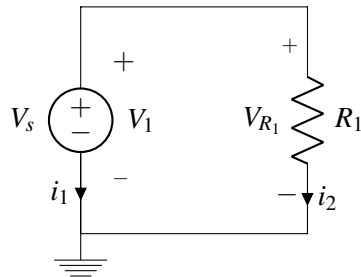# EECS 16A   Designing Information Devices and Systems I
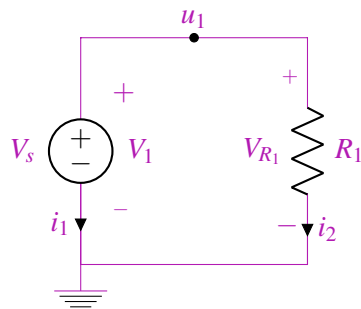## Spring 2020
<div align="right">Discussion 7A</div>

1. Passive Sign Convention and Power

(a) Suppose we have the following circuit and label the currents as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$.

Answer: We'll start by solving the circuit for the unknown node potentials and currents.

The KCL equation for the one node in this circuit is:

$$i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$u_1 - 0 = V_1 = V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$
$$i_1 = -1\,\text{A}$$
$$i_2 = 1\,\text{A}$$

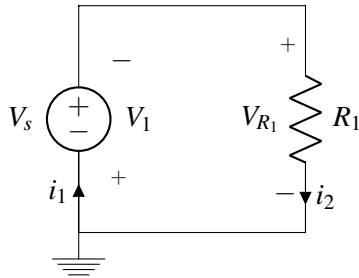From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

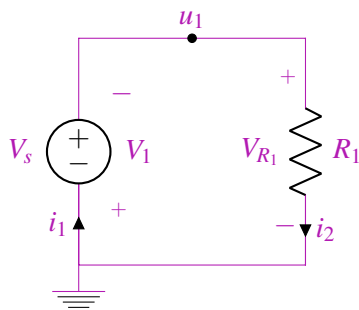Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = i_1 V_1 = i_1 V_s = -1\,\text{A} \cdot 5\,\text{V} = -5\,\text{W}$$

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is adding power to the circuit.

(b) Suppose we change the label of the currents in the circuit to be as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$.

Answer: We'll solve the circuit the same way as last time.

The KCL equation for the one node in this circuit is:

$$-i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$0 - u_1 = V_1 = -V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$

$$i_1 = 1\,\text{A}$$

$$i_2 = 1\,\text{A}$$

From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = i_1 V_1 = i_1(-V_s) = 1\,\text{A} \cdot -5\,\text{V} = -5\,\text{W}$$

Notice here that the circuit has the same power dissipated by all the elements. This is because with both labeling of currents, we followed the passive sign convention.

2. Circuit Analysis

(a) Use nodal analysis to solve for all node voltages.

(b) Find current I_{R_3} flowing through resistor R_3.
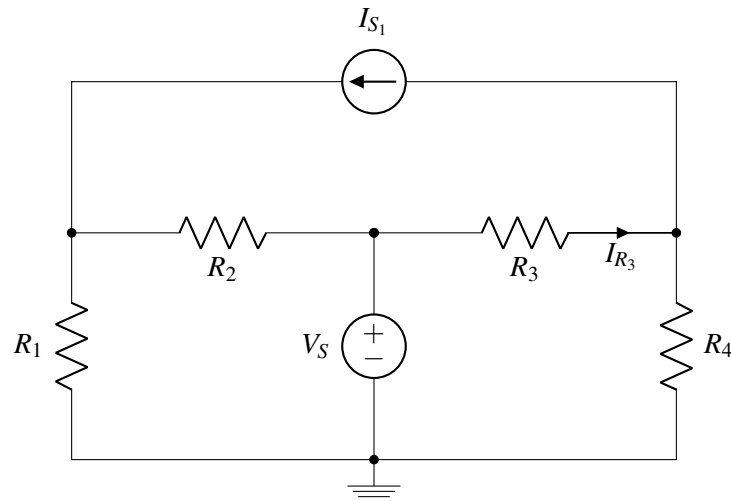
Answer:

(a) Let us follow the procedure illustrated in lecture. Step 1 (selecting a ground node) has already been done for us, we will go through steps 2-4 in the following circuit diagram. As a reminder the steps are the following:

Step 2: Label all known voltages determined by voltage sources (denoted below as u_1),

