This homework is due November 1, 2016, at 1PM.

1. **Homework process and study group**

   Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?
   Working in groups of 3-5 will earn credit for your participation grade.

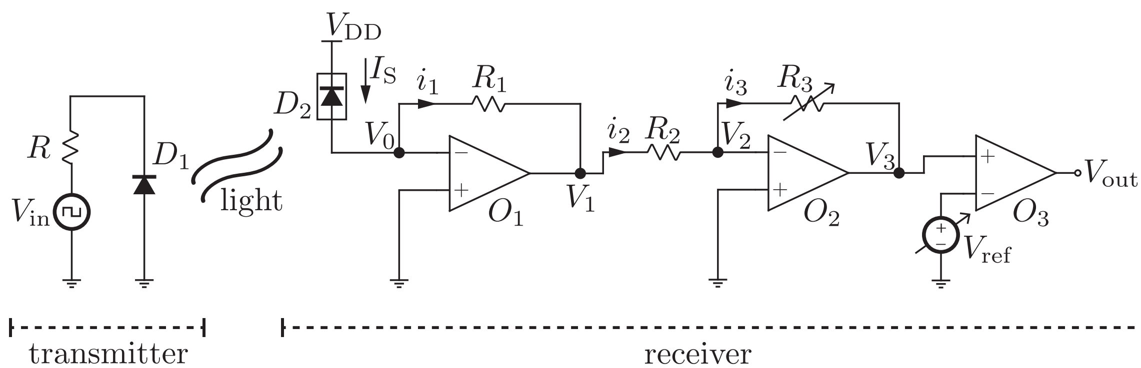
   Solution: I worked on this homework with...

   I first worked by myself for 2 hours, but got stuck on Problem 5 so I went to office hours on...

   Then I went to homework party for a few hours, where I finished the homework.

2. **Wireless communication with an LED**

   In this question, we are going to analyze the system shown in the figure below. It shows a circuit that can be used as a wireless communication system using visible light (or infrared, very similar to remote controls).



The element D_1 in the transmitter is a light-emitting diode (LED in short). An LED is an element that emits light where the brightness of the light is controlled by the current flowing through it. You can recall controlling the light emitted by an LED using your MSP430 in touch screen lab part 1. In our circuit, the current across the LED, hence its brightness, can be controlled by choosing the applied voltage V_{in} and the value of the resistor R. In the receiver, the element labeled as D_2 is a reverse biased solar cell. You can recall using a reverse biased solar cell in imaging labs 1 to 3 as a light controlled current source, by I_S we denote the current supplied by the solar cell. In this circuit the LED D_1 is used as a means for transmitting information with light, and the reverse biased solar cell D_2 is used as a receiver of light to see if anything was transmitted.

Remark: In imaging lab part 3, we have talked about how non-idealities such as background light affect the performance of a system that does light measurements. In this question we assume ideal conditions, that is, there is no source of light around except for the LED.

In our system, we define two states for the transmitter, the *transmitter is sending something* when they turn on the LED, and *transmitter is not sending anything* when they turn off the LED. On the receiver side, the goal is to convert the current I_S generated by the solar cell into a voltage and amplify it so that we can read the output voltage V_{out} to see if the transmitter was sending something or not. The circuit implements this operation through a series of op-amps. It might look look complicated at first glance, but we can analyze it a section at a time.

(a) Currents i_1, i_2 and i_3 are labeled on the diagram. Assuming the Golden Rules hold, is $I_S = i_1$? $i_1 = i_2$? $i_2 = i_3$? Treat the solar cell as an ideal current source.

Solution:

We use the op-amp golden rules, which says that in an op-amp, no current flows into or out of V_+ or V_-. Therefore we can use KCL at node V_0 and V_2 to conclude that $\boxed{I_S = i_1}$ and $\boxed{i_2 = i_3}$. However, if $I_S \neq 0$ and $R_1 \neq R_2$, then $\boxed{i_1 \neq i_2}$. This is because $V_0 = V_2 = 0V$ and V_1 is some non-zero voltage. If $R_1 \neq R_2$, then the currents flowing through them are different.

