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# Math 55, Handout 14.

**GENERALIZED PERMUTATIONS AND COMBINATIONS.**

1.1. **Permutations with repetition.** The number of $r$-permutations of a set with $n$ elements with repetition allowed is $n^r$.

1.2 **Combinations with repetition.** The number of $r$-combinations from a set with $n$ elements with repetition allowed is $C(n+r-1, r) = C(n+r-1, n-1)$.

Q1. A croissant shop has plain croissants, apple croissants, chocolate croissants, cheese croissants, marzipan croissants, and almond croissants.

(a) How many ways are there to choose a dozen croissants?

$n = 6$
$r = 12$

$C(n+r-1, r) = C(6+12-1, 12) = C(17, 12) = \frac{17!}{12!(17-12)!} = \frac{17!}{12!\,5!} = \boxed{6188}$

(b) A dozen croissants with at least one of each kind?

subtract from (a): $\binom{16}{4} = 1820$

$6188 - 1820 = \boxed{4368}$

**Permutations with (some) indistinguishable objects.**

1.3. The number of permutations of $n$ objects where there are $n_j$ indistinguishable objects of type $j$, for $j = 1, \ldots, k$ (where $\sum_{j=1}^{k} n_j = k$), is equal to $\frac{n!}{n_1!\, n_2! \cdots n_k!}$

Q2. How many different strings can be made by reordering the letters of the word MATHEMATICS?

M - 11
A - 11
T - 11
H - 1
E - 1
I - 1
C - 1
S - 1

11! total combinations since there are 2 M's, 2 A's, 2 T's we need to divide those out

$\frac{7!}{2!2!2!} = \frac{5040}{8} = \boxed{630}$

**Putting objects into boxes.**

**Distinguishable objects and distinguishable boxes.**

2.1. The number of ways to distribute $n$ distinguishable objects into $k$ distingishable boxes so that $n_j$ objects are placed into box $j$, $j = 1, \ldots, k$ (where $\sum_{j=1}^{k} n_j = k$ ), is equal to $\dfrac{n!}{n_1! n_2! \cdots n_k!}$

Q3. How many ways are there to distribute hands of five cards to each of six players from the standard deck of 52 cards?

$n = 52$
$k = 7$
$n_1 = 5$
$n_2 = 5$
$n_3 = 5$
$n_4 = 5$
$n_5 = 5$
$n_6 = 5$
$n_7 = 22$

$$\frac{n!}{n_1! n_2! \cdots n_k!} = \frac{52!}{5!5!5!5!5!5!22!} = \boxed{\frac{52!}{(5!)^6 22!}}$$

**Indistinguishable objects and distinguishable boxes.**

2.2. The number of ways to place $n$ indistinguishable objects into $k$ distinguished boxes is the same as the number of $n$-combinations from a set with $k$ elements when repetitions are allowed.

**Distinguishable objects and indistinguishable boxes.**

2.3. The number of ways to distribute $n$ distinguishable objects into $k$ indistinguishable boxes is given by the formula $\sum_{j=1}^{k} S(n, j)$ where the $S(n, j)$'s are called stirling numbers of the second kind

Each $S(n, j)$, in turn, satisfies the formula $\displaystyle\sum_{j=1}^{k} S(n,j) = \sum_{j=1}^{k} \frac{1}{j!} \sum_{i=0}^{j-1} (-1)^i \binom{j}{i} (j-i)^n$

Q4. How many ways are there to assign 3 indistinguishable offices to 5 employees, where each office can accommodate any number of employees?

distinguishable objects: 5 employees
indistinguishable boxes: 3 offices

using 1 office: ABCDE
using 2 offices: [(A)(BCDE)] + 4 for each employee, [(AB)(CDE)] ... = $\binom{5}{2}$
using 3 offices: [(A)(B)(CDE)], [(C)(D)(ABE)] ... = $\binom{5}{2}$
[(AB)(C)(DE)], [(A,D)(C)(BE)], [(AE)(C)(BD)] × 5 for each

**Indistinguishable objects and indistinguishable boxes.**

2.4. The number of ways to distribute $n$ indistinguishable objects into $k$ indistinguishable boxes is the same as the number of **partitions** of $n$ into $k$ parts. It is denoted by $p_k(n)$.

Q5. In how many ways can six identical DVDs be wrapped into wrapping paper if any number of DVDs can be wrapped together?

(1, 5) (2, 4) (3, 3)
(1, 1, 4) (2, 2, 2)
(1, 1, 1, 3) (2, 2, 1, 1)
(1, 1, 1, 1, 2) (2, 3, 1)
(1, 1, 1, 1, 1, 1)

$p_6(6)$

$\boxed{\text{10 ways}}$