Lecture - 12

Recap:

Continuous random variables

Cumulative distribution

Expectation & variance

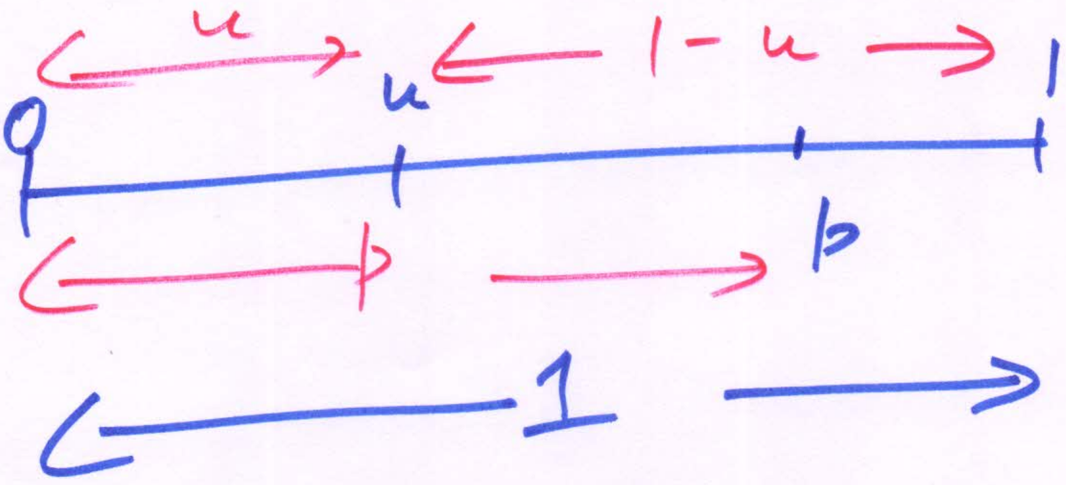
function of a random variable

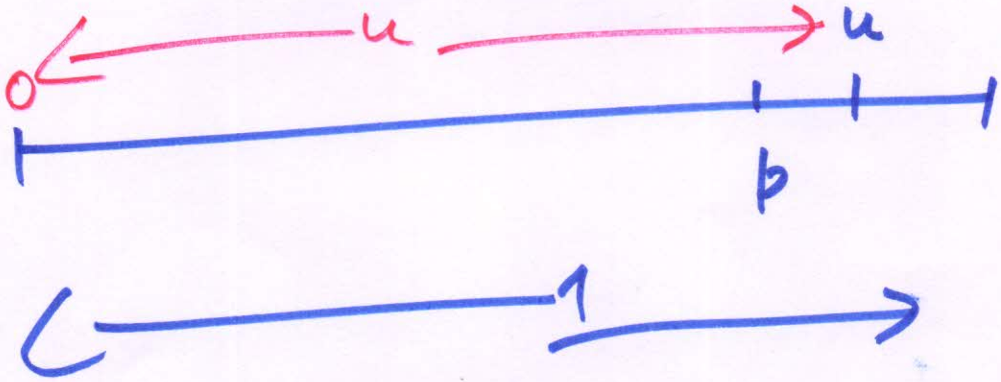
$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

Lemma: Y is non-negative

$$E[Y] = \int_{0}^{\infty} P(Y > y)\, dy$$

e.g.





$$f(u) = \begin{cases} 1 & \text{if } 0 < u < 1 \\ 0 & \text{otherwise} \end{cases}$$

