Solving LPP: Graphical method

The extreme point/corner point method

Step 1: Finding the feasible region

Step 2: Finding the optimal solution

method 1

method 2

Iso profit / Iso cost method

Let us consider the LPP as

$$\text{optimise } Z = c_1 x + c_2 y$$

$$\text{s.t.} \quad a_{11} x + a_{12} y \ (\leq = \geq) \ b_1$$

$$a_{21} x + a_{22} y \ (\leq = \geq) \ b_2$$

$$\vdots$$

$$x, y \geq 0$$

The method is applied after getting the feasible region.

Step 2.1: consider the objective function

$$Z = c_1 x + c_2 y$$

draw a line $c_1x + c_2y = k$, where k is any constant (value)

Step 2.2: optimize $\begin{cases} \text{minimize} \\ \text{maximize.} \end{cases}$

For a maximization objective.

- Draw the line $c_1x + c_2y = k$ farthest from the origin
- This line should contain at least one point of the feasible region
- Find the coordinate of this point by solving the equations of the lines on which the point lies.

For a minimization objective

- Draw the line $c_1x + c_2y = k$ nearest to the origin.
- Same as before
- same as before

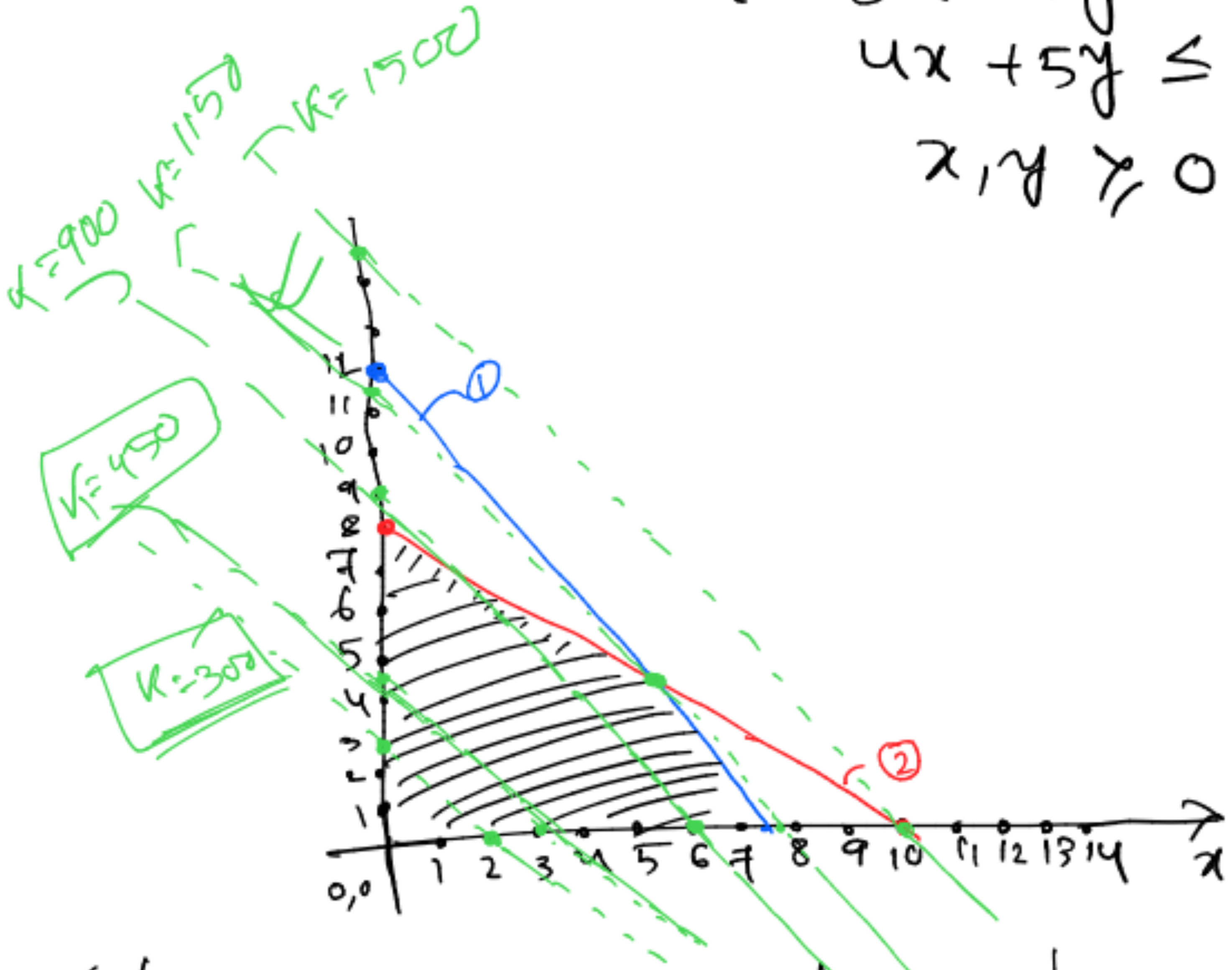
Then the optimum solution is the coordinate of the point and value of the solution is found by putting these solution to the objective function.

