NAME:

SID:

Pledge: I pledge to work alone using only the textbook (including its software) and the lecture notes for the course. Signed:

Philosophy 12A. Introduction to Logic.
Spring 2021

Prof. Paolo Mancosu.

FINAL EXAM – Due Wednesday May 12, by 2:30 p.m. (California time)

Instructions: This exam consists of 15 problems plus one bonus problem over seven pages. Write your name and SID at the top of this question form. Sign the pledge. Questions 1, 2, 3, 4, 5, 11, 12, and 13 should be answered on this form itself. Use additional paper for the other questions. You may type or use pencil or pen, but whatever you do it is your responsibility to make sure that we receive a readable document. The final will have to be scanned as a single document and submitted electronically through **bCourses**. You can include in your pdf screen shots of work carried out in Tarski's World, Boole, and Fitch. Thus, it is up to you whether you want to write down your truth tables by hand or include a screen shot with your work in Boole. Same for proofs in Fitch etc. Of course, a screen shot might not be sufficient to fully answer the question you have been asked.

When you submit your final, first ensure that you have scanned your solutions into a **single .pdf file**. Then, from **https://bcourses.berkeley.edu/** click **Courses**, navigate to **Introduction to Logic (Spring 2021)**, then click on the **Assignments** button. Once there, click the assignment that says **"Final SpeedGrader"** and click on the **Submit Assignment** button to submit your pdf document. The total number of points is 100 (+6 points for the bonus).

Remarks:

a) FO stands for First-Order; FOL stands for First-Order Logic.
b) The symbol $\perp$ (you should know what this means and how to use it) is considered part of FOL.
c) You will be expected to include line numbers in Fitch proofs regardless of whether you use the program or not. In case you use screen shots from Fitch, you can display the line numbers in Fitch by doing the following: select Proof > Show Step Numbers.
d) Should you need in some of the translation problems to translate definite descriptions (you should know what this means) please use the Russellian analysis.

1. Answer the following questions by circling T or F as appropriate. (1.5 points each) Let 'f' be a function symbol, 'c' a constant symbol, and 'A(x)' stand for a well-formed formula (wff). Then:

(a) f(A(x)) is also a well-formed formula.	T	F
(b) f(c) = c can never be true.	T	F
(c) f(c) is either true or false.	T	F
(d) $\exists x\ (x \neq c)$ is true in every Tarski's world.	T	F
(e) A(x)=A(x) is a well formed formula.	T	F
(f) $\exists x\bot \rightarrow \bot$ is a sentence.	T	F

2. Underline the scope of the quantifiers in the following well-formed formulas. Use a continuous line for the scope of $\forall$ and a dotted line for the scope of $\exists$. (1 point each for a), b) and c))

(a) $\forall x H(x) \vee \exists z G(z,x)$

(b) $\forall z(H(z) \rightarrow (H(c) \vee a \neq b))$

(c) $\exists y \forall x(H(y) \rightarrow G(x)) \vee G(x)$

3. Write F (for 'free') or B (for 'bound') below each occurrence of each variable in the following well-formed formulas. (1 point each for a), b) and c))

(a) $\forall w H(w) \rightarrow \exists z L(z, z, w)$

(b) $H(z) \vee \exists z\ (\neg H(z) \rightarrow x \neq y)$

(c) $\forall y \forall x\ (A(y) \vee B(x))$

4. Answer the following questions by circling T or F as appropriate. (1 point each)

(a) There are tautologies which are not FO-validities.	T	F
(b) All arguments with true conclusions are valid.	T	F
(c) If P is a tautology then $P \vee Q$ is an FO-validity.	T	F
(d) If P is truth-table contradictory then $Q \rightarrow P$ is a tautology.	T	F

(e) Some atomic sentences are tautologies.	T	F
(f) If an argument is sound then the conclusion is true.	T	F
(g) There are sound arguments which are not valid.	T	F
(h) Every FO-validity must contain at least one connective.	T	F

5. Translate each of the following sentences into FOL using the blocks language. Don't use quantifiers. (1.5 points for each)

(a) Neither a nor b are cubes of the same size as c.

(b) a is a cube only if b is a cube.

(c) Both a and c are tetrahedra, although neither of them is smaller than c.

(d) a is a cube provided b is not a cube.

