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Math 55, Handout 25.

REVIEW: GRAPHS.

1. Name the following bits of knowledge in an approximate order of importance within each category:

   (a) at least 20 graph-theoretic notions:

   - bipartition
   - complete
   - Hamiltonian paths/circuits
   - directed
   - Neighborhood
   - multigraph
   - Eulerian paths/circuits
   - simple
   - undirected
   - vertex/edges
   - trees
   - loops
   - matching
   - wheels
   - cube
   - hypercube
   - in degree
   - out degree
   - connected components
   - cycle

   (b) at least 5 classes of graphs:

   Bipartite, Complete, Simple, Cycle, Cube

   (c) at least 1 algorithm:

   Fleury Algorithm

   (d) 3 most important theorems:

   Handshake Theorem, Hall's Theorem, Ore's Theorem

Q1. Imagine you get the following problem on the final: "Suppose that G is a connected multigraph with $2k$ vertices of odd degree. Show that there exist k subgraphs that have G as their union, where each of these subgraphs has an Eulerian path and where no two of these subgraphs have an edge in common."

(a) Is its setup clear? Or does anything need to be clarified? Do you need any new notation?

No, it's not clear. Are there 2k vertices or just 2k odd degree vertices and some number of even degree.

(b) What formal statements (Theorems, Lemmas, etc.) do you envision using to solve this problem?

Theorem 2: A connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree

(c) What proof method will you use?

Proof method that can be used is induction on k using 2·k-1 for strong induction.

Q2. Imagine you get the following problem on the final: "Suppose there is an integer k such that every man on a desert island is willing to marry exactly k of the women on the island and that every woman on the island is willing to marry exactly k men. Also, suppose a man is willing to marry a woman if and only iff she is willing to marry him. Show that it is possible to match the men and the women on the island so that everyone is matched with someone he or she is willing to marry."

(a) Does anything need to be clarified? Do you need any new notation?

Yes, does "every man on a desert island is marrying exactly k women" mean that a man marries multiple women.

(b) What formal statements (Theorems, Lemmas, etc.) do you envision using to solve this problem?

Bipartition, Hall's Marriage Theorem

(c) What proof method will you use?

Can use direct proof and explain with Hall's theorem.

REVIEW: ADVANCED COUNTING.

2. Name the following bits of knowledge in an approximate order of importance within each category:

(a) at least 10 counting-related notions:

- inclusion - exclusion
- combinations
- permutations
- derangement
- generation
- distinguishable
- Stars and bars
- partitions
- distribution
- indistinguishable

(b) at least 10 pieces of notation related to counting:

- factorial
- combinations
- difference equation
- cardinality
- permutation
- pascal's triangle
- product rule
- Boole's equality
- deviation
- complement

(c) at least 2 algorithms/techniques:

- Inclusion - Exclusion technique
- Stars and Bars technique

(d) at least 1 theorem:

- Inclusion - Exclusion theorem

Q3. Imagine you get the following problem on the final: "Solve the recurrence relation $a_k = a_{k-1} + 3a_{k-2} + 4^k + 6$ with initial conditions $a_0 = 20$, $a_1 = 60$."

(a) Does anything need to be clarified? Will you need any further notation?

Nothing needs to be clarified because we have the necessary initial conditions

(b) What techniques do you have at your disposal to solve it?

Characteristic polynomial or create generating functions

(c) What formal statements (if any) do you envision using to solve this problem?

The theory of constant coefficient recurrence relations or theory of generating functions

Q4. Repeat this process for the problem: "An integer is called square-free if it is not divisible by the square of a positive integer greater than 1. Find the number of square-free integers less than 100."

(a) Yes, it doesn't say for non-negative integers

(b) The technique that can be used is Inclusion - Exclusion technique

(c) A formal statement that you can use to solve this problem is By Inclusion - Exclusion in the Alternative Form

Q5. Repeat this process for the problem: "Find a recurrence relation for the number of sequences a_1, a_2, $\ldots$, a_k such that $a_1 = 1$, $a_k = n$, and $a_j < a_{j+1}$ for all $j = 1, \ldots, k-1$."

(a) We can ask if the recurrence relation is based on n or k, it is based on both.

(b) We can use strong induction on n+k

(c) Formal Statement: 'By the of strong induction' and describe the basis and inductive step.