Conditional Probability

note: 0, 1, 2, 3, 4, 5 — probability always lies between 0 and 1

$P(A|B)$

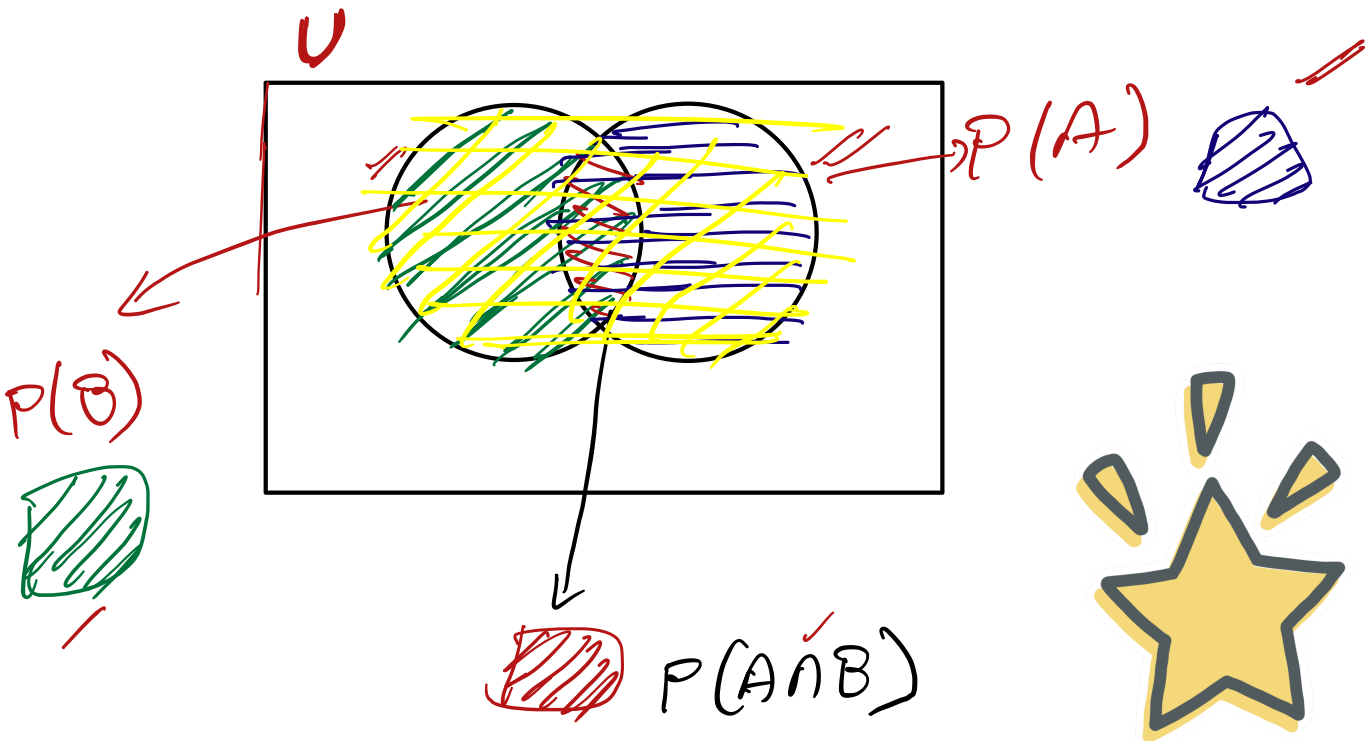
given ⇒ probability of (A) given (B)

AGENDA:

⇒ (event 'B' has (already occured))

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

DON'T FORGET



Q: The % of adults who are men & alcoholics is 2.25%. what is the probability of being an alcoholic, given being a man?

