

1. Calculate the laplacian of the following:

(i) $F = x^2 + 2xy + 3z + 4$ (ii) $F = \sin(\hat{\mathbf{k}} \cdot \vec{\mathbf{r}})$ (iii) $F = \frac{1}{r}$

2. Find the curl of the following:

(a) $\vec{\mathbf{A}} = y\hat{\mathbf{i}} - x\hat{\mathbf{j}}$

(b) $\vec{\mathbf{A}} = \frac{1}{\sqrt{x^2+y^2}}(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})$

(c) $\vec{\mathbf{A}} = \frac{1}{x^2+y^2}(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})$

(d) $\vec{\mathbf{A}} = (x^2 + y^2)\hat{\mathbf{k}}$

3. For any vector field $\vec{\mathbf{A}}$ and any scalar field F show that

(i) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{\mathbf{A}}) = 0$; (ii) $\vec{\nabla} \times (\vec{\nabla} F) = 0$.

4. Can we find a scalar function F such that $\vec{\nabla} F = y\hat{\mathbf{i}} - x\hat{\mathbf{j}}$?

What about $\vec{\nabla} F = \frac{1}{x^2+y^2}(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})$?

5. Using the expressions for $\vec{\nabla} \cdot (\vec{\mathbf{A}} \times \vec{\mathbf{B}})$ and $\vec{\nabla} \times (\vec{\mathbf{A}} \times \vec{\mathbf{B}})$ evaluate $\vec{\nabla} \cdot (\vec{\omega} \times \vec{\mathbf{r}})$ and $\vec{\nabla} \times (\vec{\omega} \times \vec{\mathbf{r}})$ where $\vec{\omega}$ is a constant vector.

6. Find the equation of the tangent plane to the ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point (x_0, y_0, z_0) on the ellipsoid.