Transportation Problem

Solving Transportation Problem

Two steps.

1) step 1

Finding initial basic feasible solution

2) step 2.

i) verifying whether the initial basic feasible solution is optimal or not

ii) a) If it is optimal then we are done.

b) If not then we modify the initial basic feasible solution towards an optimal solution.

Step 1: Finding initial basic feasible solution

Methods to find an initial basic feasible solution.

1) North-west corner rule
2) Row minimum method
3) column minimum method
4) matrix minimum method.
5) Vogel's approximation method (VAM)

or,

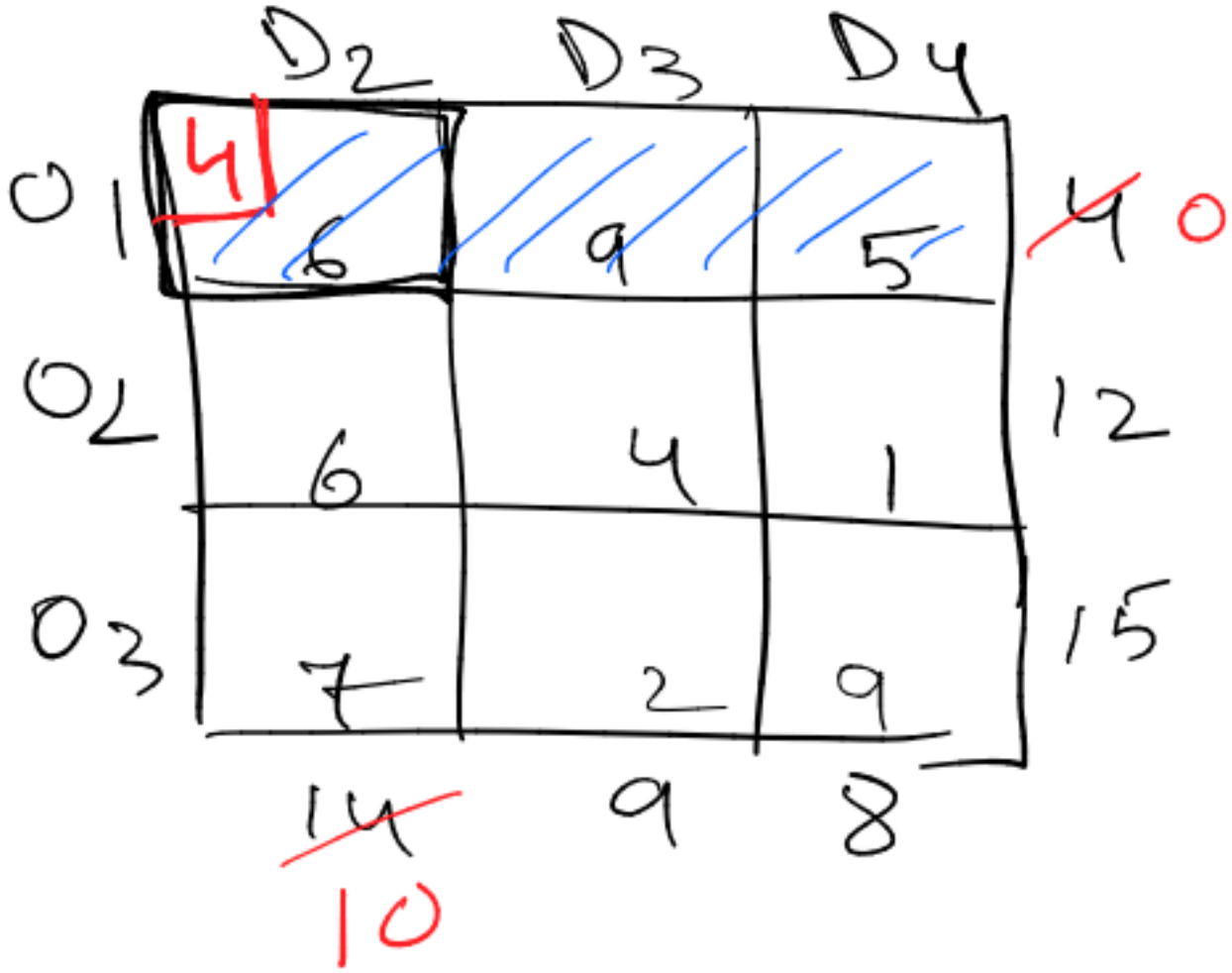
Unit penalty method.

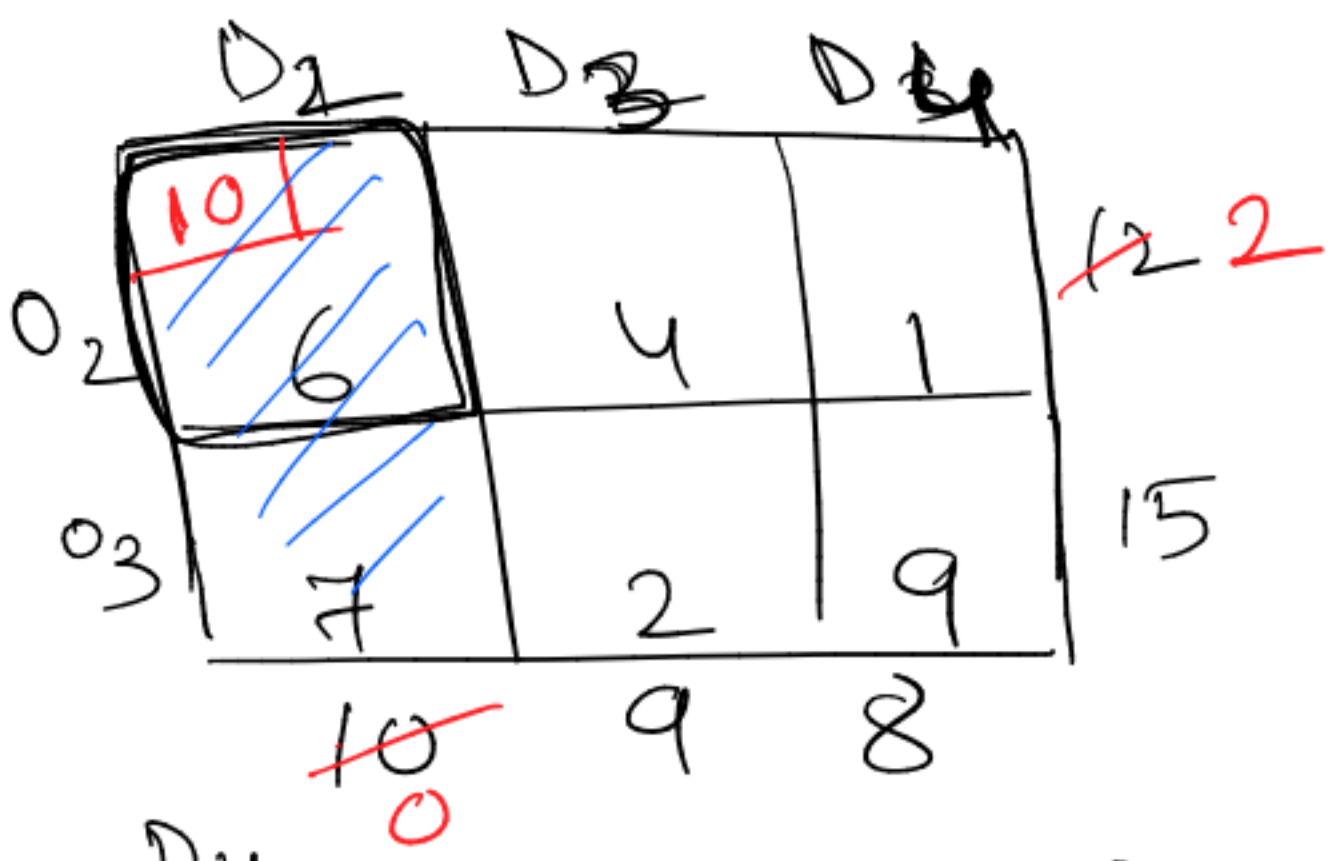
1) North-west corner rule:

