CT303 Lecture 21 - 11 November 2020

- Lecture 20 - Recap
- Transmit $p(t)$, receive $q(t)$. Transmitted message $g(t) = \sum_m a_m p(t - mT)$, received signal $r(t) = \sum_m a_m q(t - mT) + \eta(t)$, $\eta \sim$ AWGN.
  - The sampled output of the filter matched to q is given by

$$z_k = \sum_m a_m x_{k-m} + v_k,$$

$$V_k = \int_{-\infty}^{\infty} \eta(t) q(t - kT) dt$$

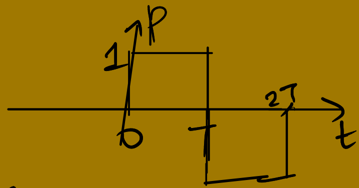
AWGN

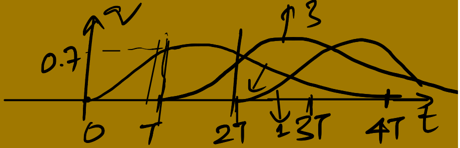
where $x_k = \int_{-\infty}^{\infty} q(t)q(t + kT)\, dt$, and $\mathbb{E}[v_k v_m] = N_0 x_{k-m}$.

Gaussian

$$E[v_k v_m] = E\Big[\int \eta(t) q(t-kT)dt \cdot \int \eta(t) q(t-mT) dt\Big]$$

$$= N_0 x_{k-m}.$$

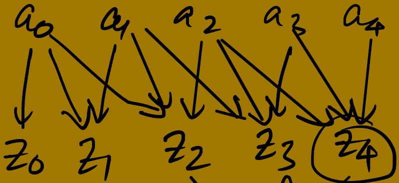




- If q is such that $x_{k-m} = \int_{-\infty}^{\infty} q(t-mT)q(t-kT) = 0$ for $|k-m| > L$, ← L symbols interfere
- $L \to a_m$'s values of $\to$ $z_k = \boxed{\sum_m a_m x_{k-m}} + v_k$. $\longrightarrow$ discrete LTI system
- Also note, $\mathbb{E}[v_k v_m] = N_0 x_{k-m} \neq 0, |k-m| \le L$.
- MLSE: $\max -J = \max_{\{a_0,\dots,a_{L-1}\}} \boxed{2\sum_k^N a_k z_k - \sum_k^N \sum_l^N a_k a_l x_{k-l}}$. $\to$ Max Likelihood Sequence estimate (MLSE)
- Recursive estimation using Viterbi algorithm, but is computationally expensive.
- ~~Cheaper solution:~~ Equalizing filter.

$J(N) = J(N-1) + a_N\{z \dots\}\{x \dots\}$

$L = 3$



$z_4 \to$ $a_2 \; a_3 \; a_4$

$2 \quad 2 \quad 2 \to 2^L \quad a_2$

M-ary $L \to M^L$

Convolutional codes.

$a_0 \; a_1 \; a_2$ $\to$ $b_0 \; b_1$ $\longrightarrow$ $C_0 \; C_1 \; C_2 \; \dots$

$d_0 \; d_1 \; d_2 \dots$

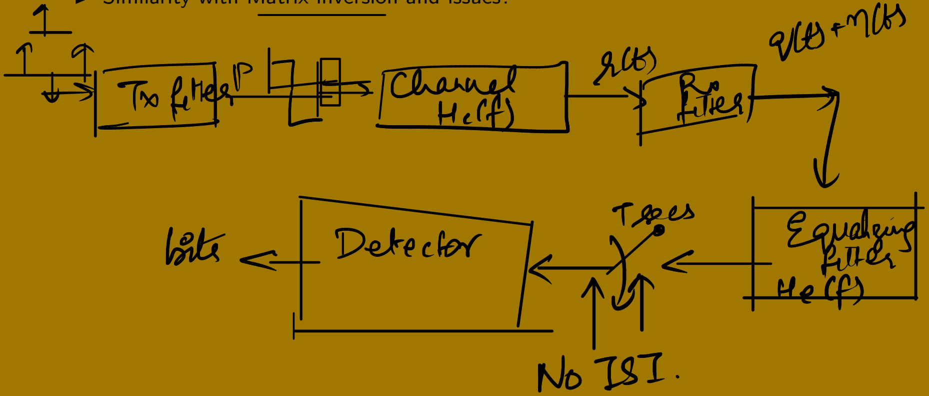
$C_3 \dots$

Equalizing filters

Equalizing filter

$He(f) = \frac{1}{H_c(f)}$

- We want $H_e(f)$ such that $H_r(f)H_t(f)H_c(f)H_e(f) = H_{RC}(f)$. Assume $H_r(f)H_t(f) = \boxed{H_{RC}(f)}$.
  - Given that $H_c(f) = |H_c(f)|\exp(j\theta_c(f))$, $\boxed{H_e(f) = \frac{1}{|H_c(f)|}\exp(-j\theta_c(f))}$.
  - Similarity with Matrix inversion and issues?



