CT303 Lecture 16: 14 October 2020

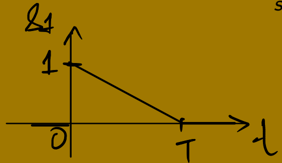
- Recap of Lecture 15.
  - Receiving filter maximizes the SNR every T secs.
  - Impulse response of the matched filter: $h(t) = s_i(T - t)$, where T is the symbol waveform duration.
  - Example:

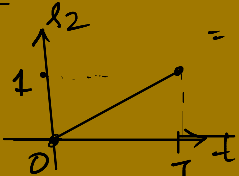
$$SNR_T = \frac{2E}{N_0}$$

$$s_1(t) = 1 - \frac{t}{T}, t \in [0, T]$$

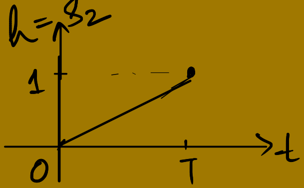
$$s_2(t) = \frac{t}{T}, t \in [0, T].$$

$$s_1(T-t) = 1 - \frac{(T-t)}{T} = 1 - 1 + t/T = t/T$$

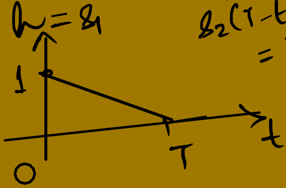




$s_1(T-t)$



$$s_2(T-t) = \frac{T-t}{T} = 1 - \frac{t}{T}$$



- Convolution with the receiving filter:

$r(t)$ - received signal

$h(t) = s_i(T-t)$

$$z(t) = \int_{-\infty}^{\infty} r(\tau)h(t-\tau)\, d\tau = r * h.$$

$$= \int_{-\infty}^{\infty} r(\tau)s_i(T-t+\tau)\, d\tau$$

Both r and s_i exists only for $[0, T]$, and at $t = T$

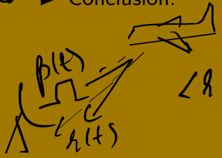
$$z(T) = \int_{0}^{T} r(\tau)s_i(\tau)\, d\tau \quad \text{[Correlation]}$$

$$:= \langle r, s_i \rangle = \langle r, \tilde{h} \rangle$$

↳ template

① Radar

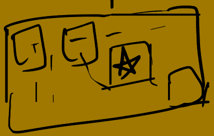
- Conclusion:



$\langle r, p \rangle$

② Image Processing

- Object detection



Rx filter → Matched filter

- Notation: $\underline{\int_0^T f(t)g(t)dt} = \boxed{\langle f, g\rangle}.$ [Projection]

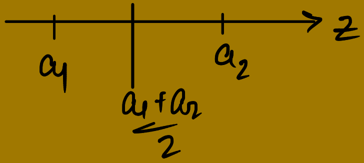
$f := \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}$ $g = \begin{pmatrix} g_1 \\ \vdots \\ g_n \end{pmatrix}$ $\Rightarrow \langle f, g\rangle = \sum_{i=1}^{n} f_i g_i$

$f := \begin{pmatrix} f_1 \\ \vdots \\ f_n \\ \vdots \end{pmatrix}$ $g = \begin{pmatrix} g_1 \\ \vdots \\ g_n \\ \vdots \end{pmatrix}$ $\Rightarrow \langle f, g\rangle = \lim_{n\to\infty} \sum_{i=1}^{n} f_i g_i$

$f : \mathbb{Z}^+ \to \mathbb{R}$ $g : \mathbb{Z}^+ \to \mathbb{R}$

$f : \mathbb{R} \to \mathbb{R}$ $g : \mathbb{R} \to \mathbb{R}$ $\Rightarrow \langle f, g\rangle = \int_{-\infty}^{\infty} f(t)g(t)dt.$

- Notation: $\int_0^T f(t)g(t)dt = \langle f, g \rangle$.
  - Aim is to minimize $P_B = Q\left(\frac{a_2 - a_1}{2\sigma_0}\right)$.



$\uparrow$ SNR such that $P_B \downarrow$

- Notation: $\int_0^T f(t)g(t)dt = \langle f, g \rangle$.
  - Aim is to minimize $P_B = Q\left(\frac{a_2 - a_1}{2\sigma_0}\right)$.
  - Let $\tilde{h}$ denote the correlation function of the matched filter.

