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Math 55, Handout 5.

SETS.

1.1. A **set** is a (possibly unordered) collection of objects.

Q1. List five most commonly used sets in mathematics and explain where our notation in class will differ from the notation in the book.

Common sets include the complex numbers, $\mathbb{C}$, real numbers, $\mathbb{R}$, the rationals $\mathbb{Q}$, the integers $\mathbb{Z}$, and the natural numbers $\mathbb{N}$. The book defines the natural numbers as all nonnegative integers, $\mathbb{N} = \{0, 1, 2 \ldots\}$. However, our class will define the natural numbers as all positive integers $\mathbb{N} = \{1, 2, 3 \ldots\}$.

1.2. Two sets are **equal** if each is a subset of the other, i.e., $A = B$ if $A \subseteq B$ and $B \subseteq A$.

1.3. A is a **subset** of B means that if x is an element of A, then x must also be in B, or $x \in A \to x \in B$.

1.4. A is a **proper subset** of B means that A is a subset of B, but $A \neq B$. What this means is that there exists $x \in B$ such that $x \notin A$.

THE SIZE OF A SET.

2.1. All sets are either finite or infinite.

The **cardinality** of a finite set is the number of its elements.

2.2. The **power set** $\mathcal{P}(S)$ of a set S is the set containing all subsets of S.

Q2. If $|S| = n$, what is $|\mathcal{P}(s)|$? Why?

$|\mathcal{P}(s)| = 2^n$. We see this because every element in S has two possibilities: eithr it is in S or it is not in S. As there are n elements, we see that there are 2 possibilities for the first element, 2 possibilities for the second element $\ldots$ and 2 possibilities for the nth element. This means there are 2^n total possible subsets of S.

2.3. The **Cartesian product** of sets $S_1, \ldots, S_k$ is

the set $S_1 \times S_2 \times \cdots \times S_k = \{(s_1, s_2, \ldots, s_k) | s_1 \in S_1, s_2 \in S_2 \ldots s_k \in S_k\}$.

Q3. How does $|S_1 \times S_2 \times \cdots \times S_k|$ depend on $|S_1|$, $|S_2|$, …, $|S_k|$? Why?

We see that $|S_1 \times S_2 \times \cdots \times S_k| = |S_1||S_2| \ldots |S_k|$. Observe that if we had just $|S_1 \times S_2|$, then we can count every element by pairing each element of S_1 with the first element in S_2. This gives us $|S_1|$ different ordered pairs. Then we can pair each element in S_1 with the second element in S_2. We repeat this until we reach the last element of S_2, which gives us a total of $|S_1||S_2|$ different ordered pairs. To extend this argument beyond two sets, we simply repeat the process by fixing the next element in the next set and create all possible ordered pairs for that fixed element, then move on.

TRUTH SETS.

Q4. Let $P(x)$ be "$|x| < |x-1|$". What is the truth set $\{x \in \mathbb{R} \mid P(x)\}$?

The truth set is $\{x \in \mathbb{R}|x < 0.5\}$. To see this we split up the absolute values.

$$|x| = \{x \in \mathbb{R}|x \geq 0\} \cup \{-x \in \mathbb{R}|x < 0\}$$

$$|x-1| = \{x-1 \in \mathbb{R}|x \geq 1\} \cup \{-x+1 \in \mathbb{R}|x < 1\}.$$

We immediately see that for $x \geq 1$, $|x| \neq |x-1|$ because the possible values that satisfy $x < x-1$ is empty.

For $0 \leq x < 1$, we see that $|x| < |x-1|$ is satisfied when $x < -x+1$, or when $x < .5$.

For $x < 0$, we see that $|x| < |x-1|$ is satisfied when $-x < -x+1$, which is true for all $x < 0$.

So as $0 \leq x < .5$ and $x < 0$ cause $|x| < |x-1|$, our truth set for this question is $\{x \in \mathbb{R}|x < 0.5\}$.

3.1. The **union** of two sets is every element that is in one set, the other set, or both.

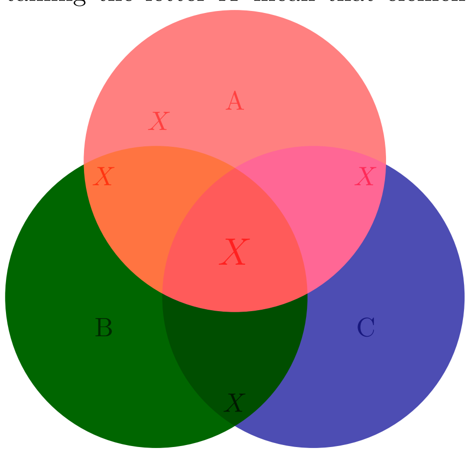
The set $A \cup B$ represents $A \cup B = \{x|x \in A \vee x \in B\}$.

3.2. The **intersection** of two sets is every element that is in both sets at the same time.

The set $A \cap B$ represents $A \cap B = \{x|x \in A \wedge x \in B\}$.

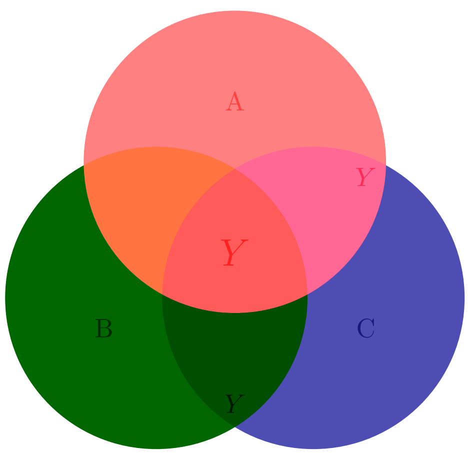
Q5. Draw a Venn diagram for $A \cup (B \cap C)$ and for $(A \cup B) \cap C$. Are these necessarily the same sets?

These are not the same set. For the first set it is possible that we can have $x \in A$, $x \notin B$, and $x \notin C$, but that would not be possible in the second set as all elements in the second set must be in C. Here is my attempt at a picture in LaTeX. For the first set, let all areas containing the letter X mean that elements in X are also in $A \cup (B \cap C)$.



As is seen, $A \cup (B \cap C)$ involves all elements in A or all elements in the intersection of B and C.

For the second set, let all areas containing the letter Y mean that elements in Y are also in $(A \cup B) \cap C$.



We see here that $(A \cup B) \cap C$ is the set of all elements in A and in C or in B and in C.

3.3. The **difference of two sets** is every element in the first set that is not in the second set.

The set $A \backslash B$ represents $A \backslash B = \{x \in A | x \notin B\}$.

3.4. The **complement** of a set is all the elements not in that set.

The set A^c represents $A^c = \{x | x \notin A\}$.