Linear algebra:

def: it deals with vectors and matrices and linear transformations.

Scalar

=> value
↳ magnitude of a point

$\boxed{x = \boxed{2}}$

Vector

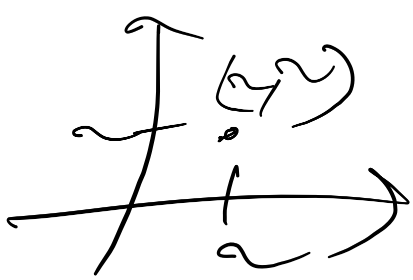
=> value (or) magnitude and its direction of a point

$x = 2\vec{i} + 2\vec{j}$

$(x, y) = (2, 2)$

x

y



matrices

↳ values in terms of a row and column and we can also find the direction of all those values.

$$
\begin{array}{c|c|c}
\hat{i} & \hat{j} & \hat{k} \\
\hline
1 & 6 & 8 \\
4 & 2 & 9 \\
5 & 7 & 3
\end{array}
$$

Transpose:

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \xrightarrow{\text{transpose}} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

orthogonal Matrix:

def: it is a square matrix which, when multiplied with their transpose matrix results in an identity matrix.

$$\boxed{A \cdot A^T = I}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \qquad A^T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} (-1 \times -1) & (0 \times 0) \\ (0 \times 0) & (1 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \boxed{A \cdot A^T = I}$$

$\Rightarrow$ What is the |orthogonal matrix|

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow -1 \rightarrow \boxed{\pm 1}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \qquad \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

det

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\alpha \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}$$

αI

$$|A| = (a \times d) - (c \times b)$$

$$\begin{bmatrix} 1 & 6 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$1\,(12 - (5))$

- → linear alg ✓
- → vector, scalar ✓
- → matrix ✓
- → transpose ✓
- → orthogonal ✓
- → det matrix ✓

Eigen value?

→ these are also known as characteristic roots of scalars associated with the system of linear equations......

$$A = \begin{pmatrix} 0 & -2 \\ 3 & 4 \end{pmatrix} \qquad \boxed{E\cdot V}$$

$$\circledast \quad |A - \lambda I| = 0 \rightarrow \boxed{E\cdot V}$$

$$\begin{pmatrix} 0 & -2 \\ 3 & 4 \end{pmatrix} - \lambda \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$= \begin{vmatrix} -\lambda & -2 \\ 0 & 4-\lambda \end{vmatrix}$$

$$-\lambda(4-\lambda) - (-2)(0) = 0$$

$$-4\lambda + \lambda^2 = 0$$

$\lambda^2 - 4\lambda = 0$

$\lambda(\lambda - 4) = 0 \Rightarrow$ $\boxed{\lambda = 0 \text{ (or) } 4}$

$\lambda = 0$, $\lambda - 4 = 0 \rightarrow 4$

for the given matrix, there are

two eigen values 0 and 4

$\Rightarrow$ how many EV does a 2×2 matrix can have?

Sol. 2 eigen values

Eigen vector:

$\Rightarrow$ vector representation of e·values

$$A = \begin{pmatrix} 1 & 4 \\ -4 & -7 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 4 \\ -4 & -7-\lambda \end{vmatrix}$$

$$(1-\lambda)(-7-\lambda) - 4(-4) = 0$$

$$\boxed{\lambda = -3, -3}$$

$\boxed{AX = \lambda X}$

$AX = -3X$ → matrix

$(A - \lambda I)X = 0$

$$\left(\begin{bmatrix} 1 & 4 \\ -4 & -7 \end{bmatrix}_{2\times 2} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$4x + 4y = 0$ (or) $\boxed{x + y = 0}$

Let, $\boxed{x = k}$

$k + y = 0$

$\boxed{y = -k}$

$$X = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Eigen vector for matrix A

⇒ what do E Values and E vectors tell us?

Sol. An eigen value is a number that tells us how much variance exists in the data in that direction.

E. vector is a number that tells us how spread of the data.

Transformation matrix :-

def: it is used to transform one vector into another vector by the process of matrix multiplication.

eg: Find the new vector formed for the vector $5\vec{i} + 6\vec{j}$, with the help of the transformation matrix

Sol.

$T = \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}$ $= \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}$

$A = 5\vec{i} + 4\vec{j}$

$\boxed{A = \begin{pmatrix} 5 \\ 4 \end{pmatrix}}$

let, the new matrix after transf... be $\boxed{B}$

$\boxed{TA = B}$

$\Rightarrow \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

$B = \begin{pmatrix} -2 \\ 13 \end{pmatrix}$

$\boxed{-2\vec{i} + 13\vec{j}}$

→ linear algebra