- Ideal filters are not realizable

- PSD and Autocorrelation of a WSSRP

$G_X(\omega) := \mathcal{F}\{R_X\}$. $\quad P_X = R_X(0) = E[X_t^2]$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} G_X(\omega)\,d\omega = \int_{-\infty}^{\infty} G_X(f)\,df$$

1. Generate only samples of CTRP.
2. $\{X_t\}_{t\in\mathbb{R}}$: Truncation $\{X_n\}_{n\in[0,N-1]}$.

Estimate the PSD of $\{X_t\}$.

$x^1(t)$
⋮
$x^S(t)$ } Sample functions, $t\in\mathbb{R}$ $\xrightarrow{\text{Truncation}}$

$x_T^1(t) = x^1(t)$, $t\in[-T/2, T/2]$; $= 0$, $t\notin[-T/2, T/2]$
⋮
$x_T^S(t) = x^S(t)$, $t\in[-T/2, T/2]$

$\downarrow \mathcal{F}$

$X_T^1(\omega)$
⋮
$X_T^S(\omega)$

$\xleftarrow[\text{Divide by } T]{\text{Expectation}}$

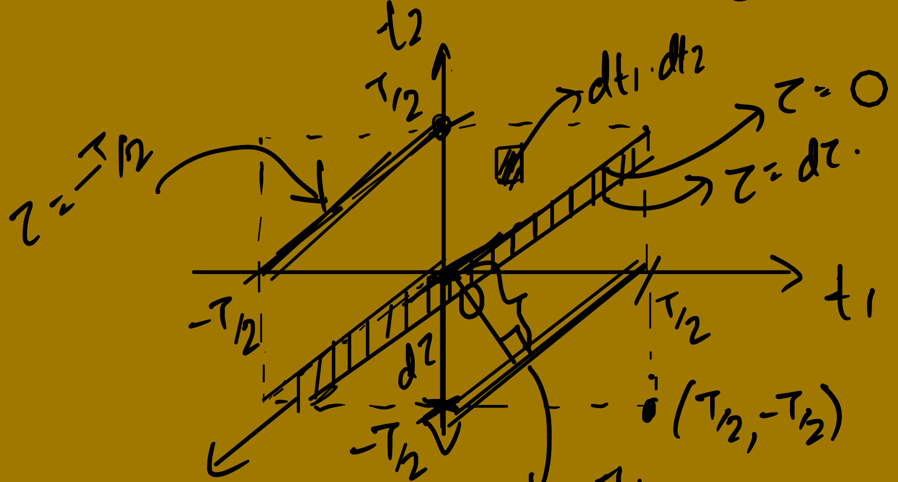
$$\boxed{\frac{E\left[|X_T(\omega)|^2\right]}{T}}$$?

$$E\left[|X_T(\omega)|^2\right] = E\left[X_T(\omega)\, X_T(\omega)^*\right]$$

$$= E\left[\int_{-T/2}^{T/2} x(t_1)\, e^{-j\omega t_1}\, dt_1 \int_{-T/2}^{T/2} x(t_2)\, e^{+j\omega t_2}\, dt_2\right]$$

$$= \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} \underbrace{\underbrace{E\left[x(t_1)\, x(t_2)\right]}_{R_x(t_1-t_2)}\, e^{-j\omega(t_1-t_2)}}_{R_x(\tau)\, e^{-j\omega\tau}}\, dt_1\, dt_2$$

$t_1 - t_2 = \tau$



$\Rightarrow \tau \in [-T, T]$

Approximate the strip by a rectangle:

$\tau = 0 \quad l(\tau) = \sqrt{2}\, T$

$\tau = T/2 \quad l(\tau) = \frac{T}{\sqrt{2}}$

$\tau = T \quad l(\tau) = 0$

$\tau = -T/2 \quad l(\tau) = T/\sqrt{2}$

$$l(\tau) = \sqrt{2}\, T - \sqrt{2}\, |\tau|$$

$$a(\tau) = \left(\sqrt{2}\, T - \sqrt{2}\, |\tau|\right) \frac{d\tau}{\sqrt{2}}$$

$$= (T - |\tau|)\, d\tau$$

$$= T\left(1 - \frac{|\tau|}{T}\right) d\tau$$

$$E\left[|X_T(\omega)|^2\right] = T\int_{-T}^{T} R_x(\tau)\, e^{-j\omega\tau}\left(1-\frac{|\tau|}{T}\right) d\tau$$

$$G_x(\omega) = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j\omega\tau}\, d\tau = \boxed{\lim_{T\to\infty} \frac{E\left[|X_T(\omega)|^2\right]}{T}}$$

— Wiener-Khintchine-Einstein Theorem

Lab3: Generate AWGN RP.

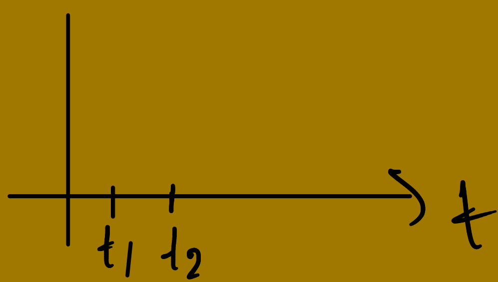
Additive White Gaussian noise

→ Random Process is Gaussian.

