1. Flip a fair coin 3 times. Let X be the number of heads in the first 2 flips and let Y be the number of heads on the last 2 flips. Compute $COV(X, Y)$ and $\rho_{X,Y}$.

2. X and Y are random variables with joint density given by

$$f_{X,Y}(x, y) = \begin{cases} cx^2 + \frac{xy}{3}, & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0, & otherwise \end{cases}$$

   (a) Find c,

   (b) $P[X + Y \geq 1]$,

   (c) marginal PDFs,

   (d) are x and Y are independent? and

   (e) $COV(X, Y)$.

3. Let $X = (X1, X2)$ have covariance matrix

$$C_X = \begin{bmatrix} 1 & \frac{1}{4} \\ \frac{1}{4} & 1 \end{bmatrix}$$

   (a) find eigenvalues and eigenvectors of C_X

   (b) find the orthogonal matrix P that diagonalize C_X. Verify that P is orthogonal and that $P^T C_X P = \Lambda$.

4. Let X and Y be the random variable with joint PDF given by

$$f_{X,Y}(x, y) = \begin{cases} 2e^{-x}e^{-y}, & 0 \leq y \leq x \leq \infty \\ 0, & elsewhere. \end{cases}$$

   Find (a) $E[XY]$, (b) $COV(X, Y)$ and (c) $\rho_{X,Y}$.