Tutorial 13

1. If 10 fair dice are rolled, find the approximate probability that the sum obtained is between 30 and 40, inclusive.

2. Let $X_i, i = 1, \ldots, 10$, be independent random variables, each uniformly distributed over $(0, 1)$. Calculate an approximation to $P\left\{\sum_{i=1}^{10} X_i > 6\right\}$.

3. An instructor has 50 exams that will be graded in sequence. The times required to grade the 50 exams are independent, with a common distribution that has mean 20 minutes and standard deviation 4 minutes. Approximate the probability that the instructor will grade at least 25 of the exams in the first 450 minutes of work.

4. A set of 200 people consisting of 100 men and 100 women is randomly divided into 100 pairs of 2 each. Give an upper bound to the probability that at most 30 of these pairs will consist of a man and a woman.

5. The number of automobiles sold weekly at a certain dealership is a random variable with expected value 16. Give an upper bound to the probability that

   (a) next week's sales exceed 18;

   (b) next week's sales exceed 25.

6. If

$$E[X] = 75 \quad E[Y] = 75 \quad \text{Var}(X) = 10$$
$$\text{Var}(Y) = 12 \quad \text{Cov}(X, Y) = -3$$

   give an upper bound to $P\{|X - Y| > 15\}$.

7. The servicing of a machine requires two separate steps, with the time needed for the first step being an exponential random variable with mean .2 hour and the time for the second step being an independent exponential random variable with mean .3 hour. If a repair person has 20 machines to service, approximate the probability that all the work can be completed in 8 hours.

8. On each bet, a gambler loses 1 with probability .7, loses 2 with probability .2, or wins 10 with probability .1. Approximate the probability that the gambler will be losing after his first 100 bets.

9. Each of the batteries in a collection of 40 batteries is equally likely to be either a type A or a type B battery. Type A batteries last for an amount of time that has mean 50 and standard deviation 15; type B batteries last for an amount of time that has mean 30 and standard deviation 6.

   (a) Approximate the probability that the total life of all 40 batteries exceeds 1700.

   (b) Suppose it is known that 20 of the batteries are type A and 20 are type B. Now approximate the probability that the total life of all 40 batteries exceeds 1700.