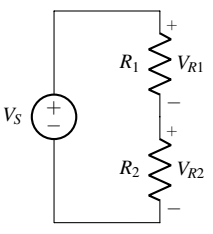
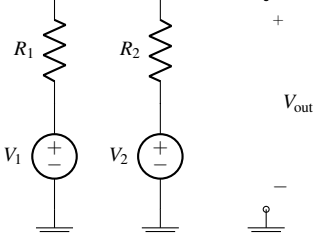
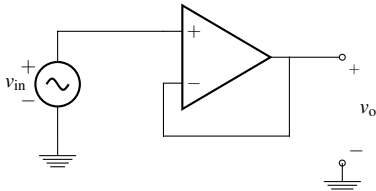
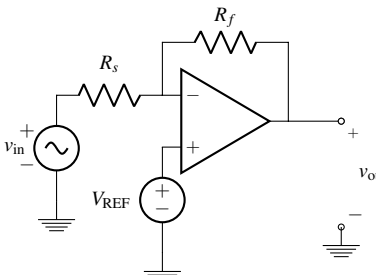
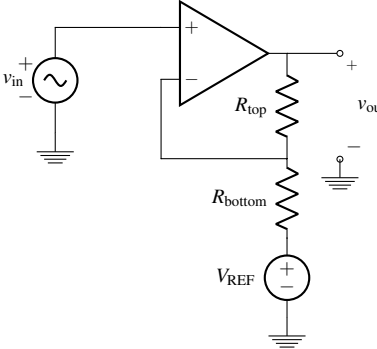
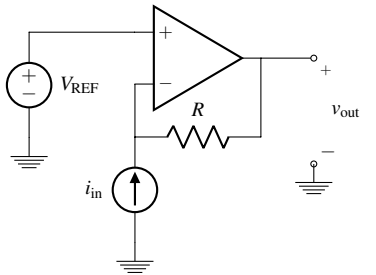


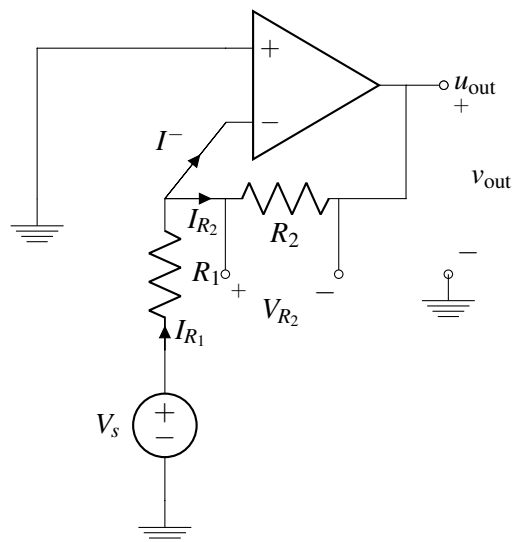
# EECS 16A Designing Information Devices and Systems I

## Spring 2020 Discussion 9B

### For Reference: Example Circuits

<p style="text-align: center;"><b>Voltage Divider</b></p>  $V_{R2} = V_S \left( \frac{R_2}{R_1 + R_2} \right)$	<p style="text-align: center;"><b>Voltage Summer</b></p>  $V_{out} = V_1 \left( \frac{R_2}{R_1 + R_2} \right) + V_2 \left( \frac{R_1}{R_1 + R_2} \right)$	<p style="text-align: center;"><b>Unity Gain Buffer</b></p>  $\frac{v_{out}}{v_{in}} = 1$
<p style="text-align: center;"><b>Inverting Amplifier</b></p>  $v_{out} = v_{in} \left( -\frac{R_f}{R_s} \right) + V_{REF} \left( \frac{R_f}{R_s} + 1 \right)$	<p style="text-align: center;"><b>Non-inverting Amplifier</b></p>  $v_{out} = v_{in} \left( 1 + \frac{R_{top}}{R_{bottom}} \right) - V_{REF} \left( \frac{R_{top}}{R_{bottom}} \right)$	<p style="text-align: center;"><b>Transresistance Amplifier</b></p>  $v_{out} = i_{in} (-R) + V_{REF}$

