

NAME:

SID:

Pledge: I pledge to work alone using only the textbook (including its software) and the lecture notes for the course. Signed:

**Philosophy 12A. Introduction to Logic.
Spring 2021**

Prof. Paolo Mancosu.

MIDTERM EXAM -- Due Monday, March 8, 2021 by 4 p.m.

Instructions: This exam consists of 9 problems over four pages. Write your name and SID at the top of this question form. Sign the pledge. Questions 1, 2, 3b, 4b, and 8 should be answered on this form itself. Use additional paper for the other questions. The midterm will have to be scanned as a single document and submitted electronically through **bCourses**. You can include in your pdf screen shots of work carried out in Tarski's World, Boole, and Fitch. First, ensure that you have scanned your solutions into a **single .pdf file**. Then, from <https://bcourses.berkeley.edu/> click **Courses**, navigate to **Introduction to Logic (Spring 2021)**, then click on the **Assignments** button. Once there, click the assignment that says **"Midterm SpeedGrader"** and click on the **Submit Assignment** button to submit your solutions. The total number of points is 80.

(1) Using the predicate and function symbols given, plus the identity symbol "=", translate the following five sentences into FOL. (3 points for each)

Individual constants:

j: Jerry
e: Elaine
k: Kramer
g: George
n: New York

Predicates:

C(x): x is a comedian
T(x, y): x is taller than y
D(x, y): x dislikes y
WorksIn (x, y): x works in y

Function symbol:

bf(x): the best friend of x

- (a) Jerry and Kramer work in New York, but Elaine does not.
- (b) Although Elaine dislikes New York, Jerry is a comedian who works in New York.
- (c) Neither Kramer is a comedian working in New York nor is George not taller than Kramer.

(d) George is Elaine's best friend, yet he dislikes Jerry.

(e) George is taller than Elaine's best friend or Elaine is taller than George's best friend, but not both.

(2) For each of the following questions, put an "x" in front of the answer (answers) you think is (are) right. Note that, for each question, there might be more than one correct answer.

(a) Suppose you have an argument in which one of the premises is a logical contradiction. Which of the following is then true? (6 points)

- ☐ The argument is logically valid.
- ☐ The argument could be logically invalid.
- ☐ The argument is logically sound.
- ☐ The argument could be logically sound.
- ☐ The argument is logically unsound.

(b) Suppose P is a tautology. Which of the following is the case? (6 points)

- ☐ $\neg(P \wedge \neg P)$ is contradictory.
- ☐ $\neg P$ can be true.
- ☐ $\neg P$ can be false.
- ☐ P is true in all Tarski's worlds but not in every circumstance.

Note: Questions (3) and (4) require the construction of truth tables. All columns (including the reference columns) should be filled in. Answer 3b and 4b by circling the appropriate letter (Y (yes) or N (no)).

(3) Consider the following sentence of FOL:

$$(P \wedge \neg(Q \vee R)) \vee ((Q \wedge R) \wedge \neg P)$$

(a) Construct a truth table for this sentence; (5 points)

(b) Is the sentence a tautology? (5 points) **Y** **N**

(4) Consider the following two sentences of FOL:

$$P \vee \neg(Q \vee \neg P) \qquad Q \vee (P \wedge \neg Q)$$

a. Construct a joint truth table for these sentences; (5 points)

b. On the basis of your joint truth table determine the answers to the following questions. (3 points each)

Is $Q \vee (P \wedge \neg Q)$ a tautological consequence of $P \vee \neg(Q \vee \neg P)$? **Y** **N**

Is $P \vee \neg(Q \vee \neg P)$ a tautological consequence of $Q \vee (P \wedge \neg Q)$? **Y** **N**

(5) Consider the English connective “it is a logical contradiction that”. If you think it is truth-functional, provide a truth table for it. If you think it is not truth-functional, explain why not. (8 points)

Note: Questions 6 and 7 require the construction of formal proofs in system *F*. You may use the same rules and their “generous application” as in Fitch. However, you cannot appeal to Taut-Con, FOL-Con, or Ana-Con. Make sure you provide accurate justifications for your inference rules.

(6) Give a formal proof of the following argument. (7 points)

$A \vee B$
 $\neg D \wedge E$
 $\neg A \vee D$

 B

(7) Give a formal proof of the following argument. (7 points)

$\neg A \vee E$

 $\neg(A \wedge \neg E)$

(8) Give a truth value assignment (circle the appropriate values for A, D, E, and H) that will show the invalidity of the following argument. (5 points)

$\neg D \vee E$
 $\neg A \vee E$

 $D \vee H$

A:	T	F
D:	T	F
E:	T	F
H:	T	F

(9) Describe a Tarski World and interpret A, B, and C as specific atomic sentences in the blocks language so as to show that the following argument is invalid. (5 points)

$A \vee (B \wedge \neg C)$
 $\neg B$

 $(A \wedge B) \vee \neg C$