1. Let $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$. Find $\vec{\nabla} f$. Find the rate of change of f at the point $(1, 1, 0)$ along a direction specified by the unit vector $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$.

2. Let $\vec{\mathbf{r}}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z). Show that
(a) $\vec{\nabla}(1/r) = -\hat{\mathbf{r}}/r^2$
(b) Evaluate $\vec{\nabla}(r^n)$

3. A real square matrix M is orthogonal if $M^{-1} = M^T$. Using the fact that the magnitude of a vector doesn't change under rotation prove that a rotation matrix is orthogonal.

4. *This question tries to give an idea of what a scalar quantity is.*
The electric potential at a point on a horizontal plate with respect to a given coordinate system is given as $V(x, y) = xy$. If someone work with a coordinate system that is rotated by $45°$, the new coordinates (x', y') are given in terms of the old ones as $x' = \frac{x+y}{\sqrt{2}}$ and $y' = \frac{y-x}{\sqrt{2}}$. Let's write this as $\vec{r'} = R\vec{r}$. Potential is a scalar quantity. If $V'(x', y')$ is the functional form of the potential function in the new coordinate system then $V'(x', y') = V(x, y)$.
   (a) Find the form of the function $V'(x', y')$.
   (b) Verify that $\vec{\nabla}' V' = R\vec{\nabla} V$, i.e., components of a gradient transform as a vector quantity.

5. Let $\vec{\mathbf{A}} = \vec{\omega} \times \vec{\mathbf{r}}$ where $\vec{\omega}$ is a fixed vector in space. Find $\vec{\nabla} \times \vec{\mathbf{A}}$.

6. Prove that for any vector field $\vec{\mathbf{A}}$, $\vec{\nabla} \cdot \vec{\mathbf{A}}$ is a scalar.

7. Find the divergence of the following:
   (a) $\vec{\mathbf{A}} = \hat{\mathbf{r}}$,
   (b) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r}$ in 2 dimension
   (c) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r}$ in 3 dimension

(d) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r^2}$ **in 3 dimension. Plot this field.**

(e) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r^3}$ **in 3 dimension**

(f) $\vec{\mathbf{A}} = y\hat{\mathbf{i}} - x\hat{\mathbf{j}}$

8. Let

$$D = \begin{pmatrix} \frac{\partial A_x}{\partial x} & \frac{\partial A_y}{\partial x} \\ \frac{\partial A_x}{\partial y} & \frac{\partial A_y}{\partial y} \end{pmatrix}$$

Under a rotation of the coordinate system

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = R \begin{pmatrix} x \\ y \end{pmatrix}$$

show that

$$D' = \begin{pmatrix} \frac{\partial A'_x}{\partial x'} & \frac{\partial A'_y}{\partial x'} \\ \frac{\partial A'_x}{\partial y'} & \frac{\partial A'_y}{\partial y'} \end{pmatrix} = RDR^T$$