

Analysis of Multi-Disciplinary Problems
Syllabus – Winter Semester

The following items indicate the broad subjects covered, along with their specific sub-topics.

- **First-order autonomous linear differential equations:**

1. Introduction and overview. The basic principles of differential equations.
2. Ordinary and partial differential equations. Orders of differential equations.
3. First order linear systems of one variable. Rate \propto state.
4. Transformation of variables, separation of variables, rescaling into dimensionless forms.
5. Scales, approximations and basic plotting techniques.

- **Examples of first-order autonomous linear systems:**

1. Stokes's law of terminal velocity.
2. Problem of atomic waste disposal.
3. Kelvin's viscoelastic deformation of rocks.
4. Duckworth-Lewis method to reset targets in interrupted cricket matches.
5. The Van Meegeren art forgery case.
6. Radio-carbon dating of the age of ancient cultures.
7. The R-C circuit.

- **First-order autonomous nonlinear differential equations:**

1. The logistic equation. Rescaling, initial condition, early exponential growth.
2. The nonlinear time scale, approximations and plotting for various initial conditions.
3. Higher orders of nonlinearity and logistic-type equations.
4. Fermi-Dirac type of equations.
5. Modifications on the logistic equation. Equations of the form, $\dot{x} = a - bx^2$.

- **Examples of first-order autonomous nonlinear differential equations:**

1. Population dynamics, Malthus and Verhulst models, world population, aberrations, implications and criticisms.
2. The laws of social dynamics (Elliot Montroll).
3. Shark and salmons, population of New York, critical cases of power-law convergence.
4. Fall of a parachute.
5. Item response theory, with reference to computational developmental psychology.
6. Spread of technological innovations, agricultural as well as industrial (Edwin Mansfield's model).
7. Growth of free-living and dividing cells, Gompertz model of tumour growth.

- **Examples of non-autonomous systems:**

1. Bacteria and toxins.
2. Power-law distributions, self-organisation in complex systems (Stuart Kauffman).

- **Second-order systems of differential equations:**

1. General coupled second-order autonomous systems.
2. Higher-order autonomous systems.
3. Coupled linear second-order autonomous systems.

- **Examples of second-order systems:**

1. Richardson's theory of conflict. War-readiness, mutual disarmament with and without grievance, unilateral disarmament, arms race.
2. Lanchester's models of combat. Conventional-conventional and conventional-guerilla battles. Lanchester's square and linear laws. Bracken's generalisation.
3. The principle of competitive exclusion in population biology.
4. Volterra's predator-prey models.
5. Love affairs.
6. The threshold theorem of epidemiology.

- **Concepts of networks:**

1. The Königsberg-bridge problem and graph theory.
2. Random networks and small-world networks.
3. Power laws and scale-free networks. Error and attack tolerance.

- **Additional topics:**

1. Phase plots of first-order autonomous systems. Stability of fixed points and the critical condition.
2. Conservative and reversible systems. Dissipation and irreversibility.
3. Continuous differentials for growth of populations.
4. Power laws and their properties.
5. The Bernoulli equation, streamline and turbulent motions, the lift of an aircraft.
6. Sigmoid activation function and the Hill function.
7. Taylor expansion in multiple variables.
8. An oscillator as a second-order system.
9. Asymptotic behaviour of the equation for the spread of epidemics.

- **Assignments:**

1. Zipf's law in the network of *Debian*.
2. Bimodality in vehicular traffic of *Jackson City, Alabama*.