Sol: (1.) we use conditional probability

(2.) A = event of being an alcoholic
B = event of being a man

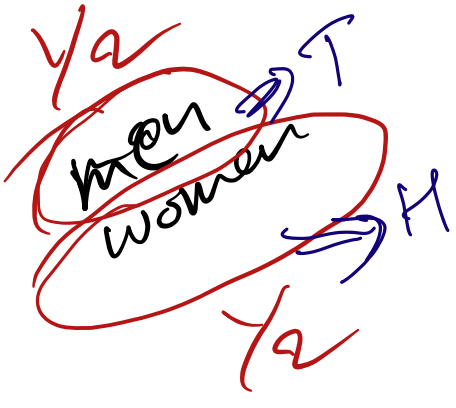
$P(A \cap B) = 0.0225$

probability of a man and alcoholic

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

!!

probability of being an alcoholic given a man



$$= \frac{0.0225}{0.5}$$

$$= 0.045$$

eg: What is the probability of two children being girls if we are told at least one is a girl?

Sol. $P(\text{2 girls} \mid \text{at least 1 girl})$

$$= \frac{P(\text{2 girls} \cap \text{at least 1 girl})}{P(\text{at least 1 girl})}$$

$$= \frac{1/4}{3/4} = \boxed{1/3}$$

eg: what is the probability of two girls given that oldest is a girl?

Sol.

$$P(2G \mid OG) = \frac{P(2G \cap OG)}{P(OG)}$$

$$= 1/4 \,/\, 1/2 = \boxed{1/2}$$

AGENDA: Qns:

Most IMP Qns

1. what is the value of $P(A \cap B)$ if $P(B)=0$

Sol. $P(A \cap B)$

↳ in this case

note: conditional probability of $P(A|B)$ is undefined when $P(B)=0$.

because if $P(B)=0$ it means that event 'B' never occurs so it does not make sense to talk about the probability of A given B.

WELL DONE

3. suppose if i toss 2 coins .....

Sol. Coin 1 = 1 Coin 2 = 0

1/4

H T H T

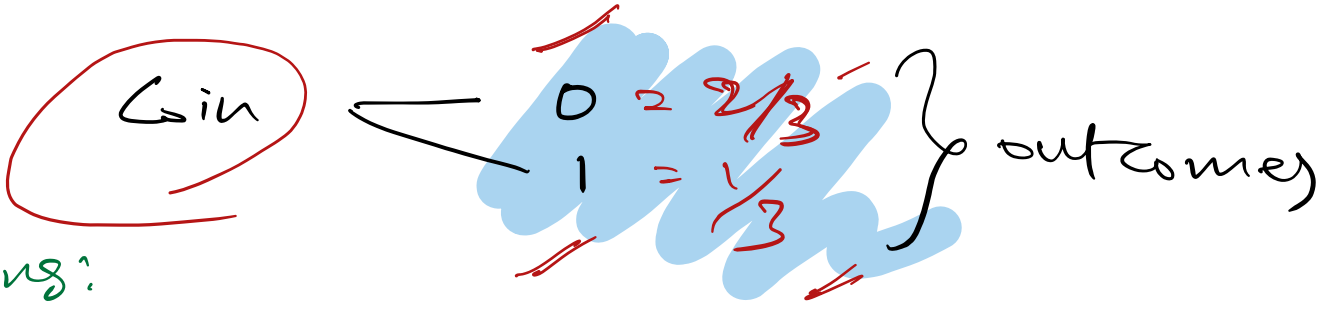
HH, HT, TH, TT

1 0, 1 0, 1 0, 1 0

= Samplespace

(e.) Suppose if I have one coin -----

Sol.



Reasoning:

↳ In a coin toss problem we have two outcomes

H — (1) T — (0)

