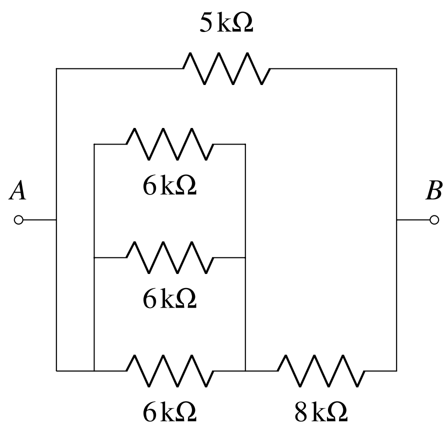
EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 7B

1. Series and Parallel Combinations

For the resistor network shown below, find an equivalent resistance between the terminals A and B using the resistor combination rules for series and parallel resistors.



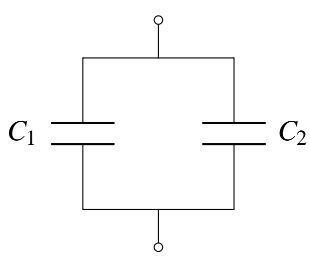
Answer:

$$5\,\text{k}\Omega \parallel ((6\,\text{k}\Omega \parallel 6\,\text{k}\Omega \parallel 6\,\text{k}\Omega) + 8\,\text{k}\Omega) = 5\,\text{k}\Omega \parallel (2\,\text{k}\Omega + 8\,\text{k}\Omega) = 5\,\text{k}\Omega \parallel 10\,\text{k}\Omega = 3.33\,\text{k}\Omega$$

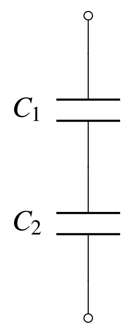
2. Series And Parallel Capacitors

Derive C_{eq} for the following circuits.

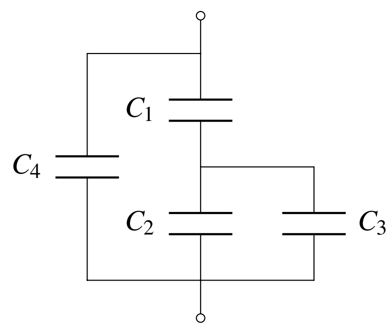
(a)



(b)



(c)

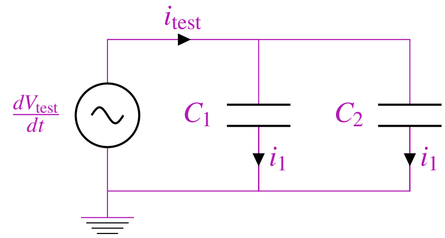


Answer:

(a)

$$C_{eq} = C_1 + C_2$$

Notice these capacitors are in parallel. We can derive their equivalent capacitance by connecting them to a voltage source with a constant derivative, as shown by the circuit below:



Since both capacitors have the same voltage across them:

$$\frac{dV_{C_1}}{dt} = \frac{dV_{C_2}}{dt} = \frac{dV_{\text{test}}}{dt}$$

$$i_1 = C_1 \frac{dV_{\text{test}}}{dt}$$

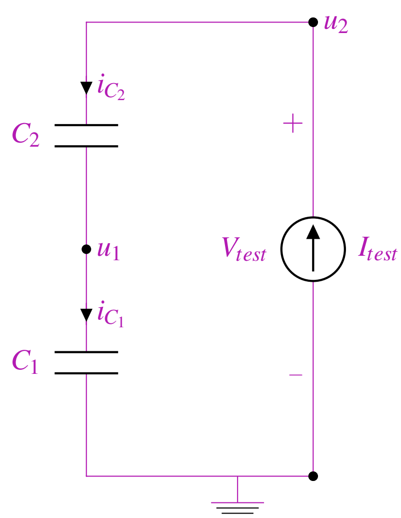
$$i_2 = C_2 \frac{dV_{\text{test}}}{dt}$$

$$i_t = i_1 + i_2 = (C_1 + C_2) \frac{dV_{\text{test}}}{dt}$$

Since we know $i_{\text{test}} = C_{\text{eq}} \frac{dV_{\text{out}}}{dt}$,

$$C_{\text{eq}} = C_1 + C_2$$

(b) In order to find the equivalence capacitance of the circuit, we plug in a test current source, and measure the rate of change of voltage across it.