Z, a_1, a_2

o/ps of Matched filter

$$Z = (r * s_i(T-t)) = r * h.$$

$$= \langle r, \tilde{h} \rangle$$

Time reversed version of h (Imp. response)

- Notation: $\int_0^T f(t)g(t)dt = \langle f, g \rangle$.
- Aim is to minimize $P_B = Q\left(\frac{a_2 - a_1}{2\sigma_0}\right)$.
- Let $\tilde{h}$ denote the correlation function of the matched filter.
- Then $a_1 = \langle s_1, \tilde{h} \rangle$, $a_2 = \langle s_2, \tilde{h} \rangle$.

$[h(t) * n(t)]$

$$s(t) = s_i(t) + n(t)$$

$$z(t) = s_i(t) * h(t) + n_o(t)$$

$$z(T) = \langle s_i, \tilde{h} \rangle + n_o$$

$$= a_i + n_o$$

$(a_2 - a_1)$ is as high as possible!

- Notation: $\int_0^T f(t)g(t)dt = \langle f, g \rangle$.
- Aim is to minimize $P_B = Q\left(\frac{a_2 - a_1}{2\sigma_0}\right)$.
- Let $\tilde{h}$ denote the correlation function of the matched filter.
- Then $a_1 = \langle s_1, \tilde{h} \rangle, a_2 = \langle s_2, \tilde{h} \rangle$.
- $\underline{a_2 - a_1} = \langle s_2 - s_1, \tilde{h} \rangle$.

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- In order to maximize $a_2 - a_1$,

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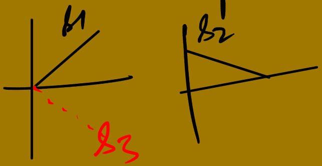
  - Let $\tilde{h}$ denote the correlation function of the matched filter.
  - Then $a_1 = \langle s_1, \tilde{h} \rangle$, $a_2 = \langle s_2, \tilde{h} \rangle$.
  - $a_2 - a_1 = \langle s_2 - s_1, \tilde{h} \rangle$.
  - In order to maximize $a_2 - a_1$,
  - either: Increase $s_2 - s_1$ $\Rightarrow \langle s_2 - s_1, \tilde{h} \rangle \uparrow$

$\Rightarrow$ Choose the 2 symbol waveforms which are as different as possible



$\{s_1, s_2\}$ $\{s_1, s_3\}$

- Notation: $\int_0^T f(t)g(t)dt = \langle f, g \rangle$.
- Aim is to minimize $P_B = Q\left(\frac{a_2 - a_1}{2\sigma_0}\right)$.
- Let $\tilde{h}$ denote the correlation function of the matched filter.
- Then $a_1 = \langle s_1, \tilde{h} \rangle, a_2 = \langle s_2, \tilde{h} \rangle$.
- $a_2 - a_1 = \langle s_2 - s_1, \tilde{h} \rangle$.
- In order to maximize $a_2 - a_1$,
- either:
- or(and):

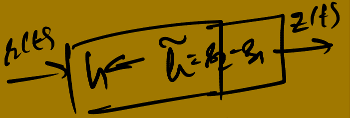
$$\tilde{h} = s_2 - s_1 \Rightarrow \text{Max } \langle s_2 - s_1, \tilde{h} \rangle$$

$$= \|s_2 - s_1\|^2$$

$$= \text{Energy of } s_2 - s_1$$

$$= E_d$$

- Notation: $\int_0^T f(t)g(t)dt = \langle f, g \rangle$.
  - Aim is to minimize $P_B = Q\left(\frac{a_2 - a_1}{2\sigma_0}\right)$.
  - Let $\tilde{h}$ denote the correlation function of the matched filter.
  - Then $a_1 = \langle s_1, \tilde{h} \rangle$, $a_2 = \langle s_2, \tilde{h} \rangle$.
  - $a_2 - a_1 = \langle s_2 - s_1, \tilde{h} \rangle$.
  - In order to maximize $a_2 - a_1$,
  - either:
  - or(and): $\rightarrow$ $\tilde{h} = (s_2 - s_1)$
- $SNR_T = \frac{(a_2 - a_1)^2}{\sigma_0^2}$.



$$z(T) = \langle \tilde{h}, r \rangle$$

$$= \langle s_2 - s_1, r \rangle$$

$$SNR_T = \frac{(a_2 - a_1)^2}{\sigma_0^2} = \frac{E_d^2}{\frac{N_0}{2} \underbrace{\int_{-\infty}^{\infty} |H(f)|^2 df}_{E_d}} = \frac{E_d^2}{\frac{N_0}{2} E_d} = \frac{2E_d}{N_0}$$

