Shivani Patel
Philos 12A
Klaus / 106

Assignment #8

Exercise 9.22

	Malcev's	Bolzano's	Boole's	Wittgenstein's
1	False	False	True	False
2	True	False	False	False
3	True	False	False	False
4	True	False	False	False
5	True	False	True	False
6	False	False	False	False
7	False	False	True	False
8	False	True	True	False
9	True	True	True	True
10	False	False	True	True

Exercise 10.1

	Annotated Sentence	Truth Functional Form	a/b/c
1	$\underline{\forall x\ x=x}$ A	A	b
2	$\underline{\exists x\ Cube(x)} \rightarrow \underline{Cube(a)}$ A B	$A \rightarrow B$	c
3	$\underline{Cube(a)} \rightarrow \underline{\exists x\ Cube(x)}$ A B	$A \rightarrow B$	b
4	$\underline{\forall x\ (Cube(x) \wedge Small(x))} \rightarrow \underline{\forall x\ (Small(x) \wedge Cube(x))}$ A B	$A \rightarrow B$	b
5	$\underline{\forall v\ (Cube(v) \leftrightarrow Small(v))} \leftrightarrow \neg\neg\underline{\forall v\ (Cube(v) \leftrightarrow Small(v))}$ A A	$A \leftrightarrow \neg\neg A$	a

Shivani Patel
Philos 12A
Klaus / 106

Assignment #8

6	$\underline{\forall x\ Cube(x)} \rightarrow \neg\underline{\exists x\ \neg Cube(x)}$ A B	$A \rightarrow \neg B$	c
7	$[\underline{\forall z\ (Cube(z) \rightarrow Large(z))} \wedge \underline{Cube(b)}] \rightarrow \underline{Large(b)}$ A B C	$[A \wedge B] \rightarrow$ C	b
8	$\underline{\exists x\ Cube(x)} \rightarrow (\underline{\exists x\ Cube(x)} \vee \underline{\exists y\ Dodec(y)})$ A A B	$A \rightarrow A \vee B$	a
9	$(\underline{\exists x\ Cube(x)} \vee \underline{\exists y\ Dodec(y)}) \rightarrow \underline{\exists x\ Cube(x)}$ A B A	$A \vee B \rightarrow A$	c
10	$[(\underline{\forall u\ Cube(u)} \rightarrow \underline{\forall u\ Small(u)}) \wedge \neg\underline{\forall u\ Small(u)}] \rightarrow \neg\underline{\forall u}$ $\underline{Cube(u)}$ A B B A	$[(A \rightarrow$ $B) \wedge \neg B] \rightarrow$ $\neg A$	a

Exercise 10.3

Truth Functional Form:

$A \rightarrow B$
$\neg B$

C
(b) logically, but not tautologically valid

Exercise 10.4

Truth Functional Form:

$A \rightarrow B$
$\neg B$

$\neg A$
(a) tautologically valid

Exercise 10.9

	Written out	**Logical Truth**	**FO Validity**
1	There is a block to the left of block b and block b is to the right of that one	Yes	No

Shivani Patel
Philos 12A
Klaus / 106

Assignment #8

2	If a small block is in the back of block c then it is a dodec	No	No
3	If there is a cube that is not b, it is then larger or smaller than block b	No	No
4	If block d is a dodec, then any block named d is a dodec	Yes	No
5	If the block is larger than block a and smaller than block b, then block a is smaller than block b	Yes	No
6	Every block larger than c is not c	Yes	No
7	Every block is either between a and d or is not between a and d	Yes	Yes
8	Every block is either between a and d or is not between d and a	Yes	Yes
9	Dodec are either small or d	No	No
10	If all cubes to the left of block e there is not a block that is a cube and not left of block e	Yes	Yes

First Order Counter Examples:

1. $\forall x\, (\square(x, b) \to \square(b, x))$

4. $\square(d) \to \forall x\, (x = d \to \square(x))$

5. $\exists y\, (\square(a, y) \land \square(y, b)) \to \square(a, b)$

6. $\forall x\, (\square(x, c) \to x \neq c)$

1. If we replaced the blue block with SameRow and orange block with SameColumn, then we would have a counterexample
4. If the blue block is replaced with Dodec then this would be a counterexample.
5. If the blue blocks are replaced with DifferentShape then this would a counterexample
6. If we replaced the orange block with SameShape then we would have a counterexample.

Shivani Patel
Philos 12A
Klaus / 106

Assignment #8

Exercise 10.12

$$\begin{array}{|l} \forall x\, \text{Cube}(x) \to \exists y\, \text{Small}(y) \\ \neg\exists y\, \text{Small}(y) \\ \hline \exists x\, \neg\text{Cube}(x) \end{array}$$

Truth Functional Form:
A → B
¬B

C

∀x Hello(x) → ∃y World(y)
¬∃y World(y)

∃x¬Hello(x)

The argument is not tautologically valid. From its truth-functional form, we cannot get the conclusion based on the given premises, thus the conclusion is not always true given the premises in truth-normal form. The argument is logically valid because we can prove the conclusion based on the premises.
(b) first-order consequences that are not tautological consequences

Exercise 10.13

$$\begin{array}{|l} \forall x\, \text{Cube}(x) \to \exists y\, \text{Small}(y) \\ \neg\exists y\, \text{Small}(y) \\ \hline \neg\forall x\, \text{Cube}(x) \end{array}$$

Truth Functional Form:
A → B
¬B

¬A

This argument is tautologically valid. From its truth-functional form, we can con get the conclusion based on the given premises as they are executed. The conclusion is always true given the premises in the truth-normal form. The argument is logically valid because we can prove the conclusion based on the premises.

Shivani Patel
Philos 12A
Klaus / 106

Assignment #8

(a) tautological consequences of the premises