Homework 2

Group 7

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# Problem 2

Since all houses are different in design, we have a permutation problem. House 1 can be placed in any of the 9 lots, House 2 in the remaining 8 lots and so on.

Therefore *n1 = 9, n2 = 8, n3 = 7* etc and by multiplication rule we get,

*Total number of ways =* *n!* = *9!* = *362880*

Another approach for the same, we can place any 6 houses from the 9 houses (*9P6*) on one side and the remaining 3 houses on the other (*3P3*).

By multiplication rule,

*Number of ways = 9P6 \* 3P3*

*= \**

*= 9!*

*= 362880*

# Problem 3

Class size = 60. Assuming a non-leap year, we have 365 days in a year. The 1st student can have a birthday on any day from 365 days, 2nd student can have it on any day from the remaining 364 days and so on. This is a permutation problem as we are arranging birth dates.

Thus, total number of ways no 2 students will have birthdays on the same day = *365P60*

=

*= 3.211830504𝐸+151*

# Problem 4

Let

Event A : Customer invests in tax-free bonds

Event B : Customer invests in mutual funds

Given:

*P(A)* = *0.6*

*P(B)* = *0.3*

*P(A ∩ B)* = *0.15*

(a) Probability that customer will invest in either tax-free bonds or mutual funds = *P(A ⋃ B)*

*P(A ⋃ B)* *=* *P(A) + P(B) - P(A ∩ B)* ------ Additive rule

*= 0.6 + 0.3 - 0.15*

*= 0.75*

(b) Probability that customer will invest in neither tax-free bonds nor mutual funds = *P(A’ ∩ B’)*

*P(A’ ∩ B’) = P((A ⋃ B)’) ------- De Morgan’s law*

*= 1 - P(A ⋃ B)*

*= 1 - 0.75*

*= 0.25*

# Problem 5

Total number of cards in deck = 52

Number of cards in poker hand = 5

Total number of ways of selecting 5 cards from 52 =

1. Probability of holding 3 aces

Number of ways of selecting 3 aces from 4 aces =

Number of ways of selecting remaining 2 cards =

By multiplication rule, number of ways of selecting these 5 cards =

Therefore *P(3 aces)* =

=

=

= *0.001736079*

1. Probability of 4 hearts and 1 club

Number of ways of selecting 4 hearts from 13 hearts =

Number of ways of selecting 1 club from 13 clubs =

By multiplication rule, number of ways of selecting these 5 cards =

Therefore *P(4 hearts and 1 club)* =

=

=

= *0.003576431*

# Problem 6

Let,

Event A: Has Canadian License plate

Event B: Is a camper

Given:

*P(A)* = 0.12

*P(B)* = 0.28

*P(Camper with Canadian License)* = *P(A ∩ B)* = *0.09*

(a) Probability that a camper entering the Luray Caverns has Canadian license plate

= *P(A | B)*

= *P(A ∩ B) / P (B)*

=

*=*

*= 0.321*

(b) Probability that a vehicle with Canadian license plates entering the Luray Caverns is a camper *= P(B | A)*

*= P(A ∩ B) / P (A) -------- Conditional probability rule*

*= 0.09 / 0.12*

*= 0.75*

(c) Probability that a vehicle entering the Luray Caverns does not have Canadian plates or is not a camper

*= P(A’ ⋃ B’)*

*= P((A ∩ B)’) --------- De Morgan’s law*

*= 1 - P(A ∩ B)*

*= 1 - 0.09*

*= 0.91*

# Problem 7

Let,

Event L: Customer purchases Latex paint

Event R: Customer purchases Rollers

Event S: Customer purchases Semigloss paint

Given:

*P(L) = 0.75*

*P(S) = 1 - P(L) = 0.25*

*P(Purchasing rollers given they purchase latex paint) = P(R | L) = 0.6*

*P(Purchasing rollers given they purchase semigloss paint) = P(R | S) = 0.3*

To find:

*P(L | R)*

By Baye’s theorem,

*P(L | R) =*

*=*

*= 0.857*

# Problem 8

Given probability density function

, 23.75 26.25

0 , elsewhere

1. Valid density function

For to be a valid density function, it should integrate to 1 from to

*=*

*= 23.75 26.25*

*= (26.25 - 23.75)*

*= \* 2.5*

*= 1*

Hence, is a valid density function.

1. Probability that weight is smaller than 24 ounces

Here, we have to calculate

*=*

*= 23.75 24*

*= (24 - 23.75)*

*= \* 0.25*

*= 0.1*

Hence, probability that weight is smaller than 4 ounces is 0.1

1. Probability that weight exceeds 26 ounces

Here we have to calculate

*=*

*= 26 26.25*

*= (26.25 - 26)*

*= \* 0.25*

*= 0.1*

Thus, the probability that weight exceeds 26 ounces is 0.1, which is not extremely rare.