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Homework 2.

Math 55, Spring 2020.

$\forall x (A \to P(x))$

$\therefore \quad A \to P(c)$

Prob 1. Establish these logical equivalences where x does not occur as a free variable in A. Assume that the domain is nonempty.

(a) $\forall x\ (A \to P(x)) \equiv A \to \forall x\ P(x)$.

If A is true, then

LHS $\forall x (T \to P(x)) = \forall x P(x)$

RHS $T \to \forall x P(x) = \forall x P(x)$

If A is false, then

LHS $\forall x (F \to P(x)) = \forall x\ F$

RHS $F \to \forall x P(x) = \forall x\ F$

(b) $\exists x(P(x) \to A) \equiv \forall x P(x) \to A$.

If A is true, then

LHS $\exists x(P(x) \to T) = \exists x P(x)$

RHS $\forall x P(x) \to T = \exists x P(x)$ ✓

truth table

if A is false, then

LHS $\exists x (P(x) \to F) = F$

RHS $\forall x P(x) \to F = F$ ✓

through the truth table

P	Q	$P \to Q$	$\neg P \vee Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

$\nmid$ = does not divide

examples

11, 13

7, 9

17, 19

Prob 2. Use predicates, quantifiers, logical connectives, and mathematical operators to express the Twin Prime Conjecture.

"there are an Infinite number of prime seperated by distance 2"

(1) P(x) = "x is prime"

(1') Q(x) = "x and x+2 are prime" $\Longrightarrow$ $Q(x) = P(x) \land P(x+2)$

(2) there are infinitely many x with property Q(x)

(2) No matter how far out I look I can find an x with Q(x)

$\underbrace{\forall y \exists x (x > y \land Q(x))}$

(1) only divisors are 1 and x

$\forall y (\underbrace{y \nmid x}_{\text{does not divide by } x} \lor \underbrace{y \leq 1}_{\text{less than } 1} \lor \underbrace{y \geq x}_{\text{greater than } x})$

$$\boxed{\begin{array}{l} \forall y \exists x (x > y \land P(x) \land P(x+2)) \\ \forall y (y \nmid x \lor y \leq 1 \lor y \geq x) \end{array}}$$

Prob 3. This argument supposedly shows that if $\forall x\,(P(x) \vee Q(x))$ is true, then $\forall x\,P(x) \vee \forall x\,Q(x)$ is true. What is wrong with it?

1. $\forall x\,(P(x) \vee Q(x))$ Premise
2. $P(c) \vee Q(c)$ Universal instantiation from (1)
3. $P(c)$ Simplification from (2)
4. $\forall x\,P(x)$ Universal generalization from (3)
5. $Q(c)$ Simplification from (2)
6. $\forall x\,Q(x)$ Universal generalization from (5)
7. $\forall x\,P(x) \vee \forall x\,Q(x)$ Conjunction from (4) and (6)

#2 is wrong because P(C) V Q(C) cannot be true/equate after $\forall_x P(x) \vee \forall_x Q(x)$ has been distributed. After that, we can stop there since the 2nd step terminates the rest of the steps

Prob 4. Show that the argument form with premises

$$\underline{p \wedge t \to r \vee s}, \quad \underline{q}, \quad \underline{q \to u \wedge t}, \quad \underline{u \to p}, \quad \underline{\neg s}$$

and conclusion

$$r$$

is valid using rules of inference from Table 1 in Section 1.6.

q

$\therefore q \to u \wedge t$ — modus podus

$u \wedge t$

$u \to p$ — hypothetical syllogism

$\therefore p \wedge t$

$p \wedge t \to r \vee s$

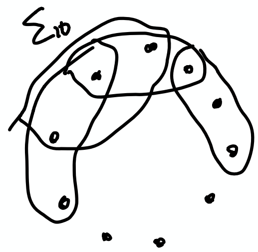
$\therefore r \vee s$ — modus podus

$\neg s$

$\therefore r$ — Disjunctive syllogism

1-10 any order

Prob 5. Let the integers $1, 2, \ldots, 10$ be placed around a circle, in any order. Show that there are 3 integers in consecutive locations whose sum is at least 17.



$$\sum_1 + \cdots + \sum_{10} = 3(1 + \cdots + 10) = 3 \cdot 5 \cdot 11 = 165$$

Average values of a $\sum = \frac{165}{10} = 16.5$

There must be some $\sum_i$ with $\sum_i > 16.5$ since $\sum_i$ is an int.

$\sum_i \geq 17.$

$\sum_1 + \cdots \sum_{10} \leq 160$

for contradiction

$\sum_i \leq 16$