↳ So, if we toss them, we will have equal probability for both such as,

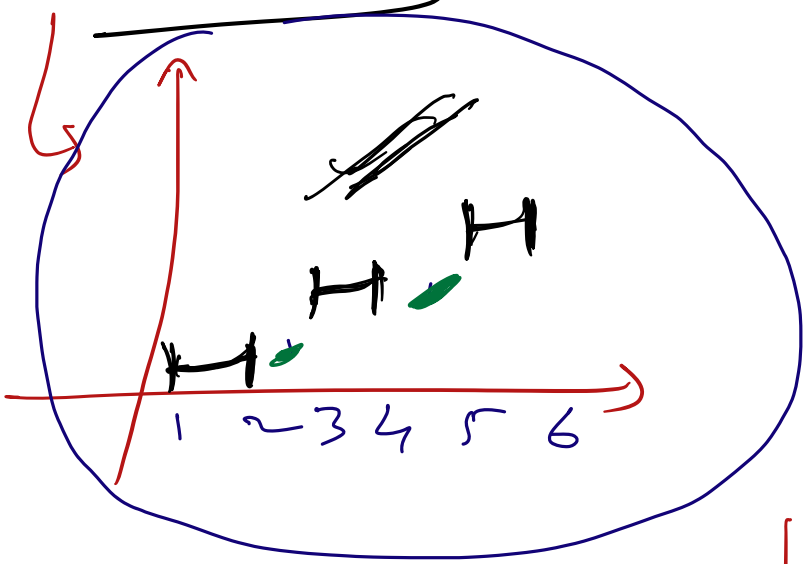
$P(H) = 1/2$, $P(T) = 1/2$

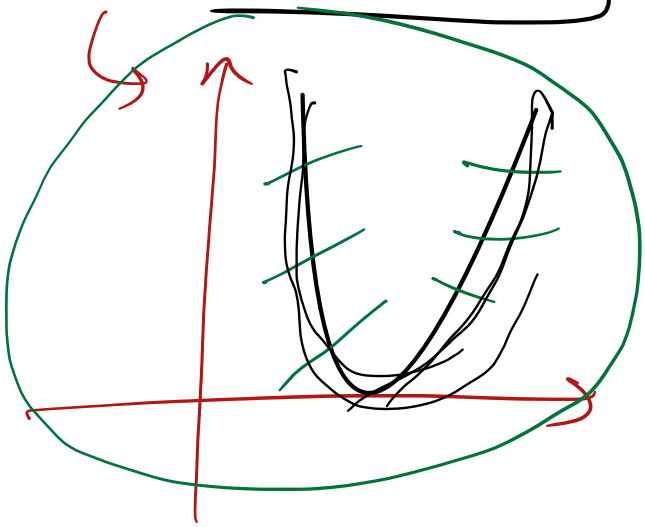
(1) (0)

So, the outcomes, given in (g.n) are not correct as (1/3) and (2/3)

(6.) Sol: what ever data we have, it will be in two forms, either in

discrete (or) Continuous





⇒ The functions (or) the values which are being plotted with break

⇒ they are called as discrete functions.

⇒ The functions (or) the values which are able to being plotted without break

they are called as Continuous functions

func^n: Step func^n

func^n: Sign function.

DONE!

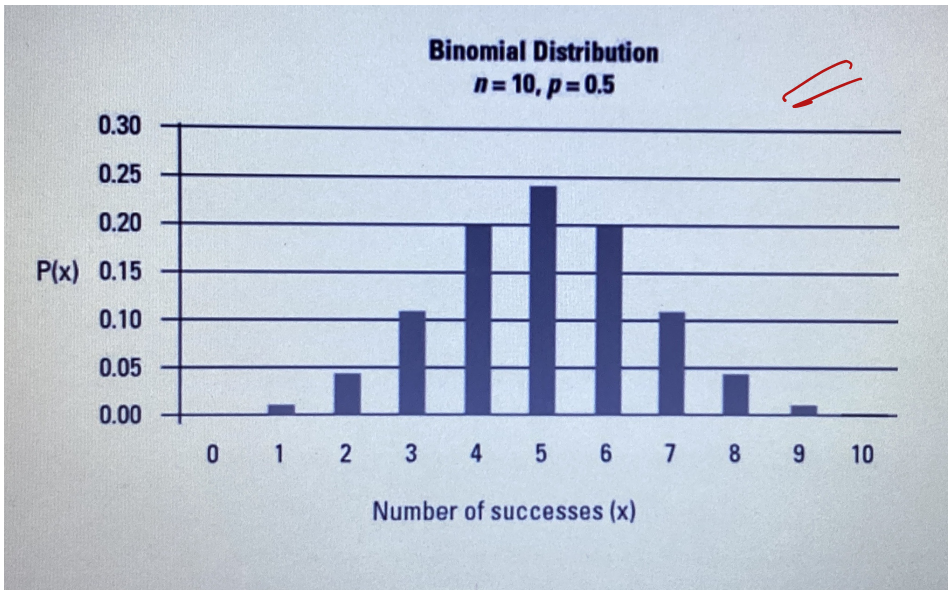
7. a distribution function which is discrete:

