Lecture - 3

Google Classroom

mqitacv

Recap:

Sample space

Event

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3 axioms

i) $0 \le p \le 1$

ii) $p(S) = 1$

iii) $p(A \cup B) = p(A) + p(B)$

if $A \cap B = \phi$

$p(\phi) = 0$

$p(\bar{E}) = 1 - p(E)$

if $E \subseteq F$, then $p(E) \le p(F)$

H: $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

W: $p(A \cup B \cup C) = \cdots$

Sample spaces with equally likely outcomes

e.g. Box X

6 white balls $w_1 \ldots w_6$

5 black balls $b_1 \ldots b_5$

draw 3 balls at random

$$P\left(\begin{array}{l}\text{1 white \& 2 black balls}\\ \text{are drawn}\end{array}\right) = ?$$

Sample space $= \{w_1 w_2 w_3, \ldots, b_1 b_2 b_3, \ldots, w_1 w_6 b_5\}$

$|S| = {}^{11}C_3$

$|Event| = {}^{6}C_1 \, {}^{5}C_2$

$$\text{Probability} = \frac{|E|}{|S|} = \frac{{}^{6}C_1 \, {}^{5}C_2}{{}^{11}C_3}$$

e.g.

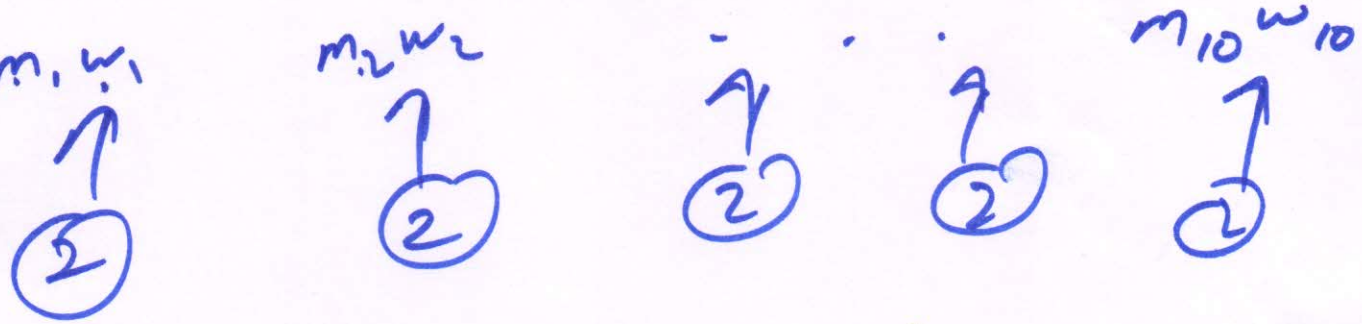
10 married couples.

Choosing 5 people at random.

what is the probability that none of them is related to each other?

$$|S| = {}^{20}C_5$$

$$|E| = {}^{10}C_5 * 2^5$$



$$p = \frac{|E|}{|S|} = \frac{{}^{10}C_5 * 2^5}{{}^{20}C_5}$$

e.g.

What is the probability of getting a straight in a hand of poker?

$|\text{sample space}| = {}^{52}C_5$

$|\text{Event}| = 10(4^5 - 4)$

10 possible straights

1 → A, 2, 3, 4, 5

2 → 2, 3, 4, 5, 6

⋮

10 → 10, J, Q, K, A

2, 3, 4, 5, 6

4 4 4 4 4 $= 4^5$

$10 * [4^5 - 4]$

e.g.

P(getting a flush)

$$= \frac{4 * {}^{13}C_5 - 40}{{}^{52}C_5}$$

choose a suit in 4 ways

P(full house) =

1. choosing 2 denominations
$^{13}C_2$

2. 66 KKK → $^4C_2$ $^4C_3$
666 KK $^4C_2$ $^4C_3$

$$\frac{{}^{13}C_2 * {}^4C_2 * {}^4C_3 * 2}{{}^{50}C_5}$$

e.g. Birthday Paradox

There are n people in a room. What is the probability that 2 (or more) people share the same birthday?.

n	p
366	1
70	0.999
23	0.5
2	people

P(2 or more people have the same birthday) =

1 − P(no two people have the same birthday)

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2 people

$= \frac{365}{365} \cdot \frac{364}{365}$

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3 people

$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \ldots$

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n people

$$\frac{365 \cdot 364 \cdot \ldots (365 - n + 1)}{365^n}$$

Program the graph