DA-IICT

CT314

TUTORIAL 4

Joint density functions

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1. For two RVs X and Y, the joint PDF is given as

$$f_{X,Y}(x,y) = \begin{cases} 4xy, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & elsewhere. \end{cases}$$

   Are X and Y independent?

2. For two RVs X and Y, the joint PDF is given as

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{2}, & 0 \le x \le y, 0 \le y \le 2 \\ 0, & elsewhere. \end{cases}$$

   Are X and Y independent? Also find $f_{\frac{Y}{X}}(\frac{y}{x})$ and $f_{\frac{X}{Y}}(\frac{x}{y})$.

3. Find the normalization factor C and the marginal PDFs for the following PDF. Are X and Y independent? Also find $P[X+Y \le 1]$.

$$f_{X,Y}(x,y) = \begin{cases} Ce^{-x}e^{-y}, & 0 \le x \le y \le \infty \\ 0, & elsewhere. \end{cases}$$

4. RVs X and Y are jointly Gaussian and their joint distribution is given by,

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{\frac{-(x^2-2\rho xy+y^2)}{2(1-\rho^2)}}, -\infty < x, y < \infty.$$

   Find the marginal PDFs. Are X and Y independent?

5. Let X and Y be two statistically independent random variables with joint PDF $f_{X,Y}(x,y)$. Let $Z = X + Y$ and $W = Y$. Find $f_Z(z)$.

6. Let X_1, X_2 and X_3 are RVs. Consider,

$$\begin{aligned} Y_1 &= X_1^2 - X_2^2 \\ Y_2 &= X_1^2 + X_2^2 \\ Y_3 &= X_3. \end{aligned}$$

   Find expression for $f_{Y_1,Y_2,Y_3}(y_1,y_2,y_3)$ for given $f_{X_1,X_2,X_3}(x_1,x_2,x_3)$. If

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3) = \frac{1}{(2\pi)^{3/2}} e^{-\frac{1}{2}(x_1^2+x_2^2+x_3^2)}, where -\infty < x_1, x_2, x_3 < \infty$$

   , find $f_{Y_1,Y_2,Y_3}(y_1,y_2,y_3)$.