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Math 55, Handout 26.

REVIEW: PROBABILITY.

1. Name the following bits of knowledge in an approximate order of importance within each category:

(a) at least 15 probabilistic notions: Bayes' Theorem, Finite probability, probability distribution, uniform distribution, probability of pairwise disjoint events, independence of events, pairwise independence, binomial distribution, geometric distribution, random variable, expected value, variance, deviation

(b) at least 5 identities: conditional probability, union of two events, complement of an event, linearity of expectations, Bienayme's Formula

(c) at least 2 inequalities: Chebyshev's inequality, Markov's inequality

(d) at least 1 method/technique: The probabilistic method

Q1. Imagine you get the following problem on the final: "What is the probability that a randomly selected bit string of length 10 is a palindrome?"

(a) Is its setup clear? Or does anything need to be clarified? Do you need any notation?

The setup is clear. Notation: $P(X)$ probability of an event

(b) What results do you envision using to solve this problem?

Formulation for permutation w/repetition:

n^r

(c) State at least one generalization of this problem.

. palindrome is a string whose reversal is identical to the original string.

REVIEW: BASIC COUNTING.

2. Name the following bits of knowledge in an approximate order of importance within each category:

(a) at least 10 number-theoretic notions or pieces of notation: tree diagrams, Ramsey theory, permutations, combinations, combinatorial proofs, binomial coefficients, binomial theorem, pascal's identity, Vandermole's identity, partitions

(b) at least 4 counting rules: product rule, sum rule, subtraction rule, division rule

(c) at least 1 principle: pigeonhole principle.

(d) at least 1 theorem: combinations w/ Repetition

Q2. Imagine you get the following problem on the final: "Show that if m and n are integers bigger than 2, then $R(m,n) \leq R(m,n-1) + R(m-1,n)$."

(a) Is its setup clear? Or does anything need to be clarified? Do you need any new notation?

No new notation is needed, and the setup is clear.

(b) What results do you envision using to solve this problem?

I'd use any formulas involving Ramsey #s.

$k \geq 2$, then $R(k,k) \geq 2^{k/2}$

(c) What proof method would you use?

mathematical induction

REVIEW: NUMBER THEORY.

3. Name the following bits of knowledge in an approximate order of importance within each category:

(a) at least 10 number-theoretic notions: representation of integers, Fundamental Thm of Arithmetic, Trial division, Twin Prime Conjecture, Greatest common Divisor, Least common Multiple

(b) at least 5 pieces of notation: quotion, remainder, div, mod, congruency

(c) at least 3 theorems: Prime Number Theorem, Bezout's Theorem, Chinese Remainder Theorem.

(d) at least 2 algorithms: division algorithm, Euclidean algorithm.

Q3. Imagine you get the following problem on the final: "Prove or disprove that there are three consecutive odd positive integers that are primes."

(a) Is its setup clear? Or does anything need to be clarified? Do you need any notation?

$x, x+2, x+4$ Every other #

(b) What results do you envision using to solve this problem?

3, 5, 7, since all other int values are divisible by 3.

(c) State at least one generalization of this problem.

- The first few natural numbers are prime.

1, 3, 5, 7

REVIEW: BASIC STRUCTURES.

Sets. Set Operations. Functions. Sequences and sums. Cardinality of sets.

REVIEW: LOGIC AND PROOFS.

Propositional logic. Predicates and quantifies. Rules of inference.

Induction. Strong induction. Well-ordering. Recursion. Structural induction.