Lecture - 2   P 1

Definitions:

Sample Space: Set of all possible outcomes of an experiment.

e.g. toss a coin

Sample Space = {H, T}

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e.g. throw a dice.

Sample space = {1, 2, 3, 4, 5, 6}

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e.g. throw 2 dice simultaneously

Sample Space = { (1,1), (1,2) . . . (1,6), . . . (6,6) }   11

Event: ~~any~~ a subset of the sample space.

let $|S| = n$, 2^n no. of events can be defined.

E_1 = first dice showing a 5
$= \{(5,1), \ldots, (5,6)\}$

E_2 = sum of 2 dice = 8
$= \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

Set operation: $\cup$, $\cap$, Complement

Commutative laws

Assosiative laws

distributive laws $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De Morgan's laws

3 axioms of probability

i) $0 \leq P(E) \leq 1$

ii) $P(\text{Sample Space}) = 1$

iii) $E_1, \dots, E_n$ are mutually ~~exect~~ exclusive,

probability of union = sum of probabilities

$$p(\cup E_i) = \sum p(E_i)$$

$$E_i \cap E_j = \phi \text{ if } i \neq j$$

null set

e.g. Show that

$$P(\phi) = 0$$

$$\bar{S} = \phi$$

$$P(\bar{S}) = P(\phi)$$

$$S \cap \phi = \phi$$

$\Rightarrow$ S & ϕ are mutually exclusive.

Using 3rd axiom,

$$P(A \cup B) = P(A) + P(B),$$

when $A \cap B = \phi$

$$\Rightarrow P(S \cup \phi) = P(S) + P(\phi)$$

$$P(S) = P(S) + P(\phi)$$

Using 2nd axiom, $P(S) = 1$

$$\Rightarrow P(\phi) = 0$$

e.g.: Prove that

$$P(\bar{E}) = 1 - P(E)$$

$$E \cap \bar{E} = \phi$$

$\Rightarrow E$ & $\bar{E}$ are mutually exclusive.

Using 3rd axiom on E & $\bar{E}$,

$$P(E \cup \bar{E}) = P(E) + P(\bar{E})$$

$$1 = P(S) = P(E) + P(\bar{E})$$

by axiom 2

$$\Rightarrow P(\bar{E}) = 1 - P(E)$$

e.g.:

if $E \subseteq F$, then

$$P(E) \leq P(F)$$

$$E' \cup E = F$$

$$\bar{E} \cap F$$

$$A = E$$

$$B = \bar{E} \cap F$$

$$A \cap B = \phi$$

Using axiom iii on A & B

$$P(A \cup B) = P(A) + P(B)$$

$$P(F) = P(E) + \underbrace{P(\bar{E} \cap F)}_{\geq 0 \text{ by axiom i}}$$

$$\therefore P(E) \leq P(F)$$

e.g.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Prove this
H.W.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

H.W.