Optimization

An optimisation is a process of maximising or minimising a quantity under given constraints.

- we are surrounded by optimization problems.
- Most of the problems in this world are optimization
- one have to mimize (proverty, grief, wars, etc.) or maximise (happiness / peace / health condition / welth etc.
- Unfortunately those problems are not solvable or partially solvable or at least we are not concentrat on those.

- we are looking at many real world problems where we need to optimise (minimise/maximise) mathematical quantities with given constraint that also be represented as mathematical functions.

- These problem may partiall solve our above mentioned problem to some extent.

- some examples may be minimizing the tour length of an optimization problem, ~~that~~ ~~we~~ maximize the number of items that can place in a bag etc that we encounter in our daily life.

- various techniques are available to model the real world problem (optimization) into ~~an~~ ~~optimisation~~ mathematical model

- various techniques are available for the solution of the optimization problems.
  ~~under the heading mathematical programming~~

Linear programming

~~A large class of~~

- optimization problems involves finding the greatest possible numerical value (maximisation) or least possible value (minimization) of some mathematical function of any number of independent variables.

- A large class of problems can be formulated as maximization of minimization a linear form whose variables may be restricted to values satisfying a system of linear equations or inequalities, known as constraints.

Formulation of a linear programming problem

Problem 1: A manufacturer has two types of machines to choose from. He must have at least 3 A type of machine and 1 B type of machine. The cost of the machine is Rs. 1000 for the type A and Rs. 1200 for the type B. The area taken by the two types of machine are $4m^2$ and $5m^2$ respectively. The total cost must not exceed Rs, 15000 and total available area is $40\ m^2$. The weekly profit from the output of type A and type B machines are Rs 120 and Rs 100 respectively. Find the maximum profit of the manufacturer.

Formulation:

Let x and y be the number of types A and B machines chosen by the manufacturer.

Cost constraints:

type A machine cost is Rs 1000
type B " " is Rs 1200
manufacturer has total cost available is Rs.15000

This implies that,

$$1000x + 1200y \leq 15000$$

$$\Rightarrow 5x + 6y \leq 75 \quad \text{—— (1)}$$

Space constraints:

type A takes $4m^2$ area
type B takes $5m^2$ area
total area available is $40m^2$

This implies that,

$$\cancel{4m^2 + 5m^2}$$

$$4x + 5x \leq 40 \quad \text{—— (2)}$$

Number of machines chosen by the manufacturer

$$\left.\begin{array}{ll} \text{type A} & \text{at least 3} \\ \text{type B} & \text{at least 1} \end{array}\right\} \Rightarrow \begin{array}{l} x \geq 3 \quad \text{—— (3)} \\ y \geq 1 \quad \text{—— (4)} \end{array}$$

objective of the manufacturer:

profit from type A machine is Rs 120

" " type B " is Rs 100

~~total~~ Total profit is $120x + 100y$.

~~the~~ manufacturer wants to maximise the total profit such that all the cost, space, number constraints are met.

So the problem becomes.

$$\text{maximise } Z = 120x + 100y$$

such that

$$5x + 6y \leq 75$$
$$4x + 5y \leq 40$$
$$x \geq 3$$
$$y \geq 1$$

We need to find the values of the variables x and y.

These variables are called decision variables.

Problem

A vaccine produce company preparing a production plan on two types of vaccines, type I and type II. There are sufficient raw material available to make 20000 doses of type I and 40000 doses of type II, but there are only 45000 bottles are available to put the doses of the vaccine.

It take 3 hours to prepare the material to make 1000 doses of type I and 1 hour to prepare the material to make 1000 ~~bottles~~ doses of type II. There are 66 hours deadline to prepare the materials. The profit of type I vaccine is Rs. 8.00 per dose and of type II vaccine is Rs 7.00 per dose. Find the maximum profit of the company.

Formulation:-

Let the company produces x thousand doses of type I vaccine and y thousand doses of type II vaccine.

Availability of bottles:-

There are only 45000 bottles are available

This implies that,

$$x + y \leq 45 \quad \text{—— (1)}$$

Time requirement:

type 1 requires 3 hours for 1000 ~~doses~~

type 2 " 1 hours " 1000 doses.

total time available is 66 hours.

This implies that,

$$3x + y \leq 66 \quad \text{—— (2)}$$

maximum doses prepare

type I - 20000

type II - 40000

This implies that,

$$x \leq 20$$

$$y \leq 40$$

Non-negativity condition

The number of produce doses must be non-negative, i.e.,

$$x \geq 0 \ , \ y \geq 0$$

Objective of the company

profit from type I Rs. 8.00 per dose

" " type II Rs 7.00 per dose

total profit,

$$8000x + 7000y.$$

company wants to maximise the total profit such that all above constraints are fulfilled.

Therefore the problem becomes.

$$\text{maximise } z = 8000x + 7000y$$

$$\begin{aligned} \text{s.t. } x + y &\leq 45 \\ 3x + y &\leq 66 \\ x &\leq 20 \\ y &\leq 40 \\ x, y, &\geq 0 \end{aligned}$$

Problem: A software company decided to hire and train programmers over the next four months to complete an ongoing project. The requirement is ~~to have~~ 8000 hours of programming in September, 9000 in october, 8000 in ~~December~~ November and 6000 in December.

It takes one month of training before a newly recruited programmer can be trained and put to the project. A programmer must be hired a month before working on the actual project. A trainee requires 100 hours of training ~~and these 100 hours are~~ from existing trained programmer so that these 100 hours are deducted from the trained programmer from their actual working hour.

Each experienced programmer can work upto 180 hours in a month. The company has 50 regular ~~employee~~ programmer at the begining of September.

If the maximum time spent by the experienced programmers satisfies a months requirement, then they are paid salary for a month.

By the end of each month, 10 % of experienced programmers quit their jobs for some unknown reasons. The company pays ~~an~~ Rs. 30000 to the experienced and Rs.15000 to trainee ~~to~~ per month.

~~The obj~~. Find the minimum salary spent by the company to complete the project.

Formulation:

- Let x_1, x_2, x_3 be the number of trainees hired by the company during September, October, and November. During December they ~~are not hire~~ don't require to hire any trainee as they required training for a month that is not desirable.
- It is given that, the ~~number of experienced programmer at the begining~~ of September is $e_1 = 50$
- Let e_1, e_2, e_3, e_4 ~~be~~ the number of experienced programmer at the begining of Sept, Oct, Nov, and Dec. respectively.
- clearly $e_1 = 50$
- ~~Since 100 new trainee~~ $e_2 = 0.9(50 + x_1)$

  as 10% experienced ~~trainee~~ programmer left and at the begining of oct there are $50 + x_1$ experienced programmer.

- Similarly, $e_3 = 0.9(e_2 + x_2)$

  and $e_4 = 0.9(e_3 + x_3)$

During September, total time spent by the experienced programmer is,

$$180e_1 - 100x_1$$

September requirement:

$$180e_1 - 100x_1 \geq 8000$$

Similarly,

October requirement: $180e_2 - 100x_2 \geq 9000$

November ,, : $180e_3 - 100x_3 \geq 8000$

December ,, : $180e_4 \geq 6000$

Total expenditure that ~~are~~ is minimise is.

$$\text{minimize } Z = 30000\sum_{i=1}^{4} e_i + 15000\sum_{i=1}^{3} x_i$$

Further, the numbers $e_1, e_2, e_3, e_4, x_1, x_2, x_3$ are non-negative,

Note that the decision variables must be integers.