Lecture 11: 28 September 2020

Natural Sampling

Pulse train.

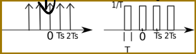

Figure: (left) Sampling using impulse train, (right) Natural sampling

- Sampled signal $x_s(t) = x(t)x_p(t)$, where $x_p(t) = \sum_{n\in\mathbb{Z}} c_n \exp(jn2\pi f_s t)$ and $c_n = \frac{1}{T_s} sinc(nTf_s)$.

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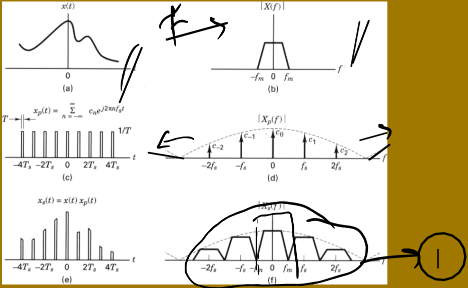
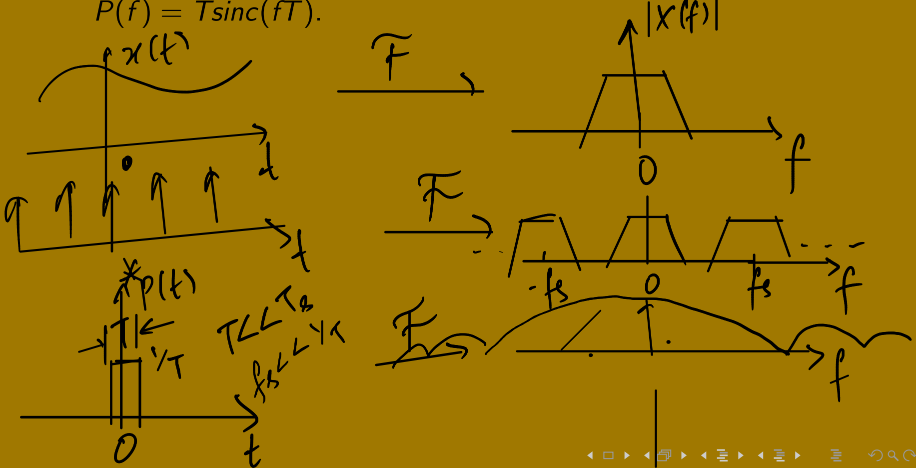
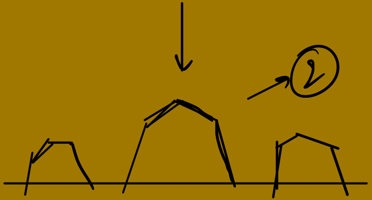


Figure: Natural sampling, Image Source: Sklar

Sample & Hold/ Flat top sampling

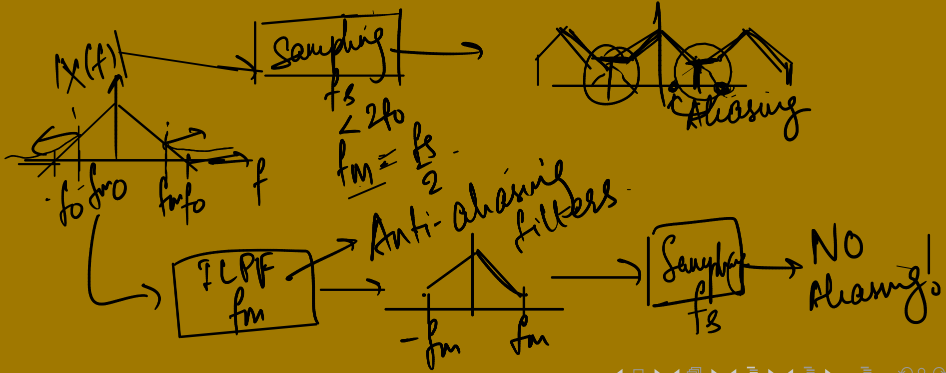
- Sampled signal $x_s(t) = p(t) * x(t)x_\delta(t)$, where $x_\delta(t) = \sum_{n\in\mathbb{Z}} \delta(t - nT_s)$, and $p(t)$ is a pulse of width T sec.
- Frequency domain: $P(f)\sum_{n\in\mathbb{Z}} X(f - nf_s)$, with $P(f) = Tsinc(fT)$.



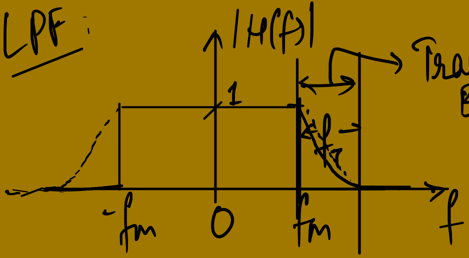


Aliasing

- Aliasing occurs if
  - Sampling rate lower than Nyquist criteria, or
  - Signal not BL.
- Solution: Anti-aliasing filter & Oversampling.
  - Advantages of oversampling?



