Tutorial 11

1. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables having distribution function F and expected value μ. Such a sequence of random variables is said to constitute a sample from the distribution F. The quantity $X = \sum_{i=1}^{n} \frac{X_i}{n}$ is called the sample mean. Compute $E[X]$.

2. If independent trials having a constant probability p of being successes are performed, determine the expected number of trials required to amass a total of r successes.

3. Ten hunters are waiting for ducks to fly by. When a flock of ducks flies overhead, the hunters fire at the same time, but each chooses his target at random, independently of the others. If each hunter independently hits his target with probability p, compute the expected number of ducks that escape unhurt when a flock of size 10 flies overhead.

4. An urn has n white and m black balls that are removed one at a time in a randomly chosen order. Find the expected number of instances in which a white ball is immediately followed by a black one.

5. Twenty individuals consisting of 10 married couples are to be seated at 5 different tables, with 4 people at each table.

   (a) If the seating is done "at random," what is the expected number of married couples that are seated at the same table?

   (b) If 2 men and 2 women are randomly chosen to be seated at each table, what is the expected number of married couples that are seated at the same table?

6. A coin that comes up heads with probability p is flipped until either a total of n heads or of m tails is amassed. Find the expected number of flips.