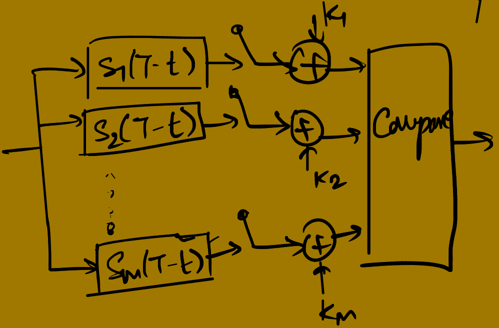
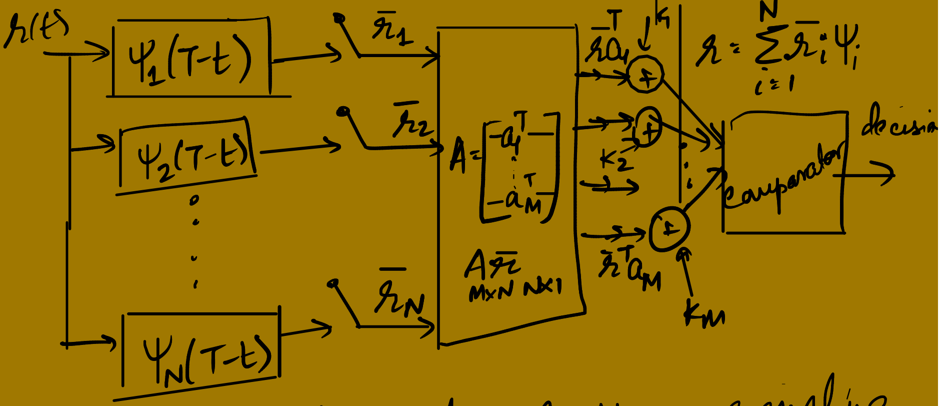
CT303 Lecture 18: 21 October 2020

- Lecture 17 recap:
  - M-ary signaling. $m = \log_2 M$ 'N'
  - Likelihood $P(r|s_i) = \mathcal{N}(a_i, \sigma I)$.
  - MAP based M-ary signaling receiver using M matched filters: $s_1(T-t), \ldots, s_M(T-t)$.
  - Decision: s_p where $p = \arg\max_i \{k_i + \bar{r}^T a_i\}$.



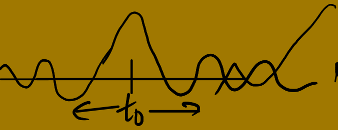
$S = \text{span}(\{s_1, \ldots, s_M\})$

$B = \{\psi_1, \ldots, \psi_N\}, N \leq M.$



N-matched filters based M-ary signaling receiver

Inter Symbol Interference (ISI)



$$\text{sinc}\, t = \frac{\sin(\pi t)}{\pi t} \sim \frac{1}{t}$$

Desire that the pulses should decay rapidly

- Due to channel response, there may be ISI at the receiver.
  - Sinc pulses are ideal, i.e., one can transmit R 1 pulses/sec over a channel with 2 bandwidth $R/2$ Hz. But these pulses are not realizable. Also sensitive to jitter.
  - Given that $p(t)$ is a realizable pulse/waveform that does not introduce ISI, will you transmit pulses of the form $p(t)$?

Determining ISI-less pulses

- Constraint 1: No ISI at decision making instants $\boxed{t = nT}$ $\Leftarrow$ $p(t)$

$$\boxed{\begin{aligned} p(0) &= 1 \\ p(nT) &= 0, \forall n \in \mathbb{Z} \quad n \neq 0 \end{aligned}}$$

- Re-writing this constraint:

$$\Downarrow$$

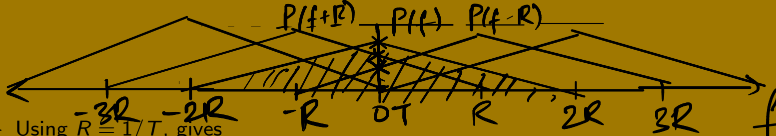
$$\boxed{p(t) \sum_{n \in \mathbb{Z}} \delta(t - nT) = \delta(t)}$$

- Taking Fourier transform on both sides:

$$P(f) * \frac{1}{T} \sum_{n \in \mathbb{Z}} \delta(f - \frac{n}{T}) = 1 \tag{1}$$

$$\frac{1}{T} \sum_{n \in \mathbb{Z}} P(f - \frac{n}{T}) = 1 \tag{2}$$

$R = \frac{1}{T}$



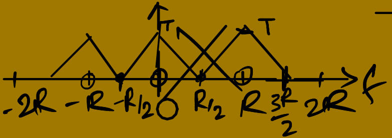
- Using $R = 1/T$, gives

Let Channel B.W = R Hz

$$\boxed{\sum_{n\in\mathbb{Z}} P(f-nR) \equiv T} \qquad \boxed{(3)}$$

- Constraint 2: Maximum bandwidth of p must be R, i.e., $P(f) = 0, \forall |f| > R$.
- $\Rightarrow$ at any f, at most two replicas of P overlap.

