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Math 55, Handout 15.

INTRODUCTION TO FINITE PROBABILITY.

0.0. **Blanket assumption (for this lecture):** all experiments have finitely many, equally likely, outcomes.

1.1. An **experiment** is a procedure that yields one of a given set of possible outcomes.

1.2. The **sample space** is the set S of all possible outcomes.

1.3. An **event** E is a subset of sample space S.

1.4. If S is a finite nonempty sample space of equally likely outcomes, and $E \subseteq S$ is an event, then the **probability** of E is

$$p(E) = \frac{|E|}{|S|}$$

Q1. What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 8?

$$p(E) = \frac{5}{36}$$

Q2. What is the probability that a randomly selected integer chosen from the first 100 positive integers is divisible

(a) by 3?

$$\frac{33}{100}$$

(b) by 5?

$$\frac{20}{100}$$

(c) by 9?

$$\frac{11}{100}$$

Q3. What is the probability that a five-card poker hand contains

(a) a flush?
Including straight flushes:

$$\frac{\binom{4}{1}\binom{13}{5}}{\binom{52}{5}}$$

(b) a straight flush?
Including royal flush:

$$10\frac{\binom{4}{1}}{\binom{52}{5}}$$

1.5. Let E be an event in a sample space S. The **complementary event** $\overline{E}$ is defined as

$$\overline{E} = S \setminus E$$

1.6. The probability of $\overline{E}$ is given by

$$p(\overline{E}) = 1 - p(E)$$

Q4. A fair coin is tossed 7 times (landing heads up or tails up every time). What is the probability that at least one of the coin tosses results in a heads up outcome?

$$P(E) = 1 - P(E^c) = 1 - (1/2)^7$$

1.7. Let E_1 and E_2 be events in the sample space S. Then

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$$

1.8. Two events E_1 and E_2 are called **independent** if

$$p(E \cap F) = p(E)p(F)$$

Q5. (a) What is the probability that a randomly selected integer chosen from the first 100 positive integers is divisible by 3 or by 5? Are the two events (drawing an integer divisible by 3 and drawing an integer divisible by 5 randomly from the first 100 positive integers) independent?

$$\begin{aligned} p(E_3 \cup E_5) &= p(E_3) + p(E_5) - p(E_3 \cap E_5) \\ &= 33/100 + 20/100 - 6/100 \\ &= 47/100 \end{aligned}$$

They are not independent.

(b) Same questions for divisibility by 3 or by 9.

$$\begin{aligned} p(E_3 \cup E_9) &= p(E_3) + p(E_9) - p(E_3 \cap E_9) \\ &= 33/100 + 11/100 - 11/100 \\ &= 33/100 \end{aligned}$$

They are not independent.