Step 3: Label all remaining unknown nodes (denoted below as u_2, u_3),

Step 4: Label element voltages and currents (V_{R_1}, I_{R_1}, etc.)
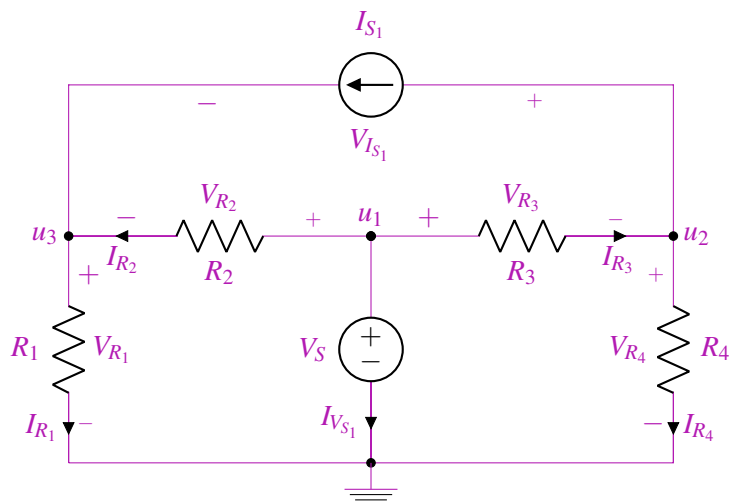
Note: You can pick any direction you want for the currents but once you have done so you the voltages you choose have to abide by the passive sign convention!! Vice versa, you can pick the element voltage sign first but then the current has to abide by the passive sign convention.

Step 5: Write KCL equations for all nodes with unknown voltages (namely u_2, u_3):

$$-I_{R_3} + I_{S_1} + I_{R_4} = 0$$
$$-I_{R_2} - I_{S_1} + I_{R_1} = 0$$

Step 6: Find expressions for all element currents in terms of element voltages and characteristics,

$$u_3 - 0 = I_{R_1} R_1$$
$$V_s - u_3 = I_{R_2} R_2$$
$$u_2 - 0 = I_{R_4} R_4$$
$$V_s - u_2 = I_{R_3} R_3$$

Where we used the fact that $u_1 = V_s$

Step 7: Substitute expressions found in 6 into the KCL equations from step 5,

$$-\frac{V_s - u_2}{R_3} + I_{S_1} + \frac{u_2}{R_4} = 0$$
$$-\frac{V_s - u_3}{R_2} - I_{S_1} + \frac{u_3}{R_1} = 0$$

Notice that we can now directly solve for u_2 and u_3 and then substitute into the equations of Step 6 to find the currents through all resistors. The node voltages are given by:

$$u_2 = \frac{V_s R_4 - I_{S_1} R_3 R_4}{R_3 + R_4}$$
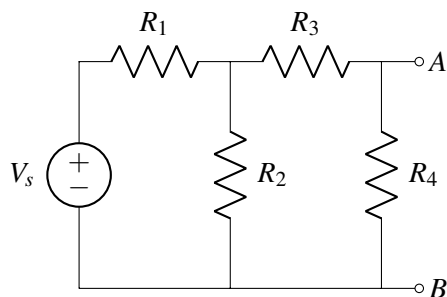$$u_3 = \frac{V_s R_1 + I_{S_1} R_1 R_2}{R_1 + R_2}$$

Finally, KCL in node V_1 will give us the current through the voltage source, V_s.

(b)

$$I_{R_3} = \frac{V_s - u_2}{R_3}$$

3. KVL and KCL

For the circuit shown below, $V_s = 5\,\text{V}$, $R_1 = R_2 = 4\,\text{k}\Omega$, and $R_3 = R_4 = 2\,\text{k}\Omega$.

(a) For the circuit above, write KVL equations for each loop and KCL equations for each node.

Answer:

KVL through both of the loops:

$$V_s - V_{R_1} - V_{R_2} = 0$$

$$V_s - V_{R_1} - V_{R_3} - V_{R_4} = 0$$

$$V_{R_2} - V_{R_3} - V_{R_4} = 0$$

KCL at each of the nodes:

$$-I_{V_s} + I_{R_1} = 0$$

$$-I_{R_1} + I_{R_2} + I_{R_3} = 0$$

$$-I_{R_3} + I_{R_4} = 0$$

$$-I_{R_4} - I_{R_2} + I_{V_s} = 0$$

The above equations combined with Ohm's law for each resistor would let us find the current through and voltage across each resistor. The system would in general be overdetermined and difficult to solve. The next part shows a quicker way to find the solution.

(b) Solve for the voltage between A and B using resistor combination rules and divider rules.

Answer:

First, let's find the equivalent reisistance:

$$R_{eq} = R_1 + R_2 \parallel (R_3 + R_4) = 4\,\text{k}\Omega + 4\,\text{k}\Omega \parallel 4\,\text{k}\Omega = 6\,\text{k}\Omega$$

Now, we can find the current out of the voltage source using Ohm's law and this equivalent resistance:

$$I = \frac{V_s}{R_{eq}} = \frac{5\,\text{V}}{6\,\text{k}\Omega} = 0.833\,\text{mA}$$

This current is divided among the two branches at the node where R_1, R_2, and R_3 meet. Let's use the currend divider formula:

$$I_{R_{34}} = \frac{R_2}{R_2 + R_3 + R_4} I_s = \frac{4\,\text{k}\Omega}{8\,\text{k}\Omega} I_s = 0.5 I_s = 0.417\,\text{mA}$$

Finally, using the current through R_4, we find the votlage across it:

$$V_{AB} = I_4 R_4 = 0.417\,\text{mA} \cdot 2\,\text{k}\Omega = 0.833\,\text{V}$$