Name: Shivani	GSI: madeline Brandt	DISC #: 103

Homework 1.

Math 55, Spring 2020.

Prob 1. (a) Find a formula of two propositional variables p and q that involves only negation and disjunction that is equivalent to the implication $\neg p \to q$ and provide its truth table to see it indeed coincides with the truth table for $\neg p \to q$.

p	q	$\neg p \to q$
T	T	T
T	F	T
F	T	T
F	F	F

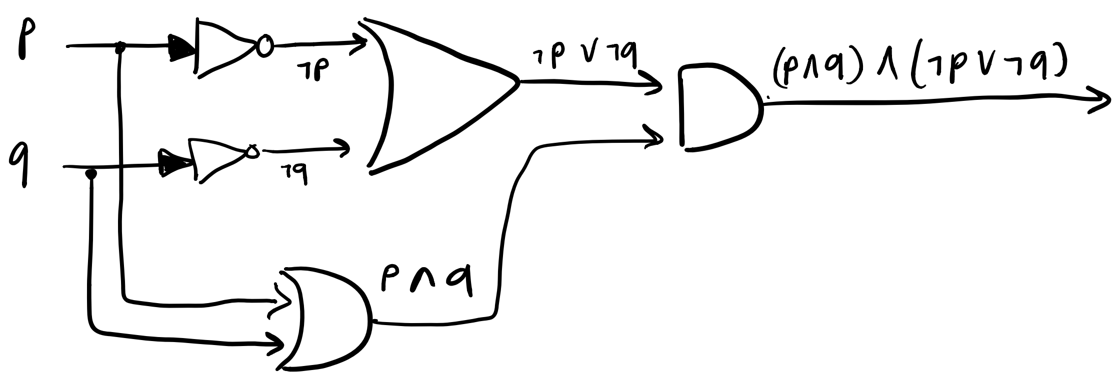
p	q	$p \vee p$
T	T	T
T	F	T
F	T	T
F	F	F

(b) Find a formula of two propositional variables p and q that exclusive involves only negation, conjunction, and disjunction that is equivalent to $p \oplus q$. Likewise, provide its truth table to see it indeed coincides with the truth table for $p \oplus q$.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$\neg p \wedge q$	$p \wedge \neg q$	$(\neg p \wedge q) \vee (p \wedge \neg q)$
T	T			F
T	F			T
F	T			T
F	F			F

Prob 2. Given the combinatorial circuit for the formula $\neg p \lor \neg q$, expand it to a larger one taking only propositional variables p and q as inputs so that the larger circuit outputs the value F (false) no matter the truth values of p and q.



$\neg p$	$\neg q$	$\neg p \lor \neg q$	$q \land \neg p \lor \neg q$	$(p \land q) \land (\neg p \lor \neg q)$
F	F	F	F	F
F	T	T	F	F
T	F	T	T	F
T	T	T	F	F

$\underline{\text{knights}}$ $\underline{\text{knaves}}$ $\underline{\text{Prize}}$

k_a, k_b, k_c $\neg k_a, \neg k_b\ \neg k_c$ P_a, P_b, P_c

Prob 3. The Island of Knights and Knaves. Mr. Smullyan visited a famous island where knights always tell the truth and knaves always lie and every inhabitant is either a knight or a knave. He was introduced to three inhabitants A, B, and C of which at least one was a knave and one a knight. One of them had a prize that Mr. Smullyan could win (if and) only if he could determine correctly which one had it. The three spoke to him.

A said: B does not have the prize.

B said: I don't have the prize.

C said: I have the prize.

Formalize the problem using six propositional variables, one for each of the locals being a knight (or not) and one for each having the prize or not. Write down a compound proposition that takes the value T (true) exactly when the values of all propositional variables describe the solution to this puzzle.

Which local has the prize? Can you argue formally using your formula?

person A has the prize

$A \longrightarrow k_a \longleftrightarrow \neg P_b$

$B \longrightarrow k_b \longleftrightarrow \neg P_b$

$C \longrightarrow k_c \longleftrightarrow P_c$

$$(\underbrace{k_a \vee k_b \vee k_c}_{\text{at least one is a knight}}) \wedge (\underbrace{\neg k_a \vee \neg k_b \vee \neg k_c}_{\text{at least one is a knave}}) \wedge (\underbrace{(P_a \oplus P_b) \oplus P_c) \wedge \neg(P_a \wedge P_c \wedge P_b}_{\text{only one prize is true}})) \wedge$$

$$(\underbrace{k_a \longleftrightarrow \neg P_b}_{\text{outcome 1}}) \wedge (\underbrace{k_b \longleftrightarrow \neg P_b}_{\text{outcome 2}}) \wedge (\underbrace{k_c \longleftrightarrow P_c}_{\text{outcome 3}})$$

witch Doctor	knights	knaves
W_a, W_b, W_c	K_a, K_b, K_c	$\neg K_a, \neg K_b, \neg K_c$

Prob 4. A harrowing adventure. Having just won his prize, Mr. Smullyan was enjoying his visit - that is, until he got captured by a ferocious gang of local brigands! They grabbed his prize too, and promised to kill him unless he could solve another puzzle: Among three gang members A, B, C, exactly one was a witch doctor. Mr. Smullyan had to point at one of the three. If he pointed at the witch doctor, he would be killed! If he pointed at one that was not a witch doctor, he would go free. But he would have his prize back only if he could give a rigorous explanation. Here is what those three said to Mr. Smullyan:

A: I am a witch doctor.
B: I am not a witch doctor.
C: At most one of us is a knight.

Whom should Mr. Smullyan point at to go free? Explain.

Mr. Smullyan should point to C because if C is a knave (lying) then there is no chance that he could be the witch doctor because then it means that A and B could either be knights or knaves but one is the witch doctor. If he is telling the truth then one of A or B will be a knight and tells the truth of being a witch doctor

$$W_a \oplus W_b \oplus W_c \wedge \neg(W_a \wedge W_b \wedge W_c) \wedge (K_a \leftarrow W_a) \wedge (K_b \leftrightarrow \neg W_b) \wedge$$

$$(K_c \leftrightarrow ((\neg K_a \vee \neg K_b) \wedge (\neg K_b \vee \neg K_c) \wedge (\neg K_a \vee \neg K_c)))$$

p = pianist   ¬p = violinists

s = steinway

Prob 5. The Island of Musica. Stressed out by his last adventure, Mr. Smullyan left the Island of Knights and Knaves and visited another island instead. There, every inhabitant is either a pianist or a violinist but no inhabitant is both (and, likewise, either male or female but not both). Moreover, it so happens that female pianists there always tell the truth and male pianists always lie, and for violinists it is the opposite. Mr. Smullyan was invited to the home or Mr. and Mrs. Smith. They owned a nice piano and Mr. Smullyan asked the wife whether it was a Steinway. She said, "I am actually a violinist but this piano is not a Steinway."

Was this piano a Steinway or not? Perform formal analysis as in Prob 3.

(I am a violinist) ∧ (piano is not a steinway)

Assume she is a pianist then the whole statement should be true:

→ she is a violinist → contractiction

→ piano not a steinway

Assume violinist:

"I am not a violinist or the piano is a steinway" isT

X ☆