EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 1A

1. Systems of Equations

Solve the following systems of equations, or if there is no solution, explain why. Plot parts (a) and (b).

substitution
elimination

(a) $y = 6 - 2x$

$3x - 2(6 - 2x) = 2$

$3x + 4x = 2 + 12$

$7x = 14$

$x = 2$

$6 - 2(2)$

$y = 2$

$$\begin{cases} 2x + y = 6 \\ 3x - 2y = 2 \end{cases}$$

$$\begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \left[\begin{array}{cc|c} 2 & 1 & 6 \\ 3 & -2 & 2 \end{array}\right] \end{array}$$

(b) $y = 7 - 3x$

$6x + 2(7 - 3x) = 15$

$6x + 14 - 6x = 15$

$-14 \qquad -14$

$6x - 6x = 1$

$0 = 1$

No solution means there is no intersection with the lines they are parallel

$$\begin{cases} 6x + 2y = 15 \\ 3x + y = 7 \end{cases}$$

$$\left[\begin{array}{cc|c} 6 & 2 & 15 \\ 3 & 1 & 7 \end{array}\right] \xrightarrow{R2 \leftarrow 2R_2} \left[\begin{array}{cc|c} 6 & 2 & 15 \\ 6 & 2 & 14 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 0 & 0 & -1 \end{array}\right]$$ ← not true

(c) *redundancy

$x = 1 + y$

$z = 1 - 2y$

$(1 + y) + y + (1 - 2y) = 2$

infinite solutions > means they are the same line

$$\begin{cases} x + y + z = 2 \\ x - y = 1 \\ 2y + z = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & 0 & 1 \\ 0 & 2 & 1 & 1 \end{array}\right]$$

$x + y + z = 2$

$- x - y \quad = -1$

$\boxed{2y + z = 1}$ ← anything that satisfies this equation is the solution because it satisfies all three equations

2. Finding The Bright Cave

Nara the one-handed druid and Kody the one-handed ranger find themselves in dire straits. Before them is a cliff with four cave entrances arranged in a square: two upper caves and two lower caves. Each entrance emits a certain amount of light, and the two wish to find exactly the amount of light coming from each cave. Here's the catch: after contracting a particularly potent strain of ghoul fever, our intrepid heroes are only able to see the total intensity of light before them (so their eyes operate like a single-pixel camera). Kody and Nara are capable adventurers, but they don't know any linear algebra – and they need your help.

Kody proposes an imaging strategy where he uses his hand to completely block the light from two caves at a time. He is able to take measurements using the following four masks (black means the light is blocked from that cave):

$m_1 = m_2 + m_4 - m_3$

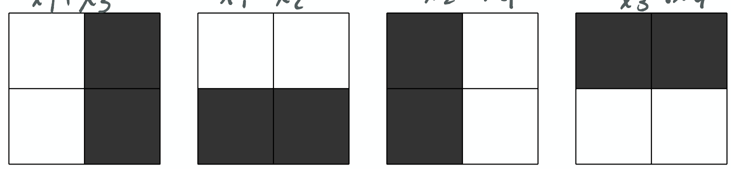
Adding $m_2 + m_4 = m_1 + m_3$ implies redundancy

$m_1 = x_2 + x_4$
$m_2 = x_1 + x_2$
$m_3 = x_2 + x_4$
$m_4 = x_3 + x_4$

Cave Labels

x_1	x_2
x_3	x_4

$x_1 + x_3$ $\quad$ $x_1 + x_2$ $\quad$ $x_2 + x_4$ $\quad$ $x_3 + x_4$



Measurement 1 $\quad$ Measurement 2 $\quad$ Measurement 3 $\quad$ Measurement 4

m_1 $\quad$ m_2 $\quad$ m_3 $\quad$ m_4

Figure 1: Four image masks.

(a) Let x_1, x_2, x_3, and x_4 represent the magnitude of light emanating from the four cave entrances shown in the image above. Write an equation for each masking process in Figure 1 which results in the four measurements of total light: m_1, m_2, m_3, and m_4.

(b) Does Kody's set of masks give us a unique solution for all four caves' light intensities? Why or why not? Does not have 4 unique solutions

(c) Nara, in her infinite wisdom, places her one hand diagonally across the entrances, covering two of the cave entrances. However, her hand is not wide enough, letting in 50% of the light from the caves covered and 100% of the light from the caves not covered. The following diagram shows the percentage of light let through from each cave:

$x_3 = m_5 - \frac{1}{2}m_2 - \frac{1}{2}m_3$

$x_4 = m_4 - x_3$

50%	100%
100%	50%

$x_2 + x_3 + \frac{1}{2}x_1 + \frac{1}{2}x_4 = m_5$

Does this additional measurement give them enough information to solve the problem? Why or why not?