Exercise 9.22

| | Malcev's | Bolzano's | Boole's | Wittgenstein's |
|----|----------|-----------|---------|----------------|
| 1 | False | False | True | False |
| 2 | True | False | False | False |
| 3 | True | False | False | False |
| 4 | True | False | False | False |
| 5 | True | False | True | False |
| 6 | False | False | False | False |
| 7 | False | False | True | False |
| 8 | False | True | True | False |
| 9 | True | True | True | True |
| 10 | False | False | True | True |

Exercise 10.1

| | Annotated Sentence | Truth Functional Form | a/b/c |
|---|---|---------------------------------|-------|
| 1 | $\frac{\forall x \ x=x}{A}$ | A | b |
| 2 | $\frac{\exists x \text{ Cube}(x)}{A} \rightarrow \frac{\text{Cube}(a)}{B}$ | $A \rightarrow B$ | С |
| 3 | $\frac{\text{Cube(a)}}{A} \to \frac{\exists x \text{ Cube(x)}}{B}$ | $A \rightarrow B$ | b |
| 4 | $\frac{\forall x (\text{Cube}(x) \land \text{Small}(x))}{A} \rightarrow \frac{\forall x (\text{Small}(x) \land \text{Cube}(x))}{B}$ | $A \rightarrow B$ | b |
| 5 | $\frac{\forall v (\text{Cube}(v) \leftrightarrow \text{Small}(v))}{A} \leftrightarrow \frac{\neg \neg}{A} \forall v (\text{Cube}(v) \leftrightarrow \text{Small}(v))}{A}$ | $A \leftrightarrow \neg \neg A$ | a |

| 6 | $ \frac{\forall x \text{ Cube}(x)}{A} \rightarrow \frac{\neg \exists x \neg \text{Cube}(x)}{B} $ | $A \rightarrow \neg B$ | С |
|----|--|---|---|
| 7 | $ \begin{array}{c c} [$ | $\begin{bmatrix} A \land B \end{bmatrix} \rightarrow C$ | b |
| 8 | $ \frac{\exists x \text{ Cube}(x) \to (\exists x \text{ Cube}(x) \lor \exists y \text{ Dodec}(y))}{A A B} $ | $A \to A \vee B$ | a |
| 9 | $ \begin{array}{ccc} (\exists x \ Cube(x) & \forall \ \exists y \ Dodec(y)) \rightarrow \exists x \ Cube(x) \\ A & B & A \end{array} $ | $A \vee B \to A$ | С |
| 10 | $ \begin{array}{c cccc} [($ | $ \begin{bmatrix} (A \to B) \land \neg B \end{bmatrix} \to \neg A $ | a |

Exercise 10.3

| Truth | Fun | ction | nal | Form | ١. |
|----------|-----|-------|------|------|----|
| 11111111 | гип | CHOI | 1111 | гон | |

 $A \rightarrow B$

 $\neg B$

C

(b) logically, but not tautologically valid

Exercise 10.4

Truth Functional Form:

 $A \rightarrow B$

 $\neg B$

 $\neg A$

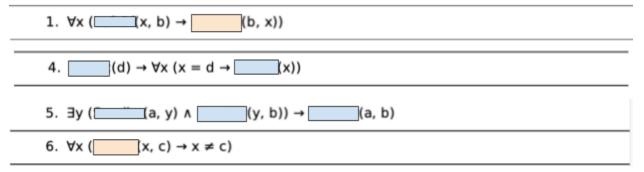
(a) tautologically valid

Exercise 10.9

| | Written out | Logical Truth | FO Validity |
|---|---|---------------|-------------|
| 1 | There is a block to the left of block b and block b is to the right of that one | Yes | No |

| 2 | If a small block is in the back of block c then it is a dodec | No | No |
|----|--|-----|-----|
| 3 | If there is a cube that is not b, it is then larger or smaller than block b | No | No |
| 4 | If block d is a dodec, then any block named d is a dodec | Yes | No |
| 5 | If the block is larger than block a and smaller than block b, then block a is smaller than block b | Yes | No |
| 6 | Every block larger than c is not c | Yes | No |
| 7 | Every block is either between a and d or is not between a and d | Yes | Yes |
| 8 | Every block is either between a and d or is not between d and a | Yes | Yes |
| 9 | Dodec are either small or d | No | No |
| 10 | If all cubes to the left of block e there is not a block that is a cube and not left of block e | Yes | Yes |

First Order Counter Examples:



- 1. If we replaced the blue block with SameRow and orange block with SameColumn, then we would have a counterexample
- 4. If the blue block is replaced with Dodec then this would be a counterexample.
- 5. If the blue blocks are replaced with DifferentShape then this would a counterexample
- 6. If we replaced the orange block with SameShape then we would have a counterexample.

Exercise 10.12

The argument is not tautologically valid. From its truth-functional form, we cannot get the conclusion based on the given premises, thus the conclusion is not always true given the premises in truth-normal form. The argument is logically valid because we can prove the conclusion based on the premises.

(b) first-order consequences that are not tautological consequences

Exercise 10.13

Truth Functional Form:

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A \rightarrow B
\neg B
----
\neg A
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This argument is tautologically valid. From its truth-functional form, we can con get the conclusion based on the given premises as they are executed. The conclusion is always true given the premises in the truth-normal form. The argument is logically valid because we can prove the conclusion based on the premises.

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Assignment #8

(a) tautological consequences of the premises