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## Math 55, Handout 5.

### SETS.

- 1.1. A set is an unordered collection of objects and can be specified/used as a roster method

Q1. List five most commonly used sets in mathematics and explain where our notation in class will differ from the notation in the book.

$\mathbb{N}$  := the set of all natural numbers

$\mathbb{Z}_+$  := the set of all non-neg ints.

$\mathbb{Z}$  :=  $\{\dots -2, -1, 0, 1, 2, \dots\}$  the set of all ints

$\mathbb{Q}$  :=  $\{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, \text{ and } q \neq 0\}$  the set of all rational #'s

$\mathbb{R}$  := the set of all real numbers

Differences - we will include 0 in  $\mathbb{Z}_+$ , we will not include 0 in the  $\mathbb{N}$

- 1.2. Two sets are equal if and only if they have the same elements. A and B are sets, then A and B are equal if and only if  $\forall x (x \in A \leftrightarrow x \in B)$ . we write  $A = B$  if A and B are equal sets.

- 1.3. A is a subset of B means every element of A is also in B (denoted as  $A \subseteq B$ )

- 1.4. A is a proper subset of B means A is a subset of B but  $A \neq B$  (denoted as  $A \subset B$ )

### THE SIZE OF A SET.

- 2.1. All sets are either finite or infinite.

The cardinality of a finite set is the number of its distinct n elements in the set (denoted as  $|S|$ )

- 2.2. The power set  $\mathcal{P}(S)$  of a set S is the set of all subsets of the set S

Q2. If  $|S| = n$ , what is  $|\mathcal{P}(S)|$ ? Why?

$n$  because  $n$  represents the cardinality of  $S$  therefore the power set of  $S$  is  $2^n$ . This is true because every element is a subset and cardinality counts the subsets

2.3. The **Cartesian product** of sets  $S_1, \dots, S_k$  is the set of ordered  $n$ -tuples  $(a_1, a_2, a_3 \dots a_n)$  where  $a_i$  belongs to  $A_i$  for  $i=1, 2, \dots, n$ . In other words:  $A_1 \times A_2 \times \dots \times A_n = \{(a_1, a_2, \dots, a_n) | a_i \in A_i \text{ for } i=1, 2, \dots, n\}$

Q3. How does  $|S_1 \times S_2 \times \dots \times S_k|$  depend on  $|S_1|, |S_2|, \dots, |S_k|$ ? Why?  
 $|S_1 \times S_2 \times \dots \times S_k| = |S_1| |S_2| \dots |S_k|$  product bc this exhausts all possible combos of elements in  $S_1, S_2 \dots S_k$

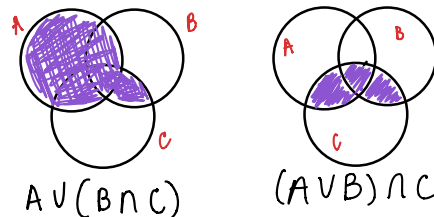
### TRUTH SETS.

Q4. Let  $P(x)$  be " $|x| < |x-1|$ ". What is the truth set  $\{x \in \mathbb{R} | P(x)\}$ ?  
 the set of real #s such that  $p(x) = |x| < |x-1|$  is true.  
 the truth set of  $\{x \in \mathbb{R} | P(x)\}$  is all  $x < 0.5$ .

3.1. The **union** of two sets is the set that contains those elements that are either in  $A$  or in  $B$  or in both

3.2. The **intersection** of two sets is the set containing those elements in both  $A$  and  $B$

Q5. Draw a Venn diagram for  $A \cup (B \cap C)$  and for  $(A \cup B) \cap C$ . Are these necessarily the same sets?



These are not the same according to the Venn diagrams to the left. the

3.3. The **difference** of two sets is the set containing those elements that are in  $A$  but not in  $B$ .

3.4. The **complement** of a set is the complement of  $A$  with respect to  $U$ . (Denoted by  $A^c$ ) Therefore, the complement of the set  $A$  is  $U - A$