Points to remember

min ratio

- For min ratio test if a negative number or zero in the divisor, then we will not evaluate the value.
    - we always consider the b value should be non-negative.
- A variable can enter the basis any number of times. Even if it leaves the basis then can again enter the basis.

various aspects of simplex algorithm

There are three stages of simplex algorithm.

I) **Initialisation:**

The algorithm starts with a basic feasible solution.
The b value is always non-negative.

II) **Iteration:**

The algorithm proceed through some intermediate iterations.

III) **Termination:**

The algorithm terminates when there is no entering variable in the table or all $z_j - c_j \geq 0$

Questions

1) Do all LPPs have optimal solution??

2) Does the simplex algorithm always terminate by providing an optimal solution??

Initialization

We shall look at a minimisation problem with "$\geq$" type constraints.

$$\min Z = 2x_1 + x_2$$

$$\boxed{P_1} \quad \begin{aligned} \text{s.t.} \quad 2x_1 + x_2 &\geq 4 \quad \text{--- } (1) \\ x_1 + 2x_2 &\geq 3 \quad \text{--- } (2) \\ x_1 + 3x_2 &\geq 6 \quad \text{--- } (3) \\ x_1, x_2 &\geq 0 \end{aligned}$$

We introduce surplus variables x_3, x_4, and x_5 to (1), (2) & (3) respectively.

Then $\boxed{P_1}$ becomes.

$$\min Z = 2x_1 + x_2 + 0x_3 + 0x_4 + 0x_5$$

$$\boxed{P_2} \quad \begin{aligned} \text{s.t.} \quad 2x_1 + x_2 - x_3 \qquad\qquad &= 4 \quad (4) \\ x_1 + 2x_2 \qquad - x_4 \qquad &= 3 \quad (5) \\ x_1 + 3x_2 \qquad\qquad - x_5 &= 6 \quad (6) \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 . \end{aligned}$$

This is a minimisation problem.
we convert $\boxed{P_2}$ into a maximisation problem by multiplying a (-1) with the objective function.

$$\underline{\underline{\max W}} = -Z = -2x_1 - x_2 + 0x_3 + 0x_4 + 0x_5$$

$$\boxed{P_3} \quad \text{s.t.} \quad \begin{aligned} 2x_1 + x_2 - x_3 \qquad\qquad &= 4 \quad \text{— (7)} \\ x_1 + 2x_2 \qquad - x_4 \qquad &= 3 \quad \text{— (8)} \\ x_1 + 3x_2 \qquad\qquad - x_5 &= 6 \quad \text{— (9)} \\ x_1, \ldots, x_5 &\geq 0 \end{aligned}$$

We start with a basic feasible solution to start an LPP.

We start by considering x_1 & x_2 as non-basic variable ie, taking $x_1 = 0$ and $x_2 = 0$.
This means considering x_3, x_4, and x_5 as basic variable.

As a result $x_3 = -4$, $x_4 = -3$ and $x_5 = -6$

so they are no more feasible solution.

For constraint type "$\geq$" surplus variables do not produce any basic feasible solution.

$\boxed{\text{How to proceed??}}$

We add 3 artificial variables x_6, x_7, and x_8 in (7), (8), and (9) respectively.

Then $\boxed{P_3}$ becomes.

$$\begin{cases} \max W = -2x_1 - x_2 + 0x_3 + 0x_4 + 0x_5 - Mx_6 - Mx_7 - Mx_8 \\ \text{s.t.}\ 2x_1 + x_2 - x_3 \quad + x_6 = 4 \quad (10) \\ x_1 + 2x_2 \quad - x_4 \quad + x_7 = 3 \quad (11) \\ x_1 + 3x_2 \quad - x_5 \quad + x_8 = 6 \quad (12) \\ x_1, x_2, \dots, x_8 \geq 0 \end{cases}$$

where M is a very big positive number.

- x_6, x_7, x_8 become initial basic variables.
- we ensure that x_6, x_7, and x_8 can not contribute to the objective function.

So we associates a very large negative value with the artificial variables.

C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	min b/a_2 ratio	operation
		c_j		-2	-1	0	0	0	$-M$	$-M$	$-M$		
$-M$	a_6	x_6	4	2	1	-1	0	0	1	0	0	$4/1=4$	
$-M$	a_7	x_7	3	1	$\boxed{2}$	0	-1	0	0	1	0	$3/2=1.5 \rightarrow$	
$-M$	a_8	x_8	6	1	3	0	0	-1	0	0	1	$6/3=2$	
Z_j-C_j				$\boxed{-4M+2}$	$-6M+1 \uparrow$	M	M	M	0	0	0	b/a_4	
$-M$	a_6	x_6	$5/2$	$3/2$	0	-1	$1/2$	0	1		0	$5/2/1/2=5$	$R_1'=R_1-R_2'$
-1	a_2	x_2	$3/2$	$1/2$	1	0	$-1/2$	0	0		0	---	$R_2'=R_2/2$
$-M$	a_8	x_8	$3/2$	$-1/2$	0	0	$\boxed{3/2}$	-1	0		1	$3/2/3/2=1 \rightarrow$	$R_3'=R_3-3R_2'$
Z_j-C_j				$\boxed{-M+3/2}$	0	M	$\boxed{-2M+\frac{1}{2}} \uparrow$	M	0		0	b/a_1	
$-M$	a_6	x_6	2	$\boxed{5/3}$	0	-1	0	$1/3$	1			$\frac{6}{5}=1.2 \rightarrow$	$R_1'=R_1-\frac{1}{2}R_3'$
-1	a_2	x_2	2	$1/3$	1	0	0	$-1/3$	0			$2/1/3=6$	$R_2'=R_2+\frac{1}{2}R_3'$
0	a_4	x_4	1	$-1/3$	0	0	1	$-2/3$	0			---	$R_3'=R_3/3/2$
Z_j-C_j				$-5/3M+5/3 \uparrow$	0	M	0	$-\frac{1}{3}M+1/3$	0				
-2	a_1	x_1	$6/5$	1	0	$-3/5$	0	$1/5$					$R_1'=R_1/5/3$
-1	a_2	x_2	$8/5$	0	1	$1/5$	0	$-\frac{2}{5}$					$R_2'=R_2-\frac{1}{3}R_1'$
0	a_4	x_4	$7/5$	0	0	$-1/5$	1	$3/5$					$R_3'=R_3+\frac{1}{3}R_1'$
z_j-c_j				0	0	1	0	0					

Here all $z_j - c_j \geq 0$. optimality reached.

$x_1 = 6/5$, $x_2 = 8/5$, $w = -2\cdot\frac{6}{5} - 1\cdot 8/5 = -4$.

$Z = (-w) = 4$