Homework 2

Group 7

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Problem 2

Since all houses are different in design, we have a permutation problem. House 1 can be placed in any of the 9 lots, House 2 in the remaining 8 lots and so on.
Therefore *n1 = 9, n2 = 8, n3 = 7* etc and by multiplication rule we get,

Total number of ways = n! = 9! = 362880

Another approach for the same, we can place any 6 houses from the 9 houses (${}_9P_6$) on one side and the remaining 3 houses on the other (${}_3P_3$).
By multiplication rule,

*Number of ways = 9P6 * 3P3*

$$= \frac{9!}{3!} * \frac{3!}{0!}$$

= 9!

= 362880

Problem 3

Class size = 60. Assuming a non-leap year, we have 365 days in a year. The 1st student can have a birthday on any day from 365 days, 2nd student can have it on any day from the remaining 364 days and so on. This is a permutation problem as we are arranging birth dates.

Thus, total number of ways no 2 students will have birthdays on the same day = ${}_{365}P_{60}$

$$= \frac{365!}{305!\ 60!}$$

= 3.211830504E+151

Problem 4

Let
Event A : Customer invests in tax-free bonds
Event B : Customer invests in mutual funds

Given:
$P(A) = 0.6$
$P(B) = 0.3$
$P(A \cap B) = 0.15$

(a) Probability that customer will invest in either tax-free bonds or mutual funds = $P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ------ Additive rule
$= 0.6 + 0.3 - 0.15$
$= 0.75$
(b) Probability that customer will invest in neither tax-free bonds nor mutual funds = $P(A' \cap B')$

$P(A' \cap B') = P((A \cup B)')$ ------- *De Morgan's law*
$= 1 - P(A \cup B)$
$= 1 - 0.75$
$= 0.25$

Problem 5

Total number of cards in deck = 52
Number of cards in poker hand = 5

Total number of ways of selecting 5 cards from 52 = $\binom{52}{5}$

(a) Probability of holding 3 aces

Number of ways of selecting 3 aces from 4 aces = $\binom{4}{3}$

Number of ways of selecting remaining 2 cards = $\binom{48}{2}$

By multiplication rule, number of ways of selecting these 5 cards = $\binom{4}{3}\binom{48}{2}$

Therefore *P(3 aces)* = $\dfrac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}}$

$= \dfrac{4 * 1128}{2598960}$

$= \dfrac{94}{54145}$

= *0.001736079*

(b) Probability of 4 hearts and 1 club

Number of ways of selecting 4 hearts from 13 hearts = $\binom{13}{4}$

Number of ways of selecting 1 club from 13 clubs = $\binom{13}{1}$

By multiplication rule, number of ways of selecting these 5 cards = $\binom{13}{4}\binom{13}{1}$

Therefore *P(4 hearts and 1 club)* = $\dfrac{\binom{13}{4}\binom{13}{1}}{\binom{52}{5}}$

$= \dfrac{715 * 13}{2598960}$

$= \dfrac{143}{39984}$

= *0.003576431*

Problem 6

Let,

Event A: Has Canadian License plate
Event B: Is a camper

Given:

P(A) = 0.12
P(B) = 0.28
P(Camper with Canadian License) = P(A ∩ B) = 0.09

(a) Probability that a camper entering the Luray Caverns has Canadian license plate

$= P(A \mid B)$

$= P(A \cap B) / P(B)$

$= \frac{0.09}{0.28}$

$= \frac{9}{28}$

$= 0.321$

(b) Probability that a vehicle with Canadian license plates entering the Luray Caverns is a camper $= P(B \mid A)$

$= P(A \cap B) / P(A)$ -------- *Conditional probability rule*

$= 0.09 / 0.12$

$= 0.75$

(c) Probability that a vehicle entering the Luray Caverns does not have Canadian plates or is not a camper

$= P(A' \cup B')$

$= P((A \cap B)')$ --------- *De Morgan's law*

$= 1 - P(A \cap B)$

$= 1 - 0.09$

$= 0.91$

Problem 7

Let,

Event L: Customer purchases Latex paint
Event R: Customer purchases Rollers
Event S: Customer purchases Semigloss paint

Given:

$P(L) = 0.75$

$P(S) = 1 - P(L) = 0.25$

P(Purchasing rollers given they purchase latex paint) $= P(R \mid L) = 0.6$

P(Purchasing rollers given they purchase semigloss paint) $= P(R \mid S) = 0.3$

To find:

$P(L \mid R)$

By Baye's theorem,

$$P(L \mid R) = \frac{P(R \mid L)\, P(L)}{P(R \mid L)\, P(L) + P(R \mid S)\, P(S)}$$

$$= \frac{0.6 * 0.75}{0.6 * 0.75 + 0.3 * 0.25}$$

$= 0.857$

Problem 8

Given probability density function

$$f(x) \;= \frac{2}{5},\;\; 23.75 \leq x \leq 26.25$$
$$0\;,\;\; \text{elsewhere}$$

(a) Valid density function

1] For $f(x)$ to be a valid density function, $f(x) \geq 0$ for all $x \in R$

We see $f(x) = \frac{2}{5}$ and 0 for all x, thus $f(x) \geq 0$ for all $x \in R$

2] For $f(x)$ to be a valid density function, it should integrate to 1 from $-\infty$ to ∞

$$\int_{-\infty}^{\infty} f(x) \;=\; \int_{-\infty}^{23.75} 0\, dx \;+\; \int_{23.75}^{26.25} \frac{2}{5} dx \;+\; \int_{26.25}^{\infty} 0\, dx$$

$$= \;0 \;+\; \int_{23.75}^{26.25} \frac{2}{5} dx \;+\; 0$$

$$= \frac{2}{5} x \Big|_{23.75}^{26.25}$$

$$= \frac{2}{5}(26.25 - 23.75)$$

$$= \frac{2}{5} * 2.5$$

$= 1$

Hence, $f(x)$ is a valid density function.

(b) Probability that weight is smaller than 24 ounces

Here, we have to calculate $P(X < 24) = \int_{-\infty}^{24} f(x)\,dx$

$$\int_{-\infty}^{24} f(x)\,dx = \int_{-\infty}^{23.75} 0\,dx + \int_{23.75}^{24} \frac{2}{5}dx$$

$$= 0 + \int_{23.75}^{24} \frac{2}{5}dx$$

$$= \frac{2}{5}x\Big|_{23.75}^{24}$$

$$= \frac{2}{5}(24 - 23.75)$$

$$= \frac{2}{5} * 0.25$$

$$= 0.1$$

Hence, probability that weight is smaller than 4 ounces is 0.1

(c) Probability that weight exceeds 26 ounces

Here we have to calculate $P(X > 26) = \int_{26}^{\infty} f(x)\,dx$

$$\int_{26}^{\infty} f(x)\,dx = \int_{26}^{26.25} \frac{2}{5}dx + \int_{26.25}^{\infty} 0\,dx$$

$$= \int_{26}^{26.25} \frac{2}{5}dx + 0$$

$$= \frac{2}{5}x\Big|_{26}^{26.25}$$

$$= \frac{2}{5}(26.25 - 26)$$

$$= \frac{2}{5} * 0.25$$

$$= 0.1$$

Thus, the probability that weight exceeds 26 ounces is 0.1, which is not extremely rare.