$L(u) =$ length of the part of the stick that contains p

$$E[L(u)] = \int_0^1 L(u) f(u)\, du = \int_0^1 L(u)\, du$$

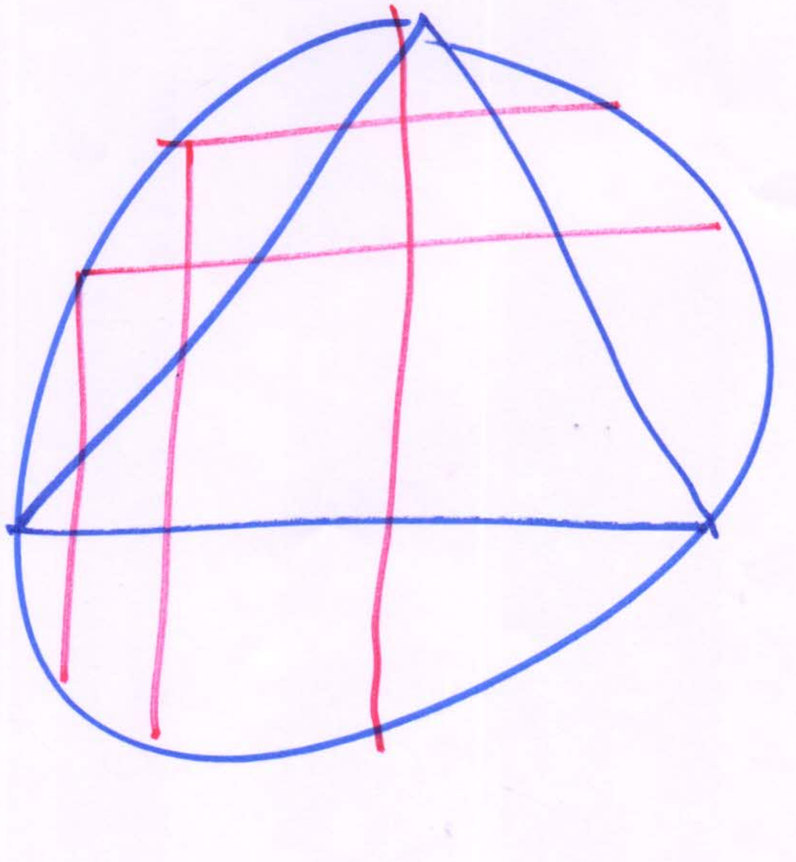
③

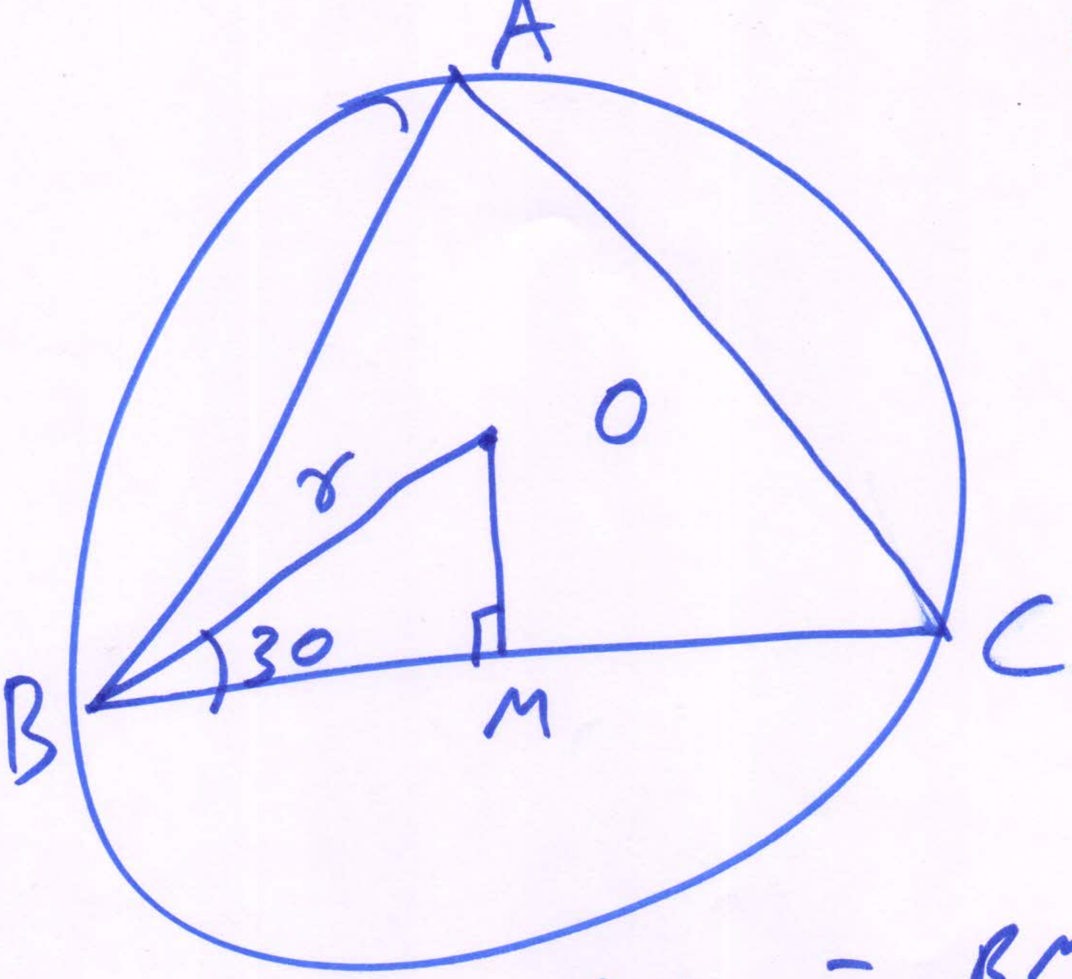
$$L(u) = \begin{cases} 1-u & \text{if } 0 < u < p \\ u & \text{if } p < u < 1 \end{cases}$$

