Game Theory

In this world of reality, there are always some conflicts of interest between two or more opponents engaging themselves in the same field of business.

Example: Two business men A and B in the same field. They are fighting for the same business.

A has executives A_1, A_2, A_3 and
B has executives B_1, B_2, B_3, B_4.
At any point of time, A can employ the service of any executive at its hand without knowing which executive B is employing.

A_1, A_2, A_3 are strategies for A

B_1, B_2, B_3, B_4 are strategies for B.

Question: What would be the outcome of the process at the end?

Definitions:

- A and B are decision makers/Players
- A_1, A_2, A_3 are strategies of A
  B_1, B_2, B_3, B_4 are strategies of B
- Profit/loss is attached with each strategy
  [What is the profit for A is loss for B and vice verse]
- If A is a maximising player then B is the minimising player and vice verse.
- The profit/loss of any player (either A or B) is shown by a matrix called the pay-off matrix.

Example

			B		
		B_1	B_2	B_3	B_4
A	A_1	3	5	9	2
	A_2	4	-5	-3	1
	A_3	-2	0	6	-5

Here, this is a pay-off matrix for player A. This means:

- If A adopts strategy A_1 and B adopts strategy B_1 the the gain/profit of player A is 3 units ($\equiv$ the loss of player B is 3 units) ($\equiv$ the profit of player B is -3 units)

- If A adopts strategy A_3 and B adopts strategy B_4 then the profit of A is -5 units (it is a loss).

  ($\equiv$ profit of B is 5 unit)

Game

A game is a solution in which two or more decision makers (players) choose course of actions (strategies) available to them and the outcome is affected by the course of actions (strategies) adopted by the players collectively.

A game is a set of rules involving

i) $n \geq 2$ decision makers with mutually opposite interests.

ii) course of actions available to each player are known to him.

iii) A clearly defined set of end states that terminate the competition (will / loss / draw)

iv) pay-off to each players at the end of each play is known.

Problem of the game:

If A is a maximising player and B is a minimising player (for a game of 2 player), then

A's problem is to

maximise his minimum gain/profit

B's problem is to

minimize his maximum loss

Both A and B are ignorant about the strategies taken by their opponent.

	B_1	B_2	B_3
A_1	2	-5	3
A_2	-2	1	6
A_3	4	1	2

If A chooses	then B chooses	Payoff
A_1	B_2	-5
A_2	B_1	-2
A_3	B_2	

Zero sum game :

Let $p_i \leftarrow$ pay-off to player i at the end.

Then if

$$\sum_{i=1}^{n} p_i = 0 \text{, then the game}$$

is called a zero - sum game.

otherwise it is called a non-zero sum game.

where n is number of players.

Two person zero sum game

If $n = 2$.

In this case $p_1 + p_2 = 0$

(profit of one is the loss of another)

course of action / strategy

- Set of strategies:
  - Each strategy is associated with a probability.

$$
\begin{array}{c|c|ccccc}
 & & & & B & & \\
 & & y_1 & y_2 & \cdots\ y_j & \cdots & y_n \\
 & & B_1 & B_2 & \cdots\ B_j & \cdots & B_n \\
\hline
 & x_1\ A_1 & & & \vdots & & \\
 & x_2\ A_2 & & & \vdots & & \\
 & \vdots & & & & & \\
A & x_i\ A_i & \cdots & \cdots & a_{ij} & & \\
 & \vdots & & & & & \\
 & x_m\ A_m & & & & & \\
\end{array}
$$

$a_{ij} \leftarrow$ payoff of player A

$x_i \leftarrow$ probability with which player A chooses strategy A_i

$y_j \leftarrow$ probability with which player B chooses strategy B_j

$$0 \le x_i, y_i \le 1$$

$$\sum_{i=1}^{m} x_i = \sum_{j=1}^{n} y_j = 1$$

$(A_1, A_2, \ldots, A_i, \ldots A_m)$ strategies of A

$X = (x_1, x_2, \ldots x_i, \ldots, x_m)$ probabilities associated with each strategy.

$(B_1, B_2, \ldots, B_j, \ldots B_n)$ strategies of B

$Y = (y_1, y_2, \ldots, y_j, \ldots y_n)$ probabilities associated with each strategy.

In particular if

$$X = \xi_i = (0, 0, \ldots, \overset{i\text{-th position}}{1}, \ldots 0)$$

$$Y = \eta_j = (0, 0, \ldots, \overset{j\text{-th position}}{1}, \ldots 0)$$

Then the game (in particular the strategy) is called pure

- The game admitting of pure strategy is called <u>strictly determined</u>.
- otherwise it is the <u>game of chance</u>.

- zero sum game
- Two person (in general n person
  player player)
- Pure strategy
- mixed strategy

$\boxed{\text{Solving a two person zero sum strictly determined game}}$

A, B : players

$A_1, A_2, \cdots, A_m$: strategies of A
$B_1, B_2, \cdots, B_n$: " " B

a_{ij} = pay-off of player A if he chooses action A_i and player B chooses action B_j

$-a_{ij}$ = pay-off of player B.

A, B are completely ignorant about each others actions taken at any point of time.

$A = (a_{ij})$: pay-off matrix is given.

Problem:

To find optimal strategies for player A and player B and the value of the game (v) ie, pay-off that player A gets at the end of each play.