- Notation: $\int_0^T f(t)g(t)dt = \langle f, g\rangle$.
  - Aim is to minimize $P_B = Q\left(\frac{a_2 - a_1}{2\sigma_0}\right)$.
  - Let $\tilde{h}$ denote the correlation function of the matched filter.
  - Then $a_1 = \langle s_1, \tilde{h}\rangle$, $a_2 = \langle s_2, \tilde{h}\rangle$.
  - $a_2 - a_1 = \langle \underline{s_2 - s_1}, \tilde{h}\rangle$.
  - In order to maximize $a_2 - a_1$,
  - either:
  - or(and):
- $\underline{SNR_T = \frac{(a_2-a_1)^2}{\sigma_0^2}}$.
  - Also $SNR_T = \frac{(a_2-a_1)^2}{\sigma_0^2} = \frac{2E_d}{N_0}$.

$s(t) \rightarrow \boxed{h(t)} \rightarrow z(t)$

$s(t) + n(t)$

$\Rightarrow P_B?$

$$\boxed{P_B = Q\left(\frac{a_2 - a_1}{2\sigma_0}\right)} =$$

$r(t) \rightarrow \boxed{\text{Rx filter}} \rightarrow \boxed{\text{Detection MAP, ML}}$

- Notation: $\int_0^T f(t)g(t)dt = \langle f, g\rangle$.
- Aim is to minimize $P_B = Q\left(\frac{a_2-a_1}{2\sigma_0}\right)$.
- Let $\tilde{h}$ denote the correlation function of the matched filter.
- Then $a_1 = \langle s_1, \tilde{h}\rangle$, $a_2 = \langle s_2, \tilde{h}\rangle$.
- $a_2 - a_1 = \langle s_2 - s_1, \tilde{h}\rangle$.
- In order to maximize $a_2 - a_1$,
- either:
- or(and):
- $SNR_T = \frac{(a_2-a_1)^2}{\sigma_0^2}$.
- Also $SNR_T = \boxed{\frac{(a_2-a_1)^2}{\sigma_0^2}} = \frac{2E_d}{N_0}$.
- Finally, $P_B = Q\left(\frac{1}{2}\sqrt{SNR_T}\right) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) \Rightarrow P_B \downarrow \quad E_d \uparrow$

$$E_d = \int_0^T (s_1 - s_2)^2(t)\,dt$$

$$P_B = Q\left(\frac{a_2 - a_1}{2\sigma_0}\right)$$

Energy in the difference of the two symbols must be as large as possible

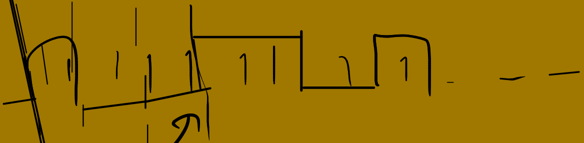
Lab 5: Binary PCM : [randi]

0 1 0 1 1 0 - - -

1000's of bit stream

$$s_1(t) = \begin{cases} +A, & t \in [0, T] \\ 0, & \text{else} \end{cases}$$

$$s_2(t) = \begin{cases} -A, & t \in [0, T] \\ 0, & \text{else} \end{cases}$$



$$P_B = \frac{1}{1000} \sum_{i=1}^{1000} \frac{N_{B_i}}{N}$$

$x(t) =$

$x(n) \quad x(nT_s) \qquad T_s = T/10$

$$r(n) = x(n) + \eta(n) \rightarrow \boxed{h(n)} \rightarrow z(n) \rightarrow z(10n)$$

Threshold.

N_B/N

Examples

- Bipolar pulses:

$$s(t) \rightarrow \boxed{h(t)} \rightarrow \boxed{\text{Detection}} \qquad \frac{a_1+a_2}{2}$$

$$s_1(t) = \begin{cases} +A, & t \in [0,T] \\ 0 & \text{else} \end{cases} \qquad s_2(t) = \begin{cases} -A, & t \in [0,T] \\ 0 \end{cases}$$

Bipolar pulses.

$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$$

- Unipolar pulses:

$$s_1(t) = \begin{cases} +A, & t \in [0,T] \\ 0, & \text{else} \end{cases} \qquad s_2(t) = 0$$

$$P_B = Q\left(\sqrt{\frac{A^2T}{2N_0}}\right)$$