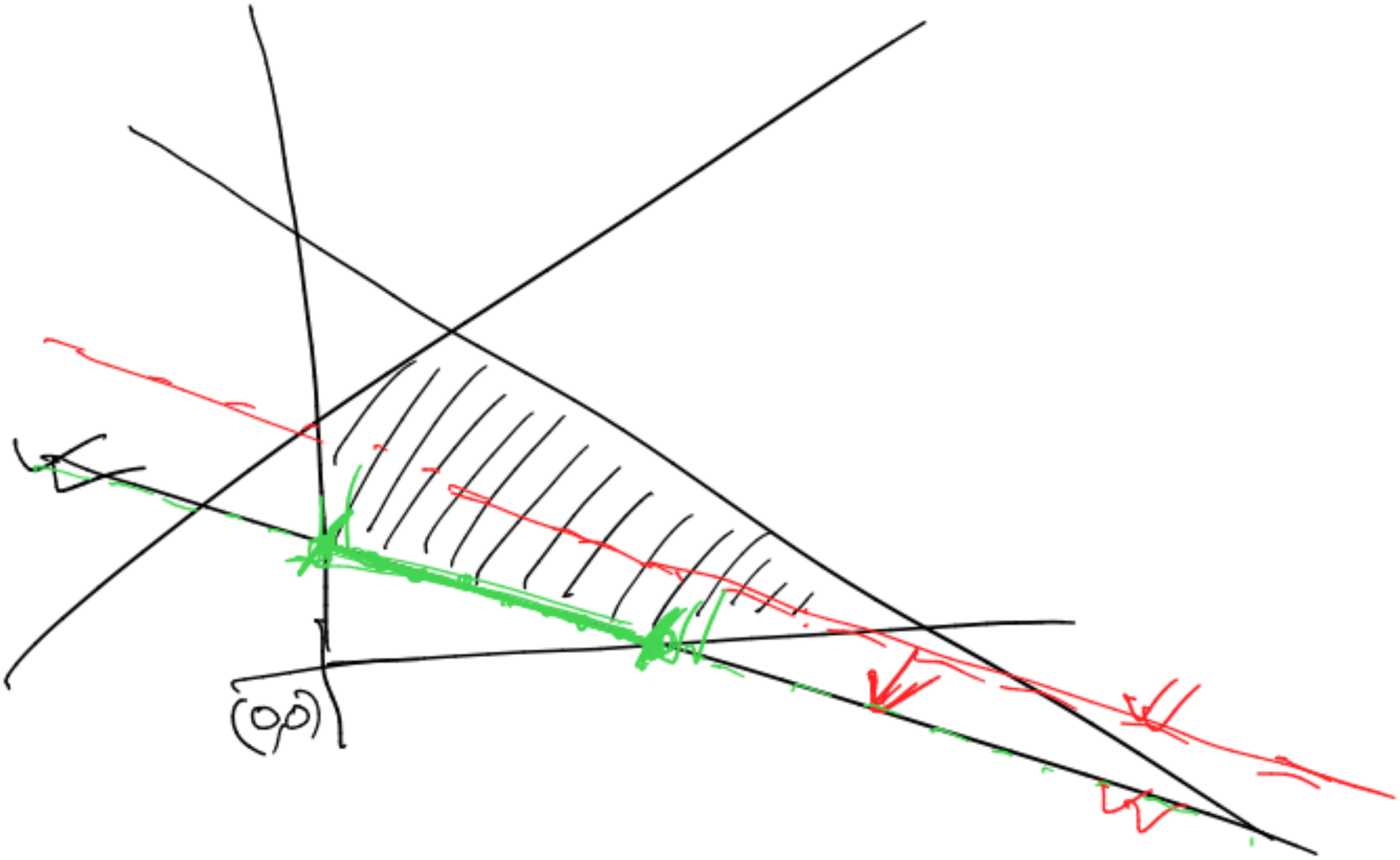


optimise $\boxed{c_1 x + c_2 y}$

$$K = Lcm(c_1, c_2)$$

Introduction to optimisation
Chong and Zak

Method 2 Iso profit / Iso cost method

we have the LPP.

optimize $Z = c_1 x + c_2 y$

minimise maximise s.t. $a_{11}x + a_{12}y \ (\leq = \geq) \ b_1$.

