

# Lecture-4

P ①

## Recap:

Sample spaces with equally  
likely outcomes

Birthday Paradox

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Tuesday : holiday: group 3  
they will come on Wednesday

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Friday: convocation rehearsal,  
group 1, come on Monday

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Timings : 2 PM to 4 PM  
even roll numbers: 2-3  
odd roll numbers: 3-4

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# Conditional Probability:

(2)

Definition:

The conditional probability of event A taking place given that event B has already taken place is

$$P(A|B) := \frac{P(A \cap B)}{P(B)} = \frac{P(AB)}{P(B)}$$

↓  
given that

$$P(AB) = P(B) P(A|B) = P(A) P(B|A)$$

$$P(B|A) = \frac{P(BA)}{P(A)} = \frac{P(AB)}{P(A)}$$



e.g.

(3)

Box = 25 bulbs

5 are good: light for 1000 hours

10 are partially defective: light for 1 hour

10 are fully defective: do not light up at all

You pick up a bulb at random. You test it, it lights up. What is the probability that it is a good bulb?

Identify the events

A: Bulb lights up

B: Bulb is good

$P(A \cap B)$ ,  $P(A)$ ,  $P(B)$ ,  $P(A|B)$ ,  $P(B|A)$ ,  $P(A \cup B)$ ,  $P(\bar{A})$ ,  $P(\bar{B})$

$$P(B|A) = \frac{P(BA)}{P(A)}$$

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$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(B)P(A|B)}{P(A)}$$

$$= \frac{5/25 \times 1}{15/25} = \frac{1}{3}$$

e.g. Assume that GoI implements NRC. They claim that it is 99% accurate. They say (the software) says that A you are not a citizen of India. Does it mean that the probability of B you not being a citizen is 0.99?



A: Slw says that you are ~~not~~ a citizen (5)

B: you are not a citizen

$$P(B|A) = \frac{P(AB)}{P(A)}$$
$$= \frac{P(A|B)P(B)}{P(A)}$$

$$P(B) = \frac{1 \text{ labh}}{1300r}$$

A, B

$$A = (A \cap B) \cup (A \cap \bar{B})$$

dis joint

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$

$$= P(B)P(A|B) + P(\bar{B})P(A|\bar{B})$$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})}$$

$$P(A|B) = 0.99, P(A|\bar{B}) = 0.01$$

⑥

$$\frac{0.99 * 1}{13000}$$

$$\frac{0.99}{13000} + 0.01 * \left(1 - \frac{1}{13000}\right)$$

$$= \frac{0.99}{0.99 + (3000 - 1) * 0.01}$$

$$= \frac{0.99}{0.99 + 130 - 0.01}$$

$$\approx \frac{1}{130}$$

e.g. You need to 7  
choose between 2  
electives: ML  
IOT

You make a choice  
by tossing a coin.

if you choose ML,

$$p(AA) = \frac{1}{2}.$$

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if you choose IOT,  
 $p(AA) = \frac{2}{3}.$

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what is the probability  
that you get AA  
in ML?



Defining events

⑧

A: you score AA

B: you choose ML

H.W.

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