Sample Midterm Solutions

Problem 1
(a)

$$\begin{aligned}
& x_2'x_0'(x_3+x_3')(x_1+x_1') + x_1'x_0(x_2+x_2')(x_3+x_3') + \\
& x_2'x_1'(x_3+x_3')(x_0+x_0') + x_3x_2x_0(x_1+x_1') \\
= \; & x_3x_2'x_1x_0' + x_3'x_2'x_1x_0' + x_3x_2'x_1'x_0' + \\
& x_3'x_2'x_1'x_0' + x_3x_2x_1'x_0 + x_3'x_2x_1'x_0 + \\
& x_3x_2'x_1'x_0 + x_3'x_2'x_1'x_0 + x_3x_2'x_1'x_0 + \\
& x_3'x_2'x_1'x_0 + x_3'x_2'x_1'x_0' + x_3x_2'x_1'x_0' + \\
& x_3x_2x_1x_0 + x_3x_2x_1'x_0 \\
= \; & \Sigma m(0,1,2,5,8,9,10,13,15) \\
= \; & E_2(x_3,x_2,x_1,x_0)
\end{aligned}$$

(b)
$\Pi M(3,4,5,7,11,12,14)$

Problem 2

$$\begin{aligned}
X &= A' + B'C' \\
z &= (XD)' \\
&= ((A'+B'C')D)' \\
&= (A'+B'C')' + D' \\
&= D' + A(B'C')' \\
&= D' + A(B+C) \\
&= D' + AB + AC
\end{aligned}$$

Problem 3
(a)

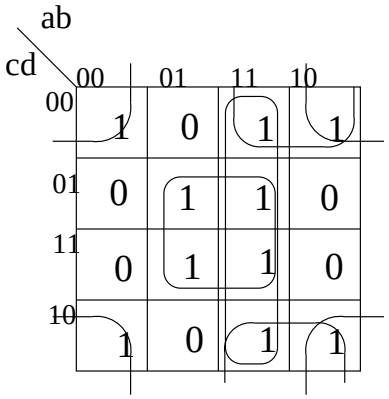
$$\begin{aligned}
T_{pHL} &= T_{pHL}(X_1) + T_{pHL}(A_2) + T_{pHL}(O_2) + T_{pHL}(X_2) \\
&= 0.32 + 0.023(2) + 0.18 + 0.019(2) + 0.22 + 0.021(1.1) + 0.32 + 0.023(1) \\
&= 1.170ns \\
T_{pLH} &= T_{pLH}(X_1) + T_{pLH}(A_2) + T_{pLH}(O_2) + T_{pLH}(X_2) \\
&= 0.32 + 0.038(2) + 0.15 + 0.039(2) + 0.14 + 0.039(1.1) + 0.32 + 0.038(1) \\
&= 1.165ns
\end{aligned}$$

(b)

	AND	OR	XOR	
size =	3(2)	+2(2)	+2(3)	=16

Problem 4

$$\begin{aligned} E &= (a'b + d)' + (b' + d')' \\ &= (a'b)'d' + (bd) \\ &= (a + b')d' + bd \\ &= ad' + b'd' + bd \end{aligned}$$



Only bd and $b'd'$ are essential prime implicants.
Sum of products is not unique: another solution is $ab + b'd' + bd$

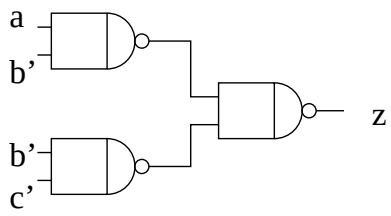
$$\begin{aligned} \text{Product of sums} &= (db' + ba'd')' \\ &= (db')'(ba'd')' \\ &= (d' + b)(b' + a + d) \end{aligned}$$

Product of sums is unique

Problem 5

$$\begin{aligned} z &= ((a + b')' + ((a' + b)' + (a + b + c)')')' \\ &= (a + b)'((a' + b)' + (a + b + c)') \\ &= (a + b')(ab' + a'b'c') \end{aligned}$$

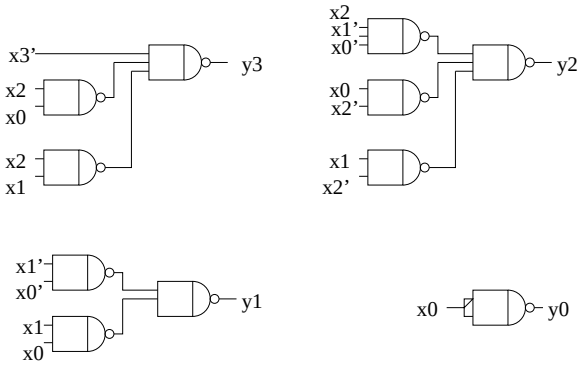
$$
\begin{aligned}
&= ab' + a'b'c' \\
&= b'(a + a'c') \\
&= b'(a + c') \\
&= ab' + b'c'
\end{aligned}
$$



Problem 6

$x_3x_2x_1x_0$	$y_3y_2y_1y_0$
0000	0011
0001	0100
0010	0101
0011	0110
0100	0111
0101	1000
0110	1001
0111	1010
1000	1011
1001	1100
1010	xxxx
1011	xxxx
1100	xxxx
1101	xxxx
1110	xxxx
1111	xxxx

$$
\begin{aligned}
y_3 &= x_3 + x_2x_0 + x_2x_1 \\
y_2 &= x_2x_1'x_0' + x_0x_2' + x_2'x_1 \\
y_1 &= x_1'x_0' + x_1x_0 \\
y_0 &= x_0'
\end{aligned}
$$


x3’
x2
x0
x2
x1
y3
x2
x1’
x0’
x0
x2’
x1
x2’
y2
x1’
x0’
x1
x0
y1
x0
y0