

Program Structures and Algorithms  
Spring 2023(SEC 01)

NAME: Shivani Datar  
NUID: 002772160

**Task:**

1.
  - a. Implement height-weighted Quick Union with Path Compression. For this, you will flesh out the class UF HWQUPC. All you have to do is to fill in the sections marked with // TO BE IMPLEMENTED ... // ...END IMPLEMENTATION.
  - b. Check that the unit tests for this class all work. You must show "green" test results in your submission (screenshot is OK).
2. Using your implementation of UF HWQUPC, develop a UF ("union-find") client that takes an integer value  $n$  from the command line to determine the number of "sites." Then generates random pairs of integers between 0 and  $n-1$ , calling `connected()` to determine if they are connected and `union()` if not. Loop until all sites are connected then print the number of connections generated. Package your program as a static method `count()` that takes  $n$  as the argument and returns the number of connections; and a `main()` that takes  $n$  from the command line, calls `count()` and prints the returned value. If you prefer, you can create a main program that doesn't require any input and runs the experiment for a fixed set of  $n$  values. Show evidence of your run(s).
3. Determine the relationship between the number of objects ( $n$ ) and the number of pairs ( $m$ ) generated to accomplish this (i.e. to reduce the number of components from  $n$  to 1). Justify your conclusion in terms of your observations and what you think might be going on.

**Relationship Conclusion:**

The relationship between the number of objects( $n$ ) and number of connections formed (Asked in step 2) is always  $n-1$ . As for all  $n$  nodes to be connected with each other with at least degree 1 is  $n-1$ . That is if we have  $n = 1000$  we can form 999 connections until all  $n$  components/nodes are connected with each other.

We are generating random pairs( $m$ ) between 0 to  $n-1$  and checking if they are connected with each other, until all the components are connected with each other. Since the generation of pairs is random in nature, I am generating ( $m$ ) for same  $n$  50 times and then taking the average of the pairs generated.

The relationship between ( $n$ ) and ( $m$ ) can be said as,  
$$m = c * n * \log n$$

Where  $c = 1.1$  approximately in this calculation and can differ machine to machine.

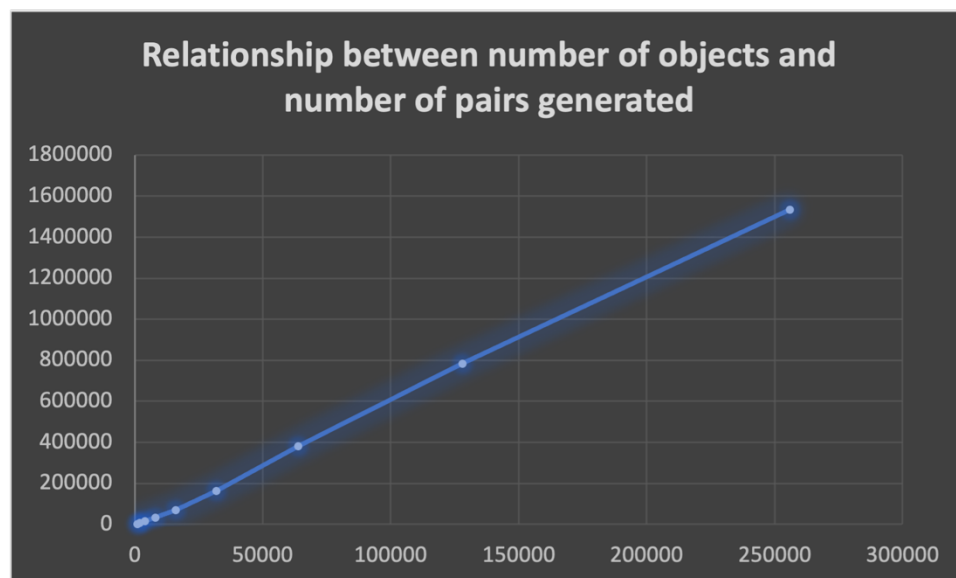
Therefore, we can say that,

$$m \propto n \log n$$

### Evidence to support that conclusion:

N(total number of objects)	M(total number of pairs generated)	$\log n$	$n \log n$	$m/n$	$c = m/n \log n$
1000	3228	3	3000	3.228	1.076
2000	7615	3.30103	6602.05999	3.8075	1.15342787
4000	16484	3.60205999	14408.24	4.121	1.14406756
8000	32687	3.90308999	31224.7199	4.085875	1.04683085
16000	70211	4.20411998	67265.9197	4.3881875	1.04378265
32000	164049	4.50514998	144164.799	5.12653125	1.13792688
64000	381878	4.80617997	307595.518	5.96684375	1.24149403
128000	784233	5.10720997	653722.876	6.12682031	1.19964136
256000	1535820	5.40823997	1384509.43	5.99929688	1.10928822

Graph :-



Code :-

Step 1.

