EECS 16A Designing Information Devices and Systems I
Spring 2016 Elad Alon, Babak Ayazifar Homework 4

This homework is due February 23, 2016, at Noon.

Optional Problems: We **do not** grade these problems. Nevertheless, you are responsible for learning the subject matter within their scope.
Bonus Problems: We **do** grade these problems. Doing them will provide an unspecified amount of extra credit; not doing them will not affect your homework grade negatively. We will specify if the problem is in or out of scope.

1. Homework process and study group

Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?
Working in groups of 3-5 will earn credit for your participation grade.

Solution: I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on Problem 5 so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.

2. Finding Null Spaces

(a) Consider the column vectors of any 3×5 matrix. What is the maximum possible number of linearly independent vectors you can pick from these column vectors?

Solution: Since the column vectors are in $\mathbb{R}^3$, there are at most 3 linearly independent vectors. Hence we can say that the column space of this matrix has dimension at most 3.

(b) Suppose we have the following 3×5 matrix after row reduction:

$$A = \begin{bmatrix} 1 & 1 & 0 & -2 & 3 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

What is the minimum number of vectors spanning the range of A. Find a set of such vectors.

Solution:

For any vector x, Ax is a linear combination of the columns of A, thus the range of A is a linear combination of its columns. We can see that there are only two linearly independent columns because the third component for each column vector is 0. Therefore a set of linearly independent vectors spanning the range of A is:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

(c) Recall that for every vector $\vec{x}$ in the null space of A, $A\vec{x} = \vec{0}$. The dimension of a the null space is the minimum number of vectors needed to span it. Find vectors that span the nullspace of A (the matrix in the previous part). What is the dimension of the nullspace of A?

Solution: Finding the null space of A is the same as solving the following system of linear equations:

$$\begin{bmatrix} 1 & 1 & 0 & -2 & 3 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0$$

$$\begin{aligned} x_1 + x_2 - 2x_4 + 3x_5 &= 0 \\ x_3 - x_4 + x_5 &= 0 \end{aligned}$$

We have 5 unknowns but only 2 linearly independent equations. Therefore there are 3 degrees of freedom in the null space. Hence the dimension of the nullspace is 3. Notice that because of the way Gaussian elimination is performed, x_1 and x_3 only appear once in each equation at the head of their respective rows/equations. Thus we let x_2, x_4 and x_5 be free variables a, b and c. Now we re-write the equations:

$$\begin{aligned} x_1 &= -a + 2b - 3c \\ x_2 &= a \\ x_3 &= b - c \\ x_4 &= b \\ x_5 &= c \end{aligned}$$

We can then write this in vector form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = a \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} -3 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Therefore the null space of A is spanned by the vectors:

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Look more closely at these vectors. Notice anything about their entries as compared to the row-reduced matrix? (Remember, here the row-reduction went both down and then back up so that the columns that actively participated in row-reduction have only one 1 in them and that 1 leads its row.) They have 1s in certain positions that represent the free variables for their respective vectors and the other vectors have 0s in those positions. Meanwhile, all the other entries have their signs flipped from the corresponding little column in the fully row-reduced matrix. This is not a coincidence and if you look closely at the pattern of the derivation above, you will see why this must always be the case.

(d) Find vector(s) that span the null space of the following matrix:

$$B = \begin{bmatrix} 1 & -2 & 2 & 4 \\ 1 & -2 & 3 & 5 \\ 2 & -4 & 5 & 9 \\ 3 & -6 & 7 & 13 \end{bmatrix}$$

Solution: Using Gaussian Elimination, we can reduce the matrix:

$$\begin{bmatrix} 1 & -2 & 2 & 4 \\ 1 & -2 & 3 & 5 \\ 2 & -4 & 5 & 9 \\ 3 & -6 & 7 & 13 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 2 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Vectors in the null space satisfy the following equations:

$$\begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

$$x_1 - 2x_2 + 2x_4 = 0$$
$$x_3 + x_4 = 0$$

We then set x_2 and x_4 to be free variables and substitute in:

$$x_1 = 2a - 2b$$
$$x_2 = a$$
$$x_3 = -b$$
$$x_4 = b$$

We then write these equations in vector form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = a \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Therefore the null space of the matrix is spanned by the vectors:

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

Notice the same pattern as before in terms of the relationship of the null-space basis found to the fully row-reduced matrix.

3. Traffic Flows

Your goal is to measure the flow rates of vehicles along roads in a town. However, it is prohibitively expensive to place a traffic sensor along every road. You realize, however, that the number of cars flowing into an intersection must equal the number of cars flowing out. You can use this "flow conservation" to determine the traffic along all roads in a network by only measuring flow along only some roads. In this problem we will explore this concept.

