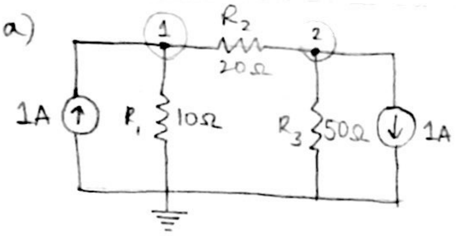
Homework #7 | EE16A

#1 a)



KCL ① (out, in)

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 1A$$

$$\frac{V_1}{10} + \frac{V_1 - V_2}{20} = 1$$

$$2V_1 + V_1 - V_2 = 20$$

$$3V_1 - V_2 = 20$$

② (out, in)

$$\frac{V_2}{R_3} + 1A = \frac{V_1 - V_2}{R_2}$$

$$\frac{V_2}{50} + 1 = \frac{V_1 - V_2}{20}$$

$$2V_2 + 100 = 5V_1 - 5V_2$$

$$5V_1 - 7V_2 = 100$$

$$3V_1 - V_2 = 20$$

$$5V_1 - 7V_2 = 100$$

$$V_1 = \frac{20 + V_2}{3}$$

$$5\left(\frac{20 + V_2}{3}\right) - 7V_2 = 100$$

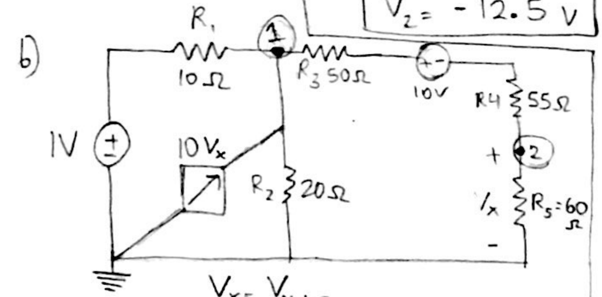
$$\frac{100 + 5V_2}{3} - 7V_2 = 100$$

$$100 + 5V_2 - 21V_2 = 300$$

$$-16V_2 = 200$$

$$\boxed{V_2 = -12.5\text{ V}} \Rightarrow \boxed{V_1 = 2.5V}$$

b)



$V_x = V_{Node\ 2}$

①

$$10V_2 + \underbrace{\frac{1V - V_1}{R_1}}_{\text{into node}} = \underbrace{\frac{V_1}{R_2} + \frac{V_1 - 10V - V_2}{R_3 + R_4}}_{\text{out of node}}$$

$$10V_2 + \frac{1 - V_1}{10} = \frac{V_1}{20} + \frac{V_1 - 10 - V_2}{105}$$

$$2100V_2 + 21 - 21V_1 = 10.5V_1 + 2V_1 - 20 - 2V_2$$

$$2102V_2 - 33.5V_1 = -41$$

②

$$\frac{V_1 - 10V - V_2}{R3 + R4} = \frac{V_2}{R_5} \Rightarrow \frac{V_1 - 10 - V_2}{105} = \frac{V_2}{60}$$

$$60(V_1 - 10 - V_2) = 105V_2$$

$$4V_1 - 40 - 4V_2 = 7V_2$$

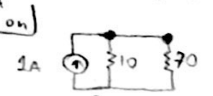
$$4V_1 - 11V_2 = 40$$

$$\left[\begin{array}{cc|c} -33.5 & 2102 & -41 \\ 4 & -11 & 40 \end{array}\right]$$

$$\boxed{\begin{array}{l} V_1 = 10.4023V \\ V_2 = 0.1463V \end{array}}$$

Verify a) w/ Superposition

w/ $1A^A{}_{on}$



$R_{eq} = 70/8\ \Omega$

current divider rule

$$i_1 = \frac{70/8}{10}(1A) = 0.875A$$

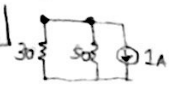
$$i_2 = \frac{70/8}{70}(1A) = 0.125A$$

$V_{1A} = (0.875A)(10\Omega) = 8.75V$

not 70!

