EECS 16A Fall 2016 — Designing Information Devices and Systems I — Babak Ayazifar, Vladimir Stojanovic — Homework 1

This homework is due September 6th, 2016, at Noon.

1. **Homework process and study group**

   Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?
   Working in groups of 3-5 will earn credit for your participation grade.

   Solution: I worked on this homework with...

   I first worked by myself for 2 hours, but got stuck on Problem 5 so I went to office hours on...

   Then I went to homework party for a few hours, where I finished the homework.

2. **Stojanovic's Optimal Smoothies**

   Solution: `#SystemsOfEquations #GaussianElimination`

   Stojanovic's Optimal Smoothies has a unique way of serving its customers. To ensure the best customer experience, each customer gets a smoothie personalized to his or her tastes. Professor Stojanovic knows that a lot of customers don't know what they want, so when the customer walks up to the counter they are asked to taste four standard smoothies that cover the entire range of flavors found in smoothies.

   Each smoothie is made of $\frac{1}{2}$ cup Greek yogurt, $\frac{1}{8}$ cup vanilla soy milk, $\frac{1}{2}$ cup crushed ice, and 1 cup mystery fruit. The four standard smoothies have the following recipes for the cup of mystery fruit:

Fruit(cups)	Banana Berry	Caribbean Passion	Mango-a-go-go	Strawberries Wild
Strawberries	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$
Bananas	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{3}$
Mangos	0	$\frac{1}{3}$	$\frac{3}{5}$	0
Blueberries	$\frac{1}{3}$	0	0	0

   Each customer is assumed to have a score (from 0 to 10) for each fruit, and the total score for the smoothie is modeled as being computed by multiplying the score for a fruit with its proportion in the smoothie. For example, if a customer's score for strawberries is 6 and bananas is 3, then the total score for the Strawberries Wild smoothie would be $6 \times \frac{2}{3} + 3 \times \frac{1}{3} = 5$.

   After a customer gives a score (from 0 to 10) for each smoothie, Professor Stojanovic then calculates (on the spot!) how much the customer likes each fruit. Then Professor Stojanovic blends up a special smoothie that will maximize the customer's score.

   Professor Ayazifar was thirsty after grading midterms, so he decided to take a drink break at SOS. He walked in and gave the following ratings:

Smoothie	Score
Banana Berry	$6\frac{1}{3}$
Caribbean Passion	$6\frac{1}{3}$
Mango-a-go-go	$6\frac{4}{5}$
Strawberries Wild	$5\frac{2}{3}$

(a) What were Professor Ayazifar's ratings for each fruit? **Work this problem out by hand.**

Solution: Using Professor Ayazifar's ratings, Professor Stojanovic mentally records the following system of equations

$$\text{Banana Berry:} \quad 6\frac{1}{3} = \frac{1}{3}x_{\text{S}} + \frac{1}{3}x_{\text{Ba}} + \frac{1}{3}x_{\text{Bb}} \tag{1}$$

$$\text{Caribbean Passion:} \quad 6\frac{1}{3} = \frac{1}{3}x_{\text{S}} + \frac{1}{3}x_{\text{Ba}} + \frac{1}{3}x_{\text{M}} \tag{2}$$

$$\text{Mango-a-go-go:} \quad 6\frac{4}{5} = \frac{2}{5}x_{\text{Ba}} + \frac{3}{5}x_{\text{M}} \tag{3}$$

$$\text{Strawberries Wild:} \quad 5\frac{2}{3} = \frac{2}{3}x_{\text{S}} + \frac{1}{3}x_{\text{Ba}} \tag{4}$$

He then multiplies each equation by the denominator of the fraction (in order to make them easier to read)

$$19 = x_{\text{S}} + x_{\text{Ba}} + x_{\text{Bb}} \tag{5}$$

$$19 = x_{\text{S}} + x_{\text{Ba}} + x_{\text{M}} \tag{6}$$

$$34 = 2x_{\text{Ba}} + 3x_{\text{M}} \tag{7}$$

$$17 = 2x_{\text{S}} + x_{\text{Ba}} \tag{8}$$

He notes that he could write the above equations in matrix form

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 2 & 3 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{\text{S}} \\ x_{\text{Ba}} \\ x_{\text{M}} \\ x_{\text{Bb}} \end{bmatrix} = \begin{bmatrix} 19 \\ 19 \\ 34 \\ 17 \end{bmatrix} \tag{9}$$

and as the augmented matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & \vdots & 19 \\ 1 & 1 & 1 & 0 & \vdots & 19 \\ 0 & 2 & 3 & 0 & \vdots & 34 \\ 2 & 1 & 0 & 0 & \vdots & 17 \end{bmatrix} \tag{10}$$

