

Assignment #10

**Exercise 10.20**

Give a chain of equivalences showing that the negation of Some P's are Q's ( $\neg \exists x (P(x) \wedge Q(x))$ ) is equivalent to No P's are Q's ( $\forall x (P(x) \rightarrow \neg Q(x))$ ).

In order to find the negation of  $\neg \exists x (P(x) \wedge Q(x))$

$$\equiv (\forall x (P(x) \rightarrow \neg Q(x)))$$

Since the negation for  $\exists$  is  $\forall$

$$\equiv \forall x (\neg P(x) \vee \neg Q(x))$$

Since  $\neg(P(x) \wedge Q(x)) = \neg P(x) \vee \neg Q(x)$  this is DeMorgans Law

$$\equiv \forall x (\neg P(x) \vee \neg Q(x))$$

$$\equiv \forall x \neg(\neg P(x) \rightarrow \neg Q(x))$$

[ $\because p \vee q \equiv \neg p \rightarrow q$  So using this logic we can apply it here:

$$\neg P(x) \vee \neg Q(x) \equiv \neg(\neg P(x) \rightarrow \neg Q(x)) ]$$

$$\equiv \forall x P(x) \rightarrow \neg Q(x)$$

Hence

$$(\neg \exists x (P(x) \wedge Q(x))) \equiv (\forall x (P(x) \rightarrow \neg Q(x)))$$

**Exercise 10.31**

1. Red, yellow, and blue in the domain of automobiles.

None of the axioms are satisfied because the car could be black and that would not satisfy everything. Thus from this, we would have a counterexample

2. Entirely red, entirely yellow, and entirely blue in the domain of automobiles.

The fourth axiom is not satisfied with the properties because the car could be entirely another color such as green or even half one color and half another color and this would give us a counterexample

3. Small, medium, and large in the domain of Tarski's World blocks.

All the axioms are satisfied in this example

4. Small, medium, and large in the domain of physical objects.

The fourth axiom is not satisfied because the object could be tiny or giant not just limiting to small, medium, or large thus giving a counterexample.

Shivani Patel  
Philos 12A  
Klaus / 106

## Assignment #10