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Math 55, Handout 2.

LOGICAL EQUIVALENCES.

1.1. A **tautology** is a compound proposition that is always true, regardless of the truth values of the propositional variables that occur in it.

1.2. A **contradiction** is a compound proposition that is always false, regardless of the truth values of the propositional variables that occur in it.

1.3. A **contingency** is neither a tautology nor a contradiction. The compound proposition is sometimes true and sometimes false.

2. The notation $p \equiv q$ means that p and q are logically equivalent.

3. De Morgan's Laws are:
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Q1. Prove the Absorption Laws: $p \vee (p \wedge q) \equiv p$ and $p \wedge (p \vee q) \equiv p$.

Proof. p $\vee(p \wedge q) \equiv p$
$Case1 : q = T$
p $\vee(p \wedge T)$
$p \vee p$
p
$p \equiv p$

Case 2: q = F
p $\vee(p \wedge q)$
$p \vee (p \wedge F)$
$p \vee p$
p
$p \equiv p$ $\square$

Proof. p $\wedge(p \vee q) \equiv p$
$Case1 : q = T$
$p \wedge (p \vee q) \equiv p$
$p \wedge T$
p
$p \equiv p$

$Case2 : q = F$
$p \wedge (p \vee F)$
$p \wedge p$
p
$p \equiv p$ $\square$

4. A proposition is **satisfiable** if there is an assignment of truth values to its variables that makes it true (i.e. it is a tautology or contingency). Otherwise it is **unsatisfiable**.

Q2. Which of these propositions is satisfiable?

$(p \vee (p \wedge q)) \vee \neg p$: **satistiable**

$(p \wedge (p \vee q)) \vee \neg q$: **satistiable**

$\neg p \vee q \to \neg(p \vee q)$: **satistiable**

PREDICATES AND QUANTIFIERS.

5. A **predicate** or **propositional function** is a property that the subject of the statement can have (**predicate**). The statement P(x) is the value of the **propositional function** P at x. It can be **unary** or n**-ary**.

5. The **universal quantification** $\forall x P(x)$ means the predicate is true for every element under consideration.

6. The **existential quantification** $\exists x P(x)$ means there is one or more element(s) under consideration for which the predicate is true.

DOMAIN AND ITS IMPORTANCE.

7. The **domain / domain of discourse / universe of discourse** means that the property of a mathematical statement is true for all values of a variable in a particular domain.

Q3. Give an example of a predicate $P(x)$ and different domains so that the statements $\forall x P(x)$ is true, $\forall x P(x)$ is false, $\exists x P(x)$ is true, $\exists x P(x)$ is false.
$\forall x P(x)$ is true: P(x): x + 1 > x, Domain: all real numbers
$\forall x P(x)$ is false: P(x): x < 2, Domain: x > 4
$\exists x P(x)$ is true: P(x): x > 3, Domain: x $\geq$ 0
$\exists x P(x)$ is false: P(x): x = x + 1, Domain: all real numbers

LOGICAL EQUIVALENCES INVOLVING QUANTIFIERS.

8. De Morgan Laws for Quantifiers are:
$\neg\exists x P(x) \equiv \forall x \neg P(x)$
$\neg\forall x P(x) \equiv \exists x \neg P(x)$

Q4. Let $P(x)$, $Q(x)$, and $R(x)$ be the predicates "x is a clear explanation", "x is satisfactory", "x is an excuse". Let the domain for x be all English text. Express these statements using the given predicates and quantifiers:

"All clear explanations are satisfactory": $\forall x(P(x) \to Q(x))$

"Some excuses are unsatisfactory": $\exists x(\neg Q(x) \land R(x))$

"Some excuses are not clear explanations": $\exists x(\neg P(x) \land R(x))$

Does the last statement follow from the first two? **Yes**

NESTED QUANTIFIERS.

Q5. Give the definition of a limit of a real-valued function $f(x)$ at point a. State its negation so that the negation sign preceeds no quantifier.
Definition: $\forall \varepsilon\ \exists N\ \forall n\ [\varepsilon > 0 \land n > N \to |a_n - a| < \varepsilon]$
Negation: $\neg[\forall \varepsilon\ \exists N\ \forall n\ [\varepsilon > 0 \land n > N \to |a_n - a| < \varepsilon]]$
$\exists \varepsilon\ \forall N\ \exists n\ \neg[\neg(\varepsilon > 0 \land n > N) \lor |a_n - a| < \varepsilon]$
$\exists \varepsilon\ \forall N\ \exists n\ [(\varepsilon > 0 \land n > N) \land |a_n - a| \geq \varepsilon]$