Assignment Problem

Structural observation of an AP

Suppose all (2) 70 and a feasible assignment exists for which all corresponding (2) are equal to some

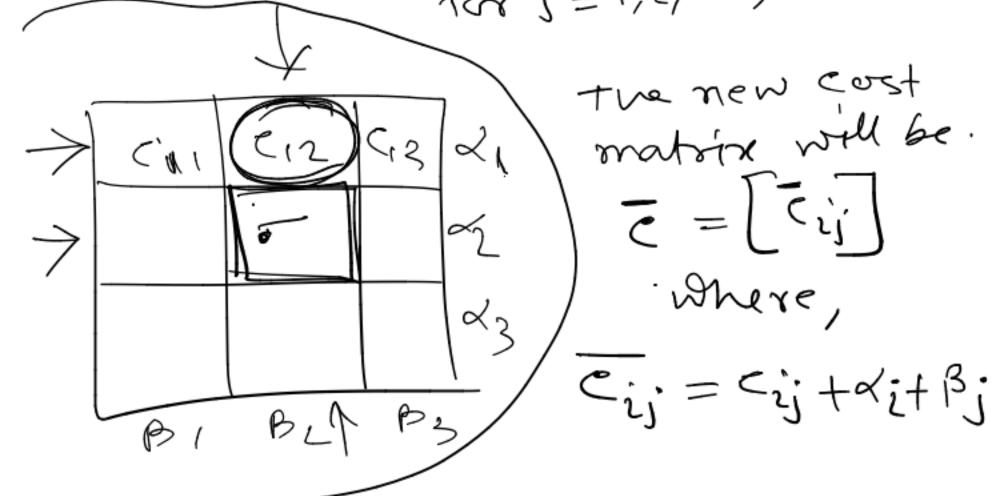
Theor this assignment will make the objective function value sero and this solution is optimal.

? ?

If a constant be added to any row and/or any column of the cost matrix of an AP, then the resulting AP has the same optimal solution as the original Problem.

Boots Let the cost matrix be c=[eij] suppose we add di to now i for [=1,2,...,m

Suppose we add Bj to column j for j=1,2,..., m.



tre new cost matrix will be.

$$\overline{c_{ij}} = c_{ij} + \alpha_{i+} \beta_{j}$$

Let I and I be the values of the original and the new problem respectively. $\overline{Z} = \sum_{i=1}^{n} \overline{\sum_{j=1}^{n}} \overline{C_{ij}} \chi_{ij}$ $= \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \left(C_{ij}^{i} + \alpha_{i}^{i} + \beta_{i}^{j} \right) \chi_{ij}^{i}$ $= \frac{\sum_{i=1}^{2} \sum_{j=1}^{2} x_{ij}^{2} + \sum_{i=1}^{2} \sum_{j=1}^{2} x_{ij}^{2}}{\sum_{j=1}^{2} \sum_{j=1}^{2} \sum_{j=1}^{2} x_{ij}^{2}} + \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{j=1}^{$ + 5 x 2 x 2 1 5 x 2 1 5 m + 3 3 [= x 2] = Z + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} = \frac{m}{2} \frac{m}{2} + \frac{m}{2} = \frac{m}{2} \frac{m}{2} = \frac{m}{2} \frac{m}{2} = \f we see that Z and Z differs that is independent by a constant of Xii

Hence the optimal solution of the original problem must be the optimal solution of the new problem.

If all eij7,0 and we find a set xij = xij such that

\[
\frac{m}{2} \frac{m}{2} \frac{m}{2} \frac{x}{2} \frac{x}{2

Proof: Cij ko and kij ko

Z = \$\frac{2}{2} \frac{2}{2} \cij \gammaij \text{ must be}

non-negative.

The minimum value Z altains is zero.

rig is a solution that cause value of 2 as sero.

Some observations

- > We can introduce sufficient number of scrops into the cost matrix. by proper choice of of and Bi
- > Then we shall find a feasible solution/assignment for which corresponding eight are sero > This gives an optimal solution.

A feasible solution of an AP will be will be will be ruch that there will be exactly one assigned eall in each row and each column.

Finding a solution of an AP Hungarian method (Konig and Egerrary)

Step 10 Subtract the minimum element For each row in the cost matrix from other elements of the respective row.

This results at least one sero in each row.

If a column does not have a 3ero, then subtract the minimum element in each column from the other elements of the respective column.

This results at best one zero in each column.

