Example

Solve the following game by LPP

$$A' = \begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

First we add 2 to each element of A

$$A = [A' + 2] = \begin{bmatrix} 1 & 0 & 10 \\ 9 & 7 & 1 \\ 8 & 2 & 14 \end{bmatrix}$$

$$\min \; p = x_1' + x_2' + \cdots + x_m'$$

$$\text{s.t.} \quad A^T x' \geq 1 \qquad\qquad x_i' = \frac{x_i}{u}$$

$$x' \geq 0$$

Let

$$\min_j E_j(x) = u$$

So, $\quad E_j(x) \geq u$ for all j

A's problem is to find a x such that $E_j(x) \geq u$.

$$\max \; u \equiv \min \frac{1}{u} = \min \frac{x_1 + x_2 + x_m}{u} \equiv \min x_1' + x_2' + \cdots + x_m'$$

s.t. $\underline{\underline{E_j(x)}} \geq u$ for all j.

$$a_{1j} x_1 + a_{2j} x_2 + \cdots + a_{mj} x_m \geq u.$$

$$a_{1j} x_1' + \cdots + a_{mj} x_m' \geq 1$$

$$\begin{cases} \min \; p = x_1' + x_2' + \cdots + x_m' \\ \text{s.t } A^T X' \geq 1 \\ X' \geq 0 \end{cases} \qquad x_i' = \frac{x_i}{u}$$

$$\begin{cases} \max \; q = y_1' + y_2' + \cdots + y_n' \\ \text{s.t. } A Y' \leq 1 \\ Y' \geq 0 \end{cases} \qquad y_j' = \frac{y_j}{w}.$$

$$\begin{bmatrix} 1 & 0 & 10 \\ 9 & 7 & 1 \\ 8 & 2 & 14 \end{bmatrix}$$

P_1

$$\min \ p = x_1' + x_2' + x_3'$$

s.t.

$$x_1' + 9x_2' + 8x_3' \geq 1$$

$$7x_2' + 2x_3' \geq 1$$

$$10x_1' + x_2' + 14x_3' \geq 1$$

$$x_1', x_2', x_3' \geq 0.$$

consider the dual.

P_2

$$\max \ q = y_1' + y_2' + y_3'$$

s.t.

$$y_1' + \quad + 10y_3' \leq 1 \quad \text{— } ①$$

$$9y_1' + 7y_2' + y_3' \leq 1 \quad \text{— } ②$$

$$8y_1' + 2y_2' + 14y_3' \leq 1 \quad \text{— } ③$$

$$y_1', y_2', y_3' \geq 0$$

since all the constraints are "$\leq$" type introduce surplus variables y_4', y_5', y_6' to ①, ②, and ③ respectively.

$$\max\ q = y_1' + y_2' + y_3' + 0y_4' + 0y_5' + 0y_6'$$

$$\text{s.t.}\quad \begin{aligned} y_1' + + 10y_3' + y_4' &= 1 \\ 9y_1' + 5y_2' + y_3' + y_5' &= 1 \\ 8y_1' + 2y_2' + 14y_3' + y_6' &= 1 \end{aligned}$$

$$y_1', \ldots, y_6' \geq 0.$$

		c_j		1	1	1	0	0	0		
C_B	B	Y'_B	b	a_1	a_2	a_3	a_4	a_5	a_6	min ratio	operation
0	a_4	y_4'	1	1	0	10	1	0	0	-----	
0	a_5	y_5'	1	9	$\boxed{7}$	1	0	1	0	1/7 →	
0	a_6	y_6'	1	8	2	14	0	0	1	1/2	
$z_j - c_j$				−1	−1 ↑	−1	0	0	0		
0	a_4	y_4'	1	1	0	10	1	0	0	1/10	$R_1' = R_1$
1	a_2	y_2'	1/7	9/7	1	1/7	0	1/7	0	1	$R_2' = R_2/7$
0	a_6	y_6'	5/7	38/7	0	$\boxed{96/7}$	0	−2/7	1	5/96 →	$R_3' = R_3 - 2R_2'$
$z_j - c_j$				2/7	0	−6/7 ↑	0	1/7	0		
0	a_4	y_4'	23/48	$-\frac{71}{24}$	0	0	1	5/24	$-\frac{35}{48}$		$R_1' = R_1 - 10R_3'$
1	a_2	y_2'	13/96	$\frac{59}{48}$	1	0	0	7/48	$-\frac{1}{96}$		$R_2' = R_2 - \frac{1}{7}R_3'$
1	a_3	y_3'	5/96	19/48	0	1	0	$-\frac{1}{48}$	7/96		$R_3' = R_3 / \frac{96}{7}$
$z_j - c_j$				24/15	0	0	0	1/8	1/16		

All $z_j - c_j \geq 0$

$$Y' = \left(0, \frac{13}{96}, \frac{5}{96}\right) \qquad \text{value } w_{max} = \frac{3}{16}$$

$$X' = \left(0, \frac{1}{8}, \frac{1}{16}\right) \qquad v_{min} = \frac{3}{16}$$

we now need to compute X and Y

$$X = \frac{x'}{u} \quad \text{and} \quad Y = \frac{y'}{w}$$

$$X = \left(0, \frac{1/8}{u}, \frac{1/16}{u}\right)$$

$$= \left(0, \frac{1/8}{3/16}, \frac{1/16}{3/16}\right)$$

$$\boxed{= \left(0, \frac{2}{3}, \frac{1}{3}\right)}$$

$$Y = \left(0, \frac{\frac{13}{96}}{w}, \frac{5/96}{w}\right)$$

$$= \left(0, \frac{13/96}{3/16}, \frac{5/96}{3/16}\right)$$

$$\boxed{= \left(0, \frac{13}{18}, \frac{5}{18}\right)}$$

value of the game,

$$\left(\frac{1}{v} - K\right) = \frac{1}{3/16} - 2 = \frac{16}{3} - 2 \; \boxed{= \frac{10}{3}}$$

$$A' = \begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

can we reduce this matrix??

B_1 is dominated by B_2

$$\begin{bmatrix} -2 & 8 \\ 5 & -1 \\ 0 & 12 \end{bmatrix}$$

✓

now use graphical here

A_1 is dominated by A_3

$$\begin{bmatrix} 5 & -1 \\ 0 & 12 \end{bmatrix}$$

now use mixed strategy. to Solve the game.

$$\begin{array}{c|cc} & y & 1-y \\ \hline x & 5 & -1 \\ 1-x & 0 & 12 \end{array}$$

$$E(x,y) = 5xy - x(1-y) + 12(1-x)(1-y)$$
$$= 5xy - x + xy + 12 - 12x - 12y + 12xy$$
$$= 18xy - 13x - 12y + 12$$

$$C + D(x-k)(y-l)$$
$$= C + Dxy - Dky - Dlx + Dkl.$$

$$D = 18$$

$$-Dk = -12 \qquad -Dl = -13$$
$$\Rightarrow k = \frac{12}{18} = \frac{2}{3} \qquad \Rightarrow l = \frac{13}{18}$$

$$C + Dkl = 12$$
$$C = 12 - \cancel{18} \cdot \frac{2}{3} \cdot \frac{13}{\cancel{18}}$$
$$= \frac{36-26}{3} = \frac{10}{3}$$

$$X = \left(\frac{2}{3}, \frac{1}{3}\right) \qquad Y = \left(\frac{13}{18}, \frac{5}{18}\right)$$

$$v = \frac{10}{3}$$