Lecture - 28

will upload scanned lecture - 26 on moodle. If its readable, then let me know & i'll scan other lecture notes.

Recap: $E[X] = E[E[X|Y]]$

Markov's inequality

Chebyshev's inequality.

M.I: if X is a random variable which is non-negative, then for any $a \geq 0$

$$P(X \geq a) \leq \frac{E[X]}{a}$$

if $E[X] = 30$

you choose $a = 20$,

then the theorem doesn't give you any <u>useful</u> information.

$$P(X \geq 20) \leq \frac{30}{20} = 1.5$$

$P(X \geq 20) \leq 1.5 \rightarrow$ True but useless

~~$P(X$~~

$P(X \geq 20) \leq 1$

We define an <u>indicator random variable</u>.

$$I = \begin{cases} 1 & \text{for } X \geq a \\ 0 & \text{for } X < a \end{cases}$$

$E[I] = 1 \cdot P(I=1) +$ (3)

$0 \cdot P(I=0)$

$= P(I=1)$

$E[I] = P(X \geq a)$ halfway done

Compare I & $\frac{X}{a}$

is $I \leq \frac{X}{a}$ (A) or

is $I \geq \frac{X}{a}$ (B)

$I = 0$ $\quad \underset{0}{I} \leq \underset{n.n.}{\frac{X}{a}}$

or

$I = 1$ then $X \geq a \Rightarrow \frac{X}{a} \geq 1$

$\Rightarrow \underset{1}{I} \leq \frac{X}{a}_{\geq 1}$

$$I \le \frac{X}{a}$$

$$E[I] \le E\left[\frac{X}{a}\right]$$

$$\boxed{P(X \ge a) \le \frac{E[X]}{a}}$$

Markov's inequality

<u>e.g.</u> No. of items produced in a factory in a week is a random variable with mean $E[X] = 50$. What can be said about the probability of producing more than 75 items this week?

X = no. of items produced in a week

$a = 75$, $E[X] = 50$ | $P(X \ge 75) \le \frac{50}{75} = \frac{2}{3}$

e.g. Lets say we know the distribution.

X is a uniform random variable.

Over $(0, 10)$

$$P(X \geq 9) \leq \frac{5}{9} = 0.55$$

$$P(X \geq a) \leq \frac{E[X]}{a} \qquad 0.1 \leq 0.55$$

$$P(X \geq 9) = \int_9^{10} \underbrace{f_X(x)}_{\frac{1}{10-0}} \, dx$$

$$= \frac{1}{10} \cdot 1 = \frac{1}{10} \; 0.1$$

e.g. - marks & attendance

X: non-negative

let's say that

$P(X \geq 40) \simeq \dfrac{\text{no. of students with marks} \geq 40}{\text{total no. of students}} \leq E[X]/40$

0.12 0.70

X: total marks in insem 1 + insem 2

0 to 50

Y = attendance 0 to 1

0.59

$P(Y \geq 0.9) \simeq \dfrac{\text{total no. of students with attendance} \geq 0.9}{\text{total no. of students in the class}}$

$\leq \dfrac{E[Y]}{0.9}$

0.97

Chebyshev's inequality

X is a random variable, mean μ, variance σ^2, $b \geq 0$.

$$P(|X-\mu| \geq b) \leq \frac{\sigma^2}{b^2}$$

We prove it using Markov's inequality.

$$P(X \geq a) \leq \frac{E[X]}{a}$$

we apply M.I. on

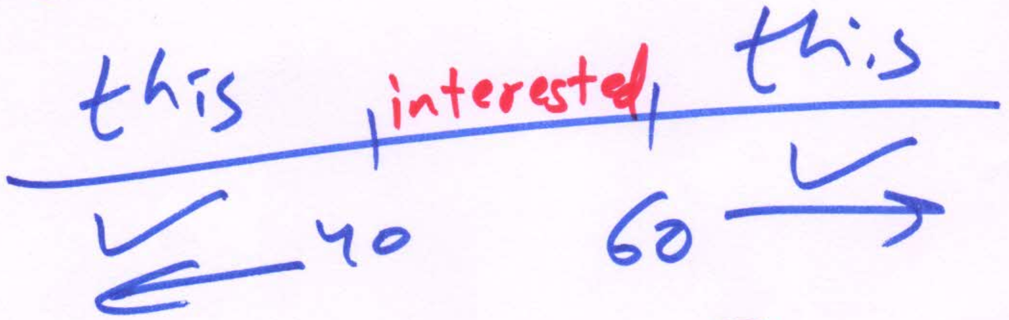
$$P((X-\mu)^2 \geq b^2) \leq \frac{E[X-\mu]^2}{b^2}$$

$$P(|X-\mu| \geq b) \leq \frac{\sigma^2}{b^2}$$

e.g. The no. of items produced in a factory in a week is a random variable with $\mu = 50$ and $\sigma = 5$. What can be said about the probability that this week's production is between 40 & 60?

$$P(\boxed{|X - 50| \geq 10}) \leq \frac{\sigma^2}{k^2} = \frac{25}{100}$$

$$P(40 \leq X \leq 60) = 1 - P(|X - 50| \geq 10)$$



$$P(40 \leq X \leq 60) \geq 1 - \frac{1}{4} = \frac{3}{4}$$

There is atleast 3/4 probability that the production is b/w 40 & 60.

e.g.: X is uniformly distributed $(0, 10)$

$\mu = 5$, $\sigma^2 = \frac{100}{12}$

$$P(\,|X-5| \geq 4) \leq \frac{\sigma^2}{b^2} = \frac{\overset{25}{\cancel{100}}}{\underset{3}{\cancel{12}} \cdot 16}$$

$b = 4$

$$= \frac{25}{48} \simeq 0.5 \quad \text{C.I.}$$

$P(X \leq 1 \text{ or } X \geq 9)$

$$= \int_0^1 f_x \, dx + \int_9^{10} f_x \, dx$$

$$= \frac{2}{10} = 0.2$$