This homework is due Nov 8, 2016, at 1PM.

1. **Homework process and study group**

   Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?
   Working in groups of 3-5 will earn credit for your participation grade.

   Solution: I worked on this homework with...

   I first worked by myself for 2 hours, but got stuck on Problem 5 so I went to office hours on...

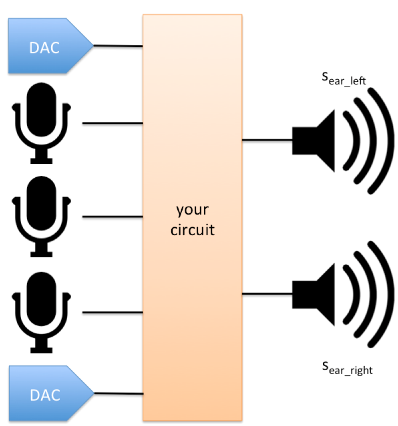
   Then I went to homework party for a few hours, where I finished the homework.

2. **Noice Cancelling Headphones**

   Implement the full noise-cancelling headphone amplifier seen in discussion. Recall that the stereo output is calculated using the following matrix equation:

$$\begin{bmatrix} s_{ear\_left} \\ s_{ear\_right} \end{bmatrix} = \begin{bmatrix} a_{1left} & a_{2left} & a_{3left} \\ a_{1right} & a_{2right} & a_{3right} \end{bmatrix} \cdot \begin{bmatrix} s_{mic1} \\ s_{mic2} \\ s_{mic3} \end{bmatrix} + \begin{bmatrix} s_{left} \\ s_{right} \end{bmatrix}$$

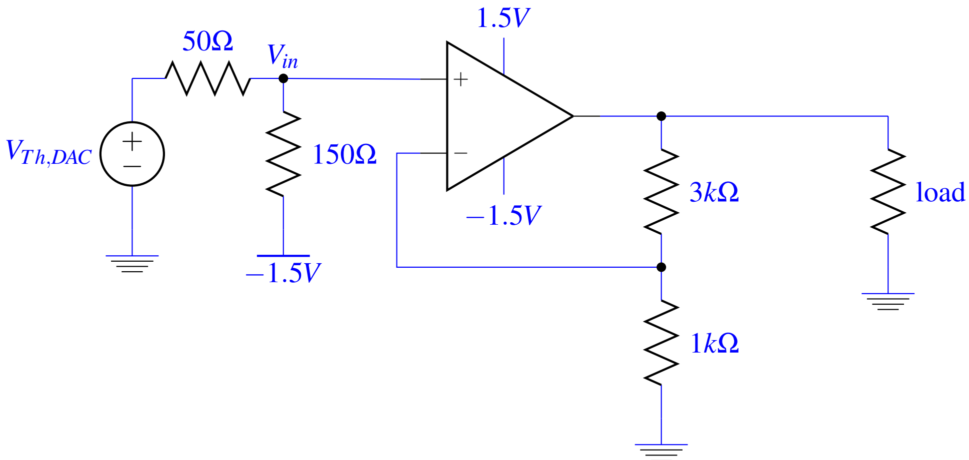
   A block diagram of the circuit is given below:



