Lecture - 16

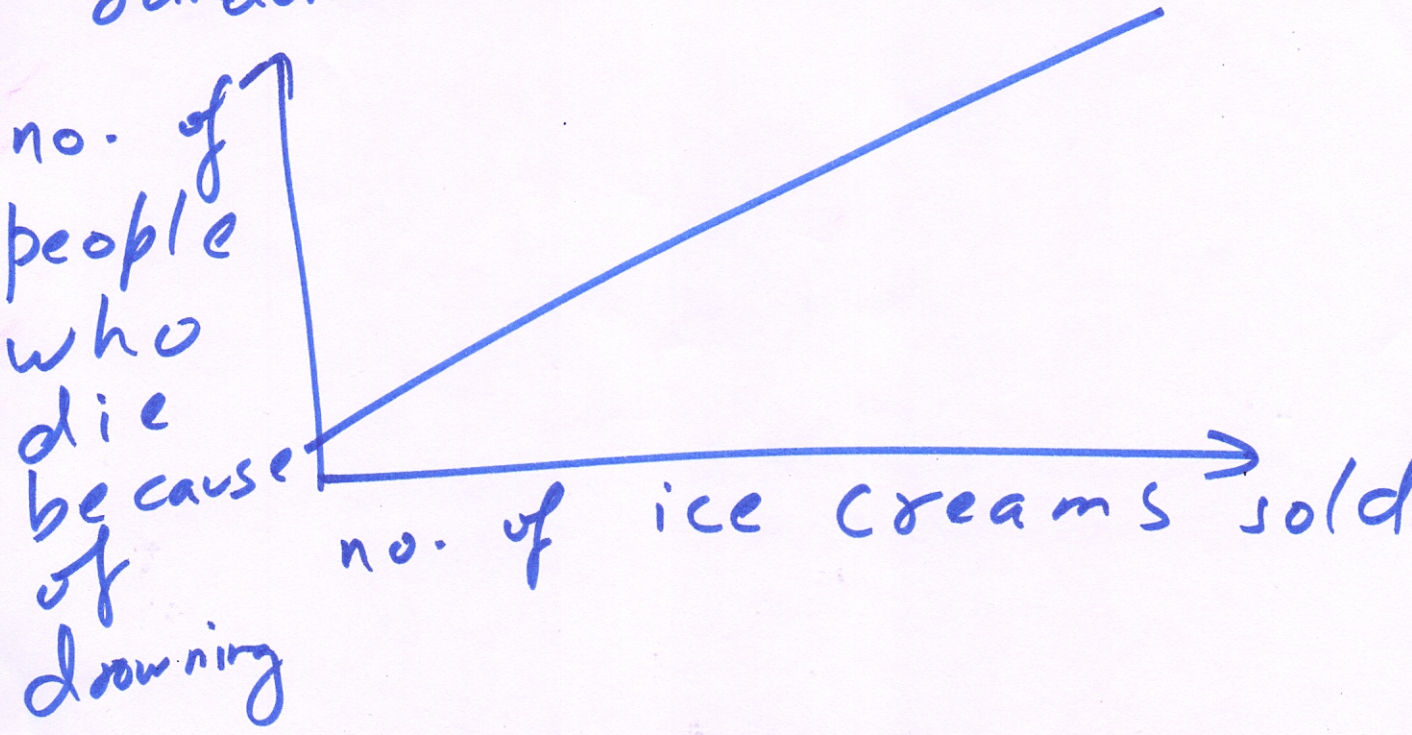
Recap

Exponential random variables

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Functions of random variables

