Lecture - 5

P 1

Recap:

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A \cap B) = P(B \cap A) = P(A|B)\,P(B) = P(B|A)\,P(A)$$

Today: groups 1 & 2

Wednesday: 3 & 4

Thursday: 5

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2 - 3 pm: even roll nos

3 - 4 pm: odd roll nos

e.g. Box has 12 balls

8 red
4 white.

you draw 2 balls at random (without replacement).

$$P(\text{both are red}) = \frac{{}^{8}C_{2}}{{}^{12}C_{2}}$$

R_1 : 1st ball drawn is red
R_2 : 2nd ball drawn is red

$$P(R_1 \cap R_2) = P(R_1)\, P(R_2 | R_1)$$
$$= P(R_2)\, P(R_1 | R_2)$$

$$P(R_1) = \frac{8}{12}$$

$$P(R_2 | R_1) = \frac{7}{11}$$

$P(A \cap B \cap C)$

$= P(A)\, P(B|A)\, P(C \mid A \cap B)$

$P(E_1 \cap E_2 \cap E_3 \cap \dots \cap E_n) =$

$P(E_1)\, P(E_2|E_1) \dots\dots P(E_n \mid E_1 \cap \dots \cap E_{n-1})$

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e.g. Insurance policy.

Accident Prone: if the probability of having an accident in the next ~~any~~ one year = 0.4

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not accident prone, if the probability is 0.2

30% of the population is accident prone

70% is not accident prone.

You take this insurance policy. What is the probability that you have an accident in the next 1 year?

Step 1: define the events

A: you are accident-prone

$\bar{A}$: you are not accident-prone

B: you have an accident in the next 1 year.

Step 2

$P(B|A) + P(B|\bar{A})$ — $10\,\alpha$

$$P(B) = P(B \cap A) + P(B \cap \bar{A})$$

$$B = (B \cap A) \cup (B \cap \bar{A})$$

$$= \frac{P(A)P(B|A)}{\cancel{P(B)P(A|B)}} + P(\bar{A})P(B|\bar{A})$$

$$0.3 * 0.4 + 0.7 * 0.2 =$$

e.g: You take a MCQ test. 4 choices.

P(know the correct answer) = 0.8

$$P(\underbrace{\text{don't know}}_{\text{You make a guess}}) = 0.2$$

You answered the question correctly. What is the probability that you knew the answer?.

Step 1: define the events

K: you know the correct answer

C: answer is correct

Step 2:

$$P(K|C) = \frac{P(K \cap C)}{P(C)} = \frac{P(C|K)\,P(K)}{P(C)}$$

⑥

$$C = (C \cap K) \cup (C \cap \bar{K})$$

$$P(C) = \underline{P(C \cap K)} + P(C \cap \bar{K})$$

$$= P(K)\,P(C|K) + P(\bar{K})\,P(C|\bar{K})$$

$$= 0.8 * 1 \qquad \cancel{0.25} * \quad 0.2 * 0.25$$

A, B, C are mutually exclusive and <u>exhaustive</u>.

$$A \cap B = B \cap C = C \cap A = \phi$$

$A \cup B \cup C =$ sample space.

E

$$P(A|E) = \frac{P(A \cap E)}{P(E)} = \frac{P(A)\,P(E|A)}{P(E)}$$

$$E = (E \cap A) \cup (E \cap B) \cup (E \cap C)$$

$$P(E) = P(E \cap A) + P(E \cap B) + P(E \cap C) \to D.$$

$$P(A|E) = \frac{P(A)\,P(E|A)}{P(A)P(E|A) + P(B)\,P(E|B) + P(C)\,P(E|C)}$$

Baye's theorem

e.g. 3 shops in your neighborhood, which sell bulbs.

Shop	p(good bulb)	p(defective)
A	0.8	0.2
B	0.4	0.6
C	0.1	0.9

You send someone to buy a bulb, it turns out to be defective.

what is the probability that the bulb was bought from shop A, B or C? 8

Step 1: defining the events

A : you bought the bulb from shop A

B : " B

C : " C

D : bulb is defective.

↳ this has already taken place.

Step 2: $P(A|D)$, $P(B|D)$, $P(C|D)$

$$P(A|D) = \frac{\overset{1/3}{P(A)}\ \overset{0.2}{P(D|A)}}{\underset{1/3}{P(A)}\ \underset{0.2}{P(D|A)} + \underset{1/3}{P(B)}\ \underset{0.6}{P(D|B)} + \underset{1/3}{P(C)}\ \underset{0.9}{P(D|C)}}$$

A, B, C are mutually exclusive & exhaustive e.g. ~~You~~ A witness (9)

sees a green-color taxi run away after an accident. The witness correctly identifies the color 90% of the time.

90% blue

10% green

What is the probability that the taxi is green?

$\bar{A} = B$

A: taxi is green

B: taxi is Blue

C: witness identified the taxi to be green

$P(A|C) =$ H.W.