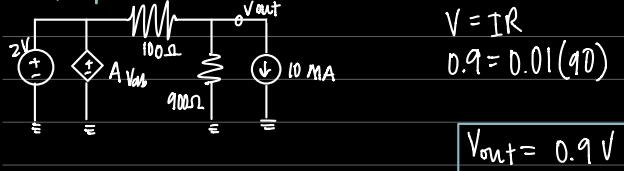


1. superposition with a dependent source



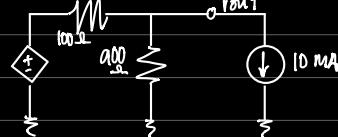
$$V = IR$$

$$0.9 = 0.01(90)$$

$$R_{eq} = \frac{100 \cdot 900}{100 + 900} = 90 \Omega$$

$$V_{out} = 0.9 V$$

eliminate independent voltage



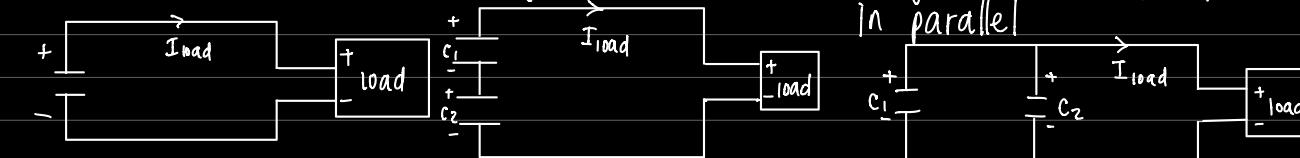
2. supercapacitors

2a) Load → the load is having constant current I_{load}

configuration 1:

configuration 2:

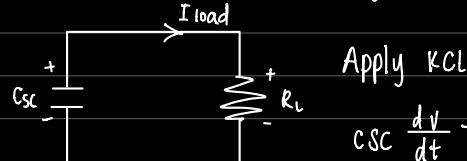
configuration 3: two supercapacitors connected in parallel

2b) Given initial voltage = V_{init}

$$C = C_1 = C_2 = C_{SC}$$

configuration 1:

- since load is taking constant current, we can assume load as sensitive load (R_L)
- so the capacitor will discharge through R_L



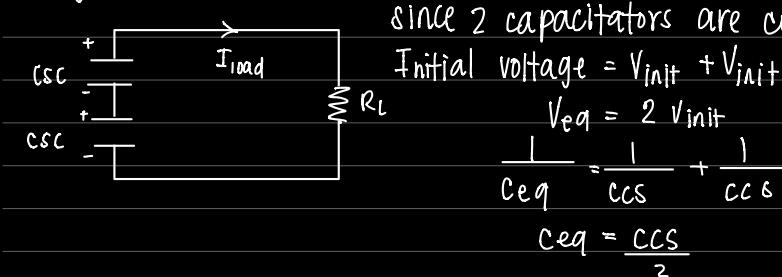
Apply KCL

$$CSC \frac{dV}{dt} + \frac{V}{R_L} = 0 \Rightarrow \frac{dV}{dt} = -\frac{V}{R_L \cdot CSC}$$

 V_{init} = Voltage across capacitor at time $t=0$

$$V(f) = V_{init} e^{-t / R_L \cdot CSC}$$

configuration 2:



since 2 capacitors are connected in series

Initial voltage = $V_{init} + V_{init}$

$$V_{eq} = 2 V_{init}$$

$$C_{eq} = \frac{1}{CCS} + \frac{1}{CCS}$$

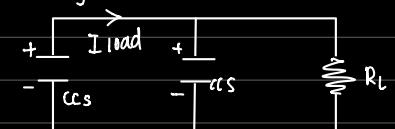
$$C_{eq} = \frac{CCS}{2}$$

Equation of capacitor voltage with time:

$$V(t) = V_{eq} e^{-t / R_L \cdot C_{eq}}$$

$$V(t) = 2 V_{init} e^{-2t / R_L \cdot CCS}$$

configuration 3:



two capacitors are connected in parallel

$$V_{eq} = V_{init}$$

$$C_{eq} = CCS + CCS$$

$$= 2 CCS$$

$$V(f) = V_{eq} e^{-t / R_L \cdot C_{eq}}$$

$$V(f) = V_{init} e^{-t / R_L \cdot CCS}$$

2c) for device to work properly capacitor must supply at least V_{min} . So $V(f) \geq V_{min}$ configuration 1 : if $V(f) = V_{min}$ $t = t_z$ (lifetime)

$$V_{min} = V_{init} e^{-t_z / R_L \cdot CSC}$$

$$\frac{t_z}{R_L \cdot CSC} = \ln \left(\frac{V_{min}}{V_{init}} \right)$$

$$t_z = R_L \cdot CSC \ln \left(\frac{V_{init}}{V_{min}} \right)$$

$$\text{configuration 2: } V_{\min} = 2V_{\text{init}} e^{-2t_L / RL \cdot CCS}$$

$$\text{configuration 3: } V_{\min} = V_{\text{init}} e^{-t_L / RL \cdot CCS}$$

$$t_L = \frac{RL \cdot CCS}{2} \ln\left(\frac{2V_{\text{init}}}{V_{\min}}\right)$$

