Homework 4

Group 7

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Problem 1

Number of delegates n = 9
The delegates can arrive by air, bus, automobile or train.
This can be modeled using multinomial distribution with
$p_1 = 0.4,\ p_2 = 0.2,\ p_3 = 0.3,\ p_4 = 0.1$ and
$x_1 = 3,\ x_2 = 3,\ x_3 = 1, x_4 = 2$

Probability of required event =
$f(x_1, x_2, x_3, x_4; p1, p2, p3, p4, n) = f(3, 3, 1, 2; 0.4, 0.2, 0.3, 0.1, 9)$

$$= \binom{n}{x_1, x_2, x_3, x_4} p_1^{x_1} * p_2^{x_2} * p_3^{x_3} * p_4^{x_4}$$

$$= \binom{9}{3, 3, 1, 2}\ 0.4^3 * 0.2^2 * 0.3^1 * 0.1^2$$

= *0.00774*

Problem 2

Probability that pipework failure is caused due to operator failure = 0.3.

a) Next 20 failures, n = 20
   This can be modeled by the binomial distribution $b(x; 20, 0.3)$
   Probability that at least 8 are due to operator failure = $P(X \geq 8)$
   = $1 - P(X \leq 7)$
   = 1 - 0.7722718
   = *0.2277282*
b) Probability that no more than 3 are due to operator failure = $P(X \leq 3)$
   = *0.1070868*
c) Probability that out of 20 failures, 5 are due to operator error is $b(5; 20, 30)$
   $P(X = 5) = 0.1788631$
   This probability value is reasonable and not very small. Thus the statement that 30% failures are due to operator failures seems acceptable. If it would have been very small,

like less than 0.05, we could've rejected it the way we reject a null hypothesis in hypothesis testing.

Problem 3

We can use the Hypergeometric distribution for this problem.
From 10 missiles, 4 are selected at random and 2 are defective.
Thus *N = 10, n = 4, k = 2*

a) Probability that all 4 will fire, here we use hypergeometric distribution and model non defectives as success event. Thus the distribution is $h(x;\ 10,\ 4,\ 8)$

$P(X\ =\ 4)$= $h(4;\ 10,\ 4,\ 8)$ = $\dfrac{\binom{8}{4}\binom{2}{0}}{\binom{10}{4}}$

= 0.333

b) Probability that at most 2 will not fire. Here we use hypergeometric distribution $h(x; 10,\ 4,\ 2)$ and consider defectives as the success event.

P(X ≤ 2) = $\sum_{x=0}^{2} h(x; 10, 4, 2)$

= $h(0; 10,\ 4,\ 2)$ + $h(1; 10,\ 4,\ 2)$ + $h(2; 10,\ 4,\ 2)$

= 0.333 + 0.5333 + 0.1333

= *1*

Problem 4

Aircrafts follow a Poisson process and $\mu\ =\ 6t$, and corresponding poisson distribution is $p(x; 6)$

a) Probability that exactly 4 aircrafts arrive in 1 hour = $P(X\ =\ 4;\ \mu t\ =\ 6)$= $\dfrac{e^{-6}6^4}{4!}$

= 0.1338526

b) Probability that at least 4 arrive in 1 hour = $P(X\ \geq\ 4;\ \mu t\ =\ 6)$= 1 - $P(X\ \leq\ 3;\ \mu t\ =\ 6)$

= 1 - 0.1512039

= 0.8487961

c) Working day is 12 hours. Thus, μt =6 * 12 = 72 and we use poisson distribution $p(x; 72)$
Probability that at least 75 small aircrafts arrive in working day =

$P(X\ \geq\ 75;\ \mu t\ =\ 72)\ =\ 1\ -\ P(X\ \leq\ 74;\ \mu t\ =\ 72)$

= 1 - $\sum_{0}^{74} p(x; 72)$ = 1 - 0.6226727

= 0.3773273

Problem 5

Average length of lye bread = μ = 30cm
Standard deviation of length of lye bread = σ= 2cm
Lengths are said to be normally distributed. Thus we use the normal distribution
$N(x;\ \mu\ =\ 30,\ \sigma\ =\ 2)$

a) Probability that length is longer than 31.7 cm = $P(X\ \geq\ 31.7,\ \mu\ =\ 30,\ \sigma\ =\ 2)$

   Converting to standard normal, $Z\ =\frac{x-30}{2}$

   $=\frac{31.7-30}{2}$= 0.85

   $P(X\ \geq\ 31.7,\ \mu\ =\ 30,\ \sigma\ =\ 2)$ = $P(Z\ \geq\ 0.85,\ \mu\ =\ 0,\ \sigma\ =\ 1)$

