Lecture-7

Recap:

Independent Events

$$P(A \cap B) = P(A)\,P(B)$$

Discrete Random Variables

Expectation of a discrete random variable:

$$E[X] = \sum_{i=1}^{n} x_i\, p(x_i)$$

discrete: countable ⟵ finite OR

non-discrete: uncountable ⟵ set of Reals

one to one correspondence with the set of natural numbers $\mathbb{N}$

E.g. discrete, infinite

$$P(X=i) = \frac{c\lambda^i}{i!}, \quad i=0,1,2,\ldots$$

$P(X=0) = ? = c = e^{-\lambda}$

$$\sum_{i=0}^{\infty} P(X=i) = 1$$

$$\sum_{i=0}^{\infty} \frac{c\cdot\lambda^i}{i!} = 1$$

$$c \left(\sum_{i=0}^{\infty} \frac{\lambda^i}{i!} \right) = 1$$

$$c \cdot e^{\lambda} = 1 \Rightarrow c = e^{-\lambda}$$

e.g.

Gamble A		Gamble B	
0.89	1 Crore	1	1 Crore
0.04	5 Crores		
0.07	Nothing		
15 people		20 people	
1.09		1	

Gamble C		Gamble D	
0.04	5 Crores	0.11	1 Crore
0.96	Nothing	0.89	Nothing
25 people		7 people	
0.20		0.11	

Allais Paradox

e.g.

Box: 90 balls.

30 red balls

60 either black or yellow

$b + y = 60$

A	B
win if you get a red ball	win if you get a black ball
40 people	15 people
$\frac{30}{90}$ $\boxed{30>b}$	$\frac{b}{90}$
C	D
win if you get a red ball or a yellow ball	win if you get a black ball or a yellow ball
$\frac{30+y}{90}$ 1 person	60 people $\frac{30+30}{90}$

$y > 30$

$60 - b > 30$

$b < 30$

e.g.

	Quiz	show
Q_1	0.6	200
Q_2	0.8	100

X = total reward

$= \{0, 100, 200, 300\}$

$E[X]$ → you want to maximize

$Q_1 Q_2$ or $Q_2 Q_1$

Σ_1	p_1	r_1
Σ_2	p_2	r_2

if Σ_1 first, then Σ_2

$E[X]$ $X =$

0	$1-p_1$
r_1	$p_1(1-p_2)$
r_2	0
$r_1 + r_2$	$p_1 p_2$

$$E[X] = r_1(p_1)(1-p_2) + (r_1 + r_2)\, p_1 p_2$$

if you answer Q_2 first.

$$X = \begin{matrix} 0 & 1-p_2 \\ r_1 & 0 \\ r_2 & p_2(1-p_1) \\ (r_1+r_2) & p_1 p_2 \end{matrix}$$

$$E[X] = r_2 p_2 (1-p_1) + (r_1+r_2) p_1 p_2$$

You will answer Q_1 first

if $r_1 p_1 (1-p_2) > r_2 p_2 (1-p_1)$

$$\frac{r_1 p_1}{1-p_1} > \frac{r_2 p_2}{1-p_2}$$

e.g.

Functions of a random variable

X	p
-1	0.2
0	0.5
1	0.3

$Y = X^2$

$= g(X)$

Y	p
0	0.5
1	0.5

$E[X] = -1(.2) + 0(.5) + 1(.3)$

$= 0.1$

$E[Y] = \underline{0.5}$

$E[Y] = \sum g(x)\, p(x)$

$= (-1)^2(0.2) + 0^2(0.5) +$

$(1)^2\, 0.3 = \underline{0.5}$

$$E[g(x)] = \sum g(x) p(x)$$

e.g. You are a wine dealer in Himachal. Your business is seasonal. Each bottle you sell, you make a profit of Rs. b. Each bottle unsold, you make a loss of Rs. l.

X = no. of bottle that you sell

How many bottles do you need to stock in order to maximize your profit?

X	
0	p_0
1	p_1
⋮	
n	p_n

Variance:

$X = 0$ with $p = 1$

$$Y = \begin{cases} -1 & \text{with } p = 0.5 \\ 1 & \text{with } p = 0.5 \end{cases}$$

$$Z = \begin{cases} -100 & p = 0.5 \\ +100 & p = 0.5 \end{cases}$$

$E[X] = E[Y] = E[Z] = 0$

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e.g.

Kohli: 30, 40, 50, 50, 55, 45, 60, 70, 45, 55

Dhoni: 10, 10, 90, 80, 10, 90, 90, 20, 20, 80

$E[\text{Kohli}] = 50 = E[\text{Dhoni}]$

Variance:

$$Var = E\left[x - E[X] \right]^2$$

$E[X]$ = mean / weighted average