- In find method, we check if the given node is connected to the parent root or not. And until its not connected we do the compression so that we can have a path compressed structure.

```

public int find(int p) {
    validate(p);
    int root = p;
    while (root != parent[root]) {
        if (this.pathCompression) {
            doPathCompression(root);
        }
        root = parent[root];
    }
    return root;
}

```

- b. Under union we call mergeComponents method. Here we first check if they are same, else we try to find which node has a smaller connected set of nodes, we point the smaller one to the taller connected set of nodes.

```

private void mergeComponents(int i, int j) {
    // FIXME make shorter root point to taller one
    if (i == j) {
        return;
    }
    if (height[i] < height[j]) {
        parent[i] = j;
        height[j] += height[i];
    } else {
        parent[j] = i;
        height[i] += height[j];
    }
    // END
}

```

- c. Under do path compression, we have to lower the size of the connected nodes, we do that by pointing a current node's child with the parent root of that node.

```

private void doPathCompression(int i) {
    // FIXME update parent to value of grandparent
    parent[i] = parent[parent[i]];
    // END
}

```

Step 2.

a. To have a UF Client I have added a main method in the same class, which takes in the number of objects and then calls the method count, and later on gives us number of connections formed and number of random pair generated till all the nodes/components are connected with each other.

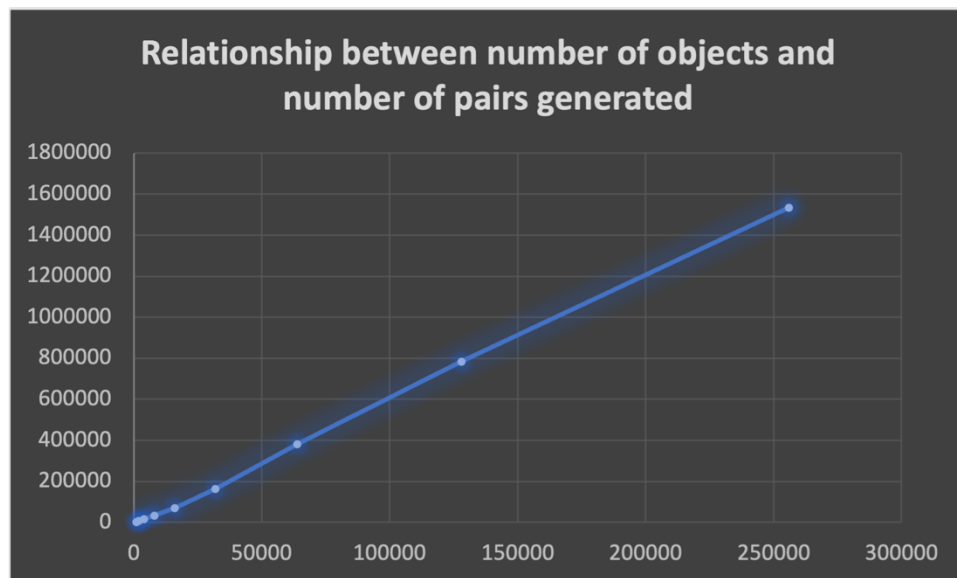
```
public static void main(String[] args) {
    {
        Scanner sc = new Scanner(System.in);
        System.out.println("Enter the number of objects");
        while(sc.hasNext()) {
            int N = sc.nextInt();
            int count[] = count(N);
            int total = 0;
            for(int i=0; i<50; i++) {
                total += count[1];
            }
            int avg = total/50;
            System.out.println("No. of objects = " + N + " with an avg pairs of " + avg + " and connections formed " + count[0]);
            System.out.println("Enter the number of objects");
        }
    }
}
```

b. Under the static count method, I am instantiating uf object of UF\_HWQUPC. Then I randomly generate pairs p and q until the total count of unconnected nodes/components is zero. The if condition checks if they are already connected, if not I increment the connections variable which gives us total number of connections. Either way, in the end the code will increment the variable pairs, which shows total number of pairs generated. And it will return an int array with both the values connections and number of pairs.

```
private static int[] count(int n) {
    UF_HWQUPC uf = new UF_HWQUPC(n);
    int connections = 0;
    int pairs = 0;
    Random r = new Random();
    while(uf.components() > 1) {
        int p = r.nextInt(n);
        int q = r.nextInt(n);
        if(!uf.connected(p, q)) {
            uf.union(p, q);
            connections += 1;
        }
        pairs += 1;
    }
    int[] connectCount = {connections, pairs};
    return connectCount;
}
```

### Graphical Representation:

N(total number of objects)	M(total number of pairs generated)	$\log n$	$n \log n$	$m/n$	$c = m/n \log n$
1000	3228	3	3000	3.228	1.076
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## Unit Test Screenshots:

