Set cover problem

<u>Input</u>: An universe U of elements $a_1, a_2, \ldots, a_n$

a collection of subsets of U

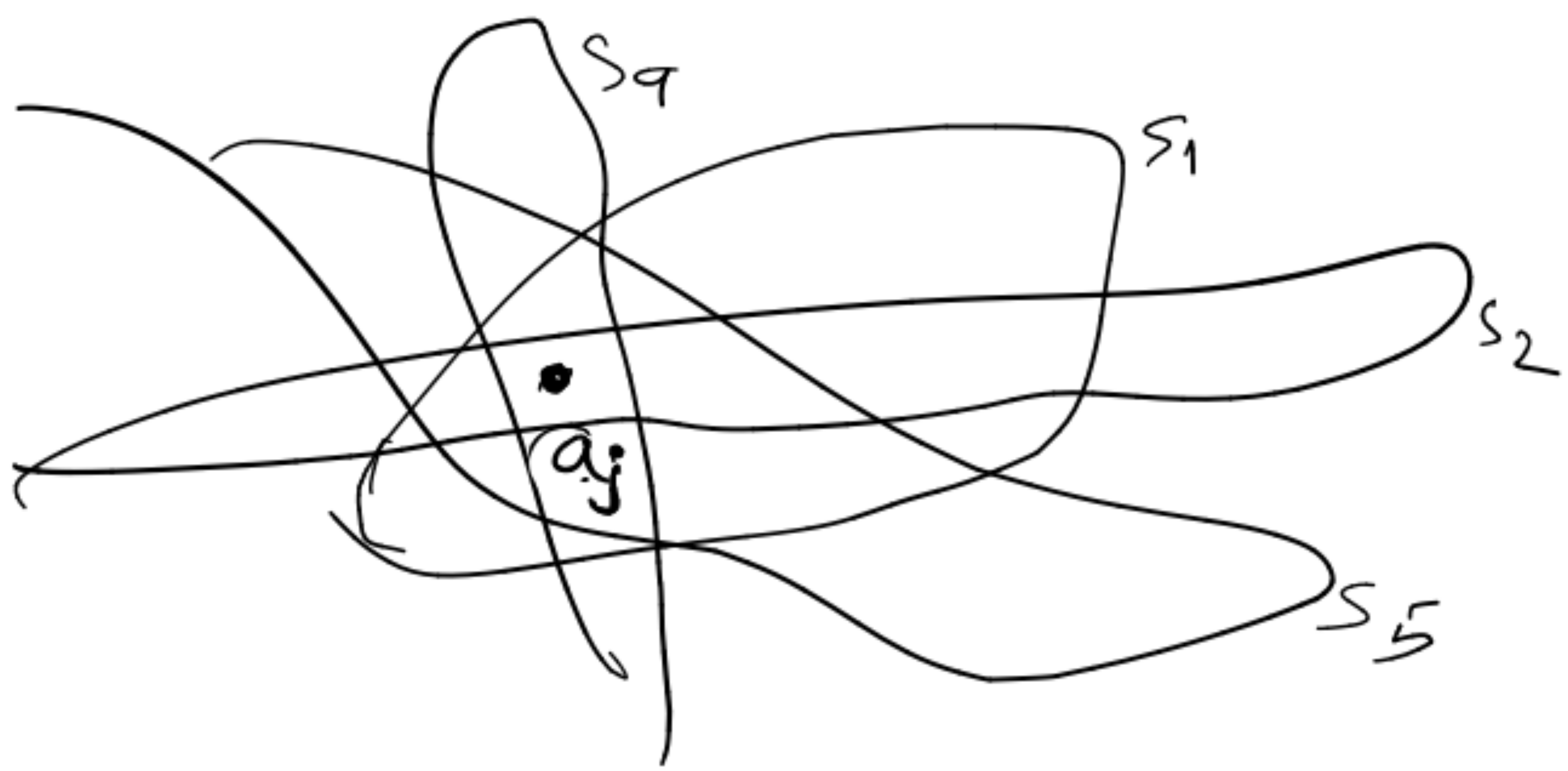
$$C = \{S_1, S_2, \ldots S_m\}$$

a weight function

$$w : S \longrightarrow \mathbb{R}^+$$

Find a minimum cost subcollection $C' \subseteq C$ such that

$$\bigcup_{S_i \in C'} S_i = U$$



ILP

For each set S_i we take a binary decision variable x_i such that

$$x_i = \begin{cases} 1 & \text{if } S_i \text{ is in the solution} \\ 0 & \text{if } S_i \text{ is not in the solution} \end{cases}$$

objective is.

