Conservative Systems

$\boxed{F = ma} \Rightarrow \boxed{m\frac{dv}{dt} = F = F(x)}$ (say)

We write F (force) as $\boxed{F = -\frac{d\psi}{dx}}$,

in which $\psi = \psi(x)$ is a potential function.

Multiplying throughout by v we get,

$\boxed{mv\frac{dv}{dt} = -\frac{d\psi}{dx}v}$ Now $\boxed{v = \frac{dx}{dt}}$

$$\Rightarrow \quad mv\frac{dv}{dt} + \frac{\cancel{dx}}{dt}\frac{d\psi}{\cancel{dx}} = 0$$

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{1}{2}mv^2 + \psi\right) = 0 \Rightarrow \boxed{\frac{1}{2}mv^2 + \psi = \mathcal{E}}.$$

in which $\mathcal{E}$ (total energy) is constant in time.

Reversible Systems

All conservative systems are reversible

Write $\boxed{m\frac{dv}{dt} = m\frac{d^2x}{dt^2}}$ since $\boxed{v = \frac{dx}{dt}}$.

$\Rightarrow \boxed{m\frac{d^2x}{dt^2} = F(x)}$ This equation is symmetric under $\boxed{t \longrightarrow -t}$.

The time reversal symmetry makes the system $\boxed{m\frac{d^2x}{dt^2} = F(x)}$ reversible in time.

Dissipation and ~~Reversal~~ Irreversibility

Friction (or viscosity) is effective in opposing motion. This dissipates energy and the conservative condition is lost. Further, reversibility is also lost. Since friction (and dissipation) acts only when there is motion, we can write dissipation as a function of velocity, $\boxed{D \equiv D(v)}$. Hence, we get $\boxed{m\frac{d^2x}{dt^2} = F(x) - D(v)}$ or

$\boxed{m\frac{dv}{dt} = F(x) - D(v)}$. The simplest possible way to write this function is by a linear formula $\boxed{D(v) = kv}$, in which k is a proportional constant. Now since $\boxed{v = dx/dt}$ we can write

$\boxed{m\frac{d^2x}{dt^2} = -k\frac{dx}{dt} + F(x)}$, with the negative sign indicating an opposition to motion. Further the transformation of $\boxed{t \longrightarrow -t}$ is no longer symmetric. The system is IRREVERSIBLE.

The Problem of Atomic Waste Disposal

The v-t equation is $\boxed{v = v_T\left(1 - e^{-t/t_0}\right)}$.

i.) When $t \ll t_0$ $\boxed{v \simeq v_T \frac{t}{t_0}}$ (linear limit).

ii.) When $t \to \infty$ $(t \gg t_0)$ $\boxed{v \simeq v_T}$ (constant)

Since $\boxed{v = dz/dt}$, when $t \ll t_0$, we get $\boxed{\frac{dz}{dt} \simeq \frac{v_T}{t_0} t}$ $\Rightarrow$ $\boxed{z \simeq \left(\frac{v_T}{t_0}\right)\frac{t^2}{2}}$ (parabolic).

And when $t \to \infty$, $\boxed{\frac{dz}{dt} \simeq v_T}$ $\Rightarrow$ $\boxed{z \simeq v_T t}$ (linear).

Fugitive Elasticity (Maxwell)

Kelvin's Viscoelastic formula: $\boxed{\frac{d\epsilon}{dt} = \frac{\sigma}{\eta} - \frac{Y}{\eta}\epsilon}$.

Or $\boxed{\sigma = Y\epsilon + \eta \frac{d\epsilon}{dt}}$. The right hand side must have dimensional compatibility in a system that shows visco-elastic behaviour.

Hence $\boxed{Y\epsilon \sim \eta \frac{d\epsilon}{dt}}$. We now write $\boxed{t = T t_0}$, in which T is dimensionless and t_0 is a time scale. Hence $\boxed{Y\epsilon \sim \frac{\eta}{t_0}\frac{d\epsilon}{dT}}$, which gives $\left[\frac{\eta}{Y}\right]$ a time dimension. Viscosity behaves like elasticity. $\boxed{\eta \sim Y t_0}$.