

1. Find the electric field a distance z above the center of a flat circular disc of radius a which carries a uniform surface charge density σ . Work out the limits $z \ll a$ and $z \gg a$.
2. If $\vec{E} = kr^3\hat{r}$ in a region find the charge density in the region.
3. A hollow spherical shell carries a charge density $\rho = k/r^2$ in the region $a \leq r \leq b$. Find the electric field in the three regions ,
 $r < a$, $a < r < b$, $r > b$.
4. Two infinite parallel planes carry equal but opposite uniform charge densities $\pm\sigma$. Find the electric field in the three regions, into which the planes partition the space.

5. Evaluate

(a) $\int (r^2 + \vec{r} \cdot \vec{a} + a^2) \delta^3(\vec{r} - \vec{a}) dV$ over the whole space where \vec{a} is a fixed vector.

(b) $\int_V |\vec{r} - \vec{b}|^2 \delta^3(5\vec{r}) dV$ over a cube of side 2, centered at the origin, and $\vec{b} = 4\hat{y} + 3\hat{z}$

6. We have seen that $\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) = 4\pi\delta^3(\vec{r})$. In a similar manner justify that

$$\vec{\nabla} \cdot \begin{pmatrix} \hat{s} \\ - \\ s \end{pmatrix} = 2\pi\delta^2(\vec{s})$$

Here s is the distance from the z axis in cylindrical coordinates and $\delta^2(\vec{s})$ is a two dimensional delta function on the xy plane.

7. Prove that $\delta(r) = 4\pi r^2 \delta^3(\vec{r})$ and $\delta(s) = 2\pi s \delta^2(\vec{s})$.

Here $\int_0^\epsilon \delta(r) dr = 1$ for any $\epsilon > 0$. The integral is 0 otherwise. $\delta(s)$ is defined likewise.

8. The electric field in a region is given as

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s} \hat{s}$$

Find the charge densities in the region.