(b) Use the Golden Rules to find V_0, V_1, V_2 and V_3 in terms of I_S, R_1, R_2 and R_3.
Hint: Solve for them from left to right, and remember to use the op-amp golden rules.

Solution:

Using the Golden rules, we know that $\boxed{V_0 = 0V}$. Using Ohm's law, we know that $V_0 - V_1 = i_1 R_1$. From the previous part we know that $i_1 = I_S$. Thus we get $\boxed{V_1 = -I_S R_1}$. Using the Golden rules again, we get $\boxed{V_2 = 0V}$. Using Ohm's law and the KCL result from the previous part, we get the following equations:

$$\begin{aligned} V_1 - V_2 &= i_2 R_2 \\ V_2 - V_3 &= i_3 R_3 \\ i_2 &= i_3 \end{aligned}$$

Solving them, we get $\boxed{V_3 = I_S R_1 R_3 / R_2}$.

(c) In the previous part, how could you check your work to gain confidence that you got the right answer?

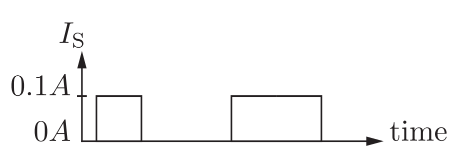
Solution:

One sanity check is checking that your answer has the right units (Voltage = Amperes $\times$ Ohms $\times$ Ohms / Ohms = Amperes $\times$ Ohms).

Also notice that R_2 and R_3 form a voltage divider since no current flows into the negative terminal of O_2. Thus we can check that the voltage divider equation holds:

$$V_2 - V_1 = (V_3 - V_1)\frac{R_2}{R_2 + R_3}$$

(d) Now, assume that the transmitter has chosen the values of V_{in} and R to control the intensity of light emitted by LED such that when *transmitter is sending something* I_S is equal to 0.1 A, and when the *transmitter is not sending anything* I_S is equal to 0 A. The following figure shows a visual example of how this current I_S might look like as time changes (note that this is just here for helping visualizing the form of the current supplied by the solar cell).



For the receiver, suppose $V_{\text{ref}} = 2V$, $R_1 = 10\,\Omega$, $R_2 = 1000\,\Omega$, and the supply voltages of the op-amps are $V_{\text{DD}} = 5V$ and $V_{\text{SS}} = -5V$. Pick a value of R_3 such that V_{out} is V_{DD} when the *transmitter is sending something* and V_{SS} when the *transmitter is not sending anything*?

Solution:

We want $V_{\text{out}} = V_3 - V_{\text{ref}} > 0V$ when $I_S = 0.1A$, and $V_{\text{out}} = V_3 - V_{\text{ref}} < 0V$ when $I_S = 0A$. We plug the known resistor values into the equation in the previous part to get $V_3 = I_S R_3/100$. When $I_S = 0$, $V_{\text{out}} = -2V < 0V$. When $I_S = 0.1A$, $V_{\text{out}} = R_3/1000 - V_{\text{ref}} > 0V$. Thus $\boxed{R_3 > 2000\Omega}$.

(e) In the previous part, how could you check your work to gain confidence that you got the right answer?

Solution:

We can check if the answer makes sense. We know that O_2 serves as an inverting amplifier and V_1 is negative (since there is a voltage drop from $V_0 = 0V$), thus we want it to amplify V_1 until it is higher than V_{ref}, so R_3 should be bigger than R_2.

3. Island Karaoke Machine

After a plane crash, you're stuck on a desert island and everyone is bored out of their minds. Fortunately you have your EE16 lab kit with op-amps, wires, and resistors, and your handy breadboard. You decide to build a Karaoke machine. You recover one speaker from the crash remains and use your iPhone as your source. You know that many songs put instruments on either "left" or "right" channel, but the vocals are usually present on both channels with equal strength.