$$\int_0^p (1-u)\,du + \int_p^1 u\,du$$

$$= p - p^2 + \frac{1}{2}$$

e.g: Consider a circle of radius r. What is the probability that ~~a rand~~ the length of a random chord is more than the length of the side of an equilateral triangle inscribed in the circle.





$$\frac{\sqrt{3}}{2} = \cos 30 = \frac{BM}{BO} = \frac{BM}{r}$$

$$BM = \frac{\sqrt{3}}{2} r$$

$$BC = 2BM = \underline{\underline{\sqrt{3} r}}$$

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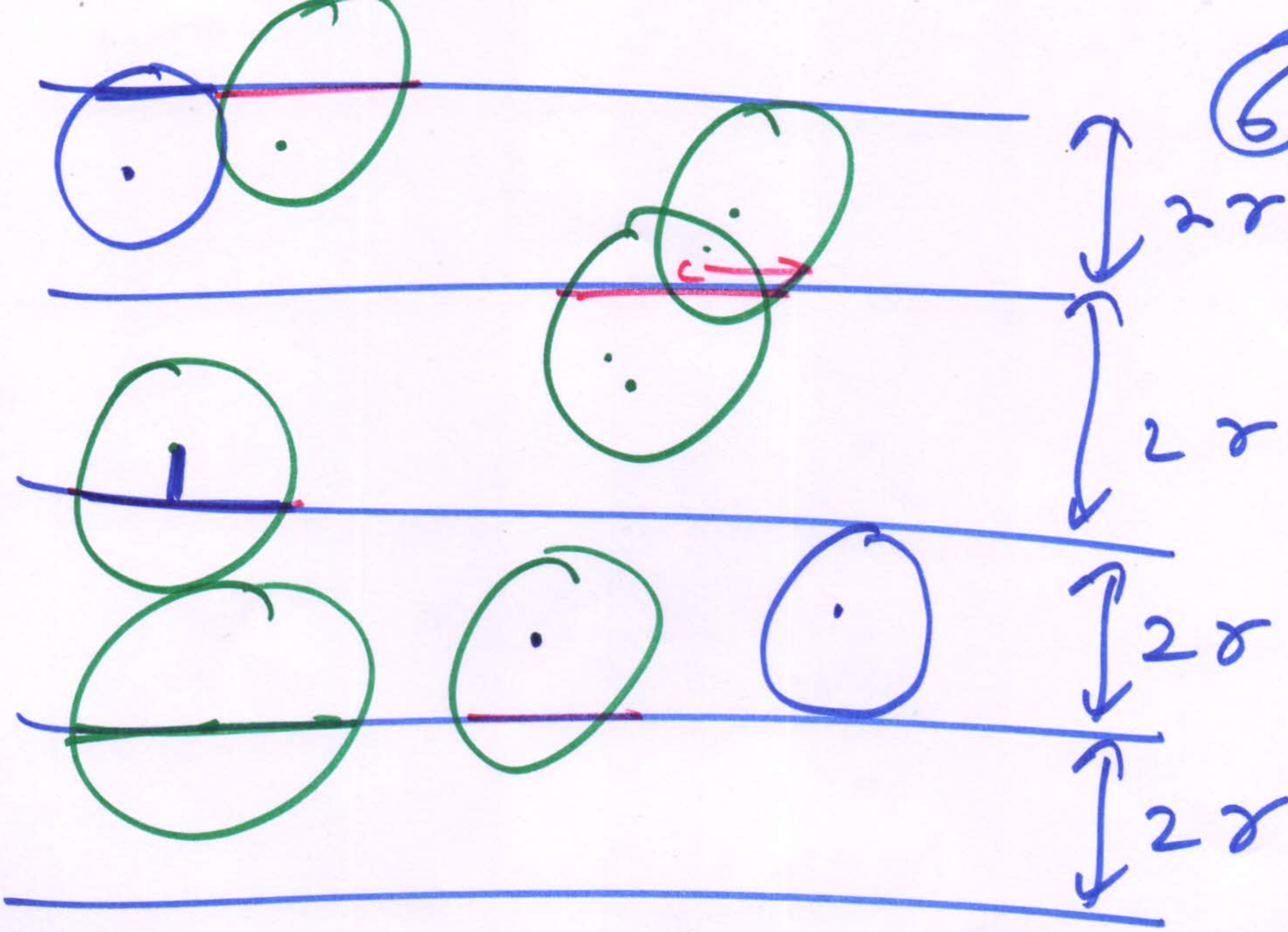


