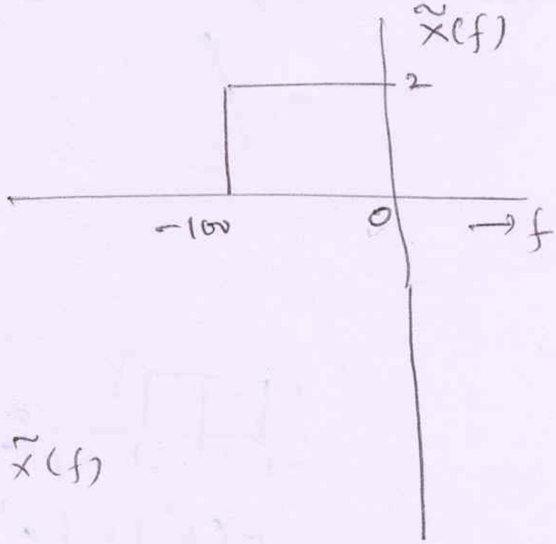
Following is the conclusion on the performance of AM demodulation using synchronous detection: Unlike envelope detection, the performance of the synchronous detection for AM signal (in the presence of noise) is the same irrespective of small noise or large noise case). (4 marks)

BEST OF LUCK

Endsem Answers
CT314 27th April 2011

Q.1

(a)



$$x_p(t) = x(t) + j\hat{x}(t)$$

$$X_p(f) = X(f) + j\hat{X}(f)$$

$$= 2X(f), \quad f > 0$$

$$= 0 \quad \text{otherwise}$$

$$x_p(t) = \tilde{x}(t)e^{j2\pi f_c t}$$

$$\therefore \tilde{x}(t) = x_p(t)e^{-j2\pi f_c t}$$

$$= X_p(f+f_c)$$

$$\angle\, \tilde{x}(t) \leftrightarrow \tilde{X}(f)$$

(b)

$$\tilde{x}(t) = \int_{-\infty}^{\infty} \tilde{X}(f)e^{+j2\pi ft}\,df = \int_{-100}^{0} 2\, e^{j2\pi ft}\,df$$

$$= 2\left[\frac{e^{j2\pi ft}}{j2\pi t}\right]_{-100}^{0}$$

$$= \frac{2}{\pi t}\left[\frac{1 - e^{-j2\pi 100t}}{2j}\right] = \frac{2}{\pi t}\left[\frac{e^{j2\pi(\frac{100}{2})t} + e^{-j2\pi(\frac{100}{2})t}}{2j}\right]e^{-j2\pi(\frac{100}{2})t}$$

$$= \frac{2}{\pi t}\left[\frac{\sin\pi(100)t}{100}\right]e^{-j\pi(100)t} \times 100$$

$$= 200\,\mathrm{sinc}(100t)\,e^{-j\pi(100)t}$$

(c) Expand this

$$\tilde{x}(t) = 200\,\text{sinc}(100)t\left[\cos 100\pi t - j\sin 100\pi t\right]$$

$$= 200\,\text{sinc}(100)t\cos 100\pi t - j\,200\,\text{sinc}(100)t\sin(100)\pi t$$

$$= x_R(t) + j x_I(t)$$

So $x_R(t) =$ real part

& $x_I(t) =$ imag. part.

(d) Envelope of $x(t) = |\tilde{x}(t)| = 200|\text{sinc}(100t)|$

absolute or mod has to be there.
(envelope is +ve)

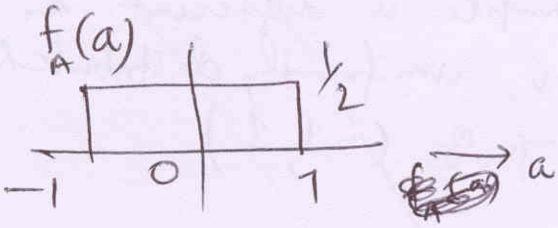
Q.2 (a) Yes, psd is always +ve, not complex.

