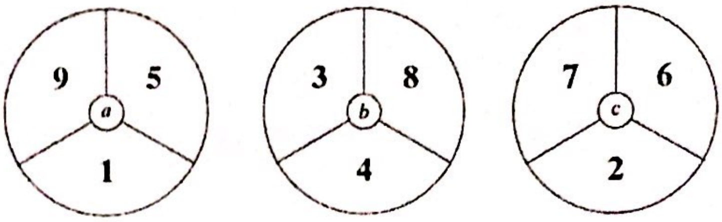
Tutorial 2 Solutions

1. Two players play the following game: Player A chooses one of the three spinners pictured in Figure below, and then player B chooses one of the remaining two spinners. Both players then spin their spinner, and the one that lands on the higher number is declared the winner. Assuming that each spinner is equally likely to land in any of its 3 regions, would you rather be player A or player B? Explain your answer!



Sol^n

- I would like to be a Player B!
- Explaination :-

A has 3 choices initialy

Case 1 : A chooses spinner 'a', Note that for every pair of the spinners there are 9 possible cases

Now, for B there are 2 cases

(i) B chooses b

$\Rightarrow P(A \text{ wins}) = \frac{3}{9} + \frac{2}{9} + 0 = \frac{5}{9}$

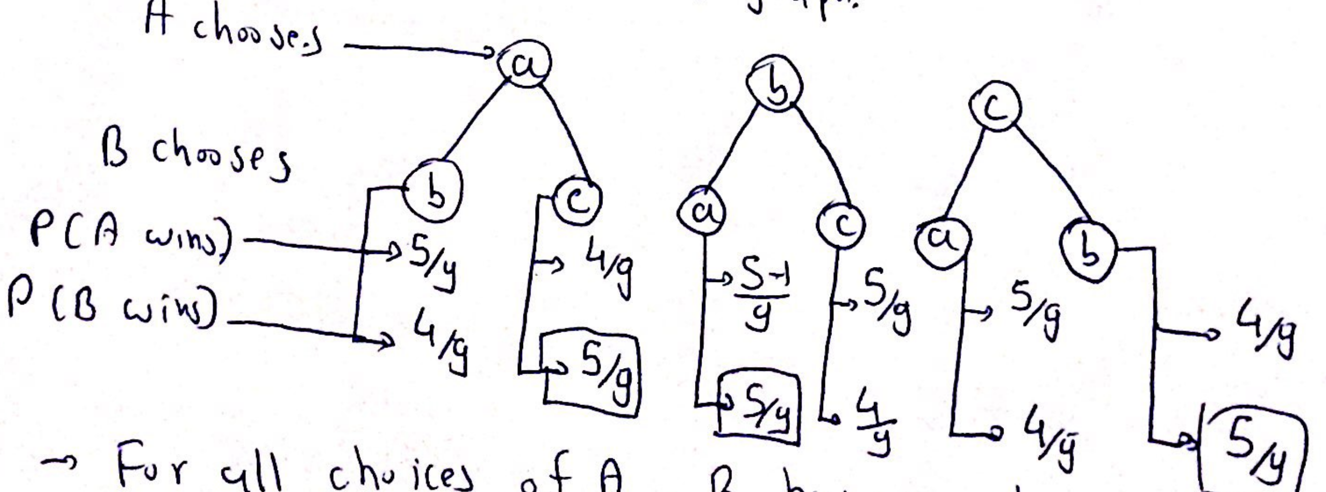
$\Rightarrow P(B \text{ wins}) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9} \; (= 1 - \frac{5}{9})$.

(ii) B chooses c

$\Rightarrow P(A \text{ wins}) = \frac{3+1+0}{9} = \frac{4}{9}$

$\Rightarrow P(B \text{ wins}) = 1 - \frac{4}{9} = \boxed{\frac{5}{9}}$.

- All the cases in a graph.



→ For all choices of A, B has a choice to take a spinner with high winning probability ⇒ B has high chance to win.

2. A bin contains 3 different types of disposable flashlights. The probability that a type 1 flashlight will give over 100 hours of use is .7, with the corresponding probabilities for type 2 and type 3 flashlights being .4 and .3, respectively. Suppose that 20 percent of the flashlights in the bin are type 1, 30 percent are type 2, and 50 percent are type 3.
   a. What is the probability that a randomly chosen flashlight will give more than 100 hours of use?
   b. Given that a flashlight lasted over 100 hours, what is the conditional probability that it was a type *j* flashlight, *j* = 1, 2, 3?

Soln. a] A: Flashlight will give more than 100 hours of use.

F_1: Randomly choosen Flash light is of type 1.
F_2: " " " 2.
F_3: " " " 3.

