Lecture - 8 P (1)

<u>Recap</u>:

Expectation of a discrete random variable

$$E[X] = \sum x_i \, P(X = x_i) = \mu$$

Variance:

$$Var(X) = E[X - E[X]]^2$$

$$= E[X - \mu]^2$$

$$= E[X^2 + \mu^2 - 2\mu X]$$

$$= E[X^2] + E[\mu^2] - 2\mu \underline{E[X]}$$

$$= E[X^2] + \mu^2 - 2\mu.\mu$$

$$= E[X^2] - \mu^2$$

$$= E[X^2] - (E[X])^2$$

e.g. You throw a dice.

what is $Var(X)$, where X is the outcome?.

$$Var(X) = E[X^2] - [E(X)]^2$$

$$E[X] = \frac{7}{2} = \frac{1}{6}(1+2+\dots+6)$$

$$E[X^2] = \frac{1}{6}(1+2^2+3^2+\dots+6^2)$$

$$Var(aX+b) = a^2 Var(X)$$

$$Var(X+10)$$

$$\boxed{Var(X) = E[X - E[X]]^2}$$

$$E[(aX+b) - E(aX+b)]^2$$

$$= E[aX + \cancel{b} - aE(X) - \cancel{b}]^2$$

$$= E[\underline{a}X - \underline{a}E[X]]^2 = a^2 E[X - E[X]]^2$$

$$= a^2 Var(X)$$

Bernoulli random variable

Doing an experiment has 2 outcomes

$P(X=1) = p$ success

$P(X=0) = 1-p$ failure

Binomial random variable.

Repeating an experiment n times, wherein $p(\text{success}) = p$ for each trial. Trials are independent

$X =$ no. of times you succeed.

Probability distribution ④

for $\underline{\underline{X}}$.

$X \in \{0, 1, 2, \cdots, n\}$

$\neq (0, \cdots, n)$

$$P(X = i) = \binom{n}{i} p^i (1-p)^{n-i}$$

$$\underset{1}{\underline{S}} \; \underline{F} \; \underset{2}{\underline{S}} \; \underline{F} \; \underset{i}{\underline{S}} \; \underline{F}\,\underline{F}$$

$\xleftarrow{\quad\quad} n \xrightarrow{\quad\quad}$

$(a+b)^n$

$$\sum_{i=0}^{n} P(X=i) = 1$$

$$= \sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i} = [p + (1-p)]^n$$

e.g.
SK is on a jury trial.
There are 12 people in the jury. Atleast 8 people need to say that he is guilty for him to be sentenced.

$P(\text{juror makes the correct decision}) = 0.9$

$P(\text{SK is guilty}) = 0.7$

What is the probability that the jury makes the correct decision? J

G: SK is guilty

Success: a particular juror making the correct decision.

$P(G) = 0.7 \quad P(\text{Success}) = 0.9$

J: jury makes correct decision

$$P(J) = P(J \cap G) + P(J \cap \bar{G})$$

$$= \underset{0.7}{P(G)}\, P(J|G) + \underset{0.3}{P(\bar{G})}\, P(J|\bar{G})$$

8 or more people make the correct decision. $\Big\}$ $P(J|G)$

$\Downarrow$

8 jury members say that he is guilty

$$\sum_{i=8}^{12} \binom{12}{i} p^i (1-p)^{12-i}$$

$p = 0.9$

$$P(J|\bar{G}) = \sum_{i=5}^{12} \binom{12}{i} p^i (1-p)^{12-i}$$

no. of people making

right decision	wrong decision
5	7
6 . . . 12	6

Binomial random variable 8

$E[X] = np$

$Var[X] = np(1-p)$

Proofs are in the book

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success probability $= p$

For what value of k

is $P(X = k)$ highest?

$p = 0.7, \quad n = 50$

Maximize

$$\binom{n}{k} p^k (1-p)^{n-k} \quad \text{over } k$$

$$k = 0, 1, \cdots, n$$

$k = 0$

$$\binom{n}{0} p^0 (1-p)^{n-0}$$

$$= (1-p)^n$$

$k = 1$ $$\binom{n}{1} p^1 (1-p)^{n-1}$$

$$= np(1-p)^{n-1}$$

$$\frac{P(X=k)}{P(X=k-1)} \geq 1$$

$$\frac{\binom{n}{k} p^k (1-p)^{n-k}}{\binom{n}{k-1} p^{k-1} (1-p)^{n-k+1}} \geq 1$$

H.W.