More aspects of initialisation

- 1) The right hand side value or the b value
- 2) Types of variables
- 3) Types of objective function
- 4) Types of constraints.

1) Right hand side or b value

The right hand side of each constraint should be non-negative. otherwise multiply a -1 to both sides of the constraint.

Exm

$$\begin{cases} \max z = x_1 + 2x_2 \\ \text{s.t.} \quad 3x_1 + 2x_2 \geq -5 \quad \text{——} ① \\ \quad x_1, x_2 \geq 0. \end{cases}$$

① can be written as

$$-3x_1 - 2x_2 \leq 5 \quad \text{——} ②$$

2) Types of variables

- variables are of 3 types.

  $\geq$, $\leq$, and unrestricted in sign

- we need $\geq$ type variables.

- For $\leq$ type variables we do the following.

  Let x_K be a variable such that $x_K \leq 0$

  for x_K we take another variable x_t with $x_t \geq 0$ and $\underline{\underline{x_t = -x_K}}$

  This change is made in all occurences of x_K in the LPP.

$\underline{\underline{Ex^m}}$

$$
P_1 \left.\begin{aligned}
&\max Z = 2x_1 + 3x_2 \\
&\text{s.t.} \quad x_1 + 2x_2 \geq 5 \\
&\qquad x_1 \geq 0, \ \underline{\underline{x_2 \leq 0}}.
\end{aligned}\right\}
$$

take $\boxed{x_3 = -x_2}$ and P_1 becomes

$$
\left.\begin{aligned}
&\max Z = 2x_1 - 3x_3 \\
&\text{s.t.} \quad x_1 - 2x_3 \geq 5 \\
&\qquad x_1, x_3 \geq 0
\end{aligned}\right\}
$$

- For unrestricted type variables. we do the following.

unrestricted — it can take positive — negative — or a zero value.

Let x_k be an unrestricted variable. then replace x_k by two variables x_p and x_q such that $x_p, x_q \geq 0$ and $\boxed{x_k = x_p - x_q}$

Ex^m $\max z = 2x_1 + 5x_2$

P_1 s.t. $x_1 + 3x_2 \geq 5$

$x_1 \geq 0$, x_2 unrestricted.

$x_2 = x_3 - x_4$ then P_1 becomes,

$\max z = 2x_1 + 5(x_3 - x_4)$

s.t. $x_1 + 3(x_3 - x_4) \geq 5$

$x_1, x_3, x_4 \geq 0$.

3> Types of objective function.

- Two types < maximisation / minimization.

we solve the maximization problem using simplex algorithm.

For minimization problem, convert it to a maximization problem and then solve it.

How??

multiply a -1 with Z.

4) <u>Types of constraints</u>

- There are 3 types of constraints. "$\geq$", "$\leq$" and "$=$"

- "$\leq$" type we introduce slack variables.

- "$\geq$" type we introduce surplus and artificial variables.

- "$=$" type, we introduce artificial variables.

- Artificial variables are used to identify the initial basic feasible solution.

Iteration

conside the problem.

$$\max Z = 70x_1 + 40x_2$$

$$P_1 \quad \text{s.t.} \quad 4x_1 + 2x_2 \leq 80 \quad \text{--- (1)}$$

$$3x_1 + 2x_2 \leq 60 \quad \text{--- (2)}$$

$$x_1, x_2 \geq 0$$

Introduce slack variables x_3 and x_4 to (1) & (2) resp. then P_1 becomes.

$$\max Z = 70x_1 + 40x_2 + 0x_3 + 0x_4$$

$$\text{s.t.} \quad 4x_1 + 2x_2 + x_3 = 80$$

$$3x_1 + 2x_2 + x_4 = 60$$

$$x_1, x_2, x_3, x_4 \geq 0$$

			c_j	70	40	0	0		
C_B	B	x_B	b	a_1	a_2	a_3	a_4	min ratio	operations.
0	a_3	x_3	80	$\boxed{4}$	2	1	0	$80/4 = 20$ →	
0	a_4	x_4	60	3	2	0	1	$60/4 = 20$	
$z_j - c_j$			$\boxed{0}$	-70 ↑	-40	0	0		
70	a_1	x_1	20	1	$1/2$	$1/4$	0	$20/1/2 = 40$	$R_1' = R_1/4$ ✓
0	a_4	$\boxed{x_4}$	0	0	$\boxed{1/2}$	$-3/4$	1	$\boxed{0/1/2 = 0}$ →	$R_2' = R_2 - 3R_1'$
$z_j - c_j$ ✓			$\boxed{1400}$	0	-5 ↑	$35/2$	0		
70	a_1	x_1	20	1	0	1	-1		$R_1' = R_1 - \frac{1}{2}R_2'$
40	a_2	x_2	0	0	1	$-3/2$	2		$R_2' = R_2/1/2$
$z_j - c_j$			$\boxed{1400}$	0	0	10	10		

all $z_j - c_j \geq 0$ so optimality reached

optimal solution is $x_1 = 20$, $x_2 = 0$, and $Z = 1400$

Observations

Observations

- There is a tie in the leaving variable.
- In the next iteration a basic variable becomes zero.
- The optimal solution is found in the second iteration. However one more iteration is used to get optimal solution without increasing the value of the objective function.

Let change the leaving variable in the tie at the first iteration.

		c_j		70	40	0	0		
C_B	B	X_B	b	a_1	a_2	a_3	a_4	min ratio	operation
0	a_3	x_3	80	4	2	1	0	$80/4 = 20$	
0	a_4	x_4	60	$\boxed{3}$	2	0	1	$60/3 = 20$ →	
$z_j - c_j$				-70 ↑	-40	0	0		
0	a_3	x_3	0	0	$-2/3$	1	$4/3$		$R_1' = R_1 - 4R_2'$
70	a_1	x_1	20	1	$2/3$	0	$1/3$		$R_2' = R_2/3$
$z_j - c_j$			$\boxed{1400}$	0	$20/3$	0	$70/3$		

- choosing x_4 instead of x_3 reduce one iteration.