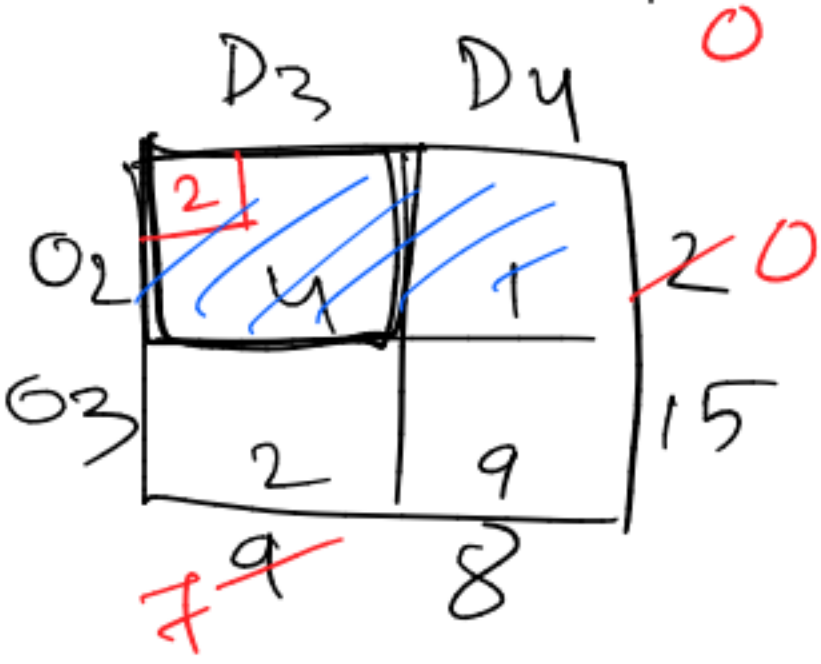
Example

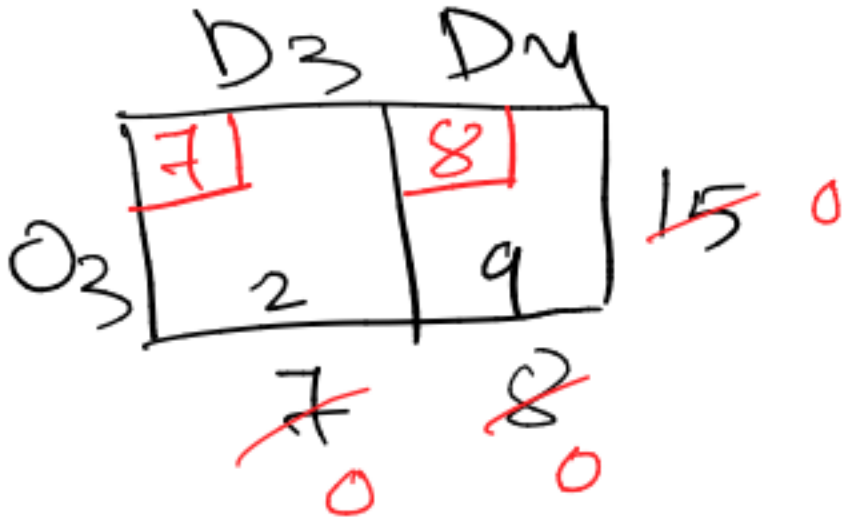
	D_1	D_2	D_3	D_4	a_i
O_1	4	6	9	5	16
O_2	2	6	4	1	12
O_3	5	7	2	9	15
b_j	12	14	9	8	43 / 43

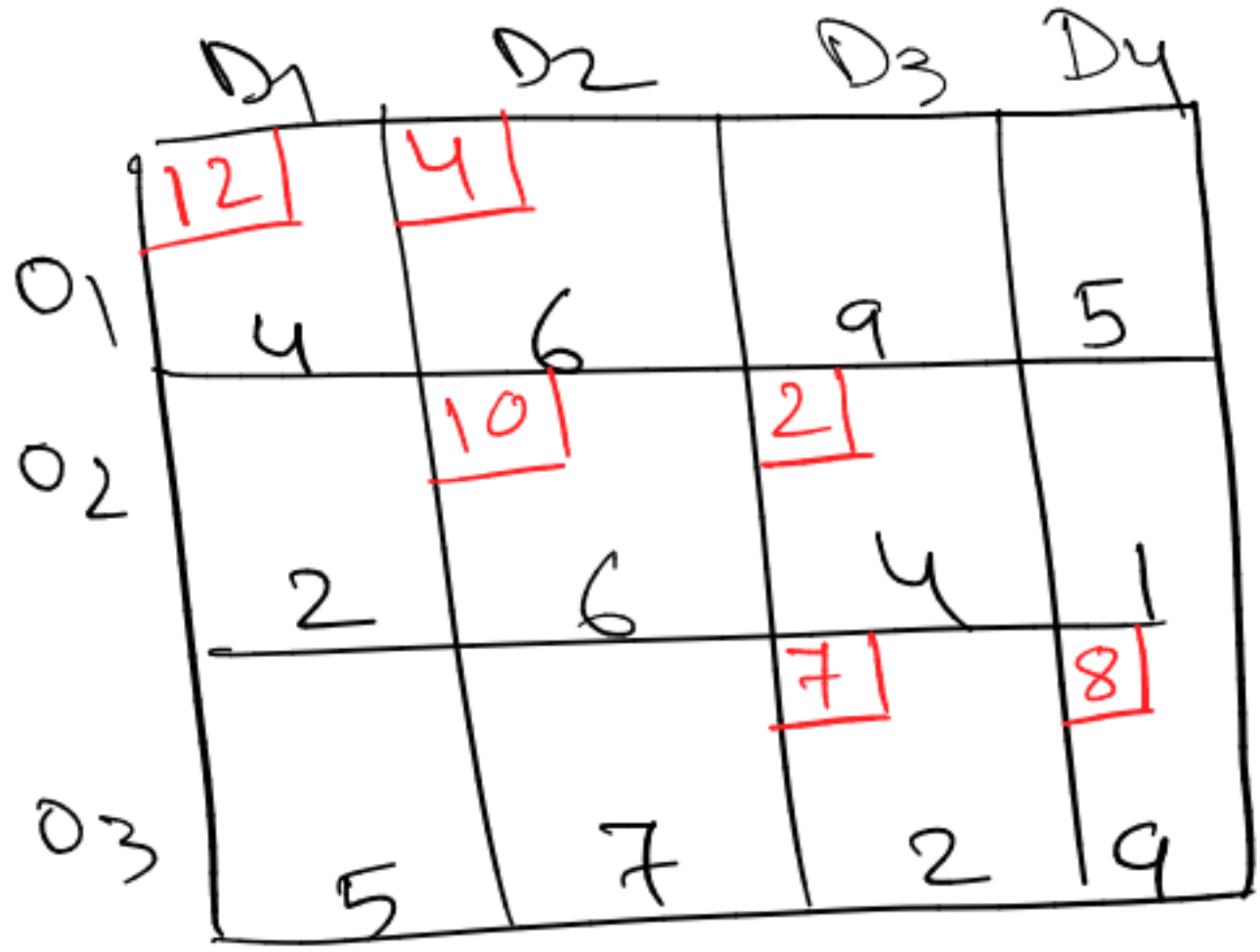
	D_1	D_2	D_3	D_4	a_i
O_1	12 → 4	6	9	5	~~16~~ 4
O_2	2	6	4	1	12
O_3	5	7	2	9	15
b_j	~~12~~ 0	14	9	8	


D2 D3 D4
O1
O2
O3
4
6 9 5
6 4 1
7 2 9
14
10
9
8
4
0
12
15


D2 D3 D4
O2
O3
10
6 4 1
7 2 9
10
0
9
8
12
2
15


D3 D4
O2
O3
2
4 1
2 9
7
9
8
20
15


D3 D4
O3
7
8
2 9
7
0
8
0
15
0



solution is.

