Homework 1 EE16a Shivani Patel 3033800943

1. Counting solutions

1a) $2x + 3y = 5$

$x + y = 2$

$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 1 & 1 & 2 \end{array}\right]$$

$R1 \leftrightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 3 & 5 \end{array}\right]$$

$R_2 \leftarrow -2R_1 + R_2$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

one solution

$x + y = 2$

$$\boxed{\begin{array}{l} y = 1 \\ x = 1 \end{array}}$$

1b) $x + y + z = 3$

$2x + 2y + 2z = 5$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 2 & 2 & 5 \end{array}\right]$$

$R_2 \leftarrow 2R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

No solution because the last row cannot contain a value for 0's $0 \neq 1$

1c) $-y + 2z = 1$

$2x + z = 2$

$$\left[\begin{array}{ccc|c} 0 & -1 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 2 \\ 0 & -1 & 2 & 1 \end{array}\right]$$

$R_1 \leftarrow R_1/2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1 \\ 0 & -1 & 2 & 1 \end{array}\right]$$

$R_2 \leftarrow -R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1 \\ 0 & 1 & -2 & -1 \end{array}\right]$$

Infinite solutions
$x = 1 - (1/2)z$
$y = -1 + 2z$
z = free variable (ex: 1)
if $z = 1$ then $x = -1/2$ and $y = 1$

1d) $$\begin{aligned} x + 2y &= 3 \\ 2x - y &= 1 \\ 3x + y &= 4 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 4 \end{array}\right]$$

$R_2 \leftarrow -2R_1 + R_2$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 3 & 1 & 4 \end{array}\right]$$

$R_3 \leftarrow -3R_1 + R_3$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & -5 & -5 \end{array}\right]$$

$R_2 \leftarrow R_2 / -5$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -5 & -5 \end{array}\right]$$

$R_3 \leftarrow 5R_2 + R_3$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$$x + 2y = 3$$
$$y = 1$$
$$x + 2(1) = 3$$
$$-2 \quad -2$$

$\boxed{x = 1,\ y = 1}$

1e) $$\begin{aligned} x + 2y &= 3 \\ 2x - y &= 1 \\ x - 3y &= -5 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 1 & -3 & -5 \end{array}\right]$$

$R_2 \leftarrow -2R_1 + R_2$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 1 & -3 & -5 \end{array}\right]$$

$R_3 \leftarrow -1R_1 + R_3$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & -5 & -8 \end{array}\right]$$

$R_2 \leftarrow R_2 / 5$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -5 & -8 \end{array}\right]$$

$R_3 \leftarrow 5R_1 + R_3$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{array}\right]$$

$\boxed{\text{No solution because the last row cannot have a value for zero, } 0 \neq -3}$

2. Dutta's optimal Boba

x_b = black $\quad x_g$ = green

x_o = oolong $\quad x_{eg}$ = earl grey

combination Drunk	score
classic	7
Roasted	7
mountain	7 2/5
okinawa	6 1/3

$$\text{classic} = \tfrac{1}{3}x_b + \tfrac{1}{3}x_o + \tfrac{1}{3}x_{eg} = 7 \quad = 7$$

$$\text{Roasted} = \tfrac{1}{3}x_b + \tfrac{1}{3}x_o + \tfrac{1}{3}g = 7 \quad = 7$$

$$\text{mountain} = \tfrac{2}{5}x_o + \tfrac{3}{5}x_g = 7\tfrac{2}{5} \quad = \tfrac{37}{5}$$

$$\text{okinawa} = \tfrac{2}{3}x_b + \tfrac{1}{3}x_o = 6\tfrac{1}{3} \quad = \tfrac{19}{3}$$

$$\begin{array}{cccc} b & o & g & eg \end{array}$$
$$\left[\begin{array}{cccc|c} \tfrac{1}{3} & \tfrac{1}{3} & 0 & \tfrac{1}{3} & 7 \\ \tfrac{1}{3} & \tfrac{1}{3} & \tfrac{1}{3} & 0 & 7 \\ 0 & \tfrac{2}{5} & \tfrac{3}{5} & 0 & 7\tfrac{2}{5} \\ \tfrac{2}{3} & \tfrac{1}{3} & 0 & 0 & 6\tfrac{1}{3} \end{array}\right]$$

normalize all digits

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 21 \\ 1 & 1 & 1 & 0 & 21 \\ 0 & 2 & 3 & 0 & 37 \\ 2 & 1 & 0 & 0 & 19 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 21 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 2 & 3 & 0 & 37 \\ 0 & -1 & 0 & -2 & -23 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 23 \\ 0 & 2 & 3 & 0 & 37 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 23 \\ 0 & 0 & 3 & -4 & -9 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 23 \\ 0 & 0 & 1 & -2 & -9 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 23 \\ 0 & 0 & 1 & -2 & -9 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right]$$

2a) $\boxed{\begin{array}{l}\text{Professor Courtade's ratings for each tea are:}\\ \text{Black} = 7\\ \text{Oolong} = 5\\ \text{Green} = 9\\ \text{Earl Grey} = 9\end{array}}$

2b) there are infinite amount of solutions using green tea and Early Grey must add up to one cup. One possible solution is ½ cup green, ½ cup Earl Grey. A possible score that can be given is 9.