Example Maximise $Z = 150x + 100y$

s.t. $8x + 3y \le 60$

$4x + 5y \le 40$

$x, y \geq 0$



Let us assume a set of values of K and draw their corresponding lines.

$K = 300$ the objective function is $3x+2y=6$ $\quad \frac{150x}{50}+\frac{100y}{50}=\frac{300}{50}$

when $x=0$ $y=\frac{6}{2}=3$

when $y=0$, $x=\frac{6}{3}=2$

$(0,3)$ $(2,0)$

$K=450$ the objective function is $3x+2y=9$

when $x=0$ $y=\frac{9}{2}=4.5$

when $y=0$, $x=\frac{9}{3}=3$

$(0,4.5)$ $(3,0)$

$K=900$: $3x+2y=18$

when $x=0$, $y=\frac{18}{2}=9$

when $y=0$, $x=\frac{18}{3}=6$

$(0,9)$, $(6,0)$

$K=1150$: $3x+2y=23$

when $x=0$, $y=\frac{23}{2}=11.5$

when $y=0$, $x=\frac{23}{3}=7.666$

$(0,11.5)$ $(7.666,0)$

$K=1500$: $3x+2y=30$

when $x=0$, $y=\frac{30}{2}=15$

when $y=0$, $x=\frac{30}{3}=10$

$(0,15)$ $(10,0)$

Observations:

The profit Z :
is proportional to the perpendicular distance of the profit line from the origin.
Hence profit increases as the profit line translates far away from the origin.

The line when $x = 1150$ belongs to the feasible region and farthest from the origin.

Therefore the optimal solution is the coordinate of the point on the line and the feasible region ie,
$x = 5$, $y = 4$
value of the solution is $Z = 1150$

Graphical method

various types of solutions and feasible regions.

Types of solutions

i) Unique solution
ii) Infinite number of solutions
iii) Unbounded solution
iv) Infeasible solution

Types of regions

i) Bounded region
ii) Unbounded region
iii) No feasible region

Example: $\min\ Z = 3x + 5y$

$$\text{s.t.}\quad 2x + 3y \geq 12 \quad \text{—} \ (1)$$
$$-x + y \leq 3 \quad \text{—} \ (2)$$
$$x \leq 4 \quad \text{—} \ (3)$$
$$y \geq 3 \quad \text{—} \ (4)$$
$$x, y \geq 0$$

