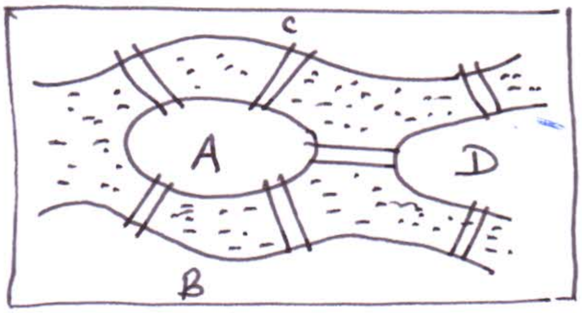
Networks and Complex Systems

The Königsberg Bridge Problem

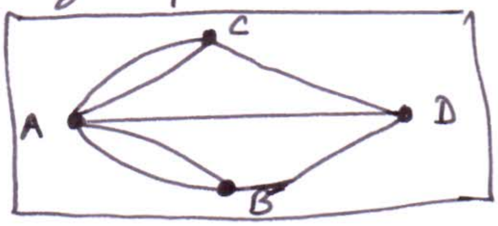
1/. Seven bridges connecting all of Königsberg City.



2/. Question: Is it possible to start at any one point in the city, and return to that same point after crossing all the seven bridges ONLY once?

Answer: No – Leonhard Euler. By introducing the concept of graph structure.

Not possible if every node on the graph has an odd number of links.

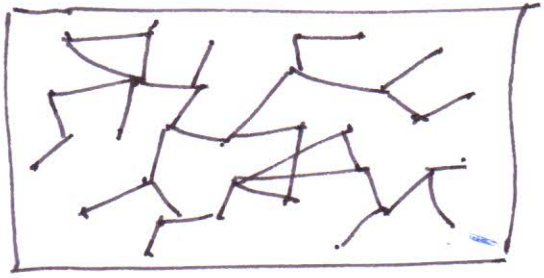


A has 5 links, B has 3, C has 3, D has 3.

3/. With this work Euler established graph theory as a branch of mathematics.

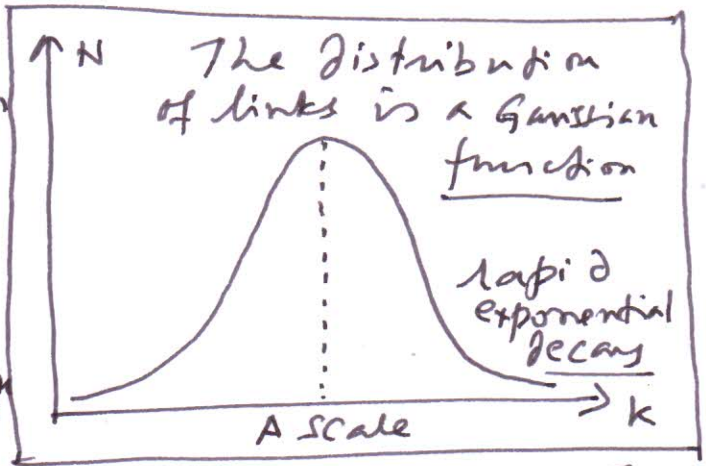
4/. Complex systems can be understood by constructing a graph or network structure.

Random Networks : (Paul Erdős, Alfréd Rényi)



1/. A graph in which the nodes are linked in a random way.

2/. The degree distribution of such a graph, in which the frequency of nodes $N(k)$ with a given number of links, k, is a Gaussian function, $\boxed{N(k) \sim e^{-k^2}}$.



Small-World Networks : (Duncan Watts, Steven Strogatz)

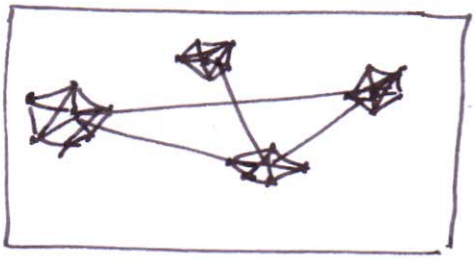
1/. In Sociology Mark Granovetter wrote about the strength of weak ties between closely-knit clusters of human social groups with common interests.