$x_{11} = 12$, $x_{12} = 4$, $x_{22} = 10$, $x_{23} = 2$

$x_{33} = 7$, $x_{34} = 8$

optimum value:

$12 \times 4 + 4 \times 6 + 10 \times 6 + 2 \times 4 + 7 \times 2 + 8 \times 9$

$= 48 + 24 + 60 + 8 + 14 + 72$

$=$

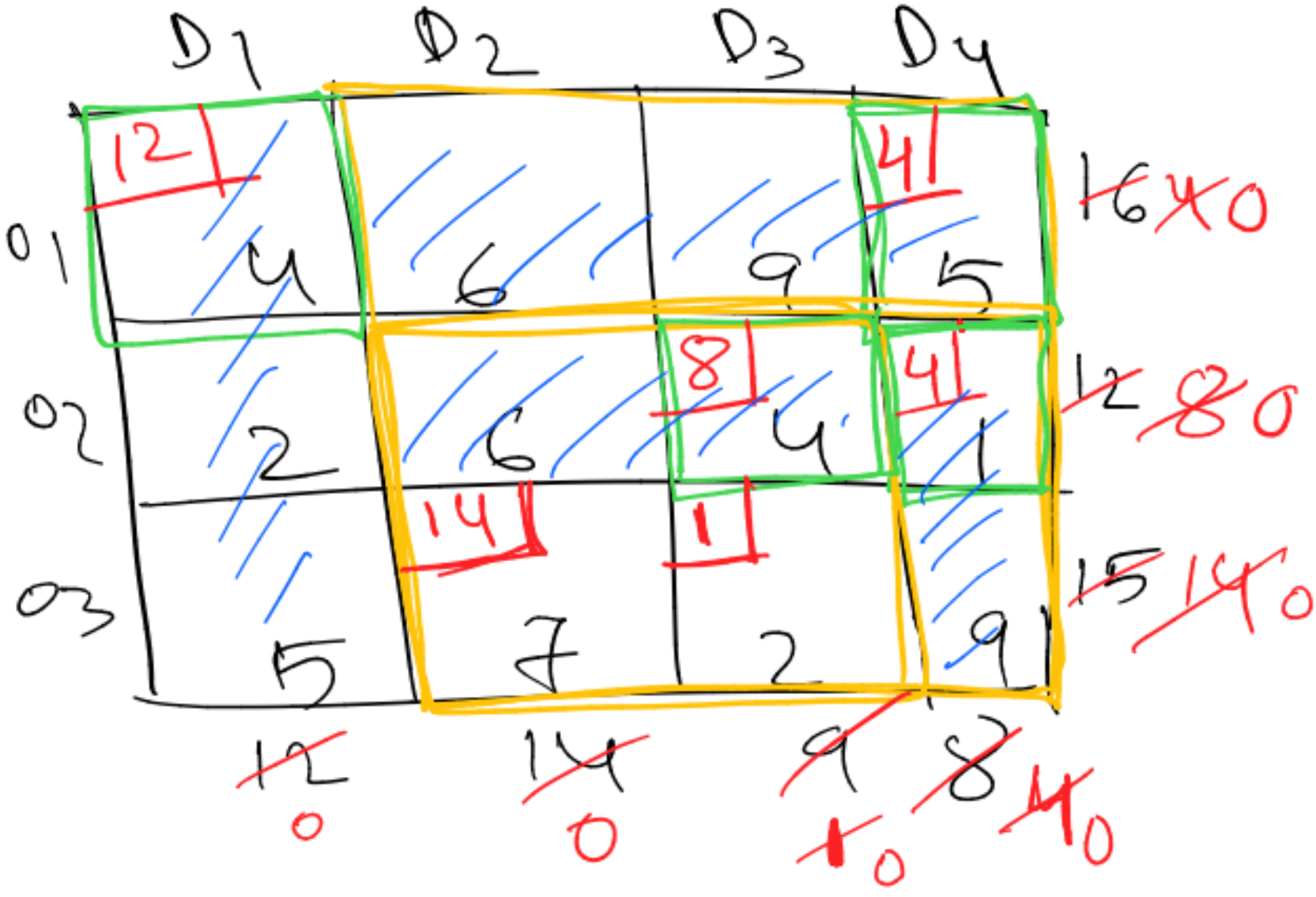
Here $m + n - 1 = 4 + 3 - 1 = 6$

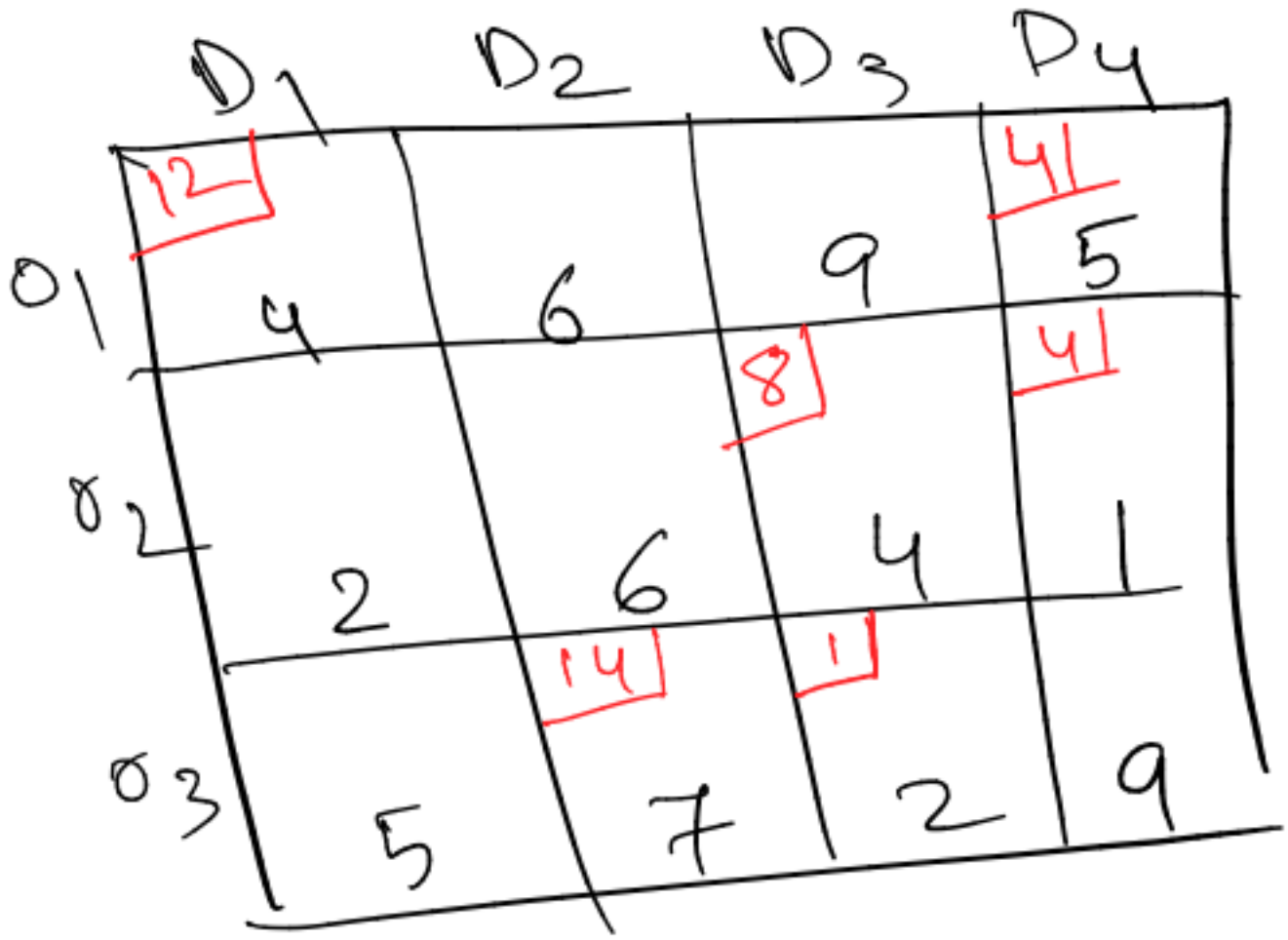
So the solution is non-degenerate.

