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Math 55, Handout 2.

LOGICAL EQUIVALENCES.

1.1. A **tautology** is a compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it

1.2. A **contradiction** is a compound proposition that is always false

1.3. A **contingency** is a compound proposition that is neither a tautology nor a contradiction

2. The notation $p \equiv q$ means the compound propositions p and q are logically equivalent

3. De Morgan's Laws are:

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Q1. Prove the Absorption Laws: $p \vee (p \wedge q) \equiv p$ and $p \wedge (p \vee q) \equiv p$.

p	q	$p \wedge q$	$p \vee q$	$p \vee (p \wedge q)$	$p \wedge (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	T	F	F
F	F	F	F	F	F

4. A proposition is **satisfiable** if there is an assignment of truth values to its variables that makes it true. Otherwise it is **unsatisfiable**.

Q2. Which of these propositions is satisfiable?

$(p \vee (p \wedge q)) \vee \neg p$: satisfiable when p = F

$(p \wedge (p \vee q)) \vee \neg q$: satisfiable when q = F

$\neg p \vee q \to \neg(p \vee q)$: satisfiable when q = F

PREDICATES AND QUANTIFIERS.

5. A **predicate** or **propositional function** is the property that the subject of the statement can have. The statement P(x) is also said to be the value of the propositional func

It can be **unary** or n-**ary**.

5. The **universal quantification** $\forall x P(x)$ means P(x) for all values of x in the domain

6. The **existential quantification** $\exists x P(x)$ means there exists an element x in the domain such that P(x)

DOMAIN AND ITS IMPORTANCE.

7. The **domain / domain of discourse / universe of discourse** means the possible values of the variable x.

Q3. Give an example of a predicate $P(x)$ and different domains so that the statements $\forall x P(x)$ is true, $\forall x P(x)$ is false, $\exists x P(x)$ is true, $\exists x P(x)$ is false.

Predicate "$x > 10$"

Domain: $[11, 19]$
$\forall x P(x) = $ True
Domain: $[0, 10]$
$\forall x P(x) = $ False

Domain: $(-\infty, \infty)$
$\exists x P(x) = $ True
Domain: $(-\infty, 100)$
$\exists x P(x) = $ False

LOGICAL EQUIVALENCES INVOLVING QUANTIFIERS.

8. De Morgan Laws for Quantifiers are:

Negation	Equivalent statement	when is Negation True?	when False?
$\neg \exists x P(x)$	$\forall x \neg P(x)$	For every x, P(x) is False	There is an x for which P(x) is true
$\neg \forall x P(x)$	$\exists x \neg P(x)$	There is an x for which P(x) is false	P(x) is true for every x

Q4. Let $P(x)$, $Q(x)$, and $R(x)$ be the predicates "x is a clear explanation", "x is satisfactory", "x is an excuse". Let the domain for x be all English text. Express these statements using the given predicates and quantifiers:

"All clear explanations are satisfactory": $\forall x\, P(x) \rightarrow Q(x)$

"Some excuses are unsatisfactory": $\exists x\, R(x) \land \neg Q(x)$

"Some excuses are not clear explanations": $\exists x\, R(x) \land \neg P(x)$

Does the last statement follow from the first two?

yes, because the statement is true due to no contradictions. if the first two are true then the third can also be true since it doesn't contradict the rest

NESTED QUANTIFIERS.

Q5. Give the definition of a limit of a real-valued function $f(x)$ at point a. State its negation so that the negation sign preceeds no quantifier.

$\forall \varepsilon\, \exists N\, \forall n\, [\varepsilon > 0 \land n > N \rightarrow |a_n - a| < \varepsilon]$

$\neg \forall \varepsilon\, \exists N\, \forall n\, [\varepsilon > 0 \land n > N \rightarrow |a_n - a| < \varepsilon]$

$\exists \varepsilon\, \forall N\, \exists n\, \neg[\neg(\varepsilon > 0 \land n > N) \lor |a_n - a| < \varepsilon]$

$\exists \varepsilon\, \forall N\, \exists n\, [(\varepsilon > 0 \land n > N \land \neg(|a_n - a| < \varepsilon)]$

$\therefore \exists \varepsilon\, \forall N\, \exists n\, [\varepsilon > 0 \land n > N \land |a_n - a| \geq \varepsilon]$