Lecture-29 P 1

Got disconnected, reconnected, you could continue

Negative marking is there in MCQ

Not there in fill in the blanks.

True/False : no negative marking ? need to check

Free navigation : you will be able to go back & forth in the question paper.

Endsem on 11 or 12 ~~June~~. July

will let you know the time & exact date

Scan lecture notes (2)

Google form: if you don't want to take the online endsem exam.

Recap: Markov's inequality

Chebyshev's inequality:

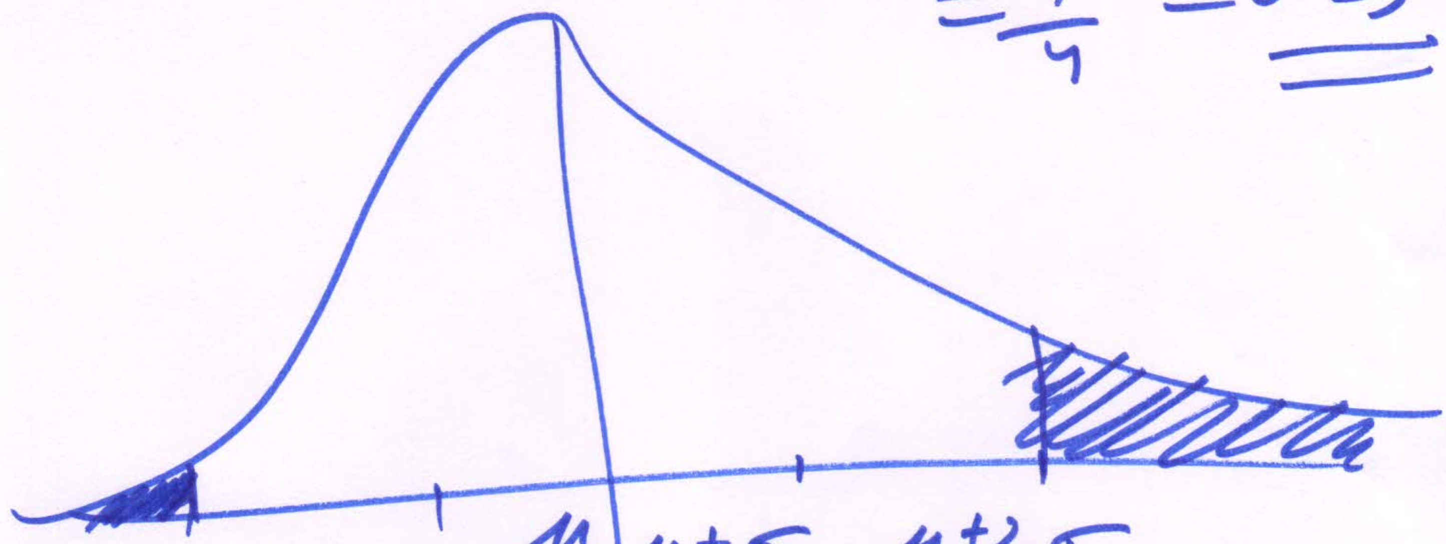
X is a random variable, mean μ, variance σ^2, $k>0$

$$P(|X-\mu| \geq k) \leq \frac{\sigma^2}{k^2}$$

e.g. $N(\mu, \sigma^2)$ Normal

3

$$P(|X-\mu| > 2\sigma) \le \frac{\sigma^2}{k^2} = \frac{\sigma^2}{4\sigma^2} = \frac{1}{4} = \underline{\underline{0.25}}$$



$$P(\underline{X < \mu - 2\sigma} \cup \underline{X > \mu + 2\sigma})$$

$$= \underbrace{P(X < \mu - 2\sigma)} + P(X > \mu + 2\sigma)$$

$$= P\left(\frac{X-\mu}{\sigma} < \frac{-2\sigma}{\sigma}\right) + P\left(\frac{X-\mu}{\sigma} > 2\right)$$

$P(Z < -2) +$

$P(Z > 2)$, where Z is

std. normal.

$= 1 - \Phi(2) + \Phi(-2)$

$= 1 - \Phi(2) + 1 - \Phi(2)$

$= 2(1 - \Phi(2))$

$= 2 - 2 * \Phi(2)$

$= 2 - 2 * 0.9772$

$= 2 - 1.9544$

$= 0.0456$

$P \leq 0.25$

e.g. if $\sigma^2 = 0$, (5)

variance $=0$.

then

$P(X = E[X]) = 1$

Prove this using Chebyshev's inequality.

$$P(|X-\mu| \geq k) \leq \frac{\sigma^2}{k^2} = 0$$

$$P(|X-\mu| \geq k) = 0 \quad , k > 0$$

$$k = \frac{1}{n}, \quad n = 1, 2, 3, \dots$$

$$P\left(|X-\mu| \geq \frac{1}{n}\right) = 0$$

what happens when $n \to \infty$?

$$\frac{1}{n} \to 0$$

$$P(|X-\mu| \neq 0) = 0$$

$$P(|X-\mu| = 0) = 1 \Rightarrow$$

$$P(X = \mu) = 1$$

Weak law of large numbers

You toss a fair coin 100 times. The no. of times you expect to get a Head is 50.
We do this experiment.

$E[X_1] = 53$

$$\underbrace{\frac{X_1+X_2+\dots+X_n}{n}}_{\text{sample mean}} \to \underbrace{E[X]}_{\text{population mean}}$$

$E[X] = 50$

as $n \to \infty$

$X_1 = 53$

$X_2 = 55$

$X_3 = 60$

$X_4 = 40$

$X_5 = 51$

$\vdots$

$X_n = 47$

$x_1, x_2, x_3, \dots, x_n$ are independent & identically distributed i.i.d.

Each has a finite mean

$E[x_i] = \mu$

Then for any $\epsilon > 0$,

$$P\left\{ \left| \frac{x_1 + x_2 + \dots + x_n}{n} - \mu \right| \geq \epsilon \right\} \to 0$$

as $n \longrightarrow \infty$