2d) single capacitor doesn't provide sufficient lifetime, so the configuration should be selected which gives high lifetime so configuration 3 will be more suitable as it gives more lifetime as compared to configuration 2 because in parallel combo. the current load get distributed between two super capacitors

2e)

$$E = \frac{1}{2} C V^2$$

$$V_C^2 = V_2 \cdot 2C_{SC} \cdot \left(V_{\text{init}} - \frac{i_{\text{load}}}{2C_{SC}} \cdot f \right)^2$$

3. Dynamic Random Access Memory (DRAM)

$$V_{\text{init}} = 1.2V$$

$$V_{\min} = V_{\text{bit}} = 0.9V$$

$$Q_{\text{bit}} = V_{\text{bit}} C_{\text{bit}} \rightarrow I_{\text{leak}} = \frac{dV_{\text{bit}}}{dt} C_{\text{bit}} \rightarrow I_{\text{leak}} = \frac{\Delta V_{\text{bit}}}{\Delta t} C_{\text{bit}}$$

$$= \frac{V_{\text{init}} - V_{\min}}{t_{\text{store}}} C_{\text{bit}}$$

$$I_{\text{leak}} = \frac{(1.2V - 0.9V) * 28fF}{10^{-3}s}$$

4. It's finally Raining

$$4a) C = \frac{\epsilon A}{d}$$

ϵ = permittivity of dielectric material
 A = Area of plates
 d = distance

Applied to physical structure

$$h_{\text{tot}} * W$$

W = distance between the plates W

$$C_{\text{empty}} = \frac{\epsilon_{\text{air}} h_{\text{tot}} W}{W} = \epsilon h_{\text{tot}}$$

$$C_{\text{full}} = \frac{\epsilon_{\text{H}_2\text{O}} h_{\text{tot}} W}{W} = 81 \epsilon h_{\text{tot}}$$

4b) capacitance of the two plates separated by water

capacitance of the two plates separated by air

$$C_{\text{water}} = \frac{\epsilon_{\text{H}_2\text{O}} h_{\text{H}_2\text{O}} W}{W} = 81 \epsilon_{\text{H}_2\text{O}}$$

$$C_{\text{air}} = \frac{\epsilon_{\text{air}} (h_{\text{tot}} - h_{\text{H}_2\text{O}}) W}{W} = \epsilon (h_{\text{tot}} - h_{\text{H}_2\text{O}})$$

• they are in parallel \Rightarrow add two results

$$C_{\text{tank}} = C_{\text{water}} + C_{\text{air}} = \epsilon (h_{\text{tot}} + 80 h_{\text{H}_2\text{O}})$$

$$4c) V_c(t) = \frac{1}{C} \int idt$$

$$V_c(t) = \frac{1}{C_{\text{tank}}} \int_0^t I_s dt$$

$$V_c(t) = \frac{I_s t}{C_{\text{tank}}}$$

4d)

1. First disconnect C_{tank} from voltage source

2. short its terminals to ensure that $V_c(0) = 0$ (that is the initial voltage is 0)

$$C_{\text{tank}} = \frac{I_s t}{V_c(t)}$$

also from part b), we have

3. Then connect its terminals to the DC current source
Is for time t

$$h_{H_2O} = \frac{C_{\text{tank}}}{\Sigma} - h_{\text{tot}} \times \frac{1}{80}$$

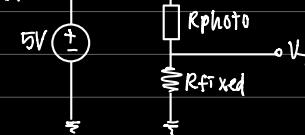
4. Note the V_C at the end of time interval t

5. Apply the following equations to compute C_{tank}
& h_{H_2O}

5. LED Alarm circuit

5a) op-amp in comparation configuration
voltage divides configuration

$$V_+ = \left(\frac{R_{\text{fixed}}}{R_{\text{fixed}} + R_{\text{photo}}} \right) \times 5$$