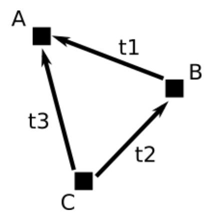


Figure 1: A simple road network.

(a) Let's begin with a network with three intersections, A, B and C. Define the flows t_1 as the rate of cars (cars/hour) on the road between B and A, t_2 as the rate on the road between C and B and t_3 as the rate on the road between C and A.

(Note: The directions of the arrows in the figure are only the way that we define the flow by convention. If there were 100 cars per hour traveling from A to C, then $t_3 = -100$.)

We assume the "flow conservation" constraints: the total number of cars per hour flowing into each intersection is zero. For example at intersection B, we have the constraint $t_2 - t_1 = 0$. The full set of constraints (one per intersection) is:

$$\begin{cases} t_1 + t_3 & = 0 \\ t_2 - t_1 & = 0 \\ -t_3 - t_2 & = 0 \end{cases}$$

As mentioned earlier, we can place sensors a road to measure the flow through it. But, we have a limited budget, and we would like to determine all of the flows with the smallest possible number of sensors.

Suppose for the network above we have one sensor reading, $t_1 = 10$. Can we figure out the flows along the other roads? (That is, the values of t_2 and t_3).

Solution: Yes, since we know that $t_1 = t_2 = -t_3$, so we must have $t_2 = 10$ and $t_3 = -10$.

(b) Now suppose we have a larger network, as shown in Figure **??**.

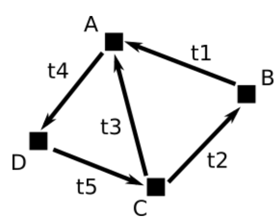


Figure 2: A larger road network.

We would again like to determine the traffic flows on all roads, using measurements from some sensors.

A Berkeley student claims that we need two sensors placed on the roads AD and BA. A Stanford student claims that we need two sensors placed on the roads CB and BA. Is it possible to determine all traffic flows with the Berkeley student's suggestion? How about the Stanford student's suggestion?

Solution: The Stanford student is wrong (obviously). Observing t_1 and t_2 is not sufficient, as t_3, t_4 and t_5 can still not be uniquely determined. Specifically, for any $t \in \mathbb{R}$, the following flow satisfies the constraints and the measurements:

$$t_4 = t$$
$$t_5 = t$$
$$t_3 = t - t_1$$

On the other hand, if we're given t_1 and t_4, we can uniquely solve for all the traffic densities as follows since we know the flow conservation constraints. We know that t_2 is the same as t_1 and t_4 is the same as t_5, since the flow going into B and D must equal to the flow going out. The flow into A, $t_1 + t_3$, must equal the flow going out, t_4, so:

$$t_2 = t_1$$
$$t_5 = t_4$$
$$t_3 = t_4 - t_1$$

This is related to the fact that t_1 and t_4 are parts of different cycles in the graph, whereas t_1 and t_2 are in the same cycle, so measuring both of them would not give additional information.

(c) Suppose we write the traffic flow on all roads as a vector $\vec{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix}$. Show that the set of valid flows (which satisfy the conservation constraints) form a subspace. Then, determine the subspace of traffic flows for the network of Figure **??**. Specifically, express this space as the span of two linearly independent vectors.

(Hint: Use the claim of the correct student in the previous part)

Solution:

Suppose we have a set of valid flows $\vec{t}$. Then for any intersection, the net flow into it is the same as the net flow out of it. If we scale $\vec{t}$ by a constant a, each t_i will also get scaled by a. The net flows into and out of the intersection would be scaled by the same amount and remain equal to each other. Thus any scaling of a valid flow is still a valid flow. Suppose now we add valid flows $\vec{t_1}$ and $\vec{t_2}$ to get $\vec{t} = \vec{t_1} + \vec{t_2}$. For any intersection I,

$$\text{net flow into } I = \text{net flow into } I \text{ from } \vec{t_1} + \text{net flow into } I \text{ from } \vec{t_2}$$
$$\text{net flow out of } I = \text{net flow out of } I \text{ from } \vec{t_1} + \text{net flow out of } I \text{ from } \vec{t_2}$$

Since the net flow into I from $\vec{t_1}$ is the same as net flow out of I from $\vec{t_1}$ and similarly for $\vec{t_2}$, the net flow into I is the same as the net flow out of I. Therefore the sum of any two valid flows is still a valid flow. Also, $\vec{t} = \vec{0}$ is a valid flow. Therefore the set of valid flows forms a subspace.

