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Math 55, Handout 6.

FUNCTIONS. (138)

Blanket assumption: A, B are nonempty sets.

1.1. A **function** $f : A \to B$ (also called mappings or transformations) is an assignment of exactly one element of B to each element of A

1.2. The domain of a function $f : A \to B$ is A domain, the **codomain** is B.
If $f(a) = b$, we say b is an image of a and a is a preimage of b.
The range of $f : A \to B$ is the set of all images of elements of A

1.3. Let $f : A \to B$, let $C \subseteq A$, and let $D \subseteq B$.
The **(full) image** of C is under A

The **(full) pre-image** of D is under f

Q1. Let $f : \mathbb{Q} \to \mathbb{Q}$ be the function $f(x) = x^2$. What is the full pre-image of the set $\mathbb{N}$ and why?

$t := \mathbb{N}$ here i.e. we are looking for elements
$x \in \mathbb{Q}$ such that $x^2 \in \mathbb{N}$

ARITHMETIC FOR REAL-VALUED FUNCTIONS.

1.4. Let $f_1, f_2 : A \to \mathbb{R}$. Then $f_1 + f_2$ and $f_1 f_2$ are **defined by**

$(f_1 + f_2) \longrightarrow (f_1) + (f_2)$

$(f_1 \cdot f_2) \longrightarrow (f_1) \circ (f_2)$

Q2. Let $f_1, f_2 : \mathbb{R} \to \mathbb{R}$ be the functions $f_1(x) = x^2 - 1$, $f_2(x) = x + 1$. Write down formulas for the functions $f_1 + f_2$ and $f_1 f_2$.

$f_1 + f_2 = x^2 - 1 + x + 1 = \boxed{x^2 + x}$ $\quad f_1 f_2 = (x^2 - 1)(x+1)$

PROPERTIES OF FUNCTIONS. (141)

1.5. A function $f : A \to B$ is called **one-to-one**, or injunction, if and only if $f(a)=f(b)$ implies that $a=b$ for all a and b in the domain of f.

1.6. A function $f : A \to B$ is called **onto**, or surjection, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a)=b$.

1.7. If a function $f : A \to B$ is both one-to-one and onto, it is called bijection

1.8. A function $f : A \to B$ is **invertible** means it has an inverse function $f^{-1} : A \to B$. A function is invertible if and only if it is a one-to-one correspondence

1.9. Let $f : A \to B$, $g : B \to C$. The composition $g \circ f : A \to C$ is defined by:

$g(f(x))$

Q3. What are the functions $f_1 \circ f_2$ and $f_2 \circ f_1$ for the functions f_1 and f_2 from Q2?

$f_1 \circ f_2 = (x+1)^2 - 1$ $\qquad$ $f_2 \circ f_1 = (x^2-1) + 1$

CARDINALITY OF SETS. (170)

2.1. Two sets A and B (written $|A| = |B|$) have the same cardinality if there is a one-to-one correspondence from A to B

2.2. A set is called **countable** if it is finite or has the same cardinality as the set of positive ints
If a set is not countable, it is called uncountable. .

Q4. Underline all of the following sets that are countable:

$$2\mathbb{N} - 1 (= \{1, 3, 5, \ldots\}), \quad \mathbb{Z}, \quad \mathbb{Q}, \quad \mathbb{R}, \quad \mathbb{C}, \quad [-1, 1].$$

Q5. Prove that the set of all finite subsets of $\mathbb{N}$ is countable.

lets suppose that subset of $\mathbb{N}$ (s_i, where i represents the # of elements in the subset)

s_0 is an empty set
s_1 is countable
s_2 is countable

through this we can prove that all finite subsets of $\mathbb{N}$ are countable