Tutorial 7

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1) Find E[X] and Var(X) when X is a normal random variable with parameters μ and σ^2

2) An expert witness in a paternity suit testifies that the length (in days) of human gestation is approximately normally distributed with parameters $\mu = 270$ and $\sigma^2 = 100$. The defendant in the suit is able to prove that he was out of the country during a period that began 290 days before the birth of the child and ended 240 days before the birth. If the defendant was, in fact, the father of the child, what is the probability that the mother could have had the very long or very short gestation indicated by the testimony?

3) To determine the effectiveness of a certain diet in reducing the amount of cholesterol in the bloodstream, 100 people are put on the diet. After they have been on the diet for a sufficient length of time, their cholesterol count will be taken. The nutritionist running this experiment has decided to endorse the diet if at least 65 percent of the people have a lower cholesterol count after going on the diet. What is the probability that the nutritionist endorses the new diet if, in fact, it has no effect on the cholesterol level?

4) Let X be an exponential random variable with parameter λ. Calculate
   a) E[X]
   b) Var(X)

5) Let X be a random variable with probability density function
$$f(x) = \begin{cases} c(1-x^2), & -1 < x < 1 \\ 0, & otherwise \end{cases}$$
   a) What is the value of c?
   b) What is the cumulative distribution function of X?

6) If X is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute
   a) $P\{X > 5\}$;
   b) $P\{4 < X < 16\}$;
   c) $P\{X < 8\}$;
   d) $P\{X < 20\}$;
   e) $P\{X > 16\}$.