$$\min \sum_{S_i \in C} w_i x_i$$

$$\text{s.t.} \quad \sum_{a_j \in S_i \in C} x_i \geq 1 \quad \forall\, a_j \in U$$

ILP for set cover.

$$\sum_{s_i \in C} w_i x_i$$

s.t.

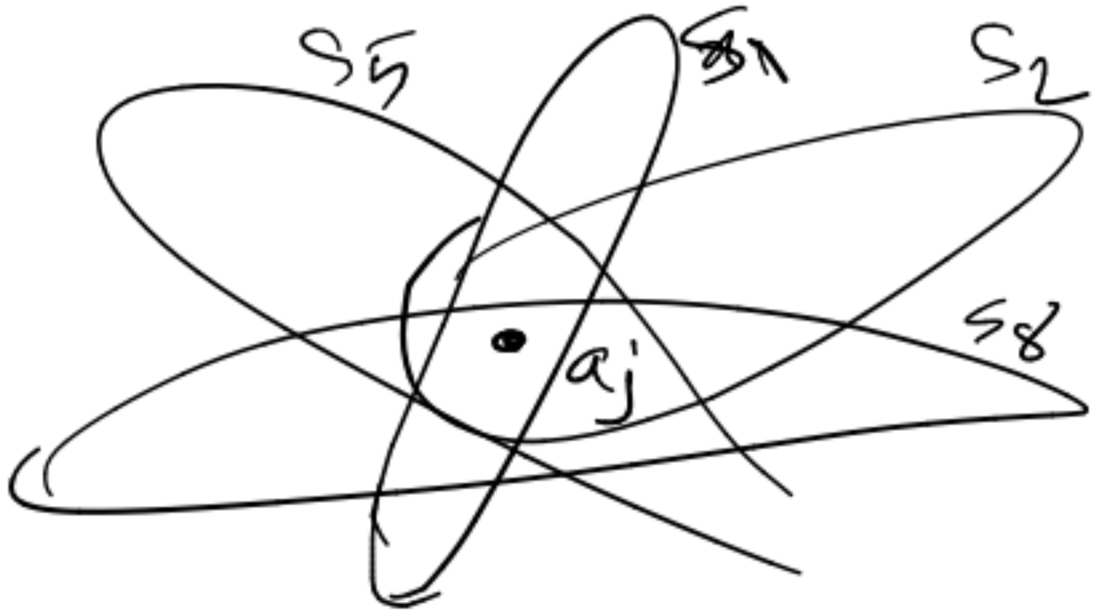
$$\boxed{\sum_{a_j \in s_i \in C} x_i \geq 1} \quad \forall\, a_j \in U$$

$$x_i \in \{0, 1\}$$

~~Consider the sets are bounded.~~
~~ie $|s_i| \leq \kappa$~~ for each $s_i \in C$

Each element a_j belongs to at most κ sets. for all $a_j \in U$.

- First formulate the SCP as a ILP.
- Relax the ILP to a LP by taking $x_i \geq 0$
- Solve the LP and let x^* be a fractional solution
- Consider $C' = \{ S_i \mid x_i^* \geq \frac{1}{K} \}$ where K defined as earlier.
- Return C' as a solution to SCP.