$a_{21}x + a_{22}y \ (\leq = \geq) \ b_2$.

$\vdots \qquad \vdots$

$x, y \geq 0$

We already computed the feasible region in step 1.

Step 2.1 consider the objective function $Z = c_1 x + c_2 y$

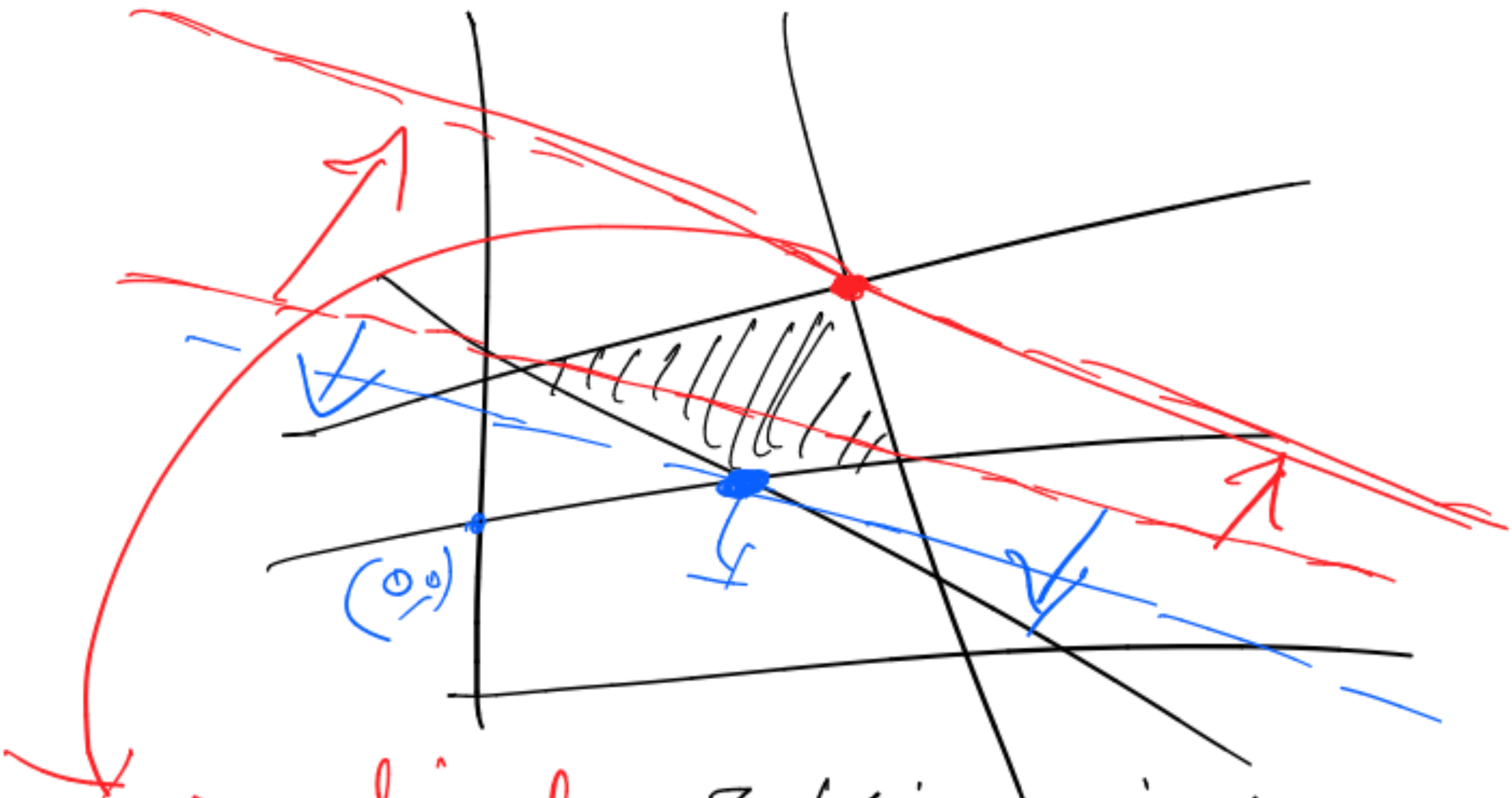
we draw the line $\boxed{K = c_1 x + c_2 y}$

