

1. Starting from the definition of a Coulomb of charge in the M.K.S system and an e.s.u of charge in the C.G.S system determine how much e.s.u of charge make one coulomb of charge.
2. If $\vec{E} = kr^3\hat{r}$ in a region find the charge density in the region.
3. A hollow spherical shell carries a charge density $\rho = k/r^2$ in the region $a \leq r \leq b$. Find the electric field in the three regions , $r < a$, $a < r < b$, $r > b$.
4. Two infinite parallel planes carry equal but opposite uniform charge densities $\pm\sigma$. Find the electric field in the three regions, into which the planes partition the space.
5. The electric field in a region is cylindrically symmetric, given as follows:

$$\begin{aligned}\vec{E}(\vec{r}) &= \frac{c\hat{s}}{s}; & \text{when } s \geq a \\ &= 0; & \text{when } s < a\end{aligned}$$

Find the charge distribution in the region using Gauss' law.

6. Evaluate

(a) $\int (r^2 + \vec{r} \cdot \vec{a} + a^2)\delta^3(\vec{r} - \vec{a})dV$ over the whole space where \vec{a} is a fixed vector.

(b) $\int_V |\vec{r} - \vec{b}|^2 \delta^3(5\vec{r})dV$ over a cube of side 2, centered at the origin, and $\vec{b} = 4\hat{y} + 3\hat{z}$

7. We have seen that $\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$. In a similar manner justify that

$$\vec{\nabla} \cdot \left(\frac{\hat{s}}{s}\right) = 2\pi\delta^2(\vec{s})$$

Here s is the distance from the z axis in cylindrical coordinates and $\delta^2(\vec{s})$ is a two dimensional delta function on the xy plane.

8. Prove that $\delta(r) = 4\pi r^2 \delta^3(\vec{r})$ and $\delta(s) = 2\pi s \delta^2(\vec{s})$.
Here $\int_0^\epsilon \delta(r) dr = 1$ for any $\epsilon > 0$. The integral is 0 otherwise.
 $\delta(s)$ is defined likewise.

9. The electric field in a region is given as

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s} \hat{s}$$

Find the charge densities in the region.