$x_1 + x_2 + x_5 + x_8 \geq 1$

Maximum flow

Given a directed graph $G(V,E)$ with two nodes s and t and positive capacities $c: E \to \mathbb{R}^+$

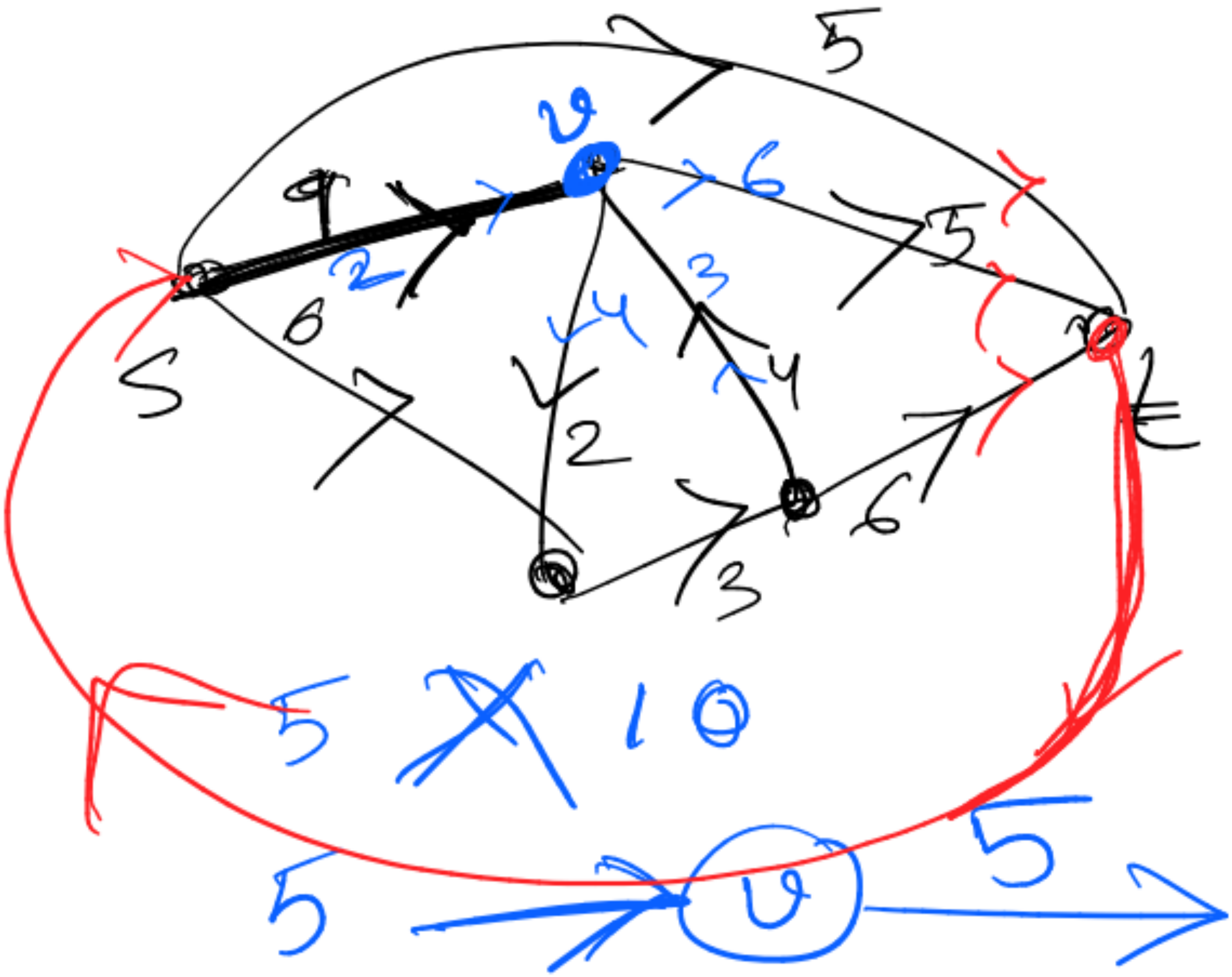
Find the maximum amount of flow that can be sent from s to t such that

i) capacity constraint:

For each edge e the flow pass through e is bounded by its capacity.

ii) flow conservation:

At each vertex v other than s and t, the total flow into v should equal to the total flow out of v.



take an edge from t to s
whose capacity is ∞.

f_{ts} as the flow from
t to s.

