Tutorial 8

1) Your company must make a sealed bid for a construction project. If you succeed in winning the contract (by having the lowest bid), then you plan to pay another firm 100 thousand dollars to do the work. If you believe that the minimum bid (in thousands of dollars) of the other participating companies can be modeled as the value of a random variable that is uniformly distributed on (70, 140), how much should you bid to maximize your expected profit?

2) Suppose that the travel time from your home to your office is normally distributed with mean 40 minutes and standard deviation 7 minutes. If you want to be 95 percent certain that you will not be late for an office appointment at 1 P.M., what is the latest time that you should leave home?

3) A roulette wheel has 38 slots, numbered 0, 00, and 1 through 36. If you bet 1 on a specified number then you either win 35 if the roulette ball lands on that number or lose 1 if it does not. If you continually make such bets, approximate the probability that
   a) You are winning after 34 bets;
   b) You are winning after 1000 bets;
   c) You are winning after 100,000 bets.

   Assume that each roll of the roulette ball is equally likely to land on any of the 38 numbers.

4) There are two types of batteries in a bin. When in use, type i batteries last (in hours) an exponentially distributed time with rate λ_i, $i = 1, 2$. A battery that is randomly chosen from the bin will be a type i battery with probability p_i, $\sum_{i=1}^{2} p_i = 1$. If a randomly chosen battery is still operating after t hours of use, what is the probability that it will still be operating after an additional s hours?

5) Evidence concerning the guilt or innocence of a defendant in a criminal investigation can be summarized by the value of an exponential random variable X whose mean μ depends on whether the defendant is guilty. If innocent, $\mu = 1$; if guilty, $\mu = 2$. The deciding judge will rule the defendant guilty if $X > c$ for some suitably chosen value of c.
   a) If the judge wants to be 95 percent certain that an innocent man will not be convicted, what should be the value of c?
   b) Using the value of c found in part (a), what is the probability that a guilty defendant will be convicted?

6) Consider a circle of radius R, and suppose that a point within the circle is randomly chosen in such a manner that all regions within the circle of equal area are equally likely to contain the point. (In other words, the point is uniformly distributed within the circle.) If we let the center of the circle denote the origin and define X and Y to be the coordinates of the point chosen (Figure 6.1), then, since (X, Y) is

equally likely to be near each point in the circle, it follows that the joint density function of X and Y is given by

$$f(x,y) = \begin{cases} c, & if\ x^2 + y^2 \le R^2 \\ 0, & if\ x^2 + y^2 > R^2 \end{cases}$$

for some value of c.

a) Determine c.
b) Find the marginal density functions of X and Y.
c) Compute the probability that D, the distance from the origin of the point selected, is less than or equal to a.
d) Find E[D].