x = distance from the center of the circle to the nearer line.

$$x \in [0, r]$$

$$f(x) = \begin{cases} \dfrac{1}{r}, & 0 < x < r \\ 0 & \text{otherwise} \end{cases}$$

Uniform distribution

$$f(x) = \begin{cases} a & 0 < x < \gamma \\ 0 & \text{otherwise} \end{cases}$$

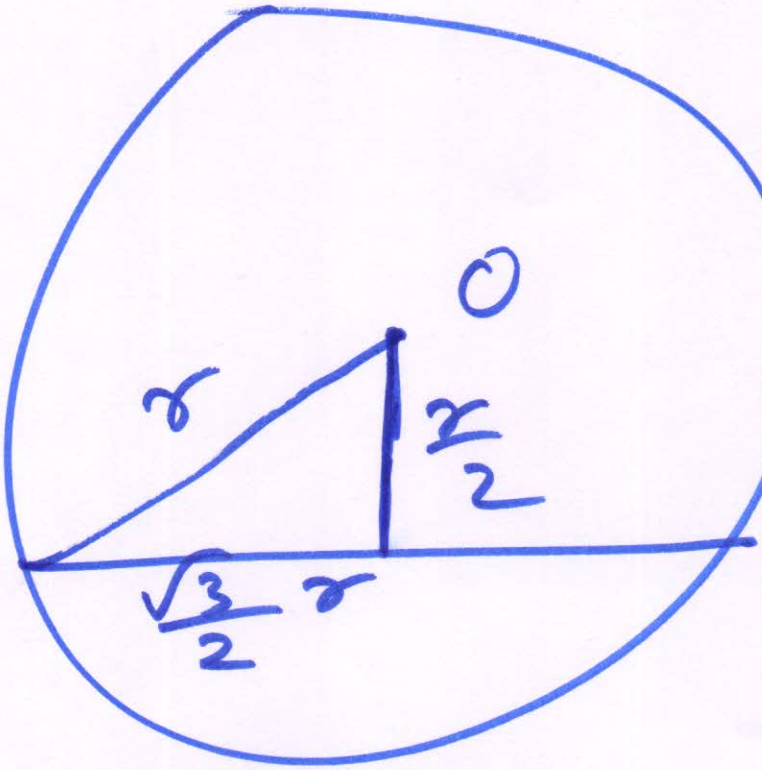
