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Math 55, Handout 14.

GENERALIZED PERMUTATIONS AND COMBINATIONS.

1.1. **Permutations with repetition.** The number of r-permutations of a set with n elements with repetition allowed is $\underline{n^r}$.

1.2 **Combinations with repetition.** The number of r-combinations from a set with n elements with repetition allowed is $\binom{n}{r}$.

Q1. A croissant shop has plain croissants, apple croissants, chocolate croissants, cheese croissants, marzipan croissants, and almond croissants.

(a) How many ways are there to choose a dozen croissants?

6^{12}

(b) A dozen croissants with at least one of each kind?

$6^{12}-1^{12}-2^{12}-3^{12}-4^{12}-5^{12}=1915328957$

Permutations with (some) indistinguishable objects.

1.3. The number of permutations of n objects where there are n_j indistinguishable objects of type j, for $j = 1, \ldots, k$ (where $\sum_{j=1}^{k} n_j = \underline{n}$), is equal to $\frac{n!}{n_1!n_2!\ldots n_k!}$

Q2. How many different strings can be made by reordering the letters of the word MATHEMATICS?

M 2; A 2; T 2; H 1; E 1; I 1; C 1; S 1

$$\frac{11!}{2!\,2!\,2!\,1!\,1!\,1!\,1!\,1!} = 4989600$$

Putting objects into boxes.

Distinguishable objects and distinguishable boxes.

2.1. The number of ways to distribute n distinguishable objects into k distingishable boxes so that n_j objects are placed into box $j, j = 1, ..., k$ (where $\sum_{j=1}^{k} n_j =$n), is equal to $\frac{n!}{n_1!n_2!...n_k!}$

Q3. How many ways are there to distribute hands of five cards to each of six players from the standard deck of 52 cards?

$$\frac{52!}{5!\,5!\,5!\,5!\,5!\,5!\,22!}$$

Indistinguishable objects and distinguishable boxes.

2.2. The number of ways to place n indistinguishable objects into k distinguished boxes is the same as the number of r-combinations from a set with n elements when repetitions are $\binom{n+r-1}{r}$.

Distinguishable objects and indistinguishable boxes.

2.3. The number of ways to distribute n distinguishable objects into k indistinguishable boxes is given by the formula $\sum_{j=1}^{k} S(n,j)$ where the $S(n,j)$' s are called Stirling numbers of the second kind.

Each $S(n, j)$, in turn, satisfies the formula

Q4. How many ways are there to assign 3 indistinguishable offices to 5 employees, where each office can accommodate any number of employees?

$$S(5,1) = 1$$

$$S(5,2) = \binom{5}{2} = 10$$

$$S(5,3) = \binom{5}{3} + \frac{\binom{5}{2}\binom{3}{2}}{2} = 10 + 15 = 25$$

1+10+25=36

Indistinguishable objects and indistinguishable boxes.

2.4. The number of ways to distribute n indistinguishable objects into k indistinguishable boxes is the same as the number of **partitions** of n into k parts. It is denoted by $p_k(n)$.

Q5. In how many ways can six identical DVDs be wrapped into wrapping paper if any number of DVDs can be wrapped together?

6 = 6
6 = 4 + 2
6 = 3 + 3
6 = 4 + 1 + 1
6 = 3 + 2 + 1
6 = 2 + 2 + 2
6 = 3 + 1 + 1 + 1
6 = 2 + 2 + 1 + 1
6 = 2 + 1 + 1 + 1 + 1
6 = 1+ 1 + 1 + 1 + 1 + 1

10 in total.