→ If PSD $\equiv k$. ($G_x(\omega) = k, \forall \omega$) → A

→ Noise gets added to the signal $r = s + n$

→ $R_x(\tau) = \delta(\tau)$

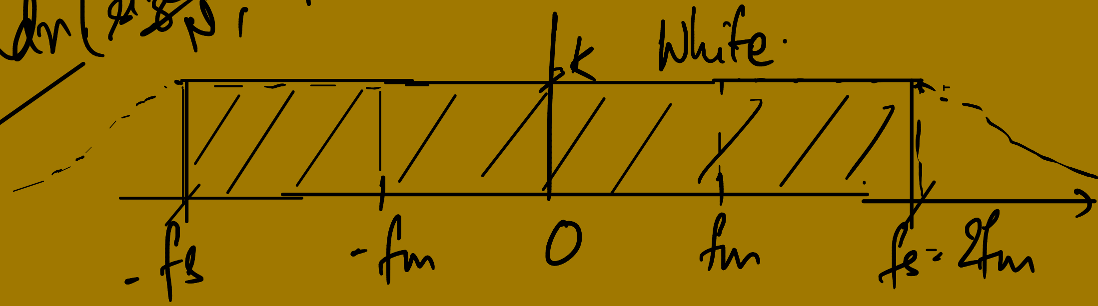


→ $P_x = \infty$

AWGN does not exist in practice!

np.random.randn(size N,) → $\sigma = 1$, mn = 0

f_m → max freq of interest.



Lab 3 : Q.3

2 RV's

randn $(2, 10^3)$

2 independent GRV's

$m = 0, \sigma = 1$.

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

randn$(n, 10^3) \rightarrow n$ ind. GRV's, 10^3 samples.

$$\Sigma = I_n$$

- Generate samples of n GRV's with specified mean vector $(n \times 1)$ and any specified $\Sigma_{n \times n}$. (SPD)
  - Verify whether Σ is SPD or not.

PCA, given samples of GRV (m, Σ)

$$\downarrow \text{PCA}$$

$$(\bar{0}, I_n)$$

Chapter 2 : Formatting & Baseband Modulation

Formatting : 1 Sampling

$\rightarrow$ Nyquist Sampling theorem: If $x(t)$ is a BL signal with max. freq. f_m Then perfect reconstruction is possible if $x(t)$ is sampled at a sampling freq $f_s \geq 2 f_m$.

Reconstruction via ILPF $\xleftrightarrow{F^{-1}}$ Interpolation via $T\,\mathrm{sinc}\left(\frac{t}{T_s}\right)$

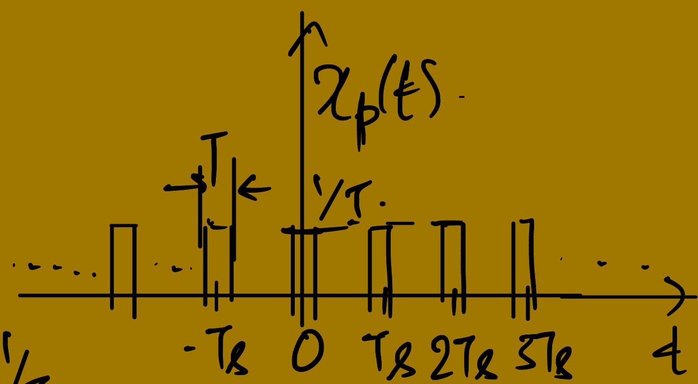
Issues: (1) ILPF cannot be realized.

(2) Any real signal is not BL (since it is TL)

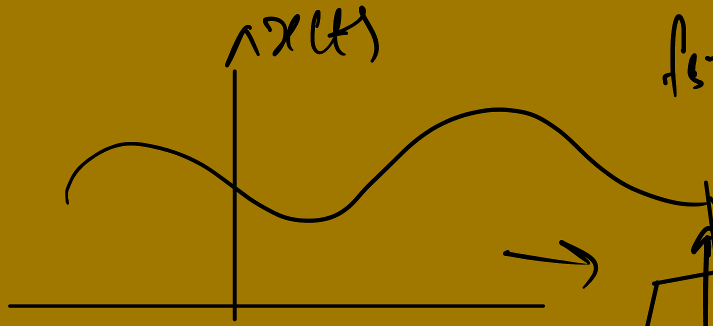
(3) One cannot generate Dirac delta's in practice!!

(3) Dirac delta

(i) Natural Sampling



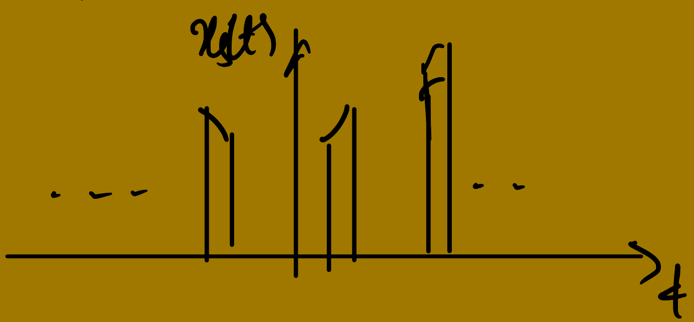
$f_s = 1/T_s$



$$x_p(t) = \sum_{n=-\infty}^{\infty} c_n e^{-jn2\pi f_s t}$$

$$c_n = \frac{1}{T_s}\mathrm{sinc}(nTf_s)$$

Sampled signal $x_s(t) = x(t)\,x_p(t)$



pulse $\longleftrightarrow$ sinc

replication in time domain $\longleftrightarrow$ sampling of sinc