Max time: 2 hours SC215: first in-sem Max marks: 25

Instructions:

- First question is for 10 marks; remaining questions are for 5 marks each.
- Please answer all parts of a question together.
- Understanding a question is part of the evaluation. The invigilators are NOT going to clarify any doubts. If you find a mistake in a question, then please state it in your answer book.

1. Each part is for 2 marks.

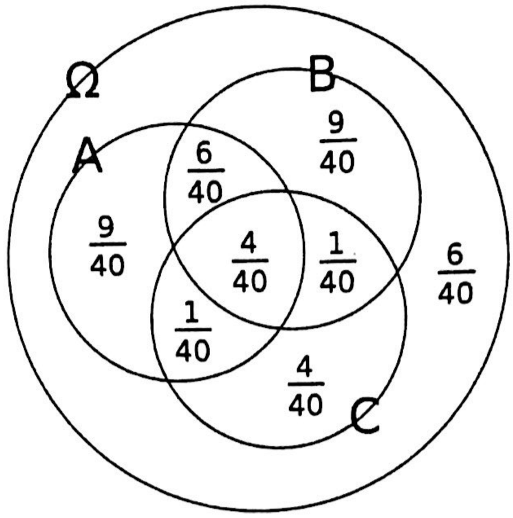
   a. Recall the iconic scene from *Sholay* where 3 men of Gabbar's gang come back after losing to Jai and Veeru. Gabbar is angry and decides to punish them. He takes a revolver from someone in his gang. The revolver has 6 bullets in it. But there are only 3 men, which is very unfair!! So Gabbar fires 3 shots consecutively in the air. Now the revolver has got 3 empty chambers and 3 bullets. After that, Gabbar spins the cylinder of the revolver several times, randomly. Gabbar then pulls the trigger 3 times, once for each of his gang members who had come back after losing. Each time, the chamber is empty and nobody dies. **What is the probability of this event?** Gabbar finds it very strange and starts laughing hysterically. Of course, he later kills all the three men, but we are not concerned with that at present! $\frac{1}{6}$

   b. Amit and Sumit play a game of marbles. Amit has 2 marbles, Sumit has 1 marble. They push a stick in the ground to act as a fixed point. They roll their marbles to see which marble comes closest to the stick. The one whose marble is closest wins the game. Assume that both Amit and Sumit are equally skilled in rolling marbles towards a fixed point. We are interested in determining the probability of Amit winning the game. Given below are two arguments to compute this probability. **You need to choose the correct argument and explain why your choice is the right choice.**

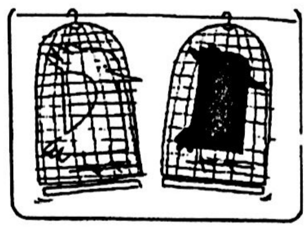
      - Since the players are equally skillful, all 3 marbles have the same chance of winning. But 2 out of 3 marbles belong to Amit. Therefore, his probability of winning is $\frac{2}{3}$.
      - Amit has 2 marbles. Now consider the position of these two marbles with respect to the position of Sumit's marble. There are 4 possible cases.
        1. Both can be better than Sumit's marble
        2. or the first can be better and the second worse
        3. or the second can be better and the first worse
        4. or both can be worse.

   Amit wins in 3 out of these 4 cases. Hence, his probability of winning is $\frac{3}{4}$.

c. See this Venn diagram. **Are the three events A, B and C independent of each other?** Explain your answer. **Ω** denotes the universal set.

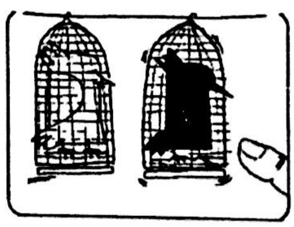


d. A lady owned 2 parrots.
One day a visitor asked:
Visitor 1: Is one bird a male?
Owner: Yes.
What is the probability that both birds are males?



$\frac{1/9}{2/9} = 1/2$

Next day, another visitor asks:
Visitor 2: Is the dark bird a male?
Owner: Yes
What is the probability that both birds are males?



Are the two probabilities the same/different? Explain your answer.

e. Three coins are tossed at once. We are interested in the probability that all three come down alike – that is to say, that either all three are heads or all three are tails. Now, we know that this probability is equal to $\frac{1}{4}$. You need to find out **what is wrong with the following argument**:

"We can say with assurance that of the three coins tossed, two of them must come down alike – both heads or both tails. It is so because it's impossible for all the three coins to show a different result (unless of course, one of the coins is standing on the edge, which we assume to be impossible). Now, what of the third coin? The probability that it's head is $\frac{1}{2}$; that it is tails is also $\frac{1}{2}$. In either case, the probability that it is the *same* as the other two is $\frac{1}{2}$. Hence, the probability that all the three are the same is $\frac{1}{2}$."

2. **Please help this doctor**, who is pondering the following dilemma. "If I am at least 80% certain that my patient has this disease, then I always recommend surgery. But if I am not quite as certain, then I recommend additional tests that are expensive and sometimes painful. Now, initially I was only 60% certain that Jones had the disease, so I ordered the series A test. This test always gives a positive result when the patient has the disease and almost never does when he is healthy. The test result was positive, and I was all set to recommend surgery when Jones informed me, for the first time, that he was diabetic. This information complicates matters because, although it doesn't change my original 60% estimate of his chances of having the disease in question, it does affect the interpretation of the results of the A test. This is so because the A test, while never yielding a positive result when the patient is healthy, does unfortunately yield a positive result 30% of the time in the case of diabetic patients who are not suffering from the disease. Now what do I do? **More tests or immediate surgery?**" Explain your solution.

3. Five men and five women are ranked according to their scores in an examination. Assume that no two scores are alike and that all the 10! possible rankings are equally likely. Let **X** denote the highest rank achieved by a man. For example, if a man tops the examination, then **X**=1. **Compute** E(**X**). Explain all the steps and show the calculations.

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4. Consider the following variant of the Monty Hall problem.

There are three closed doors. Behind one door, there is a car. Behind another door, there are car keys. And behind the third door, there is a goat. These three objects are randomly distributed behind the three doors. There are two players, A and B, who play this game together as a team. Player A enters the room first. He can open any 2 doors. If he doesn't find a car, the game stops right there and both the players lose. If player A finds a car, the doors are closed again and player A leaves the room. At this point, player A cannot communicate with player B. Now, player B enters the room. He can open any two doors. If he is able to find car keys, both players win the game and take away the car and the car keys. If player B doesn't find the car keys, both the players lose. If both the players select the doors randomly, their probability of winning the game is simply

$$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

Can you improve the winning probability? That is, can you design a strategy that will ensure that your chances of winning are more than $\frac{4}{9}$? The two players can design a strategy before the game starts. However, once the game starts, they are not allowed to communicate with each other.