APPENDIX B

SAMURAI Analysis Technique

The analysis uses an incremental form of the variational cost function that avoids the inversion of the background error covariance matrix by using a control variable $\hat{\mathbf{x}}$, similar to the forms in Barker et al. (2004, Eq. 2) and Gao et al. (2004, Eq. 7):

$$J(\hat{\mathbf{x}}) = \frac{1}{2}\hat{\mathbf{x}}^T\hat{\mathbf{x}} + \frac{1}{2}(\mathbf{HC}\hat{\mathbf{x}} - d)^T\mathbf{R}^{-1}(\mathbf{HC}\hat{\mathbf{x}} - d) \tag{B1}$$

where $\mathbf{H}$ denotes the linearized observation operator, $\mathbf{C}$ denotes the square root of the background error covariance matrix, $\mathbf{R}$ denotes the observation error covariance matrix, and $d \equiv \mathbf{y} - h(\mathbf{x}_b)$ denotes the difference between the observations $\mathbf{y}$ and the nonlinear observation operator applied to the background state estimate $h(\mathbf{x}_b)$. In the current study, h and $\mathbf{H}$ are equivalent. The cost function is minimized using a conjugate gradient algorithm (Polak 1971; Press et al. 2002) to find the atmospheric state where the gradient with respect to $\hat{\mathbf{x}}$ is 0:

$$\nabla J(\hat{\mathbf{x}}) = (\mathbf{I} + \mathbf{C}^T\mathbf{H}^T\mathbf{R}^{-1}\mathbf{HC})\hat{\mathbf{x}} - \mathbf{C}^T\mathbf{H}^T\mathbf{R}^{-1}d \tag{B2}$$

We can express the transform from the control variable to an analysis increment as an operator sequence $\delta\mathbf{x} = \mathbf{C}\hat{\mathbf{x}} = \mathbf{SDF}\hat{\mathbf{x}}$. The $\mathbf{SDF}$ matrix transforms represent the cubic B-spline transform, standard deviation of the background error, and recursive filter operators, respectively. For brevity, only the one-dimensional transform for the radial direction is illustrated here. An additional transformation in the z direction follows. The spline transform

$\mathbf{S}$ is given by:

$$
\begin{aligned}
\mathbf{S} &= (\mathbf{P}+\mathbf{Q})^{-1} \\
\mathbf{P} &= [p_{mm'}]^T, \qquad p_{mm'} = \int_D \phi_m(r)\phi_{m'}(r)dr \\
\mathbf{Q} &= [q_{mm'}]^T, \qquad q_{mm'} = \int_D \epsilon_q(r)\phi_m'''(r)\phi_{m'}'''(r)dr \\
\phi_m(r) &= \Phi\left(\frac{r-r_m}{\Delta r}\right), \quad \text{for } m \in M \text{ and } r \in D
\end{aligned}
\tag{B3}
$$

where Φ is the cubic B-spline given by:

$$
\begin{aligned}
\frac{1}{6}(1-|\xi|)^3 - \frac{2}{3}(1-|\xi|)^3 \quad &\text{if} \quad 1 \geq |\xi| \geq 0 \\
\frac{1}{6}(2-|\xi|)^3 \quad &\text{if} \quad 2 \geq |\xi| \geq 1 \\
0 \quad &\text{if} \quad |\xi| \geq 2
\end{aligned}
\tag{B4}
$$

Note that $\mathbf{P}$ becomes the identity matrix for orthogonal basis functions, but is required for computing the cubic B-Spline coefficients. Here $\mathbf{Q}$ is a third derivative constraint to reduce Gibb's oscillations in the spline transform (Ooyama 2002), with ϵ_q a specified effective spatial filter cutoff length. The resulting increment or analysis can then be evaluated at any point in the physical domain through the inner product of the basis functions and the spline coefficients. For example, for the radial wind

$$
u(r) = \boldsymbol{\phi}^T\mathbf{x} + \tilde{\boldsymbol{\phi}}^T\tilde{\mathbf{x}} \tag{B5}
$$

where the second term on the right hand side represents imposed boundary conditions (Ooyama 2002).

For the current study, the background error correlations were assumed to be Gaussian and isotropic, and were calculated using an efficient recursive filter operator that replicates the effects of this correlation (Purser et al. 2003). The operator combination $\mathbf{DF}$ is the application of the background error covariance matrix, where $\mathbf{D}$ is the standard deviation of the background errors and $\mathbf{F}$ is the recursive filter. The standard deviation of the background errors were purposefully set very high given no prior knowledge of background state other than the objective analysis from BM08. A large background error standard deviation has the detrimental side effect of making the spline analysis unconstrained in data-poor regions.The recursive filter length scale acts as both an effective distance for the influence of the observations and as a spatial filter. A large length scale helps spread the information provided by an observation across data gaps, but also removes fine-scale detail from the analysis. Sensitivity tests indicated that a 6Δ background error length scale (relative to the grid scale)was a good compromise between smoothing and data density constraints, and was adequate for the current study.

The control variable state vector is constructed in axisymmetric cylindrical space and is given by $\mathbf{x} = \{\rho r v, \psi, S', q', \rho_a'\}^T$, where ρ denotes the moist density including water vapor, ψ denotes the transverse streamfunction, S denotes the moist static energy ($c_p T + Lq + gz$), ρ_a denotes the dry air density, primes represent departures from a static, background reference state, and the remaining symbols are as defined in Section 2a. The mean tropical sounding from Jordan (1958) was used as the reference state in this study. Radial and vertical momentum were recovered from the streamfunction using $\rho u = -\partial\psi/\partial z$ and $\rho w = \partial\psi/\partial r$. The vertical and radial resolution of the analyses used in the study were 100 m and 1 km, respectively.