Feasible region bounded unique optimum solution

constraint 1

$2x + 3y = 12$

when $x = 0,\ y = 4$

when $y = 0,\ x = \frac{12}{2} = 6$

$(0, 4)$ and $(6, 0)$

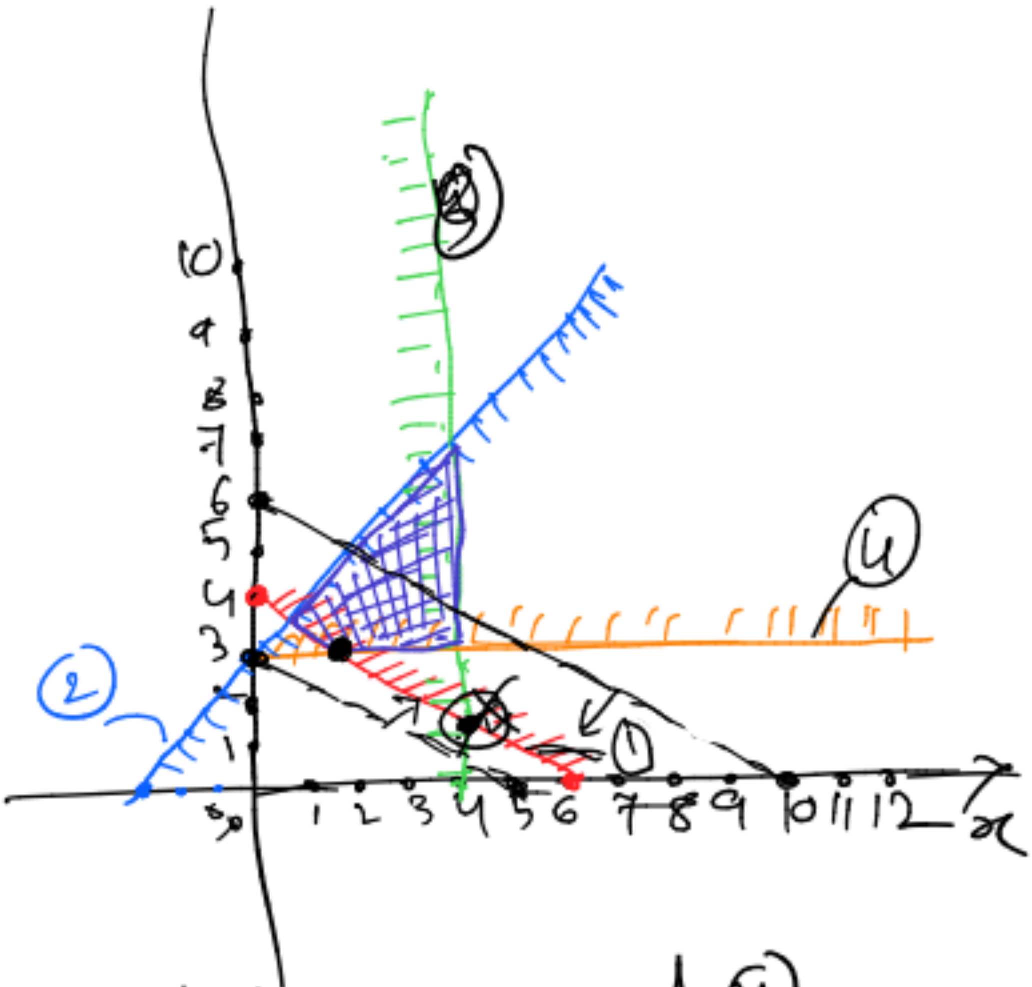
constrain 2

$-x + y = 3$

when $x = 0,\ y = 3$

when $y = 0,\ x = -3$

$(0, 3)$ and $(-3, 0)$



objective function

$3x + 5y = 15$

when $x = 0,\ y = 3$

when $y = 0,\ x = 5$

$K = 30$

$3x + 5y = 30$

$x = 0 \quad y = 6$

$y = 0 \quad x = 10$

solving (1) and (4)

$2x + 3y = 12$

$y = 3$

$2x + 9 = 12$

$\Rightarrow x = \frac{3}{2}$

solution is $x = \frac{3}{2},\ y = 3$

value is $Z = \frac{9}{2} + 15 = \frac{39}{2} = 19.5$

Example:

$$\min Z = x_1 + x_2$$

s.t. $5x_1 + 9x_2 \leq 45$

$x_1 + x_2 \geq 2$

$x_2 \leq 4$

$x_1, x_2 \geq 0$

solution:

