Lecture - 9

P 1

Recap:

Variance

Bernoulli random variable

Binomial random variable

$\downarrow$

Compute the $\underset{\text{H.W.}}{E[X]}$ & $\underset{\text{H.W.}}{Var[X]}$

Poisson random variable

Let $x = 0, 1, 2, 3, \ldots$

$$P(X=i) = e^{-\lambda} \frac{\lambda^i}{i!}$$

$$\sum_{i=0}^{\infty} P(X=i) = 1 = \sum_{i=0}^{\infty} e^{-\lambda} \frac{\lambda^i}{i!}$$

$$= e^{-\lambda} . e^{\lambda} = 1$$

$$E[X] = Var[X] = \lambda$$

H. W.

Poisson is a good approximation for Binomial for large n & small p, s.t. np is of moderate size

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Binomial random variable

$$E[X] = np = \lambda$$

$$\Rightarrow p = \frac{\lambda}{n}$$

Poisson

$$P(X=i) = \binom{n}{i} p^i (1-p)^{n-i}$$

$$= \frac{n!}{i!(n-i)!} \left(\frac{\lambda}{n}\right)^i \left(1 - \frac{\lambda}{n}\right)^{n-i} \frac{(1-\lambda/n)^i}{(1-\lambda/n)^i}$$

$$= \frac{n(n-1)(n-2)\dots(n-i+1)}{n \quad n \quad n \quad \dots \quad n} \cdot \frac{\lambda^i}{i!} \frac{(1-\lambda/n)^n \rightarrow e^{-\lambda}}{\left(1-\frac{\lambda}{n}\right)^i}$$

as $n \rightarrow \infty$, this $\rightarrow 1$

$\rightarrow 1$

e.g.

Probability of a certain item being defective is 0.1. The sample has 10 items. what is the probability that less than or equal to 1 item is defective?

$n=10$

x = no. of defective items. $p=0.1$

Binomial	Poisson
$P(X=0)+P(X=1)$	$P(X=0)+P(X=1)$
$=\binom{10}{0}p^0(1-p)^{10}+\binom{10}{1}p^1(1-p)^9$	$e^{-\lambda}\left(\frac{\lambda^0}{0!}+\frac{\lambda^1}{1!}\right)$
$=0.736$	$=e^{-1}(1+1)$
	$=\frac{2}{e}=0.7357$

$\boxed{\lambda = np = 10 \times .1 = 1}$

Geometric Random Variable

You repeat an experiment until you succeed.

$P(\text{success}) = p$

$P(\text{failure})\ 1-p$

$X =$ no. of trials until you get the 1^{st} success

$X=i$	$P(X=i)$
1	p
2	$(1-p)\,p$
3	$(1-p)^2\,p$
$\vdots$	
k	$(1-p)^{k-1}\cdot p$
$\vdots$	$\vdots$

e.g.:

Box: 20 white balls
30 black balls.

You take out a ball, note its color, then return it to the box. Repeat until you get a black ball.

X = no. of ~~balls~~ attempts until you get the 1st black ball.

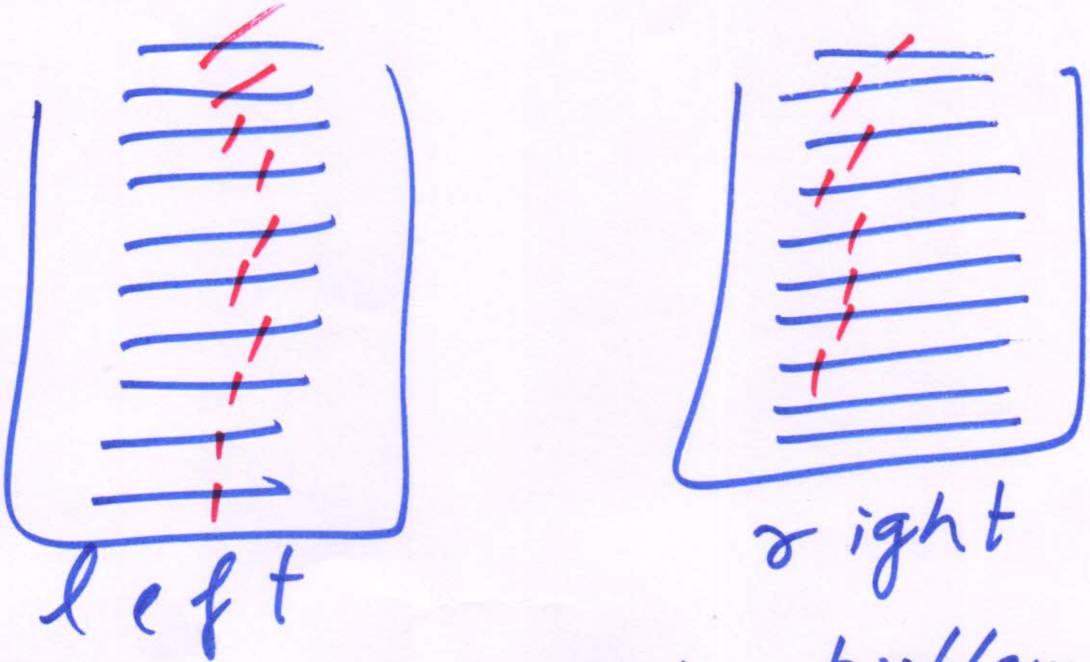
$X=i$	$P(X=i$
1	$3/5$
2	$\frac{2}{5} \cdot \frac{3}{5}$
3	$\left(\frac{2}{5}\right)^2 \cdot \frac{3}{5}$
$\vdots$	$\vdots$

What is the probability that you take ≥ 10 attempts to get the 1st black ball?

$$\sum_{i=10}^{\infty} P(X=i)$$

$$\left.\begin{aligned} E[X] &= \frac{1}{p} \\ Var[X] &= \frac{1-p}{p^2} \end{aligned}\right\} \text{H.W.}$$

e.g. An absent-minded chain-smoking mathematician



Banach match problem

Negative Binomial random variable: You keep repeating an experiment until you accumulate k successes.

~~to~~ Keep tossing a coin until you get 5 Heads,
$p(H) = 0.1$, $p(T) = 0.9$

$P(\text{success}) = p$, $P(\text{failure}) = 1-p$ 8

X = no. of trials.

Accumulate k successes

$P(X = i)$

$1 \quad 2 \quad 3 \quad \dots \quad i-1$ $\qquad i$

you get $k-1$ success

success ↓ k^{th} success

$$\binom{i-1}{k-1} p^{k-1} (1-p)^{i-k} \cdot p$$

$$P(X = i) = \binom{i-1}{4} (0.1)^4 (0.9)^{i-5} \cdot (0.1)$$

How do you relate this to the Banach match problem?

$\boxed{10}$ $\boxed{10}$

this = 0

what is the probability that the other pocket has 3 matchsticks?

experiment

success

$n = ?$

$i = ?$