The Thevenin equivalent model of the iPhone audio jack and speakers is shown below. For simplicity, we assume that the audio signals V_{Left} and V_{Right} are both DC and that the equivalent source resistance of the left/right audio channels of $R_{\text{Left}} = R_{\text{Right}} = 3\Omega$. The speaker has an equivalent resistance of 4Ω.

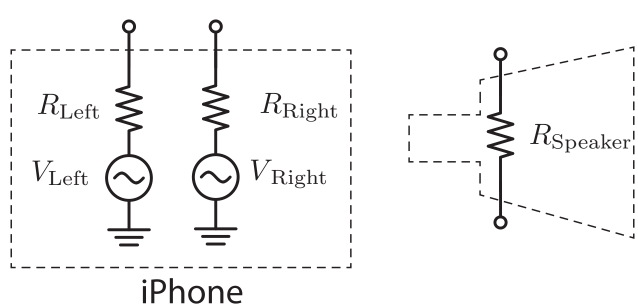
For this problem, we'll assume that

$$V_{\text{Left}} = V_{\text{vocals}}$$
$$V_{\text{Right}} = V_{\text{vocals}} + V_{\text{instrument}}$$

where $V_{\text{vocals}} = 120.524mV$ and $V_{\text{instrument}} = 50mV$.

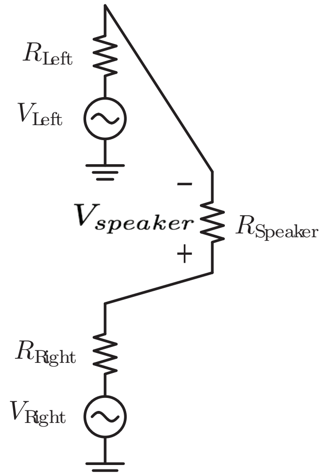
That is, the vocals are present on the left and right channel, but the instrument is present only on the right channel.

What is the goal of a karaoke machine? The ultimate goal is to *remove* the vocals from the audio output. We're going to do this by first building a circuit that takes the left and right outputs of the smartphone audio output, and takes its difference. Let's see what happens.



(a) One of your island survivors suggests the following circuit to do this. Calculate the voltage across the speaker. What do you notice? Does the voltage across the speaker depend on V_{vocals}? What do you think the islanders will hear – vocals, instruments, or both?

Solution: Let's mark the voltage across the speaker, $V_{speaker}$ from bottom to top as in the figure:



We can apply the principle of superposition to solve for $V_{speaker}$. First, we solve for the voltage across the speaker when only V_{left} is on. Let's call this $V_{speaker,left}$. Notice that the circuit becomes a voltage divider. So, we get

$$-V_{speaker,left} = \frac{V_{left}R_{speaker}}{R_{speaker}+R_{left}+R_{right}} = \frac{4V_{vocals}}{10} = 0.4V_{vocals} \tag{1}$$

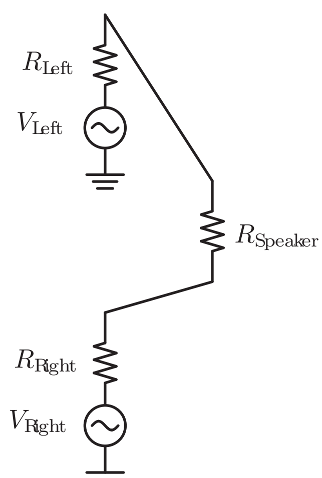
giving

$$V_{speaker,left} = -0.4V_{vocals} \tag{2}$$

Similarly, we solve for the voltage across the speaker when only V_{right} is on. Let's call this $V_{speaker,right}$. Again, notice that the circuit becomes a voltage divider. So, we get

$$V_{speaker,right} = \frac{V_{right}R_{speaker}}{R_{speaker}+R_{left}+R_{right}} = \frac{4(V_{vocals}+V_{instrument}}{10} = 0.4(V_{vocals}+V_{instrument}) \tag{3}$$

Superposition tells us that $V_{speaker} = V_{speaker,left} + V_{speaker,right} = 0.4V_{instrument} = 0.4 \times 50mV = 20mV$. What did you notice? The vocals got canceled out! The islanders will only hear the instruments, just as they wanted.



