Tutorial 03

1. A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1 percent of the healthy persons tested. (That is, if a healthy person is tested, then, with probability .01, the test result will imply that he or she has the disease.) If .5 percent of the population actually has the disease, what is the probability that a person has the disease given that the test result is positive?

2. Consider a medical practitioner pondering the following dilemma: "If I'm at least 80 percent certain that my patient has this disease, then I always recommend surgery, whereas if I'm not quite as certain, then I recommend additional tests that are expensive and sometimes painful. Now, initially I was only 60 percent certain that Jones had the disease, so I ordered the series A test, which always gives a positive result when the patient has the disease and almost never does when he is healthy. The test result was positive, and I was all set to recommend surgery when Jones informed me, for the first time, that he was diabetic. This information complicates matters because, although it doesn't change my original 60 percent estimate of his chances of having the disease in question, it does affect the interpretation of the results of the A test. This is so because the A test, while never yielding a positive result when the patient is healthy, does unfortunately yield a positive result 30 percent of the time in the case of diabetic patients who are not suffering from the disease. Now what do I do? More tests or immediate surgery?"

3. At a certain stage of a criminal investigation, the inspector in charge is 60 percent convinced of the guilt of a certain suspect. Suppose, however, that a new piece of evidence which shows that the criminal has a certain characteristic (such as left-handedness, baldness, or brown hair) is uncovered. If 20 percent of the population possesses this characteristic, how certain of the guilt of the suspect should the inspector now be if it turns out that the suspect has the characteristic?

4. Suppose that we have 3 cards that are identical in form, except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?

5. Independent trials consisting of rolling a pair of fair dice are performed. What is the probability that an outcome of 5 appears before an outcome of 7 when the outcome of a roll is the sum of the dice?

6. There are n types of coupons, and each new one collected is independently of type i with probability p_i, $\sum_i^n p_i = 1$. Suppose k coupons are to be collected. If A_i is the event that there is at least one type i coupon among those collected, then, for $i \neq j$, find (a) $P(A_i)$, (b) $P(A_i \cup A_j)$ and (c) $P(A_i|A_j)$.

7. The probability that a new car battery functions for over 10,000 miles is .8, the probability that it functions for over 20,000 miles is .4, and the probability that it functions for over 30,000 miles is .1. If a new car battery is still working after 10,000 miles, what is the probability that

   (a) its total life will exceed 20,000 miles?

   (b) its additional life will exceed 20,000 miles?

8. Two local factories, A and B, produce radios. Each radio produced at factory A is defective with probability .05, whereas each one produced at factory B is defective with probability .01. Suppose you purchase two radios that were produced at the same factory, which is equally likely to have been either factory A or factory B. If the first radio that you check is defective, what is the conditional probability that the other one is also defective?