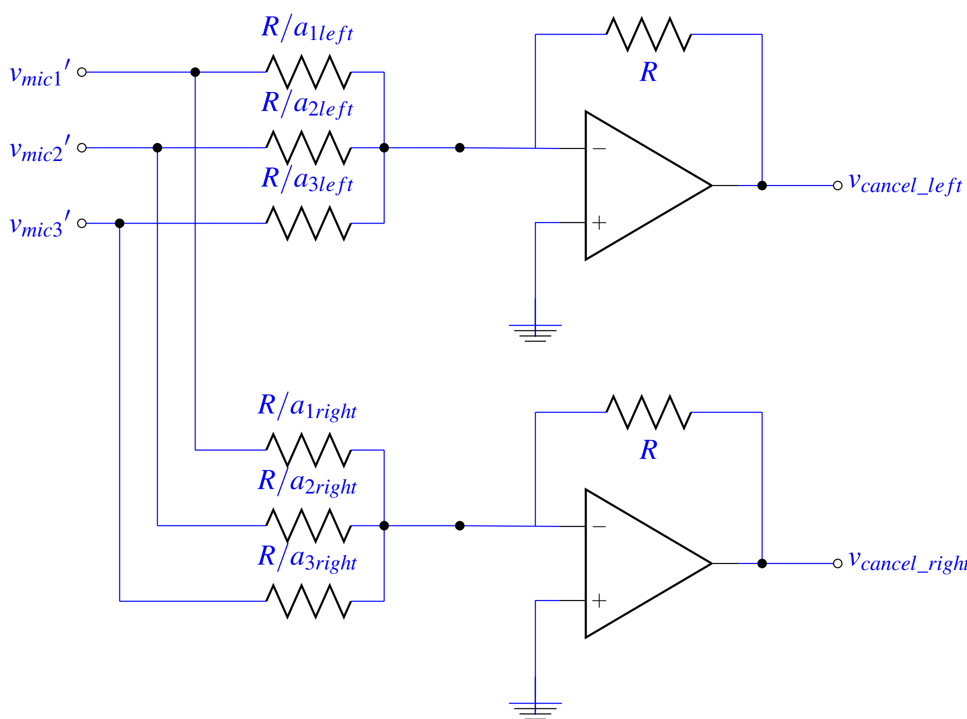
   Remember the outputs of the DAC range from $0 - 1V$ and outputs of the microphones range from $-1.5V$ to $1.5V$. The output should range from $-1.5V$ to $1.5V$. Refer back to discussion 9B and 10A for help.

   Solution: Here we are simply combining together all the pieces we have created in discussions 9B and 10A. Be sure to take a look at the solutions to each discussion if you're unsure how any piece of the circuit below works.

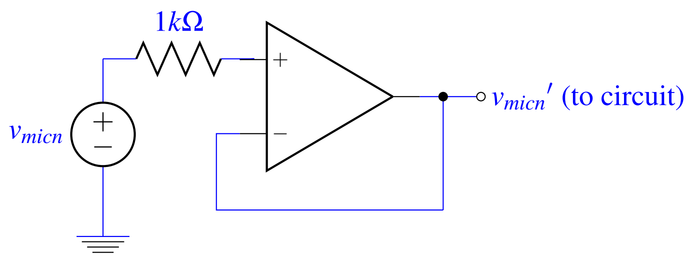
We will start back at the DAC. Recall we created a circuit to level shift the output of the DAC and amplify it in discussion 9B. Instead of connecting our amplifier to the speaker, for now we'll model the output of this as some load.



Now, we look at the microphone circuit. Recall we used the circuit below to implement the matrix multiplication and addition. Also notice we've inverted the sign of the inputs as well.

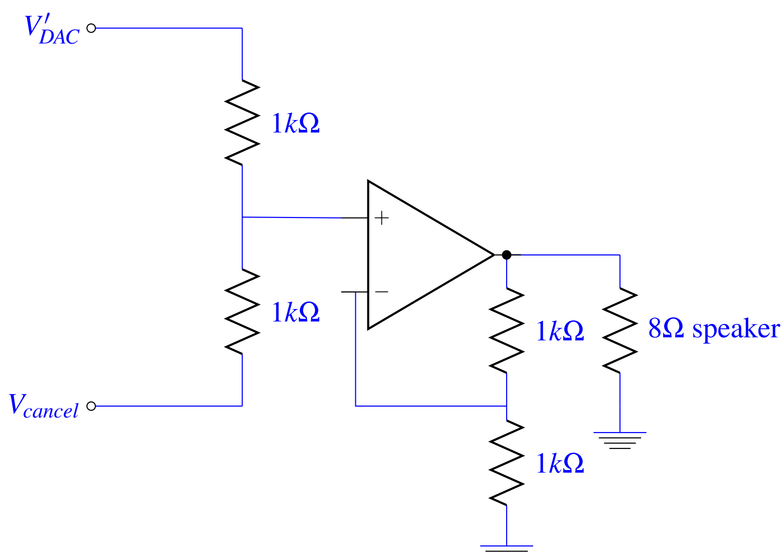


The problem with the above circuit was it was loading the microphones since they had a source resistance. The easy fix for this was the voltage buffer added to the microphones.

