EECS 16A Spring 2020 — Designing Information Devices and Systems I — Discussion 11B

Reference: Inner products

Let $\vec{x}$, $\vec{y}$, and $\vec{z}$ be vectors in real vector space $\mathbb{V}$. A mapping $\langle \cdot, \cdot \rangle$ is said to be an inner product on $\mathbb{V}$ if it satisfies the following three properties:

(a) Symmetry: $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$

(b) Linearity: $\langle \vec{x}, \vec{y} + \vec{z} \rangle = \langle \vec{x}, \vec{y} \rangle + \langle \vec{x}, \vec{z} \rangle$ and $\langle c\vec{x}, \vec{y} \rangle = c\langle \vec{x}, \vec{y} \rangle$

(c) Positive-definiteness: $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$.

We define the norm of $\vec{x}$ as $\|\vec{x}\| = \sqrt{\langle \vec{x}, \vec{x} \rangle}$.

1. Mechanical Inner Products

For the following pairs of vectors, find the Euclidean inner product $\langle \vec{x}, \vec{y} \rangle = \vec{x}^T \vec{y}$.

(a)

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

2. Inner Product Properties

Demonstrate the following properties of inner products for any vectors in $\mathbb{R}^2$, assuming we are working with the Euclidean inner product and norm.

(a) Symmetry

(b) Linearity

3. Geometric Interpretation of the Inner Product In this problem, we will explore the geometric interpretation of the Euclidean inner product, restricting ourselves to vectors in $\mathbb{R}^2$.

(a) For each of the following cases, pick two vectors that satisfy the condition and find the inner product.

i. Parallel Vectors

ii. Anti-parallel

iii. Perpendicular

(b) Now, derive a formula for the inner product of two vectors in terms of their magnitudes and the angle between them.

4. Reverse Triangle Inequality

The triangle inequality states that, for vectors $\vec{x}, \vec{y} \in \mathbb{R}^N$:

$$||\vec{x} + \vec{y}|| \leq ||\vec{x}|| + ||\vec{y}||$$

(a) First, prove the following:

$$||\vec{x} - \vec{y}|| = ||\vec{y} - \vec{x}||$$

(b) Using the triangle inequality in conjunction with the previous identity, prove the reverse triangle inequality, which states that, for vectors $\vec{x}, \vec{y} \in \mathbb{R}^N$:

$$\big| ||\vec{x}|| - ||\vec{y}|| \big| \leq ||\vec{x} - \vec{y}||$$

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