2> Row-minimum method

Example:

	D_1	D_2	D_3	D_4	
9	4	6	9	5	16
O_2	2	6	4	1	12
O_3	5	7	2	9	15
	12	14	9	8	





Solution is,

$x_{11}=12,\ x_{14}=4,\ x_{23}=8,\ x_{24}=4$

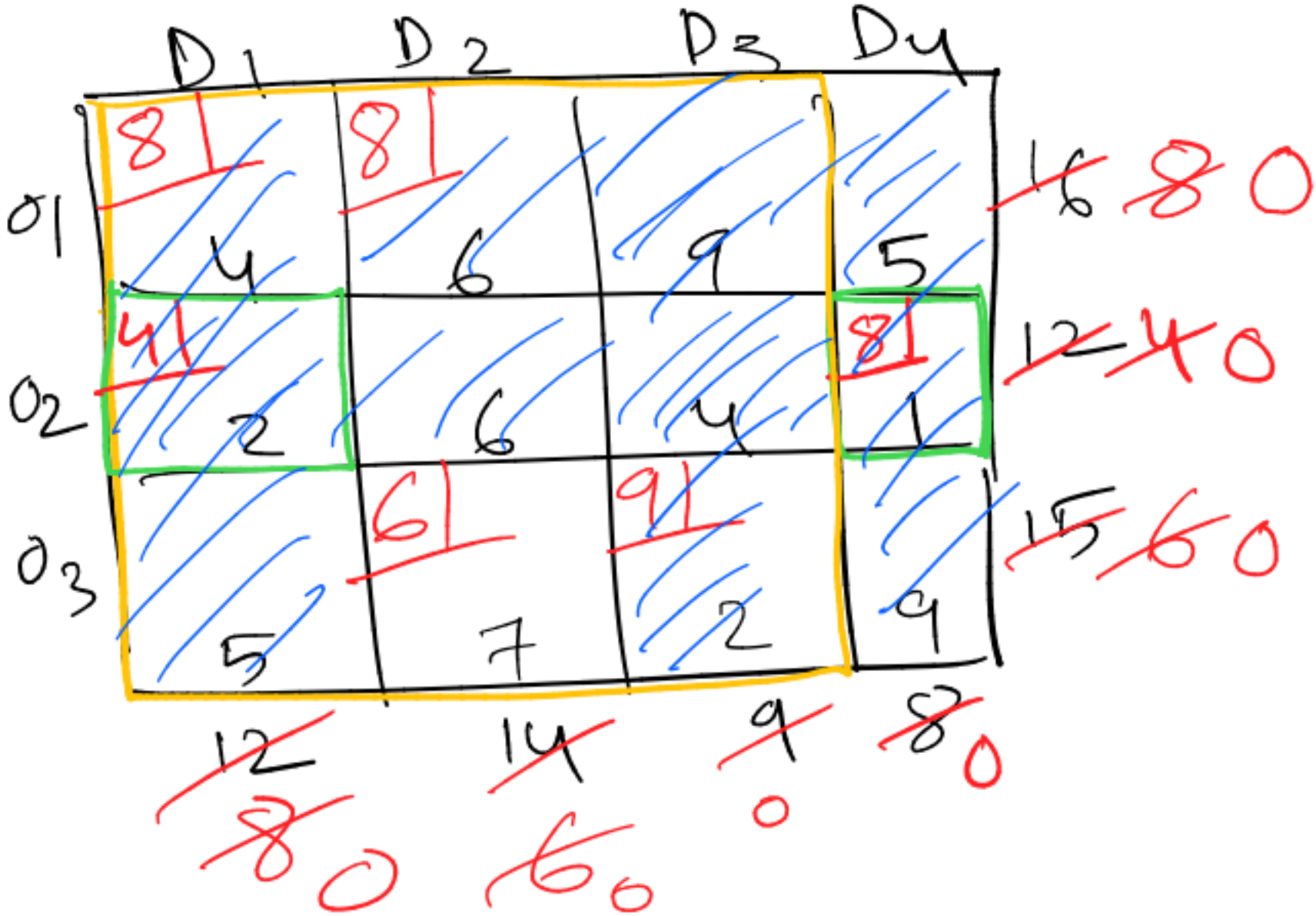
$x_{32}=14,\ x_{33}=1$

optimum value:

$12\times4+4\times5+8\times4+4\times1+14\times7+1\times2$

$=48+20+32+4+98+2$

4) Matrix minimum method



3> Vogel's approximation method (VAM)