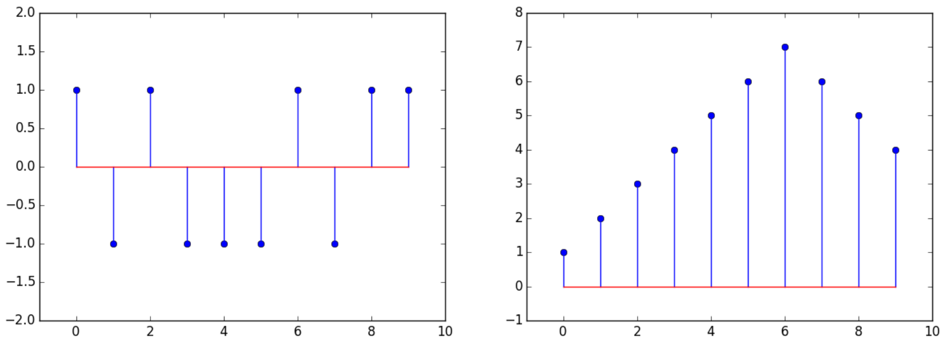


Now we have seperately implemented the two components of the noise canceling circuit, the DAC input and the canceling input from the microphone. Now we need to combine the two inputs into one. We will ofcourse use a summer to do so. However, first we need to step back and think about the sign of the voltages.

A person listening to the output of our headphones will hear the sound we send them and some noise from the outside. Through the microphone circuit, we've picked up and scaled the noise appropriately. The noise we send into the headphones must be opposite in sign to the noise heard by the person, so that the two waves cancel. Thus, the noise canceling part of the circuit must be *inverting*. Notice the above circuit is already inverting. Our summer then must be non-inverting. The circuit is show below.



3. **Mechanical: Correlation**



(a) Calculate and plot the **autocorrelation** (the inner products of one period of the signal with all the possible shifts of one period of the same signal) of each of the above signals. Each signal is periodic with a period of 10 (one period is shown).

Solution: The solution is given in `sol9.ipynb`. (It is fine if you did the calculations and plotting by hand.)

(b) Calculate and plot the **cross-correlation** (the inner products of one period of the first signal with all possible shifts of one period of the second signal) of the two signals. Each signal is periodic with a period of 10 (one period is shown).

Solution: The solution is given in `sol9.ipynb`. (It is fine if you did the calculations and plotting by hand.)

4. **Inner products**

The Cauchy-Schwarz inequality states that for two vectors $\vec{x}, \vec{y} \in \mathbb{R}^n$:

$$|\langle \vec{x}, \vec{y} \rangle| = |\vec{x}^T \vec{y}| \leq ||\vec{x}|| \cdot ||\vec{y}||$$

Use the Cauchy-Schwarz inequality to verify (i.e. prove or derive) the triangle inequality:

$$||\vec{x} + \vec{y}|| \leq ||\vec{x}|| + ||\vec{y}||$$

(Hint: Start with $||\vec{x} + \vec{y}||^2$)

Solution: We consider the Euclidean 2-norm here. By Cauchy-Schwarz inequality we have

$$|\langle \vec{x}, \vec{y} \rangle| \leq \|\vec{x}\|_2 \cdot \|\vec{y}\|_2$$

$$|\langle \vec{y}, \vec{x} \rangle| \leq \|\vec{y}\|_2 \cdot \|\vec{x}\|_2$$

Therefore,

$$\begin{aligned}
\|\vec{x} + \vec{y}\|_2^2 &= (\vec{x} + \vec{y})^T (\vec{x} + \vec{y}) \\
&= \|\vec{x}\|_2^2 + \|\vec{y}\|_2^2 + \langle \vec{x}, \vec{y} \rangle + \langle \vec{y}, \vec{x} \rangle \\
&\leq \|\vec{x}\|_2^2 + \|\vec{y}\|_2^2 + 2\|\vec{y}\|_2 \cdot \|\vec{x}\|_2 \\
&= (\|\vec{x}\|_2 + \|\vec{y}\|_2)^2
\end{aligned}$$

Taking square root on both sides, we get

$$\|\vec{x}+\vec{y}\|_2 \leq \|\vec{x}\|_2 + \|\vec{y}\|_2$$

5. **Redo the midterm.**
   Redo the midterm problems (released Thursday after the midterm). **Solution:** See midterm solutions.

6. **Your Own Problem** Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?