From KCL, we know that all of the currents are equal.

$$i_{C_1} = i_{C_2} = I_{test}$$

For each capacitor, we plug in our $I - \frac{dV}{dt}$ relationship:

$$i_{C_1} = I_{test} = C_1 \frac{du_1}{dt}$$

$$i_{C_2} = I_{test} = C_2 \frac{d(u_2 - u_1)}{dt} = C_2 \left(\frac{du_2}{dt} - \frac{du_1}{dt} \right)$$

Next, we eliminate u_1 from the equations above and rearrange.

$$\frac{du_1}{dt} = \frac{I_{test}}{C_1} \Rightarrow I_{test} = C_2 \frac{du_2}{dt} - \frac{C_2}{C_1} I_{test}$$

$$I_{test} = \frac{C_2}{1 + \frac{C_2}{C_1}} \frac{du_2}{dt}$$

Finally, we plug in that $u_2 = V_{test}$ and solve for the equivalent capacitance with $C_{eq} = I_{test} / \frac{dV_{test}}{dt}$

$$I_{test} = \frac{C_2}{1 + \frac{C_2}{C_1}} \frac{dV_{test}}{dt}$$

$$\Rightarrow C_{eq} = \frac{C_2}{1 + \frac{C_2}{C_1}} = \frac{C_1 C_2}{C_1 + C_2}$$

Note that this is the same as saying $C_{eq} = C_1 \parallel C_2$. Remember that the $\parallel$ operator is mathematical notation; in this case, the capacitors are actually in series, but *mathematically* their equivalent circuit is found via the "parallel resistor" operation.

(c) Given that we know what the relationship for capacitors in series and parallel are from the last two parts, we can just simply the capacitors step by step:

$$C_{eq} = (C_4 + (C_1 \parallel (C_2 + C_3))) = \frac{C_4(C_1 + C_2 + C_3) + C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

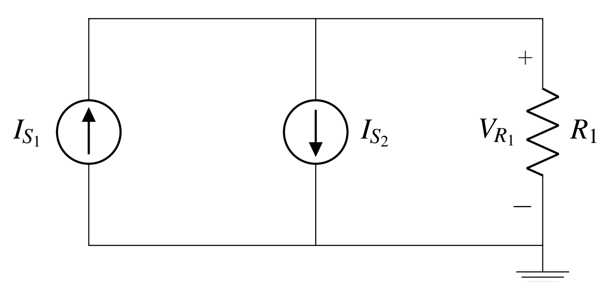
3. Superposition

For the following circuits:

i. Use the superposition theorem to solve for the voltages across the resistors.

ii. For parts (a) and (b) only, find the power dissipated/generated by all components. Is power conserved?

(a)



Answer:

i. While we could apply the algorithm we have learned in class, let's see if there's a way to find the answer quicker than before. We're looking for the voltage across the resistor, which could be found quickly using Ohm's law if we knew the current. If we were to apply KCL at the node at the top of the circuit, one source is coming in, the other source is leaving, and the current through the resistor is leaving. From KCL, we then know $i_{R_1} = I_{S_1} - I_{S_2}$. Applying Ohm's Law we find:

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

We could also solve this using superposition. Turning on I_{S_1} gives $V_{R_1} = I_{S_1}R_1$. Turning on I_{S_2} gives $V_{R_1} = -I_{S_2}R_1$. Finally, the total V_{R_1} is the sum of the individual V_{R_1}'s or

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

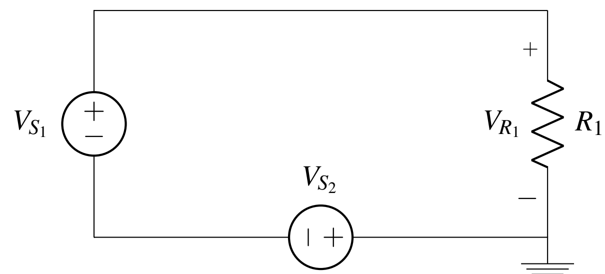
ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = (I_{S_1} - I_{S_2})^2 R_1$$
$$P_{I_{S_1}} = -I_{S_1}V_{R_1} = -(I_{S_1} - I_{S_2})I_{S_1}R_1$$
$$P_{I_{S_2}} = I_{S_2}V_{R_1} = (I_{S_1} - I_{S_2})I_{S_2}R_1$$

$$P_{R_1} + P_{I_{S_1}} + P_{I_{S_2}} = (I_{S_1} - I_{S_2})^2 R_1 - (I_{S_1} - I_{S_2})I_{S_1}R_1 + (I_{S_1} - I_{S_2})I_{S_2}R_1 = 0$$

Power is conserved.