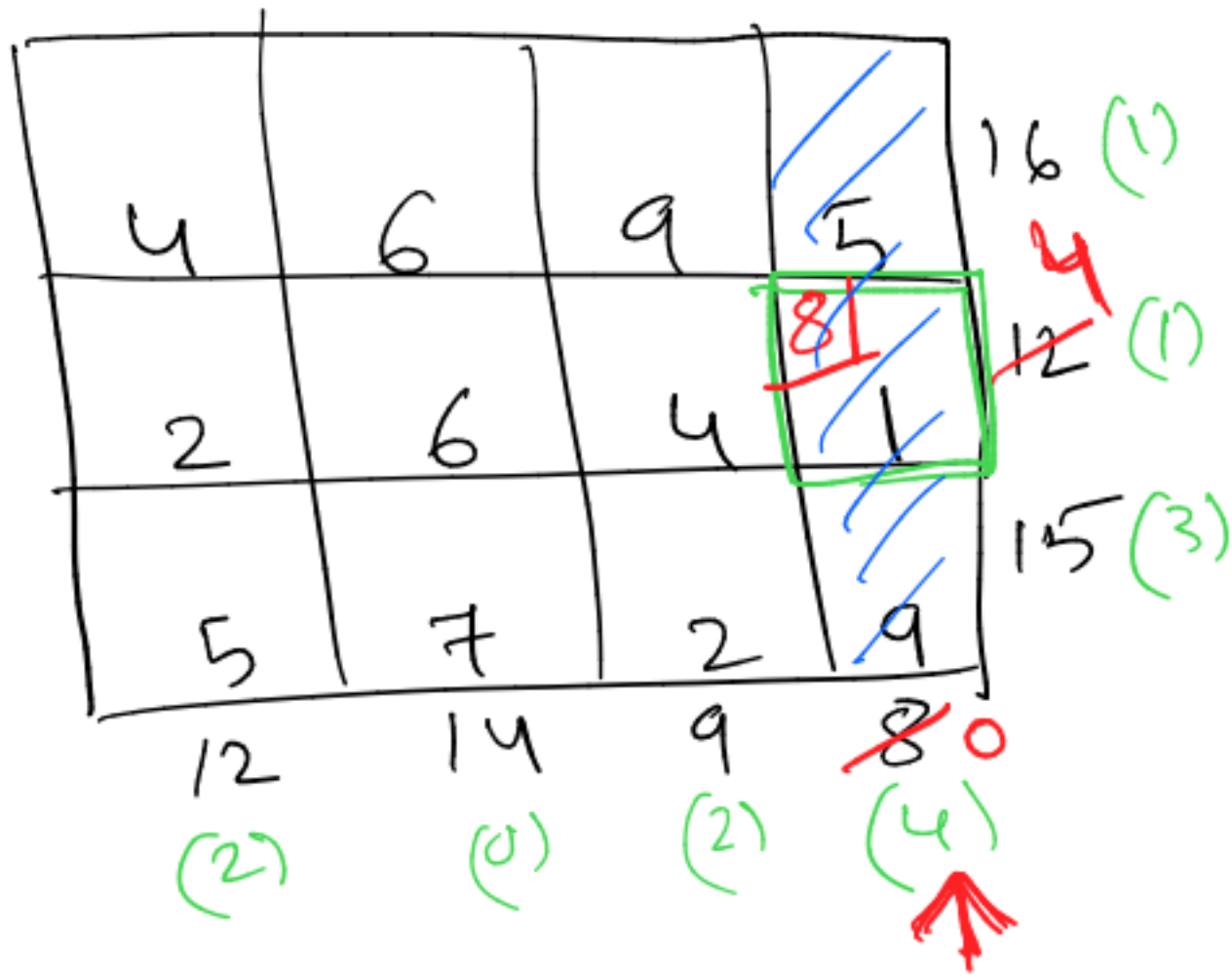
Example

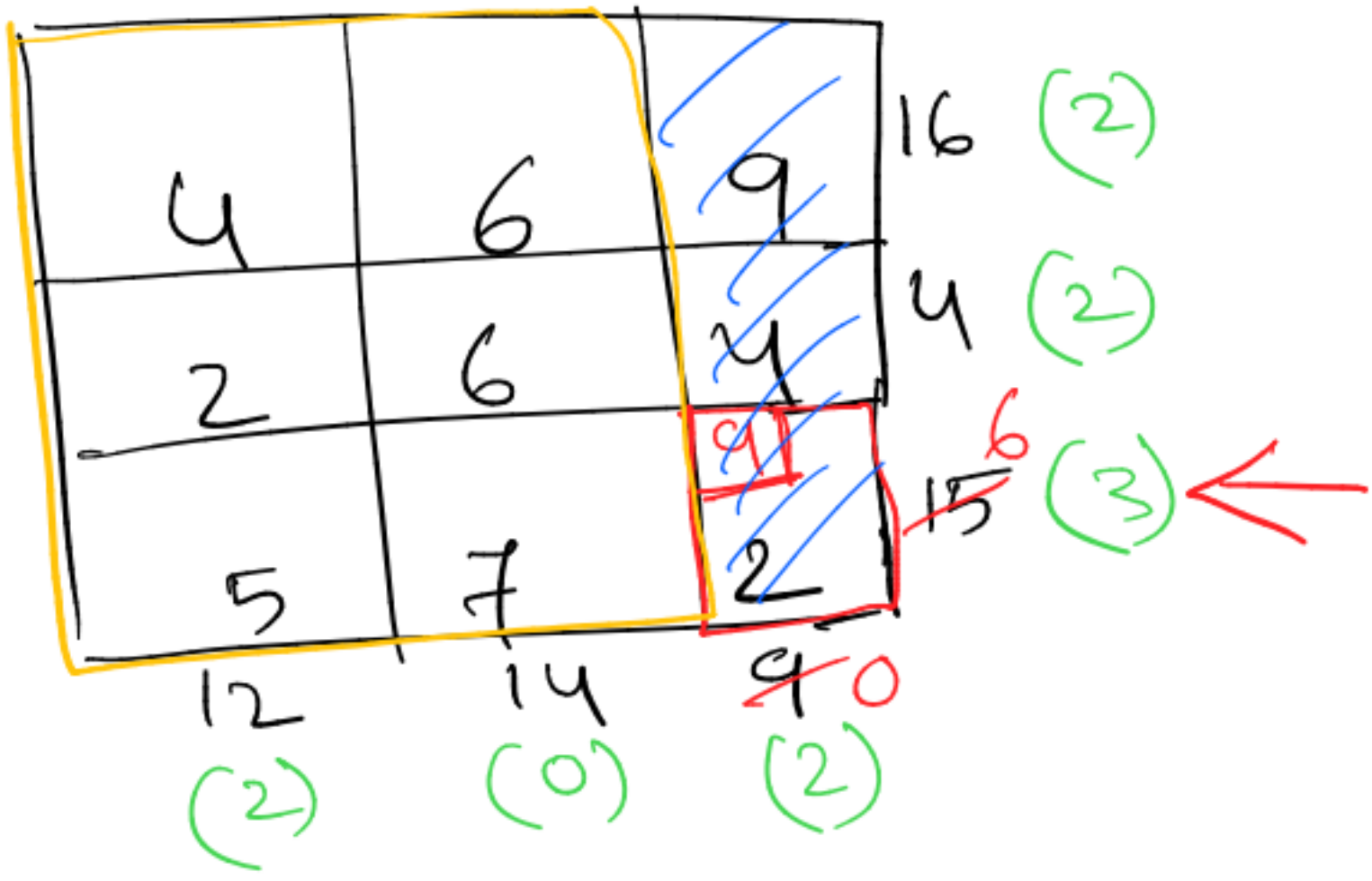
	D_1	D_2	D_3	D_4	
O_1	4	6	9	5	16
O_2	2	6	4	1	12
O_3	5	7	2	9	15
	12	14	9	8	