$\boxed{\text{Choose } K = \mathrm{Lcm}(c_1, c_2)}$

Step 2.2

Maximization

- we draw line farthest from the origin.
- The line should contain at least one point of the feasible region



coordinate of this point is the solution.

and value of the objective function at this point is optimum value.

Maximise $Z = 2x_1 + x_2$

s.t. $x_1 + x_2 \geq 5$ — (1)

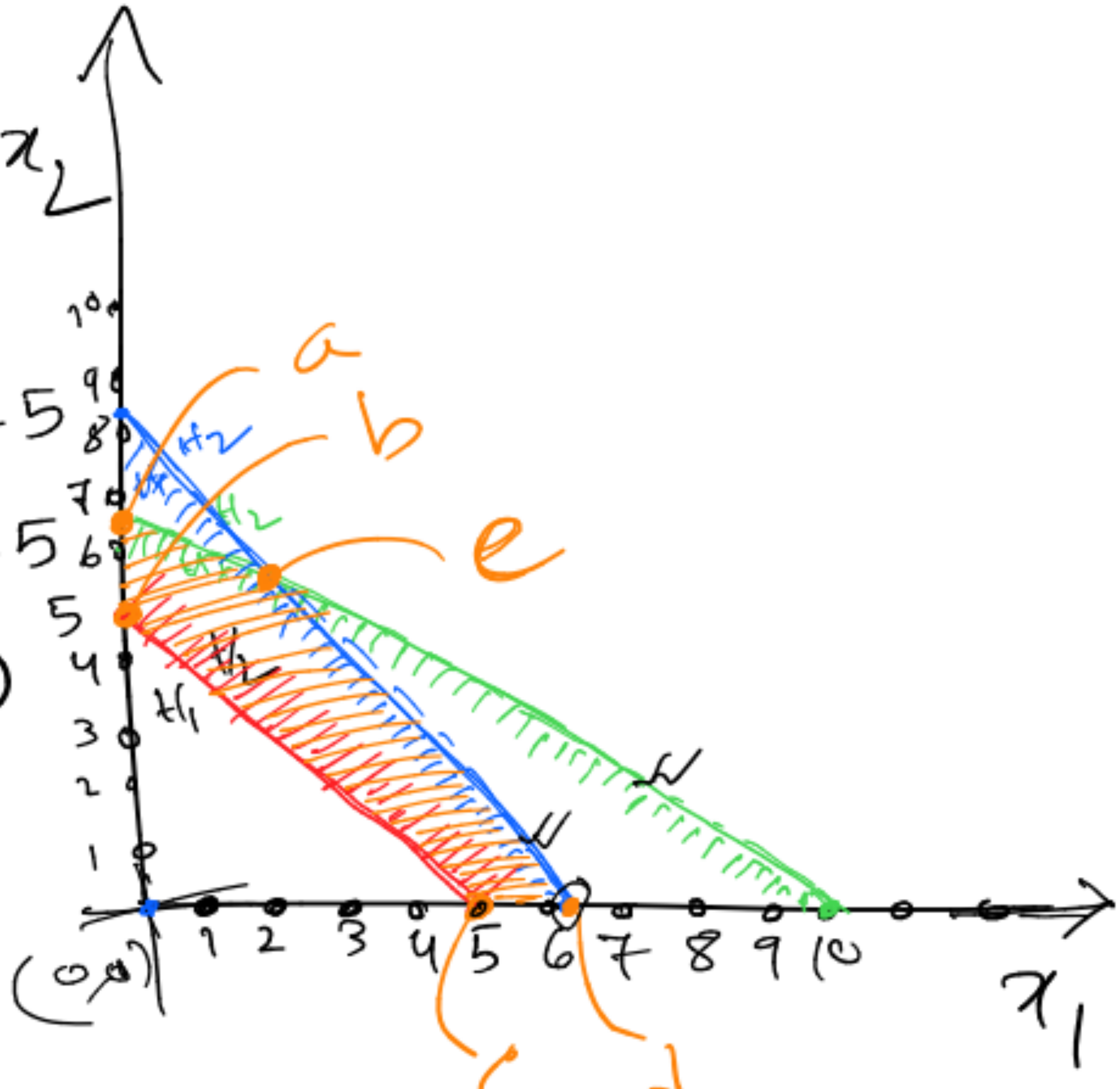
$2x_1 + 3x_2 \leq 20$ — (2) ✓✓✓

$4x_1 + 3x_2 \leq 25$ — (3) ✓✓

$\boxed{x_1, x_2 \geq 0}$

Solution

We consider the 1st quadrant of the x_1x_2-plane since both x_1 and x_2 are ≥ 0



Constraint 1

$x_1 + x_2 = 5$

when $x_1 = 0$, $x_2 = 5$

when $x_2 = 0$, $x_1 = 5$

$(0,5)$ and $(5,0)$

Find the shaded region.

$0 + 0 \geq 5$

not true.

constraint 2

$$2x_1 + 3x_2 = 20$$

when $x_1 = 0$, $x_2 = \frac{20}{3} = 6.666$

