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Math 55, Handout 23.

Q1. You are a boss of four employees: Ping, Quiggley, Ruiz, and Sitea. Ping is qualified to support hardware, networking, and wireless. Quiggley is qualified to support software and networking. Ruiz is qualified to support networking and wireless. Sitea is qualified to support hardware and software. Is there an assignment of employees so that each employee is assigned a unique area to support?

yes!

Ping: Network Ruiz: wireless

Quiggley: software Sitea: hardware

PATHS IN GRAPHS.

1.1. A **path** of length n from a vertex u to a vertex v in an undirected graph $G = (V, E)$ is a sequence of n edges $e_1, \ldots, e_n$ of G such that there exists a sequence $x_0 = u, x_1, \ldots, x_{n1}, x_n = v$ of vertices such that e_i has for $i = 1, \ldots, n$, the endpoints x_{i1} and x_i.

A path is a **circuit** if it begins and ends at the same vertex and has a greater than zero length. The path or circuit is said to **pass through** the vertices $x_1, x_2 \cdots x_n$ or **traverse** the edges $e_1, e_2, \cdots e_n$. A path or circuit is **simple** if it does not contain the same edge more than once

1.2. All these notions generalize to directed graphs.

CONNECTEDNESS IN UNDIRECTED GRAPHS.

2.1. An undirected graph is called **connected** if there is a path between every pair of distinct vertices of the graph

2.2. An undirected graph that is not connected is called **disconnected**.

2.3. A **connected component** of a graph G is a connected subgraph of G.

2.4. A **cut edge** is an edge whose removal produces a graph with more connected componets componets than in the original graph

2.5. A **cut vertex** is is a vertex whose removal from a connected graph produces a subgraph that is not connected

Q2. (a) Explain why, in the collaboration graph of mathematicians, a vertex representing a mathematician is in the same connected component as the vertex representing Paul Erdős if and only if that mathematician has a finite Erdős number. Having a finite Erdős # means the mathmateticián is connected to Erdős through some papers. Erdős is connected to himself thus allowing the zero Erdős to make sense.

(b) What does the Erdős number mean in that graph?

The Erdős # of a person m is the length of the shortest path between m and the mathematician Paul Erdős. The # is the length of the shortest chain of mathematics where each adjacent pair of mathematicians have written a joint paper

2.6. A **tree** is a connected graph not containing a simple circuit.

2.7. An undirected graph all whose connected components are trees is called a forest

Q3. If a forest has exactly 3 connected components, each with 100 vertices, how many edges does it have?

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CONNECTEDNESS IN DIRECTED GRAPHS.

3.1. A directed graph is called **strongly connected** if there is a path from a to b and from b to a whenever a and b are verticies in the graph

3.2. A directed graph is called **weakly connected** if there is a path between every two vertices in the underlying undirected graph.

Q4. Show that every strongly connected graph is weakly connected but not *vice versa*.

If a graph is strongly connected then every vertex pair can be connected by some path & can be done in both directions. this also works for a weakly connected graph

If a graph is weakly connected then there is a path between two vertices. the path may only work in one direction. since both directions may not be satisfied, the graph may not be strongly connected.

EULERIAN PATHS AND CIRCUITS.

4.1. An **Eulerian circuit** in a graph G is a simple circuit containing every edge of G.

4.2. An **Eulerian** path in a graph G is a simple path containing every edge of G.

4.3. [Theorem] A connected multigraph with at least two vertices has an Eulerian circuit if and only if each of its vertices has even degree

4.4. [Theorem] A connected multigraph with at least two vertices has an Eulerian path if and only if it has exactly two vertices of odd degree

Q5. For which values of n do the following graphs have an Eulerian circuit?

K_n : odd $n \geq 3$ C_n : $n \geq 3$ W_n : none Q_n : even $n \geq 2$