3. Filtering out the troll

3a) $\vec{m_1} = f_1(\alpha)\vec{a} + f_1(\beta)\vec{b}$ $\qquad 45\left(\frac{\pi}{180}\right) = \frac{\pi}{4} = a$

$\vec{m_2} = f_2(\alpha)\vec{a} + f_2(\beta)\vec{b}$ $\qquad 330\left(\frac{\pi}{180}\right) = \frac{33\pi}{18} = b$

$\vec{m_1} = \cos(45°)\vec{a} + \cos(-30°)\vec{b}$

$\vec{m_2} = \sin(45°)\vec{a} + \sin(-30)\vec{b}$

$$\boxed{a\begin{bmatrix}\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\end{bmatrix} + b\begin{bmatrix}\frac{\sqrt{3}}{2}\\ -\frac{1}{2}\end{bmatrix} = \begin{bmatrix}m_1\\ m_2\end{bmatrix}}$$

3b) $\vec{a} = u \cdot \vec{m_1} + v \cdot \vec{m_2}$ $\qquad u, v = \text{scalars}$

$\frac{\sqrt{2}}{2}a + \frac{\sqrt{3}}{2}b = m_1 \Rightarrow b = \frac{2}{\sqrt{3}}m_1 - \frac{\sqrt{2}}{\sqrt{3}}a$

$\frac{\sqrt{2}}{2}a - \frac{1}{2}b = m_2 \Rightarrow b = -2m_2 + \sqrt{2}a$

$\frac{2}{\sqrt{3}}m_1 - \sqrt{\frac{2}{3}}a = -2m_2 + \sqrt{2}a$

$+2m_2 \qquad +2m_2$

$\frac{2}{\sqrt{3}}m_1 + 2m_2 - \sqrt{\frac{2}{3}}a = \sqrt{2}a$

$+\sqrt{\frac{2}{3}}a \qquad +\sqrt{\frac{2}{3}}a$

$$\frac{2}{\sqrt{3}} m_1 + 2m_2 = \sqrt{2}a + \sqrt{\frac{2}{3}}a$$

$$\frac{2}{\sqrt{3}} m_1 + 2m_2 = a\left(\sqrt{2} + \sqrt{\frac{2}{3}}\right) \quad \sqrt{2} \to \sqrt{6}$$

$$= a\left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{3}}\right)$$

$$\frac{\sqrt{6}+\sqrt{2}}{\sqrt{3}}$$

$$\frac{2}{\sqrt{6}+\sqrt{2}} m_1 + \frac{2\sqrt{3}}{\sqrt{6}+\sqrt{2}} m_2 = a$$

$\boxed{\text{the values of } u = \frac{2}{\sqrt{6}+\sqrt{2}} \text{ and } v = \frac{2\sqrt{3}}{\sqrt{6}+\sqrt{2}}}$

3c. "All human beings are born free and equal in dignity and rights." - Universal declaration of human rights

4 Homework process and study group

I worked with sadia Qureshi (3034541667)

EECS16A: Homework 1

Problem 2: Filtering Out The Troll

In [1]:
```
import warnings
import wave as wv

import matplotlib.pyplot as plt
import numpy as np
import scipy
import scipy.io.wavfile
from IPython.display import Audio
from scipy import io
from scipy.io.wavfile import read

# For this to work make sure to download m1.wav and m2.wav to the same l
ocation as this jupyter notebook
warnings.filterwarnings("ignore")
sound_file_1 = "m1.wav"
sound_file_2 = "m2.wav"
```

Let's listen to the recording of the first microphone (it can take some time to load the sound file). Run the cell below, then press the play button to listen.

In [3]:
```
Audio(url="m1.wav", autoplay=False)
```

Out[3]:

0:00 / 0:00

And this is the recording of the second microphone (it can take some time to load the sound file). Run the cell below, then press the play button to listen.

In [4]:
```
Audio(url="m2.wav", autoplay=False)
```

Out[4]:

0:00 / 0:00

We read the first recording to the variable `corrupt1` and the second recording to `corrupt2`. Treat `corrupt1` and `corrupt2` as the two sound recordings picked up by microphone 1 and microphone 2 respectively.

In [5]:
```
rate1, corrupt1 = scipy.io.wavfile.read("m1.wav")
rate2, corrupt2 = scipy.io.wavfile.read("m2.wav")
```

Enter the weights of the two recordings to get the clean speech.

Note: The square root of a number a can be written as `np.sqrt(a)` in IPython.

In [6]:
```
# enter the weights u (recording 1) and v (recording 2)
u = 2/(np.sqrt(6)+np.sqrt(2))
v = (2*np.sqrt(3))/(np.sqrt(6)+np.sqrt(2))
```

Weighted combination of the two recordings:

In [7]:
```
a = u * corrupt1 + v * corrupt2
```

Let's listen to the resulting sound file (make sure your speaker's volume is not very high, the sound may be loud if things go wrong).

In [8]:
```
Audio(data=a, rate=rate1)
```

Out[8]:

0:00 / 0:10

In [ ]:
```

```