### 1. An Inverting Amplifier



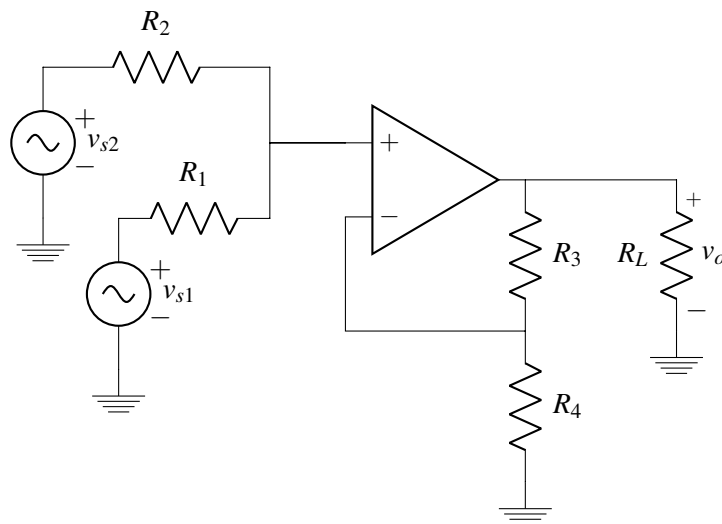
- (a) Calculate  $v_{\text{out}}$  as a function of  $V_s$  and  $R_1$  and  $R_2$ .

**Answer:**

Because the op-amp is in negative feedback, we know that  $u^+ = u^- = 0 \text{ V}$ . Therefore,  $v_{\text{out}} = u^- - V_{R_2} = -I_{R_2}R_2$ .

We also know that  $I^- = 0$ , so  $I_{R_1} = I_{R_2}$ . Thus,  $v_{\text{out}} = u^- - V_{R_2} = -I_{R_2}R_2 = -I_{R_1}R_2 = -V_s \frac{R_2}{R_1}$ .

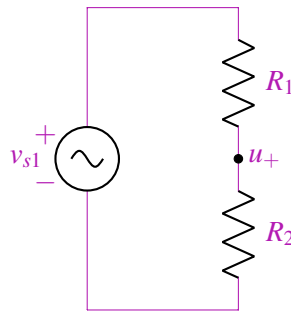
## 2. Multiple Inputs To One Op-Amp



- (a) For the circuit above, find an expression for  $v_o$ . (Hint: Use superposition.)

**Answer:**

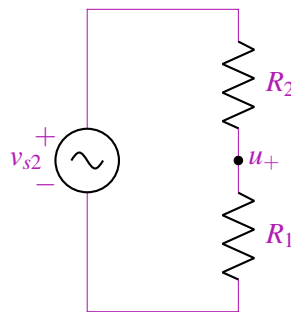
Let's call the potential at the positive input of the op-amp  $u_+$ . Using superposition, we first turn off  $v_{s2}$  and find  $u_+$ . The circuit then looks like:



We recognize the above circuit as a voltage divider. Thus,

$$u_{+,vs1} = \frac{R_2}{R_1 + R_2} v_{s1}$$

By symmetry, we expect  $v_{s2}$  to have a similar circuit and expression. The circuit for  $v_{s2}$  looks like:



The expression for  $u_+$  with  $v_{s2}$  is then:

$$u_{+,vs2} = \frac{R_1}{R_1 + R_2} v_{s2}$$

From superposition, we know the output must be the sum of these.

$$u_+ = \frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}$$

With  $u_+$  determined, we can find the output voltage directly from the formula for a non-inverting amplifier. We can also derive it using the process below.

From the negative feedback rule,  $u_+ = u_-$ . Using voltage dividers, we can express  $u_-$  in terms of  $v_o$ :

$$u_- = \frac{R_4}{R_3 + R_4} v_o$$

$$v_o = \left(1 + \frac{R_3}{R_4}\right) u_- = \left(1 + \frac{R_3}{R_4}\right) u_+$$

Now, to find the final output, we can set  $u_+$  to our earlier expression.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left( \frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2} \right)$$

- (b) How could you use this circuit to find the sum of different signals?