   = 1 - $P(Z\ \leq\ 0.85,\ \mu\ =\ 0,\ \sigma\ =\ 1)$

   Using the standard normal table,

   = 1 - 0.80234

   = *0.19766*

   Thus, *19.77%* loaves are longer than 31.7 centimeters.

b) Probability that length is between 29.3 and 33.5 cm =
   $P(29.3\ \leq\ X\ \leq\ 33.5;\ \mu\ =\ 30,\ \sigma\ =\ 2)$

   Converting to standard normal, $z_2\ =\ \frac{x_2-\mu}{\sigma}\ =\ \frac{29.3-30}{2}\ =\ -\ 0.35$

   $z_2\ =\ \frac{x_2-\mu}{\sigma}\ =\ \frac{33.5-30}{2}\ =\ 1.75$

   Thus, $P(29.3\ \leq\ X\ \leq\ 33.5;\ \mu\ =\ 30,\ \sigma\ =\ 2)$ =
   $P(-\ 0.35\ \leq\ Z\ \leq\ 1.75;\ \mu\ =\ 0,\ \sigma\ =\ 1)$

   From the standard normal table,

   = $P(Z\ \leq\ 1.75)\ -\ P(Z\ \leq -\ 0.35)$

   = 0.95994 - 0.36317

   = *0.59677*

   Thus, *59.67% lo*aves are between 29.3 and 33.5 cm

c) Probability that length is shorter than 25.5 cm = $P(X\ \leq\ 25.5;\ \mu\ =\ 30,\ \sigma\ =\ 2)$

   Converting to standard normal,

   $z\ =\frac{x-30}{2}=\frac{25.5-30}{2}=-\ 2.25$

   $P(X\ \leq\ 25.5;\ \mu\ =\ 30,\ \sigma\ =\ 2)$ = $P(Z\ \leq\ \ -\ 2.25;\ \mu\ =\ 0,\ \sigma\ =\ 1)$

   From standard normal table,

   = *0.01222*

   Thus, *1.22% l*oaves are shorter than 25.5 cm

Problem 6

Time required to repair heat pump X follows a gamma distribution with $\alpha = 2$ $\beta = 0.5$

a) Probability that at most 1 hour will be required = $P(X \leq 1)$

Gamma distribution = $\frac{1}{0.5^2\Gamma(2)} xe^{-2x} = 4xe^{-2x}$

$\int_0^1 4xe^{-2x}\,dx = (-2xe^{-2x} - e^{-2x})|_{x=0}^{x=1}$

$= [-2e^{-2} - e^{-2} + 1]$

$= [1 - 3e^{-2}]$

= *0.5939942*

b) Probability that at least 2 hours will be required = $P(X \geq 2)$

$= \int_2^\infty 4xe^{-2x}\,dx = (-2xe^{-2x} - e^{-2x})|_{x=2}^{x=\infty}$

$= (0 - 0 + 4e^{-4} + e^{-4}) = 5e^{-4}$

= *0.09157819*

Problem 7

Computer's response time follows an exponential distribution with $\beta = 3$seconds.

Thus exponential distribution is $\frac{1}{3}e^{-x/3}$, $x > 0$

a) Probability that response time exceeds 5 seconds = $P(X \geq 5)$

$= \int_5^\infty \frac{1}{3}e^{-x/3}dx = -e^{-x/3}|_{x=5}^{x=\infty}$

$= 0 + e^{-5/3}$= *0.1888756*

b) Probability that response time exceeds 10 seconds = $P(X \geq 10)$

$= \int_{10}^\infty \frac{1}{3}e^{-x/3}dx = -e^{-x/3}|_{x=10}^{x=\infty}$

$= 0 + e^{-10/3}$= *0.03567399*

Problem 8

Water usage follows lognormal distribution with $\mu = 5$ and $\sigma = 2$

To find probability that 50000 gallons of water is used for an hour

ln(X) follows a normal distribution, converting to standard normal $Z = \frac{ln(x)-5}{2}$

$P(X \geq 50000)=1 - P(X \leq 50000)$

= 1 - $\Phi(\frac{ln(50000)-5}{2})$, where Φ is the cumulative distribution function of the standard normal distribution

= 1 - $\Phi(\frac{ln(50000)-5}{2})$

= 1 - Φ(2.909889)

= 1 - 0.99813 ---- From standard normal table

= *0.00187*