↳ Binomial, poisson and Bernoulli etc....

$$= {}^{n}C_{x} \, p^{x} (1-p)^{x}$$

not happening

Let, $n = 10$, $p = 0.5$



8. how many parameters are there in gaussian distribution?

Sol.

The Gaussian distribution is also

⇒ Commonly called the "normal distribution"

⇒ and is often described as a

"bell-shaped curve"

⇒ it is a continuous funct which approximates the exact binomial distribution of events.

Formula: !!

$$f_g(x) = \frac{1}{\sqrt{2\pi(\sigma)^2}} e^{\frac{-(x-a)^2}{2(\sigma)^2}}$$

Parameters? (Sigma)

σ = Standard Deviation

a = Mean

∴ We have, two parameters.....

(9.) There are two events (A) and (B) How do you define $P(A|B)$?

Sol.

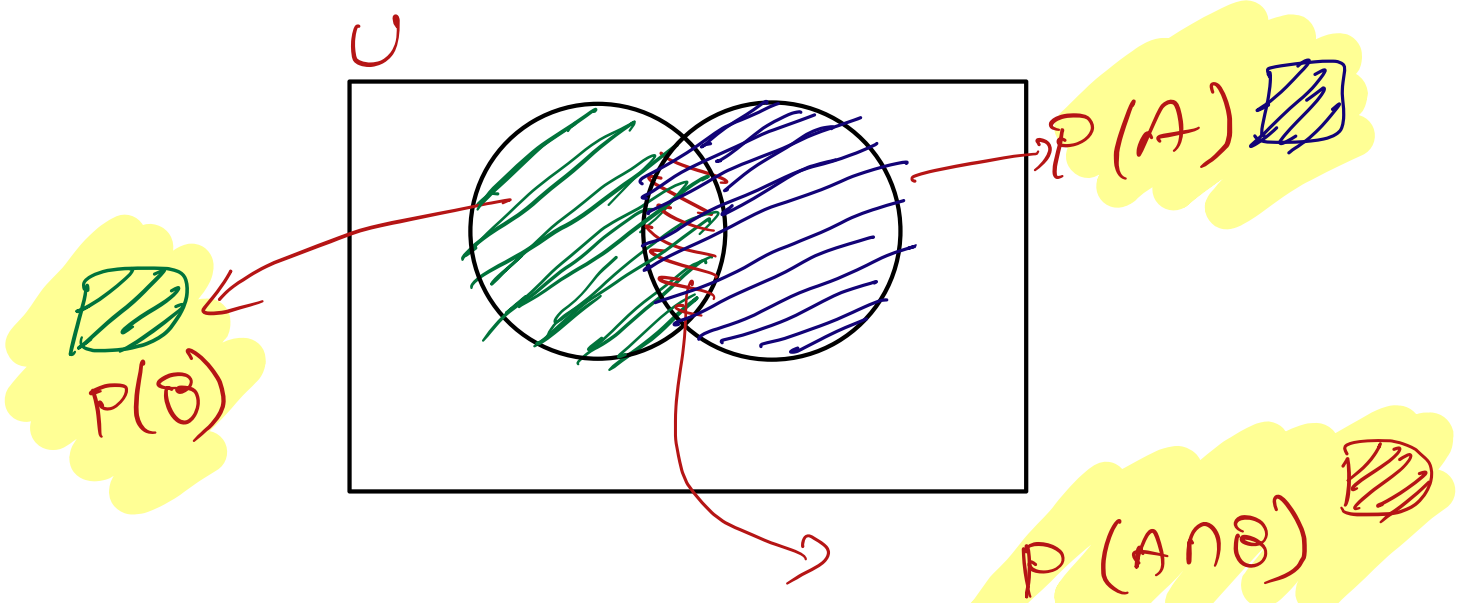
$P(A|B)$

↳ given ⇒ probability of (A) given (B)

⇒ event 'B' has already occured.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

DONE!