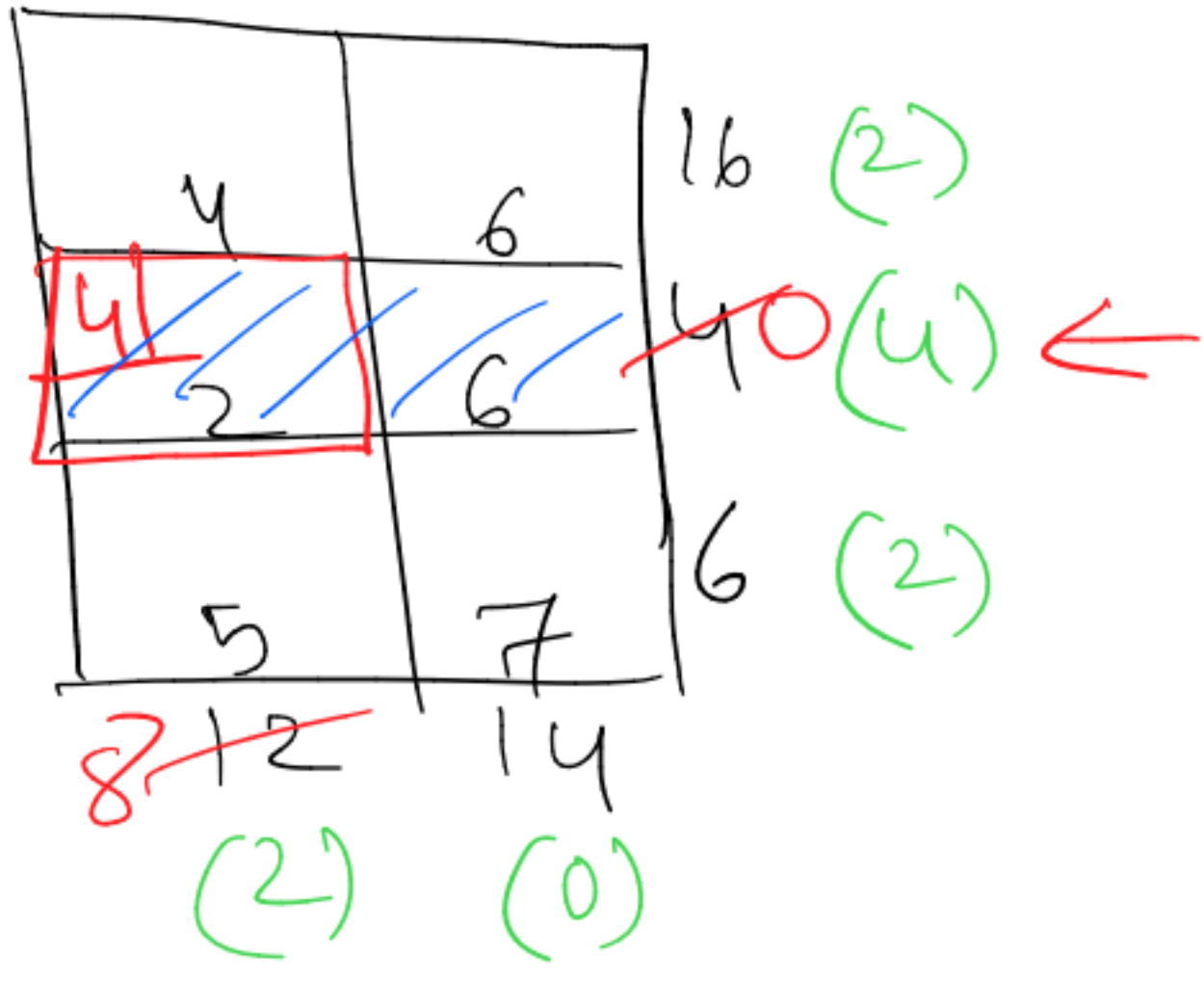
1. For each row (or column) select the lowest and second lowest costs and compute their difference.

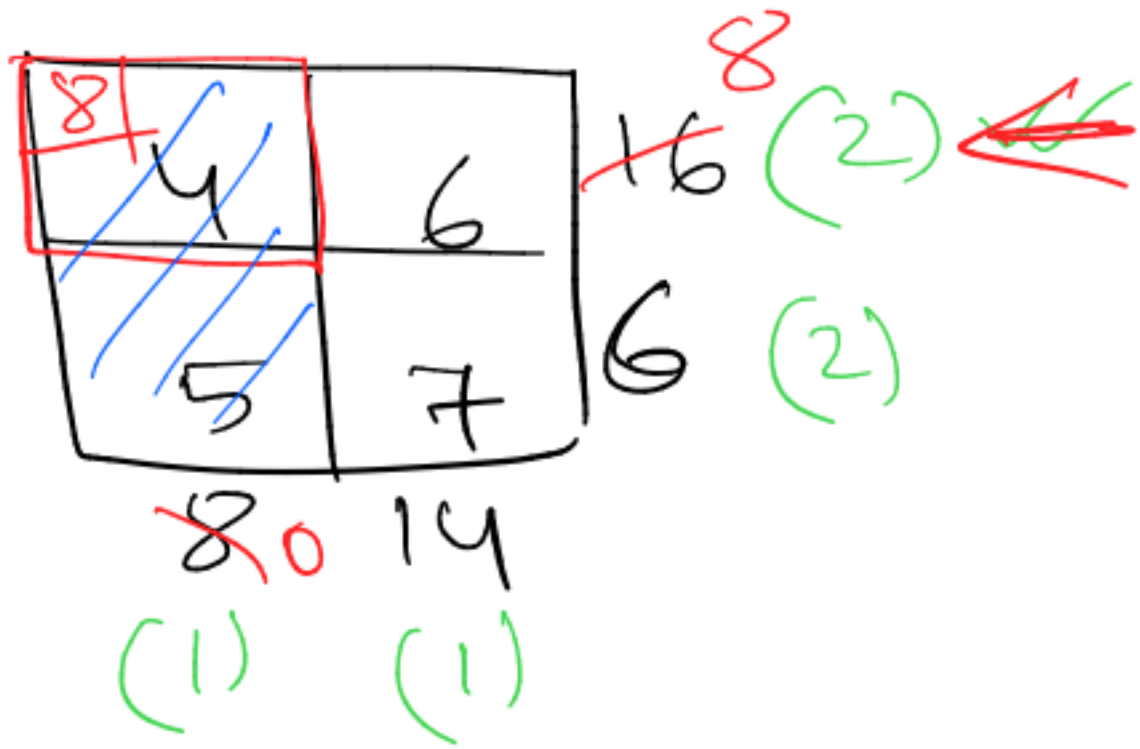
	4	6	9	5	16 (1)
	2	6	4	1	12 (1)
	5	7	2	9	15 (3)
	12	14	9	8	
	(2)	(0)	(2)	(4)	

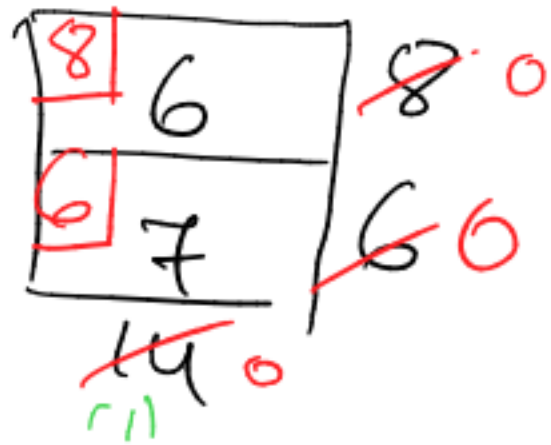
2. Find the largest difference.