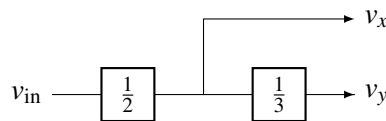
**Answer:**

The circuit already finds the weighted sum of two inputs. By setting  $R_1 = R_2$  and  $R_3 = R_4$ , we can take the exact sum of two inputs.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left(\frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}\right) = (1 + 1) \left(\frac{1}{2} v_{s1} + \frac{1}{2} v_{s2}\right) = v_{s1} + v_{s2}$$

### 3. Modular Circuits

In this problem, we will explore the design of circuits that perform a set of (arbitrary) mathematical operations in order to elucidate some of the important properties and uses of op-amps in negative feedback. We have noticed that voltage dividers are not compose-able, so we will use op-amps instead. We would like to implement the block diagram shown below:

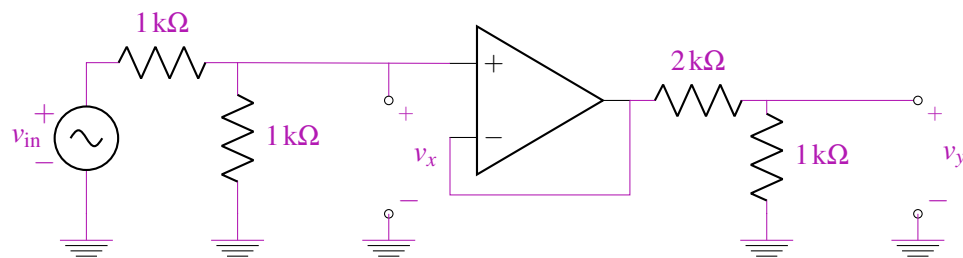


In other words, we want to implement a circuit with two outputs  $v_x$  and  $v_y$ , where  $v_x = \frac{1}{2} v_{in}$  and  $v_y = \frac{1}{3} v_x$ .

- (a) Using an ideal op-amp in negative feedback, modify the design of one of the two voltage divider circuits you built (i.e. the  $\frac{1}{2}$  block or the  $\frac{1}{3}$  block), so that the originally intended relationships between  $v_x$  and  $v_{in}$  as well as  $v_y$  and  $v_x$  are realized by the resulting overall circuit (where each block is replaced by its individual implementation). Is this configuration enough by itself to attach loads at  $v_x$  and  $v_y$ ?

**Answer:**

Use a voltage buffer. Note that this configuration's outputs would change with the addition of a load. As a follow-up, think about ways to make the outputs agnostic to the loads attached. If we used the latter half of the circuit as a fractional divider block, we would need to buffer the output.

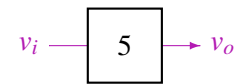
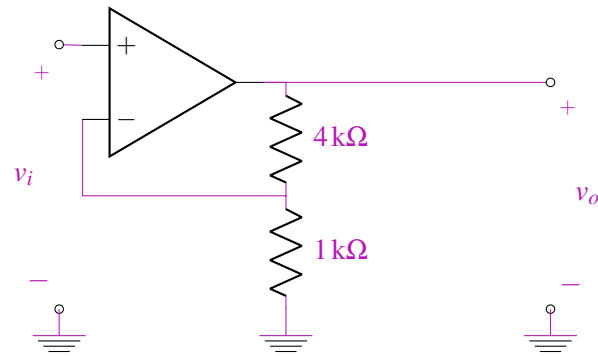


- (b) Now let's assume that we want to expand our toolbox of circuits that implement mathematical operations. In particular, design blocks that implement:
- $v_o = 5 v_i$
  - $v_o = -2 v_i$
  - $v_o = v_{i1} + v_{i2}$

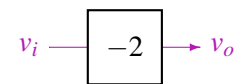
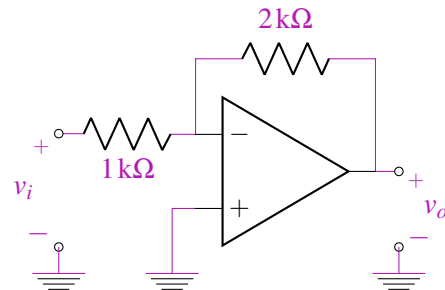
Pay careful attention to the way you design these blocks, so that connecting any one block to any other block does not modify the intended functionality of any of the blocks.

**Answer:**

i.



ii.



iii.