- Both cases the optimal value is same, i.e., 1400

Graphical method for this problem

$$\max\ Z = 70x_1 + 40x_2$$

$$\text{s.t. } 4x_1 + 2x_2 \le 80 \quad \text{--- } (1)$$

$$3x_1 + 2x_2 \le 60 \quad \text{--- } (2)$$

$$x_1, x_2 \geq 0.$$

For (1)

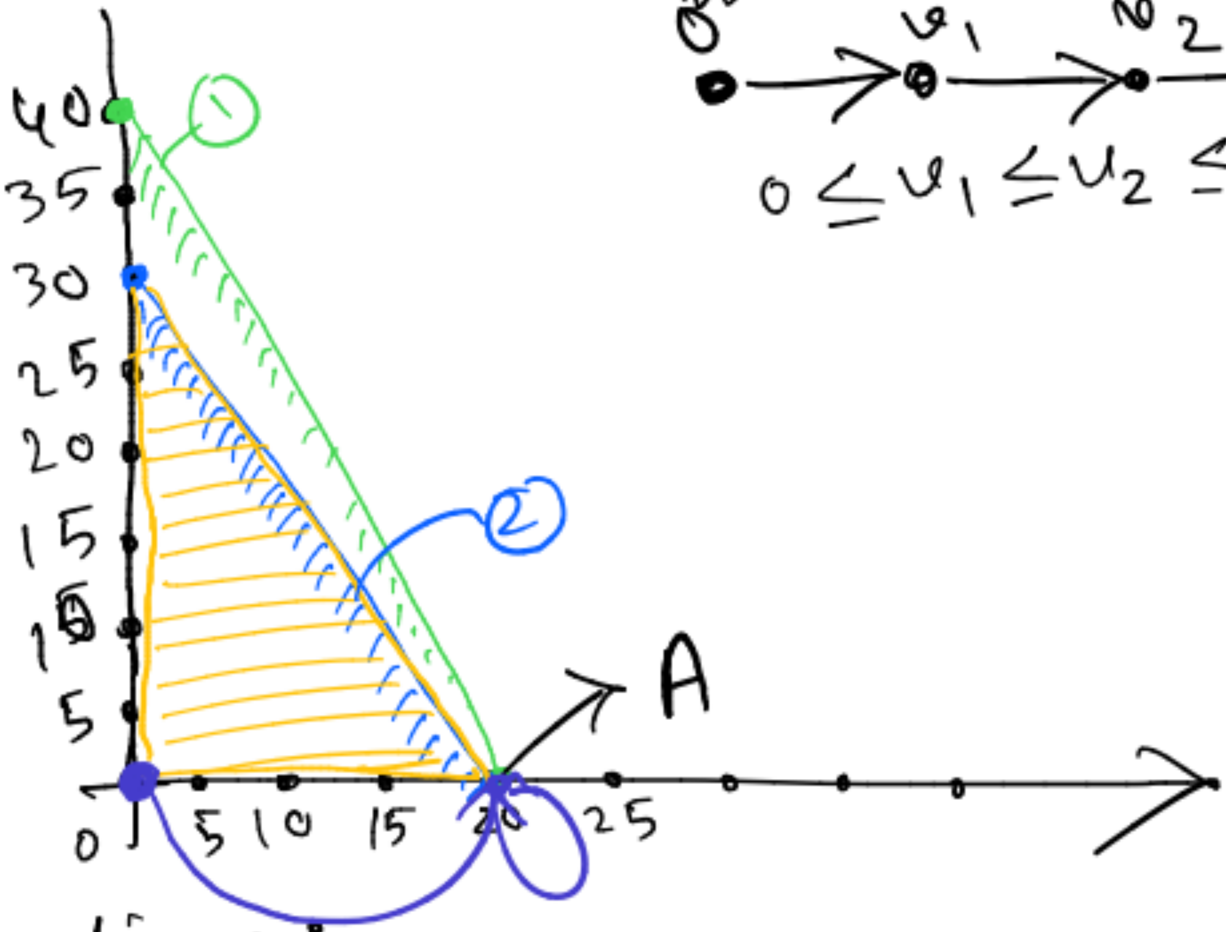
$$4x_1 + 2x_2 = 80$$

$(0, 40)$ and $(20, 0)$

For (2) $(0, 30)$ and $(20, 0)$

$$\bullet \to v_1 \to v_2 \to v_3 \to v_n$$

$$0 \le v_1 \le v_2 \le \cdots \le v_n$$



observations:

- There are 3 corner/extreme points in the feasible region.

- Point A is the intersection of

i) 1 and 2 and ✓

ii) 1 and $x_2 \geq 0$ ✓

iii) 2 and $x_2 \geq 0$

- In other way we can say that morethanonepoint is sitting on A.

- So simplex move to one cornerpoint of A to another point of A and so on. finally reaching optimal solution.

Remarks:

- Whenever there is a tie in the leaving variable, one of the basic variable takes value zero in the next iteration.
- This is called degeneracy.
- For this case we might perform some extra iterations but keeping the objective function value same.
- In our example degeneracy occurs at the final iteration. But it may happen in some intermediate iterations.

  - if optimum exists then simplex may come out of the degeneracy by itself and terminate with optimum value.

  How??

Here the entering column will have a zero or negative value against the leaving row and hence that min ration will not be computed resulting a positive value of the ~~minimum~~ min ratio.

→ A efficient tie breaking rule may leads to do a fewer iterations.