Tutorial 2

1

a) Valid. PDF $f_1(c) = \begin{cases} 0 & , c \le 0 \\ 0.5 & , 0 < c \le 1 \\ 0.25 & , 1 < c \le 3 \\ 0 & , 3 < c \end{cases}$

derivative

$f_1(c) = \frac{d F_1(c)}{dc}$

b) Not Valid. Not right continuous $F_2(c) \neq F_2(c+\delta)$

c) Not Valid. $F_3(-\infty) \neq 0$

d) Valid,

PMF is $f_4(k) = \begin{cases} 0.25 & , k = 0 \\ 0.25 & , k = 1 \\ 0.25 & , k = 3 \\ \cancel{0.25} & \cancel{, k = 3} \\ 0 & ; \end{cases}$

e) Not valid, Non decreasing

f) Not valid, $F_6(\infty) \neq 1$

2.

a) $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$

$\Rightarrow \int_0^2 cx\, dx = 1 \Rightarrow c\left[\frac{x^2}{2}\right]_0^2 = 1 \Rightarrow \boxed{c = 1/2}$

b) $P[0 \le x \le 1] = \int_0^1 \frac{1}{2}x\, dx = 1/4$

c) $P[-1/2 \le x \le 1/2] = \int_0^{1/2} 1/2\, x\, dx = 1/16$

d) $F_X(x) = \begin{cases} 0 & , x < 0 \\ x^2/4 & , 0 \le x \le 2 \\ 1 & , x > 2 \end{cases}$ $\int_0^x \frac{x}{2}\, dx = \frac{x^2}{4}$

3. $F_X(x) = \begin{cases} 0 & , x < 0 \\ kx^2 & , 0 \le x \le 10 \\ 100k , & x > 10 \end{cases}$

$F_X(\infty) = 1 \Rightarrow \cancel{F_X}\ (X \le \infty) = 1$

$P(X \le 10 + 8) = 1$

$100k = 1 \Rightarrow k = 1/100$

$\hookrightarrow P[X \le 5] = F_X(5) = \frac{1}{100} \times 25 = 1/4$

$\hookrightarrow P[5 < X \le 7]$

$= P[X \le 7] - P[X \le 5]$

$= \frac{49}{100} - \frac{25}{100}$

$= 0.24$

$\hookrightarrow f_X(x) = \frac{d}{dx} F_X(x)$

$= \begin{cases} 0 & , x < 0 \\ \frac{2x}{100} & , 0 \le x < 10 \\ 0 & , \text{Otherwise} \end{cases}$

4. $X : 1, 2, 3 \ldots$

$P(X = j) = \frac{1}{2^j} , j = 1, 2 \ldots$

a) $P(X \text{ is even}) = P(X=2) + P(X=4) \ldots$

$P(X = 2k) , k = 1 \ldots \infty$

$= \sum_{k=1}^{\infty} \frac{1}{2^{2k}} , k = 1, 2, \ldots$

b) $P(X \geq 5) = \sum_{j=5}^{\infty} 1/2^j$,

c) $P(X \text{ is divisible by } 3) = \sum_{m=1}^{\infty} \frac{1}{2^{3m}}$

5. PMF noise $f_N(n) = \begin{cases} 4/10, & n=0 \\ 3/10, & n=-1 \\ 2/10, & n=-2 \\ 1/10, & n=-3 \end{cases}$

$\underset{O/P}{Y} = \underset{\text{Signal}}{X} + N$ $\quad X = +2$

a) PMF of O/P Y $\quad f_Y(y) = \begin{cases} 4/10, & Y=2 \\ 3/10, & Y=1 \\ 2/10, & Y=0 \\ 1/10, & Y=-1 \end{cases}$

b) $P(\cancel{Y} = X) = P(Y=2) = 4/10$

c) $P(Y > 0) = 4/10 + 3/10 + 2/10 = 9/10$

6.

a) $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$

$\Rightarrow \int_{-\infty}^{\infty} a.e^{-b|x|}\, dx = 1$

$\Rightarrow \int_{-\infty}^{0} ae^{bx}\, dx + \int_{0}^{\infty} ae^{-bx}\, dx = 1$

$\Rightarrow a\left[\frac{e^{bx}}{b}\right]_{-\infty}^{0} + a\left[\frac{e^{-bx}}{-b}\right]_{\infty}^{\infty} = 1$

$\Rightarrow \frac{a}{b} + \frac{a}{b} = 1$

$\Rightarrow \boxed{2a = b}$

b) $F_X(x)$

$x < 0$

$$\Rightarrow F_X(x) = \int_{-\infty}^{x} a e^{+bx}\, dx$$

$$= \frac{a e^{bx}}{b}$$

$x > 0$

$$F_X(x) = \int_{-\infty}^{x} a e^{-b|x|}\, dx$$

$$= \int_{-\infty}^{0} a e^{+bx}\, dx + \int_{0}^{x} a e^{-bx}\, dx$$

$$= \frac{a}{b} + a\left[\frac{e^{-bx}}{-b}\right]_0^x$$

$$= \frac{a}{b} + a\left[\frac{e^{-bx}}{-b} - \frac{1}{-b}\right]$$

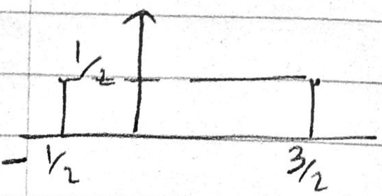
$$= 1 - \frac{1}{2} e^{-bx}, \quad x \geq 0$$

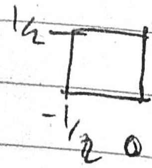
c) $$P[1 < x \leq 2] = \int_{1}^{2} a e^{-bx}\, dx$$

$$= a\left[\frac{e^{-bx}}{-b}\right]_1^2$$

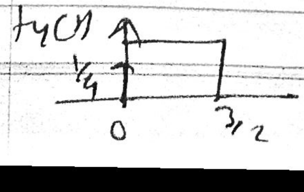
$$= \frac{1}{2}\left[e^{-b} - e^{-2b}\right]$$

7. $f_X(x)$





$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$



$$f_Y(y) = \begin{cases} \frac{1}{4}\delta(x) & x = 0 \\ \frac{1}{2}, & 0 < x \leq 3/2 \end{cases}$$