1. **In the spherical polar system:**

   (a) **Evaluate** $\frac{\partial \hat{r}}{\partial \theta}, \quad \frac{\partial \hat{\theta}}{\partial \theta}, \quad \frac{\partial \hat{\phi}}{\partial \theta}, \quad \frac{\partial \hat{r}}{\partial \phi}, \quad \frac{\partial \hat{\theta}}{\partial \phi}, \quad \frac{\partial \hat{\phi}}{\partial \phi}$

   (b) **Using the above partial derivatives evaluate** $\vec{\nabla} \cdot \hat{r}, \quad \vec{\nabla} \cdot \hat{\theta}$ **and** $\vec{\nabla} \cdot \hat{\phi}$ **where** $\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$

   Now you can evaluate the expression for $\vec{\nabla} \cdot \vec{A}$ **in spherical polar co-ordinates using the result of part (b) and using the product rules. Try it and see whether you get the expression for divergence.**

2. **Cylindrical system of co-ordinate is specified by three variables** (s, ϕ, z) **given by**

   $$x = s\cos\phi; \qquad y = s\sin\phi; \qquad z = z$$

   Find the unit vectors $\hat{s}, \quad \hat{\phi}, \quad \hat{z}$ **in this co-ordinate system. Find** $h_s, \quad h_\phi$ **and** h_z **and write down the expression for** $\vec{\nabla} F$ **for a scalar function** F **in this system.**

3. **If** $\vec{A} = s\hat{z}$ **find** $\vec{\nabla} \times \vec{A}$**.**

4. **Find the divergence of** $\vec{\mathbf{v}} = (r\cos\theta)\hat{\mathbf{r}} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}$**. Check the divergence theorem for this function, using the volume as the inverted hemispherical bowl of radius** R**, resting on the** x**-**y **plane and centred at the origin.**

5. (a) **If** $\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$ **show that there exists a vector function** $\vec{\mathbf{A}}$ **such that** $\vec{\nabla} \times \vec{\mathbf{A}} = \vec{B}$

   (b) **Show that any vector field** $\vec{\mathbf{F}}$ **can be expressed as**

   $$\vec{\mathbf{F}} = \vec{\nabla}\Phi + \vec{\nabla} \times \vec{\mathbf{A}}$$

   where Φ **is a scalar field and** $\vec{\mathbf{A}}$ **is a vector field.**
   Justify