Professor Stojanovic then proceeds to row reduce the matrix into reduced row echelon form as follows. (It's fine if you solved the system of equations by hand a different way. Here, however, we will demonstrate how to do it using Gaussian Elimination.)

Noticing that there is a 1 in the upper left hand corner, he subtracts Row 1 from Row 2 and 2×Row 1 from Row 4.

$$\begin{matrix} \text{Row2: subtract Row1} \\ \text{Row4: subtract } 2\times\text{Row1} \end{matrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 & \vdots & 19 \\ 0 & 0 & 1 & -1 & \vdots & 0 \\ 0 & 2 & 3 & 0 & \vdots & 34 \\ 0 & -1 & 0 & -2 & \vdots & -21 \end{bmatrix} \tag{11}$$

Since Row 2 has a 0 in the diagonal element, he multiplies Row 4 by -1 and then switch Rows 2 and 4

$$\begin{array}{c}\text{Multiply Row4 by -1}\\ \text{Switch Row2 and Row4}\end{array} \Rightarrow \left[\begin{array}{cccccc} 1 & 1 & 0 & 1 & \vdots & 19 \\ 0 & 1 & 0 & 2 & \vdots & 21 \\ 0 & 2 & 3 & 0 & \vdots & 34 \\ 0 & 0 & 1 & -1 & \vdots & 0 \end{array}\right] \tag{12}$$

He then subtracts Row 2 from Row 1 and $2\times$Row 2 from Row 3.

$$\begin{array}{c}\text{Row1: subtract Row2}\\ \text{Row3: subtract } 2\times\text{Row2}\end{array} \Rightarrow \left[\begin{array}{cccccc} 1 & 0 & 0 & -1 & \vdots & -2 \\ 0 & 1 & 0 & 2 & \vdots & 21 \\ 0 & 0 & 3 & -4 & \vdots & -8 \\ 0 & 0 & 1 & -1 & \vdots & 0 \end{array}\right] \tag{13}$$

He then switches Row 3 and Row 4,and then subtracts $3\times$ the new Row 3 from the new Row 4

$$\begin{array}{c}\text{Switch Row3 and Row4}\\ \text{Row4: subtract } 3\times\text{Row3}\end{array} \Rightarrow \left[\begin{array}{cccccc} 1 & 0 & 0 & -1 & \vdots & -2 \\ 0 & 1 & 0 & 2 & \vdots & 21 \\ 0 & 0 & 1 & -1 & \vdots & 0 \\ 0 & 0 & 0 & -1 & \vdots & -8 \end{array}\right] \tag{14}$$

Finally, he multiplies Row 4 by -1, and then adds Row 4 to Row 1 and Row 3 and subtracts $2\times$ Row 4 from Row 2.

$$\begin{array}{c}\text{Multiply Row 4 by -1}\\ \text{Row1,Row3: add Row4}\\ \text{Row2: subtract } 2\times\text{Row4}\end{array} \Rightarrow \left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & \vdots & 6 \\ 0 & 1 & 0 & 0 & \vdots & 5 \\ 0 & 0 & 1 & 0 & \vdots & 8 \\ 0 & 0 & 0 & 1 & \vdots & 8 \end{array}\right] \tag{15}$$

Thus Professor Stojanovic determines that Professor Ayazifar's ratings for each fruit are

Fruit	Liking Score
Strawberries	6
Bananas	5
Mangos	8
Blueberries	8

(b) What mystery fruit combination should Professor Stojanovic put in Professor Ayazifar's personalized smoothie? What score would Professor Ayazifar give for this smoothie? There may be more than one correct answer.

Solution: *Any* linear combination of mango and blueberry will be accepted, as they have equal ratings. More precisely, for any $0 \le a \le 1$, a cups of mango and $1-a$ cups of blueberries. Any such combination will yield a score of 8.

