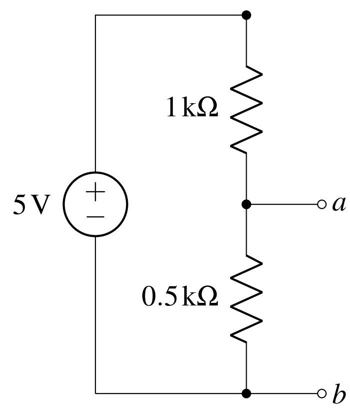
EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 9A

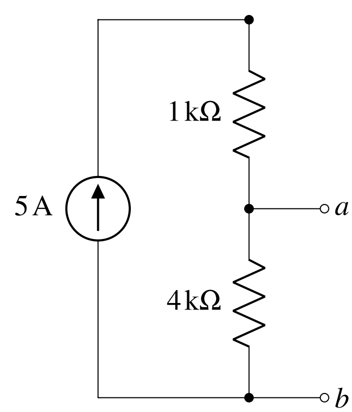
1. **Equivalence**

   Find the Thévenin and Norton equivalents across terminals *a* and *b* for the circuits given below.

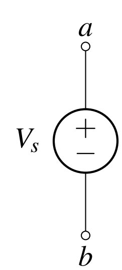
   (a)



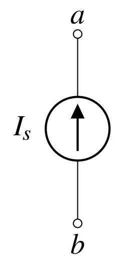
   (b)



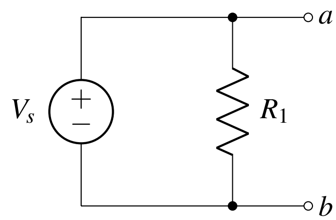
   (c)



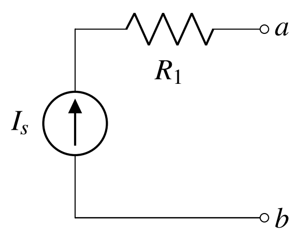
   (d)



(e) (Practice)

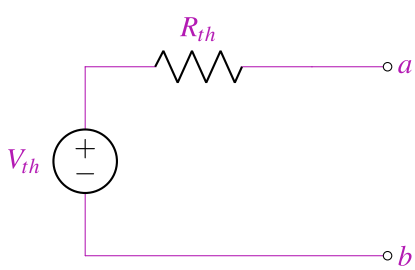


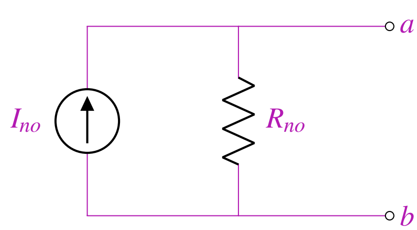
(f) (Practice)



Answer:

The general Thévenin and Norton equivalents are shown below:





(a)

$$V_{th} = 1.67\,\mathrm{V}, I_{no} = 5\,\mathrm{mA}, R_{th} = R_{no} = 333\,\Omega$$

(b)

$$V_{th} = 20\,000\,\mathrm{V}, I_{no} = 5\,\mathrm{A}, R_{th} = R_{no} = 4000\,\Omega$$

(c) A Norton equivalent of a voltage source is not necessary, since a voltage source is a basic element. The Thévenin equivalent is just a voltage source with voltage V_s, that is, $R_{th} = 0$.

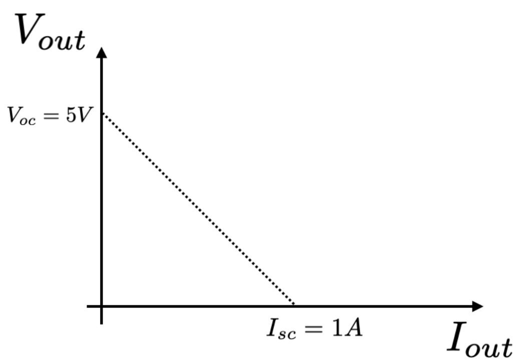
(d) A Thévenin equivalent of a current source is not necessary because a current source is a basic element and cannot be represented as a voltage source. The Norton equivalent is just a current source with current I_s, that is, $R_{no} = \infty$.

