International Institute of Information Technology, Hyderabad

(Deemed to be University)

Linear Algebra (MCS 2) Monsoon-2023

Assignment 1

Due Date: 14, Nov, 2023

Question (1) [5 Marks] Let A and B be $m \times n$ and $n \times m$ matrices. respectively. For $n < m$, prove or disprove that AB is an invertible matrix.

Question (2) [5 Marks] Let W_1 and W_2 are two subspaces of a vector space V over a field F such that $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$. Prove that for each vector x in V, there are unique vectors $x_1 \in W_1$ and $x_2 \in W_2$ such that $x = x_1 + x_2$.

Question (3) [10 Marks] Let S be the set of all 2×2 matrices over a field F. Answer the following.

(a) **[3 Marks]** Is S a vector space over F with the operations being usual matrix addition and multiplication of a matrix by a scalar?

(b) **[4 Marks]** What is the dimension of the vector space F? Does the dimension change when we change the field from $F = \mathbb{R}$ to $F = \mathbb{C}$?

(c) **[3 Marks]** Write an explicit basis for the vector space S.

Question (4) [10 Marks] Let V be a vector space of all 2×2 matrices over a field F. Let W_1 be the set of matrices of the form

$$\begin{pmatrix} x & -x \\ y & z \end{pmatrix},$$

and W_2 be the set of matrices of the form

$$\begin{pmatrix} a & b \\ -a & c \end{pmatrix},$$

where $a, b, c, x, y, z \in F$. Answer the following:

(a) **[4 Marks]** Prove that W_1 and W_2 are subspaces of V.

(b) **[6 Marks]** Find the dimension of W_1, W_2, $W_1 + W_2$, $W_1 \cap W_2$.

Question (5) [12 Marks] Let W be a subspace of $\mathbb{C}^3$ spanned by $\alpha_1 = (1, 0, i)$ and $\alpha_1 = (1 + i, 1, -1)$.

(a) **[4 Marks]** Show that α_1 and α_2 form a basis for W.

(b) **[4 Marks]** Show that $\beta_1 = (1, 1, 0)$ and $\beta_2 = (1, i, 1 + i)$ are in W and form another basis for W.

(c) **[4 Marks]** What are coordinates of α_1 and α_2 in the ordered basis $\{\beta_1, \beta_2\}$?

Question (6) [8 Marks] Let m, n be positive integers and F be a field. Suppose W is a subspace of F^n and $\dim(\mathrm{W}) \leq \mathrm{m}$. Then there is precisely one $m \times n$ row-reduced echelon matrix over F which has W as its row space.