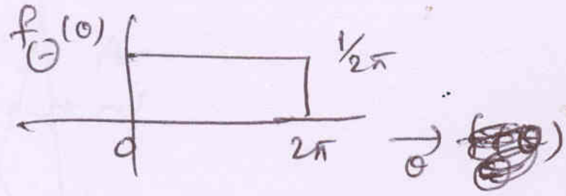
(b) no, for $\omega < 0$, psd is -ve
hence not valid psd.

(c) no, psd is real. (j makes it complex
hence not valid
psd).

Q3

$$x(t) = A\cos(\omega_c t + \Theta)$$





$$E[x(t)] = E[(A)] \cdot E[\cos(\omega_c t + \Theta)]$$ A and Θ are independent

$$E[A] = A,$$

$$E[\cos(\omega_c t + \theta)] = \frac{1}{2\pi}\int_0^{2\pi} \cos(\omega_c t + \theta)\, d\theta$$

$$= 0.$$

$$\therefore E[X(t)] = A \times 0 = 0.$$

(b) $\cancel{E[A^2] = 1/3}$ $R_X(t_1, t_2)$.

$$= E[A\cos(\omega_c t_1 + \theta) \cdot A\cos(\omega_c t_2 + \theta)]$$

$$= E[A^2]\, E[\cos(\omega_c t_1 + \theta)\cos(\omega_c t_2 + \theta)]$$ (independence of A & Θ)

Since $E[A^2] = 1/3$

$$= \frac{1}{3} E[(\cos\omega_c(t_1 - t_2)) + \cos(\omega_c(t_1 + t_2) + 2\Theta)]$$

$$= \frac{1}{3}\cos\omega_c(t_1 - t_2) + 0$$

$$= \frac{1}{3}\cos\omega_c(t_1 - t_2) = \frac{1}{3}\cos\omega_c\tau; \quad \tau_1 = t_1 - t_2$$

(c) So the process is WSS

since $R_x(t_1, t_2)$ depends on time difference only and not on t_1 and t_2.

(d) Process is ergodic time average of any sample fn $= 0$

and $E[x(t)]$ = ensemble average is also zero.

This shows that time average = Ensemble average.

(e) $R_x(0) =$ Average Power $= \frac{1}{}$ $= \int_{-\infty}^{\infty} S_x(f)\, df$.

Q4:

$N_p(t) = N(t) + j\hat{N}(t)$.

$N_p(t) = \tilde{N}(t)\, e^{j2\pi f_c t}$.

$= \left[N_R(t) + j N_I(t)\right] e^{j2\pi f_c t}$.

or $\left[N_R(t) + j N_I(t)\right] = \left[N(t) + j\hat{N}(t)\right] e^{-j2\pi f_c t}$

$= \left[N(t) + j\hat{N}(t)\right]\left(\cos 2\pi f_c t - j\sin 2\pi f_c t\right)$

$\therefore N_R(t) = N(t)\cos 2\pi f_c t + \hat{N}(t)\sin 2\pi f_c t$

$N_I(t) = \hat{N}(t)\cos 2\pi f_c t - N(t)\sin 2\pi f_c t$

taking

$\therefore$ Expectation

$E[N_R(t)] = E[N(t)]\cos 2\pi f_c t - E[\hat{N}(t)]\sin 2\pi f_c t$

But $E[\hat{N}(t)]$ is zero

Since $\hat{N}(t)$ can be obtained by passing $N(t)$ through a linear filter with impulse response as $\frac{1}{\pi t}$

because $\hat{n}(t) = \frac{1}{\pi t} * n(t)$

Hence if the i/p $N(t)$ has zero mean

o/p $\hat{N}(t)$ also has zero mean.

Since $E[\hat{N}(t)] = E[N(t)]\, H(0)$

$= 0$.

Q.5 For DSB/SC $\frac{S_o}{N_o} = \frac{S_i}{NB} = \gamma$ (Proved in the class).

~~The answer~~ when we consider $\cos\omega_c t$ instead of $\sqrt{2}\cos\omega_c t$, the answer is the same.