How could you see this? At one level, it is somewhat obvious and it is fine if you said as much — Mangoes and Blueberries are tied for being the favorite fruit and so it doesn't make a difference if you substitute one for the other in any quantity. It also doesn't make sense to substitute a less prefered fruit like Strawberries for the favorite fruits.

But if you wanted to see this mechanically, how could you do so? The calculus-inspired approach is pretty simple. We want to maximize $f(a,b,c,d) = 6a+5b+8c+8d$ subject to the constraints $a+b+c+d = 1$ and $a \geq 0$, $b \geq 0$, $c \geq 0$, $d \geq 0$. (Since we can only have one cup of fruit and negative fruit makes no sense.) So we can compute the marginal value of each fruit by differentiating f with respect to a,b,c,d respectively. This gives rise to the constants $6,5,8,8$. Now, we know because of the symmetric sum constraint $a+b+c+d = 1$ that at an optimal mix, the marginal value of each fruit will be identical unless that fruit's level is hitting its constraint. If it is hitting a lower constraint, then its marginal value should be lower. (i.e. this isn't as good a deal but we just can't reduce the amount any further) If it is hitting an upper constraint, then its marginal value should be higher. (i.e. this is a very good deal but we just aren't allowed to take any more.) The fancy name for this intuitive fact is the "Karush-Kuhn-Tucker Conditions" or KKT conditions. Anyway, this tells us immediately that at the optimal mix, we can only have c and d be nonzero and a and b must be zero.

However, most students find that an example like this is more useful as a way to remember why the KKT conditions are as they are rather than actually using the KKT or other calculus-based approaches to solve this problem. Also, don't worry if you haven't seen these KKT approaches in your calculus courses yet, when we will need them, we'll explain what we are doing carefully.

3. Finding charges from voltage measurements

Solution: `#modeling #matrixNotation`

We have three point charges Q_1, Q_2, and Q_3 whose positions are known, and we want to determine their charges. In order to do that, we take three voltage measurements V_1, V_2 and V_3 at three different locations. The positions of the charges and voltages are shown in Figure 1.

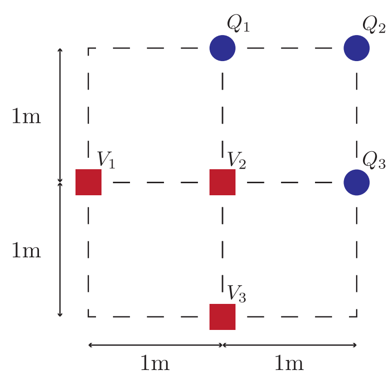

Figure 1: A

For the purpose of this problem, the following equation is true:

$$V = k\frac{Q}{r}$$

at a point r meters away (for some fixed physical constant k; this problem does not require its numerical value).

Furthermore, the voltage contribution from different point charges add up linearly. For example, in the setup of Figure 1, the voltage measured at point V_2 is

$$V_2 = k\frac{Q_1}{1} + k\frac{Q_2}{\sqrt{2}} + k\frac{Q_3}{1}$$

Given that the actual voltage measurements in the setup of Figure 1 are

$$\begin{aligned} V_1 &= k\frac{4+3\sqrt{5}+\sqrt{10}}{2\sqrt{5}}, \\ V_2 &= k\frac{2+4\sqrt{2}}{\sqrt{2}}, \\ V_3 &= k\frac{4+\sqrt{5}+3\sqrt{10}}{2\sqrt{5}} \end{aligned}$$

write the system of linear equations relating the voltages to charges. Solve the system to find the charges Q_1, Q_2, Q_3. You may use your IPython notebook to solve the system.

IPython hint: For constants a_i, b_i, c_i, y_i, you can solve the system of linear equations

$$\begin{aligned} a_1x_1 + a_2x_2 + a_3x_3 &= y_1, \\ b_1x_1 + b_2x_2 + b_3x_3 &= y_2, \\ c_1x_1 + c_2x_2 + c_3x_3 &= y_3, \end{aligned}$$

in IPython with the following code

```
import numpy as np
a = np.array([
    [a1, a2, a3],
    [b1, b2, b3],
    [c1, c2, c3]
])
b = np.array([y1, y2, y3])
x = np.linalg.solve(a, b)
```

The square root of a number a can be obtained as `np.sqrt(a)` in IPython.