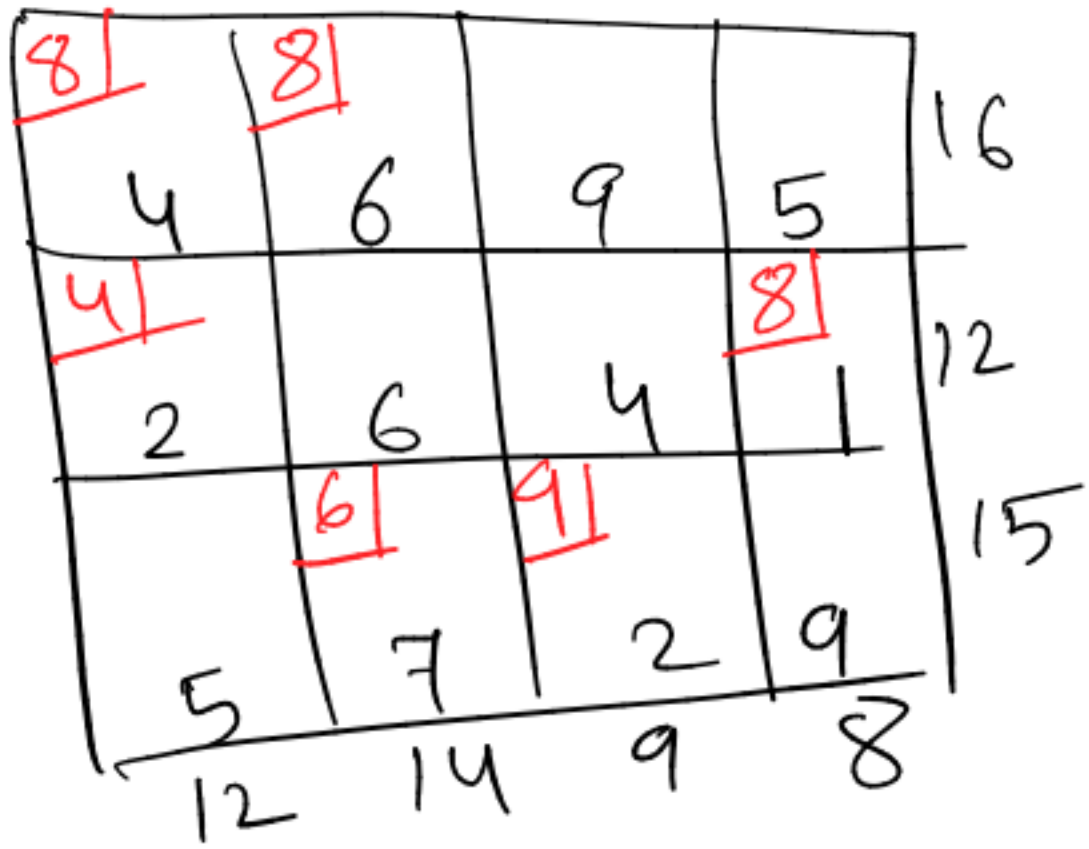





4
6
4
2
6
5
7
16
(2)
40
(4)
6
(2)
8 12
14
(2)
(0)


8
4
6
5
7
16
8
(2)
6
(2)
8 0
14
(1)
(1)


8
6
6
7
8 0
6 0
14 0
(1)



Solution:

$x_{11}=8, x_{12}=8, x_{21}=4, x_{24}=8, x_{32}=6, x_{33}=9$

Optimum value is,

$8\times4+8\times6+4\times2+8\times1+6\times7+9\times2$

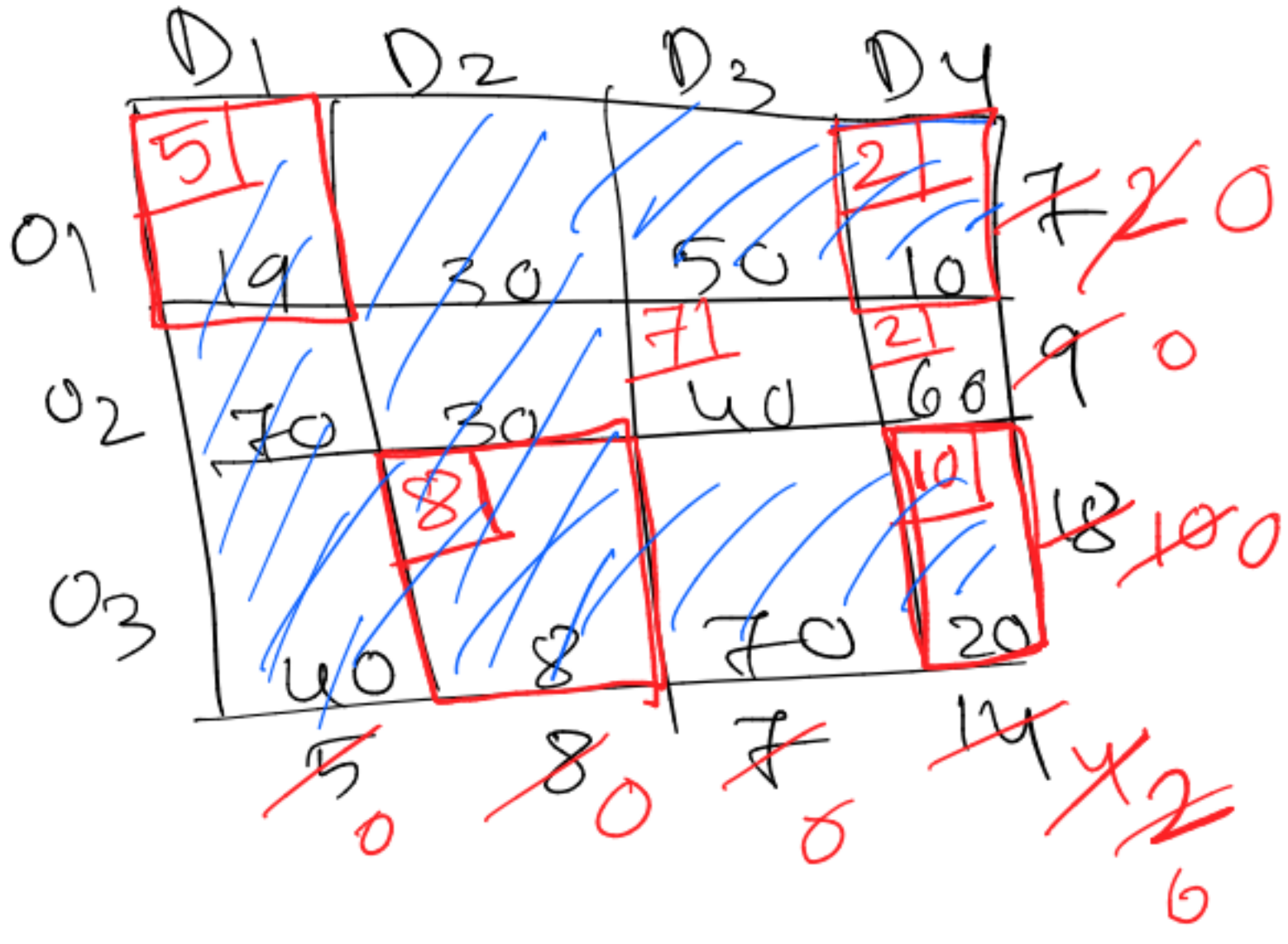
$=32+48+8+8+42+18$

Example use VAM to get an ibfs.

	D_1	D_2	D_3	D_3	
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
	5	8	7	14	

Solve it

Solution



Transportation Problem: Loop

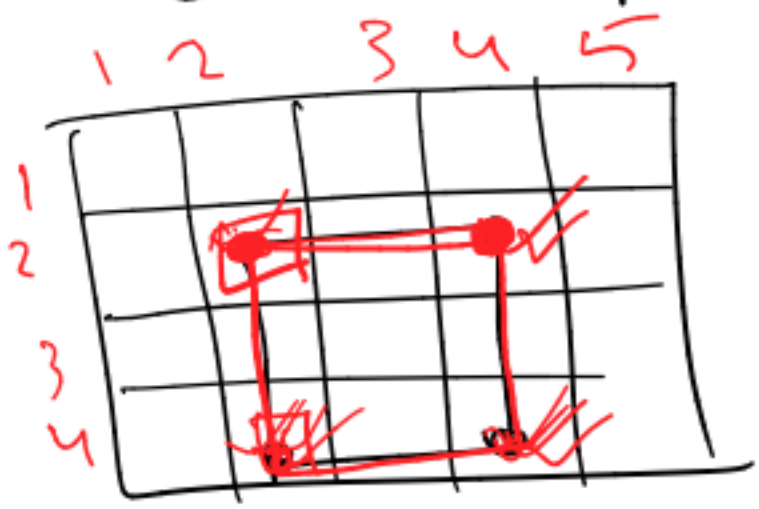
A loop is an ordered sequence of at least 4 different cells if it satisfies the following conditions.

i) Any two consecutive cells lie in either the same row or same column.

ii) No three or more consecutive cells lie in the same row or same column.