To prove this:

The signal plus noise available at the receiver input after BPF (before detection)

$$y_i(t) = m(t)\cos\omega_c t + n_R(t)\cos\omega_c t - n_I(t)\sin\omega_c t$$

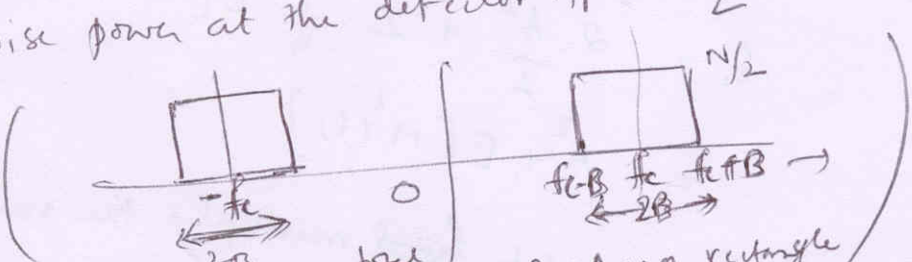
$$\therefore \text{Signal power} = \frac{1}{4}\times 2E[M^2(t)] = \frac{1}{2}E[M^2(t)]$$

($\because$ $\underbrace{M(t)}_{rp}\cos(\omega_c t + \underbrace{\Theta}_{rv\,(0,2\pi)\ \text{uniform}})$ ~~has~~ psd of $\frac{1}{4}[M(f-f_c)+M(f+f_c)]$

So the power $= \frac{1}{4}\times 2$ of power in the message)

Noise power at the detector i/p $= \frac{N}{2}\times 4B = 2NB = N_i$

not required for calculations



S_i total Power $= 2\times$ Area of rectangle $= 2\times\frac{N}{2}\times 2B = 2NB$

Now at the o/p of demodulator before LPF we have

$$y(t) = \left(m(t)\cos\omega_c t + n_R(t)\cos\omega_c t - n_I(t)\sin\omega_c t\right)\cos\omega_c t$$

$$= m(t)\cos^2\omega_c t + n_R(t)\cos^2\omega_c t - n_I(t)\sin\omega_c t\cos\omega_c t$$

After LPF

$$= \frac{m(t)}{2} + \frac{n_R(t)}{2}$$

So the o/p signal power $S_o = \frac{1}{4} E[M^2(t)]$

and the o/p noise power $= \frac{1}{4} E[N_R^2(t)] = \frac{1}{4}[N^2(t)]$

$$= \frac{1}{4}\left[\frac{N}{2} \times 4B\right] = \frac{NB}{2}$$

$$\therefore \frac{S_o}{N_o} = \frac{E[M^2(t)]}{4 \cdot NB} \cdot 2 = \frac{E(M^2(t))}{2NB} = \frac{S_i}{NB} = \gamma.$$

Q.6 AM with synchronous demodulation.

Signal + noise at the input of the synchronous demodulation

$$= \sqrt{2}\cos w_c t\,(A + m(t)) + n(t), \quad A > m(t)$$

So the signal part $= \sqrt{2}A\cos w_c t + \sqrt{2}\, m(t)\cos w_c t$.

$$\therefore S_i = 2 \cdot \frac{A^2}{2} + 2\,\frac{1}{4} E[M^2(t)]$$

$$= A^2 + E[M^2(t)]$$

To find S_o, we need to ~~feed~~ multiply the available i/p with $\sqrt{2}\cos w_c t$ & then LPF it.

So after multiplication we get $\left(\sqrt{2}A\cos w_c t + \sqrt{2}\cos w_c t\, m(t) + n_R(t)\cos w_c t - n_I(t)\sin w_c t\right)\sqrt{2}\cos w_c t$ — ①

After passing through LPF (DC removed) we get

So the signal power as $E(M^2(t))$

and to noise power = consider $\sqrt{2}\, n_R(t)\cos^2 w_c t$ (as the last term is in ① 0 because of LPF)

So Noise power $= E\left[\left(\frac{1}{\sqrt{2}}\right)^2 N_R^2(t)\right] = E\left[\frac{N^2(t)}{2}\right]$

$$= \frac{1}{2} \times \frac{N}{2} \times 4B = NB.$$

So $\frac{S_o}{N_o} = \frac{E(M^2(t))}{NB} = \frac{E[M^2(t)]}{A^2 + E[M^2(t)]} \times \frac{A^2 + E[M^2(t)]}{NB}$ → S_i/NB

Hence same as AM for large signal case.