Roll No._____

Winter Semester Academic Year 2019-20

Introductory Computational Physics (CS201)

End-Semester Examination

Dhirubhai Ambani Institute of Information and Communication Technology

Time: 2 Hours Total Marks: **35**

One-page question paper, 16 compulsory questions. Answer all the sub-parts of a question together. Marks for each question are indicated next to it. All terms and symbols carry their standard textbook meaning.
Physical constants: $\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F m^{-1}}$, $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H m^{-1}}$, $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$, $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$, $G = 6.67 \times 10^{-11}\,\mathrm{kg^{-1} m^3 s^{-2}}$, $k_\mathrm{B} = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$, $N_\mathrm{A} = 6.022 \times 10^{23}\,\mathrm{mol^{-1}}$, $e = 1.6 \times 10^{-19}\,\mathrm{C}$, $m_\mathrm{e} = 9.11 \times 10^{-31}\,\mathrm{kg}$, $m_\mathrm{n} \simeq m_\mathrm{p} = 1836 m_e$

1. Find the semi latus rectum and the eccentricity of $r = 24/(3 + 5\cos\theta)$. Identify the conic section. **[2]**

2. If $\mathbf{V} = (-4x - 3y + az)\hat{x} + (bx + 3y + 5z)\hat{y} + (4x + cy + 3z)\hat{z}$ is irrotational, then find the values of the constants a, b and c. **[2]**

3. Find the discriminant of $u_{xx} + 2u_{xy} + 5u_{yy} = 0$. Classify the partial differential equation. **[2]**

4. The acceleration due to gravity on the surface of a planet is $11\,\mathrm{m\,s^{-2}}$ and its radius is $5.82 \times 10^4\,\mathrm{km}$. Find the average density of the planet in c.g.s. units upto 2 places of decimal. **[2]**

5. The semi-major axis of a planetary orbit is $780 \times 10^6\,\mathrm{km}$. Find the orbital time period in earth years, if the mass of the Sun is $2 \times 10^{30}\,\mathrm{kg}$. **[2]**

6. The speed of an electron is measured with an uncertainty of $2 \times 10^4\,\mathrm{m\,s^{-1}}$. What is the uncertainty in locating the electron? **[2]**

7. Is the function $\psi(x) = (Ae^{-kx})/x$ a valid solution of Schrödinger's equation? Give two justifications for your answer, based on the form of ψ. (A and k are positive constants.) **[2]**

8. A gamma-ray photon is emitted from a nuclear state that has a lifetime of $1.2 \times 10^{-9}\,\mathrm{s}$. What is the uncertainty in the energy of the emitted photon in eV unit? **[2]**

9. The operator $\mathrm{d}^3/\mathrm{d}x^3$ operates on the eigenfunction $\psi = \exp(-ax)$. What is the eigenvalue? **[2]**

10. A travelling wave has the form $\psi(x,t) = a/[b + (2x + t)^2]$, in which a and b are constants, the unit of x is metre and the unit of t is second. Find the speed of the wave and its direction of propagation. **[2]**

11. State the criteria of attraction and repulsion for a force function $\mathbf{F} = F_0 \ln(r/r_0)\hat{r}$. The constants F_0 and r_0 have positive values. **[2]**

12. Find the eccentricity, the focal distance and the semi latus rectum of $9x^2 + 16y^2 = 144$. **[2.5]**

13. A particle moves under a central force $\mathbf{F} = -Kr^2\hat{r}$. If the total energy of the particle is E, derive the formula of its speed. **[2.5]**

14. Given $\dot{x} = x^2 + t^2$, $0 \le t \le 0.2$, $x(0) = 1$, $\Delta t = 0.1$, estimate $x(0.2)$ up to 3 places of decimal, by Taylor's method of the second order. **[2.5]**

15. Given $\ddot{x} + 6\dot{x} + 10x = 0$, determine the eigenvalues, classify the damping and write the general solution of $x(t)$ with arbitrary constants. **[2.5]**

16. Given $\mathbf{E} = (ar/\epsilon_0)\hat{r}$ (in which the constant $a = 5 \times 10^{-5}$) in the region $r \le 2\,\mathrm{m}$, find both the charge density and the total charge enclosed within a spherical shell of radius $r = 2\,\mathrm{m}$. **[3]**