Lecture -22

Recap:

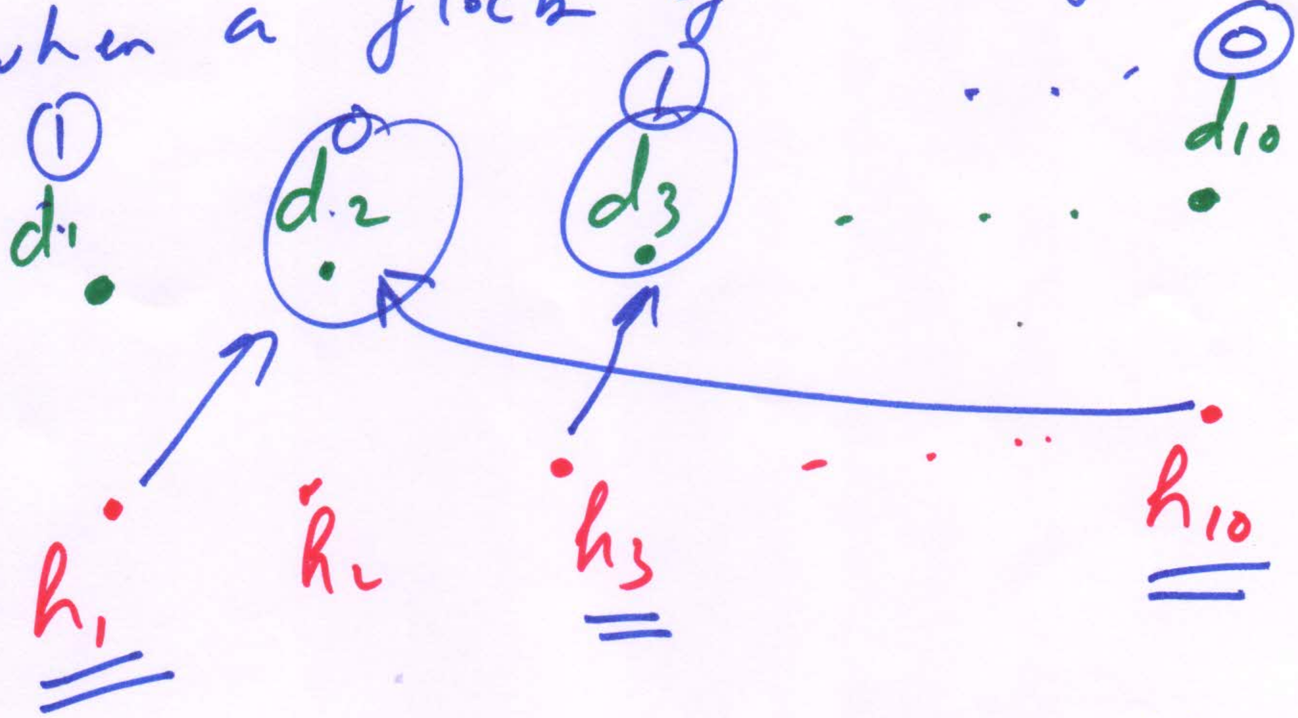
Expectation of Sum =
Sum of Expectations

Random variables X, Y, Z
They maybe independent,
they maybe dependent.

$$E[X+Y+Z] = E[X] + E[Y] + E[Z]$$

E.g. Ten hunters, who are waiting for ducks to fly by. All the 10 hunters shoot at the same time. Each hunter chooses his target at random and independently of other hunters.

Each hunter hits his target with probability $1/3$.
Compute the expected no. of ducks that escape unhurt, when a flock of 10 flies by.



Define random variables
$\{X_1, X_2, \dots, X_{10}$

$X_1 = 1$ if duck 1 is unhurt
$X_1 = 0$ if duck 1 is hurt/killed.

Y = total no. of ducks that escape unhurt.

$$\underbrace{y = x_1 + x_2 + \cdots + x_{10}}$$

This is essentially the 1st step in solving this problem.

$$E[Y] = E[X_1 + X_2 + \cdots + X_{10}]$$

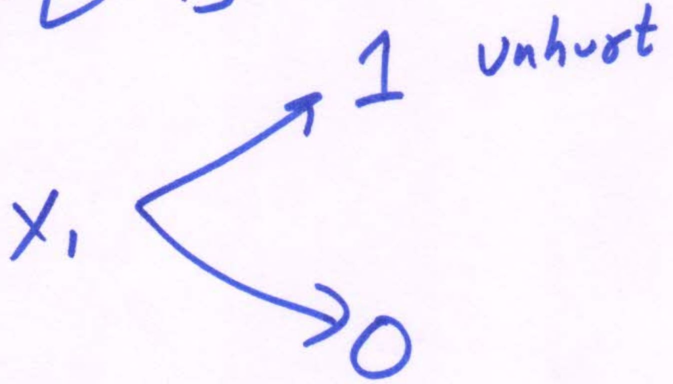
sum of expectations = expectation of sum

$$E[Y] = E[X_1] + E[X_2] + \cdots + E[X_{10}]$$

$$= \sum_{i=1}^{10} E[X_i]$$

What is $E[X_1]$?

$$E[X_1] = ? = 1 \cdot p(X_1 = 1) + 0 \cdot p(X_1 = 0)$$



$= p(X_1 = 1) =$ This is the probability that duck no. 1 is unhurt.

$P($ duck no. $\underline{\underline{1}}$ is unhurt$)$

$$= \left(1 - \underbrace{\frac{1}{10} \cdot \frac{1}{3}}\right)^{10}$$

Each hunter

$$E[X_1] = \left(1 - \frac{1}{30}\right)^{10} = \left(\frac{29}{30}\right)^{10}$$

$$E[Y] = 10 \cdot E[X_1]$$
$$= 10 \cdot \left(\frac{29}{30}\right)^{10} \approx 7.$$

Coupon Collecting Problem

N types of coupons that you want to collect, atleast 1 of each type.

$\underline{X}$ = no. of items that you purchase in order to collect N coupons. $E[X]$

X is clearly a random variable, with a minimum value of N.

A, B, C, D, E

$X = 14$

$X_1 \cancel{X} = 1$

A A A B A B A C A B C A D E

X_1 = no. of ~~items~~ that one needs to ~~buy~~ in order to get coupon no. ~~1~~

$X_1 = 1$, ~~$X_2 = 4$~~, $X_3 = 8$, $X_4 = 13$, $X_5 = 14$

~~A 5t~~

Assume that you already have i distinct coupons.

X_i = no. of ADDITIONAL coupons that you need to buy in order to get a new type of coupon

$\cancel{X_1 = 1}$

	Already have
$X_0 = 1$	none
$X_1 = 3$	A
$X_2 = 4$	A, B
$X_3 = 5$	A, B, C
$X_4 = 1$	A, B, C, D

$X = \sum X_i = 14$

$$E[X] = E[X_0 + \dots + X_{N-1}]$$

$$= \sum_{i=0}^{N-1} E[X_i]$$

X_1

What is $E[X_1]$?

You have 1 type of coupon. How many more do you need to buy in order to get a coupon of a different type?

$X_0 = 1$

$E[X_0] = 1$

A A A A A A A A - - - B

X_1	probability	Event
1	$\frac{N-1}{N}$	AB, AC, AD, ..., A...
2	$\frac{1}{N} \cdot \frac{N-1}{N}$	AAB, AAC, AAD, ...
3	$\frac{1}{N} \cdot \frac{1}{N} \cdot \frac{N-1}{N}$	AAAB, AAAC

Geometric Random Variable