Tutorial 01

1. There are six runners in the 100-yard dash. How many ways are there for three medals to be awarded if ties are possible? (The runner or runners who finish with the fastest time receive gold medals, the runner or runners who finish with exactly one runner ahead receive silver medals, and the runner or runners who finish with exactly two runners ahead receive bronze medals).

2. How many positive integers less than 1,000,000 have the sum of their digits equal to 19?

3. Find the solution to the recurrence relation

$$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$$

with initial conditions $a_0 = 2, a_1 = 5$, and $a_2 = 15$.

4. Find the solution to the recurrence relation

$$a_n = 5a_{n-1} - 6a_{n-2} + 7^n.$$

5. Four red, 8 blue, and 5 green balls are randomly arranged in a line.

   (a) What is the probability that the first 5 balls are blue?

   (b) What is the probability that none of the first 5 balls are blue?

   (c) What is the probability that the final 3 balls are differently colored.

   (d) What is the probability that all the red balls are together?

6. Balls are randomly removed from an urn that initially contains 20 red and 10 blue balls.

   (a) What is the probability that all of the red balls are removed before all of the blue ones have been removed?

   Now suppose that the urn initially contains 20 red, 10 blue, and 8 green balls.

   (b) Now what is the probability that all of the red balls are removed before all of the blue ones have been removed?

   (c) What is the probability that the colors are depleted in the order blue, red, green?

   (d) What is the probability that the group of blue balls is the first of the three groups to be removed?