Autumn2017

DA-IICT, B.Tech, Sem III

1. Prove that for any vector field \vec{A} , $\vec{\nabla} \cdot \vec{A}$ is a scalar.

- 2. Let $\vec{A} = \vec{\omega} \times \vec{r}$ where $\vec{\omega}$ is a fixed vector in space. Find $\vec{\nabla} \times \vec{A}$.
- 3. Find the divergence of the following:
 - (a) $\vec{\mathbf{A}} = \hat{\mathbf{r}}$,
 - (b) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r}$ in 2 dimension
 - (c) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r}$ in 3 dimension
 - (d) $\vec{\mathbf{A}} = \frac{\hat{\mathbf{r}}}{r^2}$ in 3 dimension. Plot this field.
 - (e) $\vec{A} = \frac{\hat{\mathbf{r}}}{r^3}$ in 3 dimension
- 4. Find the curl of the following:
 - (a) $\vec{\mathbf{A}} = y\hat{\mathbf{i}} x\hat{\mathbf{j}}$
 - **(b)** $\vec{\mathbf{A}} = \frac{1}{\sqrt{x^2 + y^2}} (y\hat{\mathbf{i}} x\hat{\mathbf{j}})$
 - (c) $\vec{\mathbf{A}} = \frac{1}{x^2+y^2}(y\hat{\mathbf{i}} x\hat{\mathbf{j}})$
 - (d) $\vec{\mathbf{A}} = (x^2 + y^2)\hat{\mathbf{k}}$
- 5. For any vector field \vec{A} and any scalar field F show that
 - (i) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{\mathbf{A}}) = 0$; (ii) $\vec{\nabla} \times (\vec{\nabla} F) = 0$.
- 6. Can we find a scalar function F such that $\vec{\nabla} F = y\hat{\mathbf{i}} x\hat{\mathbf{j}}$? What about $\vec{\nabla} F = \frac{1}{x^2 + y^2} (y \hat{\mathbf{i}} - x \hat{\mathbf{j}})$?
- 7. Find the equation of the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point (x_0, y_0, z_0) on the ellipsoid.