constraint 1

$5x_1 + 9x_2 = 45$

when $x_1 = 0$, $x_2 = \frac{45}{9} = 5$

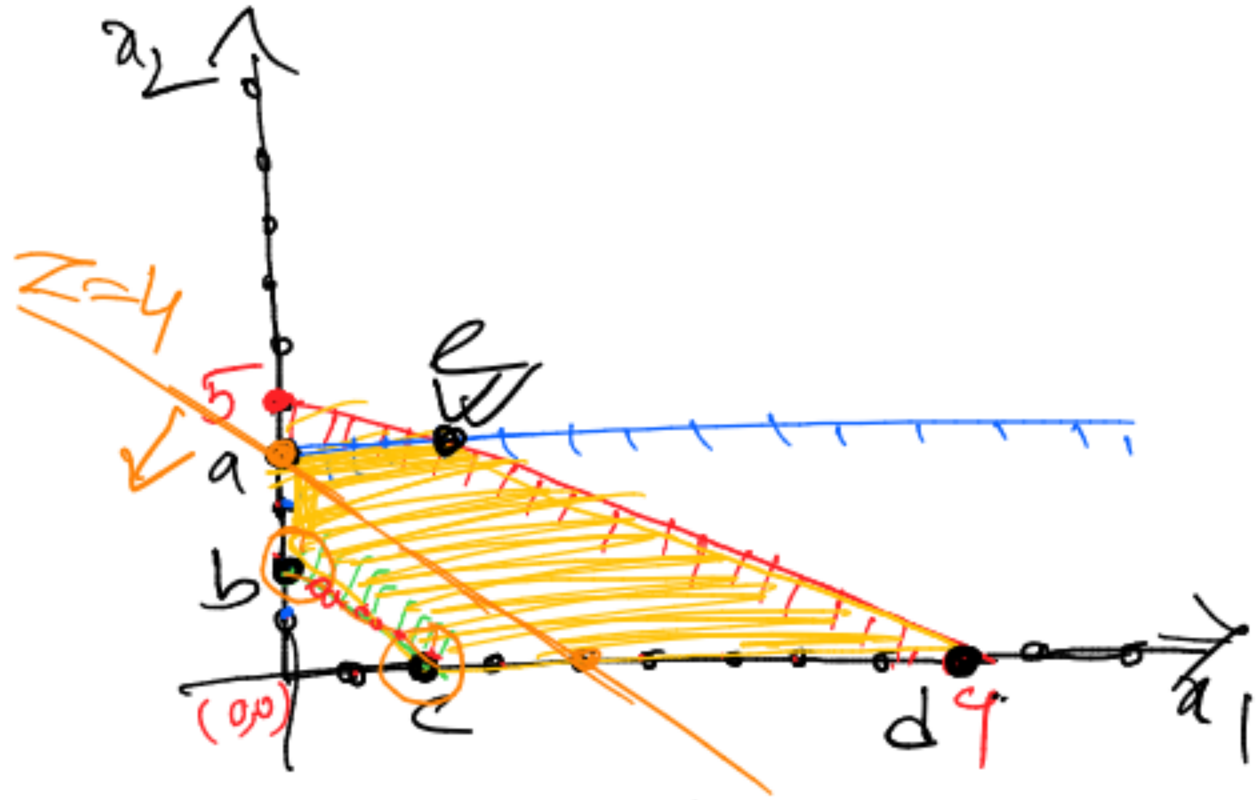
when, $x_2 = 0$, $x_1 = \frac{45}{5} = 9$

constraint 2

$x_1 + x_2 = 2$

when $x_1 = 0$, $x_2 = 2$

when $x_2 = 0$, $x_1 = 2$



$5x_1 + 9x_2 = 45$

$x_2 = 4$

$5x_1 = 45 - 36$

$x_1 = \frac{9}{5}$

corner points are,

$\underset{a}{(0,4)}, \underset{b}{(0,2)}, \underset{d}{(9,0)}, \underset{c}{(2,0)}, \underset{e}{\left(\frac{9}{5}, 4\right)}$

obj fun: $x_1 + x_2$

values at the corner points

a : 4

$\Rightarrow$ b : 2

d = 9

$\Rightarrow$ c = 2

$e = \frac{9}{5} + 4 = \frac{29}{5} = 5.8$

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objective function line.

$x_1 + x_2 = 4$

when $x_1 = 0$, $x_2 = 4$

$x_2 = 0$, $x_1 = 4$

It is evident that from the nature of the profit line that the minimum value of Z will be 2 when the profit line coincides with the line of constraint 2.

Hence any point on the line of constraint ② gives an optimal solution.

Hence the value of the optimum solution is $Z = 2$

However there are an infinite number of solutions.

Feasible region is bounded Infinite number of solutions

We also say that there are alternative optimum solution.

Example:

$$\max Z = 3x_1 + 4x_2$$

$$\text{s.t.} \quad x_1 - x_2 \geq 0$$

$$-x_1 + 3x_2 \leq 3$$

$$x_1, x_2 \geq 0$$