when $x_2 = 0$, $x_1 = \frac{20}{2} = 10$

$(0, 6.666)$ and $(10, 0)$

$0 + 0 \leq 20$ true.

constraint 3

$$4x_1 + 3x_2 = 25$$

when, $x_1 = 0$, $x_2 = \frac{25}{3} = 8.333$

when $x_2 = 0$, $x_1 = \frac{25}{4} = 6.25$

$(0, 8.333)$ and $(6.25, 0)$

$0 + 0 \leq 25$ true.

Finding the optimal solution

Method 1 extreme/corner point method

corner points	value at the obj. fun. $\boxed{(2x_1+x_2)}$
a $\left(0, \frac{20}{3}\right)$	$\frac{20}{3} = 6.666$
b $(0, 5)$	5
c $(5, 0)$	10
d $\left(\frac{25}{4}, 0\right)$	$\frac{25}{2} = 12.5$ maximum
e $\left(\frac{5}{2}, 5\right)$	10

$$2x_1 + 3x_2 = 20$$
$$4x_1 + 3x_2 = 25$$
$$\Rightarrow 3x_2 = 25 - 4x_1$$

$$2x_1 + 25 - 4x_1 = 20$$
$$\Rightarrow -2x_1 = -5$$
$$\Rightarrow x_1 = 5/2$$
$$x_2 = \frac{25 - \frac{20}{2}}{3} = \frac{\frac{30}{2}}{3} = \frac{15}{3} = 5$$