$P(A)$ = probability of event 'A'

$P(B)$ = probability of event 'B'

(10.) binomial distribution?

Sol.

Def: A frequency distribution of the possible number of successful outcomes in a given number of trails in each of which there is the same probability of success.

$$P_x = {}^{n}C_x \, p^x \, (1-p)^{n-x}$$

!!

⇒ P = binomial probability

⇒ n = no. of trails

⇒ x = no. of times for a specific outcome within n trails

(*) ${}^{n}C_x$ = no. of combinations

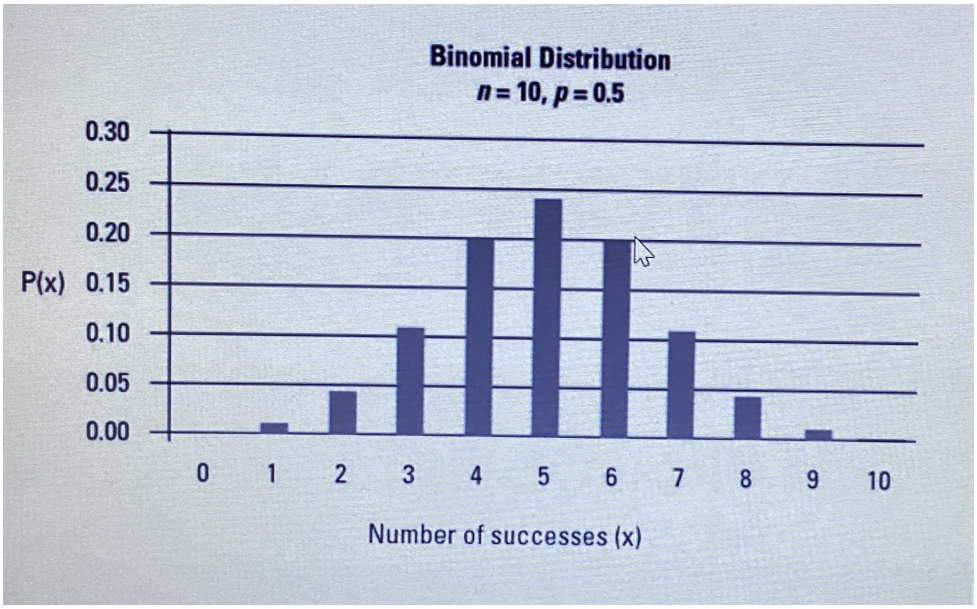
⇒ p = probability of success on a single trial

⇒ $(1-p)$ = probability of failure

$$nC_x = \frac{n!}{x!\,(n-x)!}$$

where, $n! = (n)(n-1)(n-2)\cdots 1$

let, $n = 10$ and $p = 0.5$



(11.) what is the probability of two independent events?

Sol.

The Multiplication Rule for Independent Events:

⇒ let (A) and (B) are two events

⇒ And they are independent of each other.

↳ note: They dont affect each other with respect to other when they occur

⇒ In short, there is no relation b/w them.

⇒ then,

$$P(A \text{ and } B) = P(A) * P(B)$$

!!

ex!

Blood type	O	A	B	AB
prob	0.44	0.42	0.10	0.04

⇒ two people selected randomly

⇒

what is the probability that both have blood type O?

Sol. Let O_1 = person 1 has blood type O

and

O_2 = person 2 has blood type O

we find,

$$P(O_1 \text{ and } O_2)$$

note!

Since, they were chosen simultaneously and at random, the blood type of one has no effect on the blood type of other.

