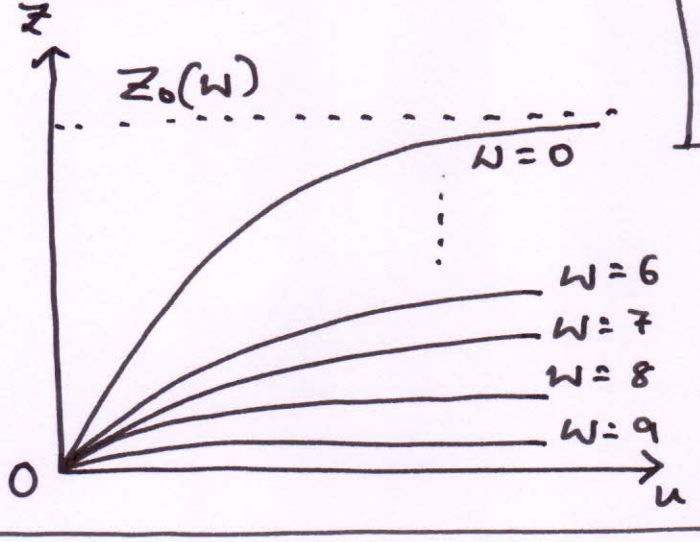
The Graph of the Duckworth-Lewis Equation



$$\boxed{Z(u,w) = Z_0(w)\left[1 - e^{-b(w)u}\right]}$$

With larger values of w (wickets lost), values of b increase. Convergence is quicker.

Changes in Population (Discrete/Continuous)

Population changes in discrete step of unity (1). If a population size is x, and it changes (grows) by Δx, then the per capita growth is $\boxed{\dfrac{\Delta x}{x}}$ and the per capita growth rate is $\boxed{\dfrac{1}{x}\dfrac{\Delta x}{\Delta t}}$, in which Δt is the time taken for the growth.

If x is very large and $\boxed{\Delta x \ll x}$, then the discrete quantities can be replaced by continuously changing quantities.

$\Rightarrow \boxed{\dfrac{1}{x}\dfrac{\Delta x}{\Delta t} \equiv \dfrac{1}{x}\dfrac{dx}{dt}}$ Now x is continuously differentiable with respect to t.

Plotting of Equations like $\boxed{\frac{dx}{dT} = -x(1-x)}$

$$\boxed{\frac{d^2x}{dT^2} = \frac{df}{dx}\frac{dx}{dT}}$$

in which $f(x) = -x + x^2$

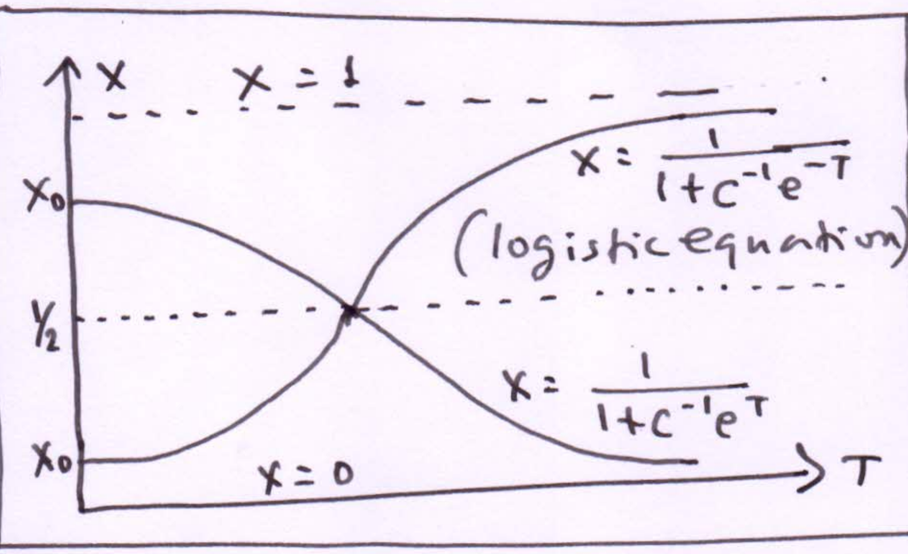
$$\Rightarrow \boxed{\frac{df}{dx} = f'(x) = -1 + 2x}$$

i.) If $\boxed{x < 1/2}$, $\boxed{\frac{d^2x}{dT^2} > 0}$ $\left(\because \frac{dx}{dT} < 0 \ \& \ \frac{df}{dx} < 0\right)$

This means $x(T)$ decreases at an increasing rate.

ii.) If $\boxed{x > 1/2}$, $\boxed{\frac{d^2x}{dT^2} < 0}$ $\left(\because \frac{dx}{dT} < 0 \ \& \ \frac{df}{dx} > 0\right)$,

i.e. $x(T)$ decreases at a decreasing rate



For $\frac{dx}{dT} = x(1-x)$.

