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Math 55, Handout 3.

RULES OF INFERENCE.

1.1. An **argument form** is a sequence of compound propositions involving propositional variables

1.2. An argument form is **valid** ~~is~~ no matter which particular propositions are substituted for the propositional variables

1.3. A **fallacy** is a form of incorrect reasoning which leads to invalid arguments

1.4. Common fallacies are fallacy of affirming the conclusion, fallacy of denying the hypoth-esis

Q1. Draw the truth table for the tautology behind the Rule of Resolution. $((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$

p	q	r	$((p \vee q) \wedge (\neg p \vee r)$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$
F	F	F	F	T
F	F	T	F	T
F	T	F	T	T
F	T	T	T	T
T	F	F	F	T
T	F	T	T	T
T	T	F	F	T
T	T	T	T	T

Q2. Give a non-mathematical example of **universal modus tollens**.

1. $P(x)$ = "x is a programmer"
2. $Q(x)$ = "x knows how to code
3. Then, $\forall x\,(P(x) \to Q(x))$: All programmers know how to code
4. $\neg Q(a)$, babies do not know how to code
5. $\therefore \neg P(a)$: Babies are not programmers

INTRODUCTION TO PROOFS.

2.1. A **theorem** is a statement that can be shown to be true.

A **lemma, proposition, result, corollary** are also theorems.

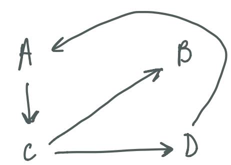
2.2. A **conjecture** is a statement that is being proposed to be a true statement, usually on the basis of some partial evidence, a heuristic argument, or the intution of an expert.

2.3. Common proof methods are

- Direct proofs
- proof by contraposition
- proof by contradiction

There are also **vacuous** and **trivial** proofs.

Q3. A theorem states that conditions A, B, C, D are equivalent. What is the minimal number of implications you need to prove to prove the theorem?



you need a minimum of 4 implications

Q4. Prove directly that a product of two rational numbers is rational.

given: $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers with a, b, c, d integers ($b, d \neq 0$), Prove: $\frac{a}{b} \times \frac{c}{d}$ is a rational #

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ } $b \neq 0, d \neq 0$, ac and bd are integers. thus, $\frac{ac}{bd}$ is a fraction with integers in the numerator & denominator making it a rational number

Q5. Given the true fact that $\sqrt{2}$ is irrational, what method of proof will you use to show that $\sqrt{2}+1$ is irrational? Give your proof.

- proof by contradiction
- Assume that $\sqrt{2}+1$ is rational such that $\sqrt{2}+1 = \frac{m}{n}$ where $m, n \in \mathbb{N}$ and m and n are relatively prime
- Then, $\sqrt{2}+1 = \frac{m}{n}$

$$\sqrt{2} = \frac{m}{n} - 1$$

$$\sqrt{2} = \frac{m-n}{n}$$ ← contradiction part

$\sqrt{2}$ is irrational and $\frac{m-n}{n}$ is rational by definition.

F and T = False

Therefore $\sqrt{2}+1$ cannot be rational due to $\sqrt{2}$ being irrational