Groups and linear algebra (SC220) Autumn 2018
In Sem -I Time: 1hr 30 min

Name: ________________________________

Student I.D.: ________________________________

Section 1. True/False (2 pts. each)

Print "T" if the statement is true, otherwise print "F". In either case give a justification or a counter example.

______ If G and H are cyclic groups then $G \times K$ is also cyclic.

______ D_6 (Group of Symmetries of a hexagon) is isomorphic to A_4 (Group of even permutations of 4 letters)

______ Z_{11}^* is a cyclic group.

______ The remainder when 3^{47} is divided by 23 is 9

______ In S_4 let $\sigma = (123)(34)$ then σ^{2018} is $(13)(24)$

______ In D_n the subgroup generated by r is a Normal subgroup.

______ Every abelian group is cyclic

______ The matrices of type $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$ with $a, b, d \in \mathbb{R}, ad \neq 0$ form a subgroup of $GL_2(\mathbb{R})$

______ The group $(\mathbb{Q}, +)$ is isomorphic to $(\mathbb{Q}^+, \times)$

______ Let α and β be any two permutations in S_n then $\alpha^4\beta^{-2}\alpha$ is an odd permutation if α is an even permutation.

Section 2. Short Answer (10 pts each)

Answer all problems in as thorough detail as possible.

1. Prove that the subgroup of S_4 generated by (12) and $(12)(34)$ is isomorphic to D_4.

2. Let G be a group and $|G| = pq$ where p and q are primes. Show that any proper subgroup of G is cyclic.

3. Let H and K be Normal subgroups of a group G such that $H \cap K = e$. Show that every element of H commutes with every element of K.

Answer Key for Exam $\boxed{\text{A}}$

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