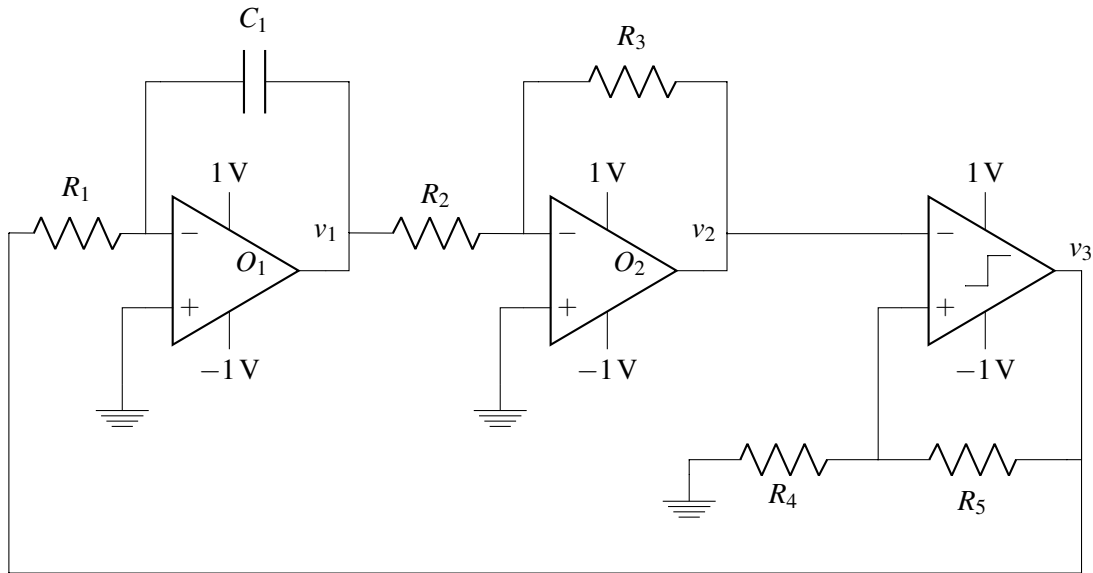


# EECS 16A Designing Information Devices and Systems I Discussion 11A

## 1. Timer Circuit

In this problem, we will walk through another useful, real-world circuit, the timer circuit. The circuit is shown below. All resistors have a resistance of  $1\text{ k}\Omega$  and  $C_1 = 1\text{ }\mu\text{F}$ .



- (a) Find the current through the capacitor  $C_1$  in terms of the voltage  $V_3$  and the resistor  $R_1$ .

**Answer:**

For an op-amp, no current flows into the input terminals. Therefore, all the current through  $R_1$  must flow through  $C_1$ . Applying the Golden Rules, we know that  $v_+ = v_- = 0\text{ V}$ .

$$i_{R_1} = i_{C_1} = \frac{v_3}{R_1}$$

- (b) Suppose that at time  $t = 0$ ,  $C_1$  is uncharged. Find the voltage  $v_1$  in terms of  $t$ ,  $v_3$ , and  $R_1$ . What is the maximum  $|v_1|$  could be?

**Answer:**

Recall the voltage across a capacitor is related to the charge on the capacitor, that is  $Q = CV$ . Current is related to charge with the equation  $I = \frac{dQ}{dt}$ .

$$v_{C_1} = \frac{Q}{C_1} = \frac{It}{C_1} = \frac{v_3}{R_1 C_1} t = \frac{v_3}{1\text{ ms}} t$$

Note that a  $\Omega\text{F}$  is a second. Using the formula for  $v_{C_1}$  from above and the fact that  $v_- = v_+ = 0$ , the voltage at node  $v_1$  can be calculated as:

$$v_{C_1} = 0 - v_1 \implies v_1 = -\frac{v_3}{1\text{ ms}} t$$

The maximum or minimum for  $v_1$  is the top or bottom supply rail, so either  $+1\text{ V}$  or  $-1\text{ V}$ . Therefore, the maximum  $|v_1| = 1\text{ V}$ .

