Q1. A Bag contains 15 Balls distinguishable only by their colors, 10 are Blue and 5 are Red. I Reach into the Bag with Both hands and pull out 2 Balls (one with each hand) and Record their colors.

Q1. what is the Random phenomena?

Q2. what is the sample space?

Q3. Express the event that the Ball in my left hand is red as a subset of the sample space.

(a) Reaching the Bag and picking 2 Balls is the Random phenomena here.

(b) sample space is set of all possible outcomes so 105 possible outcomes.

Q2. Three unbiased coins are tossed. what is the probability of getting at most 2 heads?

sol: The possible outcomes are —

HHH   HTH
HTT   THH
THT   TTT
TTH
HHT

$\dfrac{7}{8}$ is the probability of getting at most 2 Heads when 3 coins are tossed.

Q3. Throw 2 dice. A and B events are defined.

A = {max is 2}, B = {min is 2}. Are A and B independent?

Sol: Two events are independent when

$$P(A \cap B) = P(A) * P(B)$$

Here, $P(A) = 4/36 = 1/9$

$P(B) = 5/6$

$P(A \cap B) = 1/36$

clearly,

$$\frac{1}{9} \times \frac{5}{6} \neq \frac{1}{36}$$

So we conclude that A and B are not independent.

Q4. You call 2 uber and 3 ola cars. If the time that each takes to reach you is IID, what is the probability that uber arrives first?
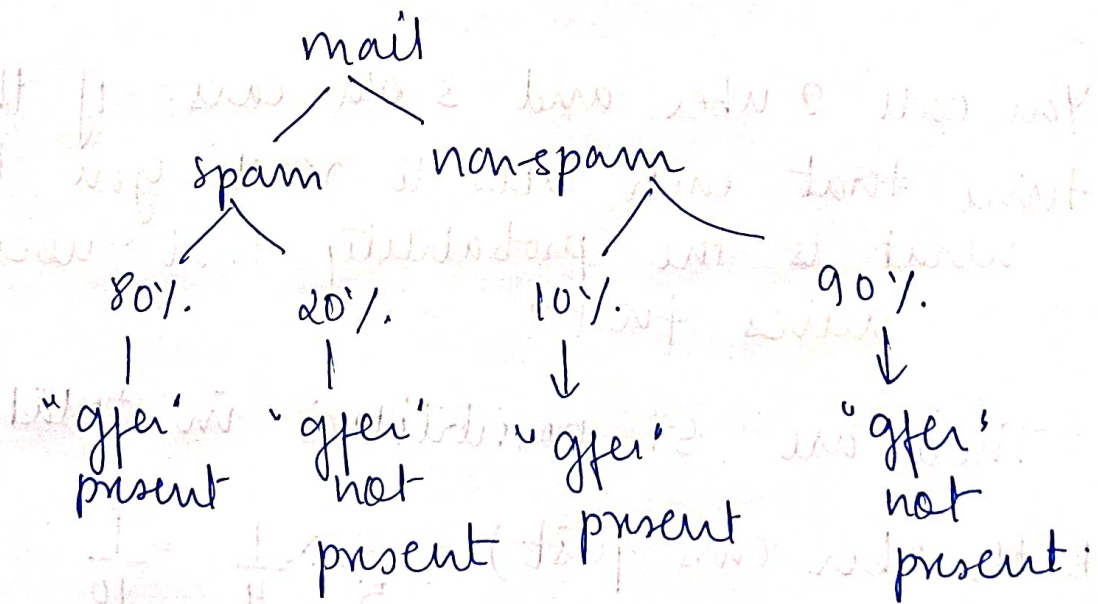
Sol: There are 5! possibilities in total.

$$P(\text{Both uber cars first}) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$$

$$P\left(\begin{array}{c}\text{1 uber then other uber and} \\ \text{then ola}\end{array}\right) = \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3}.$$

so, combined probability $= \dfrac{1}{10} + \dfrac{1}{10}$

$$= \dfrac{1}{5}.$$

Q5. By using NLP, I can detect spam e-mails in my inbox. Assume that the word "offer" occurs 80% of the spam messages in my a/c. Also let's assume "offer" occurs in 10% of my desired e-mails. If 30% of the received e-mail are considered as a spam, and I will receive a new message which contains "offer". what is the probability that it is a spam?

mail
/   \
spam   non-spam
/ \   /   \
80% 20%   10%   90%
| | ↓   ↓
"offer" "offer" "offer" "offer"
present not   present not
         present   present

	Spam	ham	Total
"offer"	240	70	310
no "offer"	60	630	690
Total	300	700	1000

Probability of new spam message having "offer" word is

$$\frac{240}{1000} = 0.24.$$

Q7. A salesperson from an automobile firm XYZ believes that the probability of making a sale is 38%. If he talks to 5 customer on a particular day, what is the probability that he will make exactly 2 sales.

using Binomial probability formula is

$$C(n, x) \times p^x \times (1-p)^{(n-x)}.$$

here,

$n = 5$

$x = 2$

$p = 0.38$

$$C(n,x) \times p^x \times (1-p)^{(n-x)}$$

$$C(5,2) \times (0.38)^2 \times (1-0.38)^{5-2}$$

$$\frac{5!}{2!\,(5-2)!} \times 0.1444 \times \overset{0.238}{0.3294}$$

$$10 \times 0.1444 \times \cancel{0.3294}\ 0.238$$

$$\cancel{0.476} = 0.344.$$

Q8. A machine produces items of which 1% at Random are defective. How many items can be packed in a box while keeping the chance of one or more defectives in the box to be no more than 0.5? what are the expected value and standard deviation of the number of defectives in a Box of that size?

sol: expected value $= n \times p$

$$p = \frac{1}{100} = 0.01.$$

Let's calculate n,

$$P(d \geq 1) \leq 0.5$$

$$1 - P(x = 0) \leq 0.5$$

$$1 - (1-p)^n \leq 0.5$$

$$1 - 0.5 \leq (1-p)^n$$

$$\frac{1}{2} \leq (1-p)^n$$

$$\log\left(\frac{1}{2}\right) \leq \log\left[(1-p)^n\right]$$

$$\log\left(\frac{1}{2}\right) \leq n \times \log(1-p)$$

$$n \geq \frac{\log \frac{1}{2}}{\log(1-0.01)}$$

$$n \geq \frac{\log \frac{1}{2}}{\log 0.99}$$

$$n \geq 68.5$$

$$n = 69$$

so, expected value $= 0.69.$