

This homework is due March 29, 2016, at Noon.

We hope you have/had a lovely Spring Break!! Relax and recuperate :)

1. Homework process and study group

Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?

Working in groups of 3-5 will earn credit for your participation grade.

Solution: I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on Problem 5 so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.

2. IoT4eva Revisited

After guiding them to make an intelligent selection for their super-capacitors, IoT4eva was so happy with your performance that you got a promotion! The good news is that you're getting paid more, but the "bad" news is that you have more responsibilities too. In particular, you are now responsible not only for selecting the super-capacitors used to power the device, but also for building the rest of the circuitry associated with the power supply.

In practice, many real circuits (especially sensors that are trying to detect very small signals) don't like to operate with supply voltages that vary substantially over time. Remembering that the voltage on our super capacitors drops linearly as we pull current out of them, this means that if we want to use these super capacitors for our device, we need to build another circuit. This circuit is powered by the super-capacitor and produces a constant voltage at its output, where this voltage will then be used to supply power to rest of the device. These circuits are often referred to as "voltage regulators", and in this problem we'll explore how to build the simplest form of such a voltage regulator.

- (a) The first problem we may have had to solve to realize such a voltage regulator is to figure out how to build a reference that would allow us to set the voltage at the output of our regulator to a known absolute value. Fortunately someone else in the company has already built one of those and made it available to you - you can model this circuit as a voltage source whose value is $0.8V$ with a source resistance of $1k\Omega$. (The internals of this voltage reference circuit aren't important for this problem, but as you should see shortly, this circuit by itself is not appropriate for supplying power to the rest of the device.)

Now that we have a reference we can focus on the core of the voltage regulator itself. Using this reference circuit, an op-amp, and resistors, design a circuit that is powered by the super-capacitor voltage V_{sc} (which for now you can assume is always high enough for the circuit to work) and that would produce a constant $1.2V$ supply voltage for the rest of the device. Note that you can model the load from the rest of the device as a $10mA$ current source; please be sure to choose specific values for any resistors you use in your circuit as well.

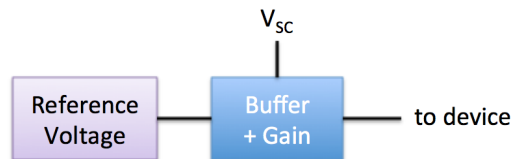
Solution: Let's practice the design method.

Step 1:

In this problem the ultimate objective is to output a $1.2V$ node that is capable of driving the load modeled as a $10mA$ current source. We are also required to power up the voltage regulator using the super-capacitor.

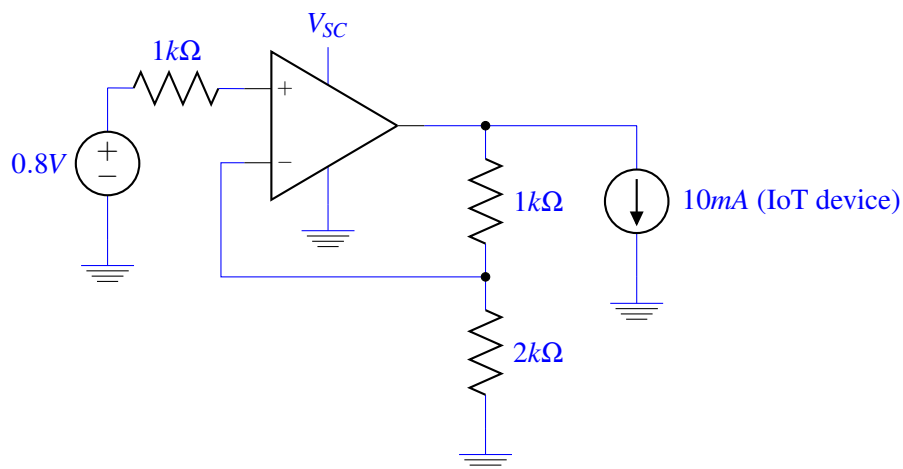
Step 2:

We are given a reference voltage source that we can use to power the IoT device. However, the voltage of the reference is not high enough and it has a high source resistance (in fact, if we run $0.8mA$ of current through the source resistance the voltage drop across the resistor would be the same as the voltage source itself). The only other source of voltage is the super-capacitor, but the problem is it has a variable voltage. Thus, we need to build a circuit that buffers (provides low output resistance) and amplifies the reference voltage which is powered by the super-capacitor.



Step 3:

Now that we have a high-level block diagram of the circuit, we can think about how to implement it. We need some form of buffer, so we will definitely need an op-amp. Moreover, we also know that the output of the op-amp must be directly connected to the device for it to act as a buffer. We also know that the circuit can only be powered by the super-capacitor, so we power the op-amp using V_{SC} . The other functionality we have not dealt with is the gain. We need a gain of $1.2V/0.8V = 1.5$, and we have seen that we can implement this using a non-inverting amplifier. Since the gain of a non-inverting amplifier is $\frac{R_2+R_1}{R_1}$, we can set any value to R_1 and R_2 such that this ratio is 1.5. For example, we can use $R_1 = 2k\Omega$ and $R_2 = 1k\Omega$. Note that the source resistance of the reference voltage doesn't play a role here, since there is no current flowing into the inputs of the op-amp. The circuit is shown below.