6. Give formal Fitch proofs of the following arguments. Do not use Con rules. Even if you use a screen shot make sure all the justifications are displayed or points will be taken off. (5 points for each)

(a)

$(A \leftrightarrow \neg A)$
\----------
$B \wedge \neg B$

(b)

$A \vee \neg B$
\------------
$\neg(\neg A \wedge B)$

7. Let A, B, and C be atomic sentences. Consider the following two arguments. Are they valid? If so, show validity by proving in Fitch the conclusion from the premises (in which case make sure you display the justification of the steps). If not, provide a truth value assignment to the atomic sentences to show invalidity and explain why the truth value assignment shows invalidity. (5 points each)

(a)

$A \to B$
$\neg A$

$\neg B$

(b)

$(A \wedge B) \to C$
—————————
$(A \vee B) \to C$

8. Write down a full truth table (i.e. fill out every column of the truth table) for the following sentence and determine whether it is a tautology. In order to get a full score on this question the truth table, as well as the answer to whether the sentence is a tautology, must both be correct. (4 points)

$(C \vee (A \wedge B)) \to (\neg C \leftrightarrow B)$

9. Give a formal Fitch proof of the following argument. Do not use any Con rules. Even if you use a screen shot make sure all the justifications for the steps are displayed or points will be taken off. (6 points)

$\forall y \exists x\ Likes(y, x) \wedge \forall y\ \neg\ Likes(y, b)$
$\exists x \forall y\ Likes(x, y)$

$\perp$

10. Give a formal Fitch proof without premises of the following sentence. Do not use any Con rules. Even if you use a screen shot make sure all the justifications are displayed or points will be taken off. (6 points)

$\forall x\ (\neg \exists y\ Likes(y,x) \to \neg Likes(x,x))$

11. Using the predicate and constant symbols given, plus the identity symbol "=", translate the following sentences into FOL. (1.5 for each sentence)

Domain of discourse: natural numbers (i.e. 0, 1, 2, etc.)
Function symbols: s(x):= 'the successor of x'
Individual constants: 1, 2
Predicates: D(x, y) := "x is divisible by y" (example: 6 is divisible by 3)
E(x):= "x is even"
P(x):= "x is prime"

(a) Every number has a successor.

(b) Every prime number is divisible by one.

(c) A number is even just in case it is divisible by two.

(d) Some prime numbers are not even numbers.

(e) There is exactly one even prime number.

(f) No primes besides 2 are even numbers.

(g) Some numbers are not divisible by any even number.

(h) Every even prime number is a successor of some number.

(i) Any even prime number is a successor of any prime which divides it.

(j) Every number which has a successor does not divide it.

12. Using the predicate and constant symbols given, plus the identity symbol "=", translate the following sentences into FOL (1.5 for each sentence).

Domain of discourse: people and things in the Post Office
Individual constants: b (standing for 'Bob')
Predicates: P(x) := "x a person"
F(x, y):= "x is in front of y"
M(x, y):= "x is mailing y"
L(x):= "x is a letter"
B(x):= "x is a box"
H(x):= "x is heavy"

(a) At least two people are mailing heavy boxes.

(b) Some people are mailing exactly two boxes.

(c) In front of Bob there are at most two people.

(d) Every person who is mailing a heavy box has a person in front of himself/herself.

13. With the same symbolic key as above translate the following two sentences into FOL (2 points for each sentence)

(a) Bob is mailing the heavy box.

(b) The person in front of Bob is mailing the heavy letter.

14. Provide an interpretation with domain of discourse the natural numbers (i.e. 0, 1, 2 etc.) to show that the second sentence is not an FO-consequence of the first. In addition to providing the interpretation, you will also have to explain why the interpretation you provide shows that the second sentence is not an FO-consequence of the first. (5 points)

$\forall y \exists x\, L(x,y)$ $\qquad$ $\exists x \forall y\, L(x,y)$

15. Provide an interpretation with domain of discourse the natural numbers (i.e. 0, 1, 2 etc.) to show that the pairs of sentences below are not FO-equivalent. You will also have to explain why the interpretation you provide shows that the two sentences are not FO-equivalent. (5 points)

$\exists x(P(x) \rightarrow Q(x))$ $\qquad$ $\exists x P(x) \rightarrow \exists x Q(x)$

16. (Bonus; 6 points) Consider the following connective

P	Q	P|Q
T	T	F
T	F	T
F	T	T
F	F	T

(a) Express (P|Q) using P, Q and the three connectives ∨, ∧, ¬. (1.5 points)
(b) Express (P ∧ Q) using only the | connective. (1.5 points)
(c) Express ¬P using only the | connective. (1.5 points)
(d) Is the set of connectives {|} truth-functionally complete? Choose among the following three options:

i. Yes
ii. No
iii. Insufficient information given

Explain you answer. (1.5 points)