Name: Shivani Patel	GSI: Madeline Brandt	DISC #: 103

Handout 1.

Math 55.

1. A **proposition** is a declarative sentence that is either true or false, but not both
2. A **propositional variable** is variables that represent propositions, just as letters are used to denote numerical variables
3. The **truth value of a proposition** is either true if is a true proposition or false if it is a false proposition

LOGICAL OPERATORS.

4. The **negation** of a proposition p is the result of the operation of the negation operator on a proposition

4.1 The **truth table** of negation is

p	$\neg p$
T	F
F	T

5. The **conjunction** of two propositions p and q is true when both p and q are true and is false otherwise

5.1 The **truth table** of conjunction:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

6. The **disjunction** of two propositions p and q is false when both p and q are false and is true otherwise

6.1 The **truth table** of disjunction:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

7. The **implication** connecting propositions p and q is true when both p and q are true and when p is false (no matter what truth value q has)

7.1 The **truth table** of implication:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Q1. Draw the truth table for the compound proposition $p \wedge q \vee r \to (p \to r)$.

p	q	r	$p \wedge q$	$p \wedge q \vee r$	$p \to r$	$p \wedge q \vee r \to (p \to r)$
T	T	T	T	T	T	T
T	T	F	T	T	F	F
T	F	T	F	T	T	T
T	F	F	F	F	F	T
F	T	T	F	T	T	T
F	T	F	F	F	T	T
F	F	T	F	T	T	T
F	F	F	F	F	T	T

8.1 The **converse** of $p \to q$ is $q \to p$

8.2 The **contrapositive** of $p \to q$ is $\neg q \to \neg p$

8.3 The **inverse** of $p \to q$ is $\neg p \to \neg q$

9. Two (compound) propositions are **equivalent** if they always have the same truth value

Q2. Which of the 1) implication and its 2) converse, 3) contrapositive, and 4) inverse are equivalent?

1) implication and 3) contrapositive are equivalent

2) converse and 4) inverse are equivalent

10. The **biconditional** of two propositions p and q is true when both conditional statements $p \to q$ and $q \to p$ are true and is false otherwise $(p \leftrightarrow q)$

10.1 The **truth table** of biconditional:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

LOGIC AND BIT OPERATIONS.

11.1 A **bit** is a symbol with two possible values, namely 0 and 1

11.2 **Bit operations** are logical connectives . They extend to **bit strings**.

Q3. Perform bitwise OR, bitwise AND, and bitwise XOR on the bit strings 10001011011 and 110101001.

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 10001011011
 00110101001
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 10111111011  bitwise OR
 00000001001  bitwise AND
 10111110010  bitwise XOR
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APPLICATIONS

TRANSLATION OF NATURAL SPEECH.

Q4. Form the contrapositive of: If I stay home, then it will snow tonight.

If it does not snow tonight then I will not stay home.

BOOLEAN SEARCHES.

Q5. How to search for info on universities in Mexico but not in New Mexico?

NOT NEW AND MEXICO AND UNIVERSITIES

LOGIC PUZZLES.

Q6. There are signs on the doors to two rooms. The first sign says "In this room there is a lady, and in the other room there is a tiger"; the sign on the second door treads "In one of these rooms there is a lady, and in the other room there is a tiger". Suppose you know that one of these signs is true and the other false. Behind which door there is a lady?

p: there is a lady behind door one

$\neg p$: there is a tiger behind door 1

q: there is a lady behind door 2

$\neg q$: there is a tiger behind door 2

statement 1: $p \wedge \neg q$

statement 2: $(p \wedge \neg q) \vee (\neg p \wedge q)$

If #1 is true then #2 is true which doesn't work so #2 has to be true so that #1 is false

#2 has the lady behind the door

LOGIC CIRCUITS.

Q7. Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $(p \vee \neg q) \wedge (\neg q \vee r)$ from inputs p, q, and r.