Lecture 13: 5 October 2020

- Lecture 12 review
  - Formatting: Sampling and Quantization
  - Baseband modulation
  - Binary PCM waveforms
  - Sources of corruption: Sampling jitter, Quantization error, Channel noise (AWGN), Channel distortion - ISI.

Lab-4

NRZ-L without delay → Not WSS SP?

NRZ-L with delay → WSS SP.

Analytically derive R_x.

$R_x(t_1, t_2)$

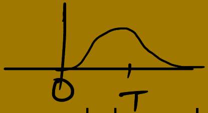
$$x(t) = \sum_{k=-\infty}^{\infty} V_k \, p(t - kT)$$

$+V$

$-V$

T → pulse duration

Bandwidth efficiency



- As demonstrated, if bits are encoded as pulses, this will lead to ISI, unless the channel BW is high enough.
  - How many symbols/sec can we accommodate in a given bandwidth?

B.W: W How much R_s can it support?

η_B $\boxed{R_s/W}$ → symbols/sec/Hz

↳ Bandwidth efficiency

$\eta_B < 1$ not good.

Binary PCM scheme with maximum η_B

- Consider a (ideal, $\boxed{\text{noiseless}}$) channel with BW f_0 Hz.

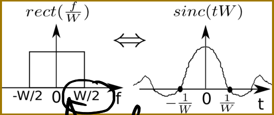
channel will be modeled as an ILPF $[-f_0, f_0]$ Hz

- Note the Fourier transform pair:

$x(t)$

sinc(tW)

$x(t-t_0)$



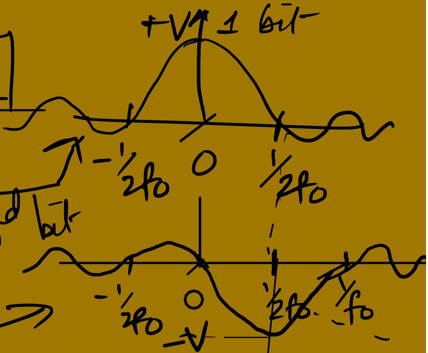
$= f_0$

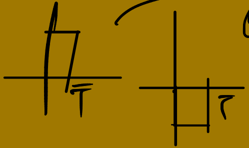
Figure: Fourier Transform pair

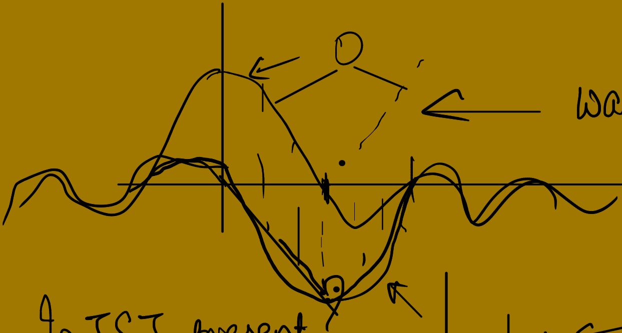
- Let $W = 2f_0$, $T_s = \frac{1}{2f_0} = \frac{1}{W}$.
  - $\boxed{\tilde{x}(t) = \sum_k x(kT_s) sinc(2f_0 t - k).}$

$+V$

or $-V$

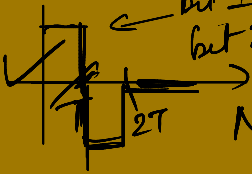






waveform representing 2 bits

Is ISI present?
(With sinc waveforms)
$(-\infty, \infty)$.
Yes, there is ISI!!



bit 1 : $[0, T]$
bit 2 : $[T, 2T]$

No ISI!

There is no ISI <u>every T seconds</u> //

- Note that $sinc(2f_0 nT_s - k) = 0, \forall n \neq k$.
- Introduces ISI, but zero ISI at decision making instants $kT_s, k \in \mathbb{Z}$.
- Thus $2f_0$ symbols/sec can be transmitted over a channel with BW f_0 Hz, without distortion (assuming no noise)

In practice generating sinc wf is not possible.

Duobinary encoding

0 1 0 0 1 1 1 0 0 - - —

$-V \; +V \; -V \; -V \; +V \; +V \; +V$. - —

- Let $\{x_k\}$ be a sequence of bi-polar amplitudes assigned to bits, each separated by T sec.
- Let $y_k = \underline{x_k + x_{k-1}}$, with a fixed x_{-1}.

$$\underline{\underline{y_k}} = \begin{cases} \underline{2V}, \text{ if } x_k = x_{k-1} = V \\ \underline{0}, \text{ if } x_k = V, x_{k-1} = -V \\ -\underline{2V}, \text{ if } x_k = x_{k-1} = -\underline{V} \end{cases}$$

- Transmit $y(t) = \underline{\sum_k y_k sinc(\frac{t-kT}{T})}$.

(No noise)

$\boxed{\text{Channel}} \longrightarrow y(t) \xrightarrow{kT_s} y_k$

$y_k = 2V \rightarrow x_k = V$

$y_k = -2V \rightarrow x_k = -V$ | $y_k = 0, \; x_k = -x_{k-1}$

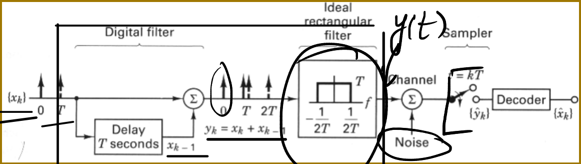


Figure: Duobinary encoding block diagram. Image Source: Sklar.

system. ILPF

- Decoding:

$$\hat{x}_k = V, \text{if } y_k = 2V$$
$$\hat{x}_k = -V, \text{if } y_k = -2V$$
$$\hat{x}_k = \overline{\hat{x}_{k-1}}, \text{if } y_k = 0$$

Realizing Duobinary signaling

- $y_k = x_k + x_{k-1} \rightarrow H_1(f) = 1 + \exp(-j2\pi fT), h_1(t) = \delta(t) + \delta(t-T)$.
  - $y_k \rightarrow \sum_k y_k sinc(Tt-k) \Rightarrow H_2(f) = T\ rect(fT), h_2(t) = sinc(\frac{t}{T})$.

    ILPF
  - Thus, the duobinary system is

$$H(f) = H_1(f)H_2(f) = T(1 + \exp(-j2\pi fT))rect(fT),$$

$$h(t) = sinc\left(\frac{t}{T}\right) + sinc\left(\frac{t-T}{T}\right)$$

$$H(f) = Te^{-j\pi fT}\underbrace{\left(e^{j\pi fT} + e^{-j\pi fT}\right)}_{2\cos(\pi fT)} rect(Tf)$$

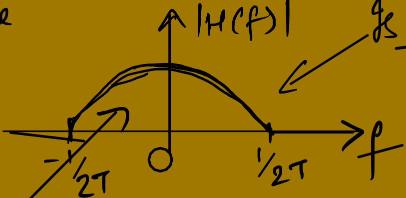
- Simplifying, rect(tT)

$$|H(f)| = \begin{cases} 2T\cos(\pi f T), & -\frac{1}{2T} \le f \le \frac{1}{2T} \\ 0, & \text{otherwise.} \end{cases}$$

× Not realizable

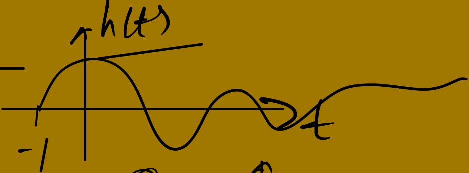
Is this realizable?

No!



- Is this free lunch?

Better approximation is possible.



h(t-1)