$V_{2A} = (0.125A)(50\Omega) = 6.25V$

w/ $1A^B$ on



opp. dir.

$R_{eq} = \frac{75}{4}\Omega$

current divider rule

$$i_1 = \frac{75/4}{30}(1A) = 0.625A$$

$$i_2 = \frac{75/4}{50}(1A) = 0.375A$$

$V_{1B} = -(0.625A)(10\Omega) = -6.25V$

$V_{2B} = -(0.375A)(50\Omega) = -18.75V$

$V_1 = 8.75 - 6.25 = 2.5V$ ✓ $V_2 = 6.25 - 18.75 = -12.5$ ✓

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#2 v2

a)

$$U_2 = \frac{R_2}{R_1+R_2} V_b \qquad U_3 = \frac{R_3}{R_3+R_T} V_b$$

$$V_{Th} = U_3 - U_2 = \frac{R_3}{R_3+R_T} V_b - \frac{R_2}{R_1+R_2} V_b$$

$$\boxed{V_{Th} = V_b\left(\frac{R_3}{R_3+R_T} - \frac{R_2}{R_1+R_2}\right)}$$

$$\boxed{R_{Th} = (R_T+R_1) \| (R_3+R_2)}$$

b) $V_0 = V_{Th} = \frac{R_3}{R_3+R_T} V_b - \frac{R_2}{R_1+R_2} V_b$

$$\frac{R_3}{R_3+R_T} V_b = V_0 + \frac{R_2}{R_1+R_2} V_b$$

$$\frac{R_3 V_b}{R_3+R_T} = \frac{V_0 R_1 + V_0 R_2 + V_b R_2}{R_1+R_2}$$

$$\boxed{R_T = \frac{R_3 (V_b R_1 - V_0 R_1 - V_0 R_2)}{V_0 R_1 + V_0 R_2 + V_b R_2}}$$

c) If $R_T = R_1 = R_2 = R_3$, then $\boxed{V_o = 0.}$

$$V_o = \left(\frac{y}{y+R_T} - \frac{y}{2y}\right)V_b$$

As temp rises, voltage at the bridge becomes higher.

$$V_o = \left(\frac{y}{y+R_T} - \frac{1}{2}\right)V_b$$

d)

$$Q = \frac{dV_o}{dT} = R_T \frac{dV_o}{dR_T} \frac{1}{R_T} \frac{dR_T}{dT}$$

$$\frac{dV_o}{dR_T} = \frac{-R_3 V_o}{(R_T+R_3)^2} \qquad \frac{dR_T}{dt} = \frac{-R_2(T_0)\,\beta e^{R\left(\frac{1}{T}-\frac{1}{T_0}\right)}}{T^2}$$

$$= \frac{-R_T\,\beta}{T^2}$$

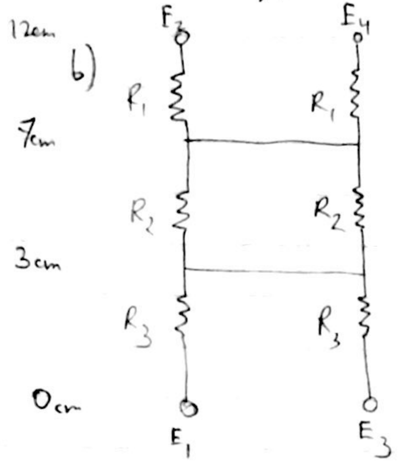
$$Q = \frac{R_T R_3 V_1}{(R_T+R_3)^2} \cdot \frac{\beta}{T_2}$$

$$= \frac{\alpha\ V_1}{(1+\alpha)^2} \frac{\beta}{T^2}$$

$$\frac{dQ}{dt} = \frac{1-\alpha}{(\alpha+1)^3}\left(\frac{V_0\beta}{T^2}\right) = 0 \qquad \frac{dQ}{dt} = 0 \text{ when } \alpha = 1$$

#3 a) $R = \rho \frac{L}{A} = (1\,\Omega_m) \dfrac{0.12m}{0.03m \cdot 0.0005m} = 8000\Omega = \boxed{8k\Omega}$

b)

