TUTORIAL 05

1. Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1-x)^2 \ ; \ -1 < x < 1 \\ 0 \ ; \ \text{otherwise} \end{cases}$$

   (a) What is the value of c?

   (b) What is the cumulative distribution function of X?

2. Let X be uniformly distributed over (α, β). Find (a) $E[X]$ and (b) $Var(X)$.

3. Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. That is, they arrive at 7, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that he waits, (a) less than 5 minutes for a bus and (b) more than 10 minutes for a bus.

4. Consider the function

$$f(x) = \begin{cases} C(2x - x^3) & ; \ 0 < x < \frac{5}{2} \\ 0 & ; \ \text{otherwise} \end{cases}$$

   Could f be a probability density function? If so, determine C. Repeat if $f(x)$ were given by

$$f(x) = \begin{cases} C(2x - x^2) & ; \ 0 < x < \frac{5}{2} \\ 0 & ; \ \text{otherwise} \end{cases}$$

5. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & : \ x > 10 \\ 0 & x \leq 10 \end{cases}$$

   (a) Find $P\{X > 20\}$.

   (b) What is the cumulative distribution function of X?

   (c) What is the probability that, of 6 such types of devices, at least 3 will function for at least 15 hours? What assumptions are you making?

6. Suppose that if you are s minutes early for an appointment, then you incur the cost cs, and if you are s minutes late, then you incur the cost ks. Suppose also that the travel time from where you presently are to the location of your appointment is a continuous random variable having the probability density function f. Determine the time at which you should depart if you want to minimize your expected cost.

7. If X is a normal variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute (a) $P\{X > 5\}$, (b) $P\{4 < X < 16\}$, (c) $P\{X < 8\}$, (d) $P\{X < 2\}$ and (e) $P\{X > 16\}$.

8. If X is a normal random variable with parameters $\mu = 3$ and $\sigma^2 = 9$, find (a) $P\{2 < X < 5\}$, (b) $P\{X > 0\}$ and (c) $P\{|X - 3| > 6\}$.

9. A randomly chosen IQ test taker obtains a score that is approximately a normal random variable with mean 100 and standard deviation 15. What is the probability that the score of such a person is (a) above 125; (b) between 90 and 110?

10. Suppose that the travel time from your home to your office is normally distributed with mean 40 minutes and standard deviation 7 minutes. If you want to be 95 percent certain that you will not be late for an office appointment at 1 P.M., what is the latest time that you should leave home?

11. The life of a certain type of automobile tire is normally distributed with mean 34,000 miles and standard deviation 4000 miles.
    (1) What is the probability that such a tire lasts over 40,000 miles?
    (b) What is the probability that if lasts between 30,000 and 35,000 miles?
    (c) Given that it has survived 30,000 miles, what is the conditional probability that the tire survives another 10,000 miles?

12. The annual rainfall in Cleveland, Ohio is approximately a normal random variable with mean 40.2 inches and standard deviation 8.4 inches. What is the probability that
    (a) next year's rainfall will exceed 44 inches?
    (b) the yearly rainfall in exactly 3 of the next 7 years will exceed 44 inches?