Linearly dependent : $c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$
if not all c_k are 0

else. linearly independent. $c_1 v_1 + c_2 v_2 + \cdots c_k v_k = 0$
iff all $c_k = 0$

rank = number of linearly independent vectors

span = set of all possible linear combinations

vector space : must be closed under addition & multiplication
must contain the zero vector

subspace : subset of vector space that is a vector space

bases : linearly indep. set of vectors that span a vector space
* bases are not unique

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$A_{12} = A_2 A_1$

Matrix mult. is not commutative

Reflection :

across x-axis $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ y-axis : $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ y=x : $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$\begin{array}{cc} & \begin{array}{cc} \%_1 & \%_2 \end{array} \\ \begin{array}{c} \%_1 \\ \%_2 \end{array} & \begin{bmatrix} \%_1 \to \%_1 & \%_2 \to \%_1 \\ \%_1 \to \%_2 & \cdots \end{bmatrix} \end{array}$$

"conserved" if cols sum to 1

$A^{-1}A = I$, $(A^{-1})^{-1} = A$, $(kA)^{-1} = k^{-1}A^{-1}$, $(A^T)^{-1} = (A^{-1})^T$, $(AB)^{-1} = B^{-1}A^{-1}$
k is nonzero A, B are invertible

Non-invertible if determinant is 0.

$$n_1 \downarrow \overset{m_1 \rightarrow}{\begin{bmatrix} & \\ & \end{bmatrix}} \times n_2 \downarrow \overset{m_2 \rightarrow}{\begin{bmatrix} & \\ & \end{bmatrix}} = n_1 \downarrow \overset{m_2 \rightarrow}{\begin{bmatrix} & \\ & \end{bmatrix}}$$

$n_1 \times \underline{m_1} \times \underline{n_2} \times m_2 = n_1 \times m_2$

$\theta = 180$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$\theta = 30$

$\sqrt{3}/2 + \frac{1}{2}$

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\frac{\sqrt{3}}{2} - \frac{1}{2}$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Proofs : Rank(AB) < Rank(A)

A is invertible, B is not $\therefore$ There exists $\underline{x}$ s.t. $B\underline{x} = 0$

$A(B\vec{x}) = A\vec{0} = \vec{0}$, since $A(B\vec{x}) = (AB)\vec{x}$,

$(AB)\vec{x} = \vec{0}$, $\therefore$ columns of AB are linearly dependent

and its rank must be $< n$.

True / False :

· $n \times n$ matrix A for which $A^2 = 0$, False

→ Left and right multiply by A^{-1} and we get $I_n = 0$, contradiction

A is invertible $n \times n$, $\forall \vec{b}\ \mathbb{R}^n$ $A\vec{x} = \vec{b}$ has a unique solution, True

→ $A^{-1}Ax = A^{-1}B$ ⇒ $\vec{x} = A^{-1}\vec{b}$

A set of n linearly dependent vectors in $\mathbb{R}^n$ can span $\mathbb{R}^n$, False

→ n linearly dep. vectors span $0 < \dim(\text{span}) < n$

Rank (5×5) > Rank (4×4) , False

→ Rank is determined by # lin. indep. rows

Proofs : 1) what do we know? → translate into math form

2) what would we like to show ? → "

3) How do we get from 1) to 2) ?

Prove : if $v_1, v_2, v_1 + v_2$ are all solutions to $A\underline{x} = \underline{b}$, then $\underline{b} = \underline{0}$

$$A\vec{v}_1 = \vec{b}$$
$$A\vec{v}_2 = \vec{b}$$
$$A(v_1 + v_2) = \vec{b}$$
$$A\vec{v}_1 + A\vec{v}_2 = \vec{b}$$
$$\vec{b} + \vec{b} = \vec{b}$$
$$\vec{b} = 0$$

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Apply row operations to T matrix to keep span.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^T \Rightarrow [x \ y \ z]$$