- (c) How is  $v_2$  related to  $v_1$ ? What is the voltage  $v_2$ ?

**Answer:**

$O_2$  is an inverting amplifier. The output voltage  $v_2$  is equal to  $-v_1$ .

$$v_2 = \frac{v_3}{1\text{ ms}}t$$

Now, let's independently analyze the circuit in the two possible outputs of the comparator, when  $v_3 = 1\text{ V}$  and when  $v_3 = -1\text{ V}$ .

- (d) Assume that the output of the comparator  $v_3$  has railed to the top rail. With this value of  $v_3$ , what is  $v_2$  as a function of time? What is the voltage at the positive input of the comparator? At what time will the two inputs of the comparator be equal?

**Answer:**

With  $v_3$  at the top rail,  $v_2$  is  $\frac{t}{1\text{ ms}}\text{ V}$ . The voltage at the positive input of the comparator is  $0.5\text{ V}$  because of  $R_5$  and  $R_4$ . Therefore, when  $t = 0.5\text{ ms}$ ,  $v_2 = 0.5\text{ V}$ .

- (e) Now assume that the reverse occurs, that is, the output of the comparator has railed to the bottom rail. Repeat part (d) with this value of  $v_3$ .

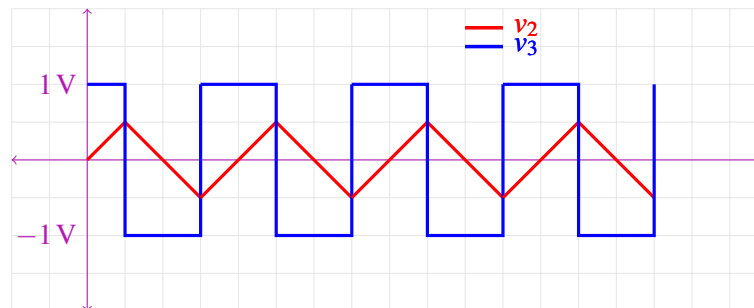
**Answer:**

With  $v_3$  at the bottom rail,  $v_2$  is  $-\frac{t}{1\text{ ms}}\text{ V}$ . Similar to part (d), the voltage at the positive input is  $-0.5\text{ V}$ . Therefore, when  $t = 0.5\text{ ms}$ ,  $v_2 = -0.5\text{ V}$ .

- (f) What is  $v_3$  as a function of time? Draw a graph of  $v_3$  and  $v_2$ . Since the graph is periodic, find its period and frequency.

**Answer:**

Notice that in each of the above cases, once  $v_2$  was equal to  $v_+$ , the output of the comparator would flip. This leads to a periodic function, where  $v_3$  is either  $+1\text{ V}$  or  $-1\text{ V}$ . The period of this function is  $T = 2\text{ ms}$ . Notice that in each of the above cases we analyzed, we always assumed that the capacitor was initially uncharged. However, when  $v_3$  switches, the capacitor will already have some charge built up on it, so it must first be drained. This is why the period is twice what we expect.



- (g) Suppose that we changed the value of  $C_1$  to be  $2\mu\text{F}$ ? What is the new period? Suppose that we change  $R_5$  to be  $2\text{ k}\Omega$ . What is the new period? What if we change  $R_5$  to be  $0\Omega$ ? Will this circuit still operate?

**Answer:**

Notice above we got the constant  $1\text{ ms}$  by multiplying  $R_1$  and  $C_1$  together. If we double  $C_1$ , the effective period would double because it would take longer to charge  $C_1$  to the same voltage with the same current.

Changing  $R_5$  affects the “flip” threshold because  $v_+$  is at a different voltage. Increasing  $R_5$  decreases the voltage at  $v_+$ , so we would expect the flip voltage to decrease. In fact, the new period is  $\frac{4}{3}$ ms. The circuit would not operate if  $R_5 = 0\Omega$ . The inverting input needs to be able to go above and below the non-inverting input, which is not possible if the non-inverting input is constant at the rail.