Solution:

Let us denote the distance to the ith voltage measurement point from the jth source as r_{ij}; we have by using the Pythagorean theorem (formula for the length of the hypotenuse of a right triangle):

$$\begin{aligned} r_{1,1} &= \sqrt{2}, & r_{1,2} &= \sqrt{5}, & r_{1,3} &= 2, \\ r_{2,1} &= 1, & r_{2,2} &= \sqrt{2}, & r_{2,3} &= 1, \\ r_{3,1} &= 2, & r_{3,2} &= \sqrt{5}, & r_{3,3} &= \sqrt{2}. \end{aligned}$$

Then, the voltages V_1, V_2 and V_3 can be expressed as

$$k\left(\frac{Q_1}{\sqrt{2}}+\frac{Q_2}{\sqrt{5}}+\frac{Q_3}{2}\right)=V_1,$$
$$k\left(Q_1+\frac{Q_2}{\sqrt{2}}+Q_3\right)=V_2,$$
$$k\left(\frac{Q_1}{2}+\frac{Q_2}{\sqrt{5}}+\frac{Q_3}{\sqrt{2}}\right)=V_3.$$

Plugging in the values of V_1, V_2 and V_3 and dividing through by k we get the linear system

$$\frac{Q_1}{\sqrt{2}}+\frac{Q_2}{\sqrt{5}}+\frac{Q_3}{2}=\frac{4+3\sqrt{5}+\sqrt{10}}{2\sqrt{5}},$$
$$Q_1+\frac{Q_2}{\sqrt{2}}+Q_3=\frac{2+4\sqrt{2}}{\sqrt{2}},$$
$$\frac{Q_1}{2}+\frac{Q_2}{\sqrt{5}}+\frac{Q_3}{\sqrt{2}}=\frac{4+\sqrt{5}+3\sqrt{10}}{2\sqrt{5}}.$$

The system can be solved by the following IPython code:

```
import numpy as np

r11 = np.sqrt(2); r12 = np.sqrt(5); r13 = 2
r21 = 1; r22 = np.sqrt(2); r23 = 1
r31 = 2; r32 = np.sqrt(5); r33 = np.sqrt(2);

y1 = (4 + 3*np.sqrt(5) + np.sqrt(10))/(2*np.sqrt(5))
y2 = (2 + 4*np.sqrt(2))/( np.sqrt(2) )
y3 = (4 + np.sqrt(5) + 3*np.sqrt(10)) / ( 2*np.sqrt(5) )

a= np.array([
    [1/r11, 1/r12, 1/r13],
    [1/r21, 1/r22, 1/r23],
    [1/r31, 1/r32, 1/r33]
])
b = np.array([y1, y2, y3])
x = np.linalg.solve(a, b)
print(x)
```

The result is $Q_1 = 1$, $Q_2 = 2$ and $Q_3 = 3$.

4. The Framingham Risk Score

Solution: `#SystemsOfEquations`

The Framingham risk score estimates the 10-year cardiovascular disease (CVD) risk of an individual. There are multiple factors (predictors) that weigh in the calculation of the Framingham score. For each individual, these factors are: gender, age, total cholesterol, level of high-density lipoprotein (HDL) cholesterol, systolic blood pressure (SBP), whether or not the individual smokes, whether or not the individual is treated for high blood pressure and whether or not the individual is diabetic. For this problem, we will focus on the algorithm

Patient ID	Age (years)	Total Chol. (mg/dL)	HDL Chol. (mg/dL)	SBP (mm Hg)	Risk (p)
1	66	198	55	132	0.1550
2	61	180	47	124	0.1108
3	60	180	50	120	0.0940
4	23	132	45	132	0.0105

Table 1: Patient records for 10-year CVD risk assessment

that estimates the CVD risk for female individuals who smoke, are not treated for high blood pressure and are not diabetic.

To calculate the 10-year CVD risk of an individual in the group described above, a score R is first assigned based on the values of age, total cholesterol, HDL cholesterol and systolic blood pressure as follows

$$\begin{aligned} R =& a \cdot \ln(\text{age (years)}) + b \cdot \ln(\text{total cholesterol (mg/dL)}) + \\ & c \cdot \ln(\text{HDL cholesterol (mg/dL)}) + d \cdot \ln(\text{SBP (mm Hg)}) \end{aligned}$$

where a, b, c and d are constant coefficients and $\ln(\cdot)$ denotes the natural (base e) logarithm.