Optimum value of the LPP is

$$\frac{25}{2} = 12.5$$

and the solution is,

$$x_1 = \frac{25}{4}, \quad x_2 = 0$$

<u>Method 2</u>: Iso cost / Iso profit method

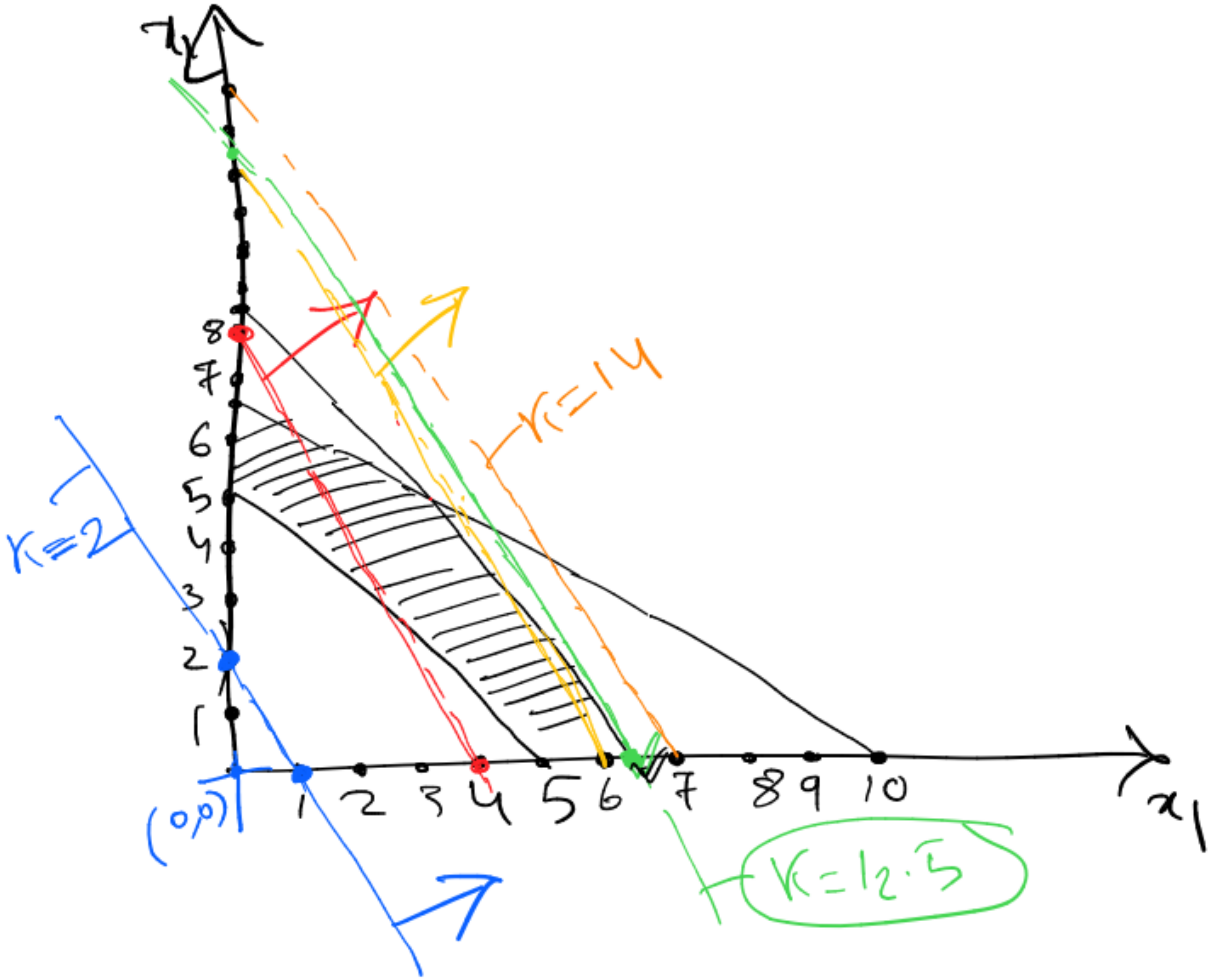
Let us consider the objective function line

$$2x_1 + x_2 = 2$$

When $x_1 = 0$, $x_2 = 2$

When $x_2 = 0$, $x_1 = 1$

$(0, 2)$ and $(1, 0)$



$K=8$ objective function line

$2x_1+x_2=8$

when, $x_1=0$, $x_2=8$

when $x_2=$, $x_1=4$

$(0,8)$ and $(4,0)$

$K = 12$: $2x_1 + x_2 = 12$

when, $x_1 = 0$, $x_2 = 12$

$x_2 = 0$, $x_1 = 6$

$(0, 12)$ $(6, 0)$

$K = 14$: $2x_1 + x_2 = 14$

$(0, 14)$ and $(7, 0)$

$K = 12.5$: $2x_1 + x_2 = 12.5$

$(0, 12.5)$, $(6.25, 0)$

optimal value is 12.5

and optimal solution is.

$(6.25, 0)$

ie, $x_1 = 6.25$ and $x_2 = 0$