LP maximise f_{ts}

f_{ij} an the flow from i-th vertex to jth vertex i.e, $(i,j) \in E$.

constraints:

capacity constraint $f_{ij} \leq c_{ij} \quad \forall (i,j) \in E$

flow conservation $\sum_{j:(i,j)\in E} f_{ji} - \sum_{j:(i,j)\in E} f_{ij} \leq 0 \quad \forall i \in V$

$f_{ij} \geq 0$

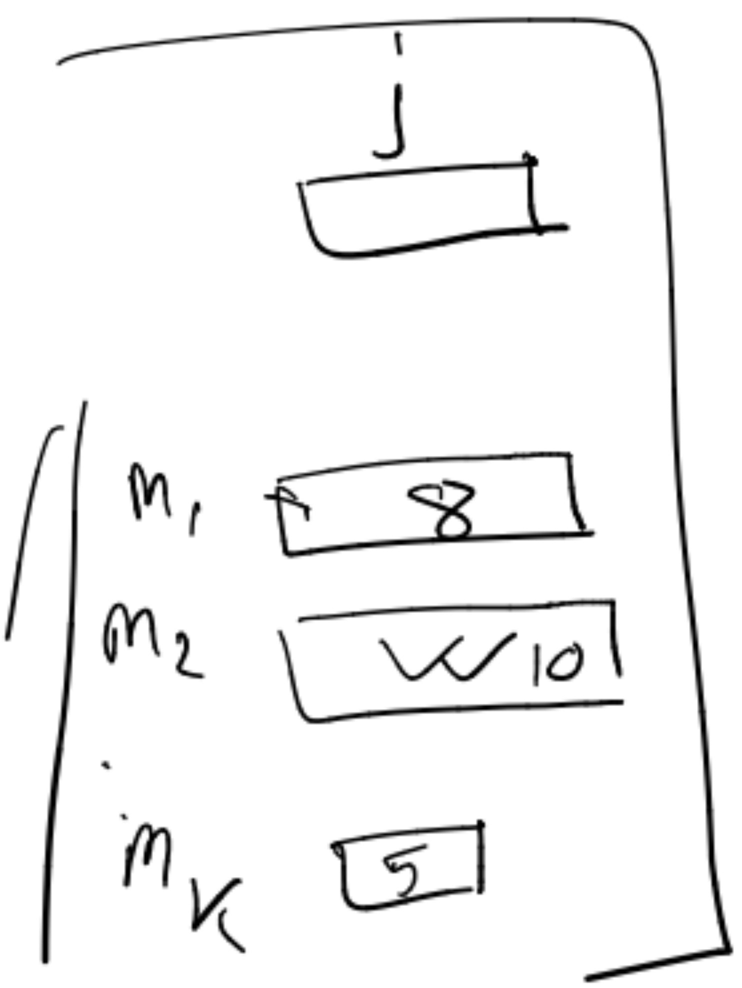
Scheduling jobs on unrelated parallel machines:

Given a set J of jobs and m machines.

p_{ij} be the process time of job j to machine i

The goal is to schedule the jobs on the machines so as to minimize the makespan

maximum process time of any machine.



Let t be the makespan.

$x_{ij} \leftarrow$ job j is scheduled on machine i

$$x_{ij} = \begin{cases} 1 & \text{job } j \text{ is scheduled on machine } i \\ 0 & \text{otherwise.} \end{cases}$$