$\Rightarrow$ Therefore, O_1 and O_2 are independent

$$\Rightarrow P(O_1 \text{ and } O_2) = P(O_1) * P(O_2)$$

$$= 0.44 * 0.44$$

$$= 0.1936$$

(12.) tossed two coins ------

Sol.

Coin 1	= 0	Coin 2	= 1

outcome is X

outcome is Y

and $Z = X + Y$ (given)

1. what is the range of Z ?

sol

Coin 1 ⇒ 0

H T

X

$P(X) = 1/2$

Coin 2 ⇒ 1

H T

Y

$P(Y) = 1/2$

range $(Z = X + Y)$

0 to 1

0 to 1

but, we cant get 1 but, we have

only two conditions H or T

$\Rightarrow$ range (Z) = 0 to 1

2. $P(Z <= 1) = 1 - P(Z > 1)$

13. how will you define $P(A|B)$?

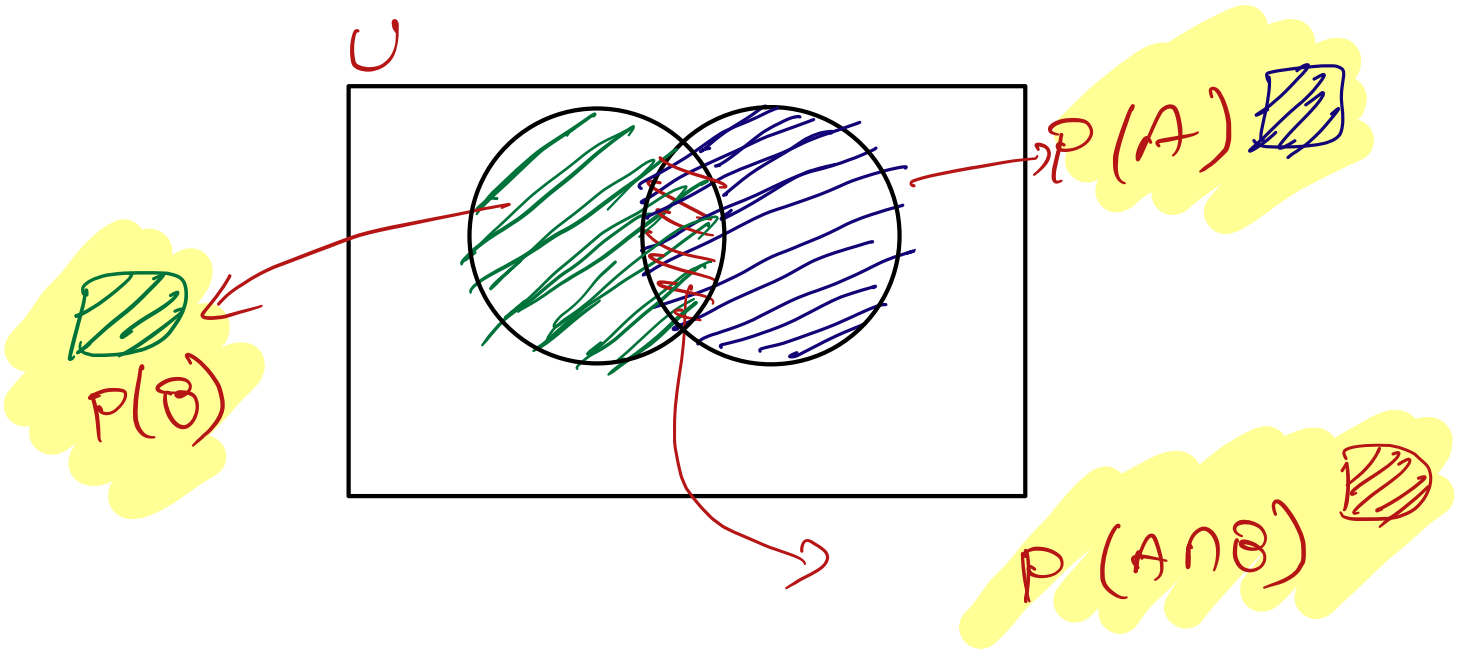
Sol.

