TUTORIAL 07

(1) The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y} & ;\ 0 < x < \infty,\ 0 < y < \infty \\ 0 & ;\ \text{otherwise} \end{cases}$$

Compute $P\{X < Y\}$.

(2) A man and a woman decide to meet at a certain location. If each of them independently arrives at a time uniformly distributed between 12 noon and 1 P.M., find the probability that the first to arrive has to wait longer than 10 minutes.

(3) Prove that the continuous random variables X and Y are independent if and only if their joint probability density function can be expressed as

$$f_{X,Y}(x,y) = h(x)g(y) \qquad -\infty < x < \infty,\ -\infty < y < \infty$$

(4) Suppose that X and Y are independent, continuous random variables having probability density functions f_X and f_Y. Prove that

$$f_{X+Y}(a) = \int_{-\infty}^{\infty} f_X(a-y)f_Y(y)dy.$$

(5) A basketball team will play a 44-game season. Twenty-six of these games are against class A teams and 18 are against class B teams. Suppose that the team will win each game against a class A team with probability .4 and will win each game against a class B team with probability .7. Suppose also that the results of the different games are independent. Approximate the probability that

(a) the team wins 25 games or more;

(b) the team wins more games against class A teams than it does against class B teams.

(6) Prove that $P\{a_1 < X \leq a_2, b_1 < Y \leq b_2\} = F(a_2,b_2) + F(a_1,b_1) - F(a_1,b_2) - F(a_2,b_1)$, whenever $a_1 < a_2,\ b_1 < b_2$.

(7) A table is ruled with equidistant parallel lines a distance D apart. A needle of length L, where $L \leq D$, is randomly thrown on the table. What is the probability that the needle will intersect one of the lines (the other probability being that the needle will be completely contained in the strip between two lines)?