Statistical Communication Theory (CT 314)

Tutorial 2.

28-01-2019

1. For each of the following functions $F_i(c)$, state whether or not $F_i(c)$ is the CDF of some random variable. If not state which of the properties of a CDF it violates. If so, find the corresponding PMF or PDF.

a) $$F_1(c) = \begin{cases} 0, & c \le 0 \\ 0.5c, & 0 < c \le 1 \\ 0.25 + 0.25c, & 1 < c \le 3 \\ 1, & 3 < c \end{cases}$$

b) $$F_2(c) = \begin{cases} 0, & c \le 0 \\ 0.5, & 0 < c \le 1 \\ 0.75, & 1 < c \le 3 \\ 1, & 3 < c \end{cases}$$

c) $$F_3(c) = \begin{cases} 0.5, & c < 1 \\ 0.75, & 0 \le c < 3 \\ 1, & 3 \le c \end{cases}$$

d) $$F_4(c) = \begin{cases} 0, & c < 0 \\ 0.25, & 0 \le c < 1 \\ 0.75, & 1 \le c < 3 \\ 1, & 3 \le c \end{cases}$$

e) $$F_5(c) = \begin{cases} 0, & c < 0 \\ 0.5, & 0 \le c < 1 \\ 0.25, & 1 \le c < 3 \\ 1, & 3 < c \end{cases}$$

f) $$F_6(c) = \begin{cases} 0, & c \le 0 \\ 0.5c, & 0 < c \le 1 \\ 0.25 + 0.25c, & 1 < c \end{cases}$$

2. The RV X has PDF

$$f_X(x) = \begin{cases} cx, & 0 \le x \le 2 \\ 0, & 0therwise \end{cases}$$

Use the PDF to find

a) Constant c,

b) $P\left[0 \le x \le 1\right]$

c) $P\left[-\frac{1}{2} \le x \le \frac{1}{2}\right]$

d) The CDF $F_X(x)$

3. A RV X has

$$F_X(x) = \begin{cases} 0, & x < 0 \\ kx^2, & 0 \le x \le 10 \\ 100k, & x > 10 \end{cases}$$

Find k, evaluate $P\left[X \le 5\right]$ and $P\left[5 < X \le 7\right]$. What is $f_X(x)$?

4. Suppose RV X takes discrete values 1, 2, 3, ... and $P(X = j) = \frac{1}{2^j}, \quad j = 1, 2, ...$

a) Find P(X is even)

b) $P(X \ge 5)$ and

c) P(X is divisible by 3)

5. A modem transmits a +2 voltage signal into a channel. The channel adds to this signal a noise term that is drawn from set {0,-1,-2,-3} with respective probabilities {4/10, 3/10, 2/10, 1/10}.
   a) Find the PMF of the output Y of the channel.
   b) What is the probability that the output of the channel is equal to the input of the channel?
   c) What is the probability that output of the channel is positive?

6. A random variable X has

$$f_X(x) = ae^{-b|x|}, \quad -\infty < x < \infty$$

Find

a) Relation between a and b

b) $F_X(x)$

c) $P\left[1 < x \le 2\right]$

7. Let input to a half wave rectifier be a random variable uniformly distributed between -1/2 to 3/2. Find the probability density function for the output.