Solution: $\boxed{x = \frac{1}{1+c^{-1}e^{-T}}}$

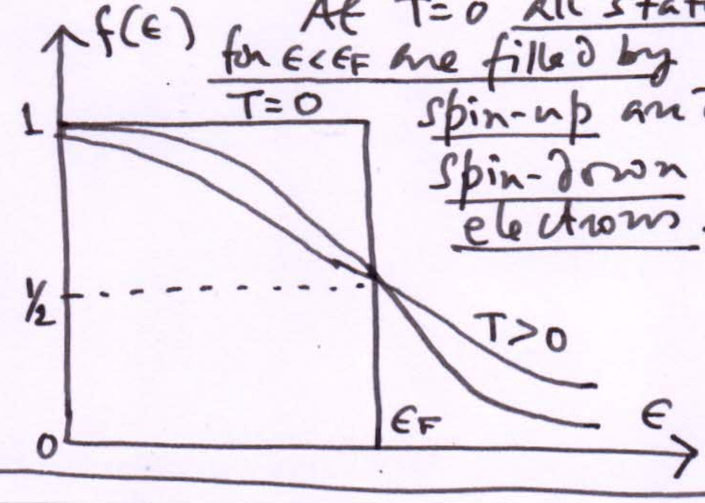
For $\frac{dx}{dT} = -x(1-x)$

Solution $\boxed{x = \frac{1}{1+c^{-1}e^{T}}}$

Fermi Function:

$$\boxed{f(\epsilon) = \frac{1}{1 + e^{(\epsilon - \epsilon_F)/k_B T}}}$$

Let $T = 0 \Rightarrow$ For $\epsilon < \epsilon_F$, $f(\epsilon) = \frac{1}{1+e^{-\infty}} = 1$. And for $\epsilon > \epsilon_F$ $f(\epsilon) = \frac{1}{1+e^{\infty}} = 0$



At $T=0$ all states for $\epsilon < \epsilon_F$ are filled by spin-up and spin-down electrons.

Power Laws and Their Properties

$\boxed{y = f(x) = Ax^{\gamma}}$ Scale $\boxed{x \longrightarrow \lambda x}$

$\therefore\; f(x) \longrightarrow f(\lambda x) = A(\lambda x)^{\gamma} = A\lambda^{\gamma} x^{\gamma} = y\lambda^{\gamma}$

$\Rightarrow$ y is scaled as $\boxed{y\lambda^{\gamma}}$ (Scale invariance)

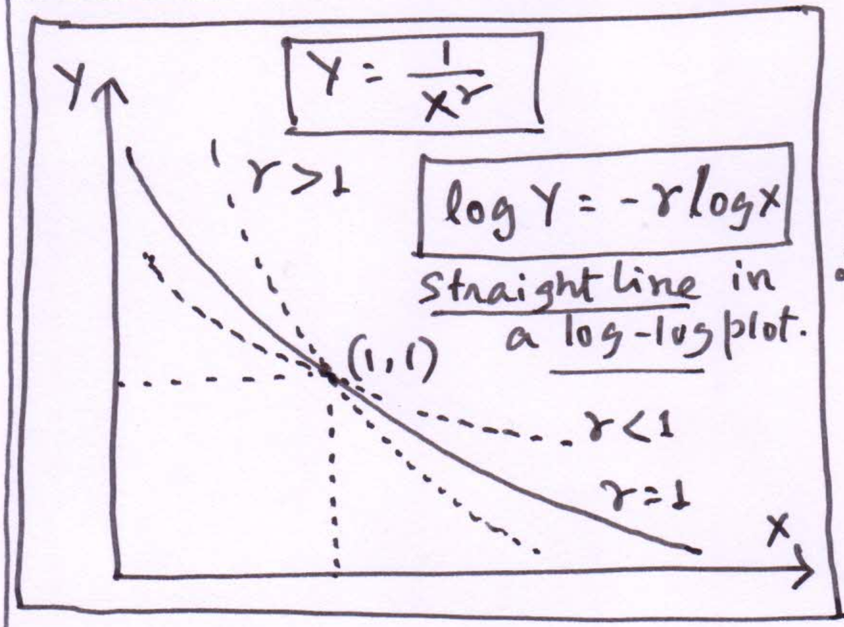
Inverse Power-Laws

$\boxed{y^m x^n = c}$ $\Rightarrow$ $\boxed{y\, x^{n/m} = c^{1/m} = a \text{ (say)}}$

$\Rightarrow \dfrac{y}{a}\, x^{n/m} = 1$ Rescale $\boxed{Y = y/a}$, $\boxed{X = x}$

and $\boxed{\gamma = n/m}$, $(\gamma > 0)$.

