The Transportation Model

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Formulation of LP problem in "R"

The objective function is $Min \quad TC = 622(x_11) + 614(x_12) + 630(x_13) + 641(x_21) + 645(x_22) + 649(x_23)$

Subject to

$$\text{supply constraints}$$
$$(x_11) + (x_12) + (x_13) \le 100$$
$$(x_12) + (x_22) + (x_23) \le 120$$

$$\text{demand constraints}$$
$$(x_11) + (x_21) \ge 80$$
$$(x_12) + (x_22) \ge 60$$
$$(x_13) + (x_23) \ge 70$$
$$(x_14) + (x_24) \ge 10$$
$$\text{Non-Negativity constraint}$$
$$x_ij \ge 0$$

where

$$i = 1, 2$$
$$j = 1, 2$$

activating the required package and creating the table

```
library(lpSolve)
tab <- matrix(c(22,14,30,600,100,
                16,20,24,625,120,
                80,60,70,"-","-"), ncol=5,byrow=TRUE)
colnames(tab) <- c("warehouse1", "warehouse2", "warehouse3", "ProductionCost", "ProductionCapacity")
rownames(tab) <- c("PlantA", "PlantB", "Demand")
tab <- as.table(tab)
tab
```

```
##        warehouse1 warehouse2 warehouse3 ProductionCost ProductionCapacity
## PlantA 22         14         30         600            100
## PlantB 16         20         24         625            120
## Demand 80         60         70         -              -
```

```r
#Creating dummy variables when supply and demand are not equal
costs <- matrix(c(622,614,630,0,
                  641,645,649,0), ncol = 4, byrow = TRUE)
colnames(costs)<-c("warehouse1","warehouse2","warehouse3","Dummy")
rownames(costs)<-c("PlantA","PlantB")
costs<-as.table(costs)
costs
```

```
##        warehouse1 warehouse2 warehouse3 Dummy
## PlantA        622        614        630     0
## PlantB        641        645        649     0
```

```r
#Supply Side
row.signs <- rep("<=",2)
row.rhs<- c(100,120)

#Demand Side
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)

#running the lp.transport function
lptrans <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)


#Getting the objective value
lptrans$objval
```

```
## [1] 132790
```

```r
#values of all the variables
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

80 AEDs *in Plant B - Warehouse1*

60 AEDs *in Plant A - Warehouse2*

40 AEDs *in Plant A - Warehouse3*

30 AEDs *in Plant B - Warehouse3 should be created in each facility, supplied to each of the three warehouses of the wholesalers, and then packaged to reduce the overall cost of manufacturing and shipment.*

Formulating the dual of the above transportation problem

Since the primary goal was to reduce transportation costs, the secondary goal would be to increase value added (VA).

$$Maximize \quad VA = \quad 80W_1 + 60W_2 + 70W_3 - 100P_A - 120P_B$$

Subject to the following constraints

$$\text{Total } Payments \ \ Constraints$$
$$W_1 - P_A >= 622$$
$$W_2 - P_A >= 614$$
$$W_3 - P_A >= 630$$
$$W_1 - P_B >= 641$$
$$W_2 - P_B >= 645$$
$$W_3 - P_B >= 649$$

$$\text{Where } \ W_1 = Warehouse \ \ 1$$
$$W_2 = Warehouse \ \ 2$$
$$W_3 = Warehouse \ \ 3$$
$$P_1 = Plant \ \ 1$$
$$P_2 = Plant \ \ 2$$

Economic Interpretation of the dual

$$W_1 <= 622 + P_A$$
$$W_2 <= 614 + P_A$$
$$W_3 <= 630 + P_A$$
$$W_1 <= 641 + P_B$$
$$W_2 <= 645 + P_B$$
$$W_3 <= 649 + P_B$$

$$\text{From } \ the \ \ above \ \ we \ \ get \ \ to \ \ see \ \ that \ \ W_1 - P_A >= 622$$

$$that \ \ can \ \ be \ \ exponented \ \ as \ \ W_1 <= 622 + P_A$$

$$\text{Here } \ W_1 \ \ is \ \ considered \ \ as \ \ the \ \ price \ \ payments \ \ being \ \ received \ \ at \ \ the \ \ origin \ \ which \ \ is \ \ nothing \ \ else,$$
$$\text{but } \ the \ \ revenue, \ \ whereas \ \ P_A + 622 \ \ is \ \ the \ \ money \ \ paid \ \ at \ \ the \ \ origin \ \ at \ \ Plant_A$$

$$\text{Therefore } \ the \ \ equation \ \ turns, \ \ out \ \ to \ \ be \ \ MR_1 >= MC_1.$$

$$\text{For } \ a \ \ profit \ \ maximization, \ \ The \ \ Marginal \ \ Revenue(MR) \ \ should \ \ be \ \ equal \ \ to \ \ Marginal \ \ Costs(MC)$$

$$\text{Therefore,} \quad MR_1 = MC_1$$

$$\text{Based on above interpretation, we can conclude that,}$$
$$\text{Profit maximization takes place if } MC \text{ is equal to } MR.$$

If MR > MC, we must lower plant expenses in order to achieve the Marginal Revenue (MR).

If MR > MC, we must increase output supply in order to achieve Marginal Revenue (MR).