Tutorial 04

1. The probability mass function of a random variable X is given by $p(i) = \frac{c\lambda^i}{i!}$, $i = 0, 1, 2, \ldots$, where λ is some positive value. Find (a) $P\{X = 0\}$ and (b) $P\{X > 2\}$.

2. A product that is sold seasonally yields a net profit of b dollars for each unit sold and a net loss of l dollars for each unit left unsold when the season ends. The number of units of the product that are ordered at a specific department store during any season is a random variable having probability mass function $p(i),\ i \geq 0$. If the store must stock this product in advance, determine the number of units the store should stock so as to maximize its expected profit.

3. Consider a jury trial in which it takes 8 of the 12 jurors to convict the defendant; that is, in order for the defendant to be convicted, at least 8 of the jurors must vote him guilty. If we assume that jurors act independently and that, whether or not the defendant is guilty, each makes the right decision with probability θ, what is the probability that the jury renders a correct decision? Let α be the probability that the defendant is guilty.

4. A communication system consists of n components, each of which will, independently, function with probability p. The total system will be able to operate effectively if at least one-half of its components function. For what values of p is a 5-component system more likely to operate effectively than a 3-component system?

5. Compute the expectation and variance of a binomial random variable with parameters (n, p).

6. Consider an experiment that consists of counting the number of α particles given off in a 1-second interval by 1 gram of radioactive material. If we know from past experience that, on the average, 3.2 such α particles are given off, what is a good approximation to the probability that no more than 2α particles will appear?

7. Prove that the expectation and variance of a Poisson random variable with parameter λ is λ.

8. Compute the expected value and the variance of a negative binomial random variable with parameters r and p.

9. Find the expectation and variance of a geometric random variable.

10. A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot and to accept the lot only if all 3 are nondefective. If 30 percent of the lots have 4 defective components and 70 percent have only 1, what proportion of lots does the purchaser reject?