$P(A|B)$

$\rightarrow$ given $\Rightarrow$ probability of (A) given (B)

$\Rightarrow$ event 'B' has already occured.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



$P(A)$ = probability of event 'A'

$P(B)$ = probability of event 'B'

(12.) Joint probability?

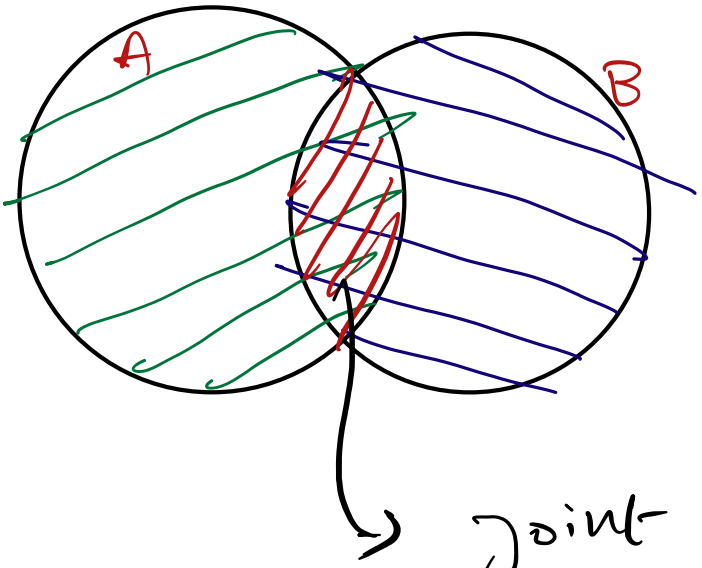
def. → it refers to the probability that two events will both occur.

→ in other words, two events occuring together.

Simultaneously

Formula:

$$JP = P(A \cap B) = P(A) * P(B)$$



Joint probability
$P(A \cap B)$

Ex:

What is the joint probability of rolling the number five twice in a fair six-sided dice?

Event "A" = The probability of rolling a 5 in the first roll is 1/6 = 0.1666.

Event "B" = The probability of rolling a 5 in the second roll is 1/6 = 0.1666.

Therefore, the joint probability of event "A" and "B" is P(1/6) x P(1/6) = 0.02777 = **2.8%**.

(18.) Bayes theorem for independent events?

Sol.

If two events A and B are independent then,

$P(A/B) = P(A)$ and $P(B/A) = P(B)$

!!

Therefore, Bayes theorem Can't be used to determine the conditional probability as we need to determine the total probability and there is no dependency of events.....

Proof:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

but, A and B are independent events

$$P(A \cap B) = P(A) * P(B)$$

$$\Rightarrow \frac{P(A)P(B)}{P(B)} \Rightarrow P(A)$$

$$\Rightarrow P(A/B) = P(A)$$

By, $P(B/A) = P(B)$

(20.) formulate Conditional prob?

Sol.

⇒ A Conditional probability is the probability that an event has occured, taking into account additional information about the result of expirement.

⇒ A Conditional probability can always be computed using the formula in the definition, sometiel it can be computed by discarding part of the Sample space

⇒ Two events A and B are independent if the probability $P(A \cap B)$ of their intersection $A \cap B$ is equal to

the product of $P(A) \cdot P(B)$ of their individual probabilities....

2. 1. mean:

⇒ It is the **average value** of all the values present in a dataset..

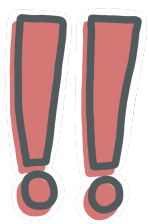
ex ⇒ [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

be values from a column in a dataset.

$$\text{mean} = \frac{1+2+3+4+5+6+7+8+9+10}{10}$$

$$= \frac{55}{10}$$

$$= 5.5$$



2. median:

⇒ After arranging all the values of a dataset in ascending order.

⇒ The middle most value is considered as the median of all the values.

ex ⇒ [5, 2, 1, 3, 4]

↳ [1, 2, 3, 4, 5]

median = 3

!!

Thanks for Your Patience