Tutorial 9

1. A table is ruled with equidistant parallel lines a distance D apart. A needle of length L, where $L \leq D$, is randomly thrown on the table. What is the probability that the needle will intersect one of the lines (the other possibility being that the needle will be completely contained in the strip between two lines)?

2. Prove that if X_i,$i = 1, ..., n$, are independent random variables that are normally distributed with respective parameters $\mu_i, \sigma_i^2, i = 1, ..., n$, then $\sum_{i=1}^n X_i$ is normally distributed with parameters $\sum_{i=1}^n \mu_i$, $\sum_{i=1}^n \sigma_i^2$.

3.A basketball team will play a 44-game season. Twenty-six of these games are against class A teams and 18 are against class B teams. Suppose that the team will win each game against a class A team with probability .4 and will win each game against a class B team with probability .7. Suppose also that the results of the different games are independent. Approximate the probability that
(a) the team wins 25 games or more;
(b) the team wins more games against class A teams than it does against class B teams.

The random variable Y is said to be a *lognormal* random variable with parameters μ and σ if $log(Y)$ is a normal random variable with mean μ and variance σ^2. That is, Y is lognormal if it can be expressed as

$$Y = e^X$$

where X is a normal random variable.

4. Starting at some fixed time, let $S(n)$ denote the price of a certain security at the end of n additional weeks, $n \geq 1$. A popular model for the evolution of these prices assumes that the price ratios $S(n)/S(n-1)$, $n \geq 1$ are independent and identically distributed lognormal random variables. Assuming this model, with parameters $\mu = .0165, \sigma = .0730$,what is the probability that
(a) the price of the security increases over each of the next two weeks?
(b) the price at the end of two weeks is higher than it is today?