Jointly distributed random variables



Joint distribution function

$$F(a,b) = P(X \le a, Y \le b)$$

$\downarrow$

AND

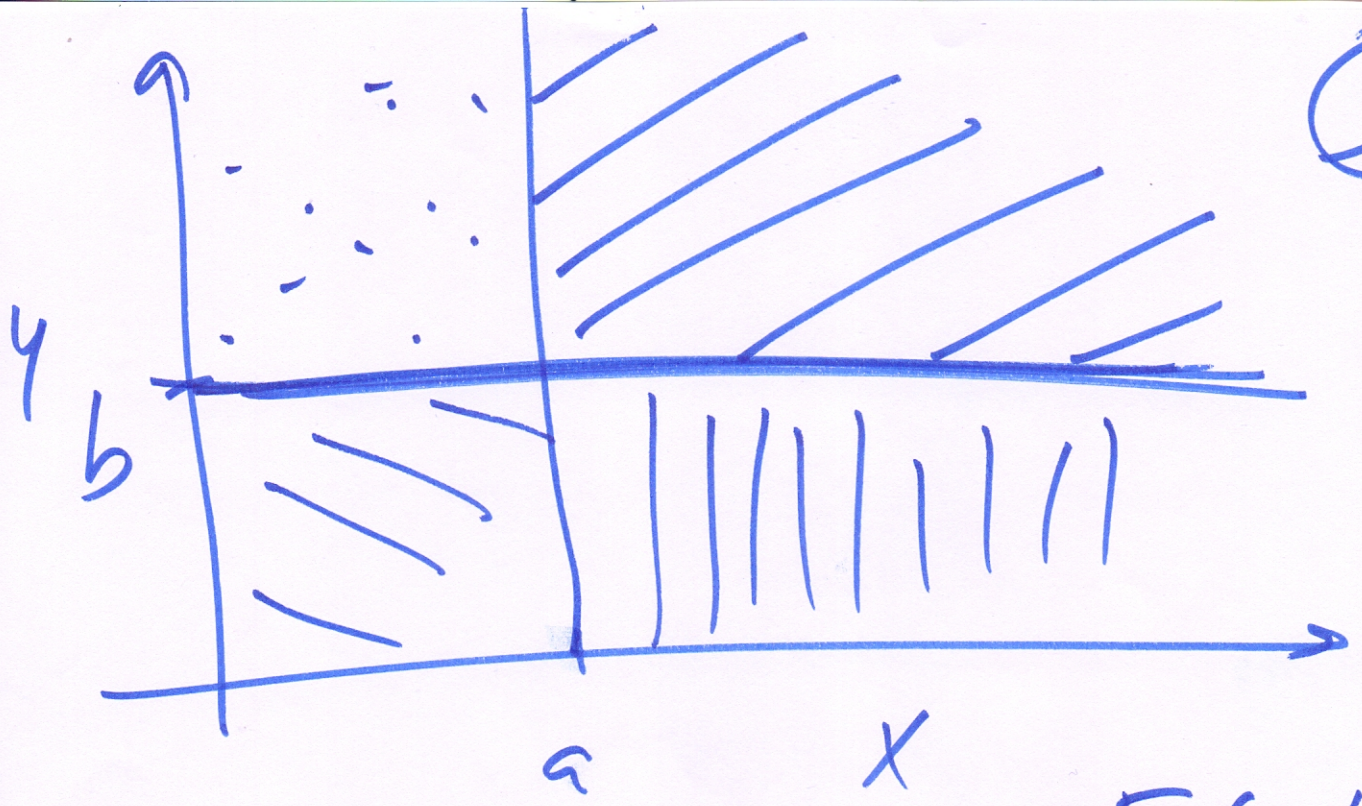
$$F_X(a) = P(X \le a) = P(X \le a, Y \le \infty)$$

$$= F(a, \infty)$$

$$F_Y(b) = F(\infty, b)$$

~~Ex~~

$P(X > a, Y > b)$ in terms of $F(a,b)$



$P(X>a, Y>b)$

$\downarrow$

AND

$=$

$1-\Big($ $F(a,b)$

$+F(\infty,b)-F(a,b)$

$+F(a,\infty)-F(a,b)\Big)$

$\rightarrow F(a,b)$

$F_X(a)$

$F_Y(b)$

$=$

$F(\infty,b)$

e.g. discrete

A bag has 3 Red,
4 white,
5 Blue balls.

You draw 3 balls at random.

X: no. of Red balls

Y: no. of white balls.

$X \in \{0, 1, 2, 3\}$

$Y \in \{0, 1, 2, 3\}$

$P(X = 1, Y = 1) = ?$

X \ Y	0	1	2	3	(5)
0	10	40	30	4	84
1	30	60	18	0	108
2	15	12	0	0	27
3	1	0	0	0	1
4.28	56	112	48	4	220

marginal for Y

marginal for X

denominator $= \binom{12}{3}$

$= \frac{12 \cdot 11 \cdot 10}{6} = 220$

eg: You have a circle of radius R. Choose a point at random within the circle uniformly. Center is origin, point (x,y)

$$f(x,y) = \begin{cases} C, & x^2+y^2<R^2 \\ 0, & \text{otherwise} \end{cases}$$

i) what is C?

$$\iint_{x^2+y^2<R^2} f(x,y)\,dx\,dy = 1$$

$$C \iint_{x^2+y^2<R^2} dx\,dy = 1$$

$$\Rightarrow C = \frac{1}{\pi R^2}$$

$$f(x,y) = \begin{cases} \frac{1}{\pi R^2}, & x^2+y^2<R^2 \\ 0, & \text{otherwise.} \end{cases}$$

ii) what is the marginal density function for X & Y?