2/. Stanley Milgram found the "six-degrees of separation" between two individuals.

3/. Human social networks are NOT formed randomly. There is order.

4/. Small-world clusters form in networks, in which all nodes are connected to one another (Watts & Strogatz)

5/. The clusters are connected to other clusters through



one or two weak links spanning across clusters. Without these weak linkages, the clusters will become isolated.

6/. Used to explain phenomena like synchrony and self-organisation.

Clustering Coefficient: Consider a cluster in which there are k_i nodes. Then the maximum number of links that can form, connecting any two nodes, is

$$\boxed{{}^{k_i}C_2} \Rightarrow \boxed{\frac{k_i!}{2!\,(k_i-2)!} = \frac{k_i(k_i-1)}{2}}.$$

If the actual number of links is T_i, then the clustering coefficient $\boxed{C = \frac{T_i}{k_i(k_i-1)/2}}$.

For a tightly-knit cluster, $\boxed{C \to 1}$. For a random graph $\boxed{C \to 0}$.

Basic Definitions :

1.) Nodes (vertices) → Points (or vertices) in the network

2.) Edge → An undirected link joining ~~too~~ two nodes.

3.) Arc → A directed link joining two nodes.

4.) Neighbours → Two directly connected nodes.

5.) Neighbourhood → A node and all its neighbours.

6.) Path → Sequence of edges (or arcs) between any two nodes.

7.) Path length → Number of edges or arcs between two nodes.

8.) Distance → Shortest path length.

9.) Diameter → The maximum distance between any two nodes.

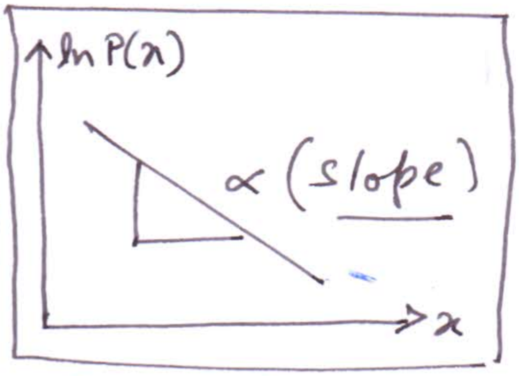
10.) Clustering → Formation of closely-knit and tight cliques.

11.) Degree distribution → Distribution plot of the number of nodes with ~~a~~ a given number of links.

Power Laws: $\boxed{P(x) = c\,x^{-\alpha}}$ $(\alpha > 0)$

$\therefore$ $\boxed{\ln P(x) = \ln c - \alpha \ln x}$

Equation of a straight line



Has no scale in this distribution function. Hence scale-free.

1/. Events with a power-law distribution:

i) Earthquakes, ii) Extinction events, iii) War/Terrorist fatalities, iv) Stock fluctuations (Crashes) v) Traffic jams.

2/. Resource distributions with a power law:

i) Income distribution, ii) Wealth distribution, iii) City-size distributions, iv) Word frequency in languages. (Both of these are Zipf's law).

Networks with Power-Law Distributions:

1/. The World Wide Web, 2/. The Internet,

3/. Movie actors and science collaboration networks.

4/. The cellular network, 5/. Ecological network,

6/. Phone call networks, 7/ Citation networks,

8/. Networks in lignistics, 9/. Power and neural networks.

10/. Protein folding, 11/. The web of human sexual contacts.

Scale-Free Networks: (Albert-László Barabási)

1/. The degree distribution is like a power law.

2/. The distribution, therefore, has no characteristic scale — Scale-free.

3/. The network is dynamically growing.

4/. Links are made preferentially with the heavily-linked nodes.

5/. The most heavily-linked nodes are the ones that bind separate clusters through weak links in a small-world network.

6/. Since the heavily-linked nodes are disproportionately small in number, and yet they are dominant in the network, they may avoid being adversely affected in a random attack or a random failure. The network is robust against random attacks.

7/. Scale-free networks with small-world structures, are vulnerable to targetted attacks (e.g. spread of AIDS, World Trade Centre, British Raj in India).