minimise t

s.t. $\displaystyle\sum_{i \in M} x_{ij} = 1 \quad \forall\, j \in J$

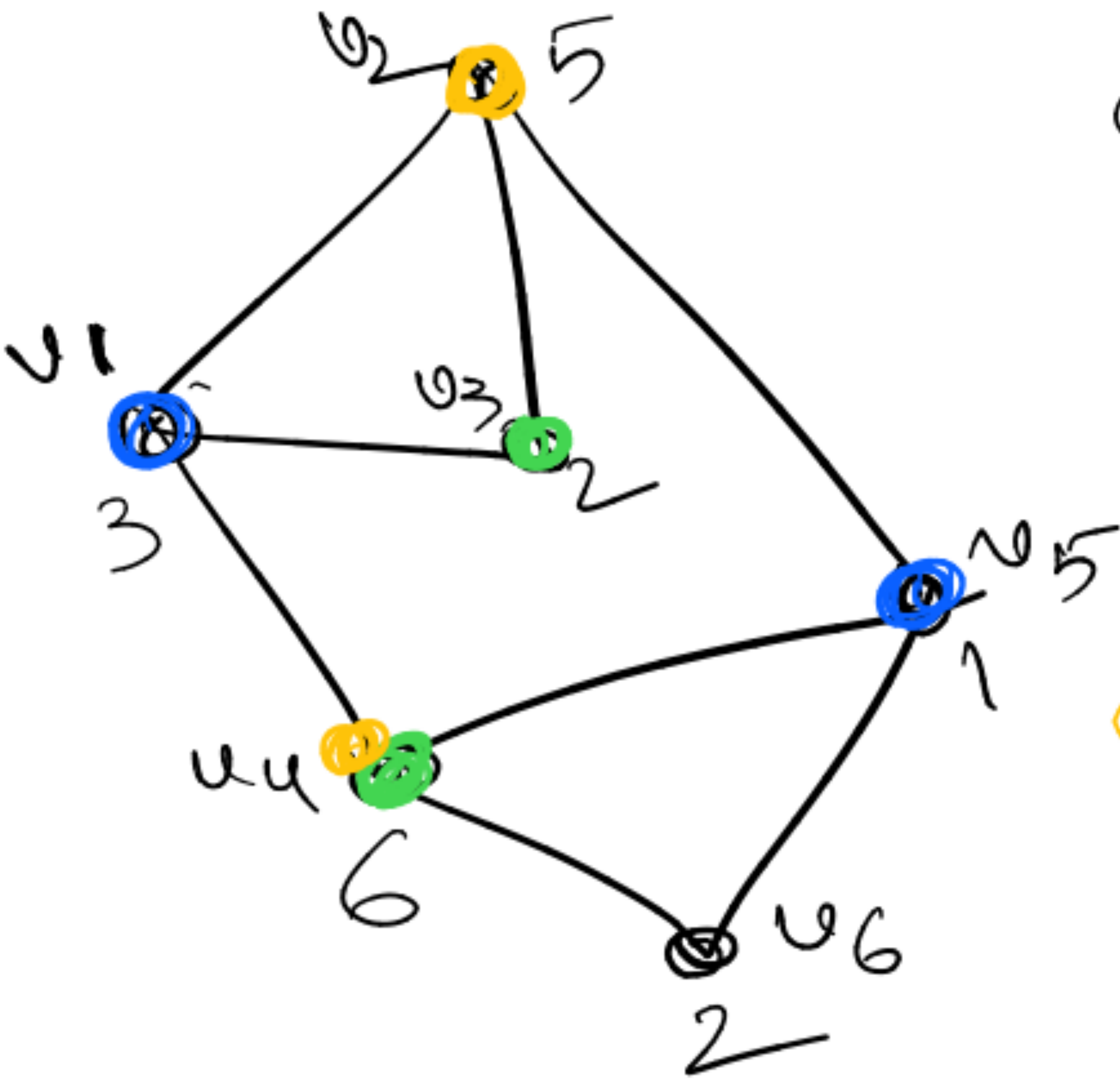
$\displaystyle\sum_{j \in J} x_{ij}\, p_{ij} \leq t \quad \forall\, i \in M$

$x_{ij} \in \{0, 1\} \quad \forall\, i \in M,\ j \in J$

Independent Set problem

Given a graph $G(V, E)$ and a weight function $w : V \to \mathbb{R}^+$

The goal is to find a maximum cost set of vertices $V' \subseteq V$ such that any two vertices in V' is non-adjacent



$\{v_1, v_5\}$ IS of cost 4

$\{v_3, v_4\}$ IS of cost 8

$\{v_2, v_4\}$ IS of cost 11