TUTORIAL 08

1. If a die is to be rolled until all sides have appeared at least once, find the expected number of times that outcome 1 appears.

2. The nine players on a basketball team consist of 2 centers, 3 forwards, and 4 backcourt players. If the players are paired up at random into three groups of size 3 each, find the expected value of the number of triplets consisting of one of each type of player.

3. Individuals 1 through n, $n > 1$, are to be recruited into a firm in the following manner: Individual 1 starts the firm and recruits individual 2. Individuals 1 and 2 will then compete to recruit individual 3. Once individual 3 is recruited, individuals 1, 2, and 3 will compete to recruit individual 4, and so on. Suppose that when individuals 1, 2, . . . , i compete to recruit individual i + 1, each of them is equally likely to be the successful recruiter. Find the expected number of the individuals 1, . . . , n who did not recruit anyone else.

4. An urn has n white and m black balls that are removed one at a time in a randomly chosen order. Find the expected number of instances in which a white ball is immediately followed by a black one.

5. Twenty individuals consisting of 10 married couples are to be seated at 5 different tables, with 4 people at each table.
    a. If the seating is done at random, what is the expected number of married couples that are seated at the same table?
    b. If 2 men and 2 women are randomly chosen to be seated at each table, what is the expected number of married couples that are seated at the same table?

6. Suppose that there are N different types of coupons, and each time one obtains a coupon, it is equally likely to be any one of the N types. Find the expected number of coupons one need amass before obtaining a complete set of at least one of each type.