LPF :



$f_s = 2f_m$

$2(f_m + f_t)$

$\underline{\underline{2f_m + f_t}}$

CD - sampling rate ~ 44.1 kHz

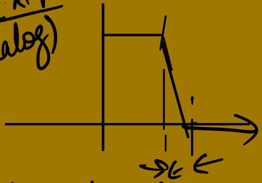
0 – 20 kHz $\xrightarrow{f_s}$ 40 kHz

+ 4.1 kHz (Transition BW)

- <u>Oversampling</u> $f_s > 2f_m$

$> (8-10) f_m$

$f_s = 2f_m + \underline{\underline{\epsilon}}$ (f_T) $\frac{A.AF}{(Analog)}$

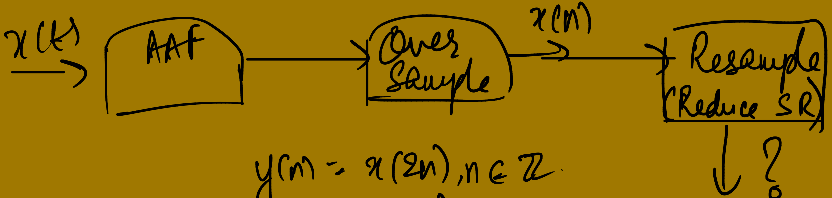


* Order of an LTI system
(Laplace transforms/z-transforms)
- order of polynomials in the Lap/z-transforms
- LCCDE - Order of the DE ~ Order the system

Higher order system/filter would be reqd in order to achieve a smaller transition B.W

- Very expensive analog filter
  - Accurate & more components.

- $f_s > 10 f_m$ (Oversampling).
- Use a cheap analog filter which has a high transition B.W.
- Handle a large no. of samples.

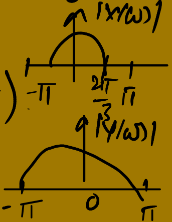


$y(n) = x(2n), n \in \mathbb{Z}$

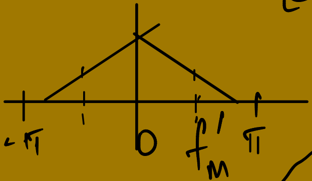
$y(n) = x(5n), n \in \mathbb{Z}$

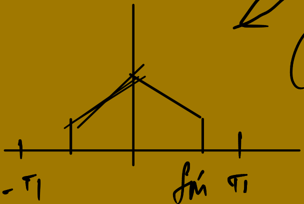
$Y(\omega)$ $X(\omega)$, $\omega \in [0, 2\pi)$

Aliasing!



$x(t)$ not BL → Analog AAF (cheap) → Over Sample → AAF (Digital filter) → Reduction in sampling rate





Can digital filters have 0 Trans. BW?

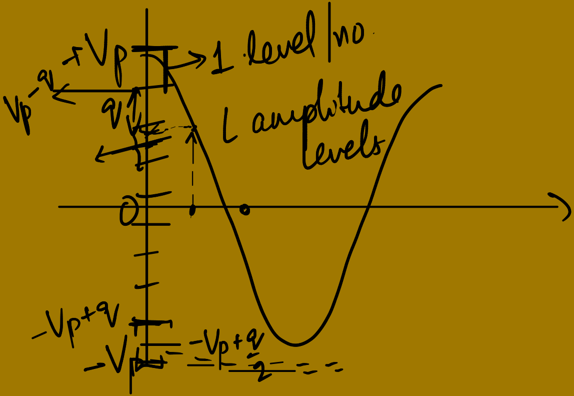
— No!

Digital LTI system: L C C Difference eqn. (Algorithm)

— A higher filter (Digital) with a lower trans. BW is easier to realize!!

Quantization

- After sampling, you have a PAM waveform, whose amplitudes can be any real number.
  - Let range of input amplitudes be $[-V_p, V_p]$. A Quantizer assigns one of the L possible amplitude levels for each sample.



Quantization error analysis

- L Quantization levels at $\{V_p - \frac{q}{2}, V_p - \frac{3q}{2}, \ldots, 0, \ldots, -V_p + \frac{q}{2}\}$. Thus, $\boxed{Lq = 2V_p}$
- Assuming uniform distribution of signal amplitudes, error variance $\sigma^2 = \frac{q^2}{12}$.

$$\int_{-q/2}^{q/2} e^2 \frac{1}{q}\, de$$

- Peak Signal power to Noise ratio: $\left(\frac{S}{N}\right) = \frac{V_p^2}{q^2/12} = 3L^2$
- Any downside to increasing L?
- Non-uniform Quantization.

$\uparrow L \quad \uparrow SNR$

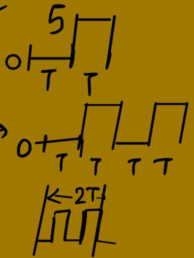
$L = 4 \Rightarrow 2$ bits/sample

$L = 16 \Rightarrow 4$ bits/sample

① $\uparrow L \rightarrow \uparrow$ delay or $\uparrow$ BW.

01 →

0101 →



Pulse Code Modulation(PCM)

PCM: Samples → bits

- PCM: Quantized samples $\{-V_p + \frac{q}{2}, -V_p + \frac{3q}{2}, \ldots, 0, \ldots, V_p - \frac{q}{2}\} \rightarrow \{0, 1, \ldots, L-1\} \rightarrow \{(0)_2, (1)_2, \ldots, (L-1)_2\}$, each of length $log_2(L)$ bits.

Bits → Analog (Pulse) Waveforms

- Baseband Modulation: PCM waveforms.
  - Binary PCM waveforms & M-ary PCM waveforms.
  - Classification: (a) Non-return to zero(NRZ), (b) Return-to-zero (RZ), (c) Phase encoded, (d) Multi-level binary.

M-ary PCM.

Quantization levels determine bits/sample

01100111 . . .

Binary PCM.

0 → 1 10V

1 → 5V

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