(b) How much power is delivered to the speaker?

Solution: The power delivered to the speaker, which we can call $P_{speaker}$, is going to be

$$P_{speaker} = \frac{V_{speaker}^2}{R_{speaker}} = \frac{20 \times 20 \times 10^{-6}}{4} W = 0.1 mW \tag{4}$$

Clearly very little power. This will translate to low volume.

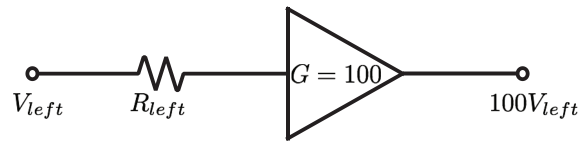
(c) Clearly, we need to boost the sound level to get the party going. We can do this by *amplifying* both V_{Left} and V_{Right}. Keep in mind that we could use the inverting or non-inverting amplifiers from Problem 3 for this – an inverting amplifier has negative gain, and a non-inverting amplifier has positive gain.

Let's assume, just for this part, that all the amplifiers we have produce a gain of 100 (if they are non-inverting), and -100 (if they are inverting). How would you take the difference of the two amplified outputs across the speaker?

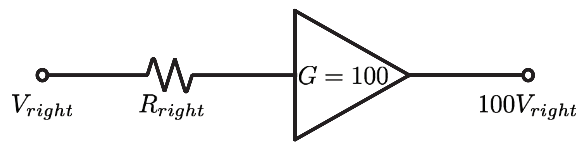
Solution:

Basically, we have three components of the circuit we want to build that we already know:

- The part of the circuit that amplifies V_{left} to $100V_{left}$, which we can draw as below:

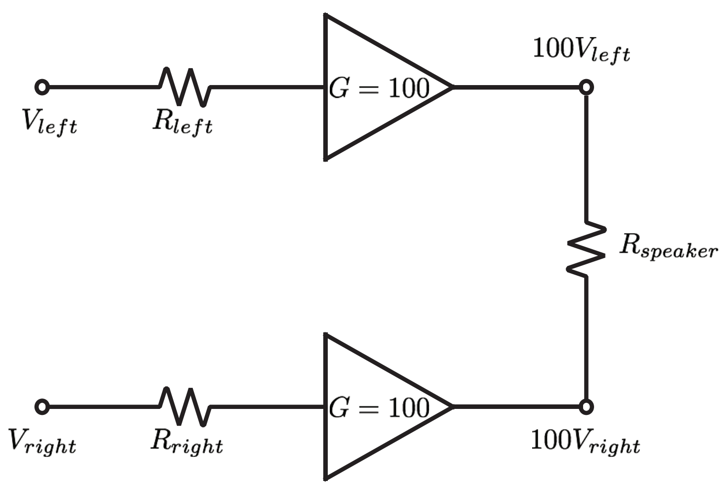


- The part of the circuit that amplifies V_{right} to $100V_{right}$, which we can draw as below:



- The speaker

If we want to take the difference of the two amplified outputs across the speaker, all we need to do is connect the output terminals of the first two components to the terminals of the speaker as shown below:



You can see this solution taking inspiration from part a). Why do we get exactly $100(V_{right} - V_{left})$ across the speaker? Why does the voltage not divide as before?

To answer this, look back to the circuit for the non-inverting amplifier in Problem 3a). If we solve for the Thevenin output resistance of this circuit, we will find that it is zero. And the Thevenin voltage will be $100V_{left}$ (or $100V_{right}$). This implies that, no matter what R_{left} or R_{right} is, we are going to only see $100V_{left}$ or $100V_{right}$ at the output.

