Name: Shivani Patel GSI: Madeline Brandt DISC #: 103

# Homework 3.

**Math 55, Spring 2020.**

**Prob 1.** Prove that there are no integer solutions to the equation

$$2x^2 + 5y^2 = 14.$$

$2x^2 > 14$ $|x| \geq 3$ $[-2, -1, 0, 1, 2]$

$5y^2 > 14$ $|y| \geq 2$ $[-1, 0, 1]$

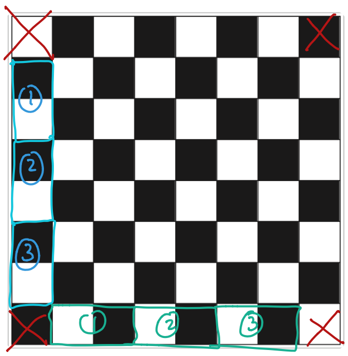
$2x^2 \longrightarrow 0, 2, 8$

$5y^2 \longrightarrow 0, 5$

largest possible sum is 13
therefore it is impossible for
$2x^2 + 5y^2 = 8$

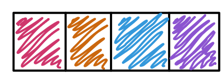
**Prob 2.** (a) Prove or disprove that you can use dominoes to tile a standard checkerboard with all four corners removed.

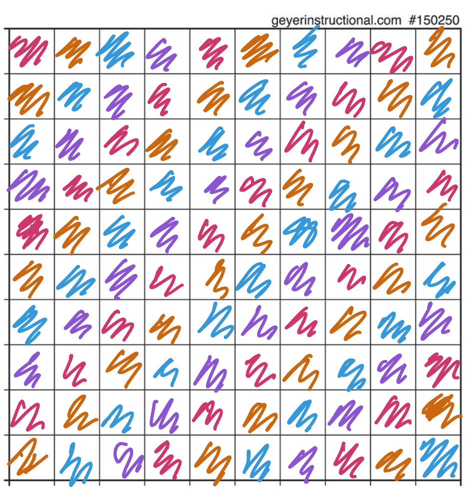
If the 4 corners are removed, we can still replace the dominos on the board because we are removing one of each color (2 blacks, 2 white) therefore either direction will still work. see below.




one tile covers one of each color

(b) Prove or disprove that you can tile a $10 \times 10$ checkerboard using straight tetrominoes.



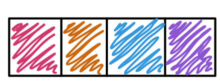



pink - 25 count

orange - 26 count

uneven count

In order for this to work you would need an equal amount of colors on the board (see above) you would not be able to cover the entire board. Because if you do 10/4 you will not be able to do this. through the visual, we can see that the middle is empty and no combination will allow for you to put all 1x4 or 4x1.

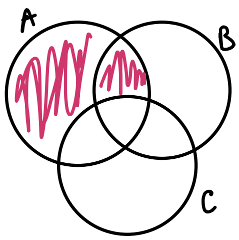
No matter where I place  it does not cover one tile of each color. There an uneven count if you tile them in the order above

**Prob 3 [Russell's Paradox].** Let $S$ be the set of all sets that do not contain themselves: $S = \{x : x \notin x\}$. Show that both $S \in S$ and $S \notin S$ lead to contradictions. This paradox shows inherent problems with Naïve Set Theory.

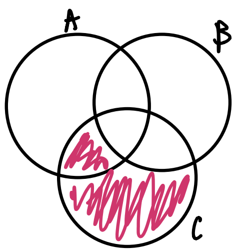
$S \in S$ means that $S$ cannot contain itself because the definition is $S = \{x : x \in x\}$ therefore $S$ cannot contain itself. $S \notin S$ shows that $S$ cannot contain itself but through the definition it would be a double negative therefore $S$ is within $S$

**Prob 4.** Prove the following set inclusions using membership tables and illustrate them using Venn diagrams. What are the logical analogues of these formulas?

(a) $(A \setminus C) \cap (C \setminus B) = \emptyset$.




$A \setminus C$



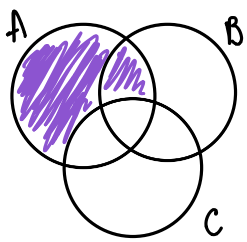

$C \setminus B$

no overlap
therefore
it equals $\emptyset$

| A | B | C | $(A \setminus C)$ | $(C \setminus B)$ | $(A \setminus C) \cap (C \setminus B)$ |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | T | F | T | F | F |
| T | F | T | F | T | F |
| T | F | F | T | F | F |
| F | T | T | F | F | F |
| F | T | F | F | F | F |
| F | F | T | F | T | F |
| F | F | F | F | F | F |

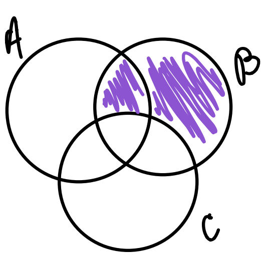
$$(A \wedge \neg C) \wedge (C \wedge \neg B) = F$$

(b) $(A \setminus C) \cup (B \setminus C) = (A \cup B) \setminus C$.




$A \setminus C$

| A | B | C | $A \setminus C$ | $B \setminus C$ | $(A \cup B) \setminus C$ |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | T | F | T | T | T |
| T | F | T | F | F | F |
| T | F | F | T | T | T |
| F | T | T | F | F | F |
| F | T | F | F | F | F |
| F | F | T | F | F | F |
| F | F | F | F | F | F |



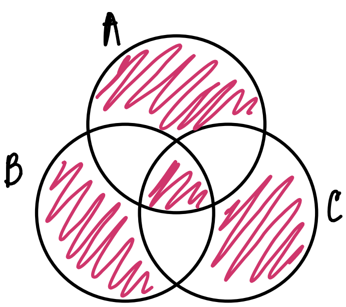

$B \setminus C$

$$(A \wedge \neg C) \wedge (B \wedge \neg C) = A \vee B \wedge \neg C$$

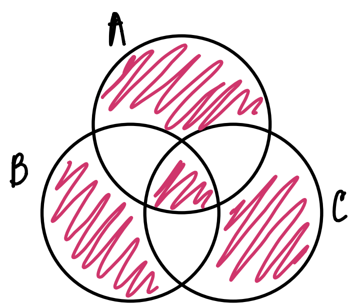
$\oplus$ exclusive or

**Prob 5.** The **symmetric difference** of sets $A$ and $B$ denoted $A \oplus B$ is the set containing all elements in either $A$ or $B$ but not in both. Is the symmetric difference associative, i.e., is it always the case that

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C?$$




$A \oplus (B \oplus C)$




$(A \oplus B) \oplus C$

The symmetric difference shows that regardless of the ordering of $A \oplus B \oplus C$ with any combo of parenthesis will equal the same relationship table