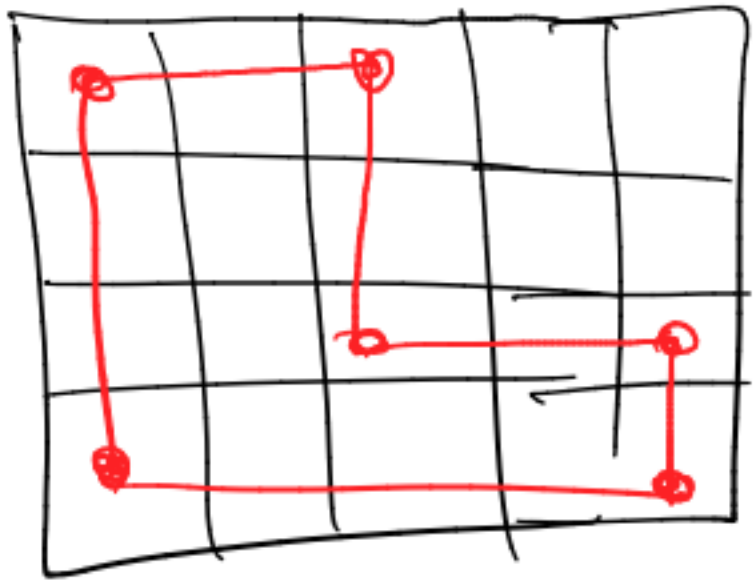
iii) The start and the end cells should be same row or same column.

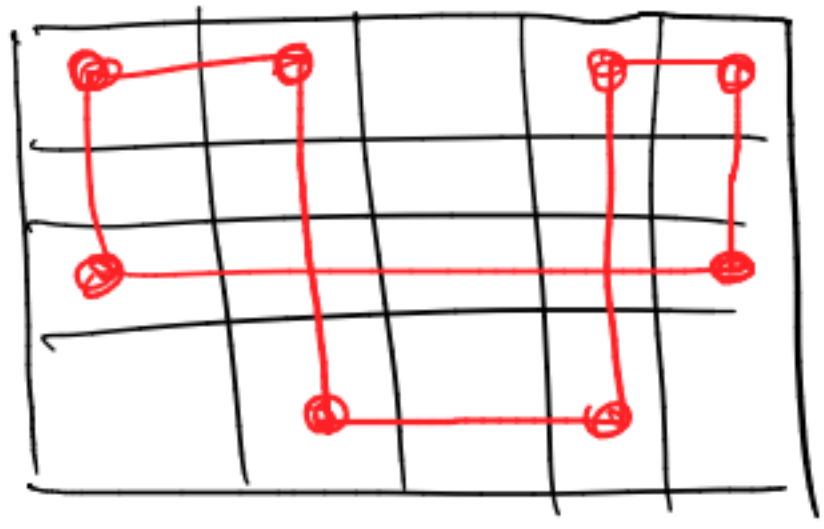
A loop can be considered as a closed path.

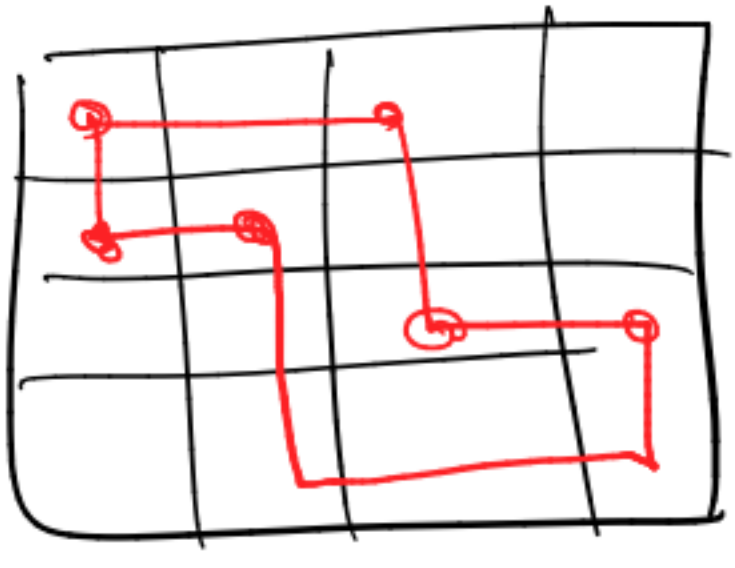


Loop:

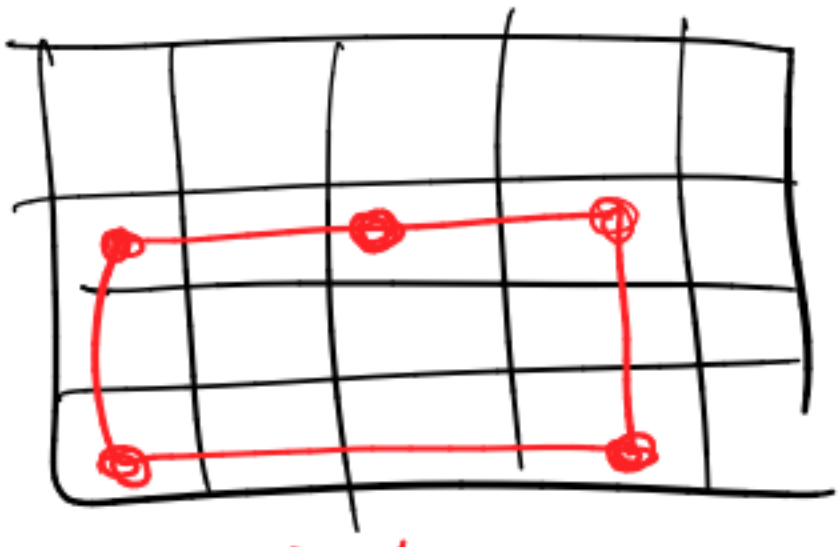
(2,2) → (2,4) → (4,4) ↓ (4,2)

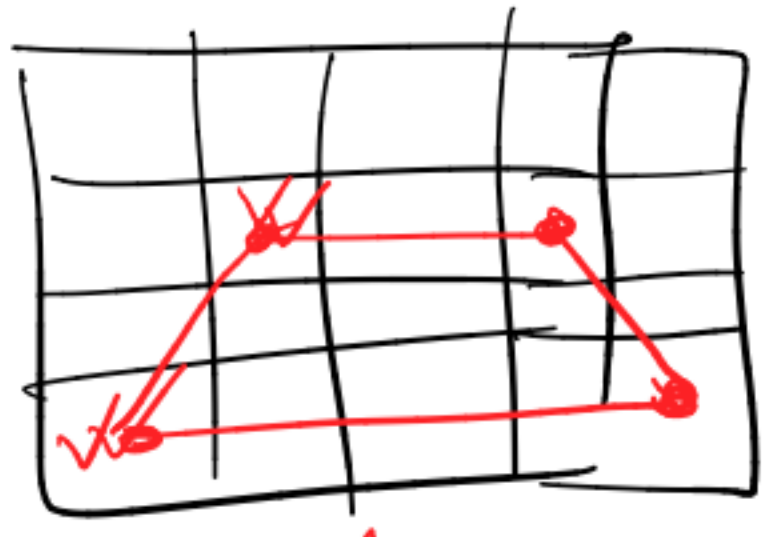






valid locks





not valid locks.