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Math 55, Handout 13.

THE PIGEONHOLE PRINCIPLE.

1.1. **The pigeonhole principle.** Let $k \in \mathbb{N}$. Suppose $k+1$ objects are placed into k boxes. Then there is at least one box containing two or more of the objects

1.2. **Equivalent formulation.** A function from a set of $k+1$ elements to a set of k elements cannot be one-to-one

Q1. Show that for every $n \in \mathbb{N}$ there is a multiple of n that has only 0s and 7s in its decimal expansion.

$a_1 = 7$

$a_2 = 77$

$a_3 = 777$

$a_i \pmod n$ where $1 \le i \le n$

If any of the a_i leaves remainder Ø then its a multiple of n containing all 7's

If $a_i \not\equiv 0 \pmod n$ for any $i \in [1,n]$

we know ai's can leave remainder $1, 2, 3, \dots, (n-1)$ since we excluded the case of Ø

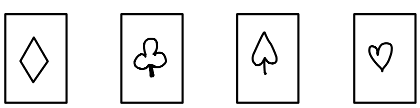
therefore there will be at least two #s with the same remainder (ex: a_p, a_q)

From this we guarentee $|a_p - a_q|$ will be divisible by n & it will only contain 0's and 7's

THE GENERALIZED PIGEONHOLE PRINCIPLE.

2.1. **The generalized pigeonhole principle.** Let $n, k \in \mathbb{N}$. Suppose n objects are placed into k boxes. Then there is at least one box containing at least $\lceil N/k \rceil$ objects

Q2. How many cards must be selected from a standard deck of 52 cards to guarantee that at least four cards of the same suit are chosen?



① worse senario: you draw 3 cards of every suit.

② the next card will satisfy the at least 4 cards of the same suit are chosen

$\underset{①}{12} + \underset{②}{1} = \boxed{13}$

Q3. Suppose that a CS lab has 16 workstations and 12 servers. A cable can directly connect a workstation to a server. For each server, only one direct connection to that server can be active at any tme. We want to guarantee that at any time any set of 12 or fewer workstations can simultaneously access different servers via direct connections. What is the minimum number of direct connections needed?

$= n + n * (m - n)$

$= 12 + 12 \times (16 - 12) =$ $\boxed{\text{96 minimum \# of direct connections}}$

APPLICATIONS OF THE PIGEONHOLE PRINCIPLE.

3.1. **[Theorem].** Every sequence of $n^2 + 1$ distinct real numbers contains a subsequence of length $n + 1$ that is either strictly increasing or strictly decreasing.

Q4. Prove or disprove that every sequence of 9 distinct real numbers contains a subsequence of length 4 that is either strictly increasing or strictly decreasing.

the theorem states that every sequence of n^2+1 distinct real numbers containing a subsequence of length $n+1$ which is either strictly $\uparrow$ or $\downarrow$

so here $n^2+1=9$, that implies $n=2\sqrt{2}$ and $n+1=4$ so $n=3$. so n in both cases are not equal so the statement is not correct

RAMSEY THEORY.

4.1. **The party problem.** At any party with at least six people, there are three people who either all know each other or all do not know each other.

4.2. **The Ramsey number** $R(m, n)$, where $m, n \in \mathbb{N}$, denotes the minimum number of people at a party such that there are either m mutual friends or n mutual enemies, assuming that every pair of people at the party are friends or enemies

Q5. Let $n \in \mathbb{N}$, $n \geq 2$. Show that the Ramsey number $R(2, n)$ equals n.

① If we have group of n people, then among them we must find either a pair of friends or a subset of n of them all of whom are mutual enemies

- If there is any pair of friends, then the condition is satisfied
- If not, then every pair of people are enemies

② there exists a group of $n-1$ people for which this is not possible

- If we have a group of $n-1$ people all of whom are enemies of each other
- there is neither a pair of friends nor a subset of n of them all of whom are mutual enemies