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Homework 12.

Math 55, Fall 2017.

simple graph : V_1 & V_2

Prob 1. Suppose that $G = (V, E)$ is a bipartite graph with bipartition (V_1, V_2) and that $A \subseteq V_1$. Show that the maximum number of vertices of V_1 that are the endpoints of a matching of G equals $|V_1| - \max_{A \subseteq V_1} \operatorname{def}(A)$ where $\operatorname{def}(A) = |A| - |N(A)|$ is called the **deficiency** of A.

subset of the edges that no two edges have a vertex in common

Given:
$G = (V, E)$ is a bipartite graph with bipartition (V_1, V_2)
$A \subseteq V_1$
$\operatorname{def}(A) = |A| - |N(A)|$

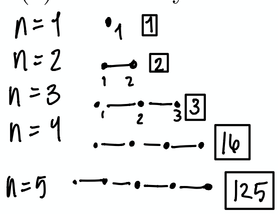
let $b = \max_{A \subseteq X_1} \operatorname{def}(A)$ and B be the subset that V_1 which achieves the maximum ($b = \operatorname{def}(B)$)
$b = |B| - |N(B)|$. $|N(B)|$ represents the # of neighbors of B. Any matching can have at most $|N(B)|$ edges with endpoints in B.

$|V_1|$ = # of elements in V_1. $|B|$ = # of elements in B. any matching can have at most $|V_1| - |B|$ edges with endpoints in $V_1 - B$
Total # of edges: $|N(B)| + |V_1| - |B| = |V_1| - b$

Now if you make a larger graph (G'), we know that there is a complete matching of V_1.
At most b of the edges cannot lie in G.
since G contains at least $|V_1| - b$ edges, the maximum number of edges is G is equal to $|V_1| - b$ which means that the maximum of # of vertices is also equal to $|V_1| - b$ □

Prob 2. The Good Will Hunting Problem(s).

(a) How many trees are there with n distinguishable/labeled vertices?



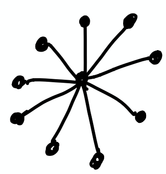
converting sequences to trees creates $n-2$ edges.

tree → sequence: remove the lowest labeled leaf, repeat until 2 remain until 2 remain. everytime you delete a leaf, add its neighbor to the sequences.

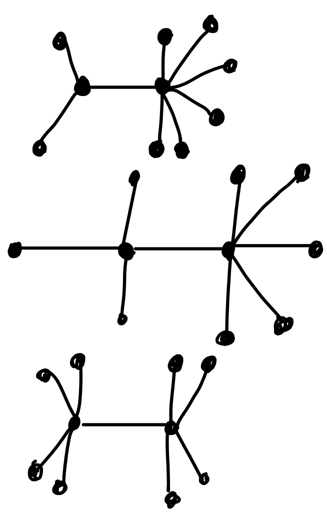
sequence → tree: $S = \{t_1, \dots, t_{n-2}\}$ let s_1 be the 1st element of $\{1, \dots, n\} \setminus S$ join s_1 to t_1. let s_2 be the smallest in $\{\{1, \dots, n\} \setminus \{s_1\}\} \setminus \{t_2, \dots, t_{n-2}\}$ join s_2 to t_2. continue until there are no elements of $\{1, \dots, n\}$ remain

(b) A homeomorphically irreducible tree is one where each vertex has either one or at least three edges incident with it. Draw all homeomorphically irreducible trees with 10 vertices.

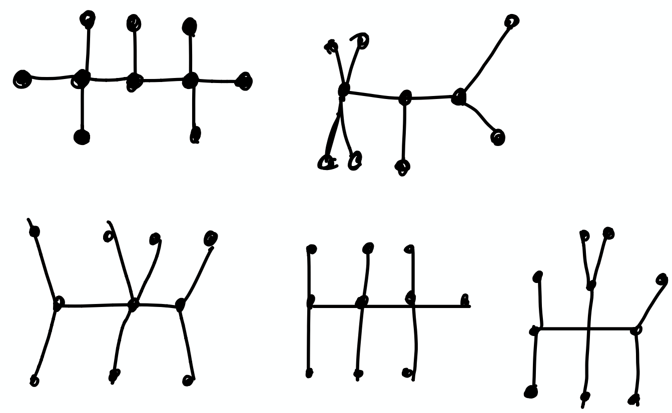
9 leaves & 1 non leaf



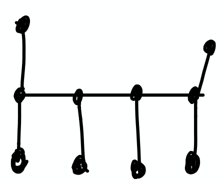
8 leaves & 2 non-leaves



7 leaves & 3 non-leaves



6 leaves & 4 non-leaves



Prob 3. Fleury's algorithm, published in 1883, constructs Eulerian circuits by first choosing an arbitrary vertex of a connected multigraph and then forming a circuit by choosing edges successfully. Once an edge is chosen, it is removed. Edges are chosen successively so that each edge begins where the last edge ends and so that this edge is not a cut edge unless there is no alternative.

Prove that Fleury's algorithm always produces an Eulerian circuit.

let G be an Eulerian graph

① chose any vertex v of G and set current vertex equal to v and circut equal to empty

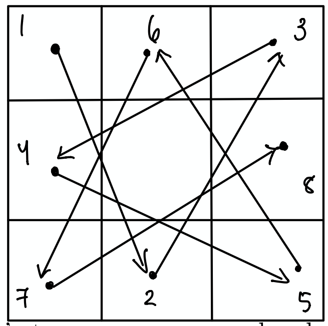
② select any edge e incident with the current vertex. Never chose a bridge of the graph unless there is no choice

③ Add e to the circuit and set the current vertex equal to the vertex at the other end of e. If e is a loop the current vertex will not move

④ Delete e from the graph. Delete any isolated verticer. Repeat 2-4 until all the edges have been deleted from G. the final circuit is an Euler circuit in G.

Prob 4. A **knight's tour** is a sequence of legal moves by a knight starting at some square of a chessboard and visiting each square exactly once. A knight's tour is called **reentrant** if there is a legal move that takes the knight from the last square of the tour to its first. One can model knight's tours via the graph that has a vertex for each square, with an edge connecting two vertices iff a knight can legally move between them.

(a) Draw the graph that represents the legal moves of a knight on a 3×3 chessboard.



(b) Prove that finding a knight's tour on an $m \times n$ chessboard is equivalent to finding a Hamiltonian path on the graph representing the legal moves of a knight on that board.

A Hamiltonian path is a graph path between two vertices of a graph that visits each vertex exactly once.

(c) Prove that finding a reentrant knight's tour on an $m \times n$ chessboard is equivalent to finding a Hamiltonian circuit on the graph representing the legal moves of a knight on that board.

A reentrant knight's tour is precisely such a circuit, since we visit each square once, making legal moves

(d) Show that there is no knight's tour on a 3×3 chessboard.

From (a), the diagram will show that we can't touch the center square ever. If we start from the center we can't move anywhere. so there is no knight's tour on a 3x3 chessboard.