$f_X(x) = ?$ $\quad f_Y(y) = ?$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

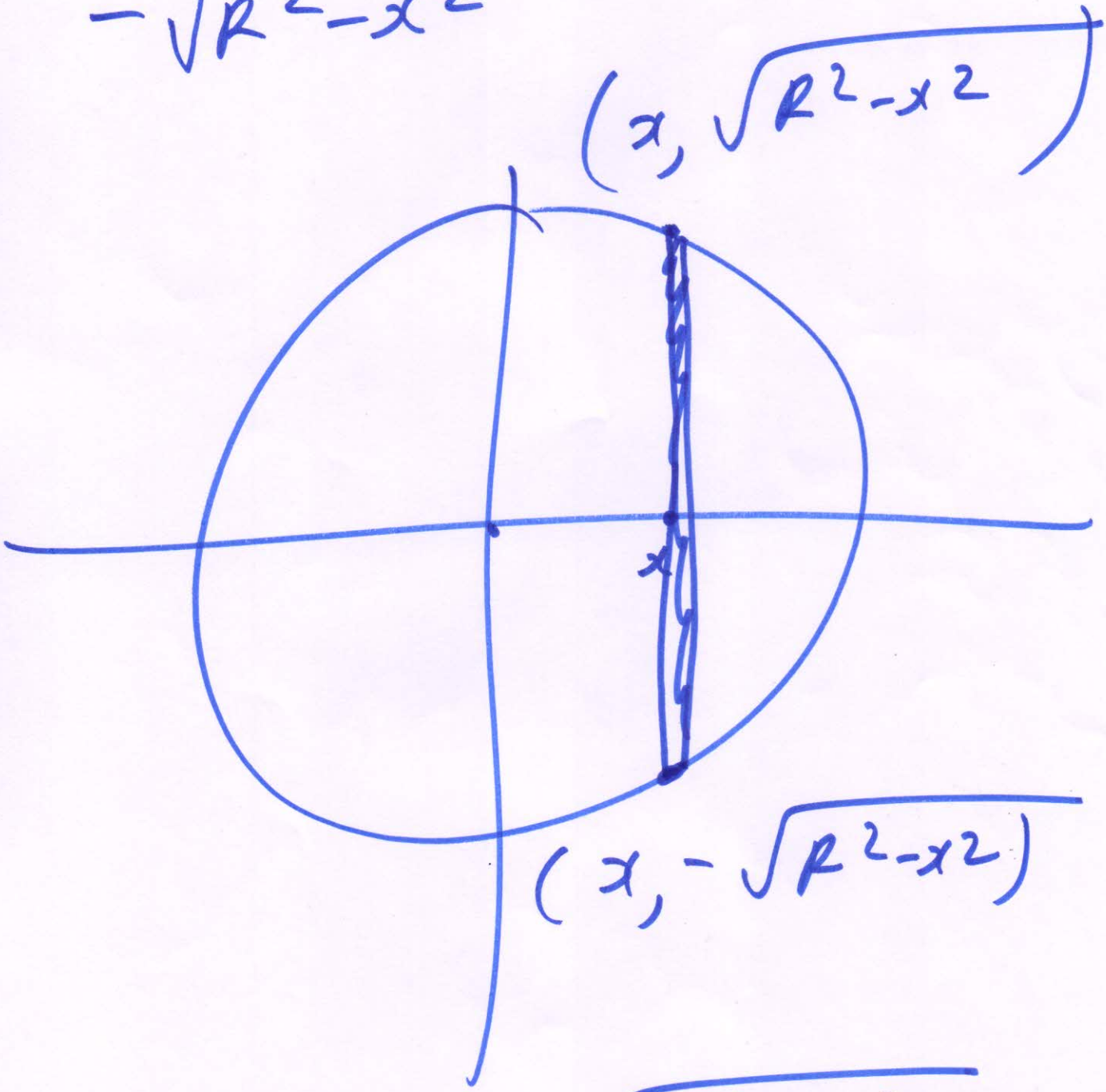
(1)

$$P(X=0) = P(X=0, Y=0) + P(X=0, Y=1) + P(X=0, Y=2) + P(X=0, Y=3)$$

$$\int_{-\infty}^{\infty} f(x,y)\, dy = f_X(x)$$

$$\frac{1}{\pi R^2} \int_{-\infty}^{\infty} dy$$

$$\frac{1}{\pi R^2} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dy$$



$$f_X(x) = \frac{2\sqrt{R^2-x^2}}{\pi R^2}$$

$$f_y(y) = \frac{2\sqrt{R^2 - y^2}}{\pi R^2}$$

iii) D is distance of this point (x, y) from the center of the circle.

what is $P(D < a) = ?$

$$P(\sqrt{x^2+y^2} < a)$$

$$= P(x^2 + y^2 < a^2)$$

$$= \iint_{x^2+y^2<a^2} f(x,y)\, dx\, dy$$

$$= \frac{\pi a^2}{\pi R^2}$$

iv) what is $\underline{E[D]}$? (11)

$$P(D < a) = \frac{a^2}{R^2}$$

$$f_D(a) = \frac{d}{da}\left(\frac{a^2}{R^2}\right) = \frac{2a}{R^2}$$

$$E(D) = \int D \cdot f_D(d)\, dd$$

$$= \int_0^R a \cdot f_D(a)\, da$$

$$= \int_0^R a \cdot \frac{2a}{R^2}\, da = \text{H.W.}$$