To determine the subspace of traffic flows for the above network, use the solution in the previous part to see what $\vec{t}$ looks like in terms of $t_1 = \alpha$ and $t_4 = \beta$:

$$\vec{t} = \begin{bmatrix} \alpha \\ \alpha \\ \beta - \alpha \\ \beta \\ \beta \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \vec{u_1} + \beta \vec{u_2}$$

Clearly, $\vec{u_1}$ and $\vec{u_2}$ are linearly independent, and the space of all possible traffic flows is the span of them.

(d) We would like a more general way of determining the possible traffic flows in a network. As a first step, let us try to write all the flow conservation constraints (one per intersection) as a matrix equation. Find a (4×5) matrix B such that the equation $B\vec{t} = \vec{0}$:

$$\begin{bmatrix} & & \\ & B & \\ & & \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

represents the flow conservation constraints for the network of Figure **??**.

(Hint: Each row is the constraint of an intersection. You can construct B using only $0, 1, -1$ *entries.)*

This matrix is called the *incidence matrix*. What does each row of this matrix represent? What does each column of this matrix represent?

Solution:

$$B = \begin{array}{c} \begin{bmatrix} +1 & 0 & +1 & -1 & 0 \\ -1 & +1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & +1 \\ 0 & 0 & 0 & +1 & -1 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix} \\ \begin{matrix} t_1 & \quad t_2 & \quad t_3 & \quad t_4 & \quad t_5 \end{matrix} \end{array}$$

(The rows of this matrix can be in any order.) Each row represents an intersection, and each column represents a road between two intersections. Each $+1$ on a row represents a road flowing into an intersection, and each -1 represents a road flowing out of an intersection. Each $+1$ in a column represents the source intersection of a road, and each -1 in a column represents the destination intersection of a road.

(e) Notice that the set of all vectors $\vec{t}$ which satisfy $B\vec{t} = \vec{0}$ is exactly the nullspace of the matrix B. That is, we can find all valid traffic flows by computing the nullspace of B. Use Guassian Elimination to determine the dimension of the nullspace of B, and compute a basis for the nullspace. (You may use a computer to compute reduced-row-echelon-form). Does this match your answer to part **??**? Can you interpret the dimension of the nullspace of the incidence matrix, for the road networks of Figure **??** and Figure **??**?

Solution: After row-reducing, we get the following matrix:

$$\begin{bmatrix} +1 & 0 & +1 & 0 & -1 \\ 0 & +1 & +1 & 0 & -1 \\ 0 & 0 & 0 & +1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the column rank is 3, the dimension of the nullspace is 2. We can find the following basis (using that observation that we had made that connects the basis vectors to the row-reduced matrix. Here, t_3 and t_5 are the free variables and so get 1s with the rest coming from sign flipping.) for the null-space:

$$a \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

This does not match the answer in the earlier part because these are two different bases. But the nullspace they span is the same.

By itself, the first vector weighted by a clearly is a vector corresponding to a small cycle in the graph. But the second one b is going around a bigger cycle. These two cycles are still independent of each other though. This is why the dimension of the nullspace can be interpreted as the number of "independent cycles" in the graph.

It is fine to give yourself full credit as long as you found a basis for the nullspace. It doesn't have to be this particular one.

(f) Now let us analyze general road networks. Say there is a road network graph G, with incidence matrix B_G. If B_G has a k-dimensional nullspace, does this mean measuring the flows along *any* k roads is always sufficient to recover the exact flows? Prove or give a counterexample.

(Hint: Consider the Stanford student.)

Solution: No, consider the network of Figure **??**. The corresponding incidence matrix has a $k = 2$ dimensional nullspace, as you showed in part (e). However, measuring t_1 and t_2 (as the Stanford student suggested) is not sufficient, as you showed in part (b).

(g) Let G be a network of n roads, with incidence matrix B_G that has a k-dimensional nullspace. We would like to characterize exactly when measuring the flows along a set of k roads is sufficient to recover the exact flow along all roads. To do this, it will help to generalize the problem, and consider measuring *linear combinations* of flows. If $\vec{t}$ is a traffic flow vector, assume we can measure linear combinations $\vec{m}_i^T\vec{t}$ for some vectors $\vec{m}_i$. Then making k measurements is equivalent to observing the vector $M\vec{t}$ for some $(k \times n)$ "measurement matrix" M (consisting of rows $\vec{m}_i^T$).