$$\int_0^{\gamma} f(x)\,dx = 1$$

$$\int_0^{\gamma} a\,dx = 1$$

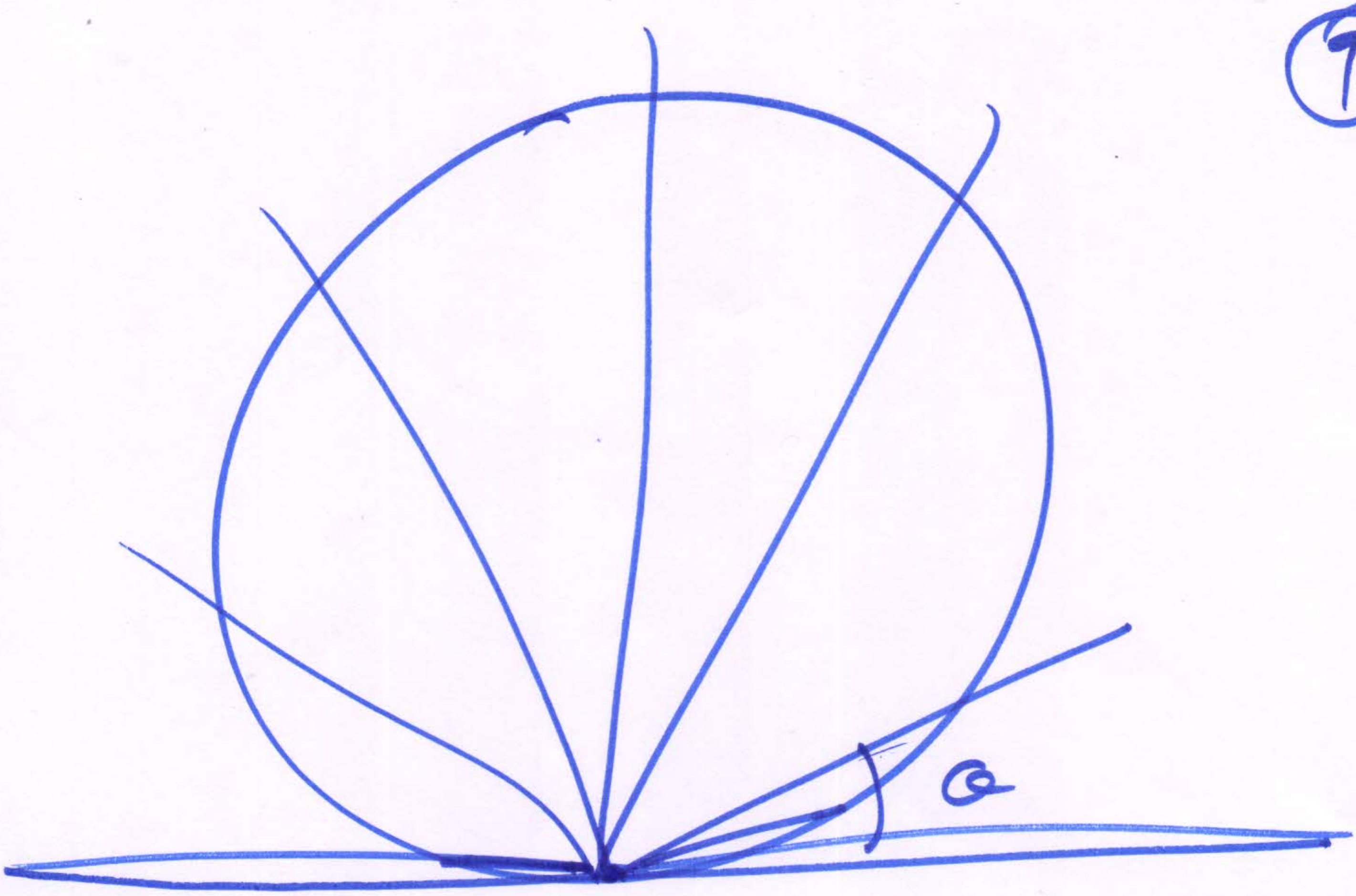
$$\therefore\ a = \frac{1}{\gamma}$$

$U\ [\alpha, \beta]$

$$f(U) = \begin{cases} \dfrac{1}{\beta - \alpha} & \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases}$$