(b)



Answer:

i. Once again, we could apply the circuit analysis algorithm or find the answer directly. Notice the circuit only has one loop, so we can use KVL to find the voltage across the resistor.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

We could also solve with superposition. Turning on V_{S_1} gives $V_{R_1} = V_{S_1}$. Turning on V_{S_2} gives $V_{R_1} = -V_{S_2}$. The overall voltage is then the sum.

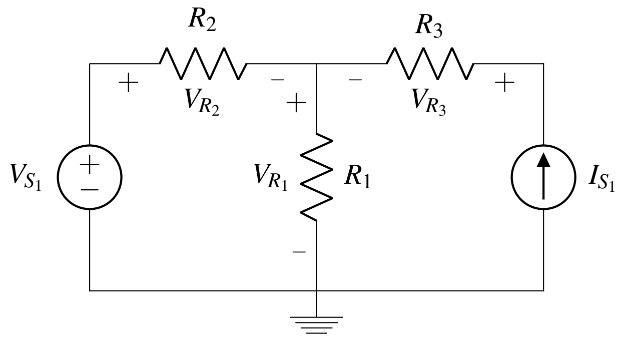
$$V_{R_1} = V_{S_1} - V_{S_2}$$

ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = \frac{(V_{S_1} - V_{S_2})^2}{R_1}$$
$$P_{V_{S_1}} = -I_{R_1}V_{S_1} = -\frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1}$$
$$P_{V_{S_2}} = I_{R_1}V_{S_2} = \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1}$$
$$P_{R_1} + P_{V_{S_1}} + P_{V_{S_2}} = \frac{(V_{S_1} - V_{S_2})^2}{R_1} - \frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1} + \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1} = 0$$

Power is conserved.

(c)



Answer: Turning on only V_{S_1}, we have the following voltages across the resistors:

$$V_{R_1} = \frac{R_1}{R_1+R_2}V_{S_1}$$

$$V_{R_2} = \frac{R_2}{R_1+R_2}V_{S_1}$$

$$V_{R_3} = 0$$

Then turning only I_{S_1}, we have the following voltages:

$$V_{R_1} = \frac{R_1R_2}{R_1+R_2}I_{S_1}$$

$$V_{R_2} = -\frac{R_1R_2}{R_1+R_2}I_{S_1}$$

$$V_{R_3} = I_{S_1}R_3$$

Using superposition we can sum up the contributions from both V_{S_1} and I_{S_1} to get:

$$V_{R_1} = \frac{R_1}{R_1+R_2}V_{S_1} + \frac{R_1R_2}{R_1+R_2}I_{S_1}$$

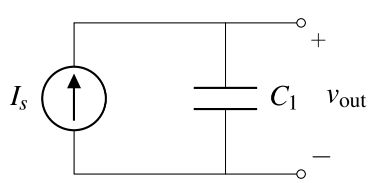
$$V_{R_2} = V_{S_1} - V_{R_1} = \frac{R_2}{R_1+R_2}V_{S_1} - \frac{R_1R_2}{R_1+R_2}I_{S_1}$$

$$V_{R_3} = I_{S_1}R_3$$

4. Current Sources And Capacitors

For the circuits given below, give an expression for $v_{\text{out}}(t)$ in terms of I_s, C_1, C_2, and t. Assume that all capacitors are initially uncharged, i.e. the initial voltage across each capacitor is $0\,\text{V}$.

(a)



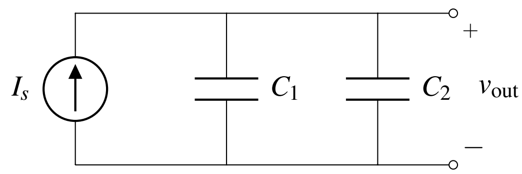
Answer:

$$I_s = C_1 \frac{dv_{\text{out}}(t)}{dt}$$

$$v_{\text{out}}(t) = \int \frac{I_s}{C_1} dt = \frac{I_s t}{C_1} + v_{\text{out}}(0)$$

Since the capacitor is initially uncharged, $v_{\text{out}}(0) = 0$, so $v_{\text{out}}(t) = \frac{I_s t}{C_1}$.

(b)



Answer:

We can combine the two capacitors into an equivalent capacitor with capacitance $C_1 + C_2$. Again, $v_{\text{out}}(0) = 0$ because all capacitors are initially uncharged.

$$I_s = (C_1 + C_2) \frac{dv_{\text{out}}(t)}{dt}$$

$$v_{\text{out}}(t) = \frac{I_s t}{C_1 + C_2} + v_{\text{out}}(0) = \frac{I_s t}{C_1 + C_2}$$