1. **Evaluate**
   (a) $\int (r^2 + \vec{\mathbf{r}} \cdot \vec{\mathbf{a}} + a^2)\delta^3(\vec{\mathbf{r}} - \vec{\mathbf{a}})dV$ **over the whole space where** $\vec{\mathbf{a}}$ **is a fixed vector.**

   (b) $\int_V |\vec{\mathbf{r}} - \vec{\mathbf{b}}|^2 \delta^3(5\vec{\mathbf{r}})dV$ **over a cube of side 2, centered at the origin, and** $\vec{\mathbf{b}} = 4\hat{\mathbf{y}} + 3\hat{\mathbf{z}}$

2. **The electric field in a region is given as**
$$\begin{aligned}\vec{E} &= \frac{\sigma}{2\epsilon_0}\hat{i}; \quad \text{for} \quad x > 0 \\ &= -\frac{\sigma}{2\epsilon_0}\hat{i}; \quad \text{for} \quad x < 0\end{aligned}$$
   Find the charge distribution in the region using the differential form of Gauss's law.

3. **The electric field in a region is cylindrically symmetric, given as follows:**
$$\begin{aligned}\vec{\mathbf{E}}(\vec{\mathbf{r}}) &= \frac{c\hat{\mathbf{s}}}{s}; \quad \text{when} \quad s \geq a \\ &= 0; \quad \text{when} \quad s < a\end{aligned}$$
   Find the charge distribution in the region using Gauss' law.

4. **We have seen that** $\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$**. In a similar manner justify that**
$$\vec{\nabla} \cdot \left(\frac{\hat{s}}{s}\right) = 2\pi\delta^2(\vec{s})$$
   Here s **is the distance from the** z **axis in cylindrical coordinates and** $\delta^2(\vec{s})$ **is a two dimensional delta function on the** xy **plane.**

5. **Prove that** $\delta(r) = 4\pi r^2\delta^3(\vec{r})$ **and** $\delta(s) = 2\pi s\delta^2(\vec{s})$**.**
   Here $\int_0^\epsilon \delta(r)dr = 1$ **for any** $\epsilon > 0$**. The integral is 0 otherwise.** $\delta(s)$ **is defined likewise.**

6. The electric field in a region is given as

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$$

Find the charge densities in the region.