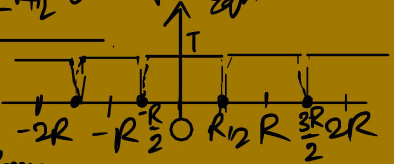
* ~~Let us denote bandwidth of p by B.~~
* Can $B < R/2$ Hz?



$B < R/2$ is not possible

Eqn 3 not satisfied

* Can $B = R/2$ Hz?

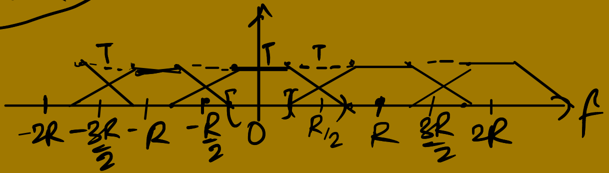


$p(f) = T \quad -R/2 < f \leq R/2$

$\Rightarrow$ = rectangular Fourier transform

$p(t) =$ sinc

- Can $B > R/2$ and $B < R$?

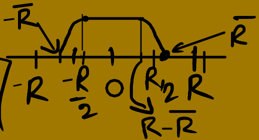


- ▶ Example: Raised Cosine pulse:

$\bar{R} \to$ RoW of p-

$R/2 < \bar{R} < R$

$$
P(f) = \begin{cases} 1, & |f| \le R - \bar{R} \\ \cos^2\left(\frac{\pi}{4}\frac{(|f| + \bar{R} - R)}{(\bar{R} - R/2)}\right), & R - \bar{R} < |f| < \bar{R} \\ 0, & |f| > \bar{R} \end{cases}
$$



- ▶ Pulse waveform:

$$
\boxed{p(t) = R\ sinc(Rt)\ \frac{\cos(2\pi(\bar{R} - R/2)t)}{1 - [4(\bar{R} - R/2)t]^2}}
$$

- ▶ The ratio $\boxed{r = \frac{\bar{R} - R/2}{R/2}}$ is called the *roll-off* factor.

$0 \le r \le 1$

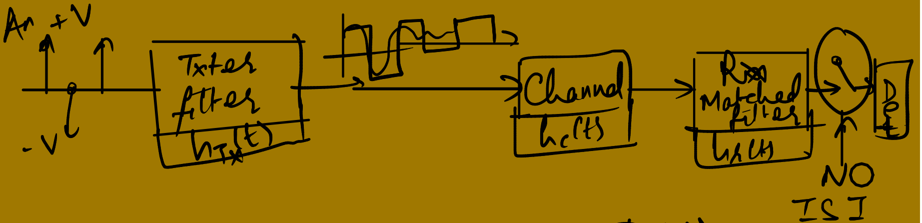
$p(t) \sim \frac{1}{t}\frac{1}{t^2} \sim \frac{1}{t^3}$

pulses that have no ISI at $t = nT$

$p(t) \rightarrow$ NO ISI at $t = nT$.

should the txted signal be

$$x(t) = \sum_n A_n p(t - nT) \; ?$$



$$H_{Tx}(f) \cdot H_c(f) \cdot H_R(f) = P_{RC}(f)$$

Let's assume $H_c(f) \equiv 1$

$\Rightarrow \quad H_{Tx}(f) \cdot H_R(f) = P_{RC}(f)$

$h_{Tx}(t) =$ Symbol wf to be Txted on the channel

$h_R(t) = h_{Tx}(T - t)$

$|H_R(f)| = |H_{Tx}(f)|$

$\Rightarrow \quad |H_R(f)|\,|H_{Tx}(f)| = |P_{RC}(f)|$

Since $|H_R(f)| = |H_{Tx}(f)|$

$\Rightarrow \quad |H_{Tx}(f)| = \sqrt{|P_{RC}(f)|}$

Square Root of RC pulses are called Root-Raised Cosine pulses.