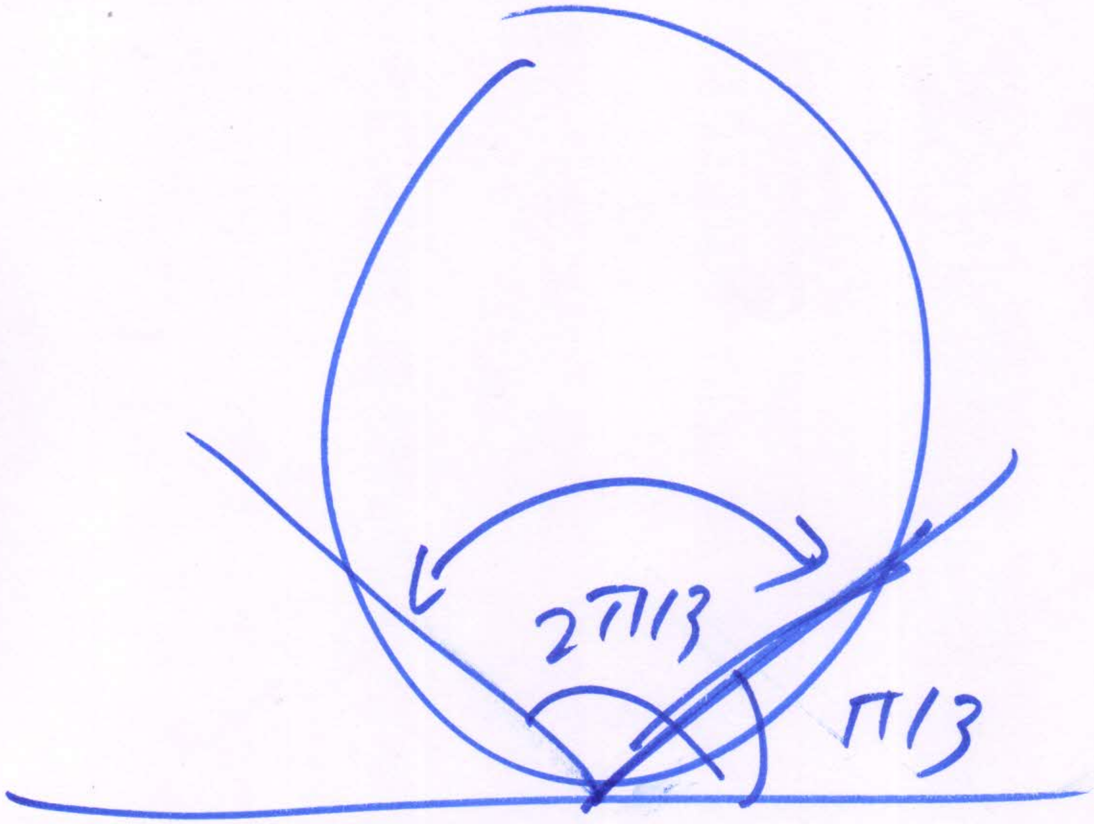
$$p = \int_{0}^{r/2} f(x)\, dx = \frac{1}{2}$$



θ is uniformly distributed

from 0 to π

$$f(\theta) = \begin{cases} \frac{1}{\pi} & 0 < \theta < \pi \\ 0 & \text{otherwise} \end{cases}$$



$$p = \int_{\pi/3}^{2\pi/3} f(\sigma)\, d\sigma = \frac{1}{3}$$

-1	-1	0	0	0	$\dots$
$-\frac{1}{2}$	$\frac{1}{2}$	-1	0	0	$\dots$
$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	-1	0	$\dots$
$-\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	-1	$0 \; 0 \; \dots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	0	0	0	0	

Sum $=0$

Sum $=-2$

Cantor's diagonalization argument: $\mathbb{R}$ is uncountable

<u>e.g.:</u> If you are 5 minutes early, the cost is 6S.

if you are 5 minutes late, the cost is 3S

Travel time is a continuous random variable X with density function $f(x)$.

You leave t minutes early for your appointment.

What is the best value of t to minimize the cost?