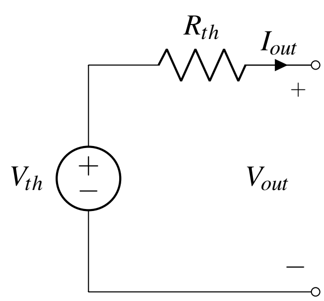
(e) The Thévenin equivalent is just a voltage source with voltage V_s, that is, $R_{th} = 0$. Notice that adding a parallel resistor does not change the Thévenin equivalent. As before, since the circuit is effectively a voltage source, a Norton equivalent is not required.

(f) The Norton equivalent is just a current source with current I_s, that is, $R_{no} = \infty$. Notice that adding a series resistor does not change the Norton equivalent. With a similar argument as before, the Thévenin equivalent for the source is not required, as it is a current source.

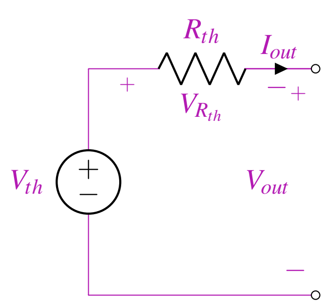
2. Thevenin equivalence

(a) You are given the following $I_{out} - V_{out}$ characteristic of the Thevenin model of a circuit. Find the Thevenin voltage and the Thevenin resistance.

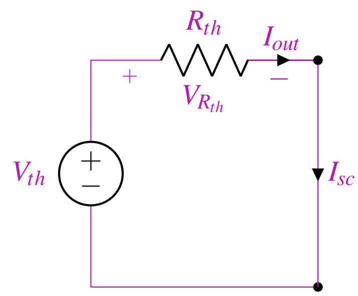




Answer: The Thevenin voltage corresponds to the open circuit voltage of the $I_{out} - V_{out}$ characteristic and is equal to 5V. By KVL, $-V_{th} + V_{R_{th}} + V_{out} = 0$. We also can see that $I_{out} = I_{R_{th}}$.

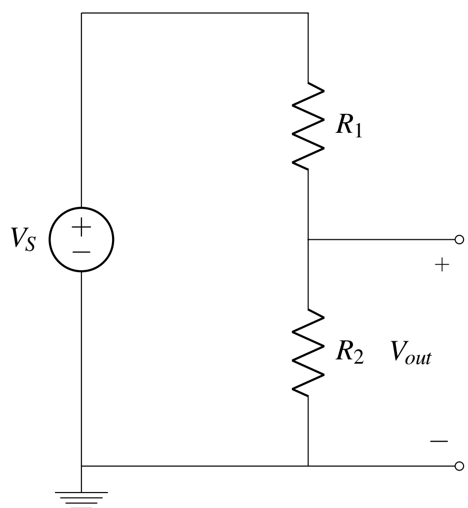


When the output is shorted, $I_{out} = I_{sc}$, and $V_{out} = 0V$, which implies that $V_{R_{th}} = V_{th}$, and $I_{R_{th}} = I_{sc}$.



Thus by Ohm's law, the Thevenin resistance is equal to $R_{th} = \frac{V_{R_{th}}}{I_{R_{th}}} = \frac{V_{th}}{I_{sc}}$.
Therefore $R_{th} = 5\Omega$.

(b) You are given a voltage divider as shown below. Find R_1 and R_2 such that the Thevenin equivalent model is the same as that of (a). You are given that $V_S = 10V$.



