Lecture-6

Recap:

Baye's theorem

lecture 1

: Monty Hall Problem

: 3 boxes : gg, gs, ss

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tutorial : group 5

Independent Events

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$\to P(E|F) = P(E)$, then

E & F are independent.

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = P(E)$$

Definition

$$\Rightarrow P(E\cap F) := P(E)\,P(F)$$

eg: toss 2 dice

A: Sum is 7

B: 1st dice shows 4

$P(A) = \frac{6}{36}$ $\quad$ $P(A)P(B) = P(A \cap B)$

$P(B) = \frac{6}{36}$ $\quad$ $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$P(A \cap B) = \frac{1}{36}$

A: Sum is 6

B: 1st dice shows 4

$P(A) = \frac{5}{36}$ $\quad$ $P(A \cap B) = \frac{1}{36}$

$P(B) = \frac{6}{36}$ $\quad$ Not independent

A: Sum is 5

B: 1st dice shows 4

A ~~x~~ A ~~&~~ B independent?

No

Independence of 3 events

Definition:

A, B & C are independent if

i) $P(ABC) = P(A)\,P(B)\,P(C)$

ii) $P(AB) = P(A)P(B)$

iii) $P(BC) = P(B)P(C)$

iv) $P(AC) = P(A)\,P(C)$

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e.g. $U = \{1, 2, 3, \dots, 8\}$

$A = B = \{1, 2, 3, 4\}$

$C = \{1, 5, 6, 7\}$

$P(A) = \frac{1}{2} = P(B) = P(C)$

$P(ABC) = P(A)P(B)P(C)$

$P(ABC) = 1/8$ $\quad P(AB) = \frac{1}{2} \neq P(A)P(B)$

$A \cap B \cap C = \{1\}$ not independent

$S = \{1, 2, 3, 4\}$ $\quad A = \{1, 2\}, B = \{1, 3\}, C = \{1, 4\}$

$P(A) = P(B) = P(C) = \frac{1}{2}$ $\quad P(BC) = P(B)P(C)$

$\underset{1/4}{P(AB)} = \underset{1/2}{P(A)}\,\underset{1/2}{P(B)}$, $P(AC) = P(A)\,P(C)$

$P(ABC) = P(A)\,P(B)\,P(C)$

$\frac{1}{4} \neq \frac{1}{2}\;\frac{1}{2}\;\frac{1}{2}$

So, A, B & C are not independent.

However, A & B are independent.

B & C are independent.

A & C are independent

A, B & C are pairwise-independent

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e.g. toss 2 coins

A: 1st coin is a Head

B: 2nd coin is a Head

C: Both show the same result

Random Variables - discrete

It is a real-valued function on the sample space.

$f : S \longrightarrow \mathbb{R}$
set of real numbers.

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e.g. toss a coin 3 times

X = no. of Heads.

Possible values that X can take: 0, 1, 2, 3

$X = i$	$P(X = i)$
0	1/8
1	3/8
2	3/8
3	1/8
total	1

6

e.g.: A box has 20 balls, numbered 1 to 20. You take out 3 balls randomly. What is the probability that at least one of balls shows a number ≥ 17?

e.g. 1, 7, 15 ✗   5, 10, 16 ✗

1, 2, 17 ✓

17, 18, 20 ✓

$P(\text{at least one ball} \geq 17) =$

$1 - P(\text{all} \leq 16)$

$$= 1 - \frac{{}^{16}C_3}{{}^{20}C_3}$$

$X =$ <u>highest</u> of the 3 balls chosen

Possible values of $X =$

$\text{Range}(X) = \{3, \ldots, 20\}$

$P(X = 17, 18, 19, 20) =$

$P(X=17) + P(X=18) + P(X=19) + P(X=20)$ iff $X = 17$, $X = 18$, $X = 19$, $X = 20$ are mutually exclusive

2, 5, 18

17, 18, 19

$\underset{i \geq 3}{P(X=i)} = \dfrac{\binom{1}{1}\binom{i-1}{2}}{{}^{20}C_3}$

$$\frac{{}^{16}C_2}{{}^{20}C_3} + \frac{{}^{17}C_2}{{}^{20}C_3} + \frac{{}^{18}C_2}{{}^{20}C_3} + \frac{{}^{19}C_2}{{}^{20}C_3}$$

$$= 1 - \frac{{}^{16}C_3}{{}^{20}C_3}$$

$${}^{20}C_3 = \underbrace{\boxed{{}^{16}C_3 + {}^{16}C_2}}_{{}^{17}C_3} + {}^{17}C_2 + {}^{18}C_2 + {}^{19}C_2$$

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e.g. Biased coin

$p(H) = p, \ p(T) = 1-p$

Keep tossing this coin until

→ you get a Head OR

→ you have tossed n times.

X = no. of tosses

$\text{Range}(X) = \{1, 2, \cdots, n\}$

$X=i$	$P(X=i)$
H 1	p
TH 2	$(1-p)\,p$
T^2H 3	$(1-p)^2\,p$
$\vdots$	
$n-1$	$(1-p)^{n-2}\cdot p$
n	$(1-p)^{n-1}\left[p+1-p\right]$
	$(1-p)^{n-1}$
Total	1 (H.W.) Verify.

e.g.: You throw 3 dice.
You bet on the no. of 6's.

if no 6, you lose 100 Rs.
if one 6, you earn 100 Rs.
if two 6, you earn 200 Rs.
if three 6, you earn 300 Rs.
would you like to
play this game?