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Math 55, Handout 3.

RULES OF INFERENCE.

1.1. An **argument form** is a sequence of statements that end with a conclusion.

1.2. An argument form is **valid** if the conclusion follows from the truth of the preceding statements of the argument.

1.3. A **fallacy** is a form of incorrect reasoning. They resemble rules of inference but are based on contingencies not tautologies.

1.4. Common fallacies are the fallacy of affirming the conclusion, which is the compound proposition $((p \to q) \wedge q) \to p$, and the fallacy of denying the hypothesis, which is the proposition $((p \to q) \wedge \neg p) \to \neg q$.

Q1. Draw the truth table for the tautology behind the Rule of Resolution. The tautology is $((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$.

p	q	r	$\neg p$	$p \vee q$	$\neg p \vee r$	$q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$
T	T	F	F	T	F	T	T
T	F	F	F	T	F	T	T
T	F	T	F	T	T	T	T
T	T	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	T	F	T	T	T
F	T	F	T	T	T	T	T
F	F	F	T	F	T	F	T

Q2. Give a non-mathematical example of **universal modus tollens**.
Universal modus tollens is the rule of inference characterized by the proposition $(\neg q \wedge (p \to q)) \to \neg p$. If q is “I will go to class” and p is “it is not snowing” then this proposition becomes “If I will not go to class and, if I go to class then it is not snowing, then it is snowing”. Or simply, “I am not going to class, I will go to class if it is not snowing, therefore it is snowing”.

INTRODUCTION TO PROOFS.

2.1. A **theorem** is a statement that can be shown to be true. A **lemma, proposition, result, corollary** are other kinds of theorems. A proposition or result is a less important theorem. A lemma is a less important theorem that is helpful in the proof of other results. A corollary is a theorem that can be established directly from a theorem that has been proved.

2.2. A **conjecture** is a statement that is proposed to be a true statement, usually on some kind of basis like partial evidence.

2.3. Common proof methods are a direct proof and indirect proofs. An example of an indirect proof is proof by contraposition. You can also use a proof by contradiction. There are also **vacuous** and **trivial** proofs.

Q3. A theorem states that conditions A, B, C, D are equivalent. What is the minimal number of implications you need to prove the theorem? We need to prove that each of A, B, C and D imply the other three. So, we need to know that A $\rightarrow$ B, A $\rightarrow$ C, A $\rightarrow$ D, B $\rightarrow$ A, B $\rightarrow$ C, B $\rightarrow$ D, C $\rightarrow$ A, C $\rightarrow$ B, C $\rightarrow$ D, and that D $\rightarrow$ A, D $\rightarrow$ B and D $\rightarrow$ C.
Starting with the four main implications (A $\rightarrow$ B, B $\rightarrow$ C, C $\rightarrow$ D, D $\rightarrow$ A), using hypothetical syllogism, we know A $\rightarrow$ C, A $\rightarrow$ D, B $\rightarrow$ D, D $\rightarrow$ B, D $\rightarrow$ C, C $\rightarrow$ A, C $\rightarrow$ B, and B $\rightarrow$ A. So the minimum number we need is four implications.

Q4. Prove directly that a product of two rational numbers is rational.
As this is a direct proof, we start with the assumption that both r and s are rational numbers. This means that $r = \frac{a}{b}$ with $b \neq 0$ and $s = \frac{c}{d}$ with $d \neq 0$. Then, we multiply r and s together and get $rs = \frac{ac}{bd}$. Since, $b \neq 0$ and $d \neq 0$, $bd \neq 0$. And rs equals an integer over an integer, making it a rational number.

Q5. Given the true fact that $\sqrt{2}$ is irrational, what method of proof will you use to show that $\sqrt{2} + 1$ is irrational? Give your proof.
Proof by Contradiction:
Supposing that r is an irrational number and s is a rational number, we want to prove that $r + s$ is irrational (using the universal generalization). Since r is irrational it cannot be written as $\frac{a}{b}$ while s can be written as $\frac{c}{d}$ where c and d are two positive integers and $d \neq 0$. Suppose the sum of r and s is rational, this means $r + s = \frac{p}{q}$ for some positive integers p and q where $q \neq 0$. This means that $r + \frac{c}{d} = \frac{p}{q}$.
This can be rewritten as $r = \frac{p}{q} - \frac{c}{d}$. Then $r = \frac{pd-cq}{qd}$, and since $q \neq 0$ and $d \neq 0$, $qd \neq 0$ and this fraction represents a postive integer over a postive integer – a rational number.
However, this contradicts our original assumption that r is irrational thus our assumption that $r + s$ is rational is false so $r + s$ must be irrational. And, if we use universal instantiation, we can say $r = \sqrt{2}$ and $s = 1$.