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Math 55, Handout 4.

PROOF METHODS.

1.1. A **proof by exhaustion** is

a proof that proceeds by exhausting all possibilities and examining a small number of examples.

1.2. A **proof by cases** is

a method that shows the original conditional statement with a hypothesis made up of propositions p1, p2, ... pn can be proved through proving each of the n conditional statements. The proof must cover all possible cases that arise in a theorem.

Q1. Write down and prove the rule of inference behind these proof methods.

$p_1 \vee p_2 \vee \dots \vee p_k \to q \equiv (p_1 \to q) \wedge (p_2 \to q) \wedge \dots \wedge (p_k \to q)$

If $p_1 \vee \dots \vee p_k \to q = F$, then $q = F$ and at least one $p_j = T$

Then $p_j \to q = F$ so the right hand side is F.

Conversely, if the R.H.S. is F, then there is at least one $p_j \to q = F$, so $q = F$ and $p_j = T$.

Thus, $p_1 \vee \dots \vee p_k = T$, $q = F$

Hence $p_1 \vee \dots \vee p_k \to q = F$

1.3. **Without loss of generality (WLOG)** means

to use an assumption in a proof that makes it possible to prove a theorem by reducing the number of cases to consider in the proof.

1.4. Common errors with these two kinds of proofs are

- drawing the incorrect conclusions from examples
- not covering every possible case
- making unwarranted assumptions

Q2. Give an example of an error with a proof by cases.

Theorem: If x is a real number, then x^2 is a positive real number.

Proof: p_1 = "x is positive"

p_2 = "x is negative"

q = "x^2 is positive"

when x is positive, x^2 is positive (product of 2 positive numbers x)

when x is negative, x^2 is positive also

This misses the case of x being 0.

2.1. A **constructive existence proof** is

an existence proof given by finding an element a such that P(a) is true.

2.2. A **non-constructive existence proof** is

an existence proof where an element a such that P(a) is true is not found, but the existence proof is proven true in another way.

Q3. Prove there is a pair of consecutive integers such that one of them is a perfect square and the other is a perfect cube. Is your proof constructive or non-contructive?

Consecutive integers: 8 and 9
(2)^3 = 8
(3)^2 = 9
This is a constructive existence proof.

3.1. A **uniqueness proof** has two parts:

Existence: show an element x with the desired property exists
Uniqueness: show that if y does not equal x, then y does not have the desired property

PROOF STRATEGIES.

4.1. **Forward reasoning** is

a proof that uses a sequence of steps that leads to the conclusion.

4.2. **Backward reasoning** is

a proof with a sequence of steps to reason backwards by finding a statement p that can rove a statement q to prove the property that p --> q.

4.3. **Adapting an existing proof** means

taking advantage of existing proofs of similar results and adapting them to prove other facts by providing clues clues for new proofs.

Q4. Prove that $(n-1)n$ is even for any integer n. What is your proof strategy? Proof by cases

case 1: n is even
$n = 2m$ for $m \in \mathbb{N}$
Thus $n(n-1) = 2m(2m-1) = 2[m(2m-1)]$ is even.
case 2: n is odd
$n = 2m-1$ where $m \in \mathbb{N}$
Thus $n(n-1) = (2m-1)(2m-2) = 2[(2m-1)(m-1)]$ is even.

Q5. Adapt your strategy from Q4 to show that $(n-1)n(n+1)$ is divisible by 3 for any integer n.

case 1: $n = 3k$
case 2: $n = 3k-1$] for some $n \in \mathbb{Z}$
case 3: $n = 3k-2$
Among 3 consecutive integers, exactly one is divisible by 3, so their product is divisible by 3