Answer:
From part (a), the Thevenin voltage corresponds to the open circuit voltage of the $I_{out} - V_{out}$ characteristic and is equal to 5V.
The Thevenin voltage V_{th} of the voltage divider is equal to:

$$\begin{aligned} V_{th} &= V_s \frac{R_2}{R_1 + R_2} \\ &= 10V \frac{R_2}{R_1 + R_2} = 5V \\ 10VR_2 &= 5VR_1 + 5VR_2 \\ 5VR_2 &= 5VR_1 \end{aligned}$$

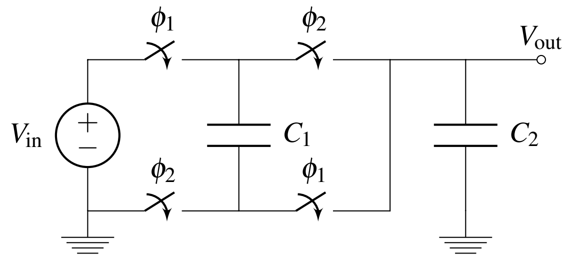
Therefore $R_1 = R_2$.
The short circuit current I_{sc} should be equal to $1A$. If we short the output of the voltage divider circuit we have:

$$I_{sc} = \frac{V_s}{R_1} = \frac{10}{R_1} = 1A$$

Therefore $R_1 = 10\Omega$ and $R_2 = 10\Omega$.

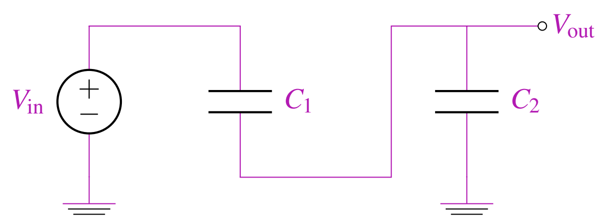
3. Charge Sharing

Consider the circuit shown below. In phase ϕ_1, the switches labeled ϕ_1 are on while the switches labeled ϕ_2 are off. In phase ϕ_2, the switches labeled ϕ_2 are on while the switches labeled ϕ_1 are off.



(a) Redraw the circuit in phase ϕ_1. Label the voltages across each capacitor and find the charge on and voltage across each capacitor as a function of V_{in}, C_1, and C_2. Assume the capacitors are uncharged before phase ϕ_1.

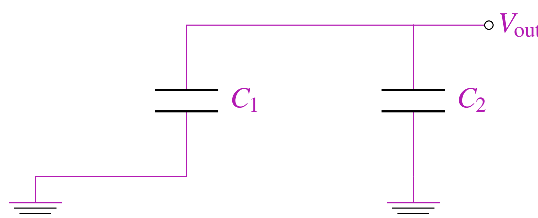
Answer:



The two capacitors in series have a total capacitance of $\frac{C_1C_2}{C_1+C_2}$. We know that there is a voltage of V_{in} across this capacitor and thus $V_{\text{in}}\frac{C_1C_2}{C_1+C_2}$ charge. The charge on C_1 must be equal to the charge on C_2. Knowing the charge on each capacitor, we know the voltage across both. Therefore, the voltage across C_1 is $\frac{C_2}{C_1+C_2}V_{\text{in}}$. The voltage across C_2 is similarly $\frac{C_1}{C_1+C_2}V_{\text{in}}$.

(b) Redraw the circuit in phase ϕ_2. Label the voltages across each capacitor and find the charge on and voltage across each capacitor as a function of V_{out}, C_1, and C_2.

Answer:



The charge on C_1 is simply C_1V_{out}. Similarly the charge on C_2 is C_2V_{out}.

(c) Find V_{out} as a function of V_{in}, C_1, and C_2.

Answer:

We know the total charge in the system is conserved between phase ϕ_1 and phase ϕ_2. There was a charge of $V_{\text{in}}\frac{C_1C_2}{C_1+C_2}$ on each capacitor, so the total charge in phase ϕ_1 was $2V_{\text{in}}\frac{C_1C_2}{C_1+C_2}$. Since we know charge is conserved, this must be equal to the total charge in phase ϕ_2.

$$2V_{\text{in}}\frac{C_1C_2}{C_1+C_2} = (C_1+C_2)V_{\text{out}}$$

$$V_{\text{out}} = 2\frac{C_1 C_2}{(C_1 + C_2)^2} V_{\text{in}}$$

(d) How will the charges be distributed in phase ϕ_2 if we assume $C_1 \gg C_2$?

Answer:

We know that the capacitors are in parallel in phase ϕ_2, so the voltage across both capacitors is the same. Considering that $Q = CV$, if $C_1 \gg C_2$, then $Q_1 \gg Q_2$.