$\Rightarrow \boxed{YX^{\gamma} = 1}$ (as in $\boxed{PV^{\gamma} = \text{constant}}$).



1/. All the curves pass through (1,1).

2/. As $x \longrightarrow \infty$, the decay is faster for higher values of γ.

3/. For finite values of X and Y, no curve touches $\boxed{X = 0}$ or $\boxed{Y = 0}$.

4/. Any part of a curve is self-similar to any other part — scale-invariant.

Fall of a Parachutist | Free Fall

The equation $\boxed{m\dfrac{dv}{dt} = mg - kv^2}$ is used to describe the free-fall of a parachutist from a height of about 30,000 ft to about 2,000 ft. After that the parachute is opened.

Bernoulli Equation

$$\boxed{\frac{V^2}{2} + \frac{P}{\rho} + gz = \text{Constant}}$$

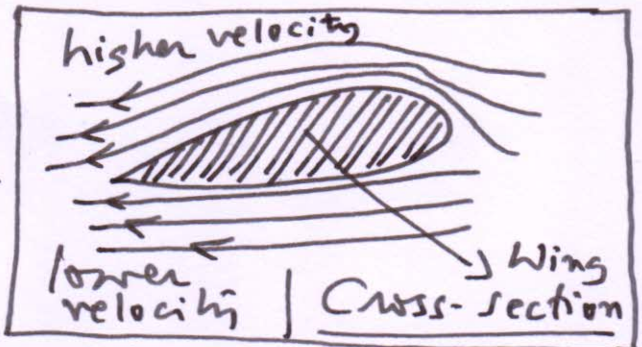
$z \rightarrow$ height
$V \rightarrow$ velocity
$P \rightarrow$ Pressure

$\rho \rightarrow$ Density, $g \rightarrow$ acceleration due to gravity.

i.) Streamline Motion:  Smooth and laminar

ii.) Turbulent Motion:  Random and chaotic

Lift of an Aircraft



i.) Above the wing closer stream-lines have higher velocity. Hence pressure is lower.

ii.) Below the wing the streamlines have lower velocity. Hence at nearly the same height the pressure is higher. This gives the lift.

Item Response Theory: Additional Points

i.) Item discrimination: $\boxed{P = c + \frac{1-c}{1+e^{-(\theta-b)/\omega}}}$.

When $\omega = 0$, for $\theta > b$, $P = c + 1 - c = 1$, and for $\theta < b$, $P = c$ (probability that a candidate with low ability responds correctly)

$\Rightarrow$ P varies between c (non-zero lowerbound) and unity (completely perfect response)

ii.) Item difficulty: The parameter b sets a scale for ability (θ). High ability to respond to an item is $\theta > b$, and low ability is $\theta < b$.

Sigmoid Activation Function

Biological neurons have a floor and ceiling of activity. This is expressed by the logistic function $\boxed{y_i = \frac{1}{1+e^{-x_j}} - a}$.

$x \rightarrow$ input to j-th unit. $y_i \rightarrow$ Activation response

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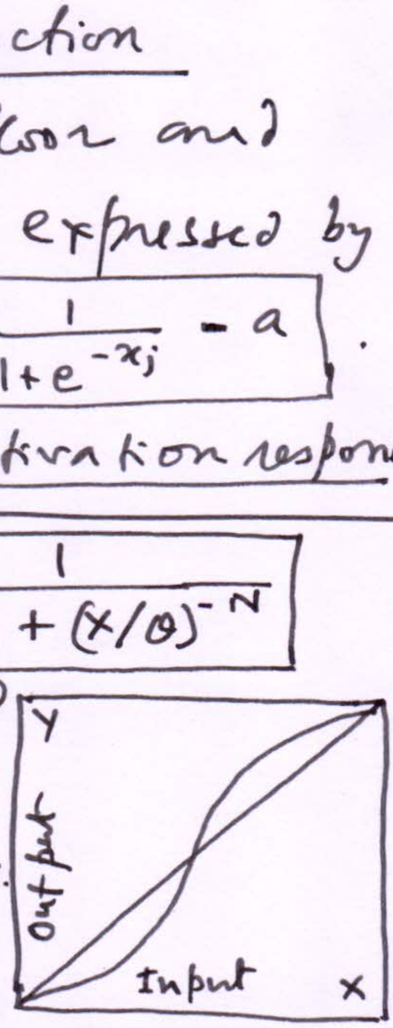
The Hill Function $\boxed{Y = \frac{1}{1+(X/\theta)^{-N}}}$

$N \rightarrow$ Hill coefficient, $\theta \rightarrow$ Threshold (constant)



Used for Positive Cooperativity, in haemoglobin, which has four monomers. Binding of one monomer with oxygen, increases the affinity for binding in the other three. After that it saturates.