Tutorial 3

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1. There are n types of coupons, and each new one collected is independently of type i with probability p_i, $\sum_{i=1}^{n}(p_i) = 1$. Suppose k coupons are to be collected. If A_i is the event that there is at least one type i coupon among those collected, then, for $i \neq j$, find
   a) $P(A_i)$
   b) $P(A_i \cup A_j)$
   c) $P(A_i|A_j)$

2. Independent trials resulting in a success with probability p and a failure with probability $1 - p$ are performed. What is the probability that n successes occur before m failures? If we think of A and B as playing a game such that A gains 1 point when a success occurs and B gains 1 point when a failure occurs, then the desired probability is the probability that A would win if the game were to be continued in a position where A needed n and B needed m more points to win.

3. A female chimp has given birth. It is not certain, however, which of two male chimps is the father. Before any genetic analysis has been performed, it is felt that the probability that male number 1 is the father is p and the probability that male number 2 is the father is $1 - p$. DNA obtained from the mother, male number 1, and male number 2 indicate that, on one specific location of the genome, the mother has the gene pair (A,A), male number 1 has the gene pair (a, a), and male number 2 has the gene pair (A, a). If a DNA test shows that the baby chimp has the gene pair (A, a), what is the probability that male number 1 is the father?

4. At a party, n men take off their hats. The hats are then mixed up, and each man randomly selects one. We say that a match occurs if a man selects his own hat. What is the probability of
   a) No matches?
   b) Exactly k matches?