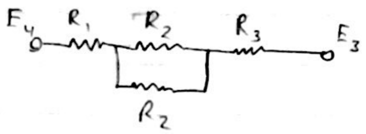


$R_1 = \dfrac{12-7}{12} \cdot R_{1\to 2} = \dfrac{5}{12} \cdot 5k\Omega = \boxed{2.083k\Omega}$

$R_2 = \dfrac{4cm}{12cm} \cdot 5k\Omega = \boxed{1.667k\Omega}$

$R_3 = \dfrac{3cm}{12cm} \cdot 5k\Omega = \boxed{1.25k\Omega}$

c)



$R_{eq} = R_1 + \overbrace{\dfrac{1}{\frac{1}{R_2} + \frac{1}{R_2}}}^{= \frac{R_2}{2}} + R_3$

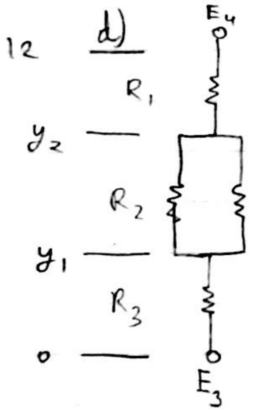
$= 2.083 + \dfrac{1.667}{2} + 1.25$

$= 4.167k\Omega$

$V = IR$

$= (0.001A)(4167\Omega) = \boxed{4.167V}$

d)



$R_1 = \dfrac{12 - y_2}{12} \cdot 5k\Omega$

$R_2 = \dfrac{y_2 - y_1}{12} \cdot 5k\Omega$

$R_3 = \dfrac{y_1}{12} \cdot 5k\Omega$

$R_{eq} = R_1 + \dfrac{R_2}{2} + R_3$

$= \left(0.12m - y_2 + \left(\tfrac{1}{2}\right)(y_2 - y_1) + y_1\right) \dfrac{5k\Omega}{0.12m}$

$= \left(0.12m + \dfrac{y_1}{2} - \dfrac{y_2}{2}\right) \dfrac{5k\Omega}{0.12m}$

$V = R_{eq} \cdot 0.001A$

$$\boxed{V = \dfrac{0.12m + \dfrac{y_1 - y_2}{2}}{0.12m} \cdot 5V}$$

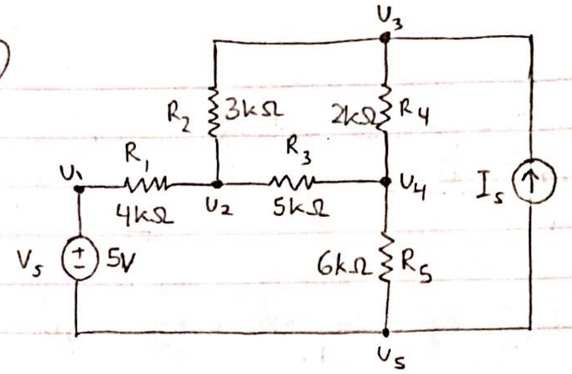
With V, we can only measure the distance between points.

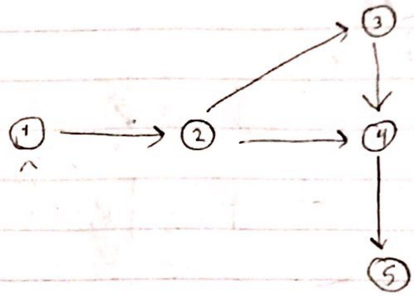
e) $V_{E4-E2} = I \cdot R_1 = \dfrac{0.12m - y_2}{0.12m} \cdot 5V$

$V_{E3-E1} = I \cdot R_3 = \dfrac{y_1}{0.12m} \cdot 5V$

#4

a)





$$
\begin{array}{cccccc}
\text{Node:} & 1 & 2 & 3 & 4 & 5 \\
F = & \multicolumn{5}{c}{\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}}
\end{array}
$$

b) $F^T\vec{i} =$

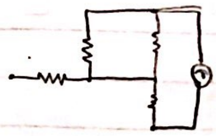
$$F^T\vec{i} = F^T \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} i_1 \\ -i_1 + i_2 + i_3 \\ -i_2 + i_4 \\ -i_3 - i_4 + i_5 \\ -i_5 \end{bmatrix} = \vec{0}$$