After the score R is calculated, it is plugged into the following formula in order to obtain the risk p (in terms of probability) of the individual suffering from a CVD in the next 10 years:

$$p = 1 - 0.95^{e^{(R-25.66)}}$$

(Note that there is a double exponent in the expression.)

When the algorithm was first devised, the only copy of the document that reported the coefficients a, b, c and d was shredded by mistake by a new intern in the hospital where the research was conducted. The intern needs to restore the values of the coefficients from existing records of hospital patients. He needs your help to do so.

Throughout the problem, you can approximate any numbers up to the fourth decimal. For example, you can approximate 0.23456789 by 0.2346 and 0.24296 by 0.2430.

(a) The intern dug up some of the records for patients in the study group who fit the criteria of the formula in question. The records are summarized in the table 1. Use these records to devise a system of linear equations where a, b, c and d are the unknowns.

Solution: Given a risk (probability) of CVD p, we can write R as a function of p as follows:

$$\begin{aligned} & p = 1 - 0.95^{e^{(R-25.66)}} \\ \iff & 0.95^{e^{(R-25.66)}} = 1 - p \\ \iff & e^{(R-25.66)} = \log_{0.95}(1-p) \\ \iff & R - 25.66 = \ln(\log_{0.95}(1-p)) \\ \iff & R = \ln(\log_{0.95}(1-p)) + 25.66 \end{aligned}$$

Then, for each patient record, we know that

$$\begin{aligned} R =& a \cdot \ln(\text{age (years)}) + b \cdot \ln(\text{total cholesterol (mg/dL)}) + \\ & c \cdot \ln(\text{HDL cholesterol (mg/dL)}) + d \cdot \ln(\text{SBP (mm Hg)}) \end{aligned}$$

which is equivalent to

$$\ln\left(\log_{0.95}(1-p)\right) + 25.66 = a \cdot \ln(\text{age (years)}) + b \cdot \ln(\text{total cholesterol (mg/dL)}) + c \cdot \ln(\text{HDL cholesterol (mg/dL)}) + d \cdot \ln(\text{SBP (mm Hg)})$$

Therefore, by plugging in the values in table 1, the system of linear equations is (one equation per patient record):

$$\begin{aligned}
a \cdot \ln(66) + b \cdot \ln(198) + c \cdot \ln(55) + d \cdot \ln(132) &= \ln\left(\log_{0.95}(1-0.1550)\right) + 25.66 \\
a \cdot \ln(61) + b \cdot \ln(180) + c \cdot \ln(47) + d \cdot \ln(124) &= \ln\left(\log_{0.95}(1-0.1108)\right) + 25.66 \\
a \cdot \ln(60) + b \cdot \ln(180) + c \cdot \ln(50) + d \cdot \ln(120) &= \ln\left(\log_{0.95}(1-0.0940)\right) + 25.66 \\
a \cdot \ln(23) + b \cdot \ln(132) + c \cdot \ln(45) + d \cdot \ln(132) &= \ln\left(\log_{0.95}(1-0.0105)\right) + 25.66
\end{aligned} \tag{16}$$

You will get full credit if you approximated the values in the system of linear equations (left hand side, right hand side or both) to the following system

$$\begin{aligned}
4.1897 \cdot a + 5.2883 \cdot b + 4.0073 \cdot c + 4.8828 \cdot d &= 26.8489 \\
4.1109 \cdot a + 5.1930 \cdot b + 3.8501 \cdot c + 4.8203 \cdot d &= 26.4883 \\
4.0943 \cdot a + 5.1930 \cdot b + 3.9120 \cdot c + 4.7875 \cdot d &= 26.3147 \\
3.1355 \cdot a + 4.8828 \cdot b + 3.8067 \cdot c + 4.8828 \cdot d &= 24.0791
\end{aligned} \tag{17}$$

(b) Solve the system of linear equations that you devised in question (a) of this problem. For this question, you can use IPython.

Note: The natural logarithm of a number a (i.e. $\ln(a)$) can be obtained as `np.log(a)` in IPython.

