Homogenity $- f(\alpha x) = \alpha f(x)$
Additivity $- f(x+y) = f(x) + f(y)$

$f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) = \alpha f(x_1, x_2) + \beta f(y_1, y_2)$

EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 1B

All linear equations
$f(x_1 \dots x_n) = a_1 x_1 + a_2 x_2 \dots a_n x_n$

- line going through the origin is linear
- line going through intercept is nonlinear

1. Linear or Nonlinear

Determine whether the following functions (f: $\mathbb{R}^2 \to \mathbb{R}$) are linear or nonlinear.

$f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) = \alpha f(x_1, x_2) + \beta f(y_1, y_2)$

(a) $f(x_1, x_2) = 3x_1 + 4x_2$
$= 3(\alpha x_1 + \beta y_1) + 4(\alpha x_2 + \beta y_2) = \alpha(3x_1 + 4x_2) + \beta(3y_1 + 4y_2)$

$$f(x_1, x_2) = 3x_1 + 4x_2$$

(b) nonlinear

$$f(x_1, x_2) = e^{x_2} + x_1^2$$

illegal

(c)

$$f(x_1, x_2) = x_2 - x_1 + 3$$

doesn't scale by α because of the constant
affine

$f(\alpha x_1, \alpha x_2) = \alpha x_2 - \alpha x_1 + 3$

2. Solving Systems of Equations

(a) Systems of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following systems of equations, state whether there is a unique solution, no solution, or an infinite number of solutions. If there are an infinite number of solutions give one possible solution.

Add
scale ①
swap

$a \in \mathbb{R}$
Infinite solutions

i. $\begin{cases} 49x + 7y = 49 \\ 42x + 6y = 42 \end{cases}$

$$\begin{array}{cc} x & y \end{array}$$
$$\left[\begin{array}{cc|c} 49 & 7 & 49 \\ 42 & 6 & 42 \end{array}\right]$$
$R_1 \leftarrow R_1/49$
$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 42 & 6 & 42 \end{array}\right]$$
$R_2 \leftarrow R_2/42$
$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 1 & 1/7 & 1 \end{array}\right]$$
infinite solutions
$R_2 \leftarrow R_1 - R_2$
$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

ii. $\begin{cases} 5x + 3y = -21 \\ 2x + y = -9 \end{cases}$

iii. $\begin{cases} 49x + 7y = 60 \\ 42x + 6y = 30 \end{cases}$

see bottom of page

iv. $\begin{cases} 2x + 2y + 4z = -1 \\ y + z = -2 \\ x + 2y + 3z = 2 \end{cases}$

v. $\begin{cases} 2x + 2y + 4z = 6 \\ y + z = 1 \\ x + 2y + 3z = 4 \end{cases}$

vi. $\begin{cases} x + y + z = 4 \\ 3z = 6 \\ y + z = 3 \end{cases}$

(b) Systems of equations can also be interpreted graphically. We will try to build a graphical intuition for the results you found in the previous part. Follow along as your TA walks through `dis1B.ipynb`.

3. Vectors Introduction to vectors and vector addition.

Definitions:
Vector: An ordered list of elements - for example:

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$$

$\mathbb{R}$ or $\mathbb{R}^1$: The set of all real numbers (i.e. the real line)
$\mathbb{R}^2$: The set of all two-element vectors with real numbered entries (i.e. plane of 2×1 vectors) - for example:

$$\vec{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \in \mathbb{R}^2$$

$\mathbb{R}^3$: The set of all three-element vectors with real numbered entries (i.e. 3-space of 3×1 vectors) - for example:

$$\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3$$

$\mathbb{R}^n$: The set of all n-element vectors with real numbered entries (i.e. n-space of $n \times 1$ vectors)

(a) Are the following vectors in $\mathbb{R}^2$?

i. $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$ yes

ii. $\begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$ no $\in \mathbb{R}^3$

(b) Graphically show the vectors:

i. $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

ii. $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

(c) Graphically show the vector sum and check your answer algebraically:

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

2i. $49x + 7y = 49$

$42x + 6y = 42$

$$\begin{array}{cc} x & y \\ \end{array}$$
$$\left[\begin{array}{cc|c} 49 & 7 & 49 \\ 42 & 6 & 42 \end{array}\right]$$

$R_1 \leftarrow R_1/49$

$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 42 & 6 & 42 \end{array}\right]$$

$R_2 \leftarrow R_2/42$

$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 1 & 1/7 & 1 \end{array}\right]$$ infinite solutions

$R_2 \leftarrow R_1 - R_2$

$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

infinite solutions

$a \in \mathbb{R}$

2ii. $5x + 3y = -21$

$2x - y = -9$

$$\begin{array}{cc} x & y \\ \end{array}$$
$$\left[\begin{array}{cc|c} 5 & 3 & -21 \\ 2 & -1 & -9 \end{array}\right]$$

$R1 \leftarrow R1/5$

$$\left[\begin{array}{cc|c} 1 & 3/5 & -21/5 \\ 2 & -1 & -9 \end{array}\right]$$

$R2 \leftarrow R_2 - 2R_1$

$$\left[\begin{array}{cc|c} 1 & 3/5 & \frac{-21}{5} \\ 0 & -1/5 & -3/5 \end{array}\right]$$

$R1 \leftarrow R1 \cdot 3/5$

$R2 \leftarrow R_2 \cdot -5$

$$\left[\begin{array}{cc|c} 1 & 0 & -6 \\ 0 & 1 & 3 \end{array}\right]$$

unique solution

$x = -6$

$y = 3$

2iv. $2x + 2y + 4z = 6$

$y + z = 1$

$x + 2y + 3z = 4$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & -1 \\ 0 & 1 & 1 & -2 \\ 1 & 2 & 3 & 2 \end{array}\right]$$

$R1 \leftarrow R1/2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1/2 \\ 0 & 1 & 1 & -2 \\ 1 & 2 & 3 & 2 \end{array}\right]$$

$R_1 \leftarrow R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1/2 \\ 0 & 1 & 1 & -2 \\ 0 & 1 & 1 & 5/2 \end{array}\right]$$

$R3 \leftarrow R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1/2 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 9/5 \end{array}\right]$$

inconsistent