Name: Shivani Patel **GSI:** Madeline Brandt **DISC #:** 103

Math 55, Handout 15.

INTRODUCTION TO FINITE PROBABILITY. 446

0.0. **Blanket assumption (for this lecture):** all experiments have finitely many, equally likely, outcomes.

1.1. An **experiment** is a procedure that yields one of a given set of possible outcomes

1.2. The **sample space** is the set of possible outcomes

1.3. An **event** is a subset of the sample space

1.4. If S is a finite nonempty sample space of equally likely outcomes, and $E \subseteq S$ is an event, then the **probability** of E is

$$p(E) = \frac{|E|}{|S|}$$

Q1. What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 8?

$$\left.\begin{matrix} 2+6 \\ 3+5 \\ 4+4 \\ 5+3 \\ 6+2 \end{matrix}\right\} 5/36 = \boxed{14\%}$$

Q2. What is the probability that a randomly selected integer chosen from the first 100 positive integers is divisible

(a) by 3?

$$\frac{33}{100} \approx \boxed{33.33\%}$$

(b) by 5?

$$\frac{20}{100} = \boxed{0.2\%}$$

(c) by 9?

$$\frac{9}{100} = \boxed{0.09\%}$$

Q3. What is the probability that a five-card poker hand contains

(a) a flush?

2 senarios:

Flushs but no straight flush:

$$(4 \text{ choose } 1) * (13 \text{ choose } 5) = \boxed{0.00198079}$$

Flush including straight flushes:

$$(4\text{-choose-}1) * (13\text{-choose-}5) - 4*10 = \boxed{0.0000138517}$$

(b) a straight flush?

straight Flush (excluding Royal)

$$4*10-4 = 36 \approx \boxed{0.0000138517}$$

straight Flush including Royal Flush

$$4 \times 10 = 40 \approx \boxed{0.0000153908}$$

1.5. Let E be an event in a sample space S. The **complementary event** $\overline{E}$ is defined as

$$P(\bar{E}) = 1 - p(E)$$

1.6. The probability of $\overline{E}$ is given by $S - E$

$$p(\bar{E}) = \frac{|S| - |E|}{|S|} = 1 - \frac{|E|}{|S|} = 1 - p(E)$$

Q4. A fair coin is tossed 7 times (landing heads up or tails up every time). What is the probability that at least one of the coin tosses results in a heads up outcome?

$1 - P(H^c) \rightarrow$ never happening

$$\left.\begin{matrix} T & T & T & T & T & T & T \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \end{matrix}\right\} \quad \boxed{1 - \frac{1}{2^7}}$$

1.7. Let E_1 and E_2 be events in the sample space S. Then

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - P(E_1 \cap E_2)$$

1.8. Two events E_1 and E_2 are called **independent** if

$$P(E_1 | E_2) = P(E_1) \Longleftrightarrow P(E_1 \cap E_2) = P(E_1)\, p(E_2)$$

Q5. (a) What is the probability that a randomly selected integer chosen from the first 100 positive integers is divisible by 3 or by 5? Are the two events (drawing an integer divisible by 3 and drawing an integer divisible by 5 randomly from the first 100 positive integers) independent?

$$\frac{33}{100} + \frac{20}{100} - \frac{6}{100} = \frac{47}{100} \Big\} \text{ Not independent}$$

(b) Same questions for divisibility by 3 or by 9.

$$\frac{33}{100} + \cancel{\frac{9}{100}} - \cancel{\frac{9}{100}} = \frac{33}{100} \Big\} \text{ Not independent}$$