Solution: The solution of the system of linear equations (16) is

$$a = 2.3099, b = 1.1696, c = -0.6945 \text{ and } d = 2.8200$$

By approximating the right hand side of the system of linear equations (16), the solution would be

$$a = 2.3096, b = 1.1701, c = -0.6945 \text{ and } d = 2.8196$$

By approximating both the left hand side and the right hand side of the system of linear equations, as in Equation (17), the solution would be

$$a = 2.3055, b = 1.1812, c = -0.6960 \text{ and } d = 2.8124$$

Full credit is given to any set of the solutions given above.

A code that solves the system of linear equations in Equation (16) is provided in **sol1.ipynb**

As a side note, notice that the coefficient c has a negative value, indicating that a higher value of the HDL cholesterol would lower the estimated risk of 10-years CVD. This is expected since HDL particles can transport fat molecules out of artery walls – and therefore lowers the risk of arterial clotting, a cause of heart failure. For that reason, HDL cholesterol is often referred to as 'the good cholesterol'. If you are interested, you can read more about the Framingham heart study at `http://circ.ahajournals.org/content/117/6/743.full`

Note: Some of the values in the algorithm were modified from the original study values.

5. **Filtering out the troll**

Solution: `#SystemsOfEquations #LinearCombination`

You attended a very important public speech and recorded it using a recording device that consists of two directional microphones. However, there was this particular person in the audience who was trolling around. When you went back home to listen to the recording you realized that the two recordings are dominated by the troll and you cannot hear the speech. Fortunately since you had two microphones, you realize that there is a way to combine the two recordings such that trolling is removed. Recollecting the scene, the locations of the speaker and the troll are shown in Figure 2.

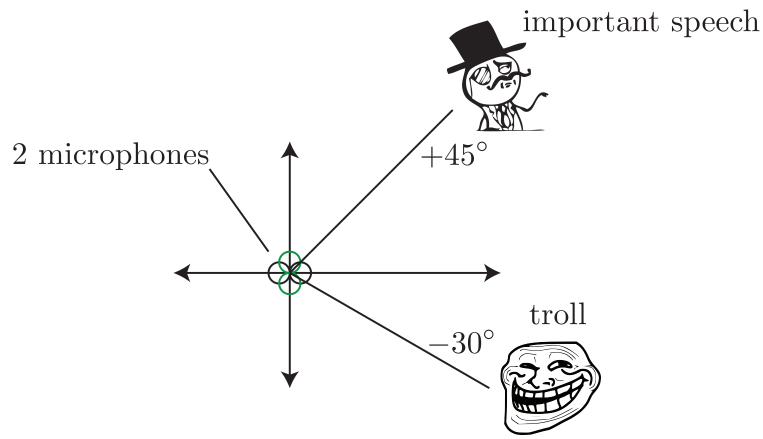

Figure 2: A

The way your recording device works is that each microphone weighs the audio signal depending on the angle of the audio source, relative to the x axis, hence the name *directional microphones*. More specifically, if the audio source is located at an angle of θ, the first microphone will record the audio signal with weight $f_1(\theta) = \cos(\theta)$, and the second microphone will record the audio signal with weight $f_2(\theta) = \sin(\theta)$. For example, an audio source that lies on the x axis will be recorded with the first microphone with weight equal to 1 (since $\cos(0) = 1$), but will not be picked by microphone two (since $\sin(0) = 0$). Note that the weights can also be negative.

Graphically, the directional characteristics of the microphones are given in Figures 3 and 4 (colors red and blue denote positive and negative values of the weight respectively). Putting all of this together, assume that there are two speakers, A and B, at angles θ and ψ, respectively. Assume that speaker A produces an audio signal represented by the vector $\vec{a} \in \mathbb{R}^n$. That is, the i-th component of $\vec{a}$ is the signal at the i-th time step. Similarly, assume speaker B produces an audio signal $\vec{b}$,

Then the first microphone will record the signal

$$\vec{m}_1 = \cos(\theta) \cdot \vec{a} + \cos(\psi) \cdot \vec{b},$$

and the second microphone will record the signal

$$\vec{m}_2 = \sin(\theta) \cdot \vec{a} + \sin(\psi) \cdot \vec{b},$$

(a) Using the notation above, let the important speaker be speaker A (with signal $\vec{a}$) and let the person trolling be "speaker" B (with signal $\vec{b}$). Express the recordings of the two microphones $\vec{m}_1$ and $\vec{m}_2$ (i.e. the signals recorded by the first and the second microphones, respectively) as a linear combination of $\vec{a}$ and $\vec{b}$.