$P(A) = ?$, $P(A) = P(F_1|A)P(F_1) + P(F_2|A)P(F_2) + P(F_3|A)P(F_3)$

$= 0.7 \times 0.2 + 0.4 \times 0.3 + 0.3 \times 0.5$

$\boxed{= 0.41}$

b] Here we need to find the $P(F_j|A)$ $j = 1, 2, 3$.

$$\therefore P(F_j|A) = \frac{P(A|F_j)P(F_j)}{\sum P(A|F_i)P(F_i)}$$

$$= \frac{P(A|F_j)P(F_j)}{\sum_{i=1}^{3} P(A|F_i)P(F_i)}$$

$$\therefore P(F_j|A) = \frac{P(A|F_j)P(F_j)}{0.41} \quad (\because \text{from a})$$

$\therefore P(F_1|A) = \frac{0.7 \times 0.2}{0.41} = \frac{14}{41}$

$\therefore P(F_2|A) = \frac{0.4 \times 0.3}{0.41} = \frac{12}{41}$

$\therefore P(F_3|A) = \frac{0.3 \times 0.5}{0.41} = \frac{15}{41}$

3. Urn A contains 2 white balls and 1 black ball, whereas urn B contains 1 white ball and 5 black balls. A ball is drawn at random from urn A and placed in urn B. A ball is then drawn from urn B. It happens to be white. What is the probability that the ball transferred was white?

Sol^n W: The ball drawn from urn B is white.

T: The ball transferred from urn A to B is white.

$P(T/W) = ?$

$$P(T/W) = \frac{P(W|T)\,P(T)}{P(W|T)\,P(T) + P(W|T^c)\,P(T^c)}$$

$$= \frac{2/7 \times 2/3}{2/7 \times 2/3 + 1/7 \times 1/3}$$

$$\boxed{= \frac{4}{5}}$$

4. A coin having probability .8 of landing on heads is flipped. A observes the result—either heads or tails—and rushes off to tell B. However, with probability .4, A will have forgotten the result by the time he reaches B. If A has forgotten, then, rather than admitting this to B, he is equally likely to tell B that the coin landed on heads or that it landed tails. (If he does remember, then he tells B the correct result.)
   a. What is the probability that B is told that the coin landed on heads?
   b. What is the probability that B is told the correct result?
   c. Given that B is told that the coin landed on heads, what is the probability that it did in fact land on heads?

Sol^n

H: Coin lands heads.

B_H: B is told that coin landed heads.

A_F: A forgets the result.

A_R: A remembers the result. $\boxed{= A_F^c}$

a] $P(B_H) = ?$. $P(B_H) = P(B_H \mid A_F)\,P(A_F) + P(B_H \mid A_R)\,P(A_R)$

$$= 0.5 \times 0.4 + 0.8 \times 0.6$$

$$\boxed{= 0.68}$$

b] C: B is told the correct result.

$$P(C) = P(C \mid A_F)\,P(A_F) + P(C \mid A_R)\,P(A_R)$$

$$= 0.5 \times 0.4 + 1 \times 0.6$$

$$\boxed{= 0.8}$$

c] $P(H \mid B_H) = ? \Rightarrow \boxed{P(H \mid B_H) = \frac{P(H \cdot B_H)}{P(B_H)}}$ ① $(P(B_H) = 0.68)$

$$P(H\,B_H) = P(H\,B_H \mid A_F)\,P(A_F) + P(H B_H \mid A_R)\,P(A_R)$$

$$= P(H \mid A_F)\,P(B_H \mid H A_F)\,P(A_F) + P(H)\,P(A_R)$$

$$= 0.8 \times 0.5 \times 0.4 + 0.8 \times 0.6$$

$$\boxed{= 0.64}$$

$$\therefore P(H \mid B_H) = \frac{0.64}{0.68} = \boxed{\frac{16}{17}}$$

5. Three players simultaneously toss coins. The coin tossed by A(B)[C] turns up heads with probability P1(P2)[P3]. If one person gets an outcome different from those of the other two, then he is the odd man out. If there is no odd man out, the players flip again and continue to do so until they get an odd man out. What is the probability that A will be the odd man? , $P_1 = 0.3$, $P_2 = 0.4$, $P_3 = 0.8$

Soln

Here, we have infinite ways ~~to~~ in which A will be the odd man, but we can write the term like...

$$P(A \text{ odd}) = P_1(1-P_2)(1-P_3) + (1-P_1)P_2P_3 + P_1P_2P_3P(A \text{ odd}) + (1-P_1)(1-P_2)(1-P_3)P(A \text{ odd})$$

here we are assuming $P(A \text{ odd})$ is a constant number that shows the probability that A will be the odd man and hence we multiply $P(A \text{ odd})$ with rest of the terms.

$$\therefore P(A \text{ odd})\left(1 - P_1P_2P_3 - (1-P_1)(1-P_2)(1-P_3)\right) = P_1(1-P_2)(1-P_3) + P_2P_3(1-P_1)$$

$$\therefore P(A \text{ odd}) \cdot (P_1 + P_2 + P_3 - P_1P_2 - P_1P_3 - P_2P_3) = P_1(1-P_2)(1-P_3) + P_2P_3(1-P_1)$$

$$\therefore \boxed{P(A \text{ odd}) = \frac{P_1(1-P_2)(1-P_3) + P_2P_3(1-P_1)}{P_1+P_2+P_3 - P_1P_2 - P_2P_3 - P_1P_3}}$$

$$= \frac{0.3 \times 0.6 \times 0.2 + 0.7 \times 0.4 \times 0.8}{0.3 + 0.4 + 0.8 - 0.12 - 0.32 - 0.24}$$

$$\boxed{= 0.3091}$$