Channel
transf of
tx pulse

$$\underline{Cx = b.}$$

$Cx = b$ ← received signal (o/p of the Rx filter)

$$H_e(f) = \frac{1}{H_c(f)}.$$

$C^{-1}Cx = C^{-1}b.$

$\Rightarrow \boxed{x = C^{-1}b}$ → Equalizing filter!

1. C is not invertible $\Longrightarrow \min \|Cx - b\|^2.$

$$x^* = \underbrace{(C^TC)^{-1}C^T}_{C^+}b.$$

C^+ → Pseudoinverse of C.

$\Rightarrow$ Channel is a Linear system.

Does $1/H_c(f)$ exist?

$$\underline{A = MBM^{-1}} \quad \left(\begin{array}{c}\text{Similarity} \\ \text{Transf.}\end{array}\right)$$

$$H_c(f) \longleftrightarrow h_c(t)$$

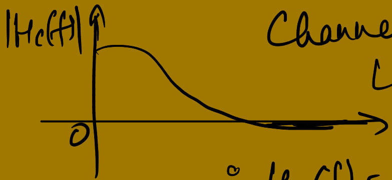
$$(h_c * x) = \mathcal{F}^{-1}\left(\mathcal{F}(h_c) \odot \mathcal{F}(x)\right)$$

$$Ax = M\,B\,M^{-1}x$$

$$\begin{bmatrix} Y(f_0) \\ \vdots \\ Y(f_n) \end{bmatrix} = \begin{bmatrix} H_c(f_0) & & 0 \\ & \ddots & \\ 0 & & H_c(f_n) \end{bmatrix} \begin{bmatrix} X(f_0) \\ \vdots \\ X(f_n) \end{bmatrix}$$

If any diagonal element $= 0$, the Mtx is not invertible.

$\Rightarrow$ If $H_c(f) = 0$ for any f, then ~~$H_c(f)$ is not inverti~~ble !!!



Channels modeled as LPF's are not invertible!

$\therefore$ $H_e(f) = \dfrac{1}{H_c(f)}$ does not exist!!

- Consider that a single root-raised cosine pulse was transmitted, and the output of the receiver filter z is as shown below:

$H_t(f) H_r(f) = H_{RC}(f)$

Discrete filter

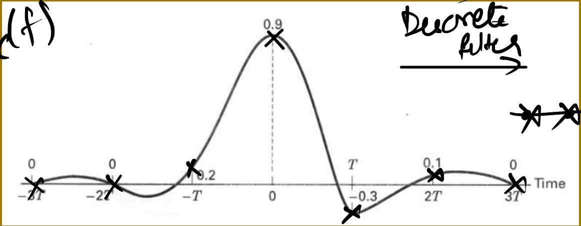


Figure: Received pulse waveform (Image Source: Sklar)

  - The aim is to eliminate ISI at multiples of T secs.

- Denoting the samples of z by z_k, can we build a discrete filter that takes in the sequence z_k and outputs a sequence x_k without ISI, i.e., $x_k = \delta_k$.

$$z_k = z(kT)$$

sklar

- Let $z_k, k = -\cancel{2}N, \dots, \cancel{2}N$ denote the $\overset{2}{\cancel{4}}N+1$ sampled receiver filter output and let $c_k, k = -N, \dots, N$ denote the filter coefficients.
  - Filter output: $x(n) = \underbrace{\sum_{k=-N}^{N} c(k) z(n-k)}_{= \delta(n)}$, $n = -2N, \dots, 2N$.

$x = c * z$

$2N+1 \quad 2N+1$

$4N+1$

$n = -2N$

$$\begin{bmatrix} z(-N) & \cdots & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} c_{-N} \\ c_{-N+1} \\ \vdots \\ c_0 \\ \vdots \\ c_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} n=-2N \\ \\ \\ n=0 \\ \\ \\ n=2N \end{array}$$

$$\underset{(4N+1)\times(2N+1)}{Z}\ \underset{(2N+1)\times 1}{c} = x_{(4N+1)\times 1}$$