For example, for the network of Figure **??**, the measurement matrix corresponding to measuring t_1 and t_4 (as the Berkeley student suggests) is:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Similarly, the measurement matrix corresponding to measuring t_1 and t_2 (as the Stanford student suggests) is:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

For general networks G and measurements M, give a condition for when the exact traffic flows can be recovered, in terms of the nullspace of M and the nullspace of B_G.

(Hint: Recovery will fail iff there are two valid flows with the same measurements. Can you express this in terms of the nullspaces of M and B_G?)

Solution: As stated in the hint, we cannot uniquely determine the flow iff there are two valid flows that yield the same set of measurements. That is, there should not be two *distinct* valid flows $\vec{t}_1$ and $\vec{t}_2$ such that $M\vec{t}_1 = M\vec{t}_2$. Or equivalently, such that $M(\vec{t}_1 - \vec{t}_2) = 0$.

The set of valid flows is the nullspace of B_G, denoted $Null(B_G)$. So recovery fails if $M(\vec{t}_1 - \vec{t}_2) = 0$ for some $\vec{t}_1, \vec{t}_2 \in Null(B_G)$, with $\vec{t}_1 \neq \vec{t}_2$. The set of valid flows is a subspace, so we can equivalently state this as: Recovery fails iff $M\vec{t} = 0$ for some $\vec{t} \neq \vec{0}$, $\vec{t} \in Null(B_G)$.

In other words, *there should be no vector $\vec{t} \neq 0$ that is both in the nullspace of B_G* and *in the nullspace of M*.

This can also be stated as: *We can recover the exact traffic flows iff the nullspace of B_G does not non-trivially intersect the nullspace of M.*

Full credit for stating any condition that is equivalent to this, using the nullspaces of M and B_G.

(h) *(Bonus)* Express the condition of the previous part in a way that can be checked computationally. For example, suppose we are given a huge road network G of all roads in Berkeley, and we want to find if our measurements M are sufficient to recover the flows.

(Hint: Consider a matrix U whose columns form a basis of the nullspace of B_G. Then $\{U\vec{x} : \vec{x} \in R^k\}$ is exactly the set of all possible traffic flows. How can we represent measurements on these flows?)

Solution: Let U be a matrix whose columns form a basis of the nullspace of B_G. Then, as in the hint, the set $\{U\vec{x} : \vec{x} \in R^k\}$ is exactly the set of all possible traffic flows.

For a given valid flow $\vec{t} = U\vec{x}$, the result of measuring this flow is $M\vec{t} = MU\vec{x}$. Now, recovering the exact flow from our measurements is equivalent to recovering $\vec{x}$ from $MU\vec{x}$. Notice that the matrix MU is $(k \times k)$, so to we can recover the exact flows iff MU is invertible. This condition can be easily checked computationally (for example, by row-reducing MU).

Remark: Notice how defining the matrix U allowed us to work with flows in terms of their low-dimensional representations ($\vec{x}$), instead of dealing directly with all their components.

(i) *(Bonus)* If the incidence matrix B_G has a k-dimensional nullspace, does this mean we can always pick a set of k roads such that measuring the flows along these roads is sufficient to recover the exact flows? Prove or give a counterexample.

Solution: Yes.

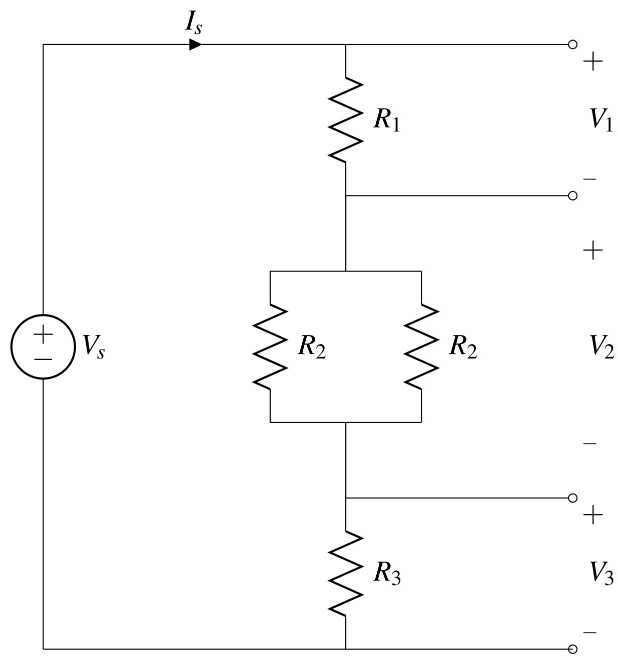
Let U be a matrix whose columns form a basis of the nullspace of B_G, as above. The k columns of U are linearly independent, since they form a basis. Since there are k linearly independent columns, when we run Gaussian-Elimination on U, we must get k pivots. (Recall that "pivot" is the technical term for being able to row-reduce and turn a column into something that has exactly one 1 in it. The pivot is the entry that we found and turned into that 1.)