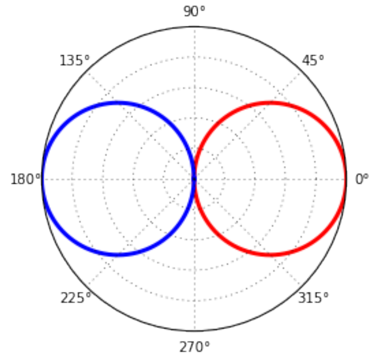

Figure 3: Directional characteristics of mic. 1

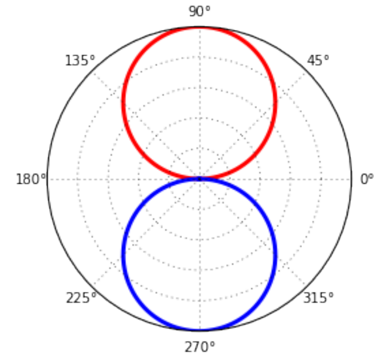

Figure 4: Directional characteristics of mic. 2

Solution:

$$\begin{aligned}\vec{m}_1 &= \cos(\pi/4)\cdot\vec{a}+\cos(-\pi/6)\cdot\vec{b}\\ &= \frac{1}{\sqrt{2}}\cdot\vec{a}+\frac{\sqrt{3}}{2}\cdot\vec{b},\\ \vec{m}_2 &= \sin(\pi/4)\cdot\vec{a}+\sin(-\pi/6)\cdot\vec{b}\\ &= \frac{1}{\sqrt{2}}\cdot\vec{a}-\frac{1}{2}\cdot\vec{b}.\end{aligned}$$

(b) Recover the important speech $\vec{a}$, as a weighted combination of $\vec{m}_1$ and $\vec{m}_2$. In other words, write $\vec{a} = u\cdot\vec{m}_1 + v\cdot\vec{m}_2$ (where u and v are scalars). What are the values of u and v?

Solution: Solving the system of linear equations yields

$$\vec{a} = \frac{\sqrt{2}}{1+\sqrt{3}}\cdot\left(\vec{m}_1+\sqrt{3}\vec{m}_2\right).$$

Therefore, the values are $u = \frac{\sqrt{2}}{1+\sqrt{3}}$ and $v = \frac{\sqrt{6}}{1+\sqrt{3}}$.

It is fine if you solved this using iPython or by hand using any valid technique. The easiest approach is to subtract the two equations from each other and immediately see that $\vec{b} = \frac{2}{\sqrt{3}+1}(\vec{m}_1-\vec{m}_2)$. Substituting that back into the second equation and multiplying through by $\sqrt{2}$ gives that $\vec{a} = \sqrt{2}(\vec{m}_2+\frac{1}{\sqrt{3}+1}(\vec{m}_1-\vec{m}_2))$ which immediatedly simplifies to the expression given above.

Notice that subtracting the two from each other is natural given the symmetry of the microphone patterns and the fact that they intersect at the 45 degree line where the important speech is happening. So we know that the result will only contain the troll. Once we have the troll contribution, we can remove it.

(c) Partial IPython code can be found in `prob1.ipynb`. Complete the code to get a clean signal of the important speech. What does the speaker say? (Optional: Where is the speech taken from?)

Solution: The solution code can be found in `sol1.ipynb`. The speaker says: “All human beings are born free and equal in dignity and rights.” and the speech was taken from the Universal Declaration of Human Rights.

Note: You may have noticed that the recordings of the two microphones sounded remarkably similar. This means that you could recover the real speech from two “trolled” recordings that sounded almost identical! Leave out the fact that the recordings are actually different, and have some fun with your friends who aren't lucky enough to be taking EE16A.

Solution: The idea of using multiple microphones to isolate speech is interesting and is increasingly used in practice. Furthermore, similar techniques are used in wireless communication both by cellular systems like LTE and increasingly by WiFi hotspots. (This is why they often have multiple antennas).

6. **Your Own Problem** Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?