Tutorial - 3

Function of RV

Date : 4th Feb '19

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1. $F_Y(y) = P[Y \le y]$

1.

$$f_X(x) = \begin{cases} 1/6 & ; 1 \le x \le 6 \\ 0 & ; \text{Otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} 1/6 & ; y = 1, 4, 9, 16, 25, 36 \\ 0 & ; \text{otherwise} \end{cases}$$

2.

$$f_X(x) = \begin{cases} 1/6 & ; -2 \le x \le 3 \\ 0 & ; \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} 1/6 + 1/6 & ; y = 4 \\ 1/6 + 1/6 & ; y = 1 \\ 1/6 & ; y = 0 \\ 1/6 & ; y = 9 \end{cases}$$

2. $F_Y(y) = P[Y \le y]$

$= P\left[\cancel{X} \le \frac{y-3}{2}\right]$

$= F_X\left(\frac{y-3}{2}\right)$

; $y = 2x+3$

; $x = \frac{y-3}{2}$

; $dx = \frac{1}{2} dy$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dx} F_X\left(\frac{y-3}{2}\right) \cdot \frac{dx}{dy}$$

$$= 3 \cdot \left(\frac{y-3}{2}\right)^2 \cdot \frac{1}{2}$$

$0 < x < 1$

$0 < 2x < 2$

$3 < 2x+3 < 5$

$\therefore 3 < y < 5$

$$= \frac{3}{8}(y-3)^2 \quad ; \quad 3 < y < 5$$

3. $F_Y(y) = P[Y \le y]$

$= P[-\sqrt{y} \le x \le \sqrt{y}]$

$= F_X(\sqrt{y}) - F_X(-\sqrt{y})$

$$f_Y(y) = f_X(\sqrt{y}) \cdot \left(\frac{1}{2\sqrt{y}}\right) + f_X(-\sqrt{y}) \cdot \left(\frac{1}{2\sqrt{y}}\right)$$

$$= \frac{1}{2\sqrt{y}}\left[f_X(\sqrt{y}) + f_X(-\sqrt{y})\right]$$

$$f_X(x) = Z \sim N(0,1)$$
$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$
$$f_Y(y) = \frac{1}{2\sqrt{y}} \left[\frac{1}{\sqrt{2\pi}} e^{-y/2} + \frac{1}{\sqrt{2\pi}} e^{-y/2} \right]$$
$$= \frac{1}{\sqrt{2\pi y}} e^{-y/2}$$

4. $F_Y(y) = P[Y \le y]$

$= P[x \le e^y]$

$= F_X(e^y)$

$Y = \ln X$

$\therefore x = e^Y$

$\therefore dx = e^Y . dy$

$$f_Y(y) = f_X(e^y) . \frac{dx}{dy}$$

$x > 1$

$\therefore Y > 0$

$$= \frac{\theta}{e^{\theta Y + \theta}} . e^y$$
$$= \theta . e^{-\theta Y} \; ; \quad \theta > 0, Y > 0$$

5. $F_Y(y) = P[Y \le y]$

$= P[x \le (b-a)y + a]$

$= F_X((b-a)y + a)$

$Y = \frac{X-a}{b-a}$

$\therefore x = (b-a)y + a$

$\therefore dx = (b-a) dy$

$$f_Y(y) = f_X((b-a)y + a) . \frac{dx}{dy}$$

$a < x < b$

$\therefore 0 < x - a < b - a$

$\therefore 0 < y < 1$

$$= \frac{(b-a)}{b-a} \; ; \quad 0 < y < 1$$
$$= 1 \; ; \quad 0 < y < 1$$