- (b) Now that we've built the voltage regulator and we know that we want its output voltage to stay fixed at $1.2V$, what is the minimum voltage we need on our super capacitors $V_{sc,min}$ to ensure that the regulator can indeed produce a fixed $1.2V$ output?

Solution: The op-amp will not be able to produce $1.2V$ at its output if $V_{SC} < 1.2V$, so $V_{SC,min} = 1.2V$.

- (c) One of the most important things to evaluate about a voltage regulator is its efficiency - i.e., the power dissipated by the load circuits (in this case, the rest of the IoT4eva device) divided by the total amount of power delivered by the power supply. Continuing to model the rest of the IoT4eva device as a $10mA$ current source, how much power is dissipated by the $10mA$ current source? As a function of V_{sc} (and assuming V_{sc} is higher than the minimum you found in part b), how much power is actually delivered by the super-capacitor? What is therefore the efficiency of your voltage regulator circuit?

Note that you can assume that the op-amp does not dissipate any power except for what is required to supply the current to its output. (Hint: The op-amp itself can't generate any power, so you should think about where this current would have to originate from.) It is also worth noting that the voltage reference circuit that was given to you would actually dissipate some power from the super-capacitor as well, but you can ignore that for this problem.

Solution: The $10mA$ current source is supplied with $1.2V$, so

$$P_{device} = 1.2V \cdot 10mA = 12mW$$

Our reference voltage does not output any current since the current into the input of an op-amp in negative feedback is $0A$, so the power associated with that source is $0W$. It is important to note that the op-amp by itself cannot generate any power. Any current flowing to the $1.2V$ output node has to come from the op-amp, and any current that flows out of the op-amp must come from the super-capacitor. The op-amp is not supplying any power, it just dissipates a part of the power it receives and passes on the rest to the output. The total power supplied by the super-capacitor is the product of the output current of the op-amp with the voltage of the super-capacitor. This output current is the sum of the current source and the current flowing through the negative feedback resistors to ground.

$$I_{op-amp} = 10mA + \frac{1.2V}{3k\Omega} = 10.4mA$$

Power from super-capacitor is thus $V_{sc} \cdot 10.4mA$. So, the efficiency is

$$\frac{1.2V \cdot 10mA}{V_{sc} \cdot 10.4mA} = \frac{12}{10.4V_{sc}}$$

Note: The efficiency might differ depending on your choice of resistor values.

Notice that V_{sc} shows up in the denominator. Thus if we increase the super-capacitor voltage our efficiency drops. This might be counterintuitive at first, but the reason this happens is that the op-amp is configured in a way that forces the output to a certain voltage. That means there is some voltage drop that happens inside the op-amp itself, and this voltage drop is wasted since it does not get delivered to the IoT device.

Even though the efficiency of this circuit is quite low, this circuit is actually used in real life quite often. This is because the circuit is small (in PCB area) and have nice properties in terms of isolating different components in a circuit from each other.

- (d) Still using only op-amps and resistors, is there anything you can do to improve the efficiency of your voltage regulator design?

Solution: We cannot really do anything with the $\frac{1.2V}{V_{sc}}$ term since $1.2V$ is required for the load and V_{sc} is basically set by the super-capacitor. Thus, we can only try to improve the $\frac{10mA}{10.4mA}$ part. Since the load current is constant, to lower this ratio, we can increase the feedback resistors; for example to $10k\Omega$ and $20k\Omega$ - this would raise the efficiency to $\frac{12}{10.4V_{sc}}$.

Note: We can never get to an efficiency of 1 (100%). This is because there is voltage drop between the super-capacitor and the output of the op-amp since V_{sc} is variable. Since at least I_{load} has to flow out of the op-amp, even without the feedback current V_{sc} has to stay exactly $1.2V$ for the efficiency to be 1, which is not possible.

3. Midterm Problem 3

Redo Midterm Problem 3.

Solution: [See midterm solutions.](#)

4. Midterm Problem 4

Redo Midterm Problem 4.

Solution: [See midterm solutions.](#)

5. Midterm Problem 5

Redo Midterm Problem 5.

Solution: [See midterm solutions.](#)

6. Midterm Problem 6

Redo Midterm Problem 6.

Solution: [See midterm solutions.](#)

7. Midterm Problem 7

Redo Midterm Problem 7.

Solution: [See midterm solutions.](#)

8. Midterm Problem 8

Redo Midterm Problem 8.

Solution: [See midterm solutions.](#)

9. Midterm Problem 9

Redo Midterm Problem 9.

Solution: [See midterm solutions.](#)

10. Midterm Problem 10

Redo Midterm Problem 10.

Solution: [See midterm solutions.](#)

11. Your Own Problem Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?