1. **Prove that for any vector field $\vec{\mathbf{A}}$, $\vec{\nabla} \cdot \vec{\mathbf{A}}$ is a scalar.**

2. **Let $\vec{\mathbf{A}} = \vec{\omega} \times \vec{\mathbf{r}}$ where $\vec{\omega}$ is a fixed vector in space. Find $\vec{\nabla} \times \vec{\mathbf{A}}$.**

3. **Find the divergence of the following:**

   (a) $\vec{\mathbf{A}} = \hat{\mathbf{r}}$,

   (b) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r}$ **in 2 dimension**

   (c) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r}$ **in 3 dimension**

   (d) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r^2}$ **in 3 dimension. Plot this field.**

   (e) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r^3}$ **in 3 dimension**

4. **Find the curl of the following:**

   (a) $\vec{\mathbf{A}} = y\hat{\mathbf{i}} - x\hat{\mathbf{j}}$

   (b) $\vec{\mathbf{A}} = \frac{1}{\sqrt{x^2+y^2}}(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})$

   (c) $\vec{\mathbf{A}} = \frac{1}{x^2+y^2}(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})$

   (d) $\vec{\mathbf{A}} = (x^2 + y^2)\hat{\mathbf{k}}$

5. **For any vector field $\vec{\mathbf{A}}$ and any scalar field F show that**
   (i) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{\mathbf{A}}) = 0$ **;** **(ii)** $\vec{\nabla} \times (\vec{\nabla} F) = 0$**.**

6. **Can we find a scalar function F such that $\vec{\nabla} F = y\hat{\mathbf{i}} - x\hat{\mathbf{j}}$?**
   What about $\vec{\nabla} F = \frac{1}{x^2+y^2}(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})$?

7. **Find the equation of the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point (x_0, y_0, z_0) on the ellipsoid.**