Tample

for fix fix fix

John 17 11

John 13 28 4 26

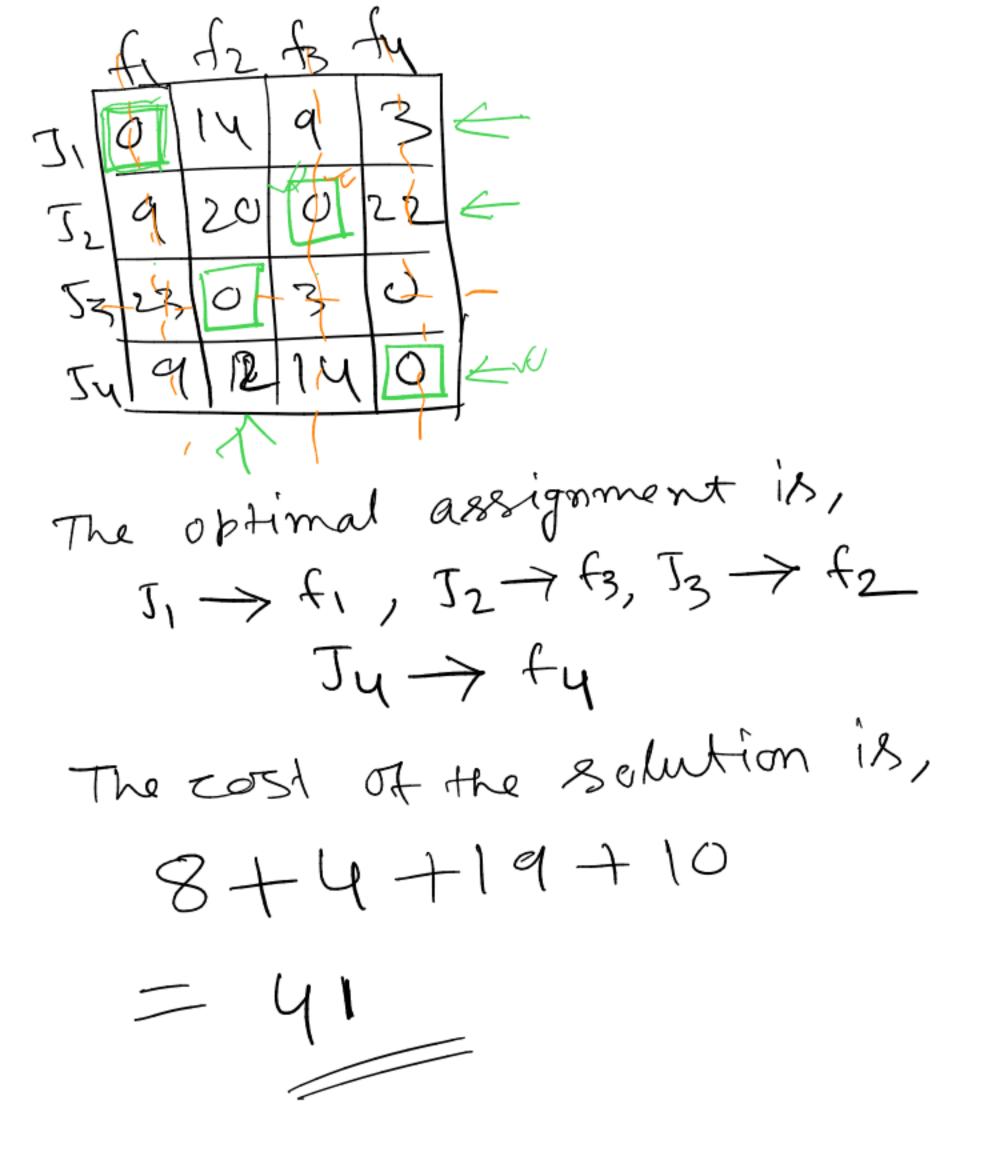
John 18 15

July 19 26 24 10

1	0	18	9	3
	9	24	0	22
	23	T	3	O
	9	16	14	0
				+

101	14	9	3
9	20	O	22
123	0	3	101
9	12	114	

Step 20 Draw the minimum possible number of horisontal and vertical lines to cover all seros in the cost matrix returned by Step 1. More there are two cases. The number of lines is equal to the order of the cost matrix In this case an optimal assignment has been reached. casse ii) the number of lines is less than the order of the matrix. we need to do something more to reach the optimality. Example number of lines = 4 order of matrix = 4 So case (1) occurres. How to get the optima assignment ??



case ii) number of lines < m le order of the matrix. we move to step 4 Step 43 -> Find the smallest element say ~ among the uncovered elements in the matrix after drawing vertical and horizontal lines. -> Subtract of from all the uneovered elements. Of the current matrix. > add & to the elements lying at whe intersections of horizontal and reatical lines. Again we proceed with step 2, with this modified matrix.

Example

	160	130	175	190	200
	135	120	130	160	175
-	140	110	155	170	185
	50	50	80	80	110
	55	35	70	80	105

After Step 1

300353015

150000

300353020

002005

200251515

no- of lines = 3 < 5

min = 15

20 15 number of lines = 5 = m we get an optimal assignment. The optimal solution is. $J_1 \rightarrow F_5$, $J_2 \rightarrow F_3$, $J_3 \rightarrow F_2$ 757 fy Ty -> +1, cost = 200+130+110+50+80 = 570