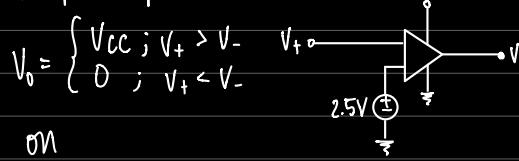


5b) light conditions $\Rightarrow R_{\text{photo}} = 1k\Omega$
dark conditions $\Rightarrow R_{\text{photo}} = 10k\Omega$
light condition $\Rightarrow V_+ = 3V$

$$V_+ = \left(\frac{R_{\text{fixed}}}{R_{\text{fixed}} + R_{\text{photo}}} \right) \times 5$$

under light conditions $3 = \frac{R_{\text{fixed}}}{R_{\text{fixed}} + 1k\Omega} \times 5$
 $3(R_{\text{fixed}} + 1k\Omega) = 5R_{\text{fixed}}$
 $3k\Omega - 5R_{\text{fixed}} = 3R_{\text{fixed}}$
 $R_{\text{fixed}} = 3/2 k\Omega$

5c) op-amp as comparation



$$V_o = \begin{cases} 5V; V_+ > 2.5V & \\ 0; V_+ < 2.5V & \end{cases}$$

$$5d) V_+ = \left(\frac{R_{\text{fixed}}}{R_{\text{fixed}} + R_{\text{photo}}} \right) \times 5V$$

$$R_{\text{fixed}} = 3/2 k\Omega \text{ from part b}$$

$$V_f = \left(\frac{R_{\text{fixed}}}{3/2 k\Omega + R_{\text{photo}}} \right) \times 5V$$

$$\text{conditions } V_+ > 2.5V \rightarrow V_o = 5V$$

$$5 \times 3/2 k\Omega > 2.5 \\ R_{\text{photo}} + 3/2 k\Omega > 2.5$$

$$1.5k\Omega > 2.5 R_{\text{photo}} + 3.75k\Omega$$

$$+ 3.75k\Omega > 2.5 R_{\text{photo}}$$

$$R_{\text{photo}} < 1.5k\Omega$$

$$\text{For } V_o = 5V \rightarrow R_{\text{photo}} < 1.5k\Omega$$

$$\text{For } V_o = 0V < 2.5V \rightarrow R_{\text{photo}} > 1.5k\Omega$$

$$\begin{cases} 5V; R_{\text{photo}} < 1.5k\Omega \\ 0; R_{\text{photo}} > 1.5k\Omega \end{cases}$$

5e) From LED portion of the circuit

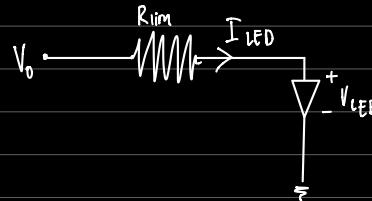
$$V_{LED} = V_o - R_{\text{lim}} \cdot I_{LED}$$

$$V_f = 3V (\text{LED to act})$$

$$I_{LED} \text{ recommend} = 20mA$$

$$3V = 5V - R_{\text{lim}} \times 20mA$$

$$\downarrow V_o$$

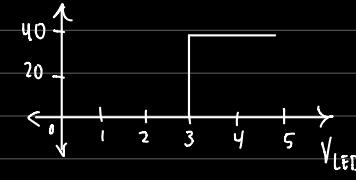


when light conditions

$$3 = 5 - R_{\text{lim}} \times 20 \times 10^{-3} I_{LED}$$

$$\Rightarrow R_{\text{lim}} \times 2 \times 10^{-2} = 2$$

$$\therefore R_{\text{lim}} = 100\Omega$$



6. DC-DC voltage divider

$$Q_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

$$V_{C1} = \frac{\frac{C_1 \cdot C_2}{C_1 + C_2} \cdot V_{in}}{C_1}$$

(6b)

$$V_{in} - \frac{C_1 \cdot C_2}{C_1 + C_2} \cdot V_{in}$$

$$Q_{C1} = \frac{C_1 \cdot C_2}{C_1 + C_2} \cdot V_{in}$$

$$V_{C2} = \frac{\frac{C_1 \cdot C_2}{C_1 + C_2} \cdot V_{in}}{C_2}$$

$$Q_{eq} = Q_{C1} V_{in}$$

$$Q_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2} \cdot V_{in}$$

$$Q_{C2} = \frac{-C_1 \cdot C_2}{C_1 + C_2} \cdot V_{in}$$

(6c) Output voltage = $V_{in}/2$; Efficiency: $E = Y_2 C_1 V_{C1}^2$

$$E_{\text{total}} = 1/2 \left(\frac{C_1 \cdot C_2}{C_1 + C_2} \right) V_{in}^2 - \left(\frac{C_1 \cdot C_2}{C_1 + C_2} \right) \left(\frac{C_1 \cdot C_2}{C_1 + C_2} \cdot V_{in} \right)^2$$

8 homework process & study group

I worked with Sadia (3034541667) but mostly by myself. This took me 6 hours. 1 hour with Sadia & 5 hours on my own