$F^T\vec{i}$ represents the KCL equations by setting the difference between current into a node and current out of a node to 0. We are missing proper equations for U_1, U_3, & U_5.

c) New KCL

U_5) $I_s + \frac{U_5 - U_4}{R_5} = \emptyset$

U_3) $\cancel{-I_S + \frac{U_3 - U_4}{R_4} + \frac{U_3 - U_2}{R_2} = \emptyset}$



$i_2 + I_S = i_4$

d)

$$\vec{b} = \begin{bmatrix} 0 \\ 0 \\ I_s \\ 0 \\ I_s \end{bmatrix}$$

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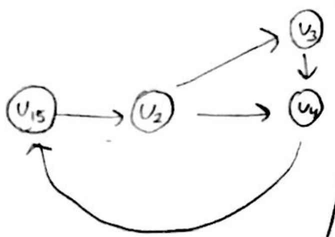
e)

$$F\vec{u} = \vec{v}$$

$$F\vec{u} = \begin{bmatrix} u_1 - u_2 \\ u_2 - u_3 \\ u_2 - u_4 \\ u_3 - u_4 \\ u_4 - u_5 \end{bmatrix}$$, which is equal to each voltage drop.

f)



$$F' = \begin{array}{c} \text{Node} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} 15 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} +1 & -1 & 0 & 0 \\ 0 & +1 & -1 & 0 \\ 0 & 0 & +1 & -1 \\ -1 & 0 & 0 & +1 \\ 0 & +1 & 0 & -1 \end{bmatrix} \end{array}$$

g)

$$F'\vec{u} + c = \vec{v}$$

$$\begin{bmatrix} u_{15} - u_2 \\ u_2 - u_3 \\ u_2 - u_4 \\ u_3 - u_4 \\ u_4 - u_{15} \end{bmatrix} + c = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

$$c = \begin{bmatrix} u_1 - V_s \\ 0 \\ 0 \\ 0 \\ V_s - u_5 \end{bmatrix}$$

h)

$$\underbrace{\begin{bmatrix} u_{15} - u_2 - V_s \\ u_2 - u_3 \\ u_2 - u_4 \\ u_3 - u_4 \\ u_4 - u_{15} - V_s \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} R_1 i_1 \\ R_2 i_2 \\ R_3 i_3 \\ R_4 i_4 \\ R_5 i_s \end{bmatrix}}_{IR}$$

$$V = IR$$

Ohm's Law

i)

$$\left.\begin{aligned} &Ri - F'\vec{u} = \vec{c} \\ &F^T\vec{i} = \vec{0} \end{aligned}\right\} \rightarrow \begin{bmatrix} R & -F' \\ F_T & \emptyset \end{bmatrix}\begin{bmatrix} \vec{i} \\ \vec{u} \end{bmatrix} = \begin{bmatrix} \vec{c} \\ \vec{0} \end{bmatrix}$$

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j) If the columns of F are linearly independent then the right & left halves will be linearly independent b/c they're made up of F. -F doesn't have full rank b/c it is an incidence matrix w/ rows summing to 0, meaning multiplying F by $\vec{1}$ would yield $\vec{0}$.

k)

$$\begin{bmatrix} R_1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & R_2 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & R_3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & R_4 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & R_5 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ u_{15} \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ I_s \\ 0 \\ I_s \\ 0 \end{bmatrix}$$

l) iPython

$i_1 = -0.006A$
$i_2 = -0.005A$
$i_3 = 0.004A$
$i_4 = -0.005A$
$i_5 = -0.006A$
$u_{15} = 4.2V$
$u_2 = 2V$
$u_3 = 3.8V$

#5 Skanda Suresh: 3031861340
Mohan Ganesan: 24199173
Addhyyan Narang: 26433045

We died together, with valor.