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Math 55, Handout 11.

BASICS OF COUNTING.

1.1. **The product rule.** Suppose that a procedure can be broken down into a sequence of independent tasks. If, for each $j = 1, \dots, k$, there are n_j ways to do the task, then there are $n_1 \cdot n_2 \cdots n_k$ ways to do the procedure.

Q1. How many vegetarian sandwiches can be made using 4 kinds of bread, 5 kinds of cheese, 3 kinds of salad leaves, 6 kinds of chopped veggies, and 7 kinds of dressing? Assume a vegetarian sandwich must be made of all these components, i.e., bread, cheese, etc.

$4 \cdot 5 \cdot 3 \cdot 6 \cdot 7 = 2520$

1.2. **The sum rule.** Suppose that all ways to perform a task can be split into k non-overlapping groups where j^{th} group has size n_j for each $j = 1, \dots, k$. Then there are $n_1 + n_2 + \dots n_k$ ways to perform a task.

Q2. Now solve a modified Q1, allowing for the possibilities that any of cheese, salad leaves, chopped veggies, and dressing (but not bread) may be omitted when making a sandwich.

all $4 \cdot 5 \cdot 3 \cdot 6 \cdot 7 = 2520$
no cheese $4 \cdot 3 \cdot 6 \cdot 7 = 504$
no salad $4 \cdot 5 \cdot 6 \cdot 7 = 840$
no veggies $4 \cdot 5 \cdot 3 \cdot 7 = 420$
no dressing $4 \cdot 5 \cdot 3 \cdot 6 = 360$
no ch. no sal. $4 \cdot 6 \cdot 7 = 168$
" no veg. $4 \cdot 3 \cdot 7 = 84$
" no dres. $4 \cdot 3 \cdot 6 = 72$
no salad no veg. $4 \cdot 5 \cdot 7 = 140$
" no dress $4 \cdot 5 \cdot 3 = 60$
only cheese $4 \cdot 5 = 20$
only salad $4 \cdot 3 = 12$
only veg. $4 \cdot 6 = 24$
only dress. $4 \cdot 7 = 24$
only bread 4

SUM all $\boxed{5376}$

1.3. **The subtraction rule (inclusion-exclusion for two sets).** Suppose that all ways to perform a task can be split into 2 groups overlapping where each group has size n_j for $j = 1, 2$ and the overlap has size 1cm . Then there are $n_1 + n_2 -$ (ways that are common to both groups) ways to perform a task.

Q3. How many bit strings of length 100 start with 10 or end with 111?

$2^{98} + 2^{97} - 2^{95}$

1.4. **The division rule.** There are n/d ways to do a task using a procedure that can be performed in n ways such that exactly d of the n ways to perform a procedure correspond to each way to do the task.

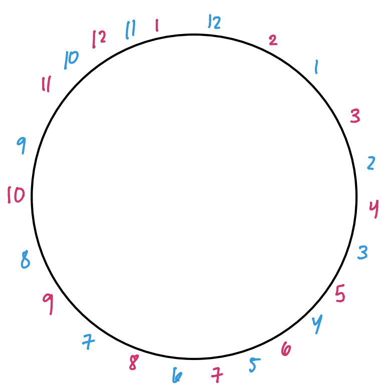
Q4. A butler at Downton Abbey is expecting 11 ladies and 11 gentlemen for dinner with the lord and lady of the house. How many seating plans at a round table are at his displosal if etiquette prescribes that no two gentlemen and no two ladies may sit next to each other? To the butler, two seatings may be considered the same if they differ only by some rotation of the table.

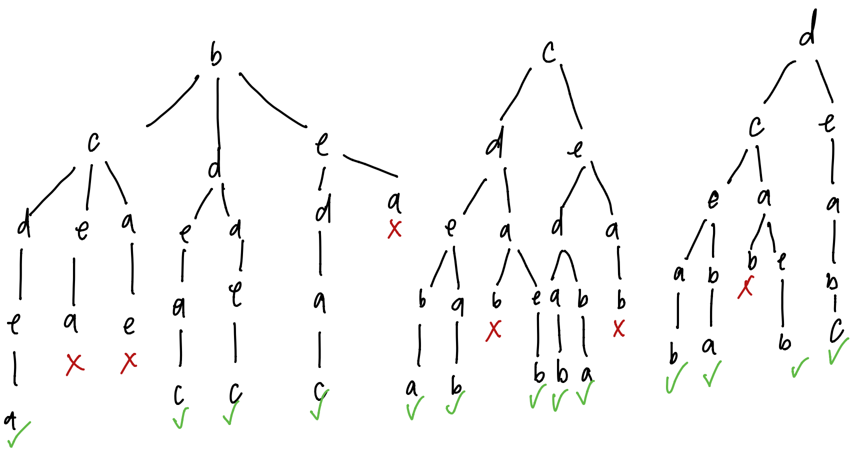
2 circles of 12 $\rightarrow$ possible arrangements for 12 $\rightarrow$ 11!
$\rightarrow$ $12!/12$ to get arrangements

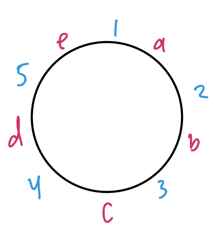
$\boxed{12 \cdot 11! \cdot 11!}$

Q5. By Jeeves! Our friend the butler was so baffled by his counting endeavors he almost forgot that the arriving guests are all married couples (of opposite gender, this being early 20th century England). And, needless to say, no married couple may be seated next to each other. How many seating plans does he now have?

[If the butler runs out of time before dinner trying to solve this, he can just exhibit a single arrangement that works and answer the same question for 2 or 3 couples of guests, in addition to the host couple.]







Five couples

$4! \times 13 = \boxed{312}$