(d) Now, design a circuit that takes in V_{Left} and V_{Right}, and outputs an amplified version of $V_{\text{instrument}}$ across the speaker load. You should be able to deliver $1W$ into the speaker load. You can use up to three op-amps, and each of them can be inverting or non-inverting.

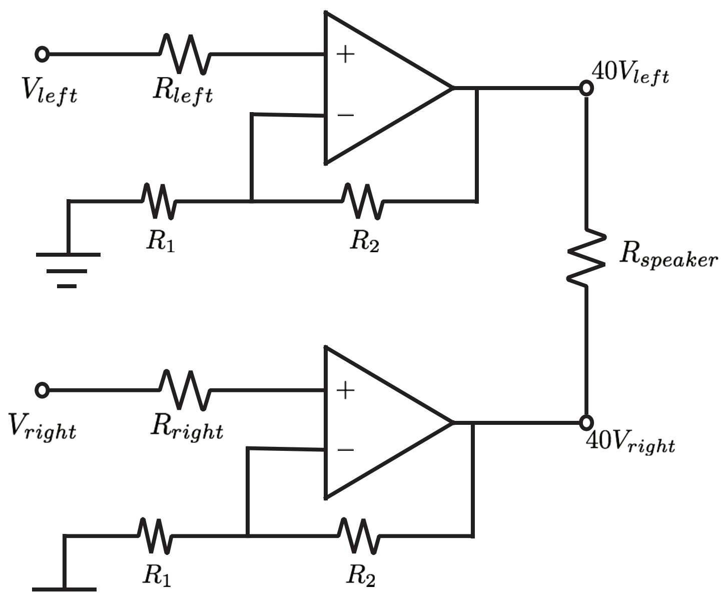
(Hint: Use the circuits from problem 3 as building blocks.)

Solution:

Solution using only two opamps: Simply feed the non-ideal voltage source $\{V_{left}, R_{left}\}$ into a non-inverting amplifier with gain G, and the non-ideal voltage $\{V_{right}, R_{right}\}$ into another non inverting amplifier with gain G. (We have a different gain from the previous part, which we need to determine.) Then connect the two outputs across $R_{speaker}$ as shown in the previous part. In this circuit, we will get $V_{speaker} = G \times 50mV = 0.05GV$, which will give us $P_{speaker} = \frac{0.0025G^2}{4}$. We want $P_{speaker} = 1W$, implying that we need $\frac{0.0025G^2}{4} = 1W$. This will give us $G = 40$.

So, we want to design for a non-inverting opamp with voltage gain 40.

We can use the circuit schematic from part c), now, we just need to design the non-inverting amplifier to have gain 40. We look at the circuit in Problem 3a) to do this, and get the equivalent circuit below:

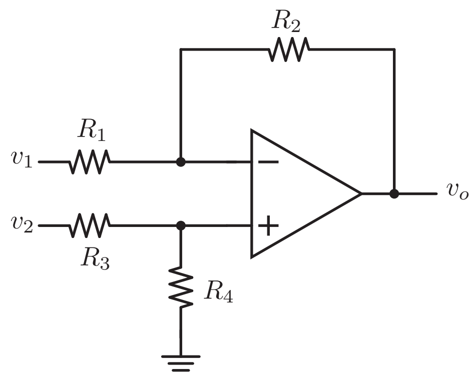


Now, we need to find R_1 and R_2. The result in Problem 3a) told us that

$$G = 1 + \frac{R_2}{R_1} \tag{5}$$

So, we can then choose any R_1 and R_2 such that $\frac{R_2}{R_1} = 39$. Note that there are multiple ways of choosing them! One such choice is $R_1 = 1k\Omega$ and $R_2 = 39k\Omega$, for instance.