Therefore the row-space of U is k dimensional, since there are some k linearly independent rows in U — namely the ones where we found pivots. Choose to measure the roads corresponding to these rows.

This will work because: For a given valid flow $\vec{t} = U\vec{x}$, the results of measuring this flow vector is $U^{(k)}\vec{x}$, where the matrix $U^{(k)}$ is some k linearly independent rows of U. By construction, the $(k \times k)$ matrix $U^{(k)}$ has all linearly independent rows, so we can invert $U^{(k)}$ to find $\vec{x}$ from $U^{(k)}\vec{x}$, and then recover the flows along all the edges as $U\vec{x}$.

This isn't the only set of k roads that will work. But it does provide a set of k roads that are guaranteed to work.

4. Faerie Mazes



Suppose $R_1 = 10$ Nolans, $R_2 = 20$ Nolans, and $R_3 = 30$ Nolans. Use the units Nolans, Vigors, and $\frac{\text{Imps}}{\text{second}}$. How much Vigor V_s must be provided such that the imp flow is $I = 2\,\frac{\text{Imps}}{\text{second}}$? With this Vigor applied, what are the vigor changes V_1, V_2, and V_3?

Solution: First write out the vigor (voltage) and imp (current) relationships:

$$V_s = V_1 + V_2 + V_3$$

All of I_s will pass through R_1 (conservation of imps), but will split into I_{Left} and I_{Right} when it reaches R_2. Then the imp flow will recombine into I_s to pass through R_3.

$$V_1 = I_s \cdot R_1$$
$$V_2 = I_L \cdot R_2 = I_R \cdot R_2$$
$$V_3 = I_s \cdot R_3$$

Since:

$$I_L \cdot R_2 = I_R \cdot R_2 \implies I_L = I_R$$
$$I_s = I_L + I_R = 2I_L \implies I_L = \frac{I_s}{2}$$
$$V_s = I_s R_1 + \frac{I_s}{2} R_2 + I_s R_3$$
$$V_s = 2\,\frac{\text{Imps}}{\text{second}} \cdot 10 \text{ Nolans} + \frac{2}{2} \cdot 20 + 2 \cdot 30 = 100 \text{ Vigors}$$

Plugging values back into the vigor relationships above:

$$V_1 = I_s \cdot R_1 = 2 \cdot 10 = 20 \text{ Vigors}$$
$$V_2 = \frac{I_s}{2} \cdot R_2 = \frac{2}{2} \cdot 20 = 20 \text{ Vigors}$$
$$V_3 = I_s \cdot R_3 = 2 \cdot 30 = 60 \text{ Vigors}$$

If you know the kinked roads (resistors) in series and parallel shortcuts, you can recognize that:

$$R_2' = \frac{1}{\frac{1}{R_2} + \frac{1}{R_2}} = \frac{R_2}{2}$$

$$V_s = I_s(R_1 + R_2' + R_3) = 2\,\frac{\text{Imps}}{\text{second}} \cdot (10N + 10N + 30N) = 2 \cdot 50 = 100 \text{ Vigors}$$

$$V_1 = I\,R_1 = 2A \cdot 10N = 20V$$
$$V_2 = I\,R_2' = 2A \cdot 10N = 20V$$
$$V_3 = I\,R_3 = 2A \cdot 30N = 60V$$

5. **Midterm Problem 3**
Redo Midterm Problem 3.

Solution: See midterm solutions.

6. **Midterm Problem 4**
Redo Midterm Problem 4.

Solution: See midterm solutions.

7. **Midterm Problem 5**
Redo Midterm Problem 5.

Solution: See midterm solutions.

8. **Midterm Problem 6**
Redo Midterm Problem 6.

Solution: See midterm solutions.

9. **Midterm Problem 7**
Redo Midterm Problem 7.

Solution: See midterm solutions.

10. **Midterm Problem 8**
Redo Midterm Problem 8.

Solution: See midterm solutions.

11. **Midterm Problem 9**
Redo Midterm Problem 9.

Solution: See midterm solutions.

12. **Your Own Problem** Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?