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Math 55, Handout 4.

PROOF METHODS.

1.1. A **proof by exhaustion** is a proof that examines a relatively small number of examples (i.e. a proof that exhausts all possibilities) An exhaustive proof is a special type of proof by cases where each case involves checking a single example.

1.2. A **proof by cases** is a method that can be used to prove a theorem by considering different cases seperately

Q1. Write down and prove the rule of inference behind these proof methods.

$p_1 \vee p_2 \vee p_3 \cdots \vee p_k \to q \equiv (p_1 \to q) \wedge (p_2 \to q) \wedge \cdots \wedge (p_k \to q)$

If $p_1 \vee \cdots \vee p_k \to q = F$, then $q = F$ and at least one $p_j = T$

Then $p_j \to q = F$, so the RHS is false

conversely, if the RHS is false then there is at least one $p_j = q = F$, so $q = F$ and $p_j = T$

so $p_1 \vee \cdots \vee p_k = T$, $q = F$, hense $p_1 \vee \cdots \vee p_k \to q = F$

this proves that $p_1 \vee p_2 \cdots \vee p_k \to q$

case 1: $p_1 \to q$
case 2: $p_2 \to q$
case 3: $p_k \to q$

1.3. **Without loss of generality (WLOG)** means we assert by proving one case of a theorem, no additional arguments is required to prove other specified cases

1.4. Common errors with these two kinds of proofs are

- Drawing conclusions from examples
- making unwarrented assumptions that lead to incorrect proofs by cases where not all cases are considered

Q2. Give an example of an error with a proof by cases.

pg. 96: it is true that every positive integer is the sum of 18 fourth powers of integers

2.1. A **constructive existence proof** is a proof that directly provides a specific example, or which gives an algorithm for producing and example

2.2. A **non-constructive existence proof** is a proof that something exists w/o actually showing how to constnuct it

Q3. Prove there is a pair of consecutive integers such that one of them is a perfect square and the other is a perfect cube. Is your proof constructive or non-contructive?

8 and 9 are a pair of consecutive integers. such that one of them is a perfect square and the other is a perfect cube ($2^3=8$ and $3^2=9$) this existence proof of $\exists x P(x)$ is true and therefore conotructive

3.1. A **uniqueness proof** has two parts:

Existance: we show that an element x with the desired property exists

uniqueness: we show that if $y \neq x$, then y does not have the desired property

PROOF STRATEGIES.

4.1. **Forward reasoning** is starting with the premise and constructing a proof using a sequence of steps

4.2. **Backward reasoning** is the opposite of forward reasoning. start from the end and end with premise

4.3. **Adapting an existing proof** means looking for possible approaches that can be used to prove a statement taking advantage of existing proofs

Q4. Prove that $(n-1)n$ is even for any integer n. What is your proof strategy?

case 1: n is even, then $n=2m$ for some $m \in \mathbb{N}$ hense

$n(n-1) = 2m(2m-1)$

$= 2[m(2m-1)]$ is even

n is odd, then $n=2m-1$ where $m \in \mathbb{N}$ hense

$n(n-1) = (2m-1)(2m-2)$

$= 2[(2m-1)(m-1)]$ is even

Q5. Adapt your strategy from Q4 to show that $(n-1)n(n+1)$ is divisible by 3 for any integer n.

$n(n-1)(n-2)$

case 1: $n = 3k$

case 2: $n = 3k-1$

case 3: $n = 3k-2$

Note that among 3 consecutive integers, exactly one is divisible by 3

Hense, their product is divisible by 3