(e) The trouble with the previous part is the number of op-amps required. Let's say you only have one opamp with you. What would you do? One night in your dreams you have an inspiration. Why not combine the inverting and non-inverting amplifier into one, as shown below!



If we set $v_2 = 0V$, what is the gain from v_1 to the output v_o? (this is the inverting path)

Solution: If we set $v_2 = 0V$, we would get $V_+ = 0$ (why?) Applying the negative feedback consequence of the Golden Rules, we will get $V_- = V_+ = 0$. Writing the node equation at the $-$ terminal of the opamp,

$$\frac{V_1 - 0}{R_1} = \frac{0 - V_{o,1}}{R_2} \tag{6}$$

which will give us

$$V_{o,1} = \frac{-V_1 R_2}{R_1} \tag{7}$$

(f) If we set $v_1 = 0V$, what is the gain from v_2 to the output v_o? (this is the non-inverting path)

Solution: If we set $v_1 = 0V$, we would get $V_+ = \frac{V_2 R_4}{R_3 + R_4} = V_-$. Writing the node equation at the $-$ terminal gives us

$$\frac{0 - V_-}{R_1} = \frac{V_- - V_{o,2}}{R_2} \tag{8}$$

which gives us

$$V_{o,2} = V_-(1 + \frac{R_2}{R_1}) = V_2(\frac{R_4}{R_3 + R_4})(1 + \frac{R_2}{R_1}) \tag{9}$$

(g) Now, determine v_o in terms of v_1 and v_2. (Hint: use superposition!) Set R_1, R_2, R_3 and R_4 such that, as before, $1W$ is delivered to the speaker load and we don't hear the vocals.

Solution: By the principle of superposition,

$$V_o = V_{o,1} + V_{o,2} \tag{10}$$

If we set $V_1 = V_{left}$ and $V_2 = V_{right}$, we'd ideally want $V_o = -40V_1 + 40V_2$. We can choose R_1, R_2, R_3 and R_4 so that this happens.

How do we do this? Let's do this in steps. First note that, looking for the expression for $V_{o,1}$, we'll want $\frac{R_2}{R_1} = 40$. So, we can choose any values of R_2 and R_1 such that this happens. One such choice is $R_1 = 1k\Omega$, $R_2 = 40k\Omega$. Then, plug that into the expression of $V_{o,2}$, and the condition we will now want is

$$(\frac{R_4}{R_3 + R_4})(1 + \frac{R_2}{R_1}) = 40$$

which gives us

$$\frac{R_4}{R_3 + R_4} = \frac{40}{41}$$

. So, we need to choose R_3 and R_4. As before, we can choose these values in many ways and one such choice is $R_4 = 40k\Omega$, $R_3 = 1k\Omega$.

NOTE: Keep in mind that, for this problem, we actually assumed that $V_1 = V_{left}$ and $V_2 = V_{right}$. Which would mean that we are ideally connecting V_{left} and V_{right} as inputs. But, in reality, we're actually connecting the outputs from the iPhone as inputs. This means that R_{left} and R_{right} will also actually affect the output.

With this effect, we will actually get

$$V_o = \frac{-V_1 R_2}{R_{1,eff}} + V_2 \left(\frac{R_4}{R_{3,eff} + R_4}\right)\left(1 + \frac{R_2}{R_{1,eff}}\right)$$

where $R_{1,eff} = R_1 + R_{left}$ and $R_{3,eff} = R_3 + R_{right}$.

So, we can just *fold in* the effect of R_{left} and R_{right} into these. For instance, we will now want to set $R_{3,eff} = 1k\Omega$. So, we can actually make $R_3 = R_{3,eff} - 3\Omega = 997\Omega$. and $R_1 = R_{1,eff} - 3\Omega = 997\Omega$.

Give yourself full credit even if you didn't notice this, but keep this in mind!

BONUS: Can you now see why we wanted to keep R_1 and R_3 in the order of $k\Omega$ or larger?

4. **Your Own Problem** Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?